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"Three Flavor Analysis of Solar Neutrinos"

A Few small Progress ($\rightleftarrows$) in Three-Flavor Mixing Scheme of Neutrinos

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Three-Flavor ν Mixing

"Good" ☺ it accomodates $\left\{ \begin{array}{l} \nu_\odot \\ \nu_{atm} \\ \text{reactor} \\ \text{accelerator} \end{array} \right.$

(except for LSND)

Almost Degenerate ν ⟵ ⊕ Hot dark matter

3 Topics today :

① $\cancel{CP}$ effects in solar ν observation ?

② Hot dark matter ν ⟷ MSW $\nu_\odot$ solutions

(almost) incompatible

③ Some Wishful thoughts on K2K experiments

$\boxed{\text{solid but negative}}$ → $\boxed{\text{postive but speculative}}$

Solar ν and $\cancel{CP}$ (hep-ph/9906530)

100 years of $\nu_\odot$ observation by Superkam.

→ Can one detect $\cancel{CP}$ effects ?

leptonic

→ No !

* Solar ν experiments are inherently "**disappearance experiments**"

∵ neutral current cannot distinguish between ν_μ and ν_τ

$$P(\nu_e \to \nu_\mu) + P(\nu_e \to \nu_\tau) = 1 - P(\nu_e \to \nu_e)$$

* There is No $\cancel{CP}$ effects in vacuum ν oscillation ∵ CPT →

$$P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\beta \to \bar{\nu}_\alpha), \text{ then}$$

$$P(\nu_e \to \nu_e) = P(\bar{\nu}_e \to \bar{\nu}_e)$$

$\cancel{CP}$ in ν conversion in matter?

$\boxed{\text{Theorem}}$

$\cancel{CP}$ phase effect disappears to 1-st order in electroweak int.

$$a(x) = \sqrt{2}\, G_F N_e(x)$$

$$i\frac{d}{dx}\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix} = \left\{ U \begin{bmatrix} m_1^2/2E & & \\ & m_2^2/2E & \\ & & m_3^2/2E \end{bmatrix} U^\dagger + \begin{bmatrix} a(x) & & \\ & 0 & \\ & & 0 \end{bmatrix} \right\} \begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix}$$

$$U = \underbrace{\begin{bmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{bmatrix}}_{U_{23}} \underbrace{\begin{bmatrix} 1 & & \\ & 1 & \\ & & e^{i\delta} \end{bmatrix}}_{\Gamma_\delta} \underbrace{\begin{bmatrix} c_{13} & & s_{13} \\ & 1 & \\ -s_{13} & & c_{13} \end{bmatrix}}_{U_{13}} \underbrace{\begin{bmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{bmatrix}}_{U_{12}}$$

Define

$$\tilde{\nu}_\alpha \equiv \left(U_{13}^\dagger \Gamma_\delta^\dagger U_{23}^\dagger \right)_{\alpha\beta} \nu_\beta \qquad \text{(Kuo-Pantaleone 87)}$$

$\rightarrow$

$$i\frac{d}{dx}\begin{bmatrix}\tilde{\nu}_e\\ \tilde{\nu}_\mu\\ \tilde{\nu}_\tau\end{bmatrix} = \left\{ U_{12} \begin{bmatrix} \frac{m_1^2}{2E} & & \\ & \frac{m_2^2}{2E} & \\ & & \frac{m_3^2}{2E} \end{bmatrix} U_{12}^\dagger + a(x) \begin{bmatrix} c_{13}^2 & 0 & c_{13}s_{13} \\ 0 & 0 & 0 \\ c_{13}s_{13} & 0 & s_{13}^2 \end{bmatrix} \right\} \begin{bmatrix}\tilde{\nu}_e\\ \tilde{\nu}_\mu\\ \tilde{\nu}_\tau\end{bmatrix}$$

δ disappears!

Physical transition ~~probability~~ amplitude

$$\langle \nu_\beta | \nu_\alpha \rangle = (\underline{U_{23}\Gamma_\delta U_{13}})_{\alpha\gamma}\,(U_{23}\Gamma_\delta U_{13})^\dagger_{\delta\beta}\,\langle \tilde{\nu}_\delta | \tilde{\nu}_\gamma \rangle$$

$$\| $$

$$\begin{bmatrix} c_{13} & 0 & s_{13} \\ -s_{23}s_{13}e^{i\delta} & c_{23} & s_{23}c_{13}e^{i\delta} \\ -c_{23}s_{13}e^{i\delta} & -s_{23} & c_{23}c_{13}e^{i\delta} \end{bmatrix}$$

$\rightarrow$

$$\langle \nu_e | \nu_e \rangle = c_{13}^2 \langle \tilde{\nu}_e | \tilde{\nu}_e \rangle + s_{13}^2 \langle \tilde{\nu}_\tau | \tilde{\nu}_\tau \rangle + c_{13}s_{13}\left(\langle \tilde{\nu}_e | \tilde{\nu}_\tau \rangle + \langle \tilde{\nu}_\tau | \tilde{\nu}_e \rangle \right)$$

$\boxed{\text{Theorem}}$

KM angle δ disappears from electron neutrino survival probability ; valid to 1-st order of electroweak interaction

<note> δ-dependence exists in $\begin{cases} P(\nu_\mu \to \nu_\mu) \\ P(\nu_\tau \to \nu_\tau) \end{cases}$

Next-to-Leading order Correction

$$\begin{bmatrix} a(x) & & \\ & 0 & \\ & & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} a(x) & & \\ & 0 & \\ & & b(x) \end{bmatrix}$$

$a(x) = \sqrt{2}\, G_\mu N_e(x)$ (Botella–Lim–Marciano '87)

$$\frac{b(x)}{a(x)} = -\frac{3\alpha}{2\pi \sin^2\theta_W}\left(\frac{m_\tau}{m_W}\right)^2\left[2\ln\frac{m_\tau}{m_W} + \frac{5}{6}\right] \simeq 5\times 10^{-5}$$

H_b : additional term in H in ν evolution eq.

$$b(x)\begin{bmatrix} c_{23}^2 s_{13}^2 & c_{23}s_{23}s_{13}e^{-i\delta} & -c_{23}^2 c_{13}s_{13} \\ c_{23}s_{23}s_{13}e^{i\delta} & s_{23}^2 & -c_{23}s_{23}c_{13}e^{i\delta} \\ -c_{23}^2 c_{13}s_{13} & -c_{23}s_{23}c_{13}e^{-i\delta} & c_{23}^2 c_{13}^2 \end{bmatrix}$$

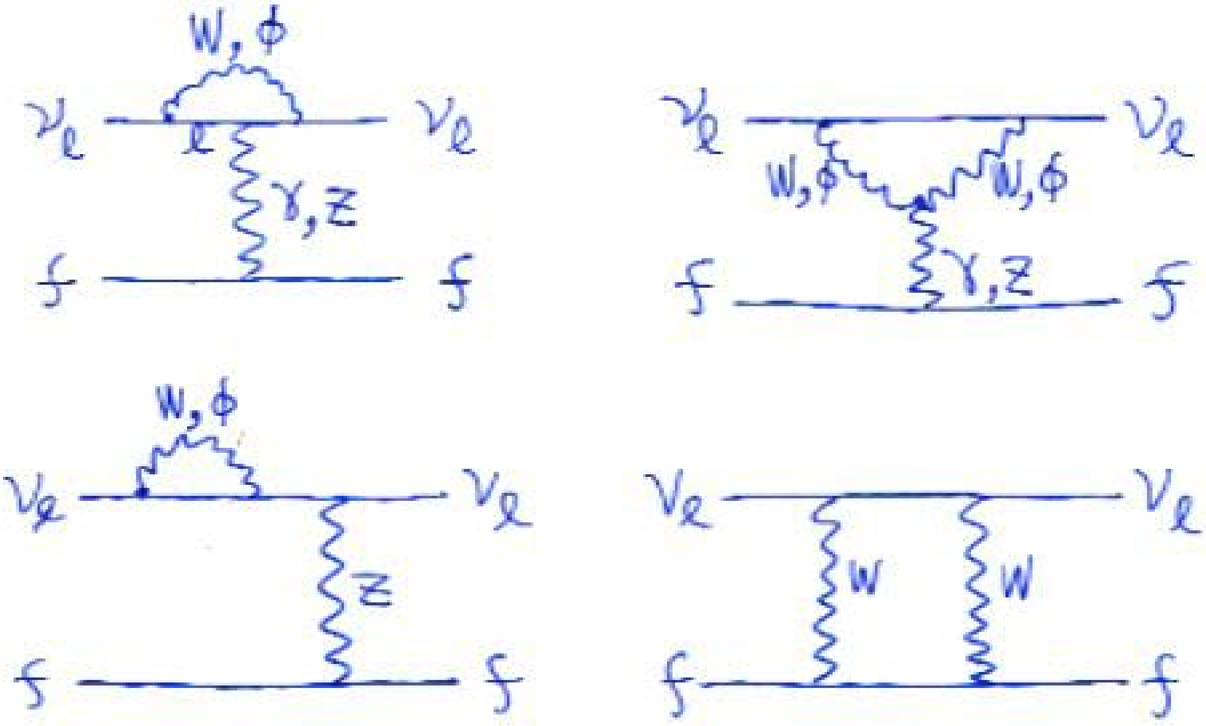
$\rightarrow$

~~CP~~ phase effect negligibly small

$$\sim \frac{b}{a}\cdot\frac{\Delta m_\odot^2}{\Delta m_{atm}^2} \simeq 5\times 10^{-5}\cdot(10^{-3}-10^{-2})$$

$$= 5\times(10^{-8}-10^{-7})$$

$\left(\begin{array}{l} \Delta m_\odot^2 = 10^{-6}-10^{-5}\ eV^2 \\ \Delta m_{atm}^2 = 10^{-3}\ eV^2 \end{array}\right.$



One-loop diagrams which gives rise to non-universal (ν_ℓ-dependent) radiative corrections to $\nu_\ell f \to \nu_\ell f$ ($f = e, u, d$)

Almost Degenerate Majorana ν
+ $0\nu\beta\beta$ Limit

$\Updownarrow$ (almost) incompatible

MSW $\begin{pmatrix}\text{small-angle}\\ \text{large-angle}\end{pmatrix}$ $\nu_\odot$ solutions

Hot dark matter $\nu \longrightarrow m_\nu \simeq$ a few eV

double β limit

$$\langle m_\nu \rangle = \left| \sum_i U_{ei}{}^2 m_i \right| < 0.2 \text{ eV}$$

(Heidelberg - Moscow 99)

$$\simeq \underset{\substack{\parallel \\ \text{a few eV}}}{m} \left| c_{12}^2 c_{13}^2 + s_{12}^2 c_{13}^2 e^{i\alpha} + s_{13}^2 e^{i\beta} \right|$$

An efficient cancellation must take place

$\rightarrow$ allowed region in solar triangle plot

Solar Triangle Plot

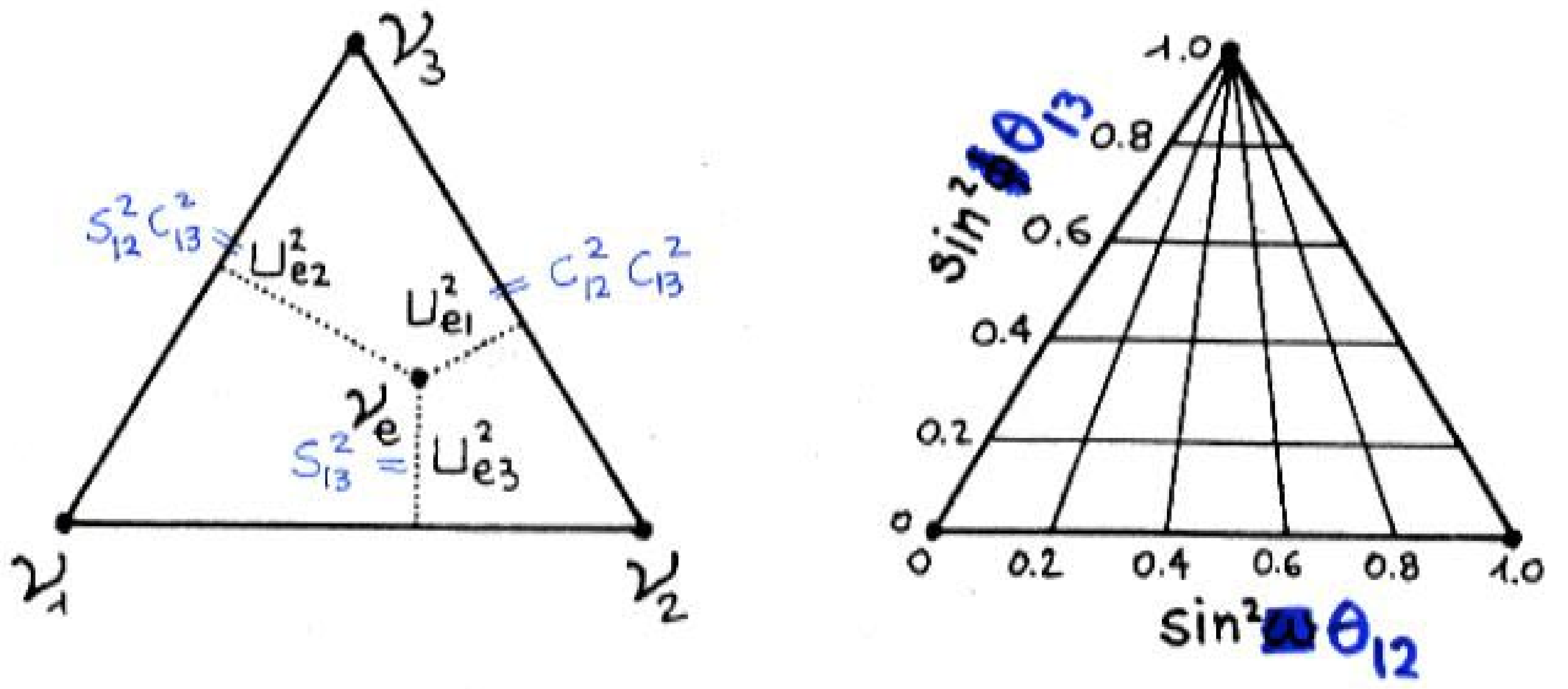


Figure 7 Triangle graph for solar neutrinos.

New!

Heidelberg-Moscow : $\langle m_\nu \rangle < 0.2$ eV

$$\frac{\langle m_\nu \rangle}{m} \equiv r \leq \frac{0.2}{4.5} \simeq 0.04$$

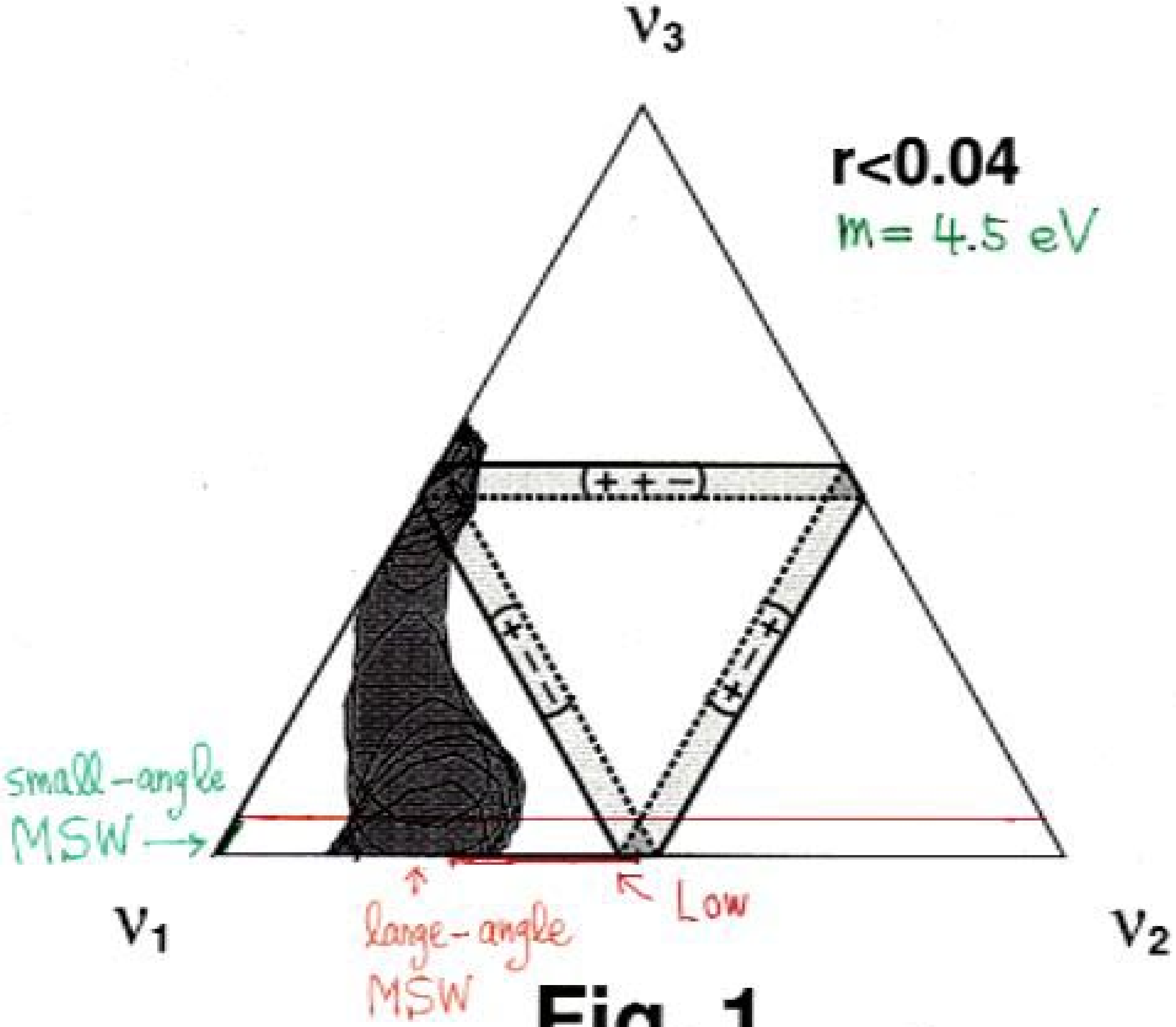


Fig. 1

(H. M. & O. Yasuda)

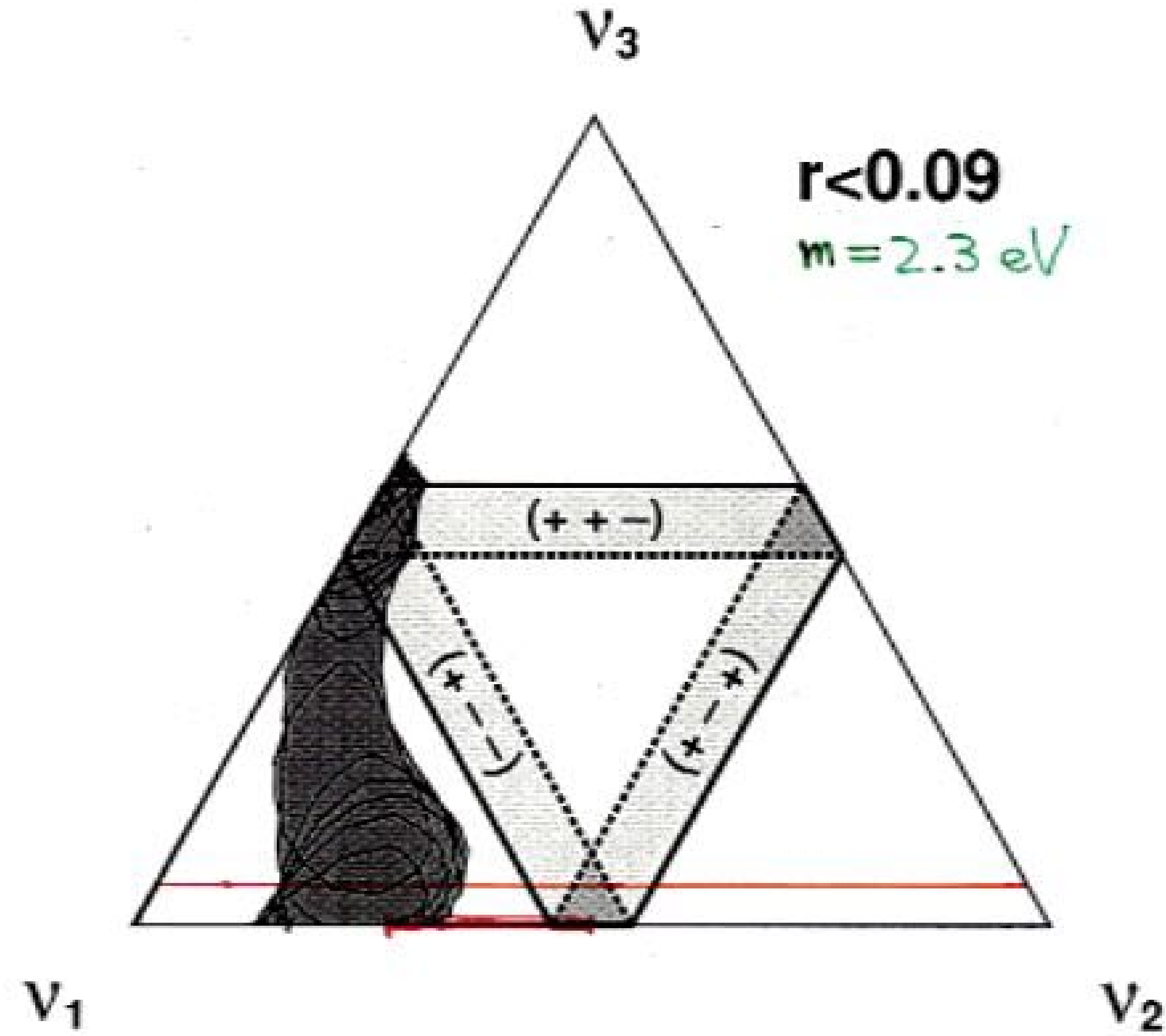


Fig. 2

In order to make contact with the more familiar two-flavor solutions, the full mass-mixing parameter space of solar neutrinos is shown in Fig. 9, together with the MSW and vacuum oscillation solutions (qualitative). The purely 2ν cases typically shown in the data analyses correspond to the left half of the front rectangles in Fig. 9 ($\phi = 0, \omega < \pi/4$). For the MSW case, notice how the usual small and large mixing solutions are connected in the 3ν space. For the vacuum case, notice how the usual 2ν solutions are symmetrized with respect to $\omega = \pi/4$ and connected in the 3ν space.

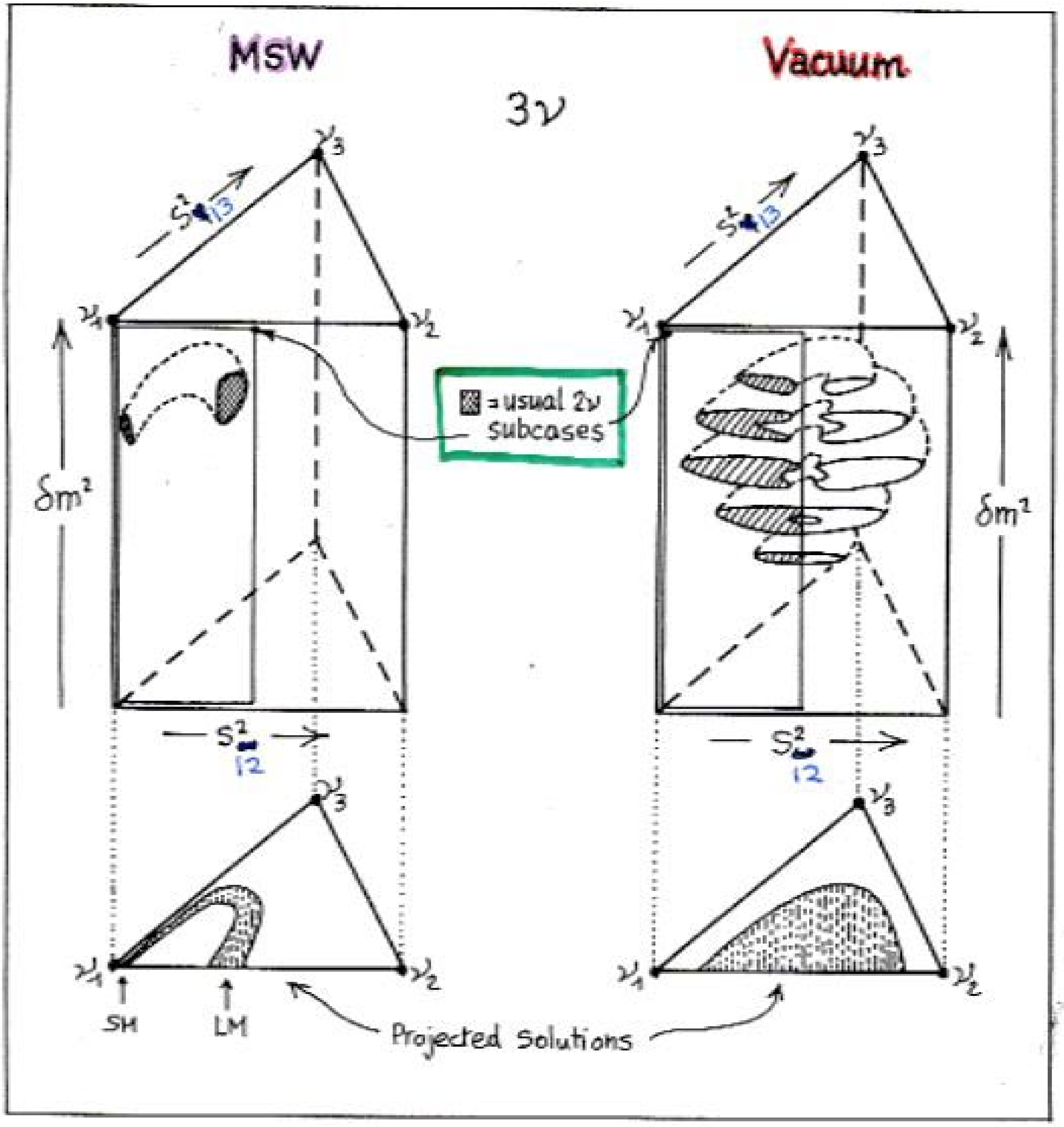


Figure 9 MSW and vacuum oscillation solutions to the solar neutrino problem in the 3ν space (qualitative).

Lower Bound on $R \equiv \dfrac{\text{No. of } \mu \text{ observed at SK}}{\text{No. of } \mu \text{ expected at SK}}$

$\updownarrow$ **or**

Less than (??) Just-so Oscillation in K2K Exp. ?

motivated by unofficial ....

K2K started mid. March (with horn) $\to$ June 27.

Total proton on target 5.3×10^{18}

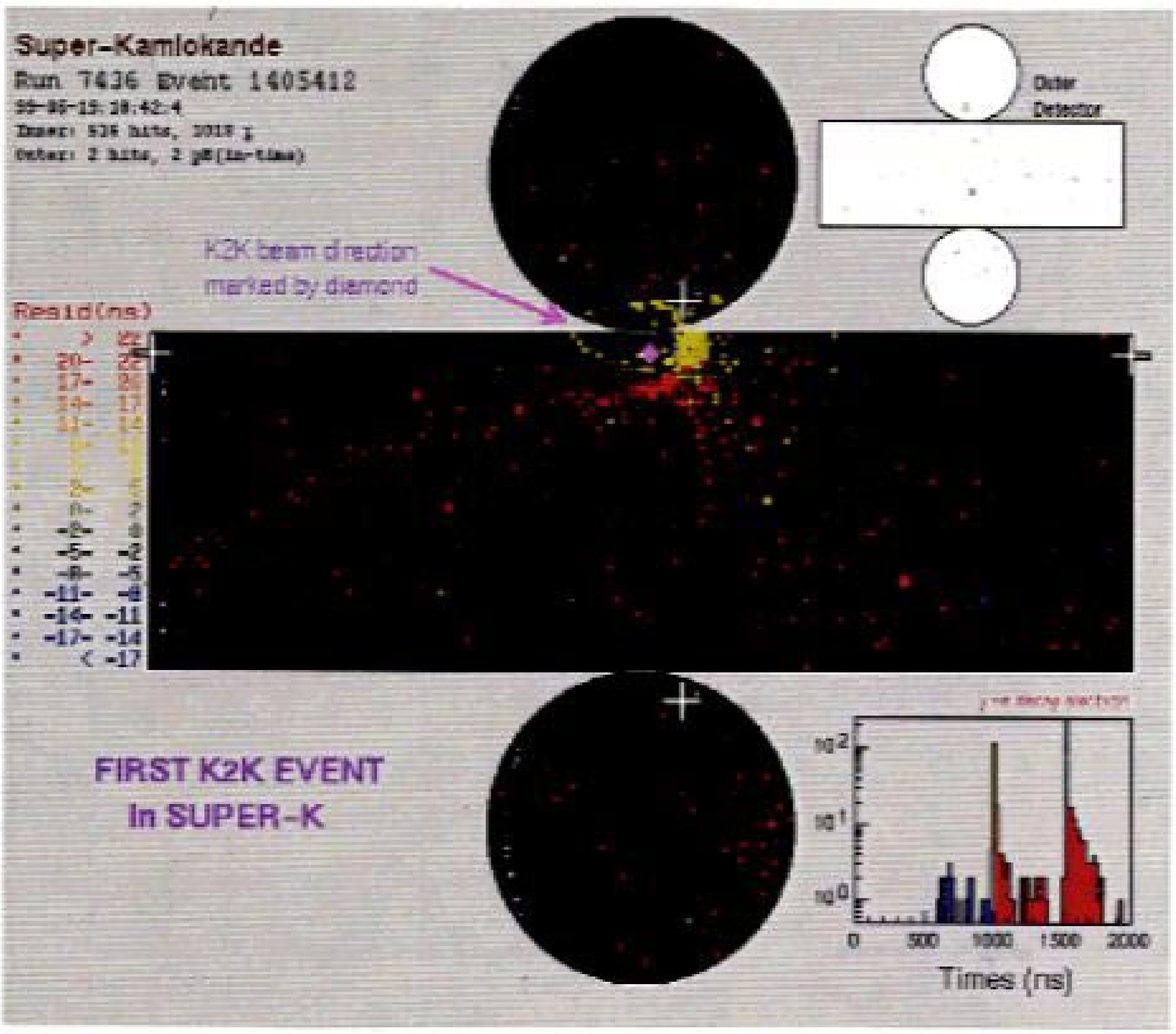
June 4-27 : stable run $\sim 3 \times 10^{18} \sim \frac{1}{30} \times 10^{20}$

$\sim$ 10 events expected

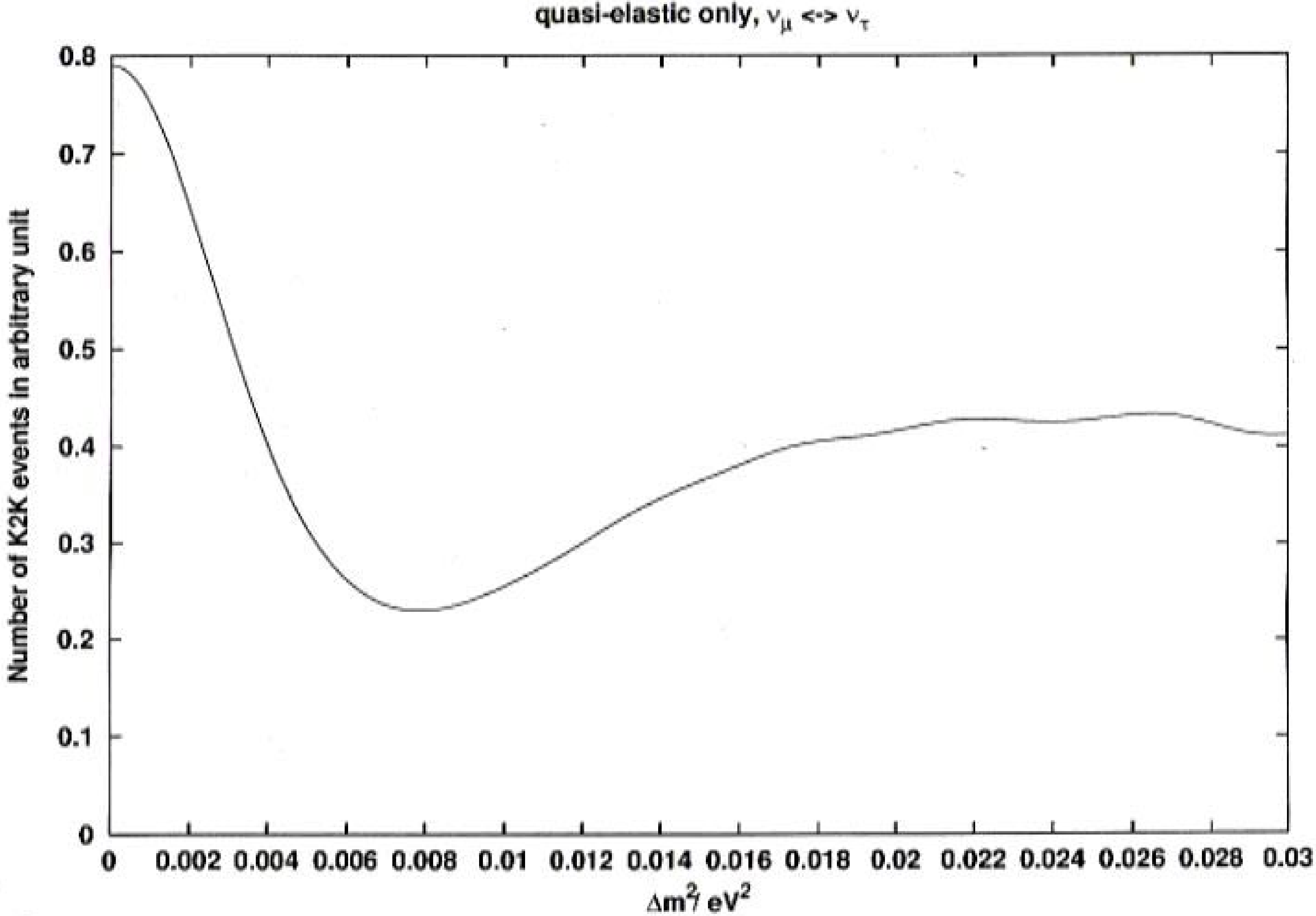
$\leftrightarrow$

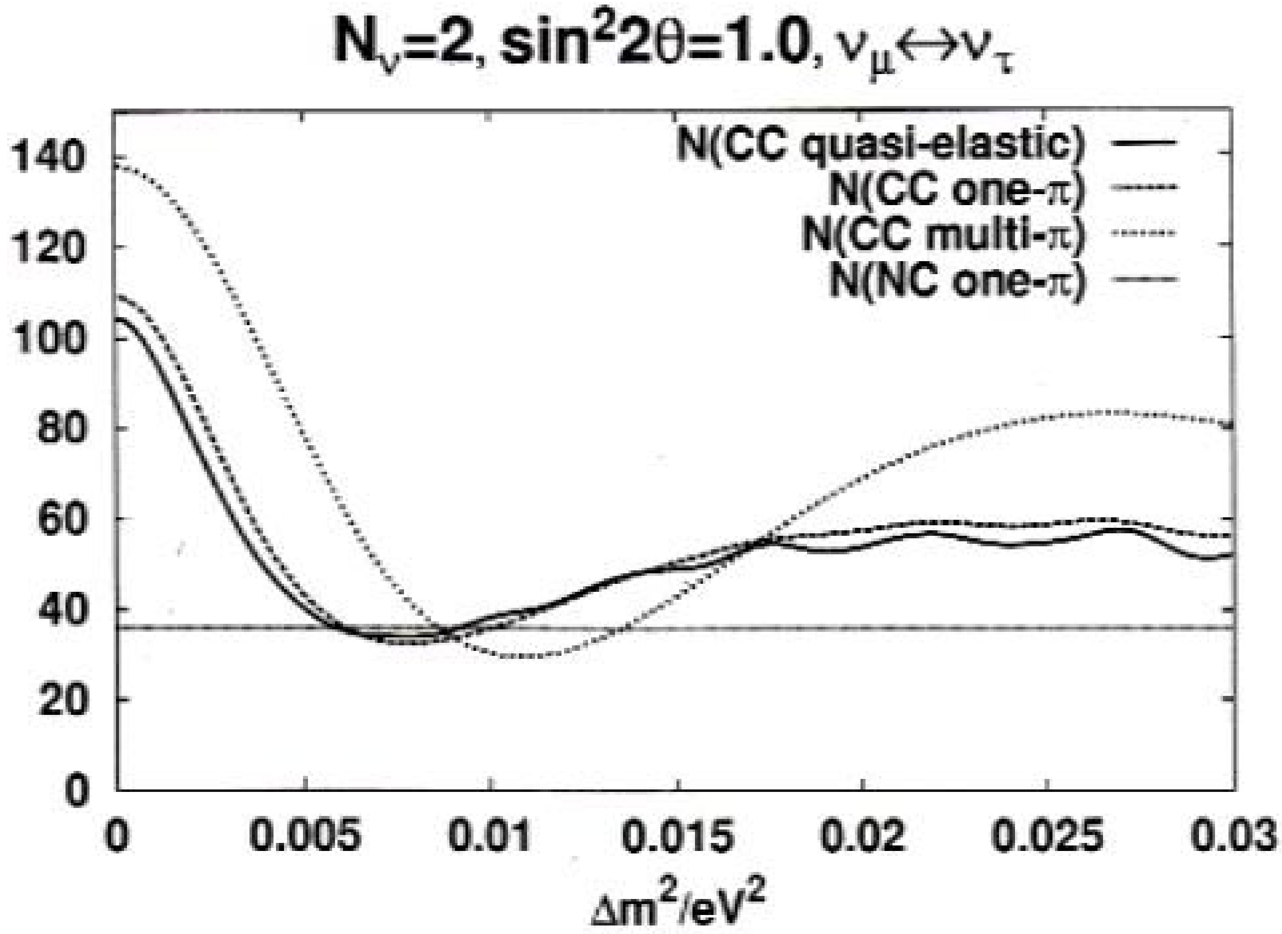
only 1 event found in the fiducial volume

Becaus of the beam energy spread R cannot be too small $\longrightarrow$ **Lower bound on R**

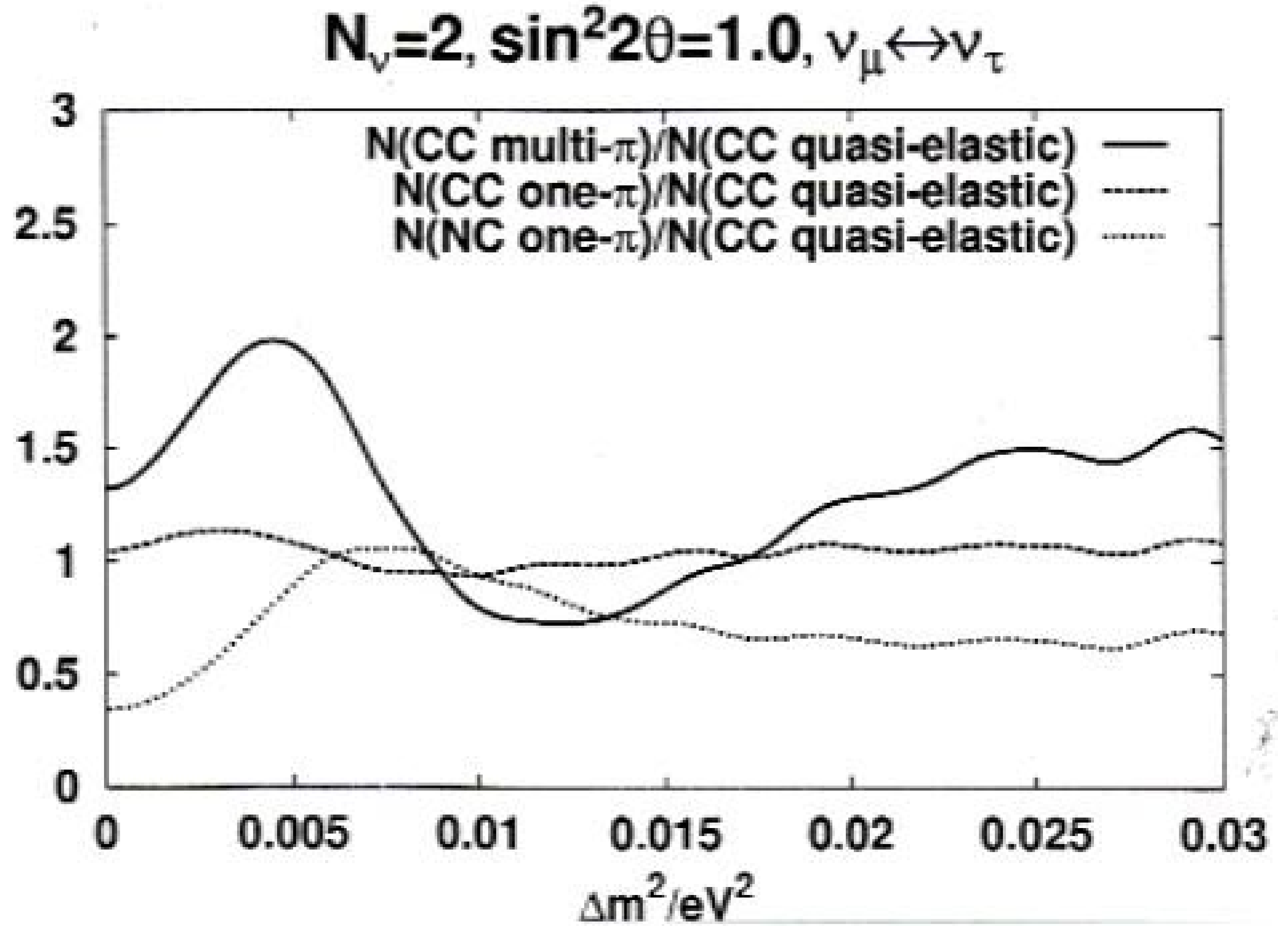


Super-Kamiokande
Run 7436 Event 1405412
Outer Detector
K2K beam direction
marked by diamond
Resid(ns)
FIRST K2K EVENT
In SUPER-K
Times (ns)


quasi-elastic only, ν_μ <-> ν_τ
Number of K2K events in arbitrary unit
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 0.002 0.004 0.006 0.008 0.01 0.012 0.014 0.016 0.018 0.02 0.022 0.024 0.026 0.028 0.03
Δm^2/ eV^2



$N_\nu=2$, $\sin^2 2\theta=1.0$, $\nu_\mu \leftrightarrow \nu_\tau$
N(CC quasi-elastic)
N(CC one-π)
N(CC multi-π)
N(NC one-π)
0
20
40
60
80
100
120
140
0
0.005
0.01
0.015
0.02
0.025
0.03
$\Delta m^2/eV^2$



$N_\nu=2$, $\sin^2 2\theta=1.0$, $\nu_\mu \leftrightarrow \nu_\tau$
N(CC multi-π)/N(CC quasi-elastic)
N(CC one-π)/N(CC quasi-elastic)
N(NC one-π)/N(CC quasi-elastic)
0
0.5
1
1.5
2
2.5
3
0
0.005
0.01
0.015
0.02
0.025
0.03
$\Delta m^2/eV^2$