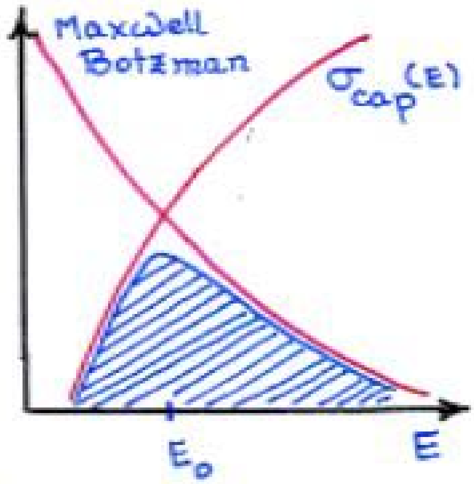
Screening Effects in Astrophysical fusion reactions



in stars:

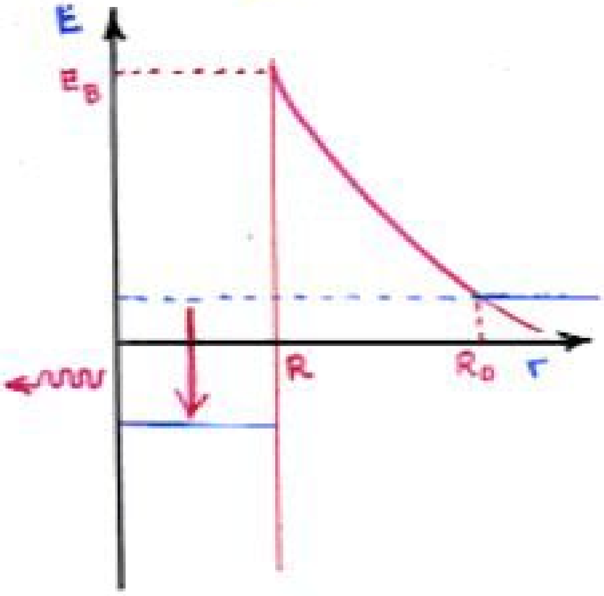
$1+2 \longrightarrow 3\,(+4)$

reaction rates:

$$r_{1,2} = \frac{n_1 n_2}{1+\delta_{12}} \int_0^\infty \phi_{MB}(E)\, v(E)\, \sigma(E)\, dE$$

Radiative capture reactions

$$\sigma_{cap}(E) = P(E)\, \sigma_{nuc}(E)$$

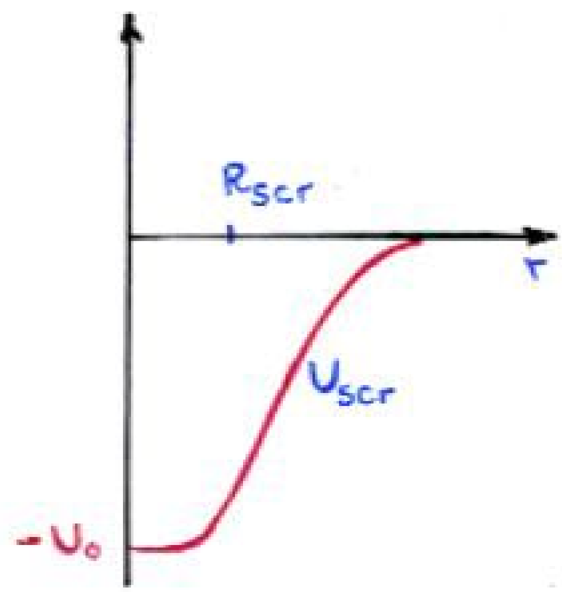


WKB, s-wave

$$P(E) = \left(\frac{E_B}{E}\right)^{1/2} \exp\left[-\frac{2}{\hbar}\sqrt{2\mu} \int_R^{R_0} \left[\frac{Z_1 Z_2 e^2}{r} + U_{scr}(r) - E\right]^{1/2} dr\right]$$

$$R_0 \approx \frac{Z_1 Z_2 e^2}{E} \sim 0.005 \text{ au}$$

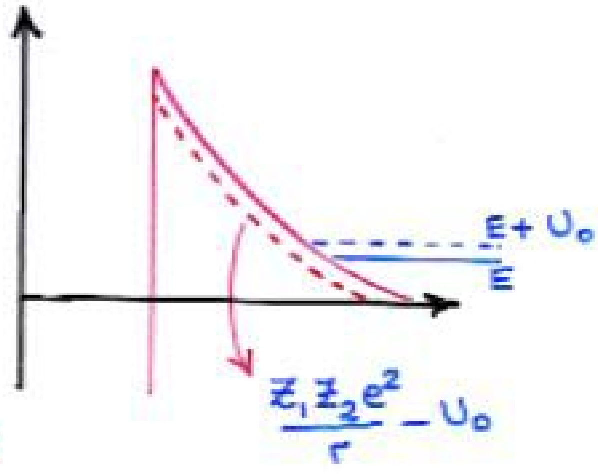
$$\ll R_{scr} \sim 0.5 \text{ au}$$



Laboratory screening

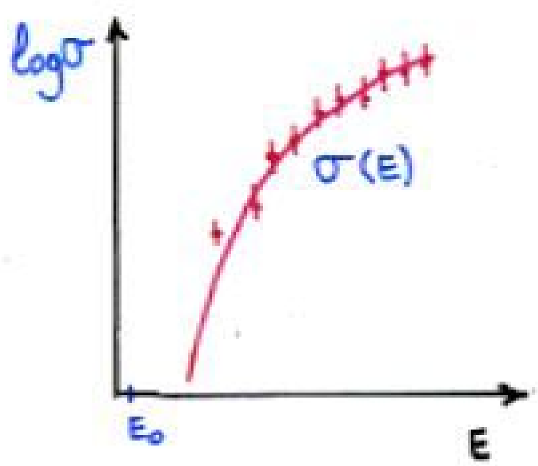
$$P(E) \cong e^{\pi \eta \frac{U_0}{E}} \, P_0(E)$$

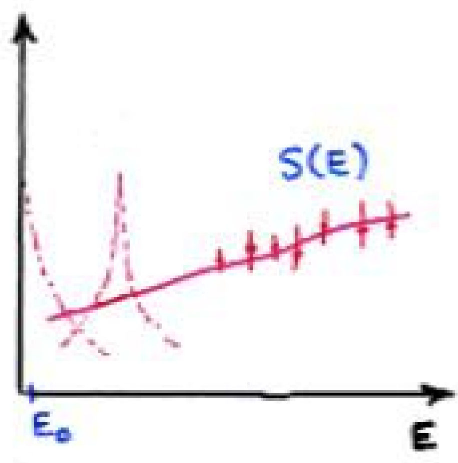
$$\eta(E) = \frac{Z_1 Z_2 e^2}{\hbar v}$$



S-factor

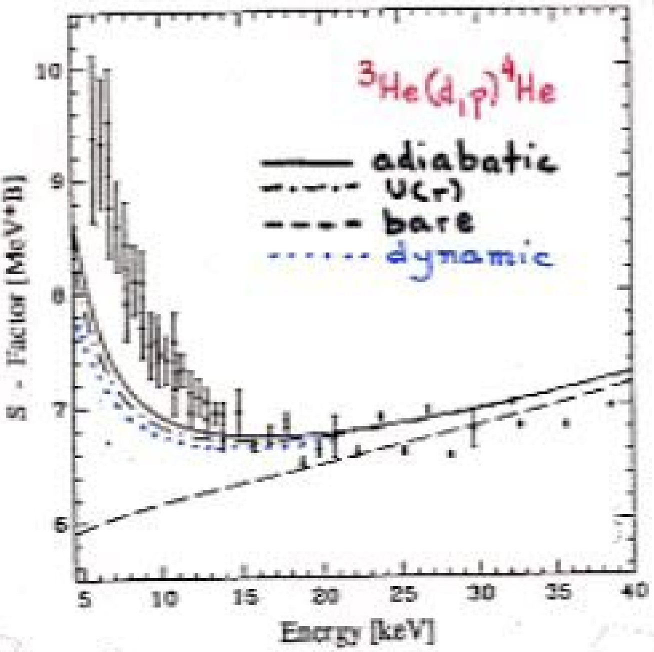
$$\sigma(E) = \frac{S(E)}{E} e^{-2\pi\eta(E)}$$





Adiabatic model (Assenbaum, Langanke, Rolfs, Z. Phys. A327 (1987) 461)

$$\boxed{U_0 = E_i - E_f}$$



dynamic TDHF

Shoppa, Koonin, Langanke, Seki, PRC 48(1993) 837

$U_0^{dyn} < U_0^{ad}$ (elect. excitations)

$U_0^{exp} = 180 \pm 30$ eV

$U_0^{ad} = 119$ eV

Langanke, Barnes, Adv. Nucl. Phys., 22(1996) 173

Reaction	ΔE (eV) experiment	ΔE (eV) adiabatic limit
$d(^3He, p)^4He$	180 ± 30	119
$^6Li(p, \alpha)^3He$	470 ± 150	186
$^6Li(d, \alpha)^4He$	380 ± 250	186
$^7Li(p, \alpha)^4He$	300 ± 280	186
$^{11}B(p, \alpha)2^4He$	620 ± 65	348

$$U_0^{exp} \sim \frac{U_0^{ad}}{2} \quad (\text{PUZZLE !})$$

Small effects in thermonuclear reactions

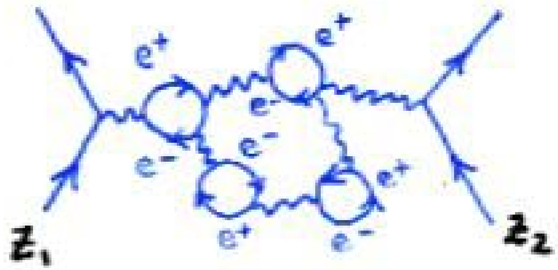
Balantekin, Bertulani, Hussein, Nucl. Phys. A627 (1997) 324

$$P(E) \propto \exp\left[-\frac{2}{\hbar}\int_R^{R_0} p(r)\,dr\right]$$

$$p(r) = \sqrt{2\mu}\left[\frac{Z_1 Z_2 e^2}{r} + U_0 + U_{small}(r) - E\right]^{1/2}$$

Vacuum polarization

$$\boxed{\begin{array}{l} \sigma_{corr} = \sigma \cdot R \\ R = \dfrac{P(U_{small})}{P(U_{sm}=0)} \end{array}}$$

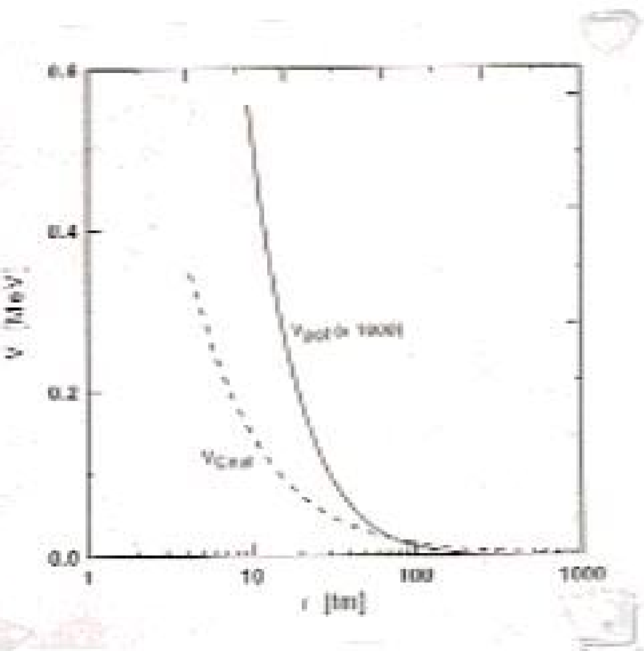


$$U_{small} \equiv U_{pol}(r)$$

$$= \frac{Z_1 Z_2 e^2}{r}\,\frac{2\alpha}{3\pi}\,I\left(\frac{2r}{\lambda_e}\right)$$

Uehling, PR 48 (1935) 55

$$I(x) = \int_1^{\infty} e^{-xt}\left(1+\frac{1}{2t^2}\right)\frac{\sqrt{t^2-1}}{t^2}\,dt$$



Relativistic effects

$$E = \frac{p^2}{2\mu} + \frac{Z_1 Z_2 e^2}{r} - \frac{p^4}{8c^2}\left(\frac{1}{m_1^3}+\frac{1}{m_2^3}\right) + \frac{Z_1 Z_2 e^2}{2m_1 m_2 c^2}\left(\frac{p^2+p_r^2}{r}\right)$$

$\Rightarrow$

$$p = (2\beta)^{-1/2}\left[-(\alpha+\gamma)+\sqrt{(\alpha+\gamma)^2+4\beta(V_c-E)}\right]^{1/2}$$

$$\alpha = 1/2\mu \;,\; \beta = (1/m_1^3 + 1/m_2^3)/8c^2 \;,\; \gamma = Z_1 Z_2 e^2/m_1 m_2 c^2 r$$

Atomic polarizability

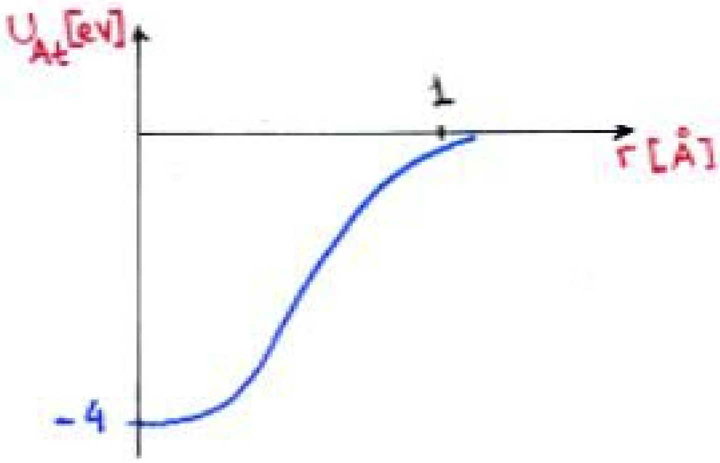
$$U_{At} = -\sum_{n\neq 0} \frac{|\langle 0|V_c(\vec{r}_i,\vec{R})|n\rangle|^2}{E_n - E_0} \;,\; V_c = \sum_i \frac{Z_1 e^2}{|\vec{R}-\vec{r}_i|}$$

$$\langle 1s|V_c^{mono}|2s\rangle = \frac{8\sqrt{2}}{27} E f(x) \;; \qquad x = \frac{3Z_2 R}{2a_0}$$

$$f(x) = (1+x)e^{-x} \;; \qquad a_0 = \frac{Z_1 Z_2 e^2}{2E}$$

$$\langle 1s|V_c^{dip}|2p\rangle = \frac{16}{9\sqrt{2}} E\, g(x)$$

$$g(x) = \frac{1}{x^2}\left(8-(8+8x+4x^2+x^3)e^{-x}\right)$$



Bremsstrahlung

$$dE_{Br}(\omega) = d\omega\, d\Omega\, \frac{\omega^2}{c^3} \left| \int_{-\infty}^{0} \frac{dt}{2\pi} e^{i\omega t} \sum_j q_j e^{-ik\hat{n}\cdot\vec{r}_j(t)} \times \left[\vec{v}_j(t) \times \hat{n}\right] \right|^2$$

$$E_{Br} = \frac{4}{3\pi^2} \hbar\omega_0 \alpha \left(\frac{v}{c}\right)^2 A_R^2 \left\{ f_1^2 + \frac{2}{5\pi^2} f_2^2 \left(\frac{v}{c}\right)^2 \times \left[\left(\frac{3}{2} - \ln(2\pi)\right)^2 + \zeta(2,2) \right] \right\}$$

$$\omega_0 = a_0/v \;; \quad f_\lambda = A_R^{\lambda-1}\left(\frac{Z_1}{A_1^\lambda} - \frac{Z_2}{A_2^\lambda}\right) \;; \quad A_R = \frac{A_1 A_2}{A_1 + A_2}$$

Table 1

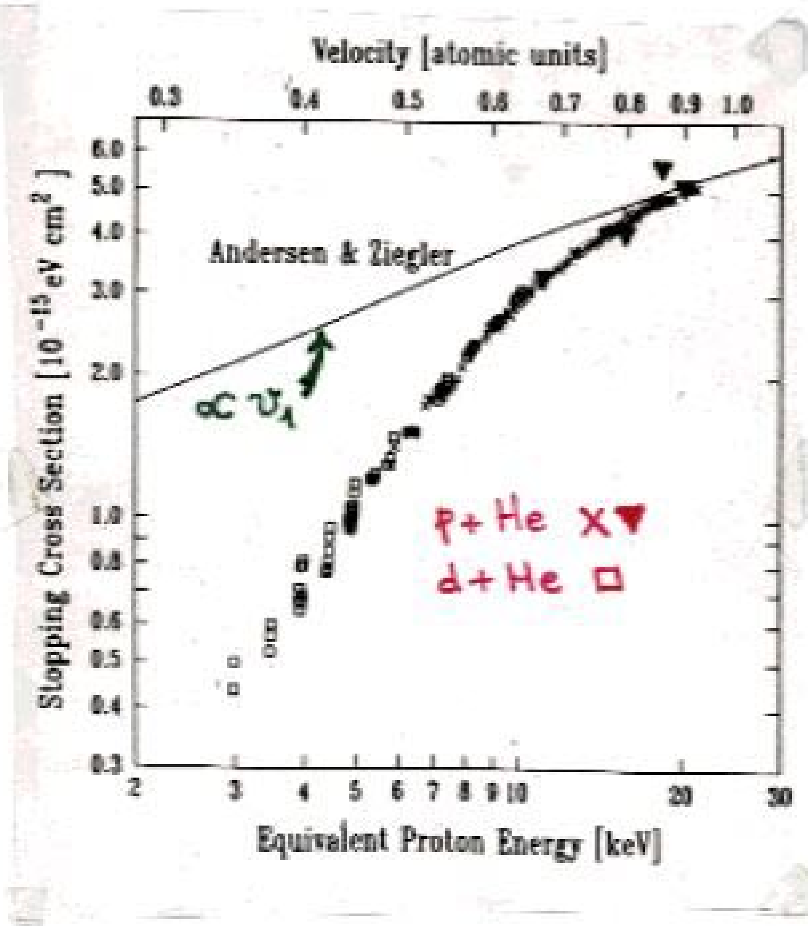
Lowest experimental energies, E_{min}, energy corrections [24] due to the screening by the atomic electrons, U_e, nuclear radii, and correction factors for the nuclear reaction: (a) due to atomic screening, $1-R_{scr}$; (b) vacuum polarization, $1-R_{VPol}$; (c) relativity, $1-R_{rel}$; (d) bremsstrahlung, $1-R_{Br}$; (e) atomic polarization, $1-R_{At}$

Reaction	E_{min} [keV]	U_e [eV]	R_n [fm]	$1-R_{scr}$	$1-R_{VPol}$ $[\times 10^{-2}]$	$1-R_{rel}$ $[\times 10^{-3}]$	$1-R_{Br}$ $[\times 10^{-3}]$	$1-R_{At}^{dip}$ $[\times 10^{-3}]$	$1-R_{At}^{mon}$ $[\times 10^{-1}]$
D(d, p)T	1.62	20	4.3	0.164	0.95	0.17	0.54	1.01	0.246
^{3}He(d, p)^{4}He	5.88	119	4.3	0.331	−1.60	0.47	1.12	0.39	0.314
D(^{3}He, p)^{4}He	5.38	113	4.3	0.364	−1.58	0.47	1.00	2.11	0.357
^{3}He(^{3}He, 2p)^{4}He	25	292	3.0	0.196	−3.14	1.75	0.58	0.35	0.321
^{6}Li(p, α)^{3}He	10.74	186	3.0	0.758	−1.82	1.07	1.36	0.30	0.360
^{7}Li(p, α)^{4}He	12.70	186	4.3	0.198	−1.88	1.04	1.28	0.17	0.284
^{6}Li(d, α)^{4}He	14.71	186	3.0	0.218	−2.32	0.72	0.71	0.15	0.313
H(^{6}Li, α)^{3}He	10.94	186	3.0	0.250	−1.82	1.07	1.23	2.55	0.350
H(^{7}Li, α)^{4}He	12.97	186	4.3	0.191	−1.88	1.04	1.17	1.42	0.275
D(^{6}Li, α)^{4}He	15.89	186	3.3	0.184	−2.34	0.71	0.35	0.91	0.262
^{10}B(p, α)^{7}Be	18.70	346	3.3	0.376	−2.38	2.05	1.45	0.57	0.758
^{11}B(p, α)^{8}Be	16.70	346	2.0	0.462	−2.36	2.00	2.13	0.85	0.906

NPA 627 (1997) 324

Stopping power

$$S(v_1) = -\frac{dE}{N\,dx} = \int_{\Delta E_{min}} \Delta E(v_1)\, d\sigma(\Delta E, v_1) = \overline{\Delta E}\ \sigma(v_1)$$



Fermi, Teller
PR 72 (1947) 399

- dynamic polarization of an electron plasma
- energy loss to plasmons

$$S \sim v_1 \rho^{1/3}$$

ρ ≡ electron density

data:
Golser, Semrad
PRL 66 (1991) 1831

"The stopping and ranges of ions in matter", eds. Andersen, Ziegler (Pergamon, 1977) Vol. 3

Janni, At. Data Nucl. Data Tables, 27 (1982) 147

E > 25 keV

$$S \sim v_1^{3-4}$$

Grande, Schiwietz, PRA 47 (1993) 1119

(i) $H^+ + He^0 \longrightarrow H^+ + He^*$ or $\longrightarrow H^0 + He^+$

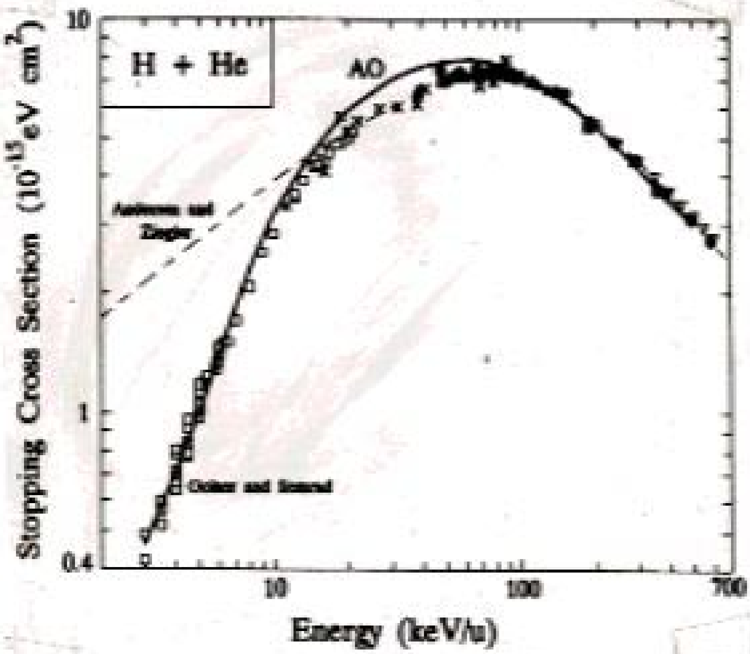
(ii) $H^0 + He^0 \longrightarrow H^0 + He^*$

(iii) $He^0 + H^0 \longrightarrow He^0 + H^*$

- time-dependent Schrödinger eq.
- active electron + frozen-core Hartree-Fock electrons
- semiclassical ion trajectories

$$S_e = \sum_i \sigma_i \, \Delta E_i$$

$n_i |a_i|^2$



$v \longrightarrow 0$

dominated by transfer

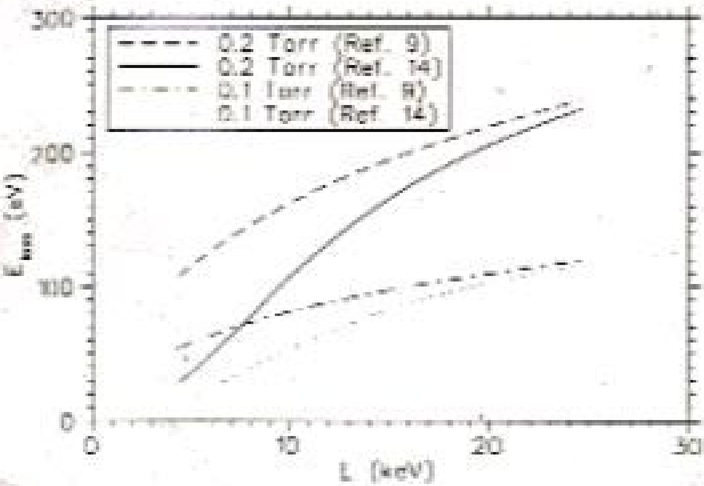
$^{3}He(d,p)^{4}He$

Bang, Ferreira, Maglione, Hansteen, PRC 53 (1996) R18

$\Delta E_{exp} = 180$ eV

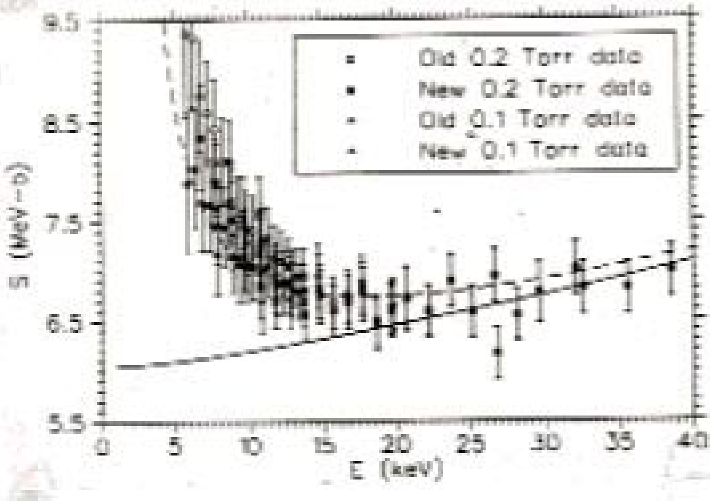
$\xrightarrow[\text{Corrected stopping power}]{}$ $\Delta E_{exp} = 126$ eV

$\sim \Delta E_{ad} = 120$ eV



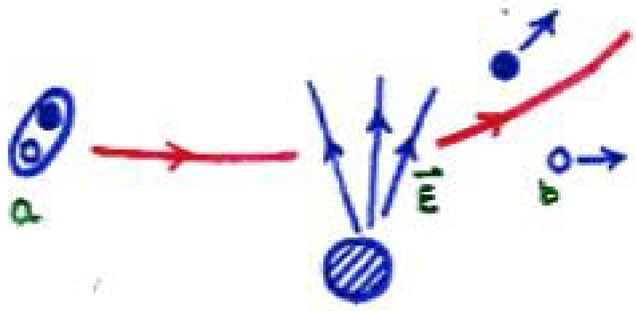
Langanke, Shoppa, Barnes, Rolfs, PLB 369 (1996) 211

Solved ?!



Baur, C. B., Rebel, NPA 458 (1986) 188

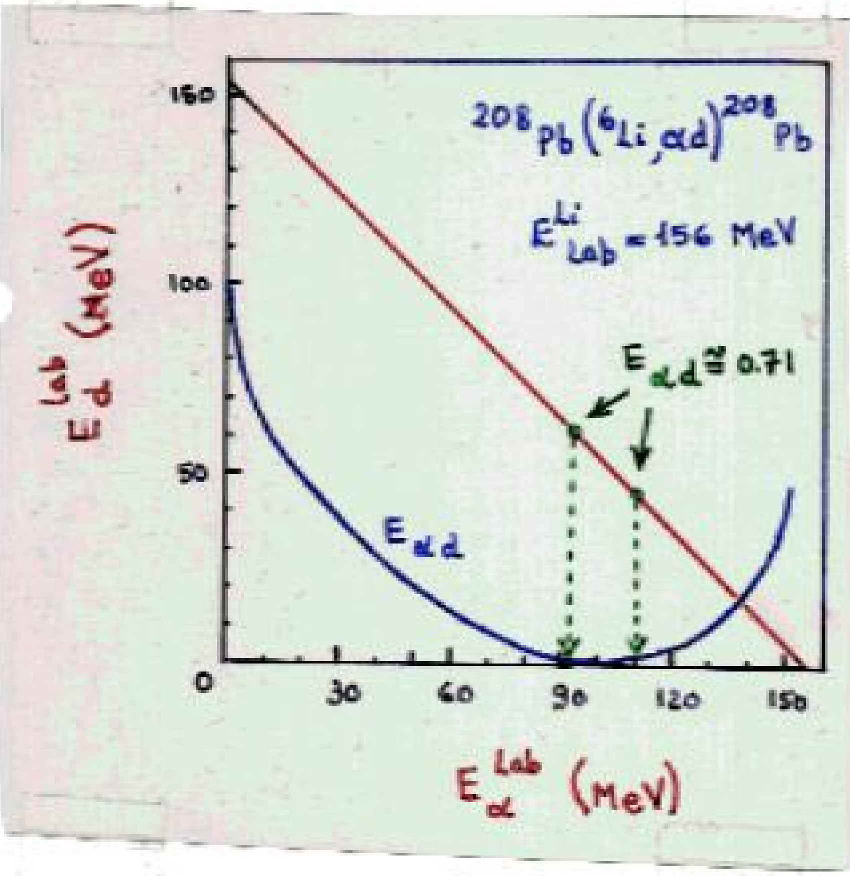
Coulomb Dissociation Method



$$\frac{d^2\sigma}{d\Omega\, dE_\gamma} = \frac{1}{E_\gamma} \sum_\lambda \frac{dn_{E\lambda}}{d\Omega}\, \sigma_{E\lambda}^{\gamma + a \to b + c}$$

Theory

$$F(\text{spins})\, \frac{k_{bc}^2}{k_\gamma^2}\, \sigma_{E\lambda}^{b+c \to a+\gamma}$$

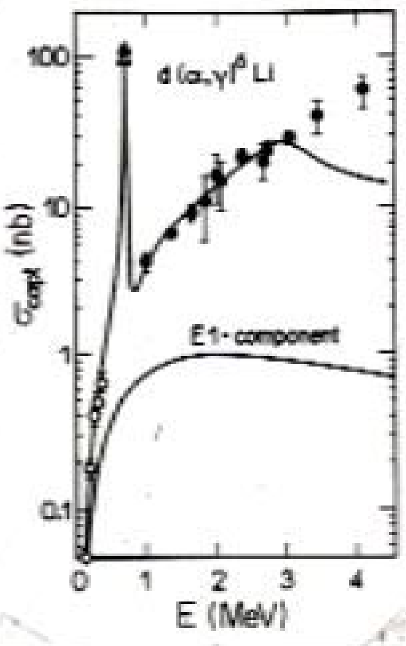


$k_{bc} \gg k_\gamma$

$\Rightarrow$ $\sigma_{photo} \gg \sigma_{cap.}$

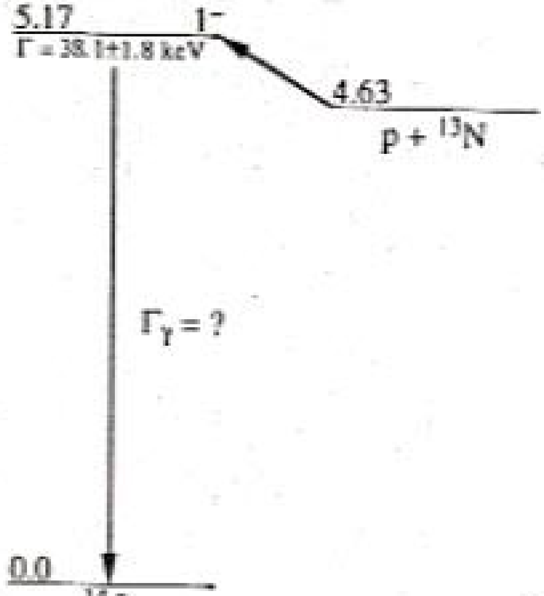
"Magnifying Glass" effect

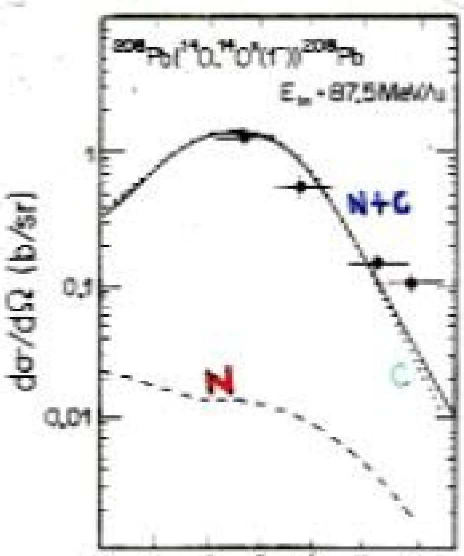
$$\Delta E_{bc} \sim 2\mu_{bc} \sqrt{\frac{E_c E_b}{m_c m_b}} \times \sin\theta_{bc}\, \Delta\theta_{bc}$$



Big Bang

Kiener et al., 1991



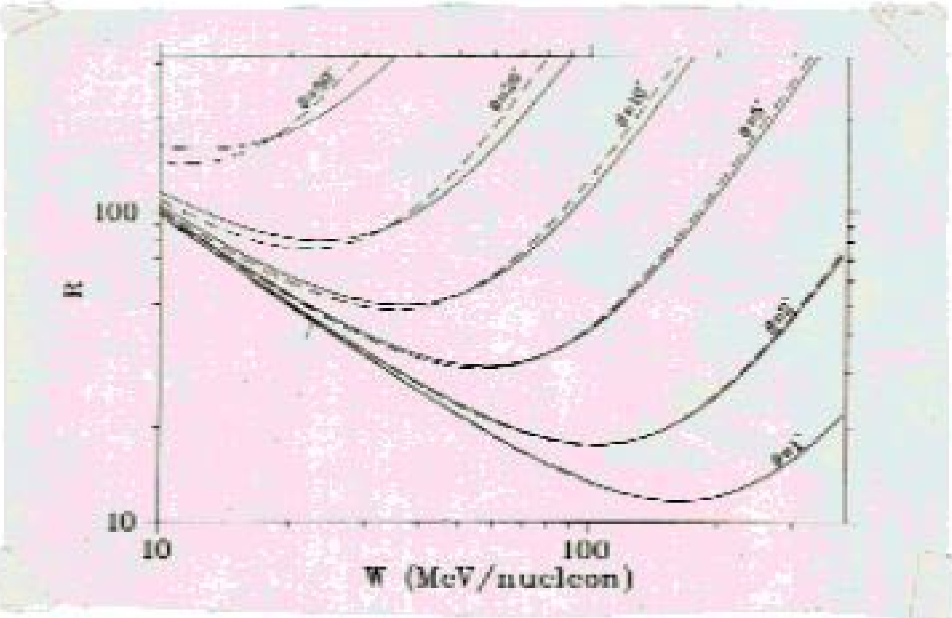


Motobayashi et al., PLB (1991)

$\Gamma_\gamma = 3.1 \pm 0.6$ eV

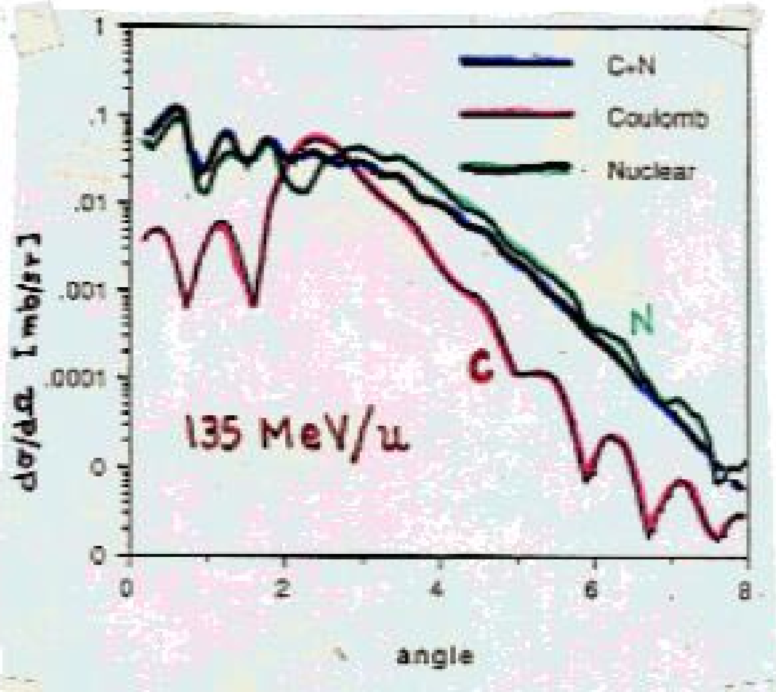
Hot CNO: $p(^{13}N,\gamma)^{14}O$

$^{12}C(\alpha,\gamma)^{16}O$



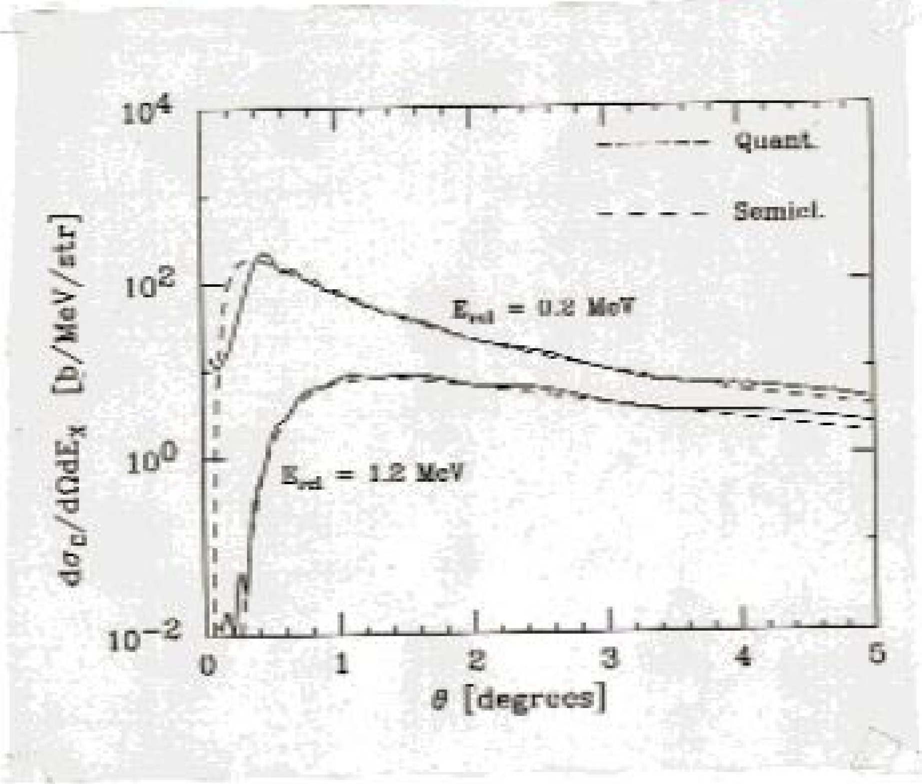
Shoppa, Koonin, PRC46(1992)382

$R = \frac{\sigma_{E2}}{\sigma_{E1}}$

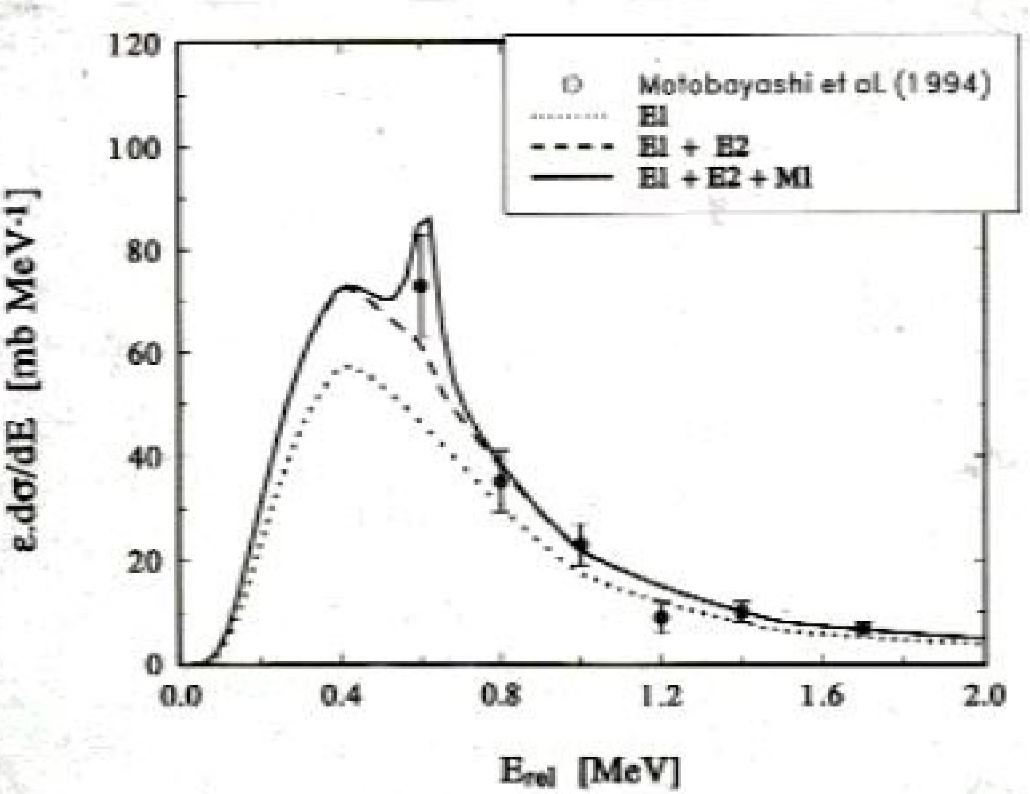


C.B., PRC49(1994)2688
NPA587(1995)318

Quantum x semiclassical

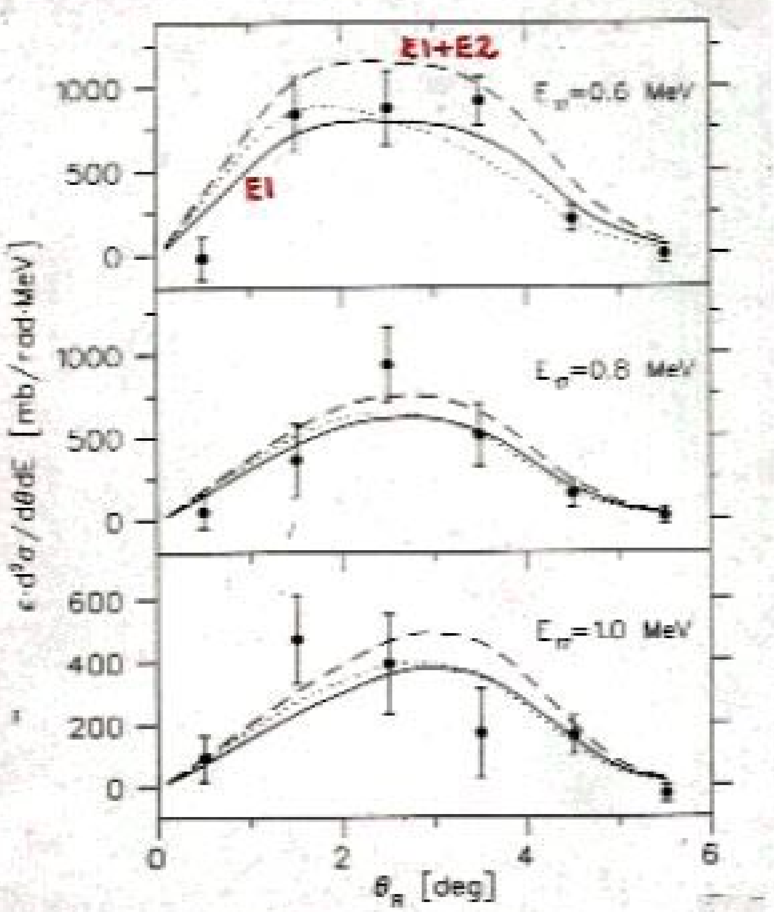


Comparison to data



$^8B + Pb \to p + ^7Be + Pb$

T. Motobayashi et al., PRL 70 (1994) 2680



Langanke, Shoppa, PRC 52 (1995) 1709

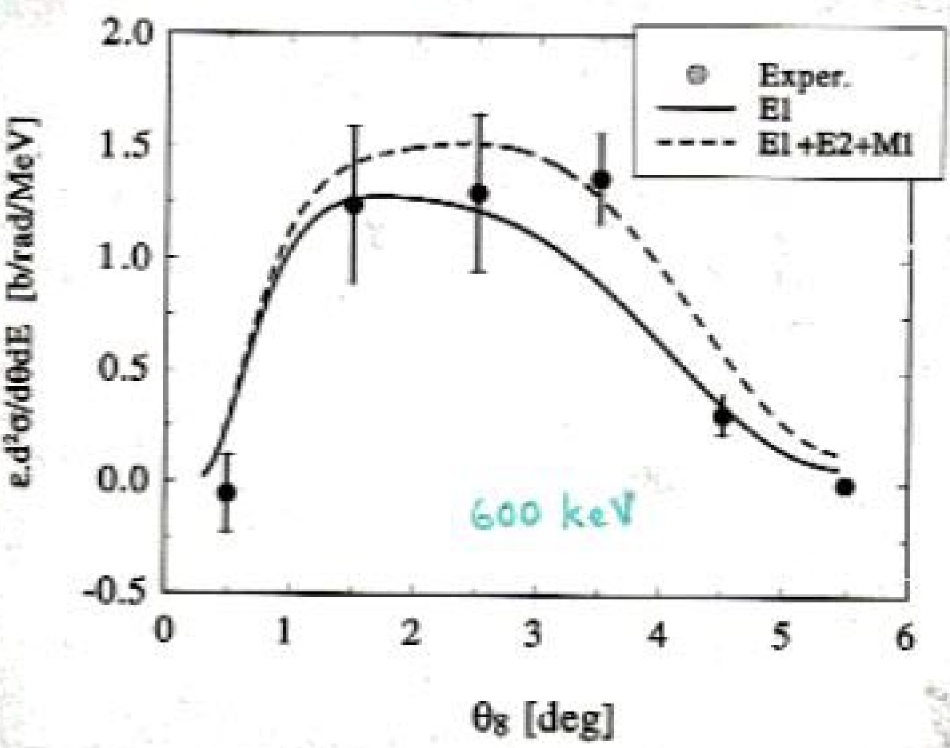
Science 266 (1994) 1157

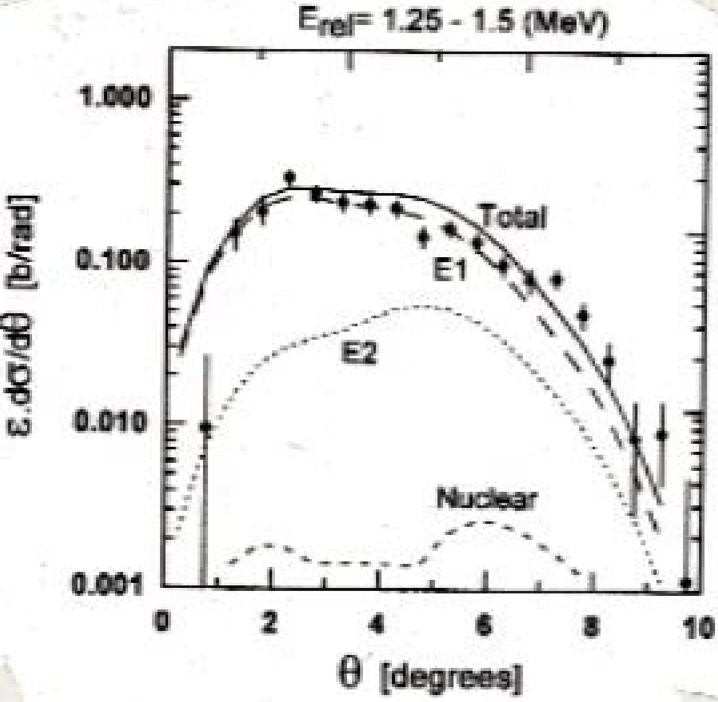
C.B., NPA 587 (1995) 318

Gai, C.B., PRC 52 (1995) 1706

χ-fit with E1, E2

$S_{E1}(0) \sim 18$ eV.b

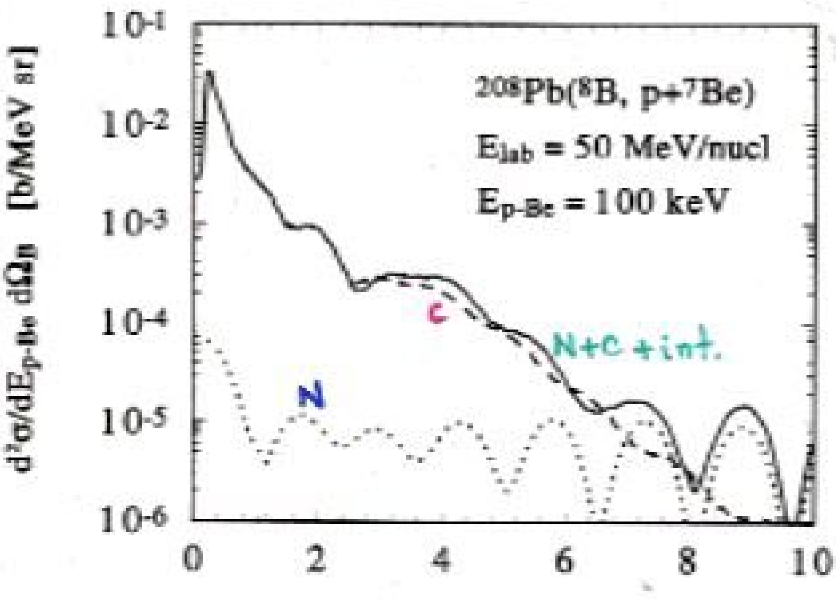




$^8B + ^{208}Pb$

50 MeV/nucl.

data: T. Kikuchi et al., Europ. Phys. J. A3 (1998) 213

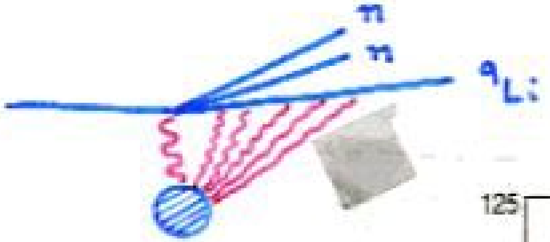


Theory:

C.B., NPA

Higher-order dynamical effects

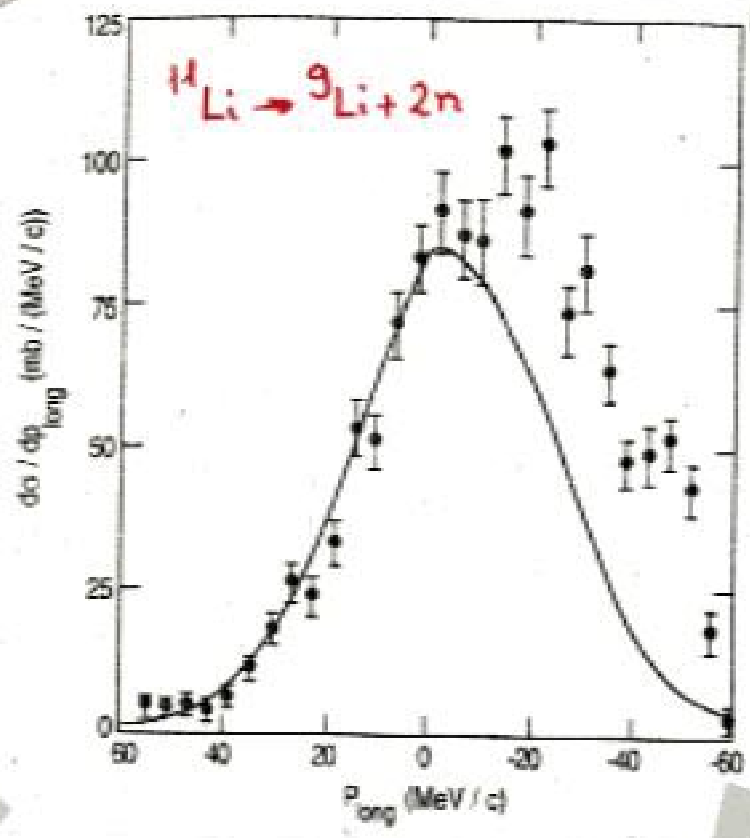
$\boxed{\sigma_c(E_r) \not\Rightarrow \sigma_\gamma(E_r)}$

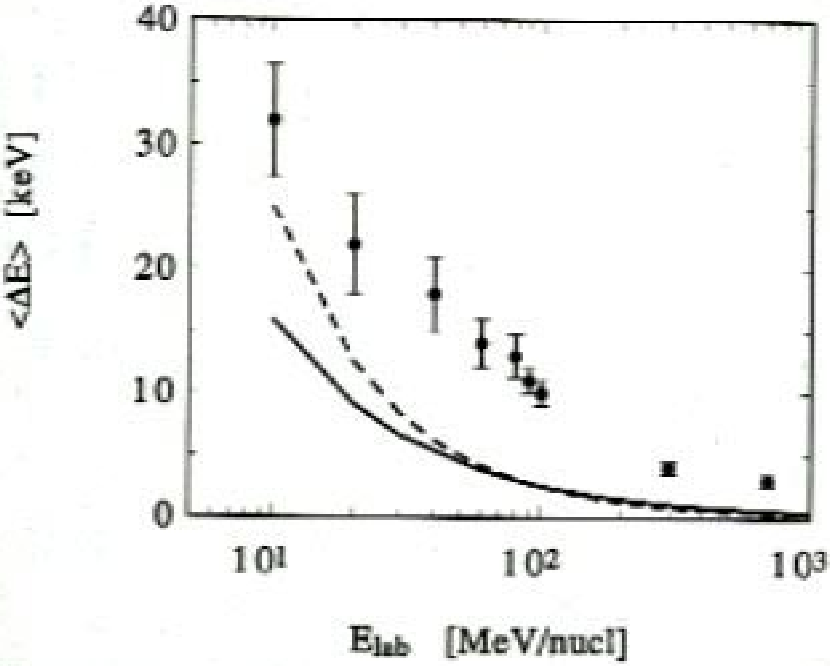


$S_{2n} = 0.3$ MeV

data:
Ieki et al.,
PRL 70 (1993) 730

theory:
C.B., G. Bertsch,
NPA 556 (1993) 136
NPA 581 (1995) 107
G. Baur, C.B., M. Kalassa
NPA 550 (1992) 527
Esbensen





^{8}B + ^{208}Pb

C.B., PRC 49 (1994) 2688
Z. Phys. A356 (1996) 293

● Monte Carlo
- - - - Classical (semi)
—— quantum

(p,n) reactions

$$V_{\pi+\rho+\text{medium corr.}} \simeq (\tau_1\cdot\tau_2)\left[v(\sigma_1\cdot\hat{q})(\sigma_2\cdot\hat{q}) + w(\sigma_1\cdot\sigma_2)\right]$$

$$\underset{}{v,w} = \underset{(L=0)}{\text{central}} + \underset{(L=2)}{\text{tensorial}}$$

Taddeucci et al., NPA 469(1987)127

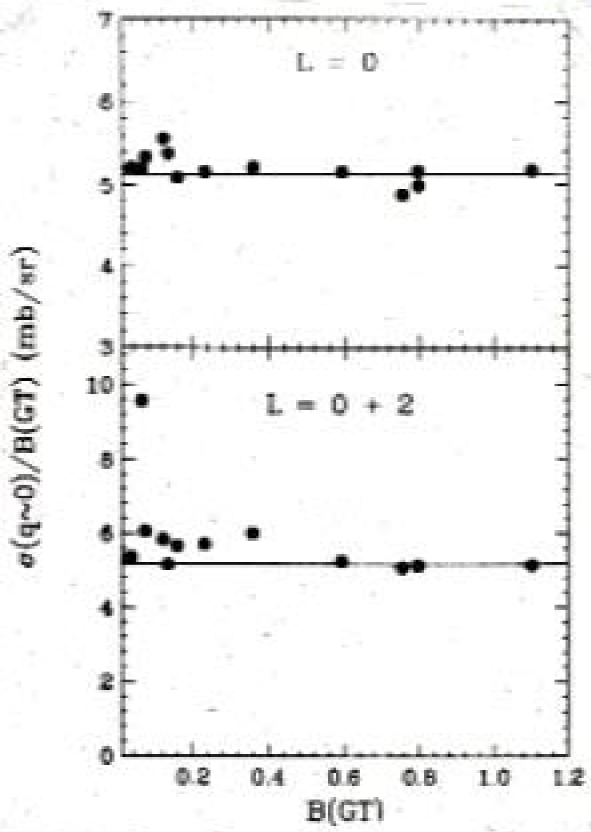
$$\boxed{\begin{aligned}&\frac{d\sigma}{d\Omega}(q\sim 0) = \left(\frac{\mu}{\pi\hbar^2}\right)^2 \frac{k_f}{k_i} N_{\sigma\tau} J_{\sigma\tau}^2 \, B(GT, i\to f)\\ &B(GT) = |\langle f|\sigma\tau|i\rangle|^2\end{aligned}}$$

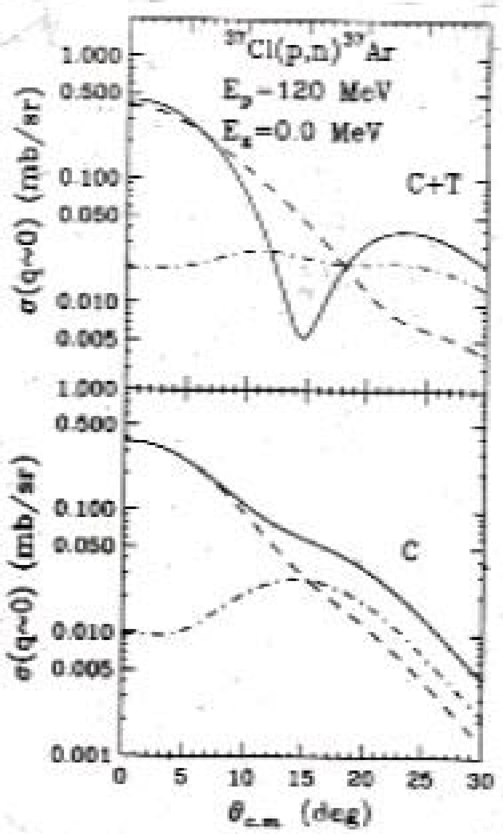
Lenske, Wolter, Bohlen, PRL 62(1989)1457
Osterfeld et al., PRC 45(1992)2854 } Heavy ion
C.B., NPA 554(1993)493

Calibration of solar neutrino detectors ?

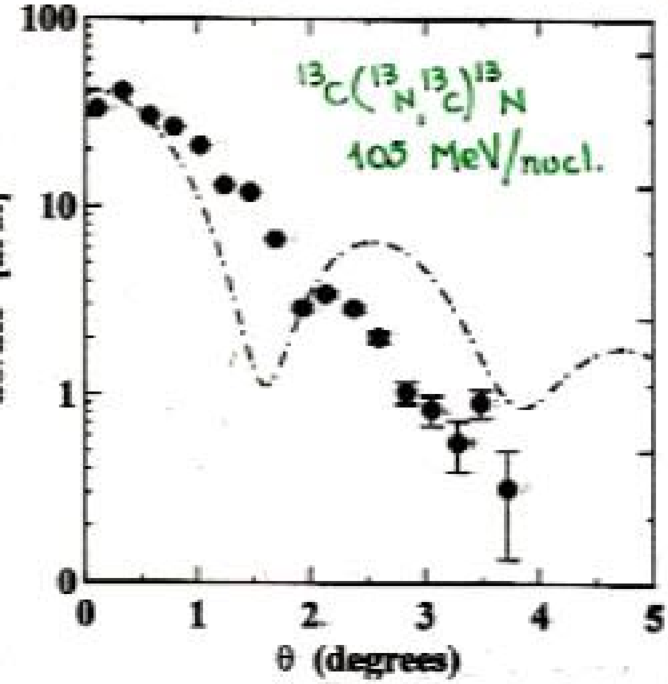
Austin et al., PRL 73(1994)30

$^{37}Cl(p,n)^{37}Ar(1^+)$





Osterfeld et al., PRC 45 (1992) 2845
Lenske, Wolter, Bohlen, PRL 62 (1989) 1457



$^{13}C(^{13}N,^{13}C)^{13}N$

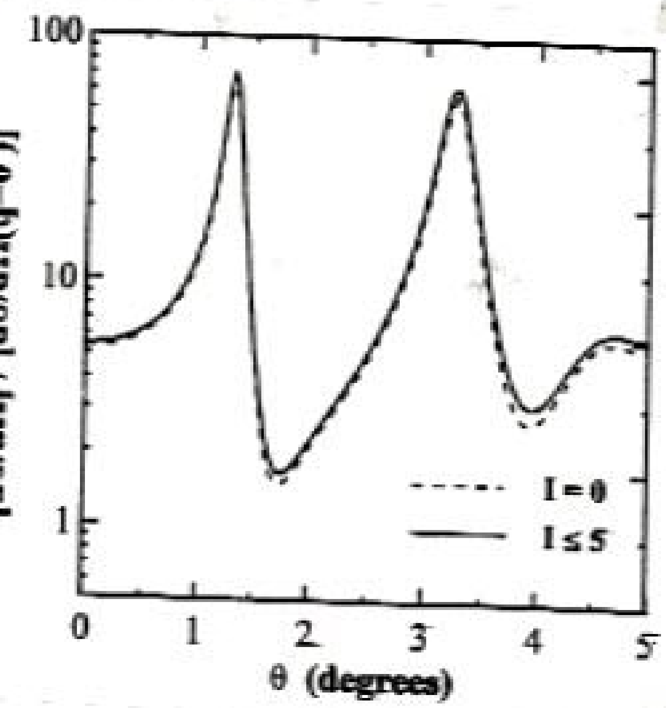
Steiner et al., PRL 76 (1996) 26

← single-particle model

$\vec{q}\cdot\vec{r} \ll 1$

$$\left(\frac{d\sigma}{d\Omega}\right)_{q=0} \simeq C_1\, B_1(GT)\, B_2(GT) + C_2\, B_1(F)\, B_2(F) + C_3 \sqrt{B_1(F) B_2(F) B_1(GT) B_2(GT)}$$

C.B., Paolo Lotti,
PLB 402 (1997) 237



$$T_{if} = \frac{1}{(2\pi)^3}\int d^3R\,\chi^{(-)*}(\vec{R})\,\chi^{(+)}(\vec{R})\int d^3q\,e^{i\vec{q}\cdot\vec{R}}\,\mathcal{M}_{exch.}(\alpha_1,\alpha_2,\vec{q})$$

$$\mathcal{M}_{exch} = \underbrace{\int d^3r_1\,d^3r_2\,\delta\rho_1(\vec{r}_1)\,\delta\rho_2(\vec{r}_2)\,e^{i\vec{q}\cdot(\vec{r}_1-\vec{r}_2)}}\,V_{exch.}(\vec{q})$$

Plane waves $\longrightarrow$ $\chi^{(-)*}\chi^{(+)} \sim e^{i\vec{Q}\cdot\vec{R}}$

$\int d^3R \ldots \longrightarrow \delta(\vec{q}-\vec{Q}) \quad \therefore \ \vec{Q}\sim 0 \longrightarrow \vec{q}\sim 0$

Not general!

In $V_{exch.}$ reduce π and ρ mass by α