

# **Pattern of Neutrino Masses and Mixings**

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*Apologies to many authors whose work is not cited herein*

## Atmospheric Nu Oscillations

- $\nu_\mu, \bar{\nu}_\mu$  disappearance, increasing with  $L$
- best fit  $\nu_\mu \rightarrow \nu_\tau$  oscillation parameters

$$\delta m_{\text{atm}}^2 = 3.5 \times 10^{-3} \text{ eV}^2$$

$$\sin^2 2\theta_{\text{atm}} = 1$$

- $\nu_\mu \rightarrow \nu_\tau$  favored over  $\nu_\mu \rightarrow \nu_s$ 
  - \* matter effects:  
up-mu data and higher energy PC data ( $2\sigma$ )
  - \* NC  $\pi^0$  production Vissani, Smirnov (1998)
- no indication of  $\nu_e, \bar{\nu}_e$  oscillations

# Neutrino Decay vs. Neutrino Oscillations for Atmospheric Neutrinos

[Barger, Learned, Pakvasa, Weiler (98) + Lipari, Lusignoli (99)]

- Mixing

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} \simeq \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$$

- Decay

$$\nu_2 \rightarrow \bar{\nu}_4 + J \quad \text{Valle (1983)}$$

- Survival probability

$$P(\nu_\mu \rightarrow \nu_\mu) = s^4 + c^4 e^{-\alpha L/E} + 2s^2 c^2 e^{-\alpha L/2E} \cos\left(\frac{\delta m_{23}^2 L}{2E}\right)$$

- Choose  $\delta m_{23}^2 < 10^{-4} \text{ eV}^2$  so oscillations play no role

$$P(\nu_\mu \rightarrow \nu_\mu) = \left(s^2 + c^2 e^{-\alpha L/2E}\right)^2$$

- Fit to SuperK data

$$\theta \simeq 57^\circ, \quad \alpha = \tau_2/m_2 = 63 \text{ km/GeV} \simeq \frac{16\pi}{g^2 \delta m_{24}^2}$$

- With constraint on  $g^2 \delta m_{24}^2$  from  $K \rightarrow \pi + \text{neutrals}$

$$\delta m_{24}^2 > (14 \text{ eV})^2$$

- So  $\nu_2, \nu_3$  nearly degenerate,  $\nu_4$  light

$$\begin{aligned} \begin{matrix} 2 \\ 3 \end{matrix} &= \nu_\mu, \nu_\tau \\ \begin{matrix} 1 \\ 4 \end{matrix} &= \nu_\tau, \nu_e \end{aligned}$$

$\nu_e \simeq \nu_1$  nearly unmixed because of more stringent bound on its Majoron coupling (SMA for solar deficit)

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Best fits of decay and oscillation models of comparable quality

neutrino decay: monotonic decrease with  $L/E$

neutrino oscillations: sinusoidal behavior in  $L/E$   
smeared by resolution  
(averages to 1/2 at large  $L/E$ )

most easily distinguishable around  $L/E \approx 400$  km/GeV

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### Long-baseline experiments

#### Decay model

$$P(\nu_\mu \rightarrow \nu_\tau) = s^2 c^2 (1 - e^{-\alpha L/2E})^2$$

$$\neq 1 - P(\nu_\mu \rightarrow \nu_\mu)$$

$$\frac{NC}{(NC)_{\text{no osc}}} = P(\nu_\mu \rightarrow \nu_\mu) + P(\nu_\mu \rightarrow \nu_\tau) = 1 - c^2 (1 - e^{-\alpha L/E})$$

K2K (250 km)

$E_\nu = 1-2$  GeV

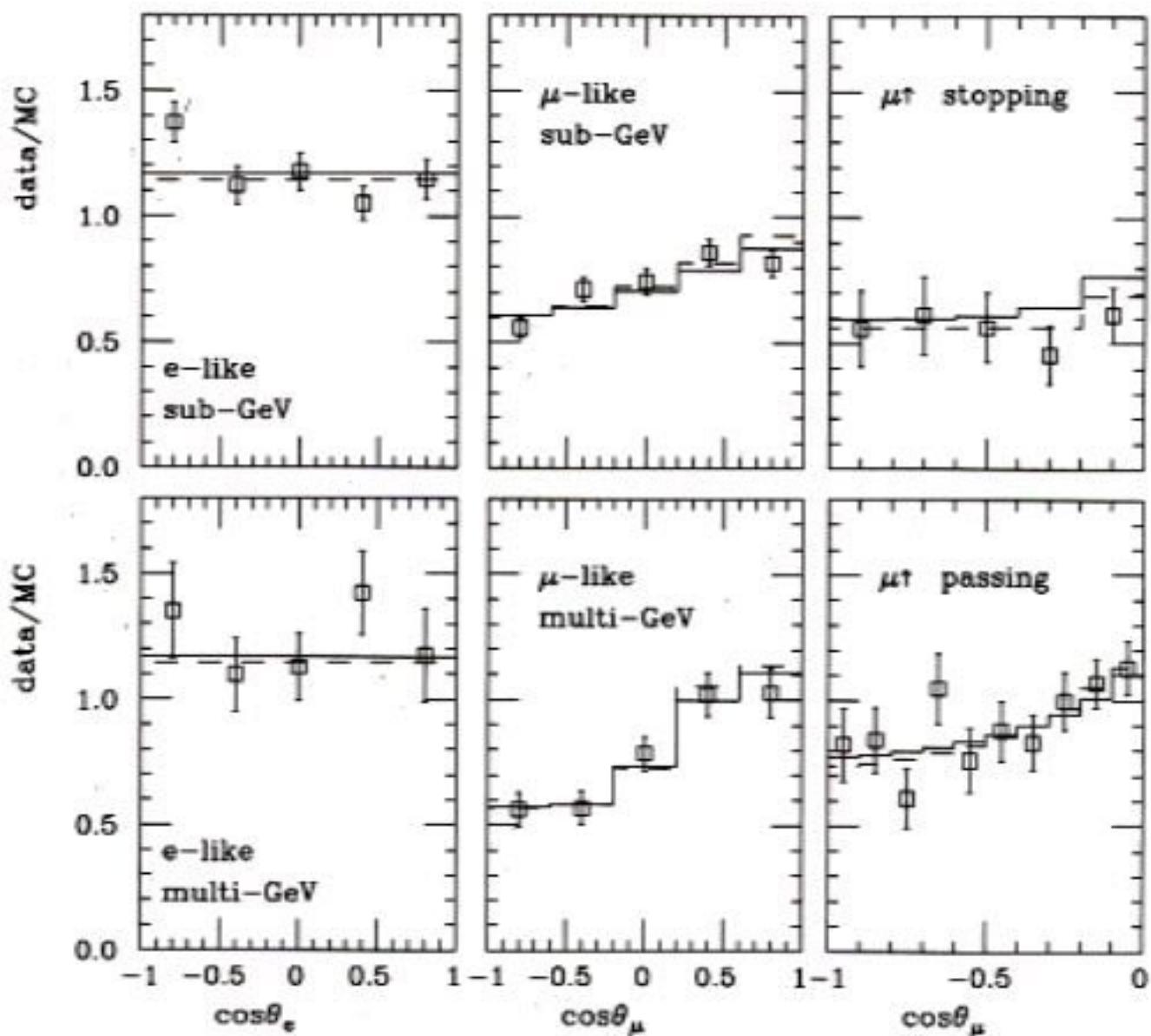
$L/E \approx 125-150$  km/GeV

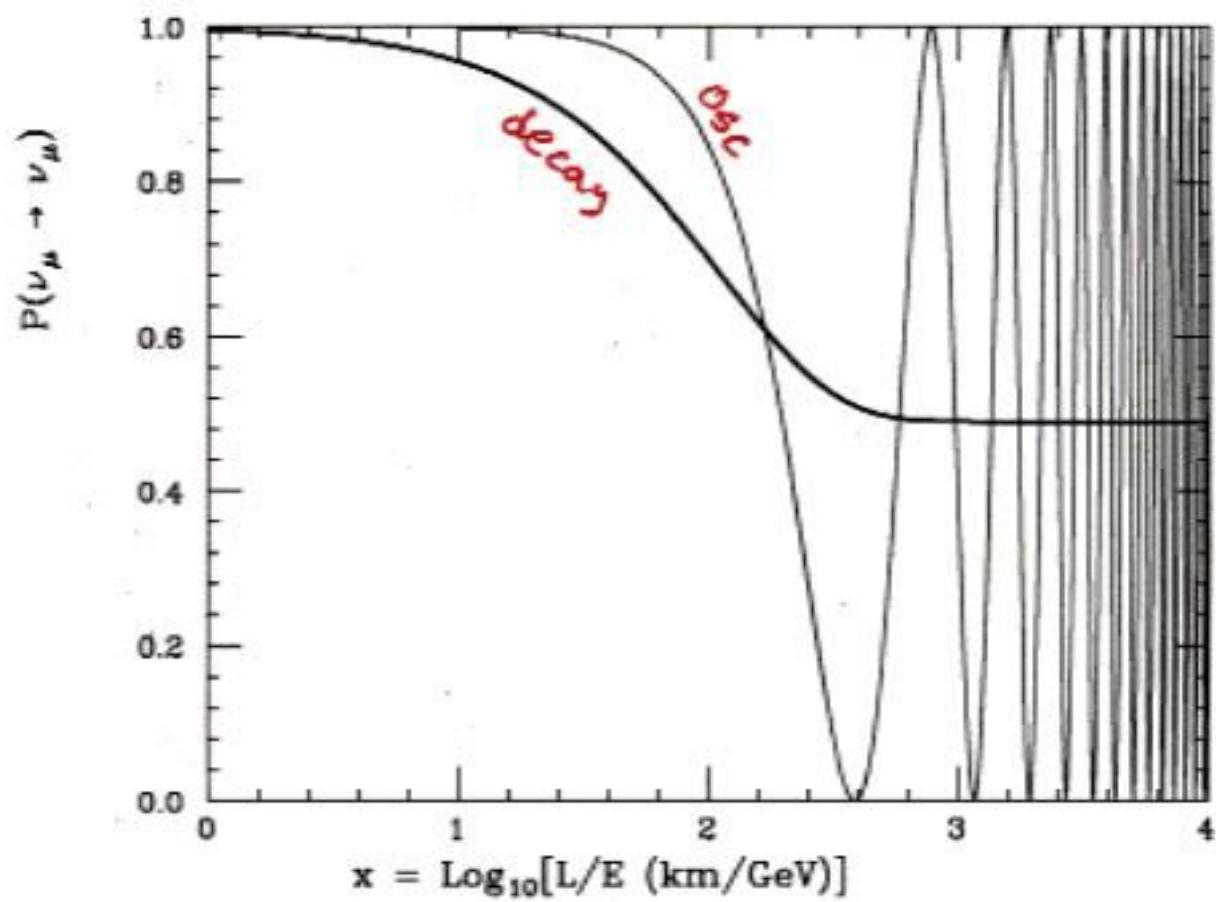
MINOS (732 km)

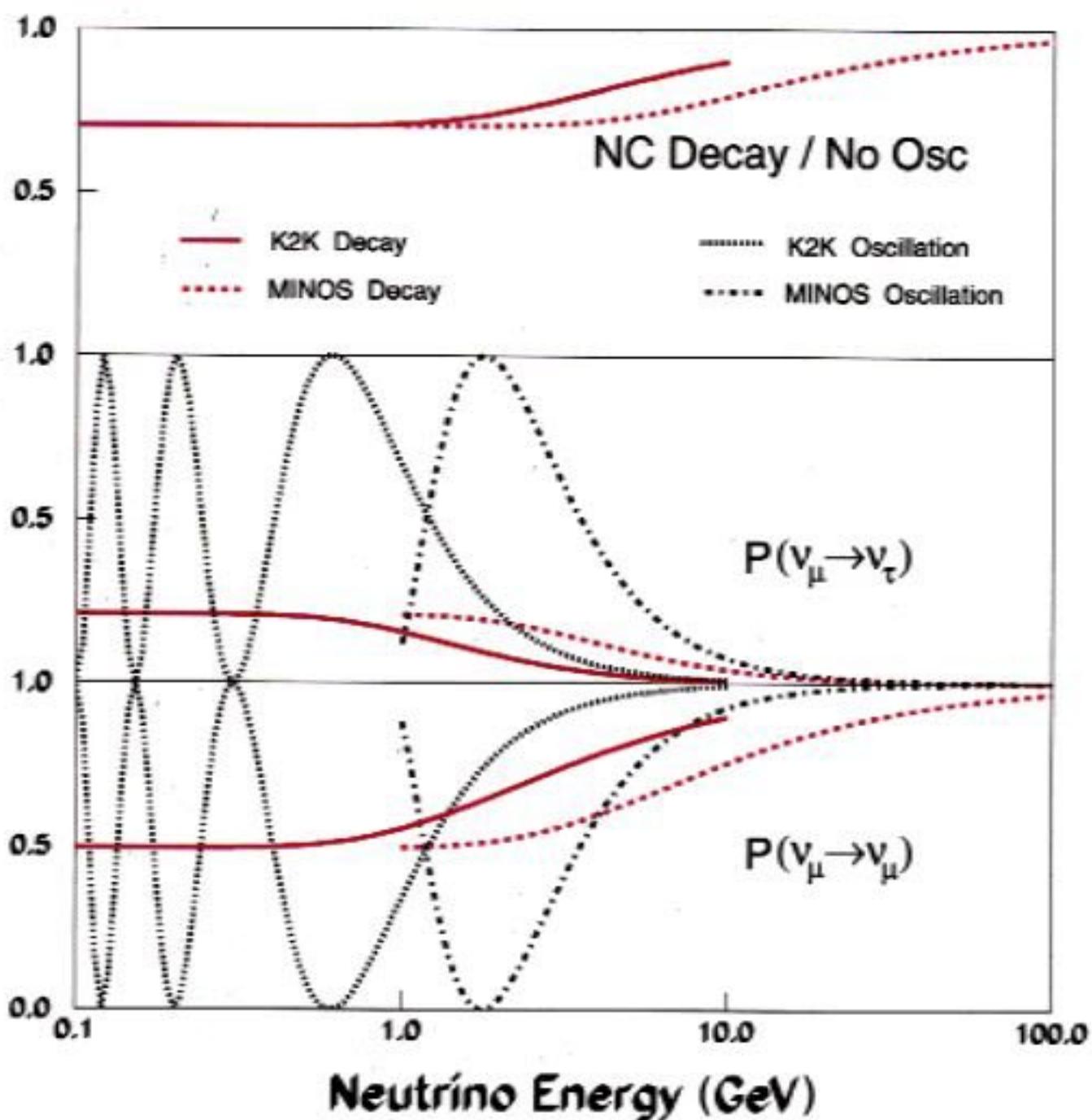
$\langle E_\nu \rangle = 3, 6, 12$  GeV

$L/E \approx 50-250$  km/GeV

Both  $P(\nu_\mu \rightarrow \nu_\mu)$  and  $NC$  can distinguish decay model from oscillations. MINOS may also detect taus.







## Solar Nu Oscillations

- Standard Solar Model flux +  ${}^8\text{B}$  normalization  
Bahcall, Basu, Pinsonneault (1998)
- Experiments sample different energy ranges and find different  $\nu_e$  flux deficits

Solutions	$\sin^2 2\theta$	$\delta m^2$
MSW (SA)	$\sim 10^{-2}$	$\sim 10^{-5} \text{ eV}^2$
MSW (LA)	$\sim 1$	$\sim 2-8 \times 10^{-5} \text{ eV}^2$
VO	$\sim 1$	$\sim 4 \times 10^{-10} \text{ eV}^2$

- $E_e$  spectrum shape
  - MSW (SA) gradual rise
  - MSW (LA) flat
  - ✓ VO sharp rise at  $E > 12 \text{ MeV}$  natural
  - SuperK data sharp rise at  $E > 12 \text{ MeV}$

LA  
Can rescue MSW by  $\sim 20 \times$  enhancement of SSM hep flux  
Bahcall, Krastev, Smirnov (1999)  
Gonzalez-Garcia et al. (1999)

- Night-Day dependence due to earth regeneration of  $v_e$

$$A_{n-d} = 2 \left[ \frac{\text{night} - \text{day}}{\text{night} + \text{day}} \right]$$

Baltz, Weneser (1994)  
 Maris, Petcov (1997)  
 Dighe, Liu, Smirnov (1999)  
 Guth, Randall, Serna (1999)

**MSW (SA)**  $> 0$  expected only for  $\cos\theta_Z = -0.8$  to  $-1$

$\checkmark$  **MSW (LA)**  $\approx 0.04 \left[ \frac{5 \times 10^{-5}}{\Delta m^2} \right]$  Bahcall, Krastev (1999)  
 uniform  $N$  excess  
 in  $\cos\theta_Z$

**VO**  $= 0$

**SuperK**  $0.060 \pm 0.036(\text{stat}) \pm 0.008(\text{syst})$   
 $(+1.6\sigma)$

- Seasonal dependence [ above  $1/L^2$  flux variation due to dependence of  $\sin^2(1.27\delta m^2 L/E)$  on  $L$  ]

Barger, Phillips, Whisnant (1981)  
 Glashow, Krauss (1987)

**MSW** = very small

$\checkmark$  **VO** natural

**SuperK** observed in  $E_e > 12$  MeV data

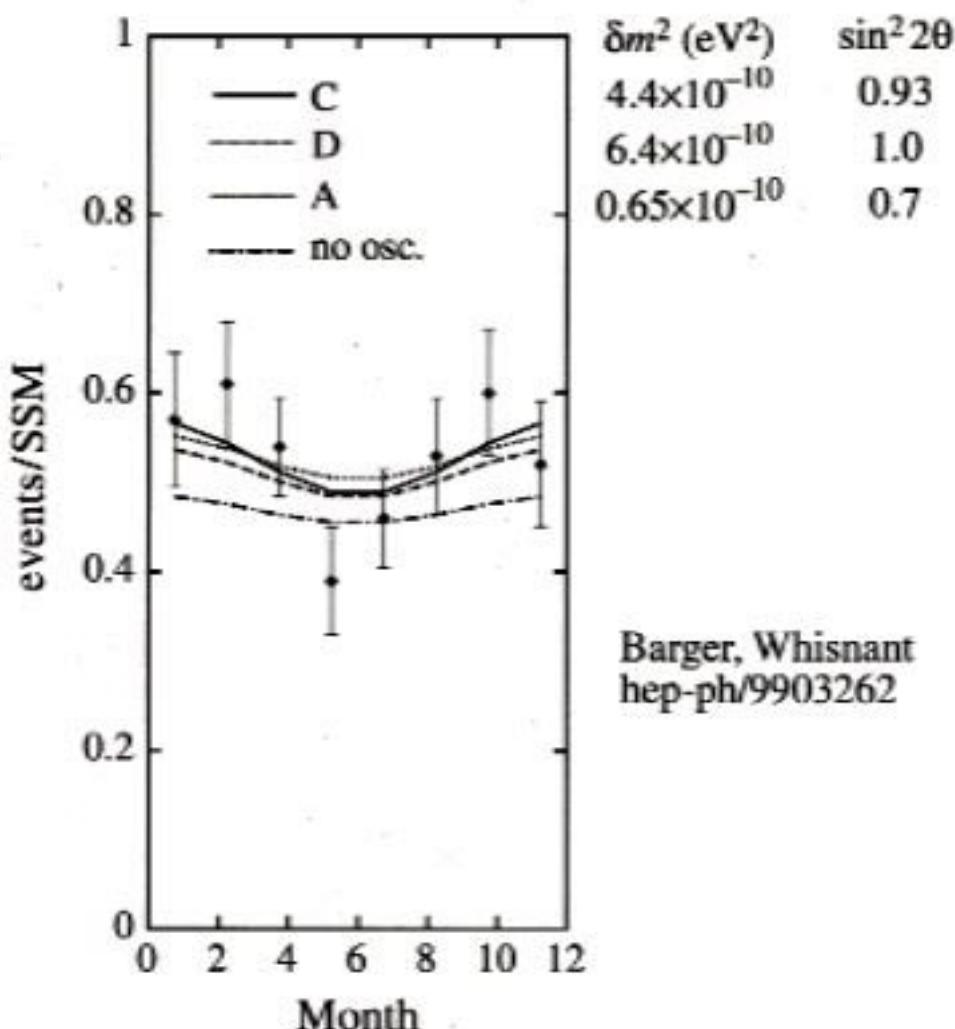
708 day SuperK data show hints of seasonal and spectrum dependences consistent with vacuum oscillations

Best global fit to all solar neutrino data

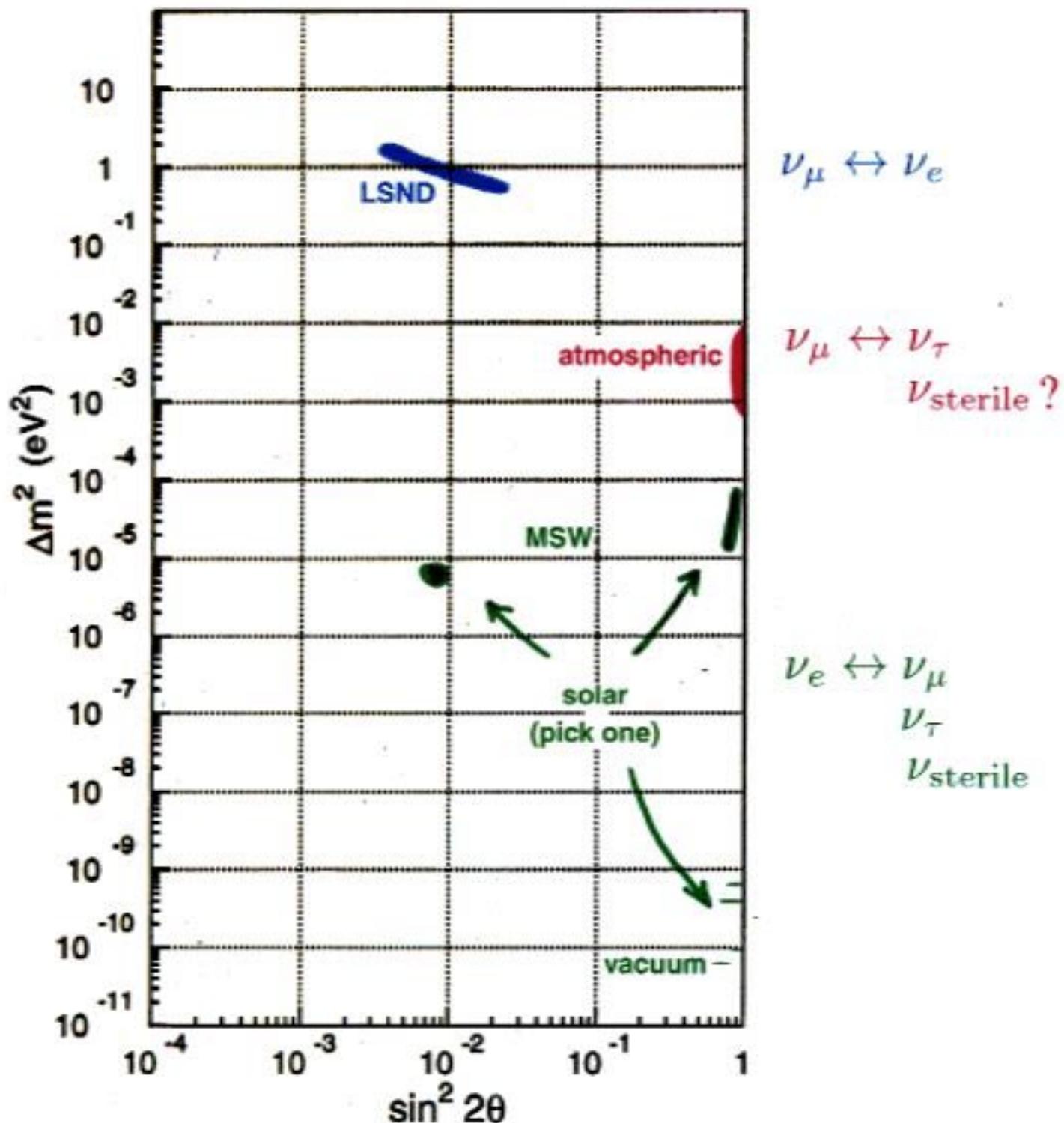
$$\delta m^2 = 4.4 \times 10^{-10} \text{ eV}^2$$

$$\sin^2 2\theta = 0.93 \quad \langle \lambda \rangle \simeq \frac{5}{2} \langle d_{\text{ES}} \rangle$$

*Seasonal*       $E_e = 11.5\text{--}20 \text{ MeV}$



# Summary of evidence



Hata, Langacker (1997)  
Bahcall, Krastev, Smirnov (1999)  
SuperK (1999)  
Gonzalez-Garcia et al. (1999)

3 neutrino mixing

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L = U_{\text{MNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Maki, Nakagawa, Sakata (1962)

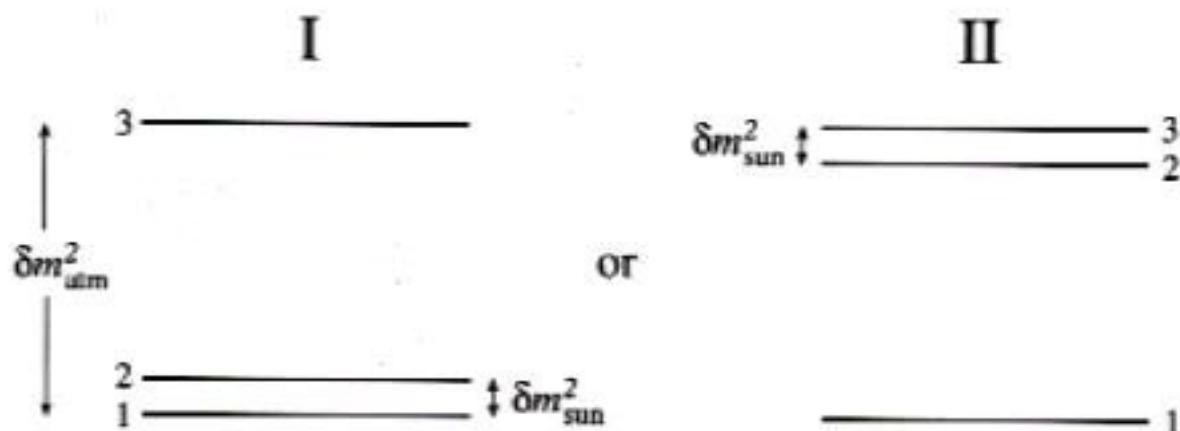
$$U_{\text{MNS}} =$$

$$\begin{pmatrix} c_1 c_3 & c_1 s_3 & s_1 e^{-i\delta} \\ -c_2 s_3 - s_1 s_2 c_3 e^{i\delta} & c_2 c_3 - s_1 s_2 s_3 e^{i\delta} & c_1 s_2 \\ s_2 s_3 - s_1 c_2 c_3 e^{i\delta} & -s_2 c_3 - s_1 c_2 s_3 e^{i\delta} & c_1 c_2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i(\phi_3 + \delta)} \end{pmatrix}$$

$$c_i \equiv \cos \theta_i \quad s_i \equiv \sin \theta_i$$

extra Majorana phases  
irrelevant to oscillations

to explain atm and solar data



## Leading oscillation: atmospheric and long-baseline

$$P(\nu_\alpha \rightarrow \nu_\beta) = A_{\alpha\beta} \sin^2 \Delta_{\text{atm}} \quad \Delta_{\text{atm}} = \frac{1.27 \delta m_{\text{atm}}^2 L}{E}$$

$$A_{\mu\tau} = 4c_1^2 \sin^2 2\theta_2$$

Same for  $\bar{\nu}$ :  
no  $\mathcal{CP}$  in leading oscillation  
BPW (1980)

$$A_{\mu e} = s_2^2 \sin^2 2\theta_1$$

$$A_{e\tau} = c_2^2 \sin^2 2\theta_1$$

### Super K atm

$\theta_2 \sim \pi/4$	maximal	$\nu_\mu \rightarrow \nu_\tau$	$2\sigma$ bounds
$\theta_1 \sim 0$	minimal	$\nu_e \rightarrow \nu_\mu, \nu_\tau$	$  \theta_2 - 45^\circ   < 13^\circ$ $\theta_1 < 17^\circ$

## Sub-leading oscillation

Solar  $\nu_e \rightarrow c_2^2 \nu_\mu + s_2^2 \nu_\tau \quad (\theta_1 \sim 0)$

$$\frac{L}{E_\nu} \sim \frac{10^{10} \text{ km}}{\text{GeV}} \quad \langle \sin^2 \Delta_{\text{atm}} \rangle \rightarrow \frac{1}{2}$$

$$P(\nu_e \rightarrow \nu_e) = 1 - \frac{1}{2} \sin^2 2\theta_1 - c_1^4 \sin^2 2\theta_3 \sin^2 \Delta_{\text{sun}}$$

$\theta_3$  determined by solar data

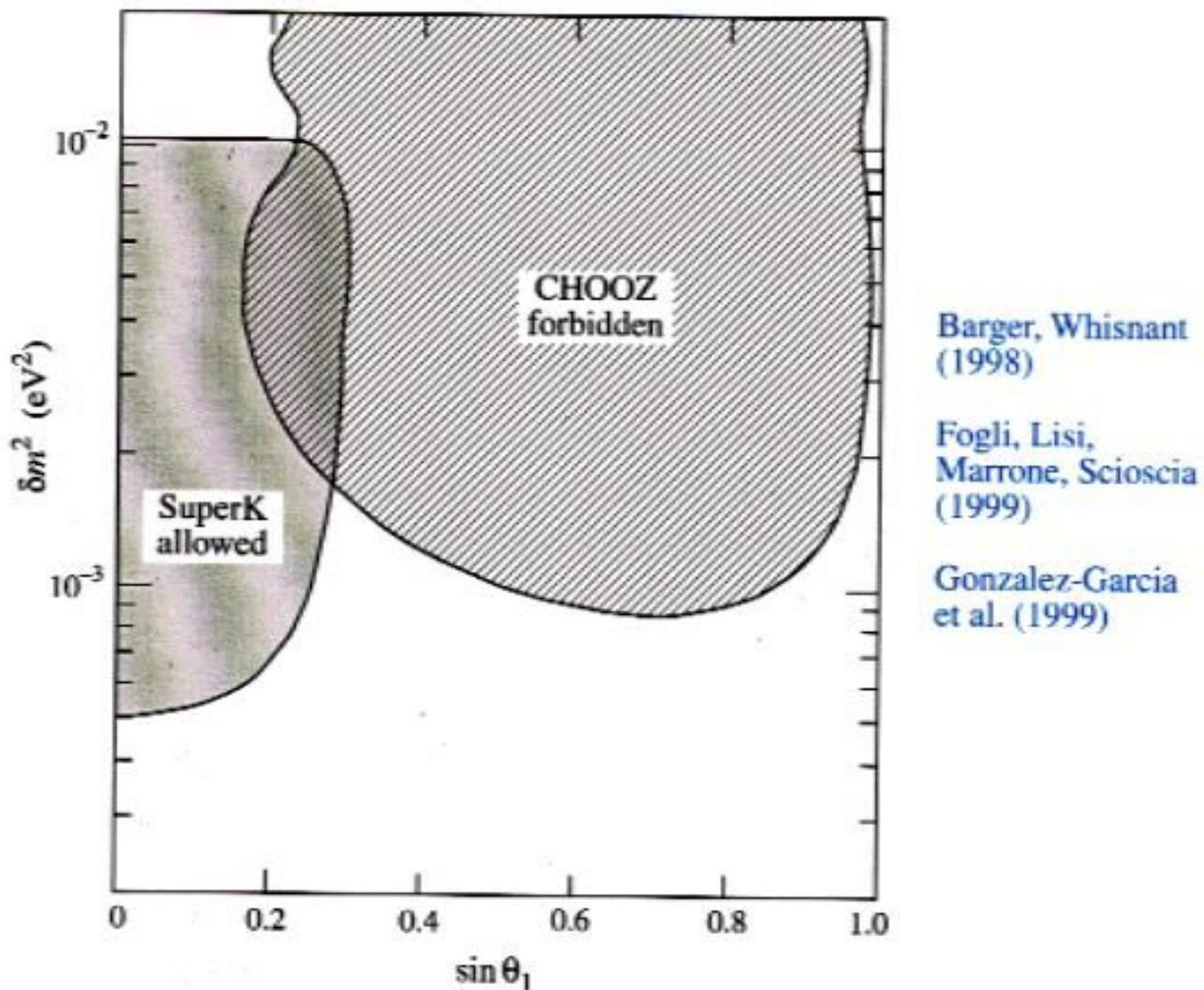
$\sin^2 2\theta_3 \sim 10^{-2}$  MSW(SA) or  $\sim 1$  MSW (LA), VO

Reconstruction of neutrino MNS matrix achieved!  
(except for phases and selection of solar solution)

Barbieri et al. (1998)

Barger, Whisnant (1998)

## Null $\bar{\nu}_e$ oscillation constraint from CHOOZ reactor



$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2 2\theta_1 \sin^2 \Delta_{\text{atm}}$$

CHOOZ limit improves  $\theta_1$  bound

ATM / SOLAR decoupling occurs at  $\theta_1 = 0$

- SOLAR  $v_e \rightarrow v_e$  depends only on  $\theta_3$
- ATM  $v_\mu \rightarrow v_\tau$  depends only on  $\theta_2$
- ATM / LBL  $v_e \rightarrow v_\tau$  vanishes at  $\theta_1 = 0$

## Bi-Maximal mixing

Hints:

$$\sin^2 2\theta_{\text{ATM}} = 1$$

$$\sin^2 2\theta_{\text{SUN}} \approx 1 \quad \text{VO or MSW(LA)}$$

Unique matrix for maximal mixing in both atm and solar sectors

$$U = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/2 & 1/2 & 1/\sqrt{2} \end{pmatrix}$$

Barger, Pakvasa,  
Weiler, Whisnant  
(1998)

Baltz, Goldhaber,  
Goldhaber (1998)

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 \Delta_{\text{ATM}} - \frac{1}{4} \sin^2 \Delta_{\text{SUN}}$$

$$P(\nu_e \rightarrow \nu_\mu) = \frac{1}{2} \sin^2 \Delta_{\text{SUN}} \quad (50\%)$$

$$P(\nu_e \rightarrow \nu_\tau) = \frac{1}{2} \sin^2 \Delta_{\text{SUN}} \quad (50\%)$$

$$[\Delta \equiv 1.27 \delta m^2 L/E]$$

No LSND Effect / no CPV

$U_{e3} = 0 \implies$  only  $\nu_\mu \rightarrow \nu_\tau$  oscillations  
at the atm scale

## Absolute neutrino mass scale (inferred limits)

$$m_3^2 - m_1^2 = \delta m_{\text{atm}}^2$$

$$m_2^2 - m_1^2 = \delta m_{\text{sun}}^2$$

$$\langle m_{\nu_e}^2 \rangle = \sum |U_{ej}^2| m_j^2 \simeq m_3^2 \sum |U_{ej}^2| + \mathcal{O}(\delta m_{\text{atm}}^2)$$

Experimental end-point limits

$$\langle m_{\nu_e}^2 \rangle < (2.5 \text{ eV})^2 \quad \text{Troitsk}$$

$$\langle m_{\nu_e}^2 \rangle < (3 \text{ eV})^2 \quad \text{Mainz}$$



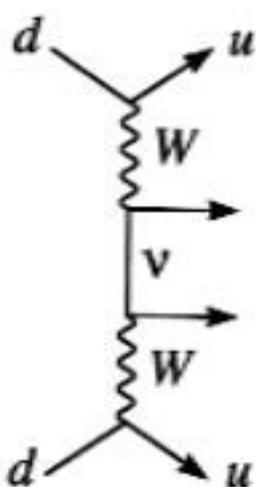
$m_j < 2.5 \text{ eV}$  for all  $j$

Barger, Weiler,  
Whisnant (1998)

Also

$$m_{\text{max}} \geq \sqrt{\delta m_{\text{atm}}^2} \geq 0.025 \text{ eV}$$

# Majorana neutrino mass in $0\nu\beta\beta$ decay



Doi et al. (1981)  
Minakata, Yasuda (1997)  
Georgi, Glashow (1999)  
BW (1999)

$0\nu\beta\beta$  decay rate determined by  $\nu_e \nu_e$  element of neutrino mass matrix ( $M_{ee}$ )

I.  $|M_{ee}| = |c_1^2 c_3^2 m_1 + c_1^2 s_3^2 m_2 e^{i\phi_2} + s_1^2 m_3 e^{i\phi_3}|$

II.  $m_1 \leftrightarrow m_3$  in I

Present limit  $|M_{ee}| < 0.2$  eV Baudis et al.  
hep-ex/9902014

Future sensitivity  $|M_{ee}| < 0.01$  eV  
GENIUS expt.  
MOON

$|M_{ee}|$  constraint (with  $\delta m_{ij}^2 \ll 1 \text{ eV}^2$ )

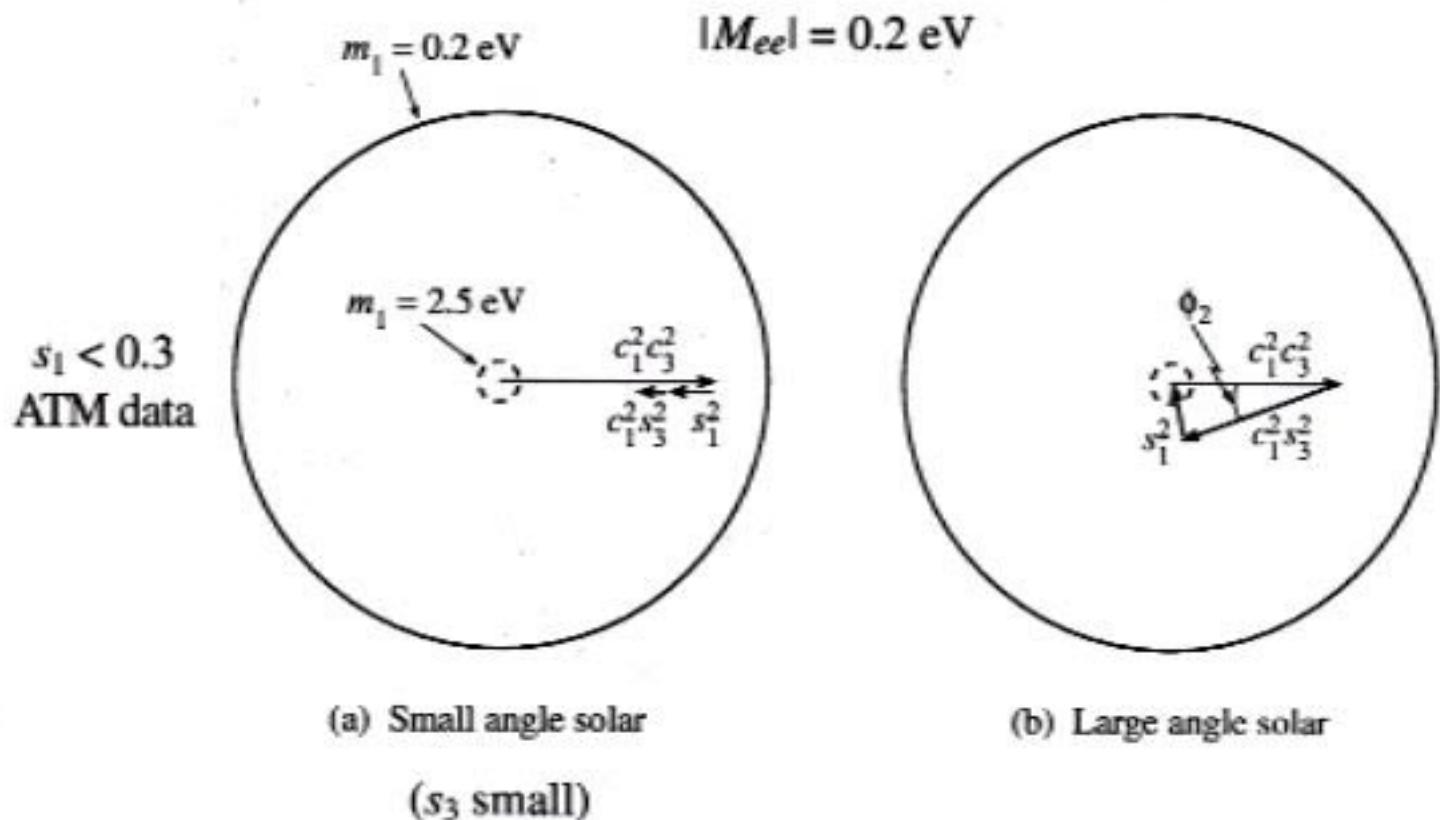
- MSW (SA)

$$\Sigma m_V < 0.75 \text{ eV}$$

(Majorana nu)

- MSW (LA) or VO

naturally satisfied if  $\nu_1$  and  $\nu_2$  have opposite CP-parity



# Beyond 3 neutrinos

## Motivations

- explain LSND effect  
or
- account for *r*-process nucleosynthesis

Bilenky, Giunti, Grimus (1998)  
Barger, Pakvasa, Weiler, Whisnant (1998)  
Gibbons et al. (1998)

McLaughlin et al. (1999)

sterile neutrinos are compatible with BBN  $N_\nu$  limit if mixing is very small

Barbieri and Dolgov (1990)

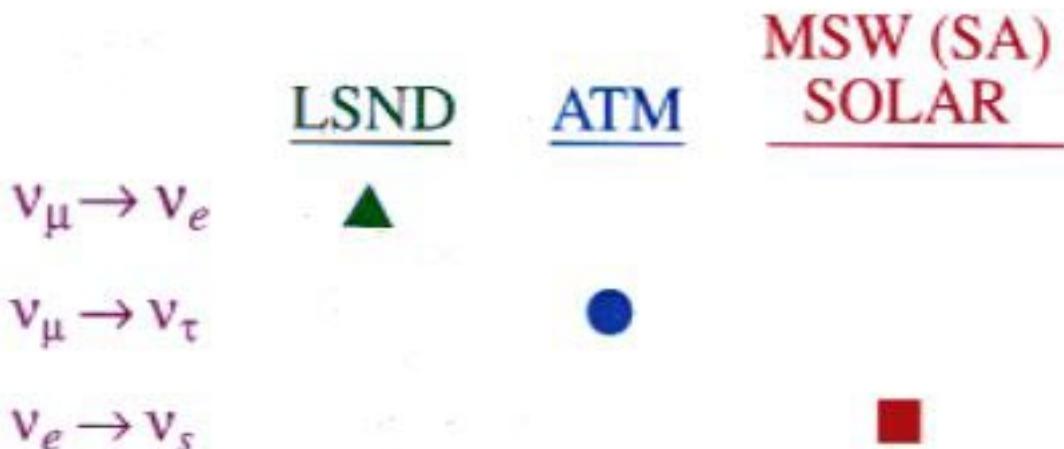
$$\delta m_{4i}^2 \sin^2 2\theta_{4i} \lesssim 10^{-7} \text{ eV}$$

Lepton-number violation partially alleviates BBN crisis ( $\delta N_{\text{eff}}^{\text{BBN}} \sim -0.5$ )

Foot, Volkas (1998)

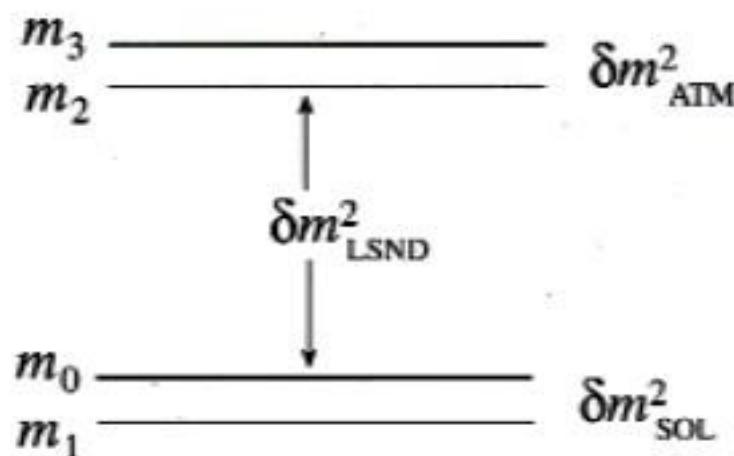
Shi, Fuller,  
Abazajian (1998)

## 4ν oscillations with LSND



$4\nu$  mass matrix can be constructed  
that consistently explains LSND, ATM,  
and SOLAR data by oscillations

2+2 mass hierarchy required for consistency  
with short-baseline limits

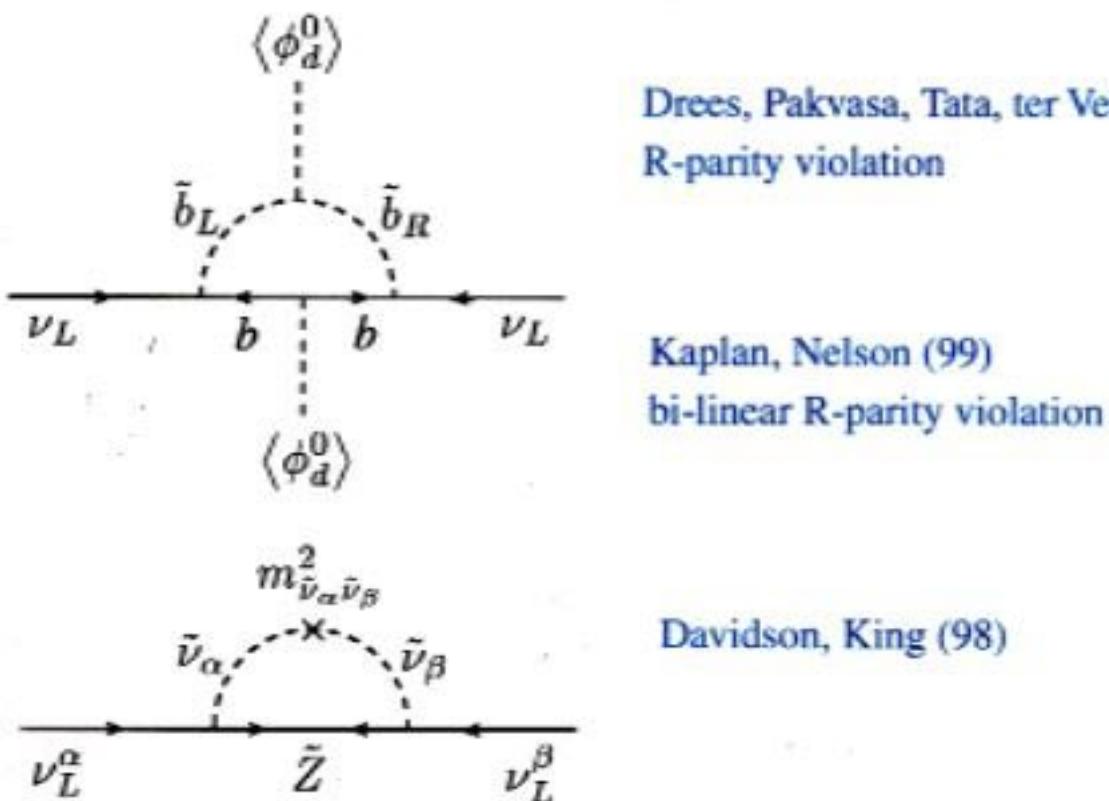


$m_0 > m_1$  for resonant matter oscillations in Sun

$$m_3 \gtrsim \sqrt{\delta m^2_{\text{LSND}}} \gtrsim 0.5 \text{ eV}$$

# Supersymmetry and $M_V$

- SUSY radiative masses



- Unification models

(assume mass matrix textures;  
usually invoke see-saw)

SO(10)

Albright, Babu, Bar (98, 99)  
Wilczek (98)

SU(5)

Hagiwara et al. (98)  
Altarelli, Feruglio (98)

Flipped SU(5)

J. Ellis et al. (98)

Anomalous U(1)

Irge, Lavignac, Ramond (98)

U(2) flavor symmetry

Barbieri et al. (98)  
Carone & Hall (97)

U(1) family symmetries

King (99), Allanach (98)

U(1)<sup>2</sup>

Froggett, Gibson, Nielsen (98)

## Typical Unified Model Predictions

- neutrinos are Majorana
  - neutrino masses are hierarchical
  - no cosmologically significant dark matter
  - large  $\nu_\mu - \nu_\tau$  oscillations are accommodated, but came somewhat as a surprise
  - mixing of  $\nu_\mu$  and  $\nu_\tau$  not fully maximal
- • small angle MSW solar solution
- no LSND effect

If either VO or MSW (LA) are the solar solution, then most proposed unified models are rejected

# Long Baseline Goals

## Vacuum osc studies

K2K (KEK → SuperK)

MINOS (Fermilab → Soudan)

(CERN → Gran Sasso)

Mu Ring (Fermilab → Soudan)

etc.

Geer (1998)

Quigg (1998)

- Precise  $\delta m^2_{\text{atm}}$  (~1% measurement)
- Reconstruction of MNS mixing matrix (determine  $\theta_1, \theta_2$ ). If MSW (LA) solar solution, may also determine  $\theta_3$

## Matter osc studies

Fermilab → Gran Sasso

Fermilab → Australia

- Observe matter effects on oscillations in very long baseline experiment (different for  $\nu_e$  and  $\bar{\nu}_e$ ); determine sign of  $\delta m^2$
- Observe matter resonance effects in propagation through earth's core
- Test for CP and T  
Apparent CP-odd asymmetry is induced by matter

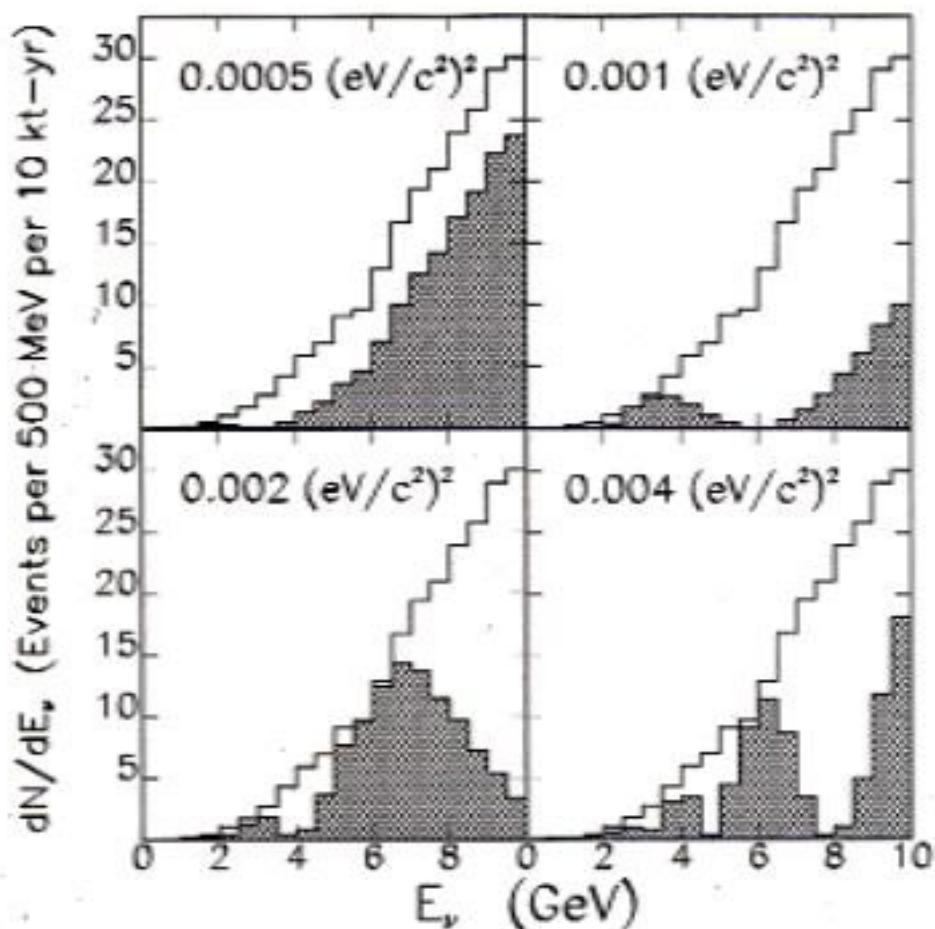


FIG. 12. Modified  $\nu_\mu$  CC interaction spectra for a 10 GeV muon storage ring neutrino source located at FNAL and a detector at the Gran Sasso underground laboratory, shown for several values of the oscillation parameter  $\Delta m^2$ , assuming  $\sin^2 2\theta = 1$ . In each of the four panels the upper curves show the unmodulated spectrum, and the lower curves the modulated spectrum corresponding to the indicated  $\Delta m^2$ .

## SUMMARY

- Neutrino physics has moved us beyond the Standard Model
- From oscillation effects, we already have a surprising amount of information about the neutrino mass hierarchy and mixing pattern
- Reconstruction of the 3 neutrino mixing matrix is on the horizon
- Long baseline experiments are the next frontier for precision measurements
- SuperK / SNO / . . . will differentiate the solar solutions

- The absolute neutrino mass scale remains to be fixed
- $0\nu\beta\beta$  decay can discern Majorana neutrinos and either bound  $\Sigma m_\nu$  or determine its value
- Cosmology (CMB, large scale structure) can tell us if  $\Sigma m_\nu > 0.4$  eV
- Convergence of particle physics, nuclear physics, and cosmology to solve neutrino mass issues
- Many models proposed for lepton and neutrino mass matrices, but no obvious synthesis yet