Partial Wave Analysis

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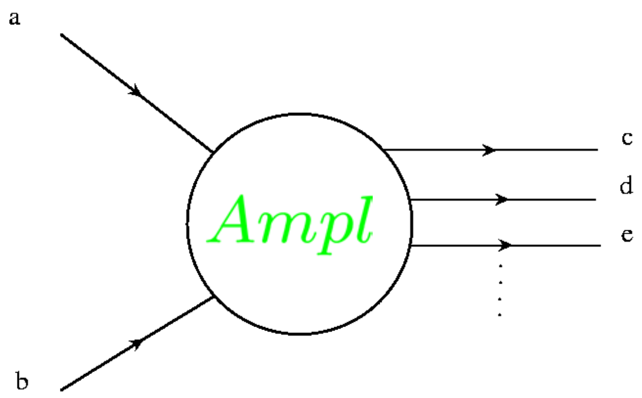
- What is it
- Relation to physical properties
- Properties of the S-matrix
- Limitation and perspectives
- Examples : peripheral production of 2- and 3-particle final states

$$\frac{d^n\sigma}{dx_1\cdots dx_n} = |Ampl|^2 = |\sum_\alpha a_\alpha(x_i) Y_\alpha(x_i)|^2$$



x_i Kinematical variables ($s_{cd}, \theta_{cd}, \phi_{cd}, \cdots$)

$a_\alpha(x_i)$ Production amplitudes depends on fit parameters (output)

$Y_\alpha(x_i)$ Production amplitudes (input)

$$\frac{d^n\sigma}{dx_1\cdots dx_n} = \frac{\Delta N_{ev}}{\Delta x_1 \cdots \Delta x_n}$$

Δx_i (mass, t, $\cdots$ bins)

… depending on how much we know about the amplitude :

- $Ampl = Y(x_i)$ Know everything !

This is best case scenario

$Y(x_i)$ includes it all :
kinematics and dynamics,
There is nothing to fit !

- $Ampl = a(x_i)$ Know nothing

Worst case scenario

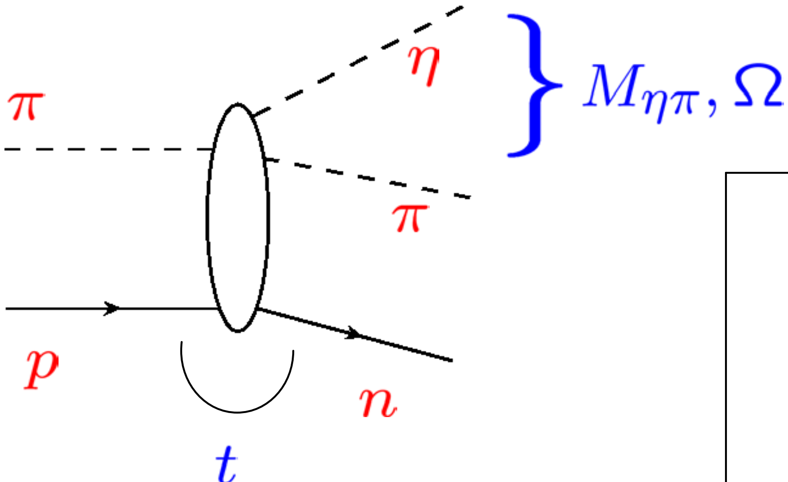
Usually somewhere in between

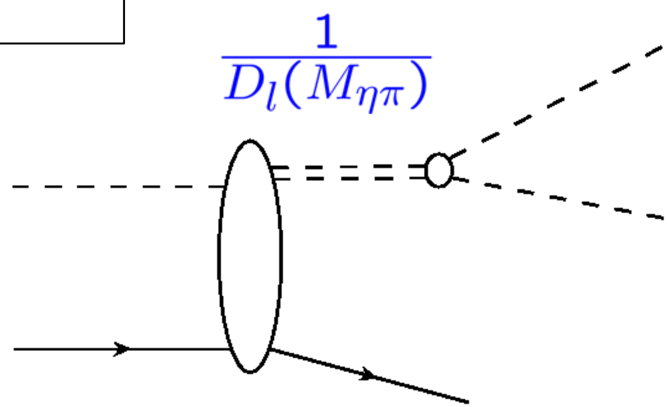
- $Ampl = \sum_\alpha a_\alpha(x_i) Y_\alpha(x_i)$

Example $\pi^- p ! \eta\pi^0 n$

α $\qquad$ x_i

- $$\frac{d\sigma}{dt\,dM_{\eta\pi}\,d\Omega} = |\sum_{lm} a_{lm}(t, M_{\eta\pi}) Y_{lm}(\Omega)|^2$$





- $$\frac{d\sigma}{dt\,dM_{\eta\pi}\,d\Omega} = |\sum_{lm} a_{lm}(t) \frac{Y_{lm}(\Omega)}{D_l(M_{\eta\pi})}|^2$$

… the less you know the more ambiguous the answer …

0 physics input
“maximal’ ambiguity

$$\frac{d^n\sigma}{dx_1\cdots dx_n} = |a(x_i)|^2 = |a(x_i)e^{if(x_i)}|^2$$

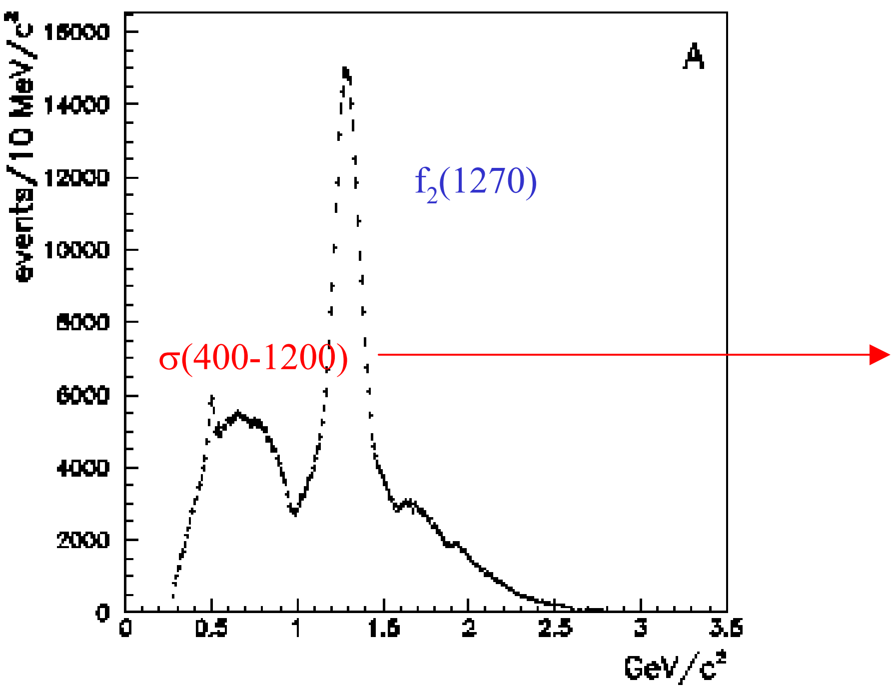
some physics input
“moderate” ambiguities

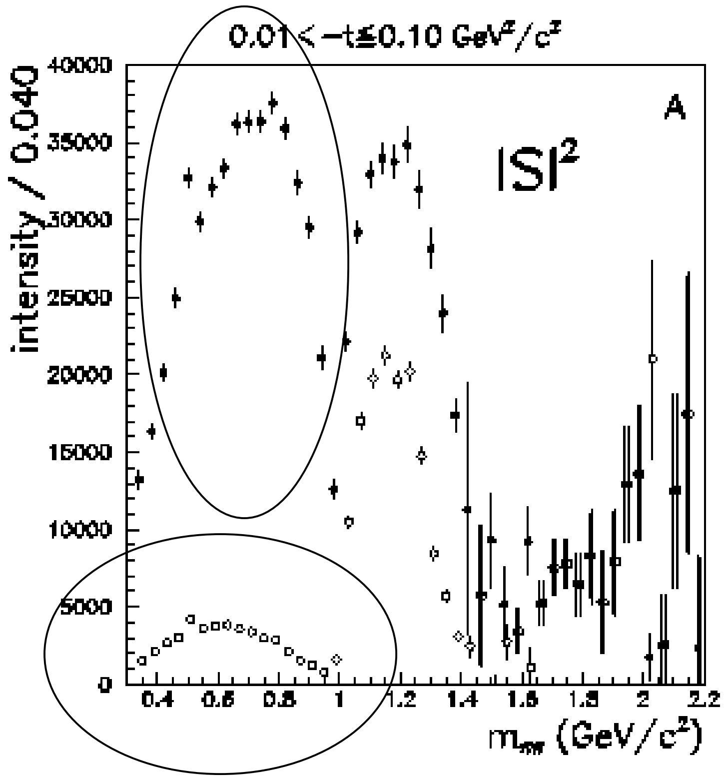
$$\frac{d^n\sigma}{dx_1\cdots dx_n} = |\sum_\alpha a_\alpha x^\alpha|^2 = |\Pi_\alpha(x - x_\alpha(a))|^2$$

know everything
no ambiguities

You do it in all possible way
to study systematics

(J. Gunter et al.) 2001 $\pi^- p$! $\pi^0\pi^0$ n



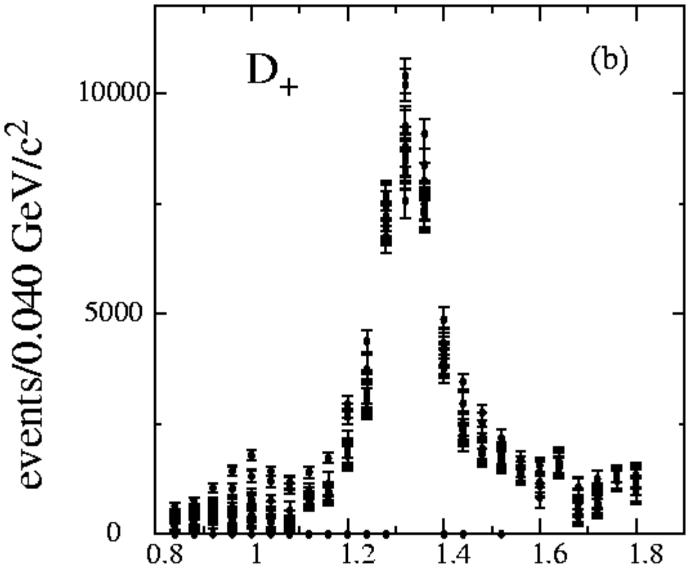


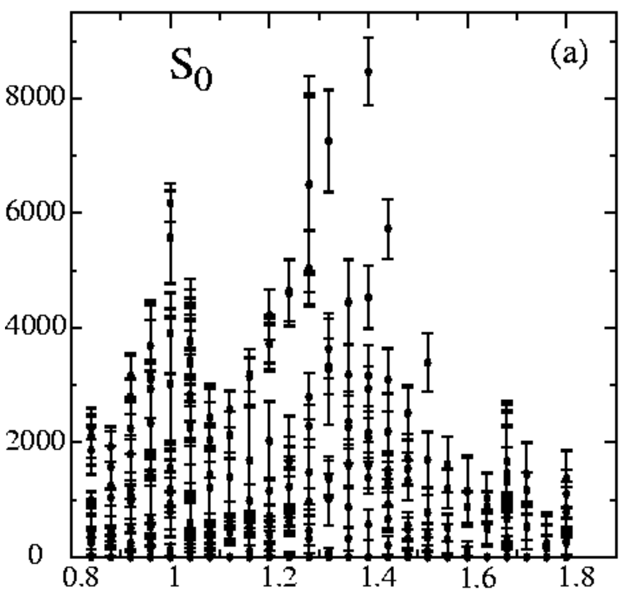
$\pi^0\pi^0$ spectrum

$$\frac{d^n\sigma}{dx_1\cdots dx_n} = |\sum_\alpha a_\alpha x^\alpha|^2 = |\Pi_\alpha(x - x_\alpha(a))|^2$$

(A.Dzierba et al.) 2003

$\pi^- p$! $\eta\pi^0 n$

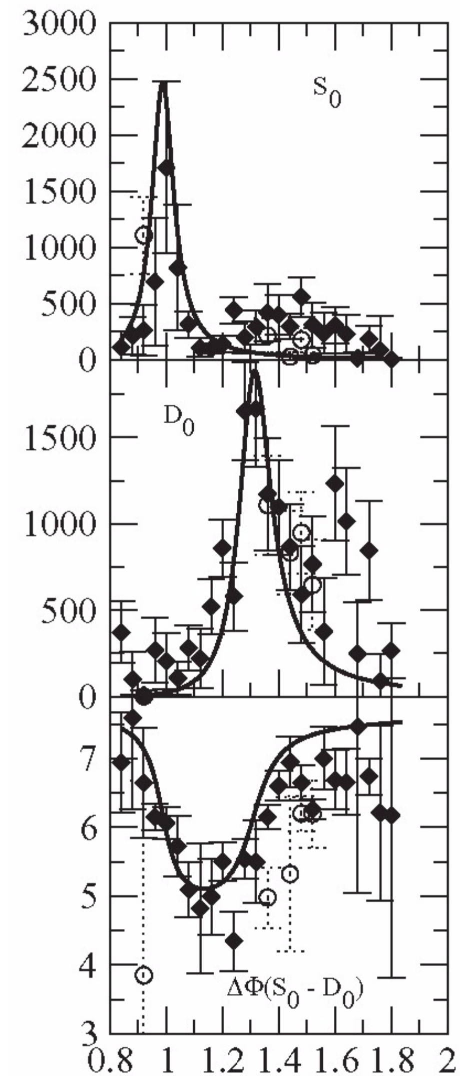




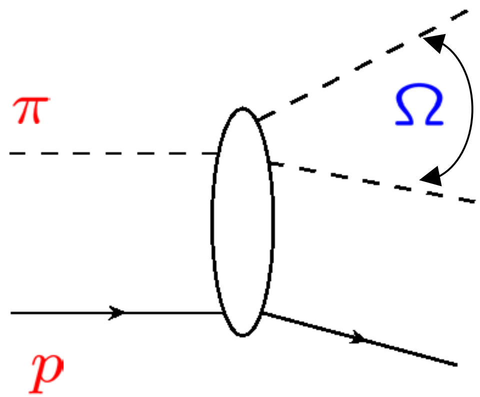
Assume a_0 and

a_2 resonances

(i.e. a dynamical assumption)



.. so, how much we know about the S-matrix …



$$Y_{lm}(\Omega)$$

- kinematical constraints

- dynamics is much harder …

❑ Heisenberg-Mandelstam program (ca. 1960-1970)

❑ QCD (ca. 1970)

❑ Jlab upgrade (ca. Now !)

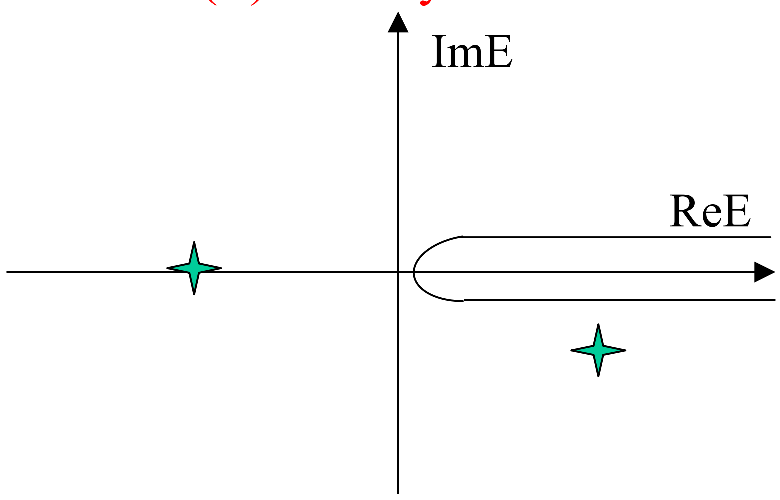
HM: reconstruct S given Mandelstam representation and the unitarity condition)

- S – matrix has specific analytic properties (causality)

$$O(t) = \int dt_0 g(t - t_0) I(t_0)$$

$$g(\tau) = 0 \text{ for } \tau < 0 \qquad G(E) = \int_0^\infty d\tau g(\tau) e^{iE\tau}$$

G(E) is analytic for ImE>0



- Unitarity (conservation of energy)

$$2iImT = T^\dagger \rho T$$

Mandelstam hypothesis :

1. There is an analytical representation for S
   (which includes poles and cuts of physical origin)

(so far unknown)

2. Given a representation a unique solution can be found

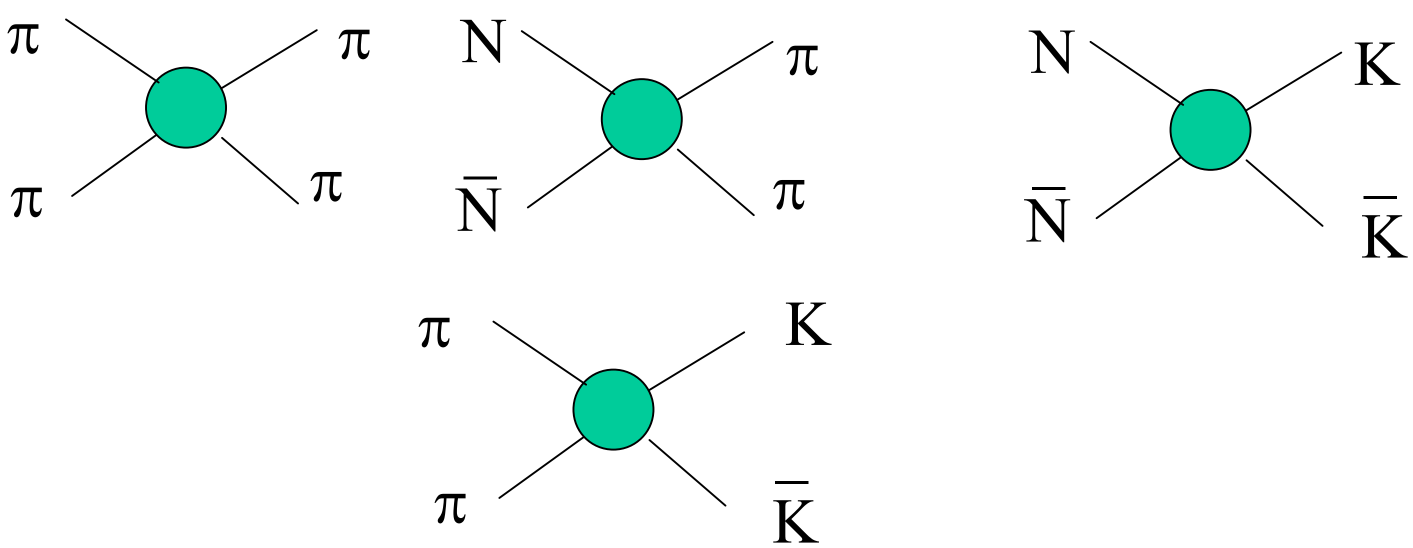
(not true)

Complete representation : complete dynamics

Non-relativistic example : need the potential

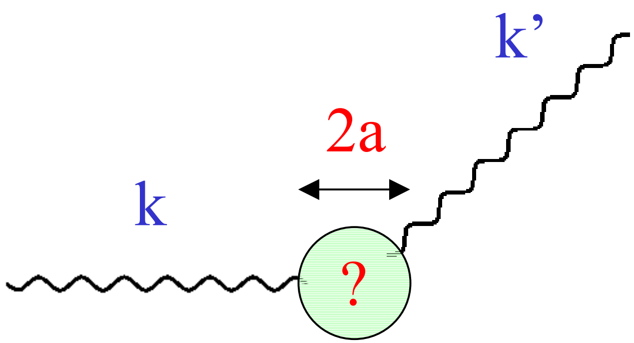
$$f(s,t) = f_B(t) + integrals$$

Relativistic example :



… a "given" representation can have multiple solutions ! (incomplete knowledge of dynamics)

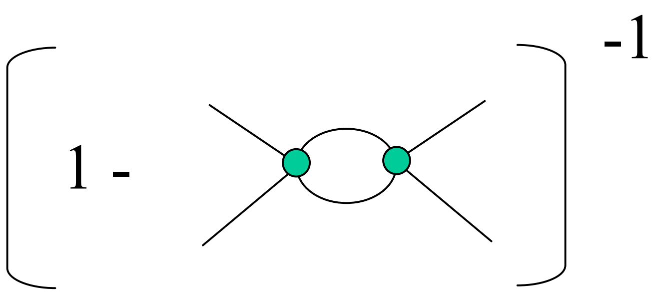
Non-relativistic example : Blaschke product



$$S(k) = e^{-2ika}\Pi_n B_n(k, k_n)$$

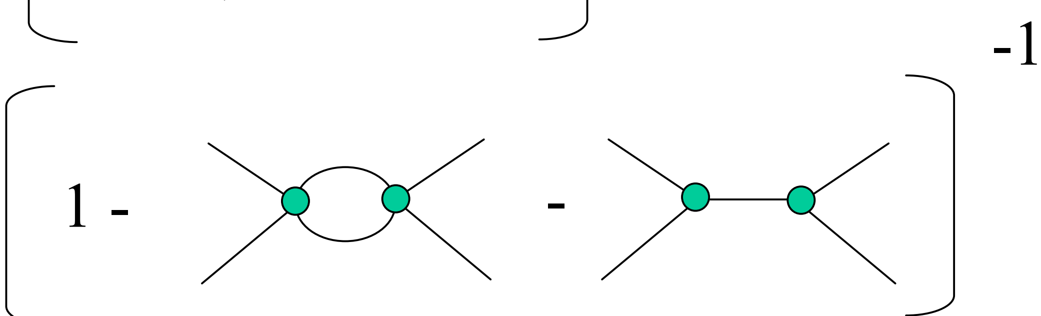
Relativistc example : (Castillejo,Daliz,Dyson (CDD) poles)

$f_l =$



Have the same representation

$f_l =$



Good news :

- Low energy :

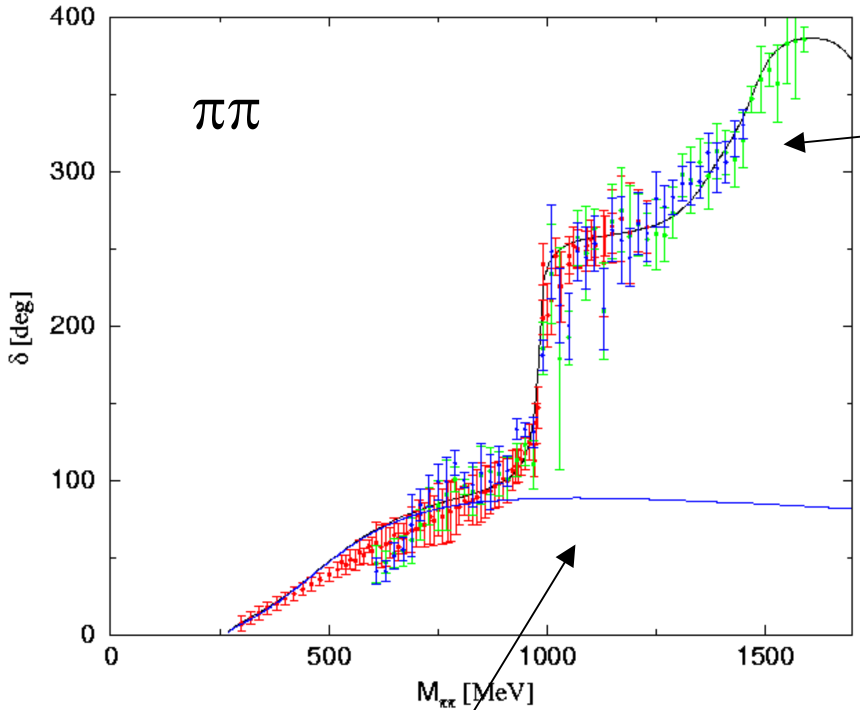
  - Effective range expansion (low energy)
  - Two body unitarity
  - Small number of (renormalized) parameters
  - QCD input

- High energy :

  - Regge behavior
  - Asymptotic freedom

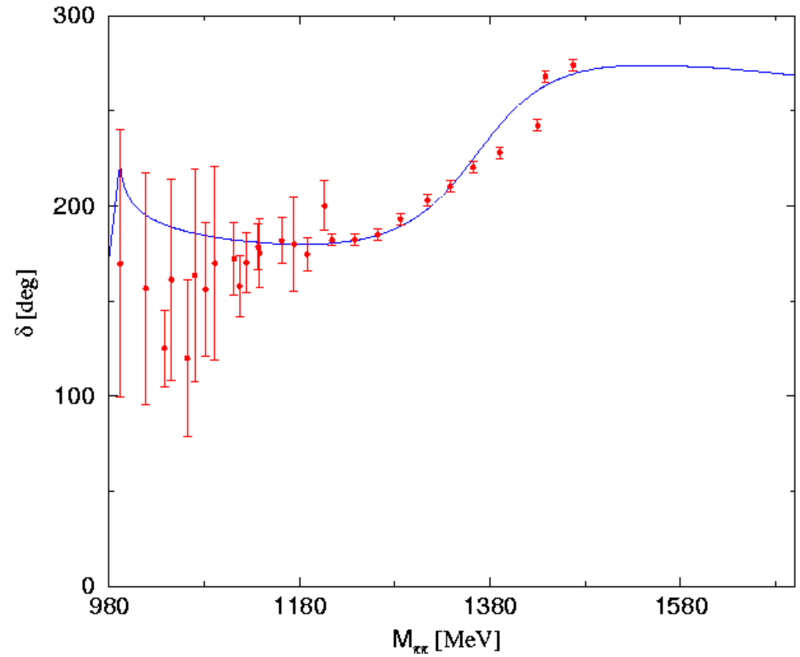
Illustration : ππ (S=I=0)

ππ



2 Resonaces @ ~1.3, 1.5 Ge

ππ+KK



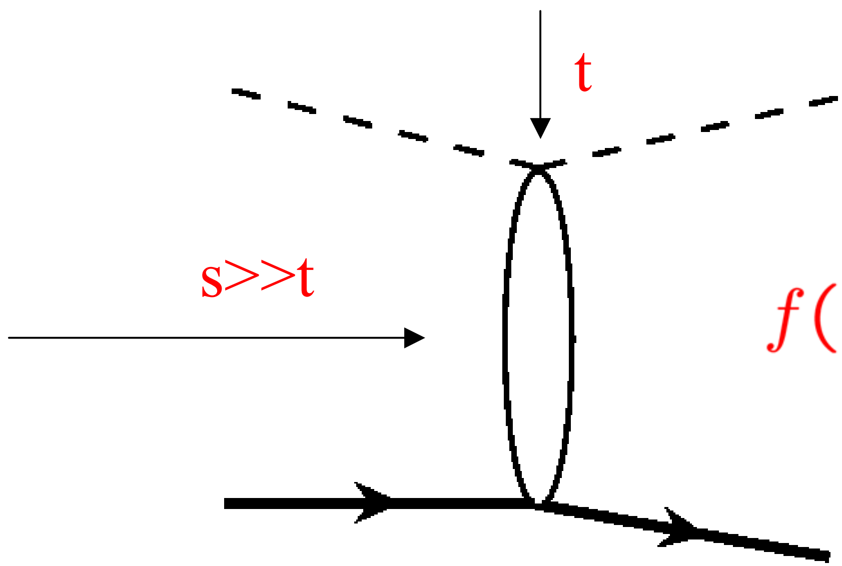
ππ only
(no KK, no resonances)

Regge poles

$$f(E, \cos\theta) \rightarrow \frac{1}{E - E_0 + i\Gamma} \rightarrow e^{-\Gamma t}$$

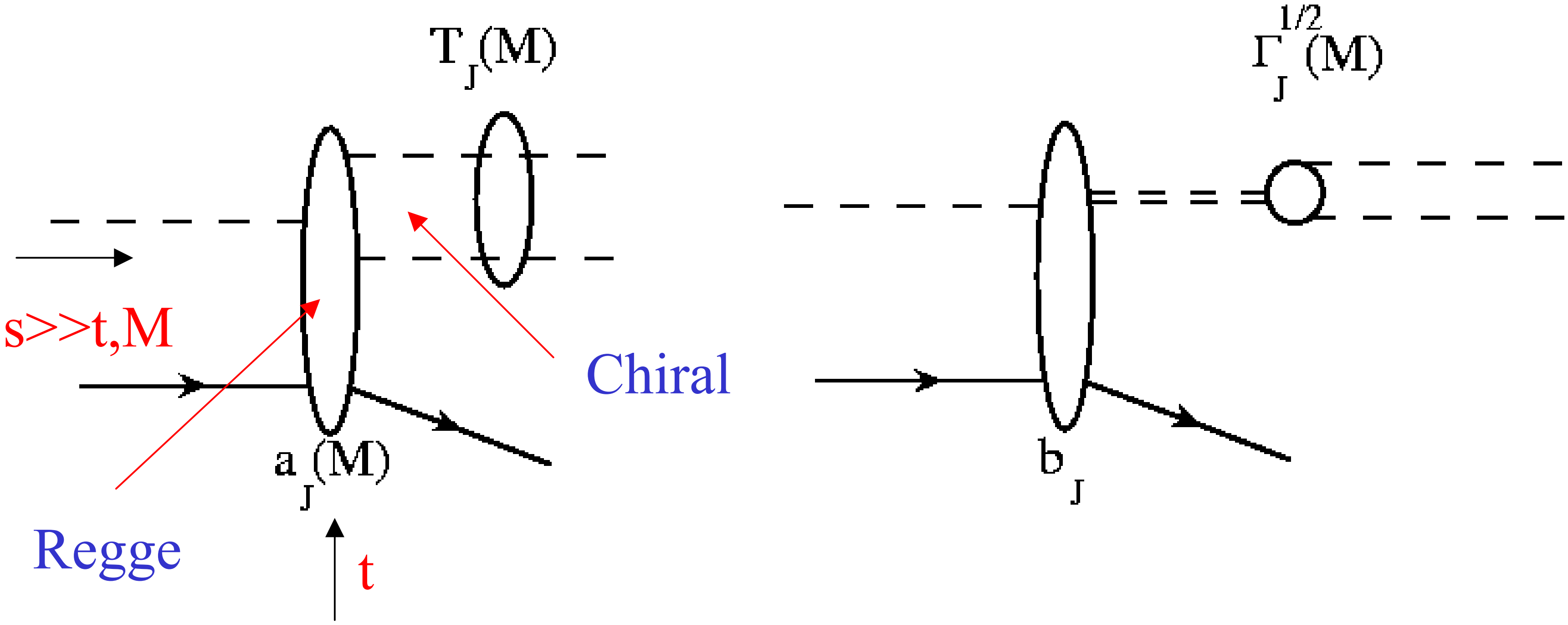
$$\downarrow \qquad\qquad \downarrow$$

$$\sum_l \frac{1}{l - \alpha(E)} P_l(\cos(\theta)) \rightarrow e^{-\theta \, Im \alpha}$$



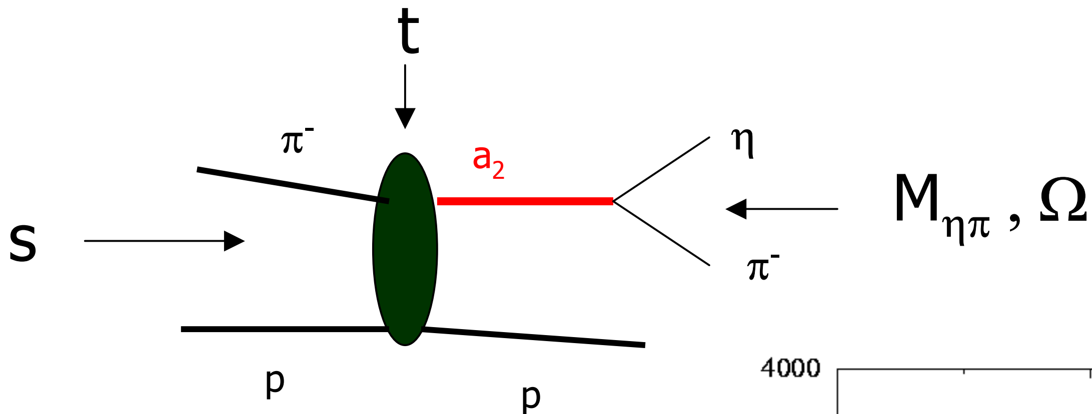
$$f(s,t) \sim s^{\alpha(t)}$$

… combine low (chiral) and high energy information

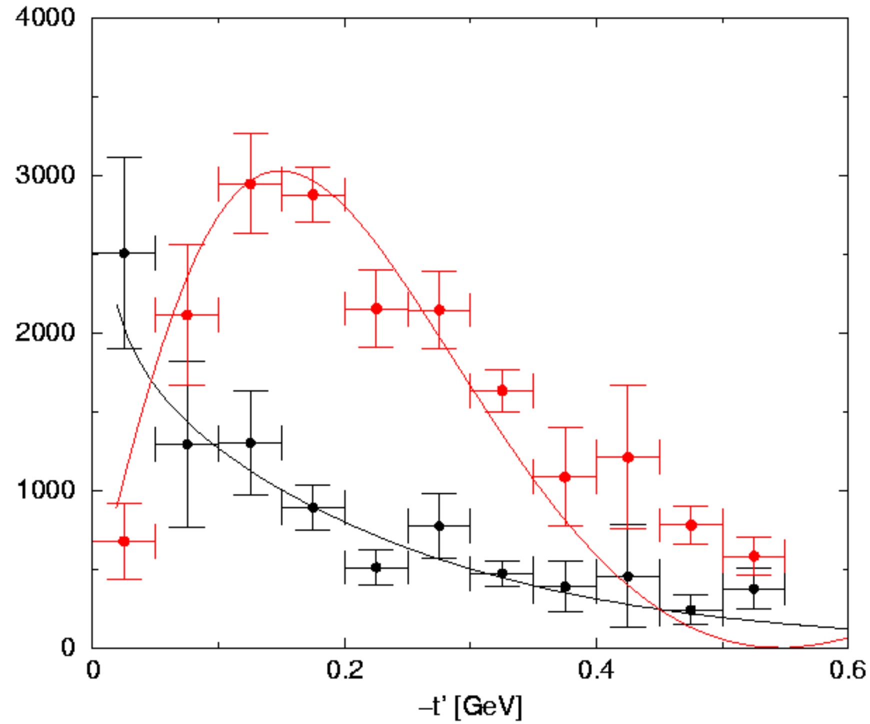


π^{-} (18GeV) p → X p → $\eta\ \pi^{-}$ p
→ $\eta'\ \pi^{-}$ p

~ 30 000 events



N_{events} = $N(s, t, M_{\eta\pi}, \Omega)$



… PWA determined by maximizing likelihood function over an even sample …

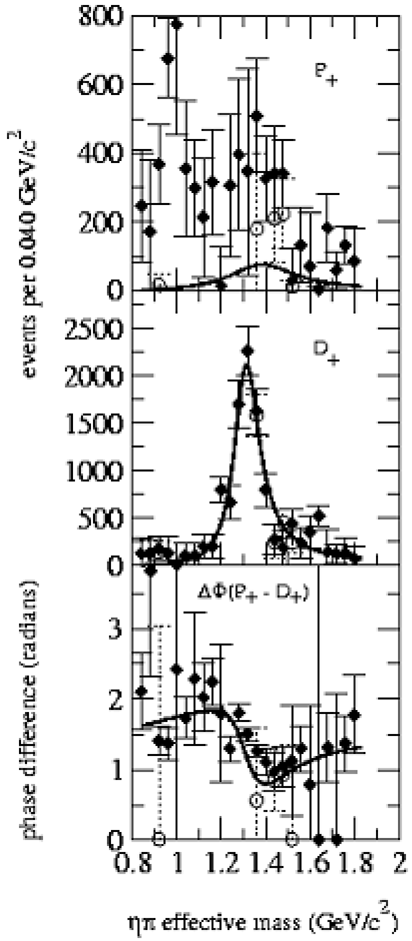
DATA (from E852) $\pi^- p ! \eta\pi^0 p$

```
[adam@mantrid00 data]$ ls -l
total 835636
-rw-r--r--    1 adam    adam    87351564 May 10 10:40 ACC
-rw-rw-r--    1 adam    adam     4882894 May 10 10:39 DAT
-rw-r--r--    1 adam    adam    762596488 May 10 10:42 RAW
[adam@mantrid00 data]$
```

	E	p_x	p_y	p_z	$(E^2 - p_i\ p_i)^{1/2}$	
Event 1						
	5.673920	-0.269088	-0.463492	5.646950	0.1345	(m_π)
	12.561666	0.458374	0.228348	12.539296	0.5471	(m_η)
	1.001964	-0.260447	0.202660	0.112333	0.9393	(m_N) (recoil)
	18.299278	-0.071161	-0.032484	18.298578	0.1396	(m_π) (beam)
Event 2						
	7.348978	-0.405326	0.602844	7.311741		
	11.217298	-0.360824	-0.456940	11.188789		
	1.255382	0.718019	-0.177694	0.382252		
	18.883385	-0.048131	-0.031789	18.882782		

Results of π^{-} p ! $\eta\pi^{0}$ n analysis (part 1)

Assume BW resonance in all, M=§1,0, P-waves

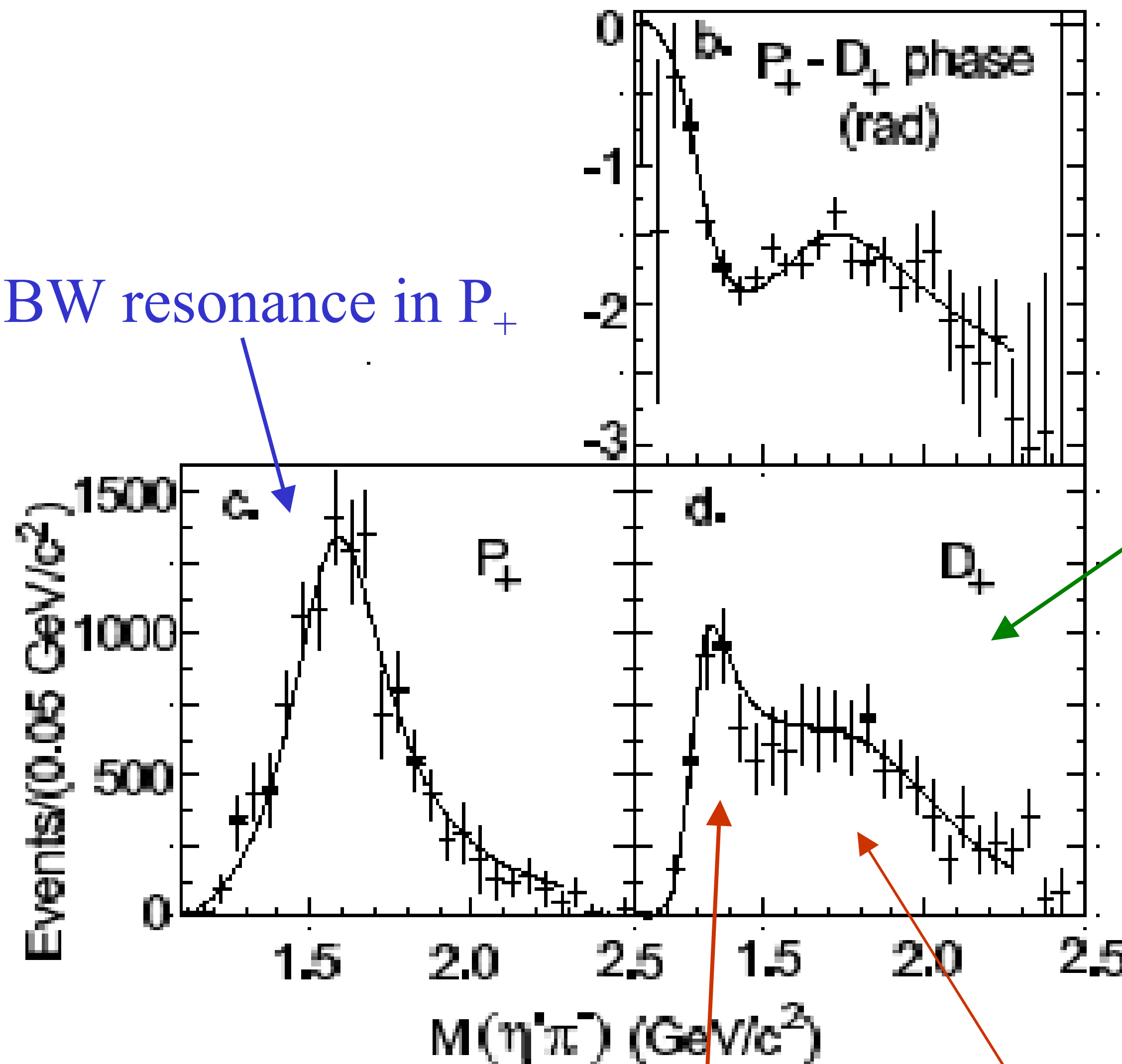


π_1(900 – 5GeV) emerges

Intensity in the weak P-waves is strongly affected by the a_2(1320), strong wave due to acceptance corrections

E852 η’π⁻ analysis

1 BW resonance in P_+



2 BW resonances in D_+

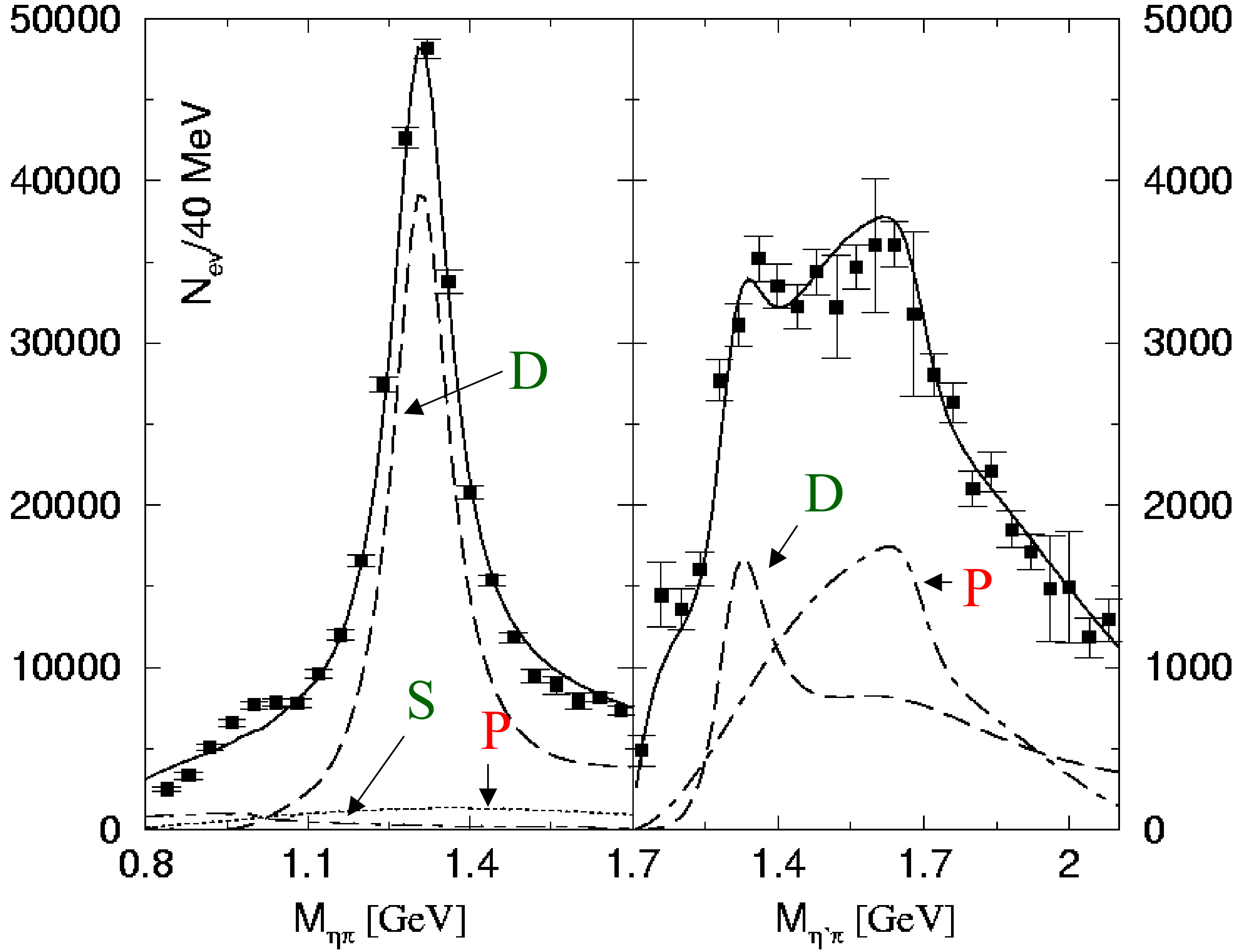
$a_2(1800)$ = ?

$a_2(1320)$

Results of coupled channel analysis of

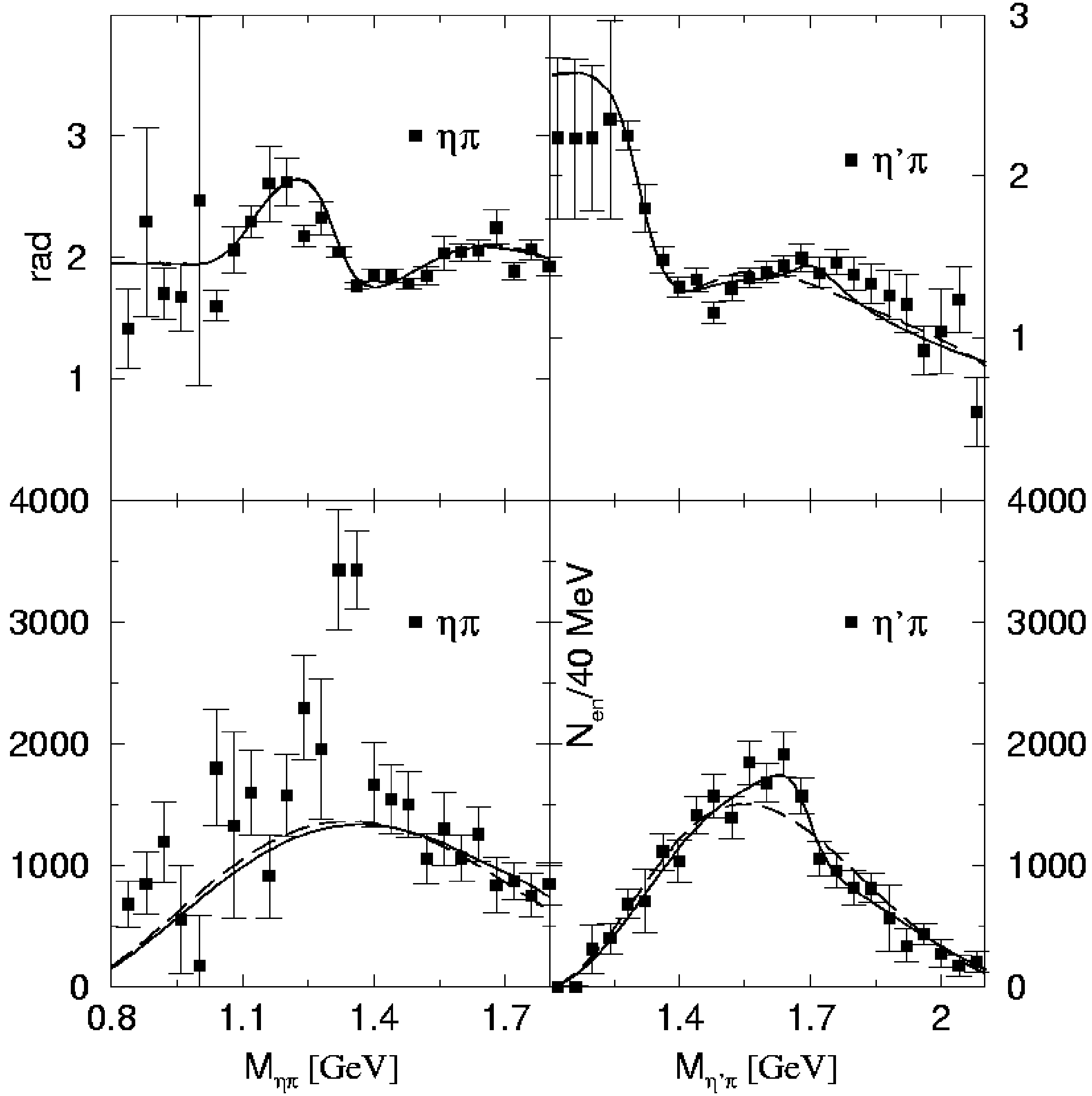
$\pi^- p ! \eta\pi^- p$

$\pi^- p ! \eta'\pi^- p$



$\phi(P^+)-\phi(D^+)$

$|P^+|^2$



rad
ηπ
η'π
N_{ev}/40 MeV
3
2
1
4000
3000
2000
1000
0
0.8
1.1
1.4
1.7
2
$M_{\eta\pi}$ [GeV]
$M_{\eta'\pi}$ [GeV]

Peripheral production of hybrid mesons

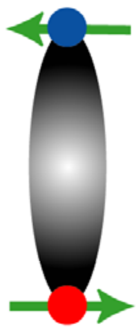
Quarks $\oplus$ **Excited Flux Tube** → **Hybrid Meson**

$S = 0$

$L = 0$

$J^{PC} = 0^{-+}$

like π, K



$$J^{PC} = \begin{cases} 1^{+-} \\ 1^{-+} \end{cases}$$

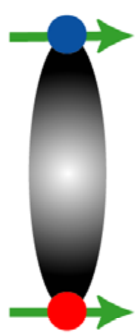
$$J^{PC} = \begin{cases} 1^{--} \\ 1^{++} \end{cases}$$

$S = 1$

$L = 0$

$J^{PC} = 1^{--}$

like γ, ρ



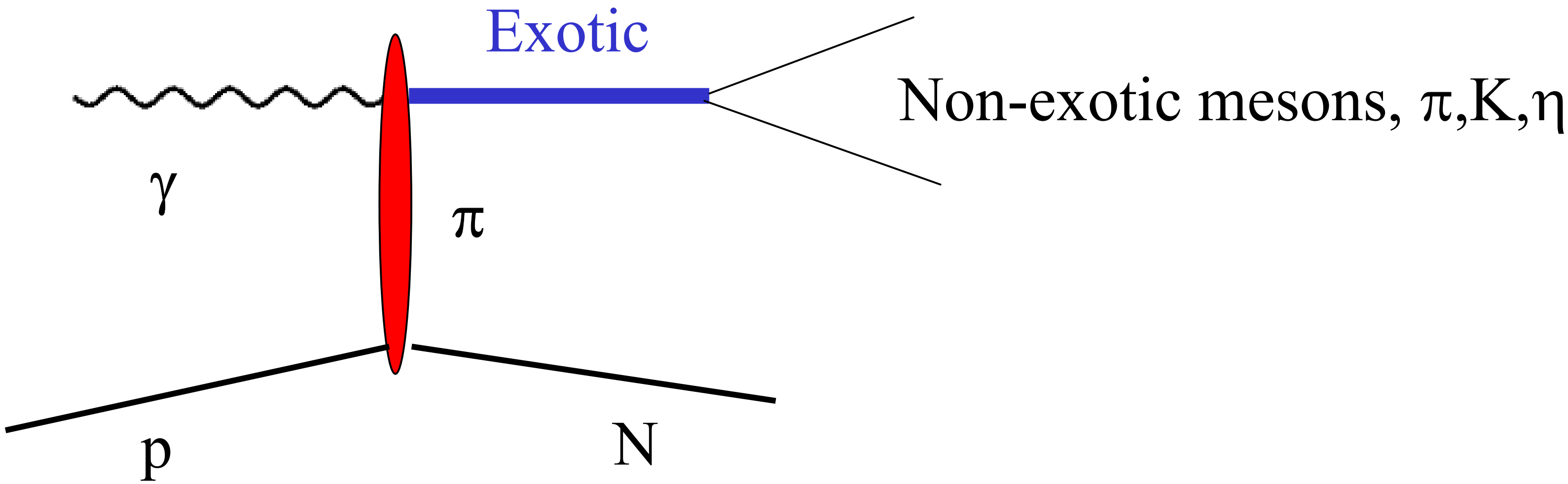
$$J^{PC} = \begin{cases} 1^{+-} \\ 1^{-+} \end{cases}$$

Exotic

$$J^{PC} = \begin{cases} 0^{-+} \quad 1^{-+} \quad 2^{-+} \\ 0^{+-} \quad 1^{+-} \quad 2^{+-} \end{cases}$$

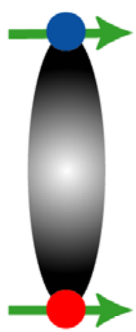
So only parallel quark spins lead to exotic J^{PC}

Photo production enhances exotic mesons



γ : quark with spins aligned

QCD ! Exotic :



π : quark with spins anti-aligned

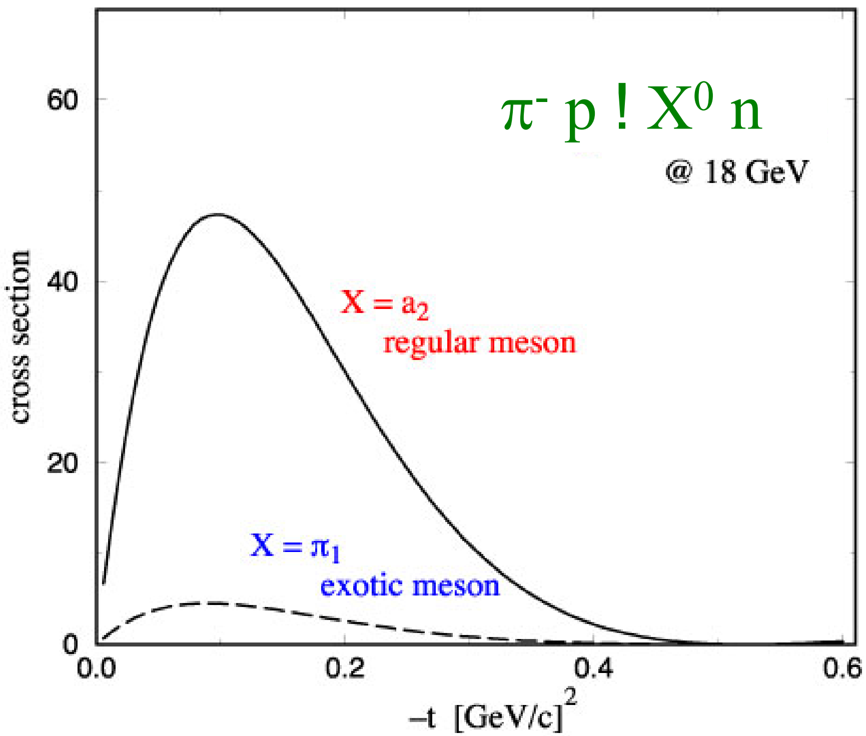
γ --> $\rho(J^{PC}=1^{--})$ --> $\pi_1(J^{PC}=1^{-+})$

VMD ↑   "pluck" the string ↑

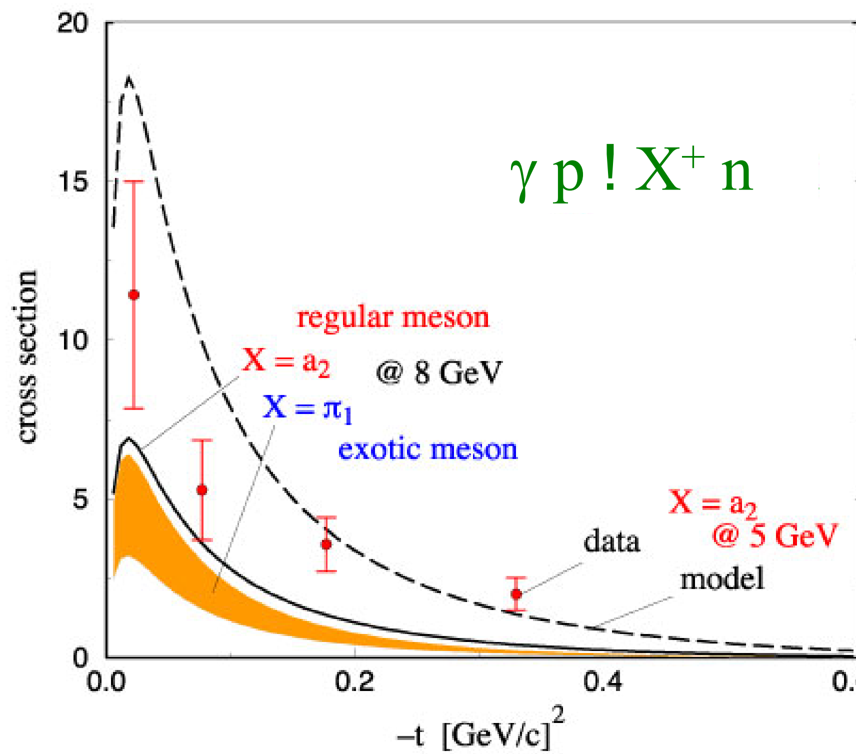
Implications for exotic meson searches

- Possible narrow QCD exotic (M=1.6 GeV)   (E852 $\pi^- p ! \pi^+\pi^- \pi^- p$)

pion



photon

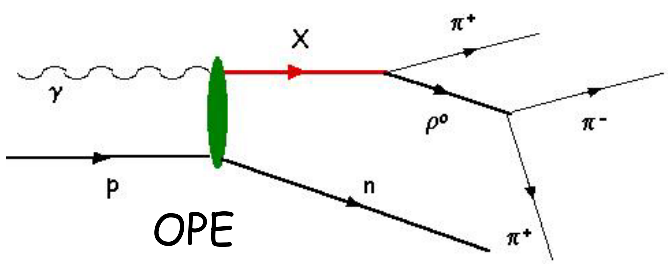


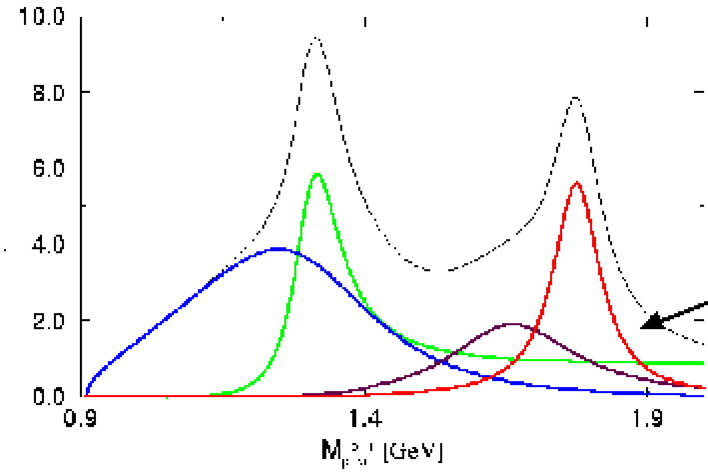
Szczepaniak & Swat (01)

exotic/non-exotic » 0.1

exotic/non-exotic » 1

Photo production enhances exotic mesons





Γ_{π_1} = 100-200 MeV,
$\Gamma_{\pi_1 \to \gamma\pi}$ = 200

1^{-+} exotic : S=1, L=1

Afanasev, AS, (00)

γ --> $\rho(J^{PC}=1^{--})$ --> $\pi_1(J^{PC}=1^{-+})$

VMD

"pluck" the string ($S=1, L_{QQ}=0 \to L_g=1$)

Agrees with Condo'93

	J^{PC}	$\rho\pi$ decay mode	Mass (MeV)	Γ (Mev)	$\Gamma_{3\pi}/\Gamma$	σ_γ (μb)
a_1	1^{++}	S D	1260	400	99% 1%	~0.03
a_2	2^{++}	D	1320	110	70%	~0.50
π_2	2^{-+}	P F	1670	260	30% 1%	~0.02
π_1	1^{-+}	P	1600	160	50%	

DARESBURY STUDY WEEKEND SERIES No. 8

THREE PARTICLE PHASE SHIFT ANALYSIS AND MESON RESONANCE PRODUCTION:

proceedings of the Daresbury Study Weekend, 1-2 February, 1975

Edited by J. B. Dainton and A. J. G. Hey

CORRECTIONS TO THE ISOBAR MODEL FOR THREE HADRON FINAL

by

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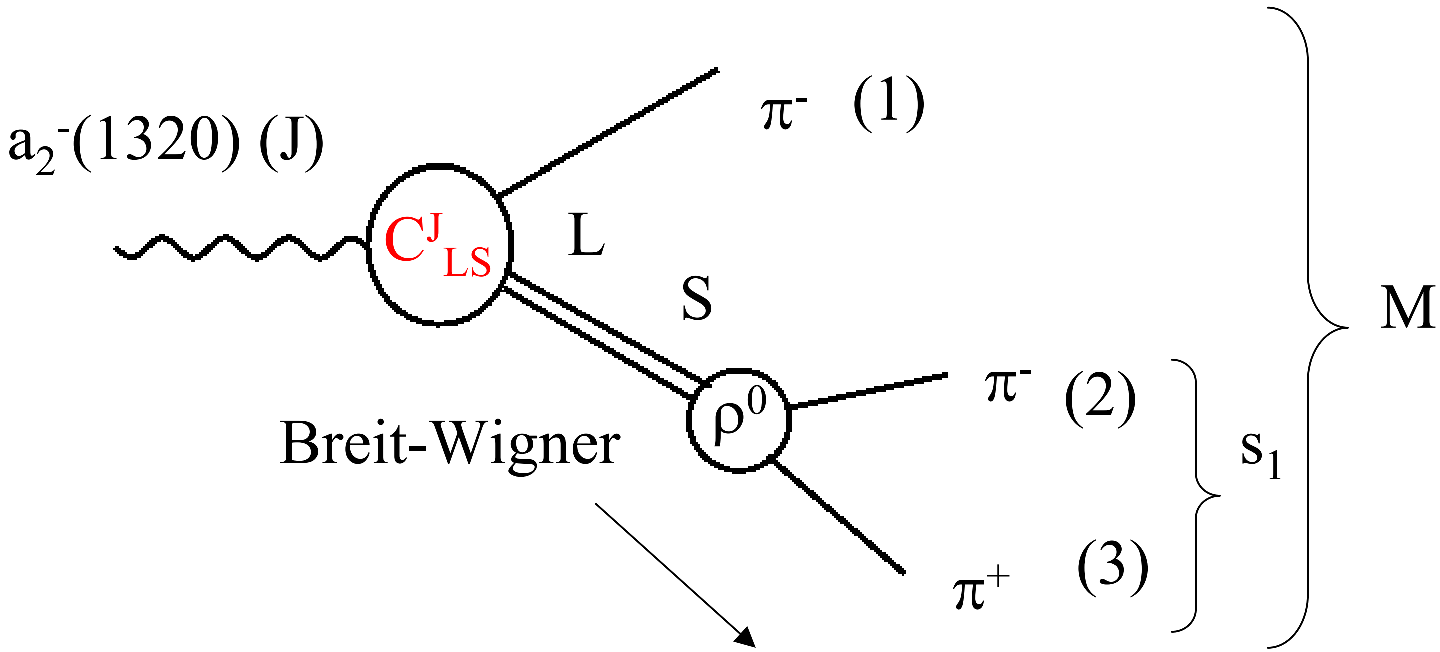
1. WHY THE ISOBAR MODEL NEEDS TO BE CORRECTED (1): UNITARITY IN FINAL STATE TWO-PARTICLE (SUB-ENERGY) CHANNELS.

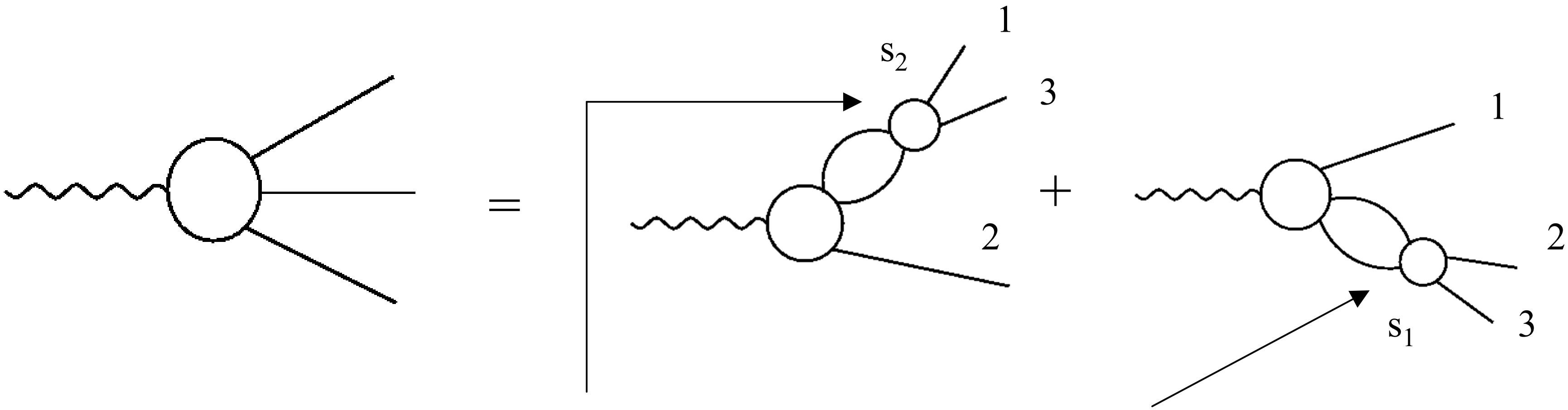
Problems with the isobar (sequential decay) model



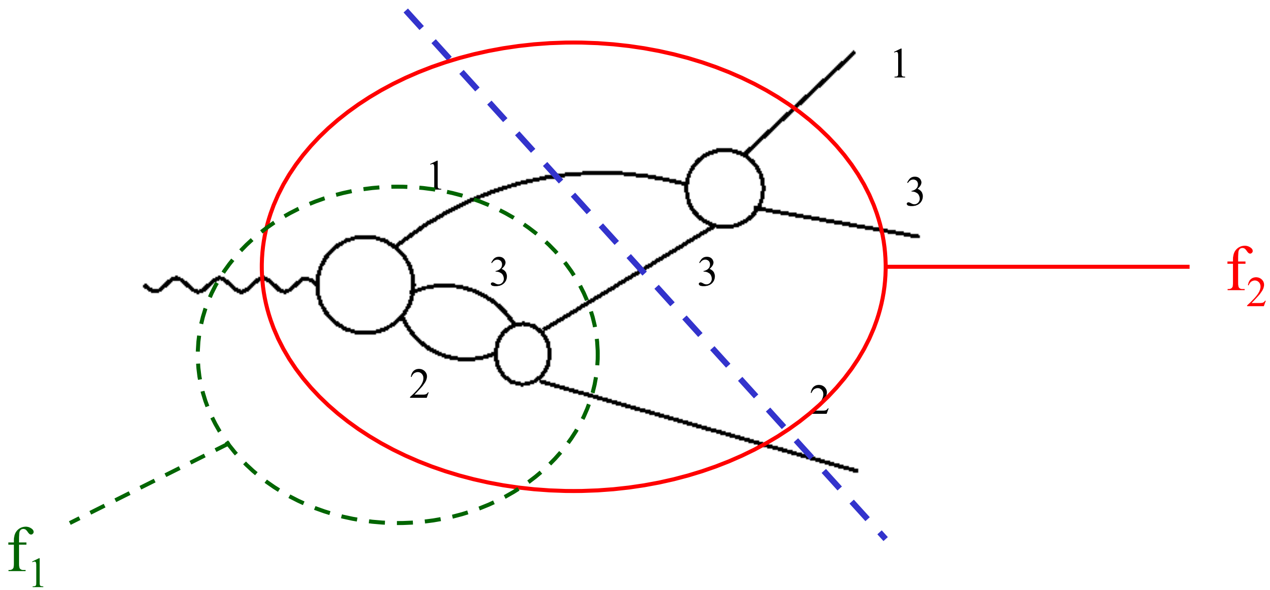
$$A(s_1,s_2,M) = C^J_{LS}\,(M) \,/\, D_1(s_1)$$

1 ←→ 2 $$F(s_1,s_2,M) = f_1(M)/D_1(s_1) + f_2(M)/D_2(s_2)$$

Independent on 2-particle sub-channel energy :
violates unitarity !



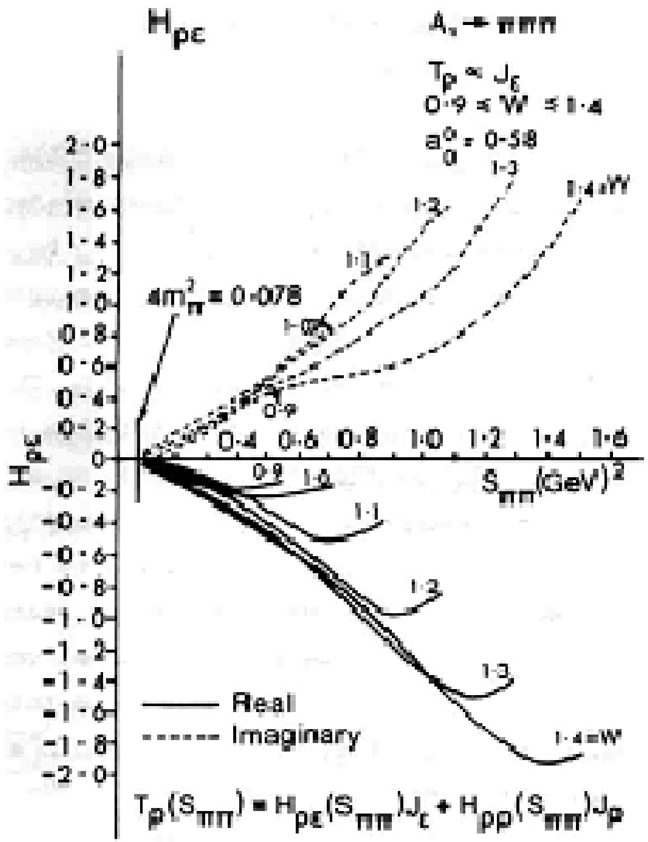
B-W : unitary in sub-channel energy

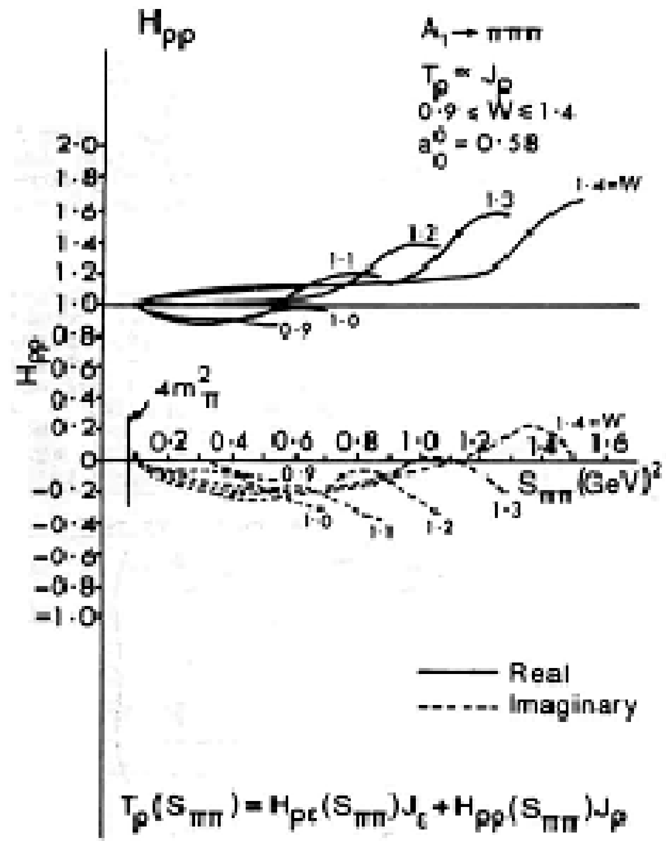


Has to depend on 2-particle energy

$f_i = f_i(s_i,M)$

$$f_2(s_{2+}) - f_2(s_{2-}) = 2i\rho(s_2)\ \text{P.V.}\mathbf{s}\ f_1(s_1)/D_1(s_1)$$

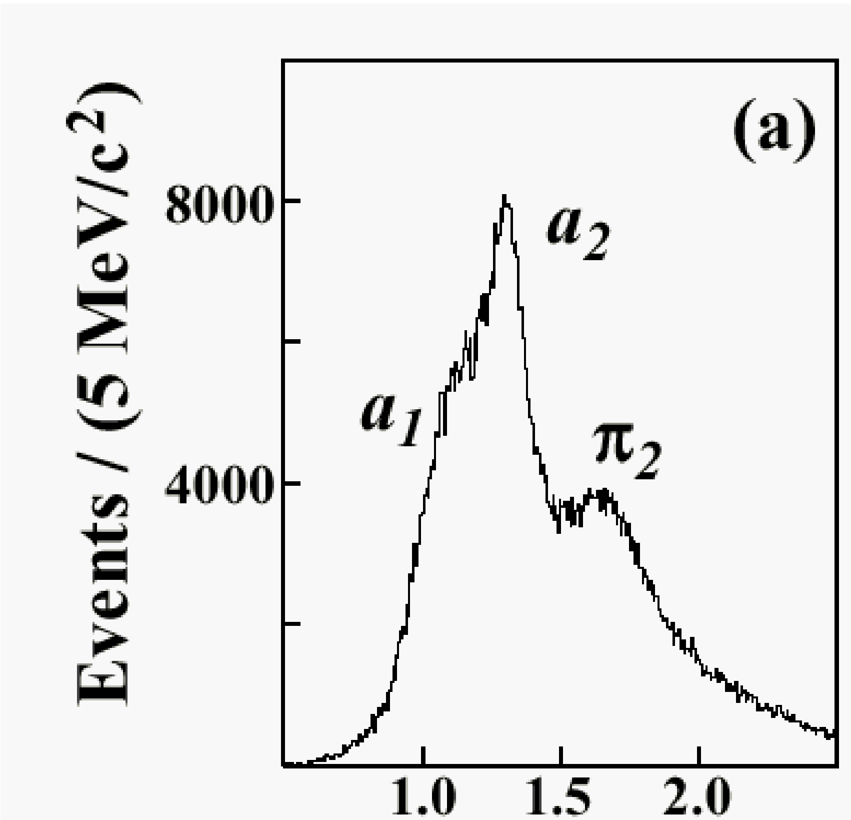




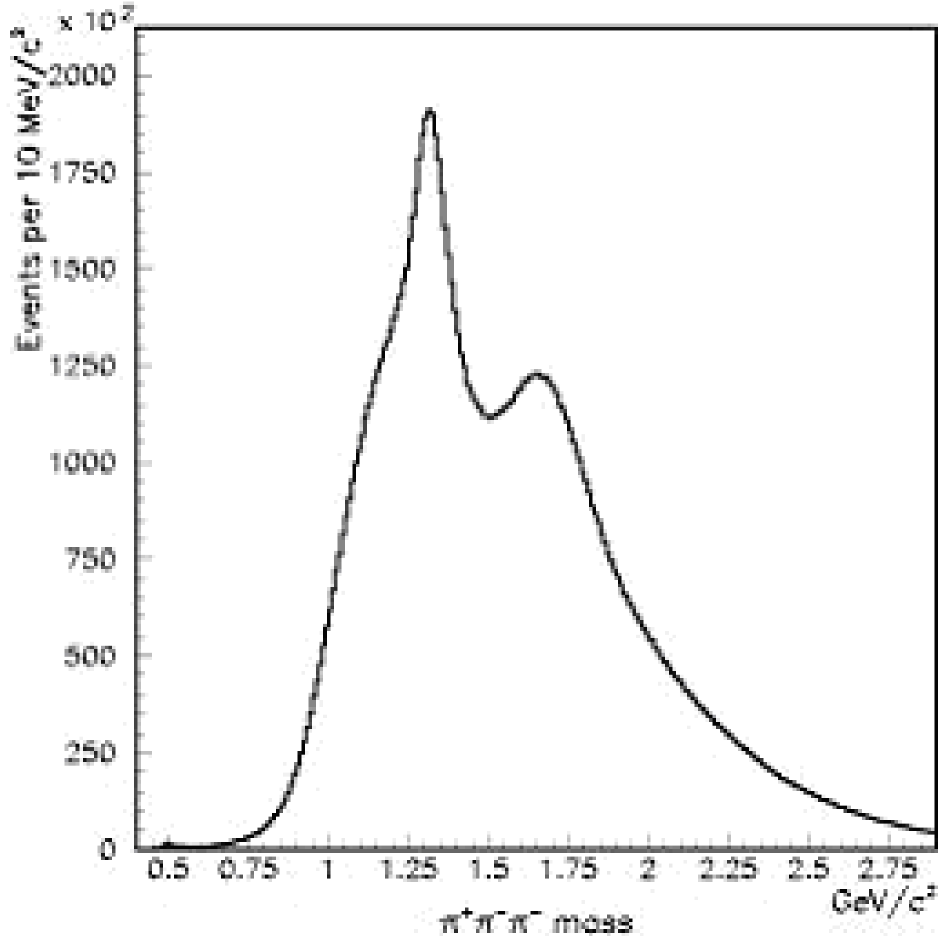
$F_1(s_1,M)_i = H(s_1,M)_{ij}\, C_j(M)$ $\quad i,j=\sigma,\rho$

3π sample

Analyzed



Available !



Summary

- Need theory input to minimize (mathematical) ambiguities and to understand systematic errors

- Need more theory input to determine physical states (coherent background vs resonances)

- There is lots of data to work with and there will be more especially needed for establishing gluonic excitations

- $\pi_1(1400)$ and $\pi_1(1600)$ in $\eta'\pi$ are most likely due to Residual interactions much like the σ meson in the $\pi\pi$ S-wave