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Target mass corrections

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I. DVCS amplitude evaluated within the operator product expansion

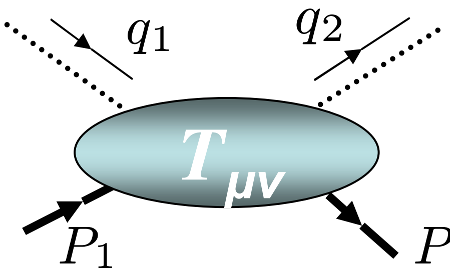
II. Systematic twist expansion

III. Target mass corrections

IV. Summary

Definitions and OPE formalism

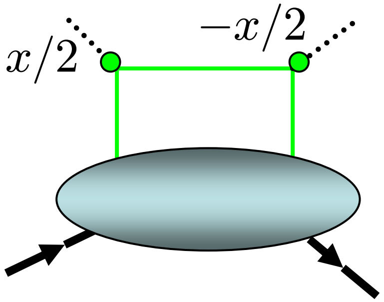
- QCD dynamics in photo leptoproduction is encoded in the hadronic tensor



$$T_{\mu\nu}(q, P, \Delta) = \int d^4x \; e^{ix\cdot q} \langle P_2 | T\{j_\mu(x/2) j_\nu(-x/2)\} | P_1 \rangle$$

$$P = P_1 + P_2,\ \Delta = P_2 - P_1,\ q = (q_1 + q_2)/2$$

- leading singularities in *1/x²* arise from the hand-bag diagram



$$T\{j_\mu(x/2) j_\nu(-x/2)\} = \frac{x_\sigma}{[-x^2 + i0]^2} \left[S_{\mu\nu}{}^{\rho\sigma} \, {}^V\!\mathcal{O}_\sigma(x/2, -x/2) + i\epsilon_{\mu\nu}{}^{\rho\sigma} \, {}^A\!\mathcal{O}_\sigma(x/2, -x/2) \right]$$

‼ however, operators contain also non-leading contributions

$${}^\Gamma\mathcal{O}_\rho(x, -x) = \bar{\psi}(-x)[-x, x]\Gamma_\rho \psi(x)$$

- the expectation value of operators might be defined in terms of DDs

$$\langle P_2 | \mathcal{O}_\rho(x, -x) | P_1 \rangle = \int_\Omega dy\, dz\, f(y, z) \mathcal{P}_\rho \; e^{-ix\cdot\mathcal{P}} + x_\rho \cdots + O(x^2)$$

- To drop non-leading terms, take the + component and replace *x* by *n* with $n^2 = 0$.
  (Fourier transform is simpler to perform with *x* rather *n*)
- Fourier-transform leads to Compton form factors, given as

$$\mathcal{F}(\xi,\eta,Q^2) = \int_\Omega dydz \frac{1}{\xi - y - z\eta - i0} f(y,z,Q^2)\,, \quad \xi = \frac{Q^2}{q\cdot P}, \quad \eta = \frac{q\cdot\Delta}{q\cdot P}$$

$$= \int_{-1}^{1} dx \frac{1}{\xi - x - i0} q(x,\eta,Q^2)$$

! power suppressed contributions are added in an uncontrolled way

⇨ for DVCS kinematics one should set

$$\eta = \xi \stackrel{\text{def}}{=} \frac{x_{\text{Bj}}}{2 - x_{\text{Bj}}}, \qquad x_{\text{Bj}} = \frac{-q_1^2}{2P_1 \cdot q_1}$$

- taking one `exact' relation into account, e.g., $\eta = \xi\left(1 - \Delta^2/2q_1^2\right)^{-1}$

  is ambiguous
  includes higher twist contributions in an uncontrolled way
  has nothing to do with target mass corrections

Systematic twist expansion

geometrical twist = dimension – spin of the operator

to separate different twist-contributions one might consider local operators

$$\mathcal{O}_\rho(x,-x) = \sum_{j=0}^{\infty} \frac{(-2i)^j}{j!} x_{\mu_1} \dots x_{\mu_j} \mathcal{O}_{\rho;\mu_1\dots\mu_j}, \quad \mathcal{O}_{\rho;\mu_1\dots\mu_j} = \bar\psi \Gamma_\rho \, i\overset{\leftrightarrow}{\mathcal{D}}_{\mu_1} \dots i\overset{\leftrightarrow}{\mathcal{D}}_{\mu_j} \, \psi,$$

General procedure, e.g., for twist-three sector

i. symmetries by means of Young-tableaus

ρ	μ_1	$\cdots$	μ_n

twist-two

μ_1	$\cdots$	μ_n
ρ		

twist-three

ii. set *x* to *n* to remove ***trace terms*** (twist-four and higher)

➢ these steps so far lead to a breaking of U(1) gauge invariance

iii. employ $\not{\mathcal{D}}\psi = 0$ to separate qq and qGq operators

iv. resum local operators and define GPDs

✓ *restoration* of U(1) gauge invariance, twist-2 is *talking* to twist-3 operators

$$q^{\mathrm{twist}-3} = C^{\mathrm{WW}} \otimes q^{\mathrm{twist}-2} + q^{\mathrm{qGq}}$$

A challenge – going beyond twist-three

- twist-two and -three contributions are distinguishable in photo electroproduction
- t/Q^2 and M^2/Q^2 corrections arise from kinematical factors and twist-four operators
- dynamical contributions might complete kinematical ones
  (e.g., the φ independent part of the interference term arises from
  kinematical and dynamical twist-3 effects and is expressed by twist-two GPDs)

- new features at twist-four (and higher) in off-forward kinematics
  - Wandzura-Wilczek terms are not uniquely defined
  - What is the appropriate basis of qGGq operators?
  - no straightforward method known to consistently evaluate twist-four contributions
  - twist-two operators know something about twist-four operators

- an idea to evaluate higher twist contributions in a straightforward way:
  - twist-decomposition + EOM in forward kinematics (should be have done)
  - use conformal symmetry to map forward operators to non-forward ones

Target mass corrections

A. Belitsky and D.M., Phys. Lett. B507 (2001) 173

geometrical twist expansion + equation of motion within *t=0*

- failure in restoration of U(1) gauge invariance
- resummation of target mass corrections is possible within DDs

$$\mathcal{F} = \int_{\Omega} dy\, dz\, f(y,z) \mathcal{C}(y,z|\xi)$$

$$\mathcal{C}\Big|_{M=0} = \frac{1}{1+\Xi^{-1}} \quad \rightarrow \quad \mathcal{C} = \frac{4\Xi - \mathcal{M}^2(1-\Xi)}{[4\Xi(1+\Xi)-\mathcal{M}^2]} + \frac{\mathcal{M}^2\left(2\Xi-\mathcal{M}^2\right)}{4\Xi(1+\mathcal{M}^2)^{3/2}} \ln\left(\frac{1-\sqrt{1+\mathcal{M}^2}+2\Xi}{1+\sqrt{1+\mathcal{M}^2}+2\Xi}\right)$$

- target mass corrections can not be absorbed by means of a \`Nachtmann' variable
- expanded version, i.e., M^2/Q^2 terms, can be expressed in terms of GPDs
  target mass corrections are in addition suppressed by ξ

$$\mathcal{F} = \int_{-1}^{1} dx \int_{-1}^{1} dx' \left[\frac{\delta(x-x')}{\xi - x - i0} + \frac{\xi M^2}{Q^2} C_1^{(1)}(x, x'|\xi)\, \hat{D}(x', \eta) + \mathcal{O}\left(\frac{\xi^2 M^4}{Q^4}\right)\right] q(x', \eta)$$

Summary

- *target mass corrections* seem to be numerical small for fixed target experiments (numerical studies are not done so far)

- *power suppressed contributions* beyond twist-three are challenging

- because of their lack one should stick to twist-two definition of variables, i.e., set $\eta=\xi$ in GPDs