

# Large-Scale Mass Table Calculations

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**J. Dobaczewski, W. Nazarewicz, J.C. Pei, N. Schunck**

- HFBTHO solver
- Benchmarking the solver
- Accelerating iterations - Broyden's Method
- Large-Scale Calculations
- Mass Table Explorer

# SciDAC UNEDF project

## Building a Universal Nuclear Energy Density Functional

### *A Low-Energy Nuclear Physics National HPC Initiative*

*George F. Bertsch*, [University of Washington](http://www.washington.edu)

The mission of the project is three-fold:

- First, to find an optimal functional using all our knowledge of the nucleonic Hamiltonian and basic nuclear properties.
- Second, to apply the EDF theory and its extensions to validate the functional using all the available relevant nuclear structure data.
- Third, to apply the validated theory to properties of interest that cannot be measured, in particular the transition properties needed for reaction theory.

The activities to be supported fall into different areas of nuclear theory and computer science, but the goal can only be achieved by working at the interfaces among these areas. They are: ab initio theory of nuclear wave functions, Effective Field Theory (EFT) and its extensions, self-consistent mean-field description of ground and excited states, large amplitude collective motion, low-energy reaction theory and computer science.

**Science Application:** Nuclear Physics

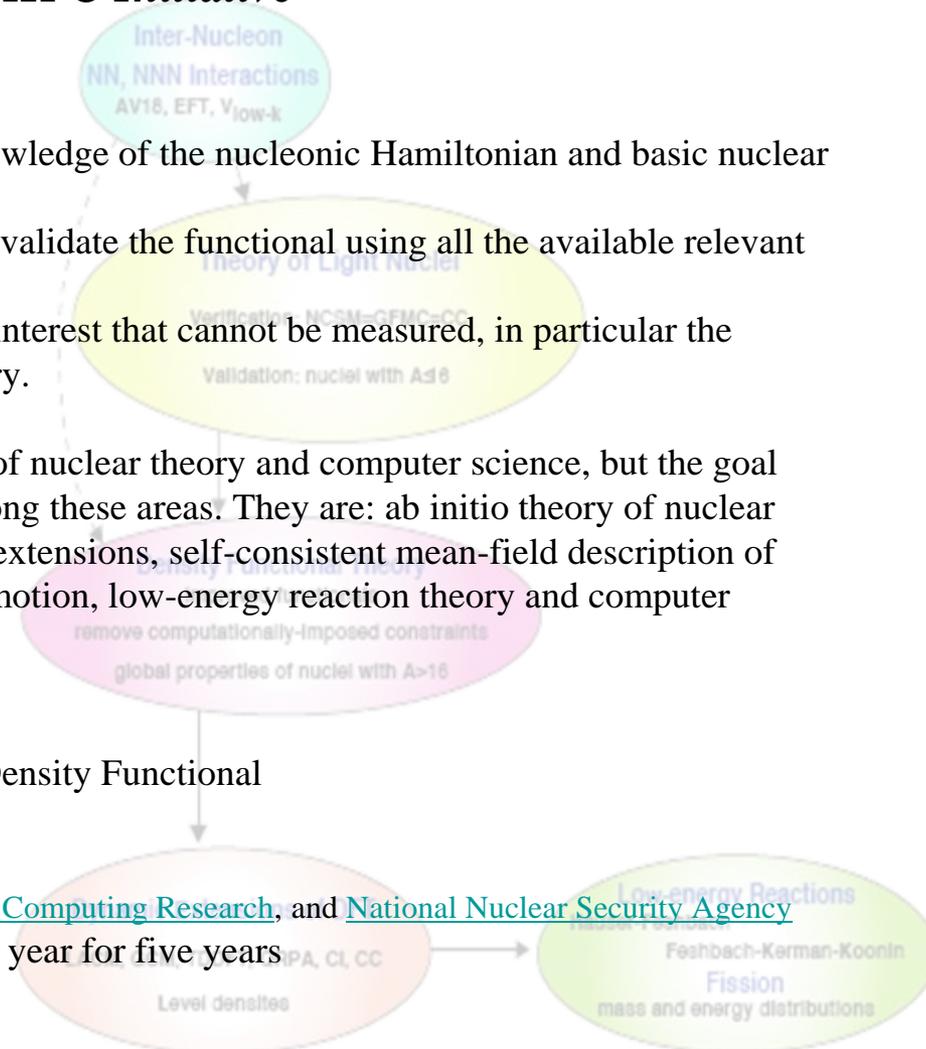
**Project Title:** Building a Universal Nuclear Energy Density Functional

**Principal Investigator:** George F. Bertsch

**Affiliation:** University of Washington

**Funding Partners:** [Office of Science](#), [Advanced Scientific Computing Research](#), and [National Nuclear Security Agency](#)

**Budget and Duration:** Approximately \$3 Million per year for five years

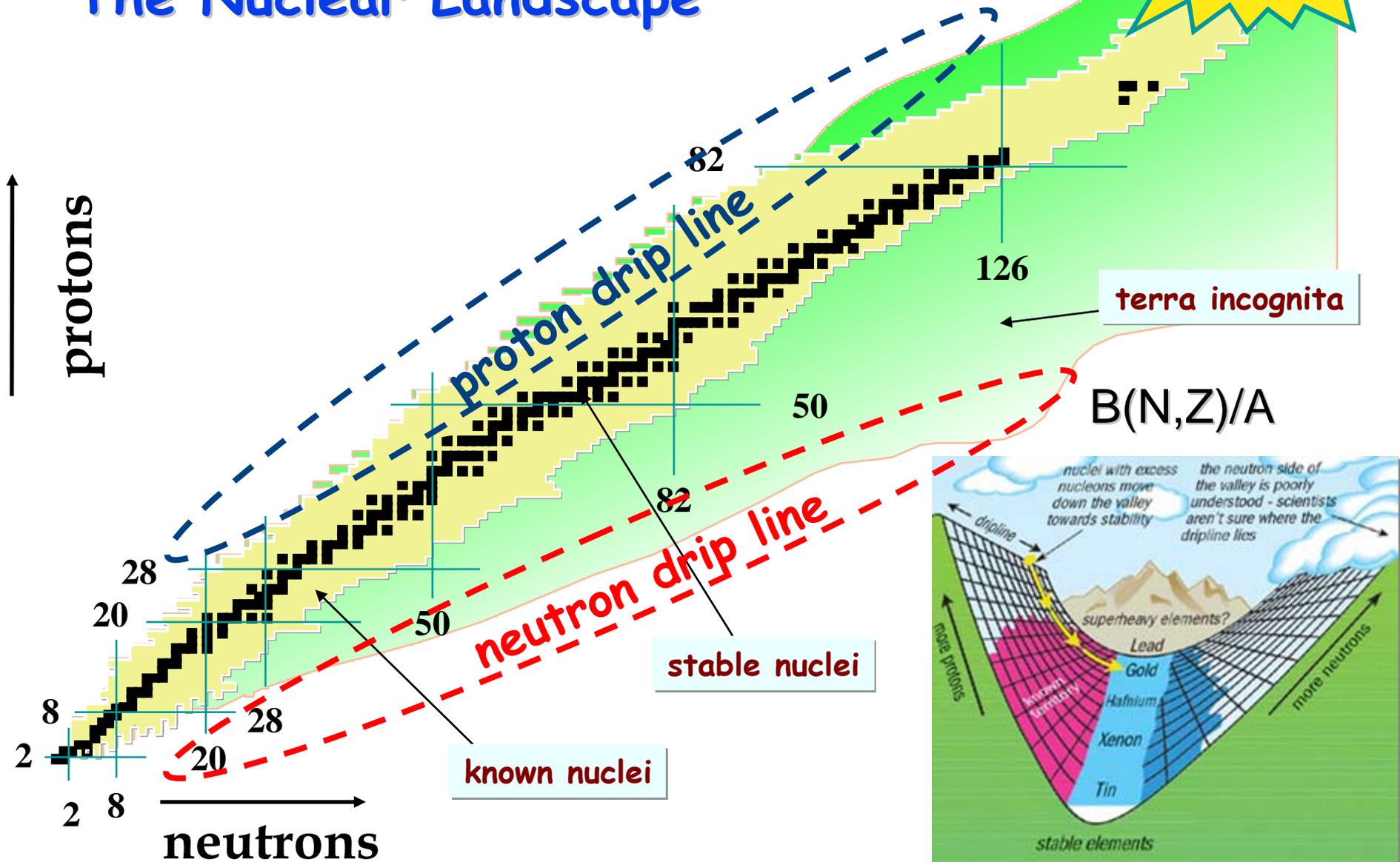


SciDAC

Scientific Discovery through Advanced Computing

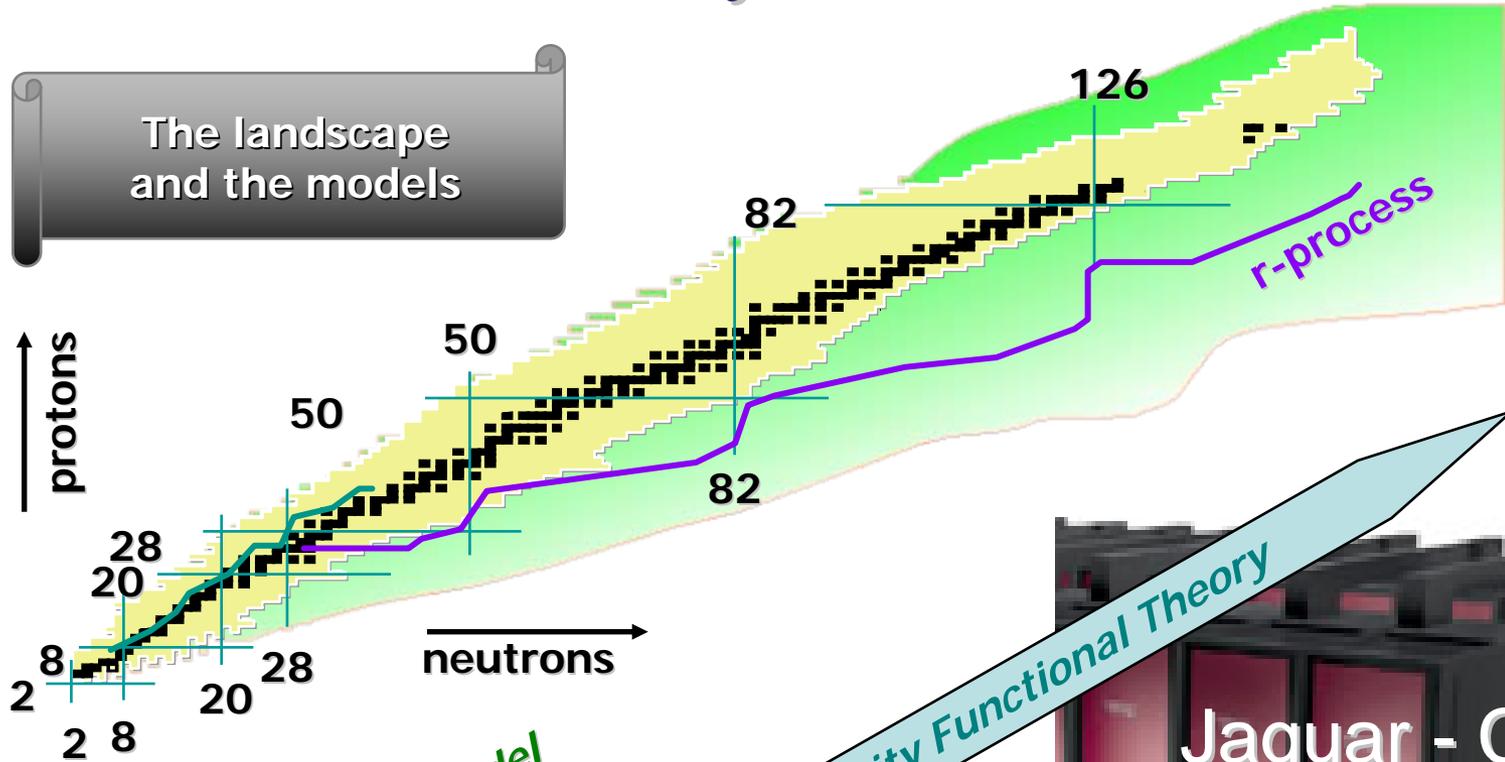
<http://unedf.org>

# The Nuclear Landscape



# Nuclear Density Functional Theory

The landscape and the models



Ab initio  
few-body  
calculations

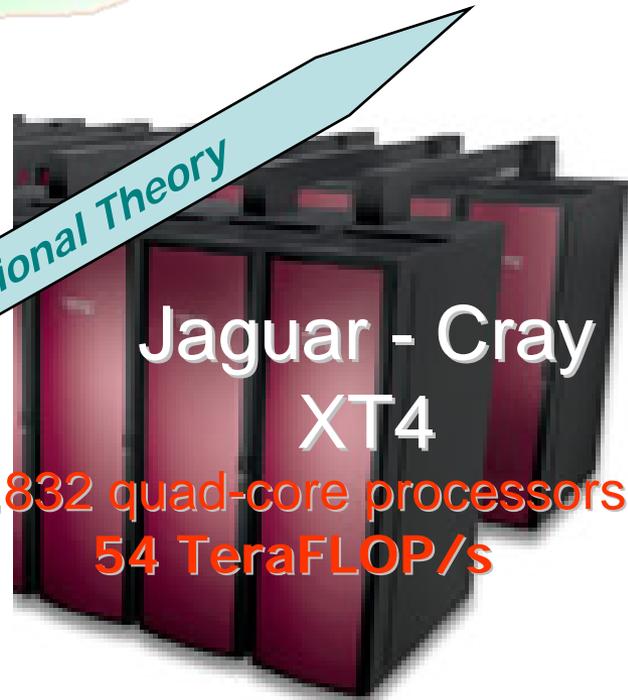
$A=12$

Shell Model

$A \sim 60$

Self-consistent Mean Field (HFB)

Nuclear Density Functional Theory



Jaguar - Cray  
XT4

7,832 quad-core processors  
54 TeraFLOP/s

# HFBTHO solver

*M.V. Stoitsov, J. Dobaczewski, W. Nazarewicz, P. Ring*

Axially Deformed Solution of the Skyrme-Hartree-Fock-Bogolyubov Equations  
Using The Transformed Harmonic Oscillator Basis. The program HFBTHO

*Computer Physics Communications Volume 167, Issue 1, 1 April 2005, Pages 43-63*

- 2D HFB in HO/THO basis (cylindrical coordinates)
- Time-reversal, axial and parity symmetries
- Skyrme type functional and contact delta pairing
- Handling even-even, odd-even and odd-odd nuclei
- Easily extendable to arbitrary functional
- Fast alternative for ground-state calculations
- ✨ Continuum discretization as coming from the diagonalization
- DFT solvers in coordinate space
  - HFBRAD – box, coordinate space, spherical symmetry
  - HFB-AX – rectangular box, B-splines, axial/parity symmetry
- DFT solvers in configurational (HO) space
  - HFODD – 3D in HO basis, symmetry unrestricted, time-odd components, cranking, projections ...
  - HFBTHO – 2D in HO/THO basis, axial/parity symmetry
- New generation HFB solvers (the talk of G. Fann)

# HFBTHO/HFODD Benchmark

<i>even-even nuclei</i>	$^{208}\text{Pb}$		$^{168}\text{Er}$		$^{120}\text{Sn}$	
	HFBTHO 2D-HO	HFODD 3D-HO	HFBTHO 2D-HO	HFODD 3D-HO	HFBTHO 2D-HO	HFODD 3D-HO
$N_0$	14	14	14	14	14	14
$N_{\text{st}}$	680	680	680	680	680	680
$b_{\perp} = b_z$	2.2348121	2.2348121	2.1566616	2.1566616	2.039048	2.039048
$\lambda_n$	-8.114 <b>078</b>	-8.114 <b>02</b>	-6.93605 <b>9</b>	-6.93605 <b>8</b>	-8.015208	-8.015208
$\lambda_p$	-8.8104 <b>77</b>	-8.8104 <b>45</b>	-7.15648 <b>6</b>	-7.15647 <b>7</b>	-8.25 <b>1999</b>	-8.24 <b>5192</b>
$\Delta_n$	0	0	0.3945 <b>72</b>	0.3945 <b>78</b>	1.24464 <b>4</b>	1.24464 <b>5</b>
$\Delta_p$	0	0	0.3906 <b>02</b>	0.3906 <b>05</b>	0	0
$E_n^{\text{pair}}$	0	0	-1.716 <b>979</b>	-1.717 <b>024</b>	-12.4263 <b>88</b>	-12.4263 <b>97</b>
$E_p^{\text{pair}}$	0	0	-1.5286 <b>16</b>	-1.5286 <b>43</b>	0	0
$R_n$	5.61975 <b>6</b>	5.61975 <b>7</b>	5.357578	5.357578	4.7330 <b>88</b>	4.7331
$R_p$	5.4600 <b>78</b>	5.4600 <b>90</b>	5.22553 <b>8</b>	5.22553 <b>9</b>	4.5962 <b>94</b>	4.5963
$Q_n$	-0.00000 <b>1</b>	6.6E-11	11.47391 <b>8</b>	11.47392 <b>0</b>	-0.000000 <b>1</b>	6.6E-11
$Q_p$	-0.00000 <b>1</b>	4.7E-11	7.88022 <b>1</b>	7.88022 <b>4</b>	-0.000000 <b>1</b>	6.6E-11
$E_n^{\text{kin}}$	2525.99 <b>2765</b>	2525.99 <b>1925</b>	1974.614 <b>008</b>	1974.613 <b>824</b>	1338.2104 <b>78</b>	1338.2105 <b>01</b>
$E_p^{\text{kin}}$	1334.85 <b>5572</b>	1334.85 <b>4465</b>	1118.31 <b>3683</b>	1118.31 <b>3442</b>	829.438 <b>221</b>	829.438 <b>221</b>
$E_{\text{SO}}$	-96.375 <b>045</b>	-96.375 <b>003</b>	-80.1868 <b>09</b>	-80.1868 <b>26</b>	-49.0023 <b>07</b>	-49.0023 <b>16</b>
$E_{\text{dir}}$	827.60 <b>7375</b>	827.60 <b>7885</b>	602.810 <b>248</b>	602.810 <b>352</b>	366.326 <b>962</b>	366.326 <b>917</b>
$E_{\text{exc}}$	-31.2484 <b>79</b>	-31.2484 <b>62</b>	-25.93591 <b>0</b>	-25.93590 <b>5</b>	-19.08958	-19.08958
$E_{\text{tot}}$	-1634.14 <b>8867</b>	-1634.14 <b>8120</b>	-1357.658 <b>500</b>	-1357.658 <b>322</b>	-1018.141 <b>626</b>	-1018.141 <b>673</b>

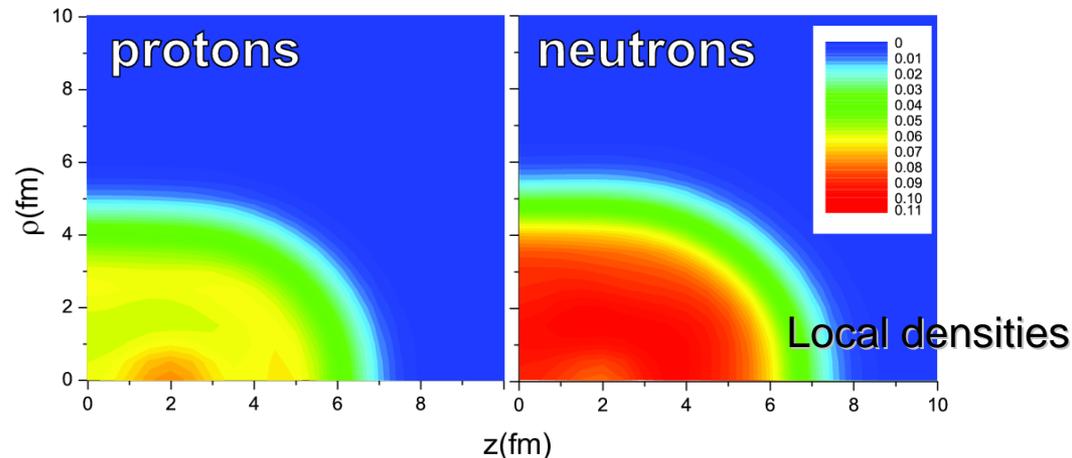
# HFODD/HFBTHO Benchmark

odd nuclei	1/2+[4,4,0]		1/2+[4,0,0]		3/2-[5,2,1]	
	HFBTHO	HFODD	HFBTHO	HFODD	HFBTHO	HFODD
$N_0$	14	14	14	14	14	14
$N_{st}$	680	680	680	680	680	680
$b_{\perp} = b_z$	2.0418697	2.0418697	2.0418697	2.0418697	2.0418697	2.0418697
$E_{qp}$	1.007 <b>644</b>	1.008	1.611 <b>961</b>	1.612	1.38 <b>8951</b>	1.387
$\lambda_n$	-7.74 <b>9566</b>	-7.74 <b>94</b>	-7.6961 <b>79</b>	-7.6962	-7.97 <b>2801</b>	-7.97 <b>42</b>
$E_n^{pair}$	-9.29 <b>4443</b>	-9.29 <b>64</b>	-10.397 <b>019</b>	-10.398 <b>3</b>	-8.703 <b>141</b>	-8.703 <b>5</b>
$\Delta_n$	1.057 <b>516</b>	1.057 <b>6</b>	1.120 <b>611</b>	1.120 <b>7</b>	1.037 <b>402</b>	1.037 <b>3</b>
$r_t$	4.6895 <b>35</b>	4.6895	4.6904 <b>59</b>	4.6905	4.6895 <b>10</b>	4.6895
$\beta$	-0.025 <b>699</b>	-0.0256	0.000 <b>000</b>	0.000 <b>1</b>	0.0 <b>15789</b>	0.0 <b>147</b>
$Q_t$	-0.86 <b>2706</b>	-0.86 <b>04</b>	0.00 <b>0000</b>	0.00 <b>36</b>	0. <b>530038</b>	0. <b>4921</b>
$E_n^{kin}$	1360.43 <b>7867</b>	1360.442 <b>751</b>	1362.40 <b>7077</b>	1362.40 <b>9601</b>	1358.9 <b>12567</b>	1358.8 <b>86614</b>
$E_p^{kin}$	827.317 <b>590</b>	827.317 <b>961</b>	827.12 <b>3364</b>	827.12 <b>3676</b>	827.19 <b>5176</b>	827.19 <b>1207</b>
$E_{SO}$	-50.4 <b>83676</b>	-50.4 <b>85916</b>	-50.92 <b>2860</b>	-50.92 <b>3940</b>	-49.6 <b>07742</b>	-49.5 <b>92026</b>
$E_{dir}$	365.7436 <b>76</b>	365.7437 <b>74</b>	365.6210 <b>13</b>	365.6210 <b>31</b>	365.736 <b>277</b>	365.735 <b>680</b>
$E_{tot}$	-1024.70727 <b>5</b>	-1024.70727 <b>2</b>	-1024.301 <b>233</b>	-1024.301 <b>252</b>	-1024.41 <b>5866</b>	-1024.41 <b>6901</b>

- **HFODD: Approximately 6 h 39 min CPU per nucleus ( $^{120}\text{Sn}$ )**
- **HFBTHO: Approximately 3 min CPU per nucleus ( $^{120}\text{Sn}$ )**

# HFBTHO/HFBAX Benchmark

$^{110}\text{Zr}$	HFB-AX	HFBTHO
	$M=13, R_{box}=19.2 \text{ fm}, \Delta r=0.6 \text{ fm}$	$N_{sh}=20$
$E_t$	-893.983	-893.840
$E_C$	226.758	226.712
$E_K^p$	632.115	631.882
$E_K^n$	1368.206	1368.201
$E_P^n$	-3.200	-3.326
$\Delta_n$	0.636	0.652
$\lambda_n$	-3.552	-3.543
$Q_2^n$	444.02	443.90



# Broyden's Method in Nuclear Structure Calculations

Andrzej Baran,<sup>1,2,3</sup> Aurel Bulgac,<sup>4</sup> Michael McNeil Forbes,<sup>4</sup> Gaute Hagen,<sup>2</sup>  
Witold Nazarewicz,<sup>1,2,5,6</sup> Nicolas Schunck,<sup>1,2</sup> and Mario V. Stoitsov<sup>1,2,7</sup>

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<sup>2</sup>*Physics Division, Oak Ridge National Laboratory,  
P.O. Box 2008, Oak Ridge, Tennessee 37831, USA*

<sup>3</sup>*Institute of Physics, University of M. Curie-Sklodowska, ul. Radziszewskiego 10, 20-031 Lublin, Poland*

<sup>4</sup>*Department of Physics, University of Washington, Seattle, WA 98195-1560*

<sup>5</sup>*Institute of Theoretical Physics, Warsaw University, ul. Hoza 69, 00-681 Warsaw, Poland*

<sup>6</sup>*School of Engineering and Science, University of the West of Scotland, Paisley PA1 2BE, UK*

<sup>7</sup>*Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia, Bulgaria*

Broyden's method, widely used in quantum chemistry electronic-structure calculations for the numerical solution of nonlinear equations in many variables, is applied in the context of the nuclear many-body problem. Examples include the unitary gas problem, the nuclear density functional theory with Skyrme functionals, and the nuclear coupled-cluster theory. The stability of the method, its ease of use, and its rapid convergence rates make Broyden's method a tool of choice for large-scale nuclear structure calculations.

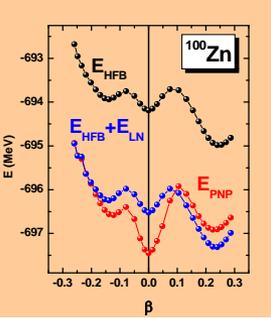
PACS numbers: 21.10.Dr, 21.60.Jz, 21.60.De, 71.15.Mb, 02.60.Cb

Phys.Rev.C 78 (2008) 014318

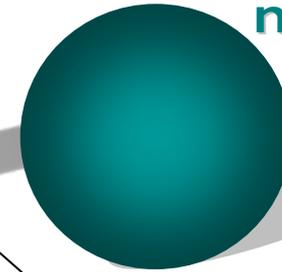
# Broyden Mixing

$$F^{(m)}(V) = V^{(m)} - V^{(m-1)}$$

$$F(V) = 0$$



Calculating new fields

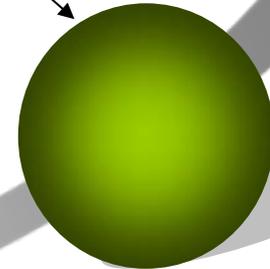


Self-Consistent HFB Solution

Calculating new densities



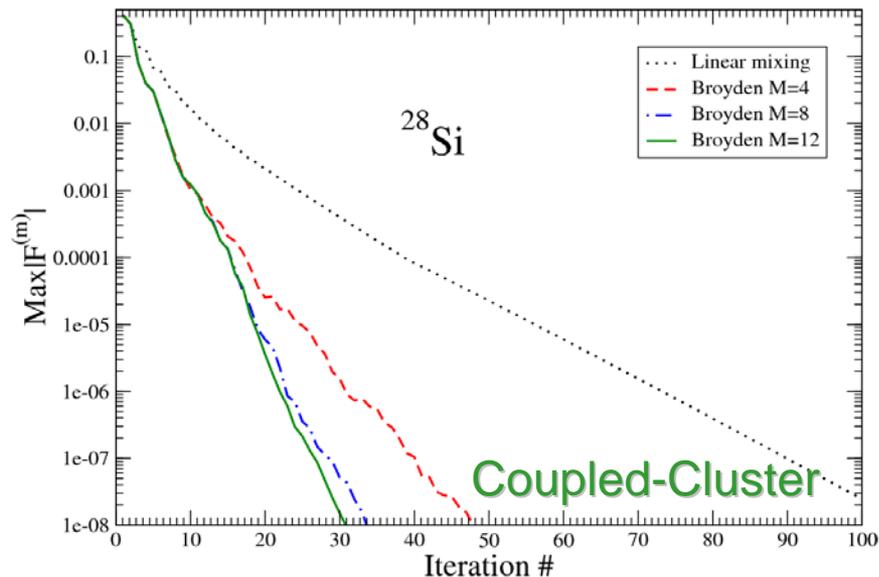
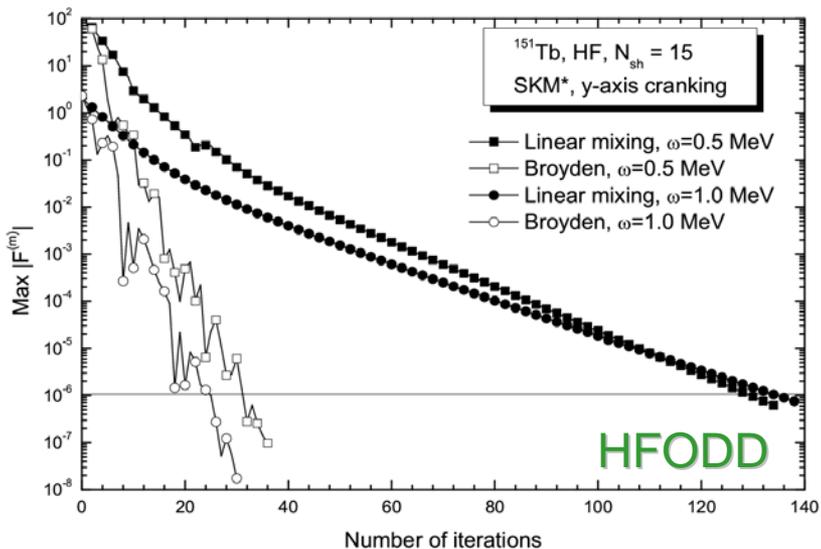
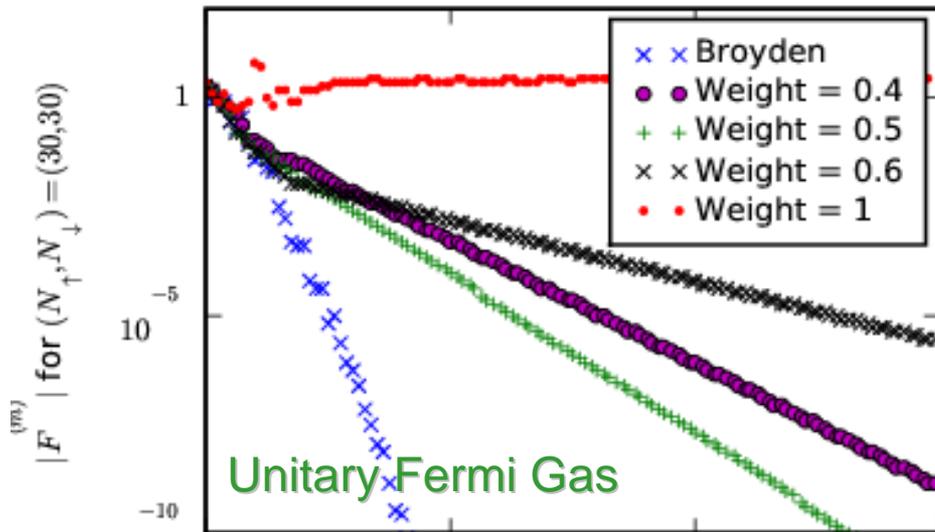
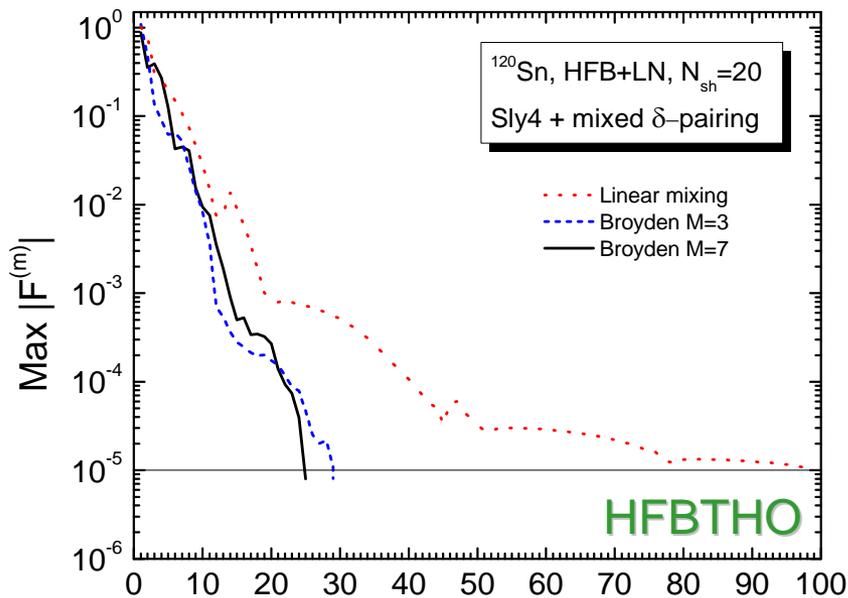
Solution of the HFB equations

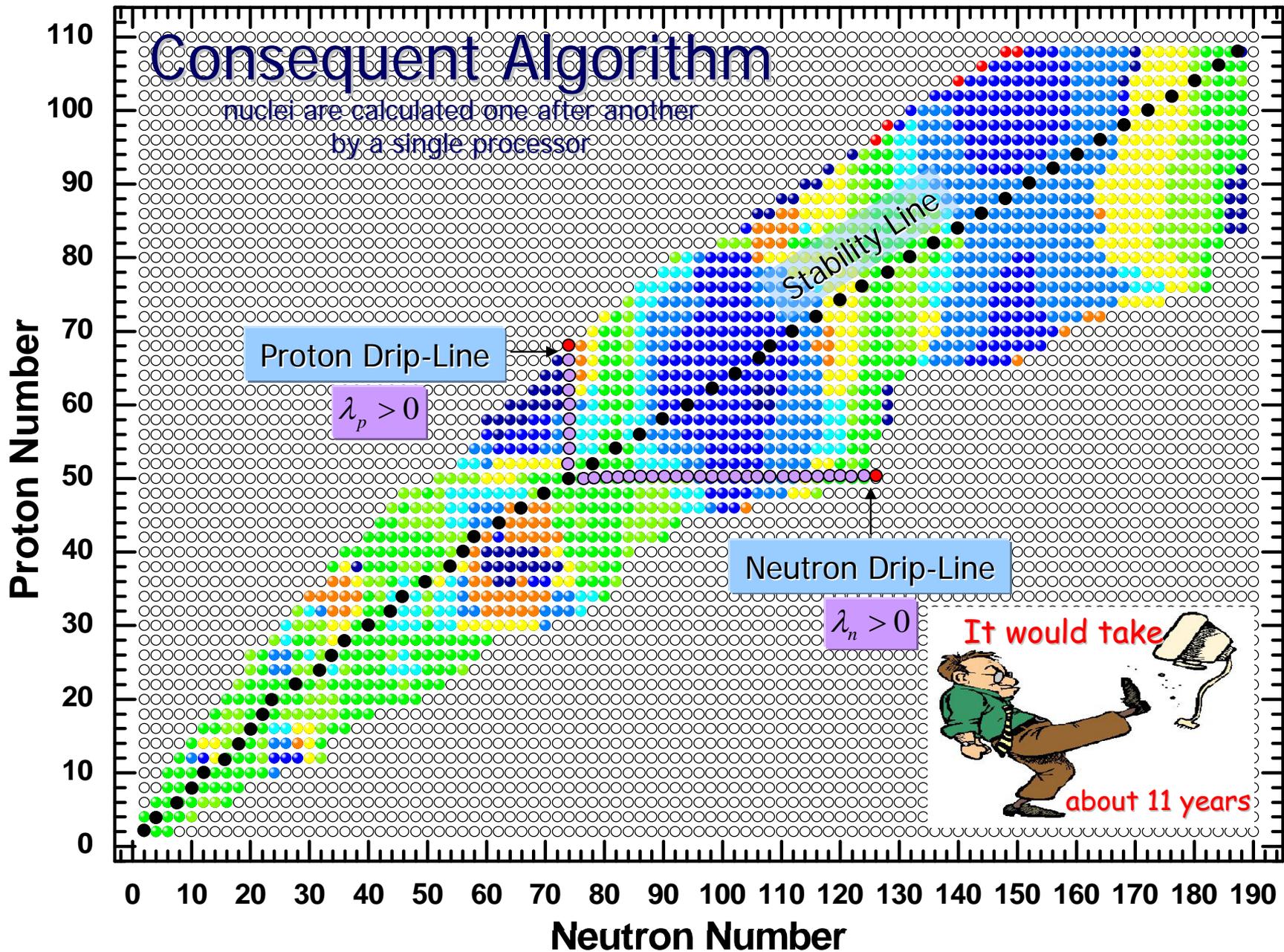


Linear mixing:  $V^{(m+1)} = V^{(m)} + \alpha F^{(m)}$

Broyden mixing:  $V^{(m+1)} = V^{(m)} - (J^{(m)})^{-1} F^{(m)}$

# Broyden Mixing





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## TOP500 List - June 2008 (1-100)

$R_{max}$  and  $R_{peak}$  values are in TFlops. For more details about other fields, check the [TOP500 description](#).

Power data in KW for entire system

[next](#)

Rank	Site	Computer/Year Vendor	Cores	$R_{max}$	$R_{peak}$	Power
1	<a href="#">DOE/NNSA/LANL United States</a>	Roadrunner - BladeCenter QS22/LS21 Cluster, PowerXCell 8i 3.2 Ghz / Opteron DC 1.8 GHz, Voltaire Infiniband / 2008 IBM	122400	1026.00	1375.78	2345.50
2	<a href="#">DOE/NNSA/LLNL United States</a>	BlueGene/L - eServer Blue Gene Solution / 2007 IBM	212992	478.20	596.38	2329.60
3	<a href="#">Argonne National Laboratory United States</a>	Blue Gene/P Solution / 2007 IBM	163840	450.30	557.06	1260.00
4	<a href="#">Texas Advanced Computing Center/Univ. of Texas United States</a>	Ranger - SunBlade x6420, Opteron Quad 2Ghz, Infiniband / 2008 Sun Microsystems	62976	326.00	503.81	2000.00
5	<a href="#">DOE/Oak Ridge National Laboratory United States</a>	Jaguar - Cray XT4 QuadCore 2.1 GHz / 2008 Cray Inc.	30976	205.00	260.20	1580.71
6	<a href="#">Forschungszentrum Juelich (FZJ) Germany</a>	JUGENE - Blue Gene/P Solution / 2007 IBM	65536	180.00	222.82	504.00
7	<a href="#">New Mexico Computing Applications Center (NMCAC) United States</a>	Encanto - SGI Altix ICE 8200, Xeon quad core 3.0 GHz / 2007 SGI	14336	133.20	172.03	861.63
8	<a href="#">Computational Research Laboratories, TATA SONS India</a>	EKA - Cluster Platform 3000 BL460c, Xeon 53xx 3GHz, Infiniband / 2008 Hewlett-Packard	14384	132.80	172.61	786.00
9	<a href="#">IDRIS France</a>	Blue Gene/P Solution / 2008 IBM	40960	112.50	139.26	315.00
10	<a href="#">Total Exploration Production France</a>	SGI Altix ICE 8200EX, Xeon quad core 3.0 GHz / 2008 SGI	10240	106.10	122.88	442.00

# Large-Scale Mass Table Calculations

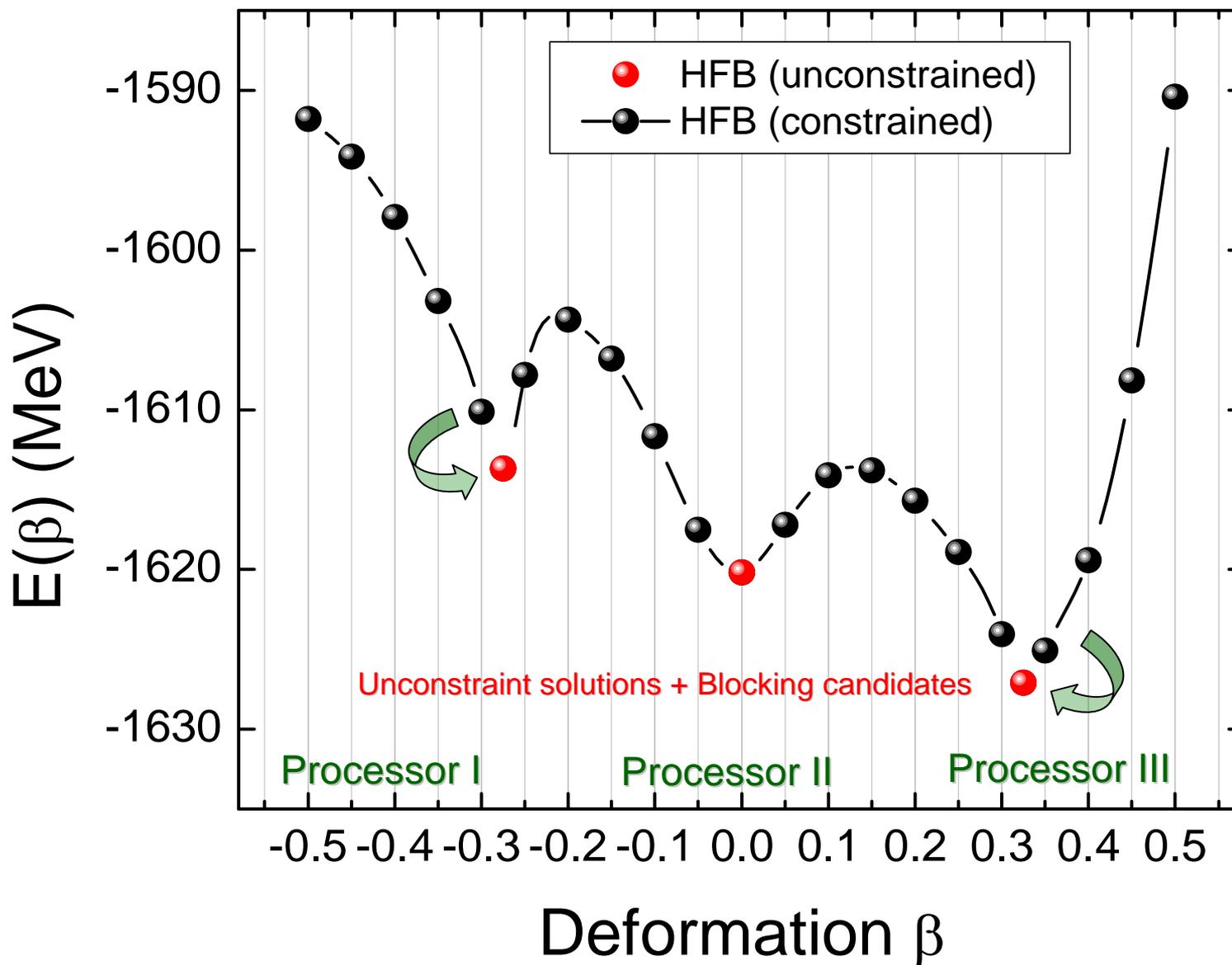


Jaguar is currently rated the 5th most powerful computer in the world with 54 teraflops and 7,832 AMD Barcelona quad-core Opteron processors.



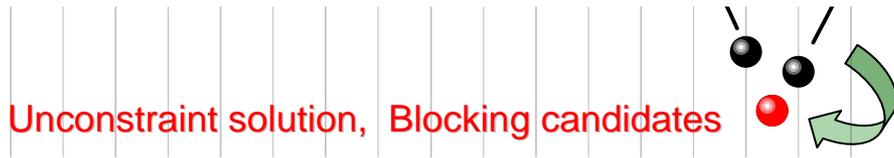
# Large-Scale Calculations

one even-even nucleus

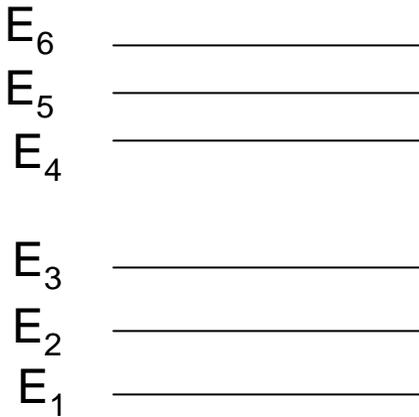


# Large-Scale Calculations

Parent even-even nucleus (N,Z)

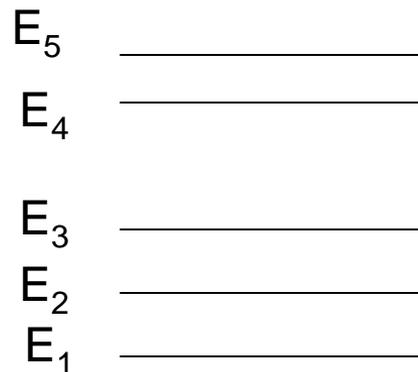


neutrons N



$E_{qp}(\min) > 0$

protons Z



$E_{qp}(\min) > 0$

Self-consistent calculations  
for odd nuclei with blocking

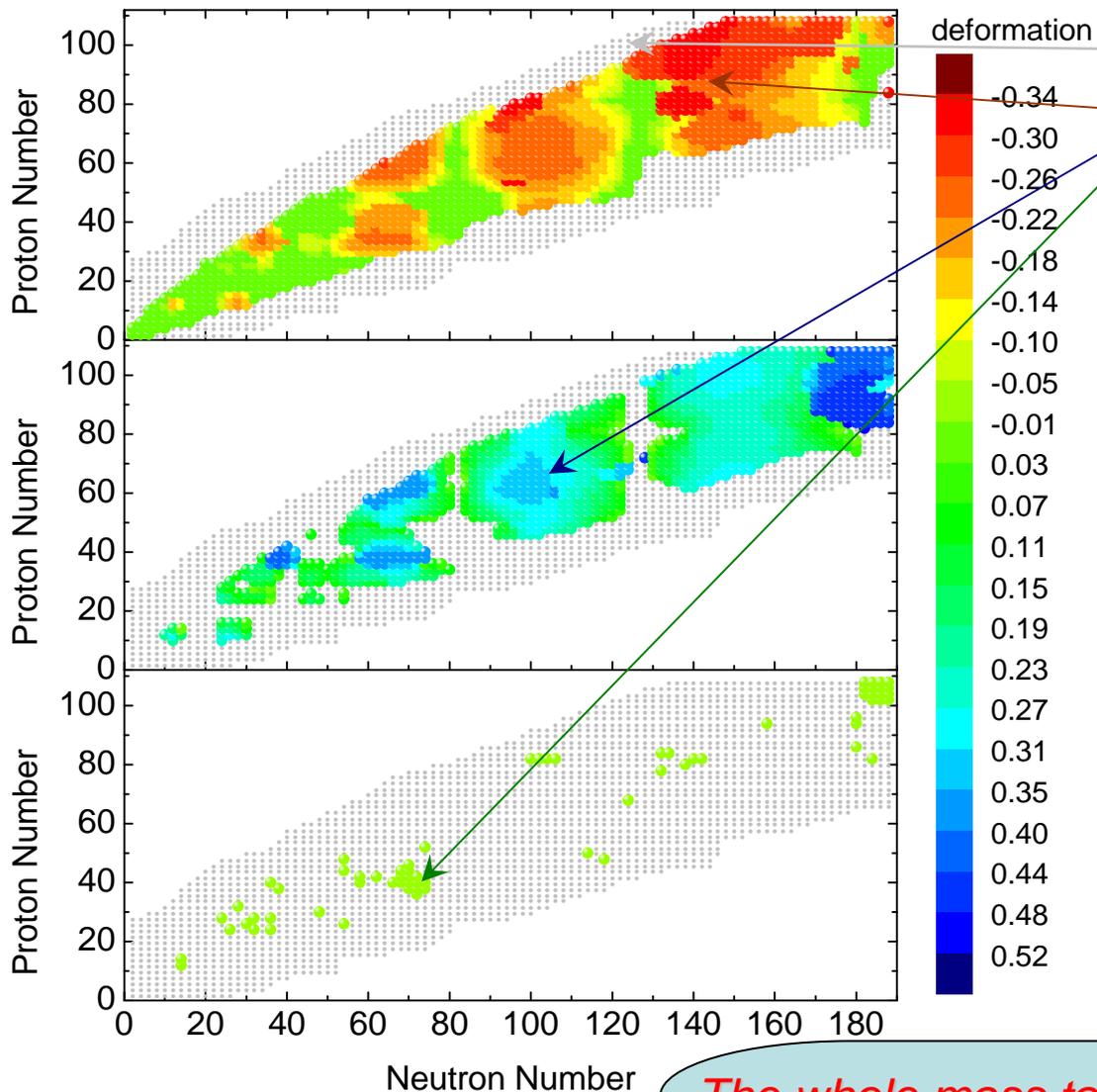
odd-even nucleus (N+1,Z)  
6 configurations

odd-even nucleus (N,Z+1)  
5 configurations

odd-odd nucleus (N+1,Z+1)  
6x5=30 configurations

41 processors

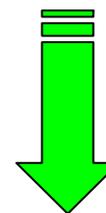
# Large-Scale Mass Table Calculations



9525 even-even states

2731 even-even bound states

2 CPU hours  
using 4000 processors



800754 odd-even and odd-odd  
states within 2 MeV  $E_{qp}$  threshold  
for the blocking candidates

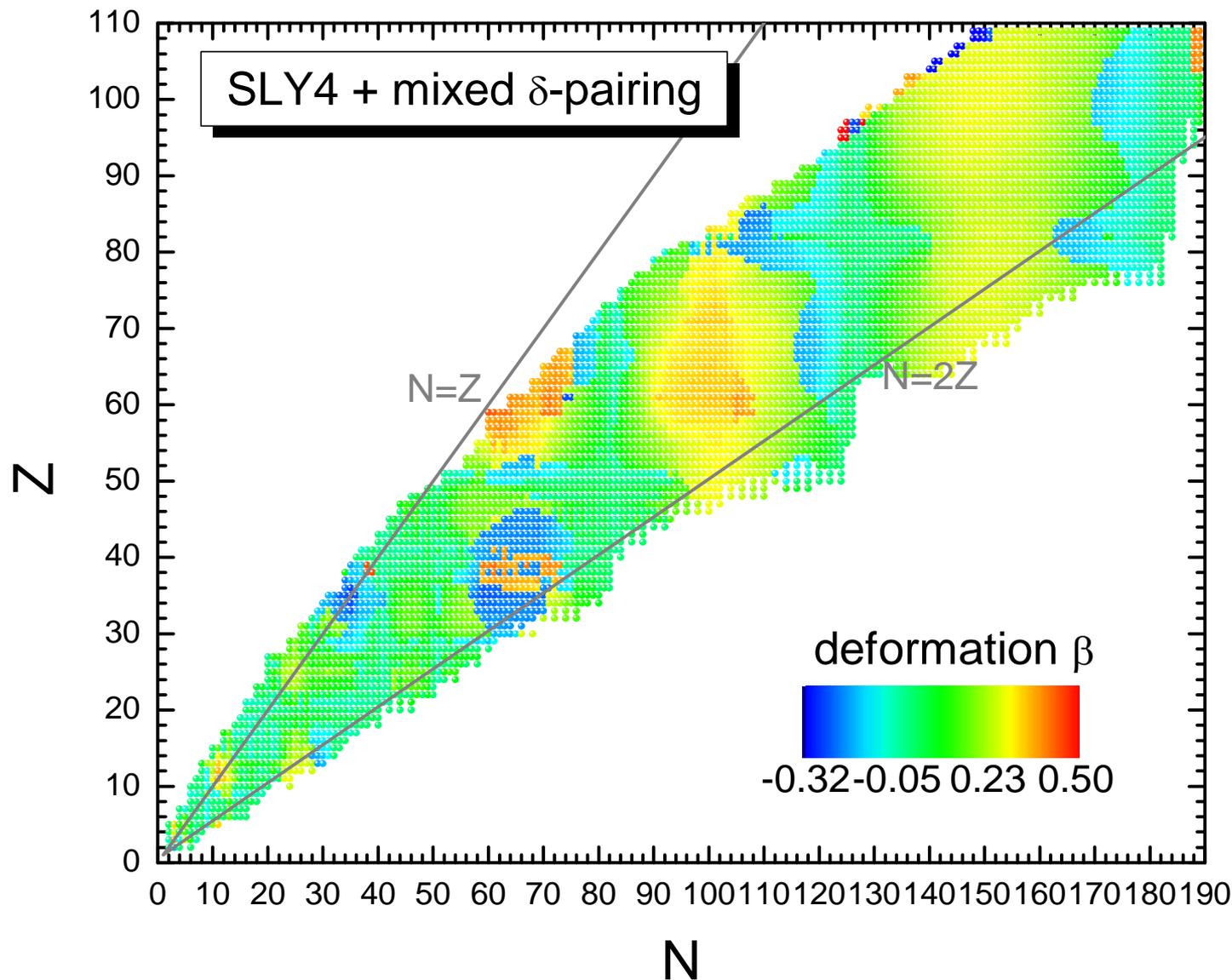
21 CPU hours  
using 4000 processors

After implementing Broyden's method

- all even-even nuclear states converge
- for odd nuclei: about 0.5%, usually high lying, configurations still diverge

*The whole mass table in a single 24 CPU hours run*

# Large-Scale Mass Table Calculations

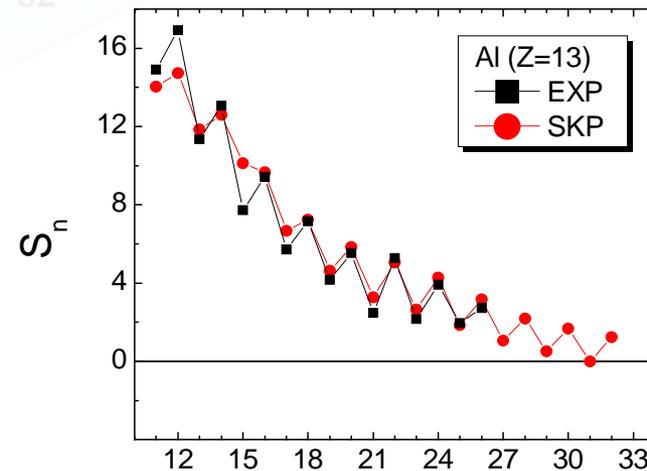
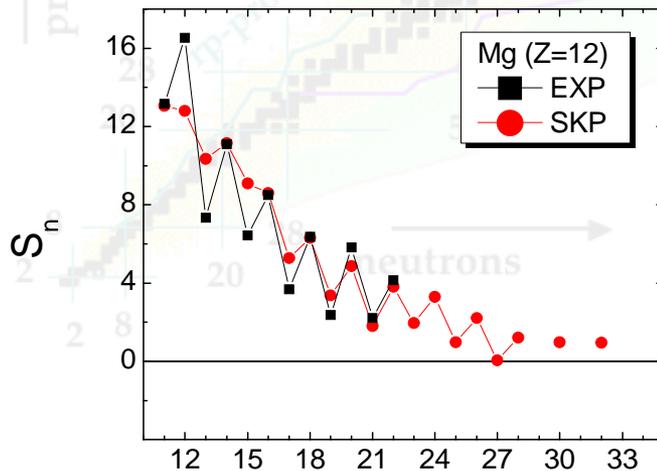


# Large-Scale Mass Table Calculations

- ✓ Complete mass-table calculations with existing (standard) energy density functionals including all even-even, odd-even and odd-odd nuclei

Data already used to compare with experiment

$$\delta V_{pn} , \Delta_0^{(3)}, {}^{141}\text{Ho}, {}^{40}\text{Mg}$$



## News

Recent MTeX 4.1beta release can be downloaded [here](#)

Additional data files available for Mass Tables calculated with different functionals can be downloaded [here](#)

ScreenShots from MTeX can be seen [here](#)

## Journal iFrames

- [Nature](#)
- [Phys Rev Lett](#)

## Mass Table explorer

*... science scales with processors ...*

Mass Table Explorer is a java application aimed to facilitate the visualization of the huge array of data coming from modern multiprocessors computers helping to understand challenging phenomena seen across the nuclear mass chart.

## Mass Tables

- [ADMC-2003](#)
- [Map of the Nuclides](#)
- [D1S Gogny](#)
- [Droplet Model](#)
- [HFB-14 BRUSLIB](#)

## Java Tools

- [Janis](#)
- [NucAstroData](#)

## Off-line Version

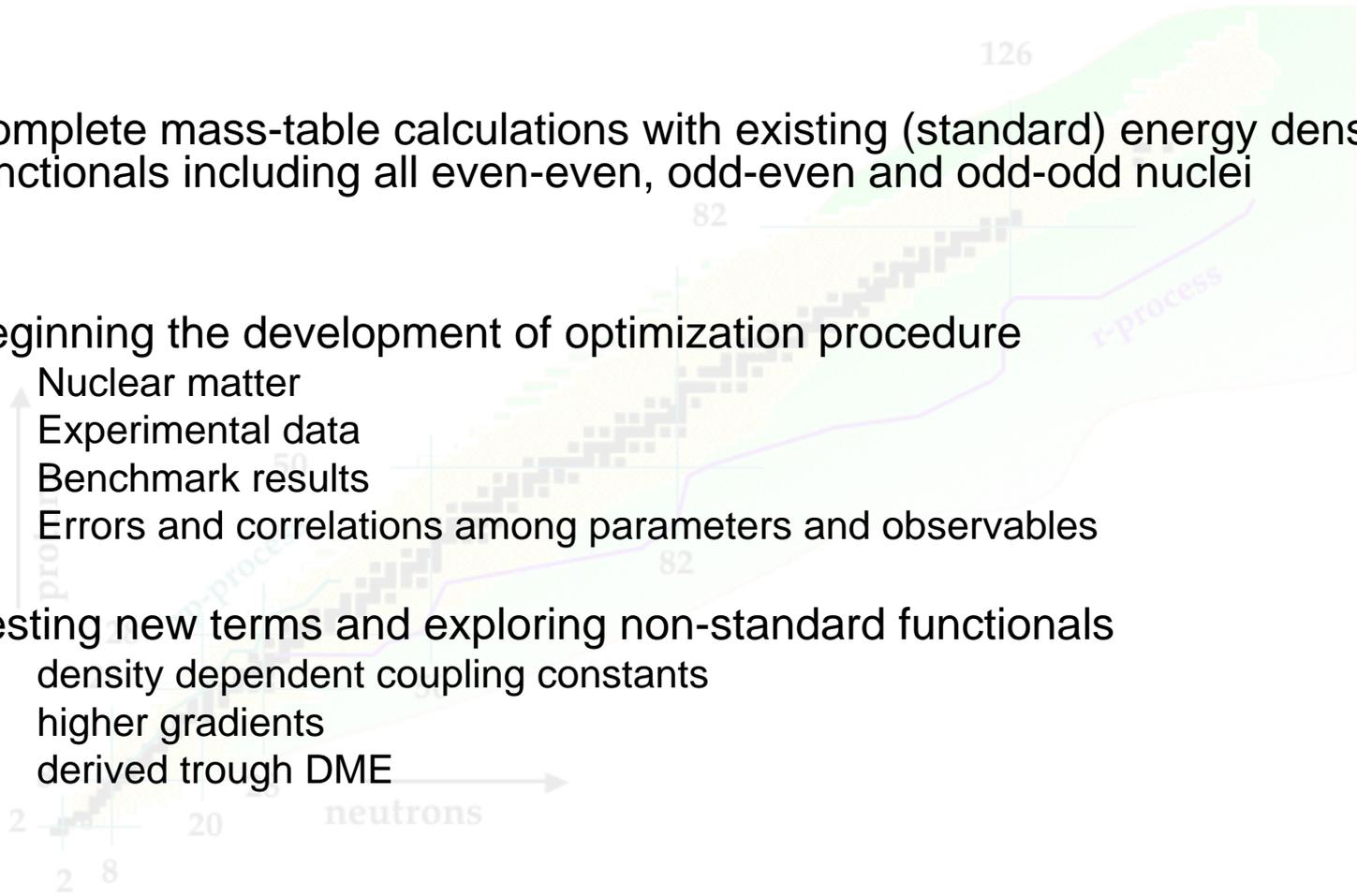
- Written in Java
- Can work on any computer
- Large sets of visualization tools
- One can compare with his own data
- Save data and images in many formats

## On-line Version

- Use Python, Gnuplot, PHP, Ajax
- Visualization right from the browser
  - one can apply mass filters
  - plot 2D and 3D charts
  - compare nuclear characteristics
  - rms errors with exp. data

# Large-Scale Mass Table Calculations

- ✓ Complete mass-table calculations with existing (standard) energy density functionals including all even-even, odd-even and odd-odd nuclei
- ☐ Beginning the development of optimization procedure
  - Nuclear matter
  - Experimental data
  - Benchmark results
  - Errors and correlations among parameters and observables
- ☐ Testing new terms and exploring non-standard functionals
  - density dependent coupling constants
  - higher gradients
  - derived trough DME



# Optimization Procedure

## standard functional with 13 ph-parameters

$$\begin{aligned}
 \mathcal{H}(\mathbf{r}) = & \frac{\hbar^2}{2m} \tau \\
 & + \frac{1}{2} t_0 \left( \left( \frac{x_0}{2} + 1 \right) \rho^2 - \left( x_0 + \frac{1}{2} \right) \sum_q \rho_q^2 \right) \\
 & + \frac{1}{4} t_1 \left( \left( \frac{x_1}{2} + 1 \right) \rho \tau - \left( x_1 + \frac{1}{2} \right) \sum_q \rho_q \tau_q \right) \\
 & + \frac{1}{4} t_2 \left( \left( \frac{x_2}{2} + 1 \right) \rho \tau + \left( x_2 + \frac{1}{2} \right) \sum_q \rho_q \tau_q \right) \\
 & - \frac{3}{16} t_1 \left( \left( \frac{x_1}{2} + 1 \right) \rho \Delta \rho + \left( x_1 + \frac{1}{2} \right) \sum_q \rho_q \Delta \rho_q \right) \\
 & + \frac{1}{16} t_2 \left( \left( \frac{x_2}{2} + 1 \right) \rho \Delta \rho + \left( x_2 + \frac{1}{2} \right) \sum_q \rho_q \Delta \rho_q \right) \\
 & + \frac{1}{12} t_3 \left( \left( \frac{x_3}{2} + 1 \right) \rho^2 - \left( x_3 + \frac{1}{2} \right) \sum_q \rho_q^2 \right) \rho^\gamma \\
 & - \frac{1}{8} (t_1 x_1 + t_2 x_2 - 5(to + te)) \sum_q \mathbf{J}_q^2 \\
 & - \frac{1}{16} (t_1 (x_1 - 1) + t_2 (x_2 + 1) - 10to) \mathbf{J}_{n..} \mathbf{J}_p \\
 & - (b_4 \rho \nabla \cdot \mathbf{J} + b'_4 \sum_q \rho_q \nabla \cdot \mathbf{J}_q),
 \end{aligned}$$

t-parameters

{t<sub>0</sub>, t<sub>1</sub>, t<sub>2</sub>, t<sub>3</sub>, x<sub>0</sub>, x<sub>1</sub>, x<sub>2</sub>, x<sub>3</sub>, to, te, b<sub>4</sub>, b'<sub>4</sub>, γ}

{C<sub>t0</sub><sup>ρ</sup>, C<sub>tD</sub><sup>ρ</sup>, C<sub>t</sub><sup>Δρ</sup>, C<sub>t</sub><sup>τ</sup>, C<sub>t</sub><sup>J</sup>, C<sub>t</sub><sup>∇J</sup>, γ}

C-parameters

$$\mathcal{H}(\mathbf{r}) = \frac{\hbar^2}{2m} \tau_0 + \mathcal{H}_0(\mathbf{r}) + \mathcal{H}_1(\mathbf{r}),$$

$$\begin{aligned}
 \mathcal{H}_t(\mathbf{r}) = & C_t^\rho \rho_t^2 + C_t^{\Delta\rho} \rho_t \Delta \rho_t + C_t^\tau \rho_t \tau_t \\
 & + \frac{1}{2} C_t^J \mathbf{J}_t^2 + C_t^{\nabla J} \rho_t \nabla \cdot \mathbf{J}_t,
 \end{aligned}$$

$$C_t^\rho = C_{t0}^\rho + C_{tD}^\rho \rho_0^\gamma.$$

$$C_{00}^\rho = \frac{3}{8} t_0, \quad C_{10}^\rho = -\frac{1}{4} t_0 \left( x_0 + \frac{1}{2} \right),$$

$$C_{0D}^\rho = \frac{1}{16} t_3, \quad C_{1D}^\rho = -\frac{1}{24} t_3 \left( x_3 + \frac{1}{2} \right),$$

$$C_0^{\Delta\rho} = \frac{1}{16} t_2 \left( x_2 + \frac{5}{4} \right) - \frac{9}{64} t_1,$$

$$C_1^{\Delta\rho} = \frac{3}{32} t_1 \left( x_1 + \frac{1}{2} \right) + \frac{1}{32} t_2 \left( x_2 + \frac{1}{2} \right)$$

$$C_0^\tau = \frac{3}{16} t_1 + \frac{1}{4} t_2 \left( x_2 + \frac{5}{4} \right),$$

$$C_1^\tau = -\frac{1}{8} t_1 \left( x_1 + \frac{1}{2} \right) + \frac{1}{8} t_2 \left( x_2 + \frac{1}{2} \right),$$

$$C_0^J = -\frac{1}{8} (t_1 (x_1 - \frac{1}{2}) + t_2 (x_2 + \frac{1}{2})) + \frac{5}{16} (3 to + te),$$

$$C_1^J = \frac{1}{16} (t_1 - t_2) + \frac{5}{16} (to - te),$$

$$C_0^{\nabla J} = -b_4 - \frac{1}{2} b'_4, \quad C_1^{\nabla J} = -\frac{1}{2} b'_4,$$

# Optimization Procedure

## t\*x problem

$$t_0 = \frac{8}{3}C_{00}^\rho, \quad t_0x_0 = -\frac{3}{4}(C_{00}^\rho + 3C_{10}^\rho),$$

$$t_1 = -\frac{4}{3}(4C_0^{\Delta\rho} - C_0^\tau),$$

$$x_1 = -\frac{2}{3t_1}(-4C_0^{\Delta\rho} - 12C_1^{\Delta\rho} + C_0^\tau + 3C_1^\tau)$$

$$t_2 = \frac{4}{3}(4C_0^{\Delta\rho} - 8C_1^{\Delta\rho} + 3C_0^\tau - 6C_1^\tau),$$

$$x_2 = \frac{2}{3t_2}(-4C_0^{\Delta\rho} + 20C_1^{\Delta\rho} - 3C_0^\tau + 15C_1^\tau)$$

$$t_3 = 16C_{0d}^\rho, \quad x_3 = -\frac{8}{t_3}(C_{0d}^\rho + 3C_{1d}^\rho),$$

$$t_0 = \frac{4}{15}(3C_0^J + 3C_1^J + 4C_0^{\Delta\rho} + 4C_1^{\Delta\rho}),$$

$$t_e = \frac{4}{15}(3C_0^J - 9C_1^J - 4C_0^{\Delta\rho} + 12C_1^{\Delta\rho} - 2$$

$$b_4 = -C_0^{\nabla J} + C_1^{\nabla J}, \quad b'_4 = -2C_1^{\nabla J},$$

TABLE I: The lower ( $v_0$ ) and upper ( $v_1$ ) limits, maximum displacement ( $d$ ) and initial values ( $v_{in}$ ) for the Skyrme parameters used to minimize the  $\chi^2$  value within the SAM.

	$v_0$	$v_1$	$d$	$v_{in}$
$t_0(\text{MeV}\cdot\text{fm}^3)$	-3000.0	-1500.0	50.0	-1603.0
$t_1(\text{MeV}\cdot\text{fm}^5)$	-500.0	500.0	20.0	515.9
$t_2(\text{MeV}\cdot\text{fm}^5)$	-500.0	500.0	20.0	84.5
$t_{31}(\text{MeV}\cdot\text{fm}^{3(\alpha_1+1)})$	1000.0	3000.0	50.0	1333.3
$t_{32}(\text{MeV}\cdot\text{fm}^{3(\alpha_2+1)})$	-1000	0.0	50.0	0.0
$t_{33}(\text{MeV}\cdot\text{fm}^{3(\alpha_3+1)})$	-500.0	500.0	20.0	0.0
$x_0$	-4.0	4.0	0.1	-0.02
$x_1$	-4.0	4.0	0.1	-0.5
$x_2$	-4.0	4.0	0.1	-1.713

TABLE III: The values of the Skyrme parameters for GSkI, GSkII and SSk interactions obtained by minimizing the  $\chi^2$ .

	GSkI	GSkII	SSk
$t_0(\text{MeV}\cdot\text{fm}^3)$	-1855.45	-1855.99	-2523.52
$t_1(\text{MeV}\cdot\text{fm}^5)$	397.23	393.08	435.00
$t_2(\text{MeV}\cdot\text{fm}^5)$	264.63	266.08	-382.04
$t_{31}(\text{MeV}\cdot\text{fm}^{3(\alpha_1+1)})$	2309.67	2307.15	2372.49
$t_{32}(\text{MeV}\cdot\text{fm}^{3(\alpha_2+1)})$	-449.01	-448.28	---
$t_{33}(\text{MeV}\cdot\text{fm}^{3(\alpha_3+1)})$	-53.31	---	---
$x_0$	0.1180	0.0909	0.6835
$x_1$	-1.7586	-0.7203	-0.4519
$x_2$	-1.8068	-1.8369	-0.9214

# Optimization Procedure

## nuclear matter hints

$$\frac{E^{NM}}{A} \equiv W(\rho_c) \approx -16 \text{ MeV},$$

$$P^{NM} \equiv \rho^2 \frac{dW(\rho)}{d\rho} \Big|_{\rho=\rho_c} = 0, \quad \rho_c \approx 0.16 \text{ fm}^{-3}$$

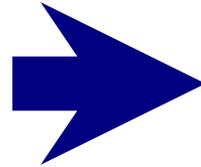
$$K^{NM} = 9\rho^2 \frac{d^2W(\rho)}{d\rho^2} \Big|_{\rho=\rho_c} \approx 220 \text{ MeV},$$

$$M_s^{*NM} = \frac{2m}{\hbar^2} \frac{dW}{d\tau} \Big|_{\rho=\rho_c} \approx 1,$$

$$a_{sym}^{NM} \equiv \frac{1}{2} \frac{d^2W(I, \rho)}{dI^2} \Big|_{\substack{\rho=\rho_c \\ I=0}} = 32.5 \text{ MeV},$$

$$L^{NM} \equiv 3\rho_c \frac{da_{sym}^{NM}}{d\rho_c} \approx 50 \text{ MeV},$$

$$M_v^{*NM} \approx 1,$$



Nuclear matter  
(7 parameters)

$$\gamma = \frac{-K^{NM} + \frac{\hbar^2}{2m} (4M_s^{*NM} - 3) \tau_c - 9 \frac{E^{NM}}{A}}{\frac{\hbar^2}{2m} (6M_s^{*NM} - 9) \tau_c + 9 \frac{E^{NM}}{A}},$$

$$C_{00}^\rho = \frac{\frac{\hbar^2}{2m} ((2 - 3\gamma) M_s^{*NM} - 3) \tau_c + 3(1 + \gamma) \frac{E^{NM}}{A}}{3\gamma\rho_c},$$

$$C_{0D}^\rho = \frac{\frac{\hbar^2}{2m} (3 - 2M_s^{*NM}) \tau_c - 3 \frac{E^{NM}}{A}}{3\gamma\rho_c^{1+\gamma}},$$

$$C_0^\tau = \frac{\hbar^2}{2m} (M_s^{*NM} - 1) \frac{1}{\rho_c}.$$

$$C_1^\tau = C_0^\tau - \frac{\hbar^2}{2m} (M_v^{*NM} - 1) \frac{1}{\rho_c}$$

$$= \frac{\hbar^2}{2m} (M_s^{*NM} - M_v^{*NM}) \frac{1}{\rho_c},$$

$$C_{10}^\rho = \frac{1}{27\gamma\rho_c} \left[ 27(1 + \gamma) a_{sym}^{NM} C - 9L^{NM} \right.$$

$$\left. - 5 \left( (1 + 3\gamma) \frac{\hbar^2}{2m} - (2 - 3\gamma) (C_0^\tau + 3C_1^\tau) \rho_c \right) \tau_c \right],$$

$$C_{1D}^\rho = \frac{1}{27\gamma\rho_c^{\gamma+1}} \left[ -27a_{sym}^{NM} C + 9L^{NM} \right.$$

$$\left. + 5 \left( \frac{\hbar^2}{2m} - 2\rho_c (C_0^\tau + 3C_1^\tau) \right) \tau_c \right],$$

# Non-standard functionals

## density dependent coupling constants

### I. Density dependence of all the coupling constants

For the time-reversal and spherical symmetries imposed, the extended EDF reads

$$\mathcal{H}_t(r) = C_t^\rho \rho_t^2 + C_t^\tau \rho_t \tau_t + C_t^{\Delta\rho} \rho_t \Delta\rho_t + \frac{1}{2} C_t^J J_t^2 + C_t^{\nabla J} \rho_t \nabla \cdot J_t \\ + C_t^{\nabla\rho} (\nabla\rho_t)^2 + C_t^{\nabla\rho'} (\nabla\rho_t) \cdot J_t$$

and depends linearly on 38 coupling constants,

$$C_t^\rho, C_t^\tau, C_t^{\Delta\rho}, C_t^J, \text{ and } C_t^{\nabla J},$$

$$\alpha_t^\rho, \alpha_t^\tau, \alpha_t^{\Delta\rho}, \alpha_t^J, \alpha_t^{\nabla J}, \alpha_t^{\nabla\rho}, \text{ and } \alpha_t^{\nabla\rho'},$$

$$\beta_t^\rho, \beta_t^\tau, \beta_t^{\Delta\rho}, \beta_t^J, \beta_t^{\nabla J}, \beta_t^{\nabla\rho}, \text{ and } \beta_t^{\nabla\rho'},$$

for  $t = 0$  and  $1$ , i.e.,

$$C_t^m(\rho_0, \rho_1) = C_t^m \left[ 1 + \alpha_t^m \left( 1 - \left( \frac{\rho_0}{\rho_{\text{sat}}} \right)^{\gamma_t^m} \right) + \beta_t^m \left( \left( \frac{\rho_1}{\rho_{\text{sat}}} \right)^2 \right)^{\eta_t^m} \right]$$

and on 28 powers  $\gamma_t^m$  and  $\eta_t^m$ .

# Large-Scale Mass Table Calculations

- ✓ Complete mass-table calculations with existing (standard) energy density functionals including all even-even, odd-even and odd-odd nuclei
- ☐ Beginning the development of optimization procedure
  - Nuclear matter
  - Experimental data
  - Benchmark results
  - Errors and correlations among parameters and observables
- ☐ Testing new terms and exploring non-standard functionals
  - density dependent coupling constants
  - higher gradients
  - derived trough DME

