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HFB mass models and the EOS of neutron-star crusts

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3. Outer crust of neutron stars
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5. Conclusions

I. Introduction

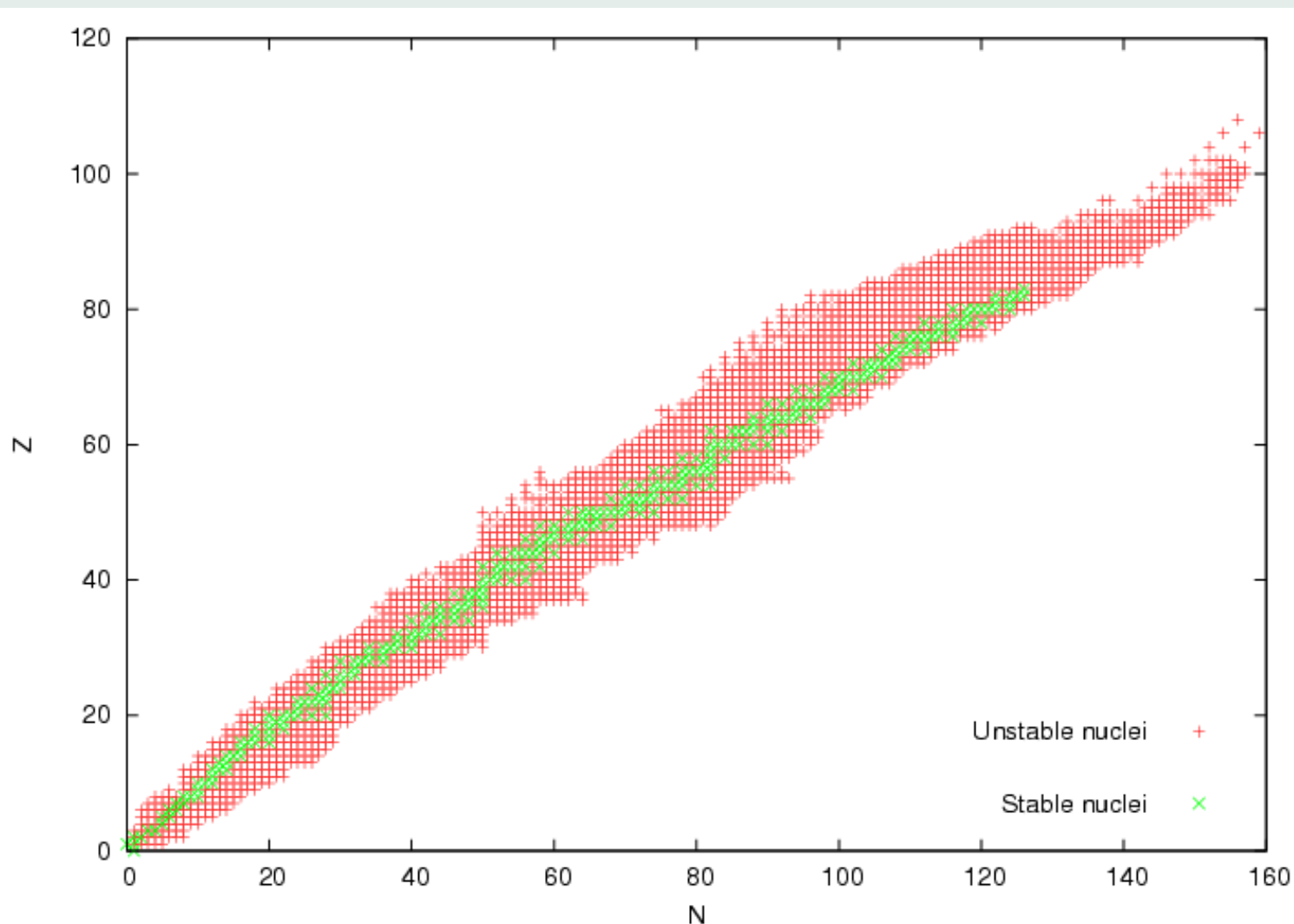


Fig. 1. Nuclei with measured masses according to 2003 AME. NOT shown: nuclei for which quoted mass is an estimate based on local systematics (indicated by # in table).

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Limits on N and Z of nuclei that can exist?

As neutrons added with Z fixed neutron separation energy S_n decreases,

$$S_n = M_{at}(N - 1, Z) - M_{at}(N, Z) + M_n$$

When $S_n = 0$ impossible to add any more neutrons.

Neutron drip line:

Z fixed, add neutrons, first nucleus with $S_n = 0$.

Likewise, when protons added with N fixed proton separation energy S_p decreases

$$S_p = M_{at}(N, Z - 1) - M_{at}(N, Z) + M_H$$

When $S_p = 0$ impossible to add any more protons.

Proton drip line:

N fixed, add protons, first nucleus with $S_p = 0$.

(Note that because of pairing isolated particle-stable nuclei might be found beyond drip lines.)

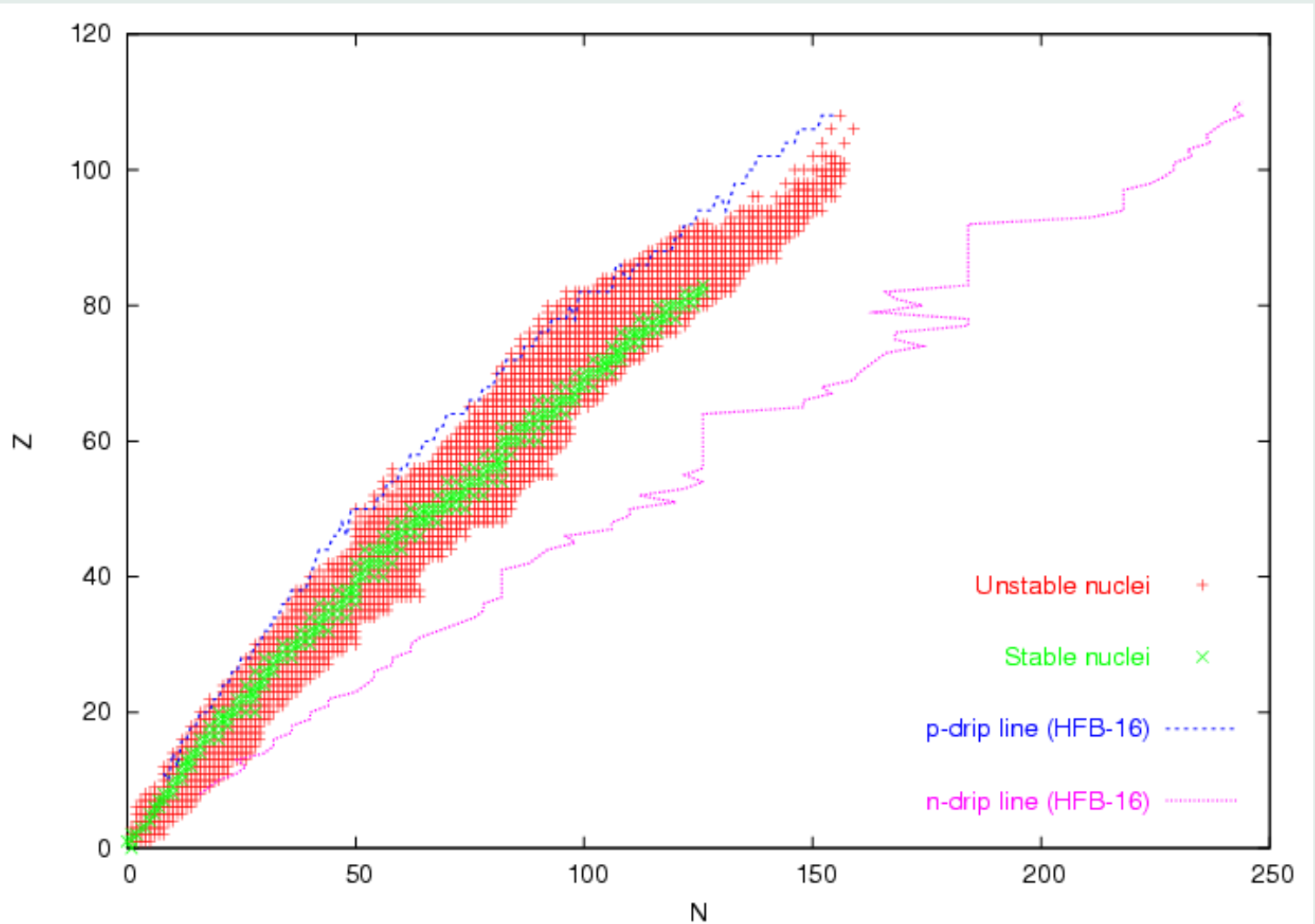


Fig. 2. HFB-16 is most recent Hartree-Fock Bogoliubov mass model.

Thousands of nuclei on neutron-rich side remain to be measured

Serious problem for **astrophysics**

- i) r-process of nucleosynthesis
- ii) EOS of neutron-star crusts

All relevant properties will have to be calculated, one way or another

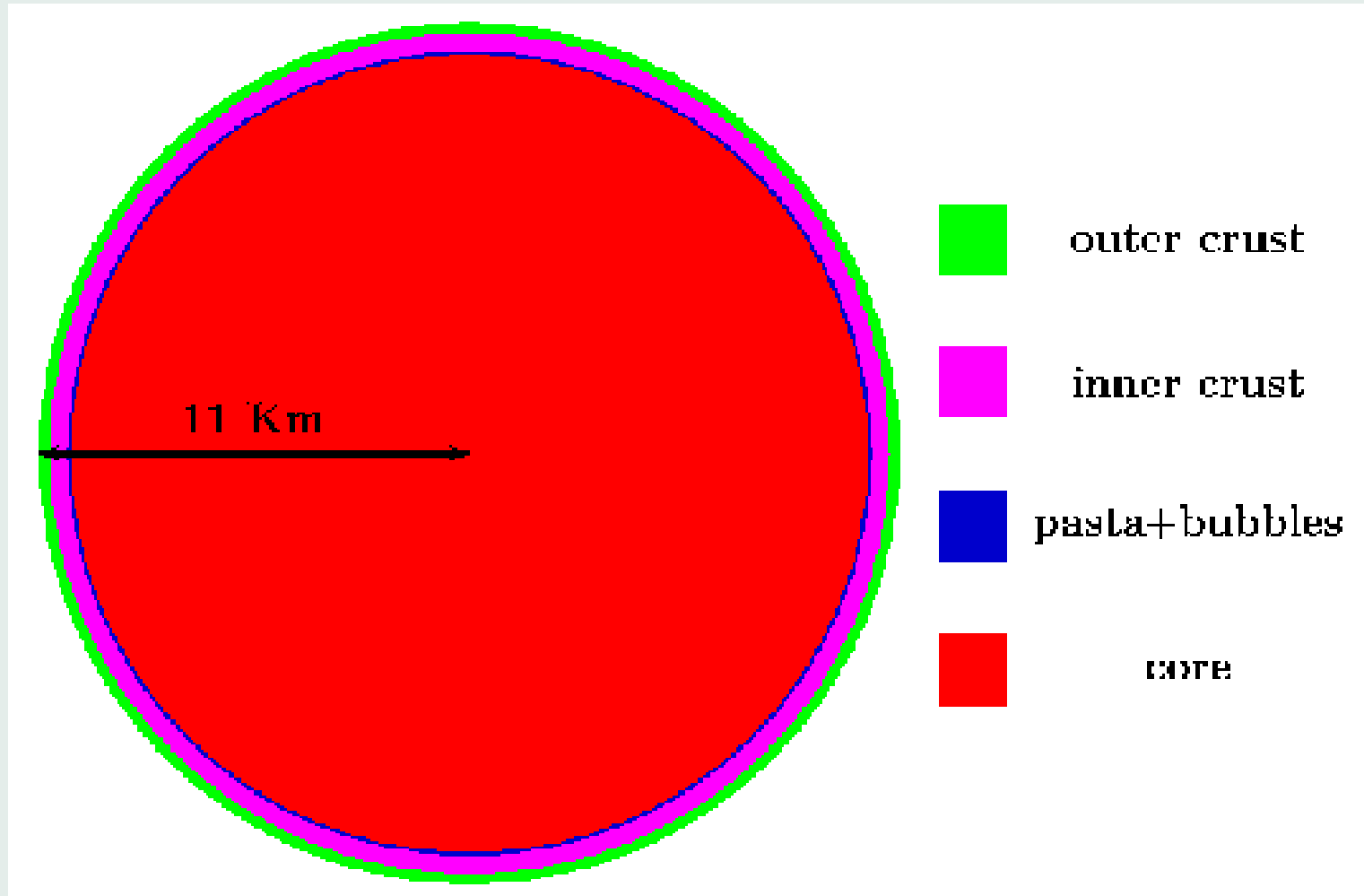


Fig. 3. Neutron star.

Outer crust. $\simeq 300$ meters thick. $0 < \rho < 1.2 \times 10^{-3} \rho_0$
 – n-rich nuclei (+ electrons); within neutron drip line.

Inner crust. $\simeq 500$ meters thick. $1.2 \times 10^{-3} \rho_0 < \rho < 0.4 \rho_0$
 – nuclear clusters (+ electrons) floating in neutron vapour;
 beyond neutron drip line.

Core. 10 km radius (roughly). ρ up to about $4 \rho_0$
 – homogeneous gas of n and p (+ electrons).
 About 97 % n at ρ around ρ_0 ; other particles
 towards centre.

Electrical neutrality everywhere assured by electron gas: beta-equilibrated with nucleons.

Outer crust

Dealing with isolated nuclei; quantity of great importance is **mass**

- use of **mass models** to extrapolate from known masses.

Inner crust

No longer dealing with isolated nuclei; **EOS** rather than masses is relevant quantity

- but mass models can be generalized to extrapolate from known masses to beyond the drip line.

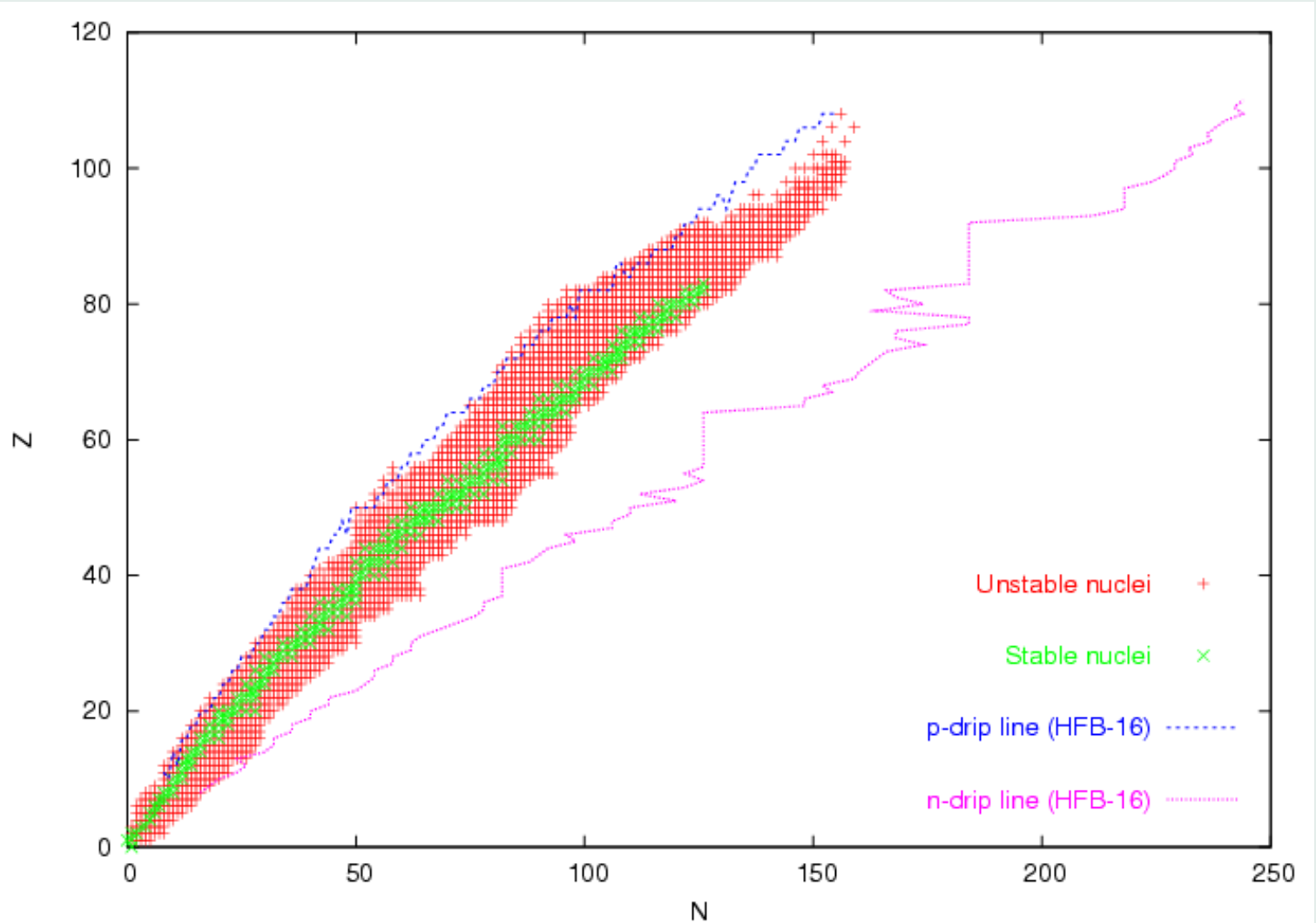


Fig. 4.

Liquid-drop(let) mass models

- all derived from Weizsäcker model of 1935
- parameters fitted to masses
- extensively refined since 1935
- latest form is FRDM - finite-range droplet model

macroscopic-microscopic approach -
shell (including deformation) and pairing
corrections grafted on to liquid-drop picture.

An intermediate form has been generalized to EOS:
“compressible liquid-drop model” (Lattimer and Swesty).

2. Skyrme-Hartree-Fock-Bogoliubov mass models

- Shell-model and pairing effects incorporated on same footing as macroscopic effects in a completely self-consistent manner.
- Based on *effective* forces: free parameters that are fitted not to two- and three-nucleon data but to mass data themselves.

– semi-empirical tradition of Weizsäcker

- A much more microscopic approach that allows for a closer conformity to reality, e.g., effective mass, neutron matter.
- More unified treatment of outer and inner crusts possible: shell effects easily incorporated into EOS of latter.

Skyrme force

10 parameters

$$\begin{aligned}v_{ij} = & t_0(1 + x_0 P_\sigma) \delta(\mathbf{r}_{ij}) \\ & + t_1(1 + x_1 P_\sigma) \frac{1}{2\hbar^2} \{p_{ij}^2 \delta(\mathbf{r}_{ij}) + h.c.\} \\ & + t_2(1 + x_2 P_\sigma) \frac{1}{\hbar^2} \mathbf{p}_{ij} \cdot \delta(\mathbf{r}_{ij}) \mathbf{p}_{ij} \\ & + \frac{1}{6} t_3(1 + x_3 P_\sigma) \rho^\alpha \delta(\mathbf{r}_{ij}) \\ & + \frac{i}{\hbar^2} W_0 (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \cdot \mathbf{p}_{ij} \times \delta(\mathbf{r}_{ij}) \mathbf{p}_{ij}\end{aligned}$$

Pairing force

$n - n$ and $p - p$ only $(T = T_z = 1)$

$$v_q^{\text{pair}}(\mathbf{r}_i, \mathbf{r}_j) = v^{\pi q} [\rho_q(\mathbf{r})] \delta(\mathbf{r}_{ij})$$

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Model not completely microscopic -

some phenomenological elements:

- **Phenomenological Wigner terms.**

If Skyrme and pairing forces are only ingredients then serious underbinding (≈ 2 MeV) for $N = Z$.

- **Coulomb exchange dropped.**

Compensation for neglected Coulomb correlations, vacuum polarization and charge-symmetry breaking.

- **Phenomenological correction for spurious collective motion.**

Form is determined by microscopic calculations, parameters fitted to fission barriers.

Not only do we require our mass models to give best possible fit to data, and to extrapolate masses as reliably as possible beyond the known region out to drip lines, but we also want to be able to extend them to the calculation of following properties of astrophysical importance:

i) EOS of inner crust of neutron stars.

ii) Fission barriers.

iii) Level densities.

iv) Beta-decay rates.

Unique effective force for all nuclear properties of astrophysical interest.

HFB-1 to HFB-16

(Force-parameter sets BSk1 to BSk16)

Imposed various constraints of physical reality:

pairing

effective mass

experimental barriers

neutron matter

Always fit not only mass data but also [charge-radii](#) data: predicted equilibrium density of symmetric INM (infinite nuclear matter) very sensitive to latter.

Rms and mean (expt. - model) deviations between data and predictions for HFB-8 and HFB-16; FRDM shown for convenience. The first pair of lines refers to all the 2149 measured masses M of nuclei with Z and $N \geq 8$ given in the 2003 AME, the second pair to the masses M_{nr} of the subset of 185 neutron-rich nuclei with $S_n \leq 5.0$ MeV, and the third pair to the 782 measured charge radii. Last line shows isoscalar effective mass in symmetric INM at equilibrium density.

	HFB-8	HFB-16	FRDM
$\sigma(M)$ [MeV]	0.635	0.632	0.656
$\bar{\epsilon}(M)$ [MeV]	0.009	-0.001	0.058
$\sigma(M_{nr})$ [MeV]	0.838	0.748	0.910
$\bar{\epsilon}(M_{nr})$ [MeV]	-0.025	0.161	0.047
$\sigma(R_c)$ [fm]	0.0275	0.0313	0.0545
$\bar{\epsilon}(R_c)$ [fm]	0.0025	-0.0149	-0.0366
M_s^*/M	0.8	0.8	-

Neutron matter

Beginning with model HFB-9 we fitted the model parameters not only to the mass data, but also require that Skyrme parameters reproduce energy-density curve of neutron matter, as calculated from realistic N-N and N-N-N forces.

This improves the reliability of the mass predictions for highly neutron-rich nuclei.

Moreover, such forces are well adapted to the calculation of the EOS of the inner crust of neutron stars with the HFB method (or approximations thereto): in addition to well representing the highly neutron-rich environment, the mass fit takes into account:

- i) presence of protons
- ii) inhomogeneities

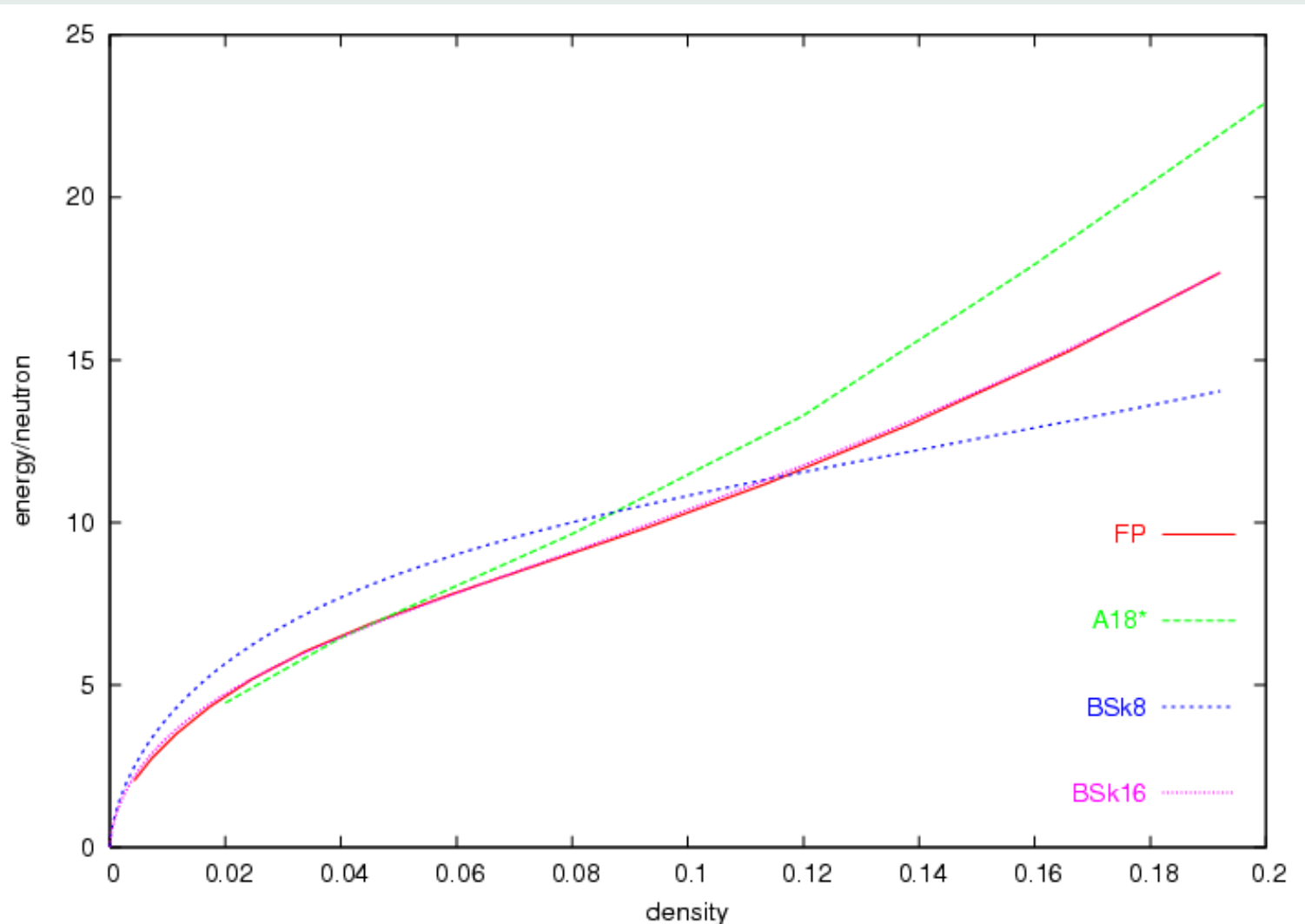


Fig. 5. FP: Friedman and Pandharipande (1981)

A18*: A18 + δv + UIX* of Akmal *et al* (1998).

If we try to constrain mass fit to A18* rms deviation increases dramatically

– seems impossible to get decent mass fits under this constraint with conventional Skyrme forces.

So which is the better neutron-matter calculation?

Calculated values of neutron-skin thickness θ_n in ^{208}Pb for different Skyrme forces.

BSk8:	$\theta_n = 0.12$ fm;	$a_{sym} = 28$ MeV
BSk16 (FP):	0.15 fm;	30 MeV
“A18*”:	0.19 fm;	32 MeV

(last force is best fit Skyrme to A18* neutron-matter curve)

Strong correlation between θ_n and a_{sym} as well.

Why this correlation between bulk properties, such as e'_n and a_{sym} , and purely surface quantity θ_n ?

Droplet model of Myers and Swiatecki

$$\theta_n = -\frac{2}{3} \sqrt{\frac{3}{5}} \frac{a_{ss}}{a_{sym}} r_0 I$$

where a_{ss} is “surface-symmetry” appearing in simple drop-model mass formula

$$e = a_v + a_{sf} A^{-1/3} + a_{sym} I^2 + a_{ss} I^2 A^{-1/3} + \text{Coulomb} + \dots$$

$$I = (N - Z)/A.$$

Mass data do not determine a_{sym} very well:

increase in a_{sym} can be compensated by a **decrease** in a_{SS}
– mass data do not yet go far enough away from stability
line.

**So measurement of θ_n will help to tie down a_{sym} ,
compensating thereby lack of mass data.**

Connection between θ_n and e'_n is more ambiguous

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HFB-16 is our best model so far.

Agrees closely with **FRDM** in the known region.

But what happens when we extrapolate
to the neutron drip line?

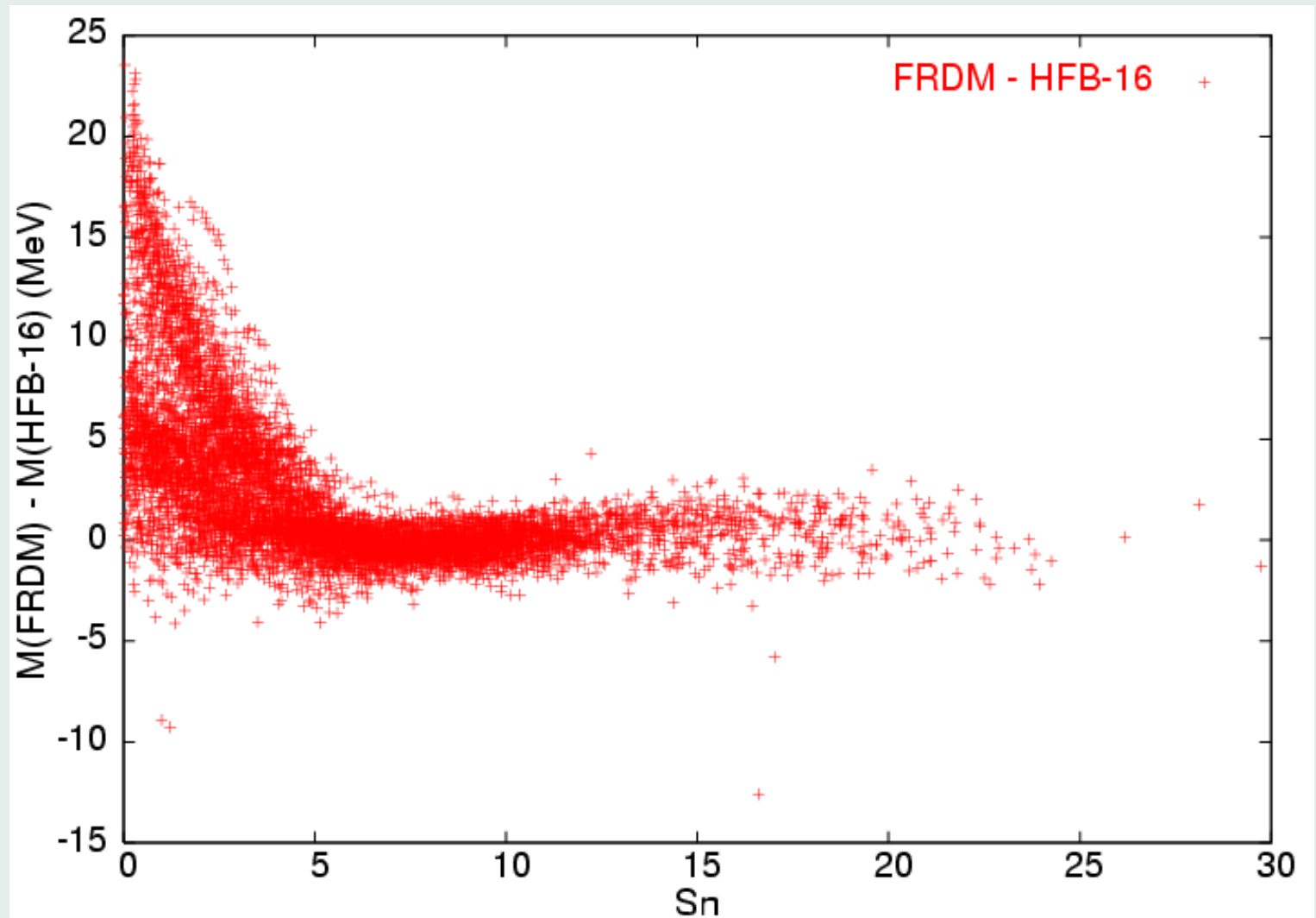


Fig. 6. FRDM masses - HFB-16 masses

3. Outer crust of neutron star

~ 300 meters thick $0 < \bar{\rho} < \sim 2.4 \times 10^{-4} \text{ nucleons.fm}^{-3}$

n-rich nuclei + degenerate electrons

but **NO FREE NEUTRONS** – within neutron drip line

– everything determined by mass model, i.e., by mass tables.

Assume $T = 0$.

Electron gas (assumed uniform) is completely degenerate; relativistic effects treated exactly. Electron energy per unit volume u_e determined uniquely by electron density

$$\rho_e = \frac{Z}{A} \bar{\rho}$$

Electron energy per nucleon

$$e_e = \frac{u_e(\rho_e)}{\bar{\rho}}$$

Total energy per nucleon for nucleus (Z, A)

$$e = e_e + \frac{1}{A} \{ E_{latt} + M'_{at}(Z, A) \}$$

E_{latt} : total energy of interaction between nucleus and electrons

$M'_{at}(Z, A)$: tabulated atomic mass - binding energy of electrons.

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Assume complete beta and nuclear equilibrium:
“COLD CATALYZED MATTER” ???

Then minimize total energy e per nucleon wrt N and Z .

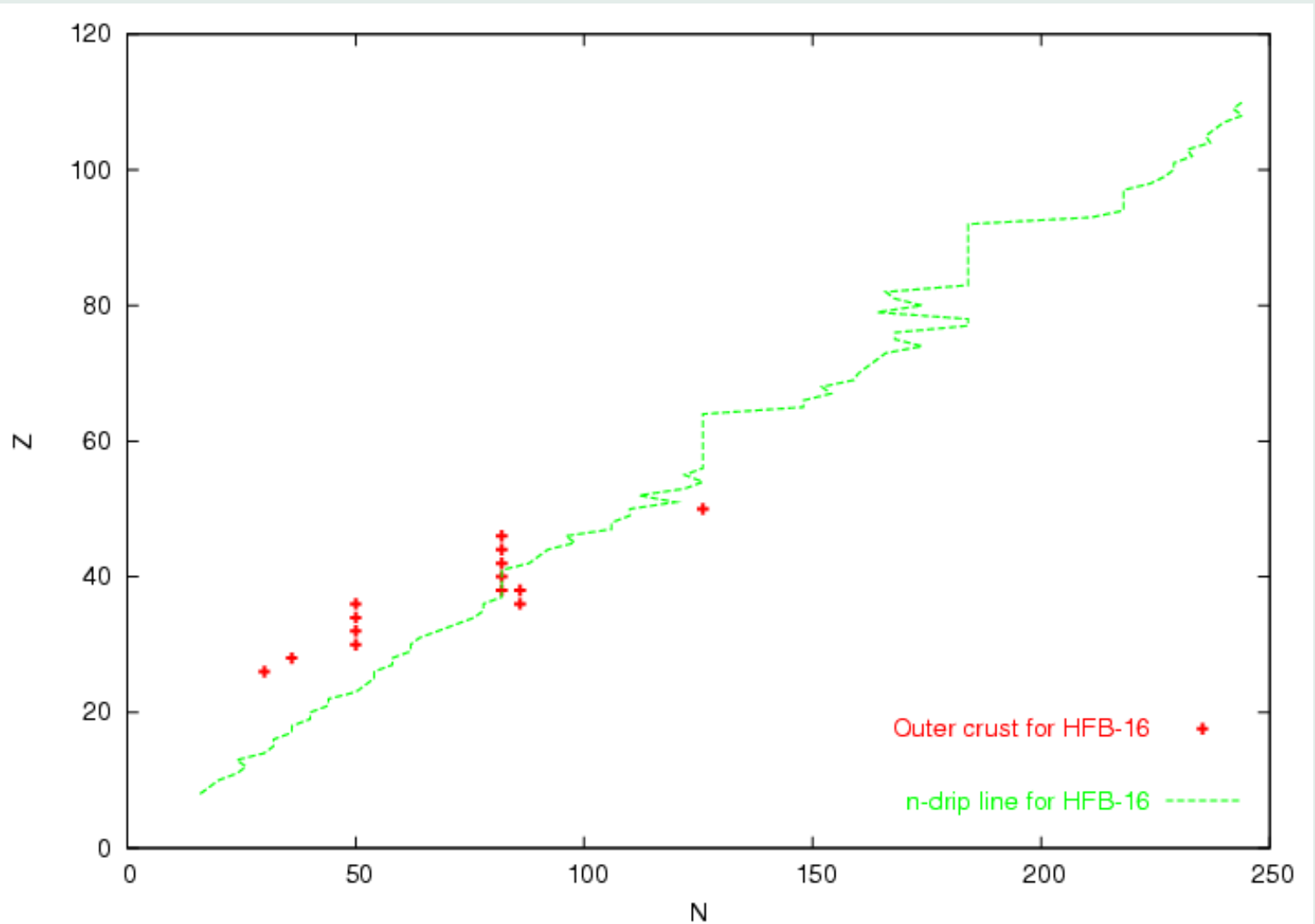


Fig. 7. Composition of outer crust for HFB-16.

4. Inner crust of neutron star

~ 500 meters thick

$\sim 2.4 \times 10^{-4} \text{ nucleons.fm}^{-3} < \bar{\rho} < \sim 0.1 \text{ nucleons.fm}^{-3}$

beyond neutron drip line

- n-p clusters in neutron vapour
- or neutron-vapour bubbles in n-p liquid

uniform gas of electrons – global neutrality

Adopt picture of **Wigner-Seitz cell** (spherical so far)

–minimize total energy per nucleon wrt N and Z

How to calculate energy with forces of our mass models?

HFB would be method of choice,

- – but neutrons populate continuum and problems with boundary conditions on their s.p. wavefunctions
- – time-consuming

So we began with ETF approximation to HF

- Extended Thomas-Fermi
- fourth-order in gradients
- semi-classical: no shell effects, no pairing

Importance of fitting Skyrme force to neutron matter

Comparison of HFB-8 and HFB-9

Until HFB-16 the best fit was found with HFB-8, but it was not fitted to neutron matter. HFB-9 was first of our models constrained by neutron matter.

We will see that this choice makes a great difference to optimal value of Z in the inner crust, and thus to observational properties of neutron star.

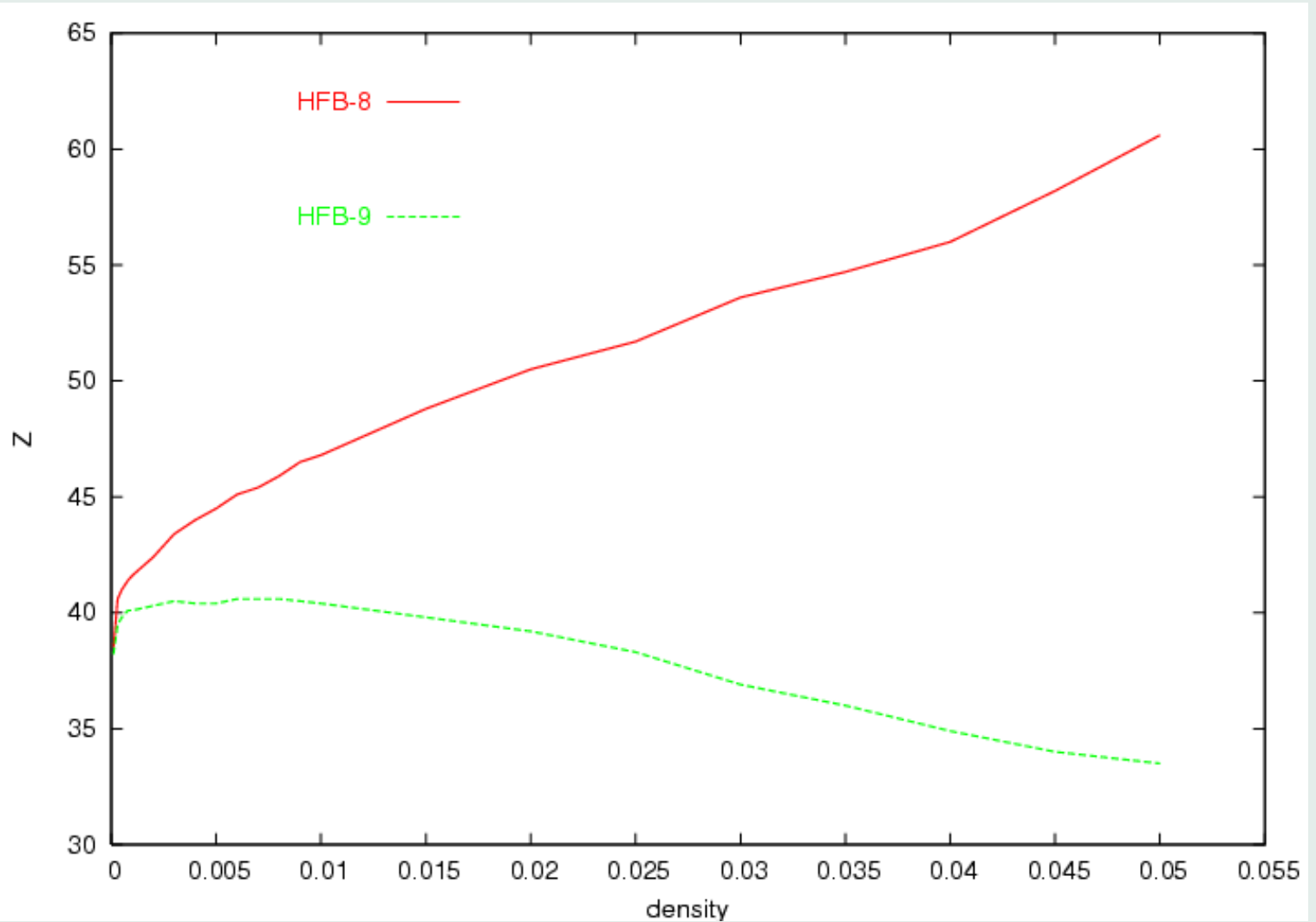


Fig. 8. Optimal Z -value in WS cell for inner crust as a function of density (nucleons. fm^{-3}) (ETF calculations).

Shell effects

- Added perturbatively and self-consistently to ETF energy by STRUTINSKY-INTEGRAL method

$$E_{ETF SI} = E_{ETF} + \sum_{q=n,p} E_q^{sc}$$

ETF SI

- Used for our first mass model and barrier calculations.

$$E_q^{sc} = \sum_i n_i \widetilde{\epsilon}_{i,q} - \int d^3\mathbf{r} \left(\frac{\hbar^2}{2M_q^*} \widetilde{\tau}_q + \widetilde{U}_q \widetilde{\rho}_q + \widetilde{\mathbf{W}}_q \cdot \widetilde{\mathbf{J}}_q \right)$$

Now extend to EOS: **TETFSI** (temperature dependent).

Neutrons in continuum, so no neutron-shell effects.

- Only proton shells, so no problem with boundary conditions.

$$F_{TETFSI} = F_{TETF} + E_p^{sc} - T(S_p^{s.p.} - S_p^{TETF})$$

E_p^{sc} as at $T = 0$, except

$$n_i = \frac{1}{1 + \exp\{(\widetilde{\epsilon}_{i,p} - \mu_p)/T\}}$$

Note also inclusion of shell effects in entropy.

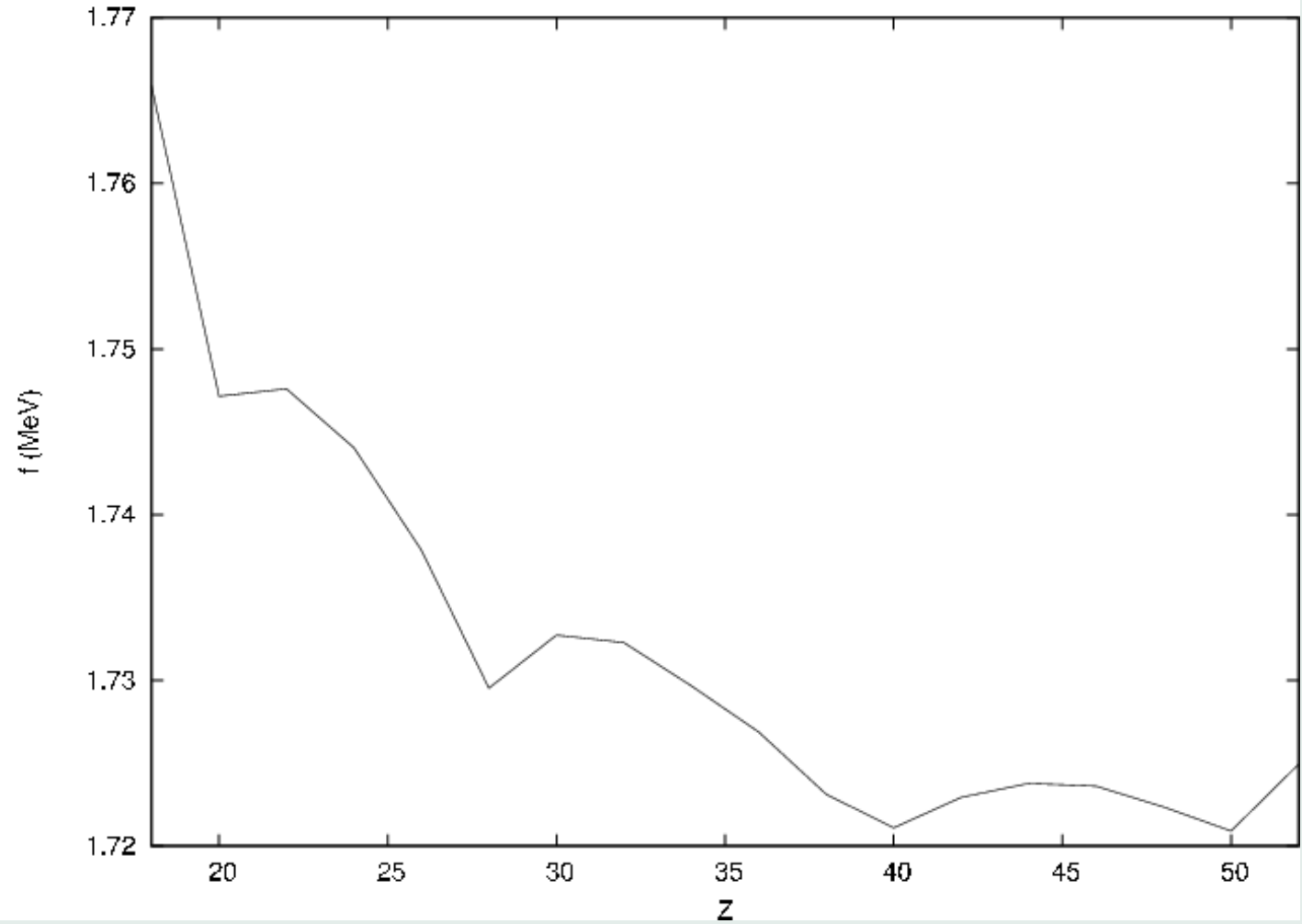


Fig. 9. Variation of $f_{TETF_{SI}}$ with Z (always for optimal value of A) at $\bar{\rho} = 0.005 \text{ fm}^{-3}$ and $T = 0.1 \text{ MeV}$ (HFB-14).

$\bar{\rho}$ (fm ⁻³)	Z	A
0.0003	50 (38)	200 (147)
0.001	50 (39)	460 (341)
0.005	50 (38)	1130 (842)
0.01	40 (38)	1210 (1107)
0.02	40 (35)	1480 (1294)
0.03	40 (33)	1595 (1303)
0.04	40 (31)	1610 (1242)
0.05	20 (30)	800 (1190)
0.06	20 (29)	765(1116)

TETFSI results for number of protons Z and total number of nucleons A in WS cell for nuclear and beta equilibrium at $T = 0.1$ MeV as a function of $\bar{\rho}$ for force of model HFB-14. TETF results in parentheses.

5. Conclusions

- Skyrme-HFB mass models can give high quality fits to the mass and charge-radius data, and to neutron matter.
- Can be extrapolated *beyond* the drip line to give EOS of inner crust of neutron-star matter; important to fit both masses and neutron-matter energy curve.
- Inner crust of neutron star shows significant shell effects.
- Measurement of neutron-skin thickness distinguishes between different realistic calculations of neutron matter. When combined with mass data determines absolute value of symmetry coefficient.