# Charge radius of $^{6}He$ and Halo nuclei in Gamow Shell Model


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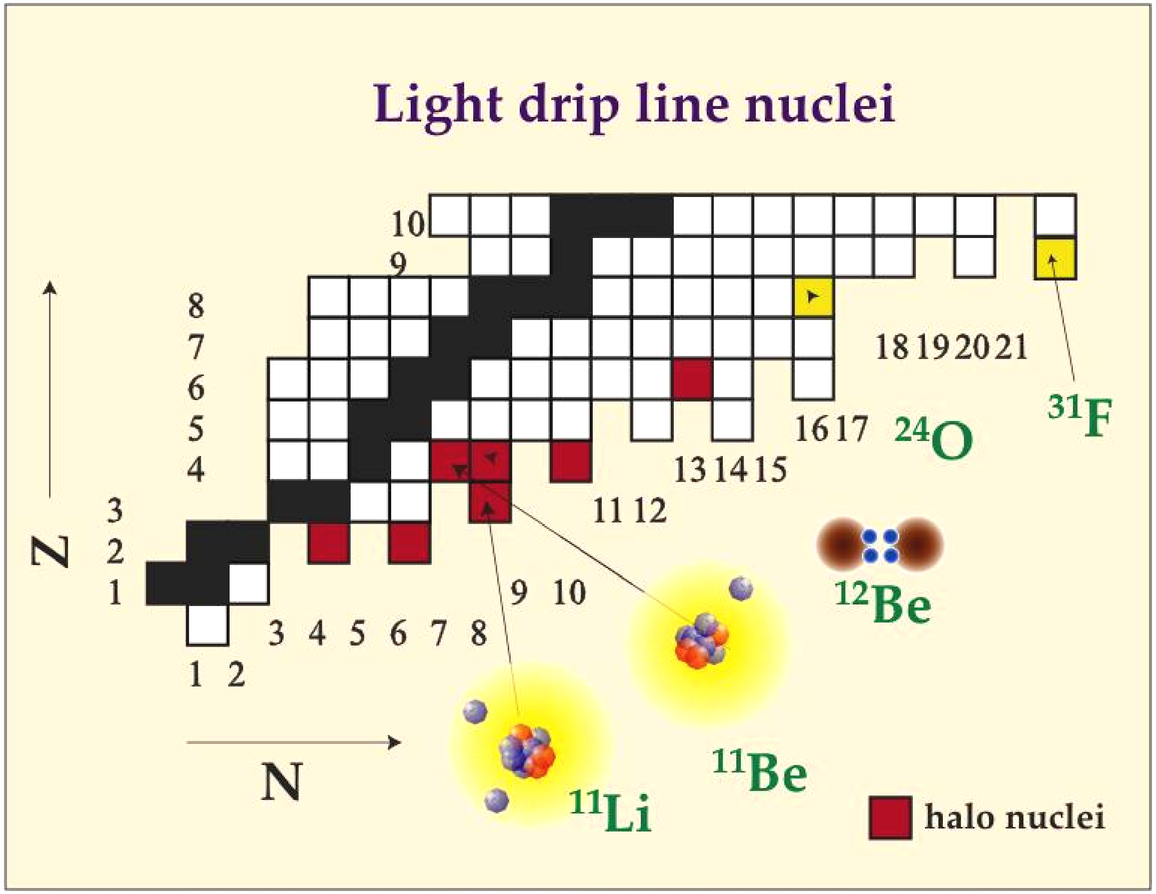
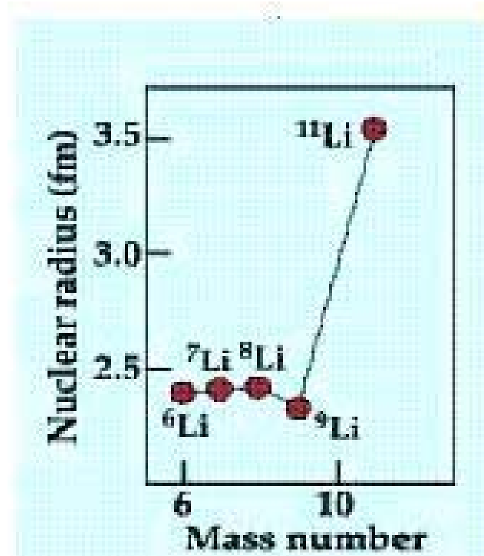
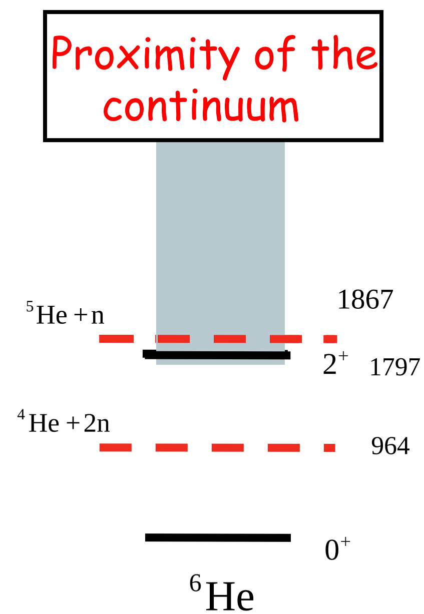
# Outline

- Drip line nuclei as Open Quantum Systems
- Gamow Shell Model Formalism
- Experimental Radii of $^{6,8}He$ ,$^{11}Li$ and $^{11}Be$
- Results on $^{6}He$ charge radius calculation
- Comparison with other models
- Conclusion and Future Plans



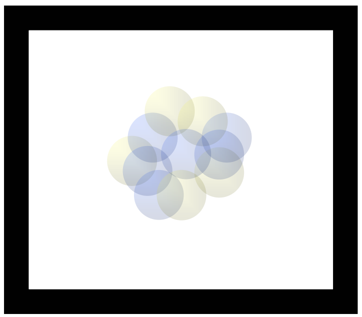

I.Tanihata *et al*
PRL 55, 2676 (1985)



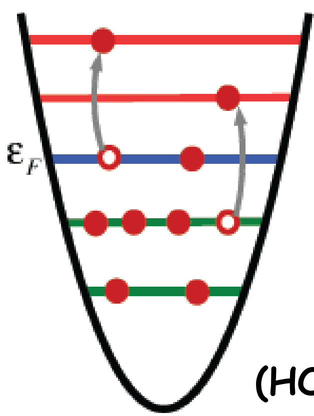

It is a major challenge of nuclear theory to develop theories and algorithms that would allows us to understand the properties of these exotic systems.

## Closed Quantum System

*(nuclei near the valley of stability)*



infinite well




discrete states

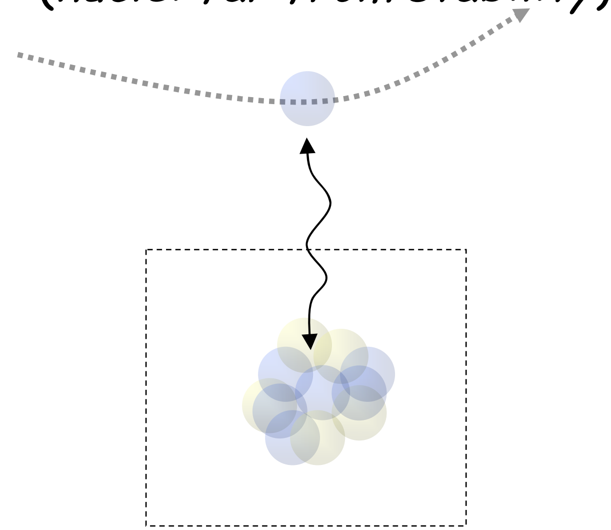
(HO) basis

nice mathematical properties:

Exact treatment of the c.m, analytical solution…

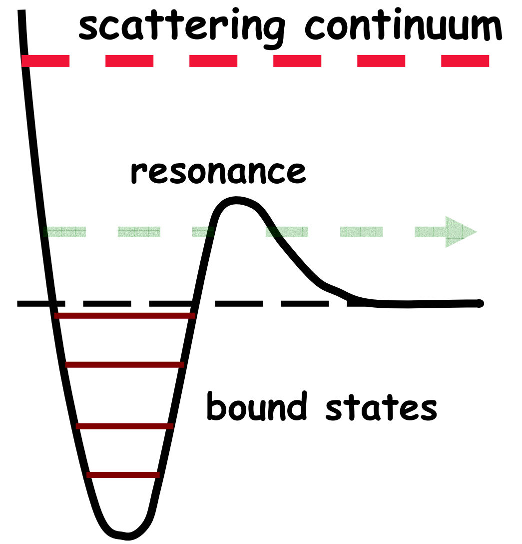
## Open quantum system

*(nuclei far from stability)*



scattering continuum

resonance



bound states

## Theories that incorporate the continuum

**Continuum Shell Model (CSM)**

- H.W.Bartz *et al*, NP A275 (1977) 111
- A.Volya and V.Zelevinsky PRC 74, 064314 (2006)

**Shell Model Embedded in Continuum (SMEC)**

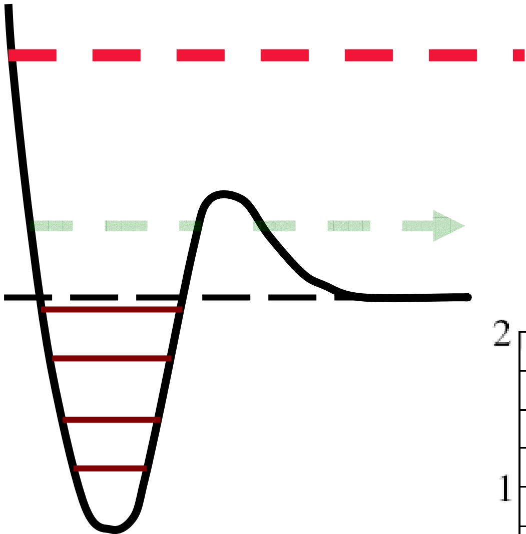
- J. Okolowicz.,*et al*, PR 374, 271 (2003)
- J. Rotureau *et al*, PRL 95 042503 (2005)

**Gamow Shell Model (GSM)**

- N. Michel *et al*, PRL 89 042502
- N. Michel *et al.*, Phys. Rev. C67, 054311 (2003)
- N. Michel *et al.*, Phys. Rev. C70, 064311 (2004
- G. Hagen *et al*, Phys. Rev. C71, 044314 (2005)
- N.Michel *et al*, J.Phys. G: Nucl.Part.Phys 36, 013101 (2009)

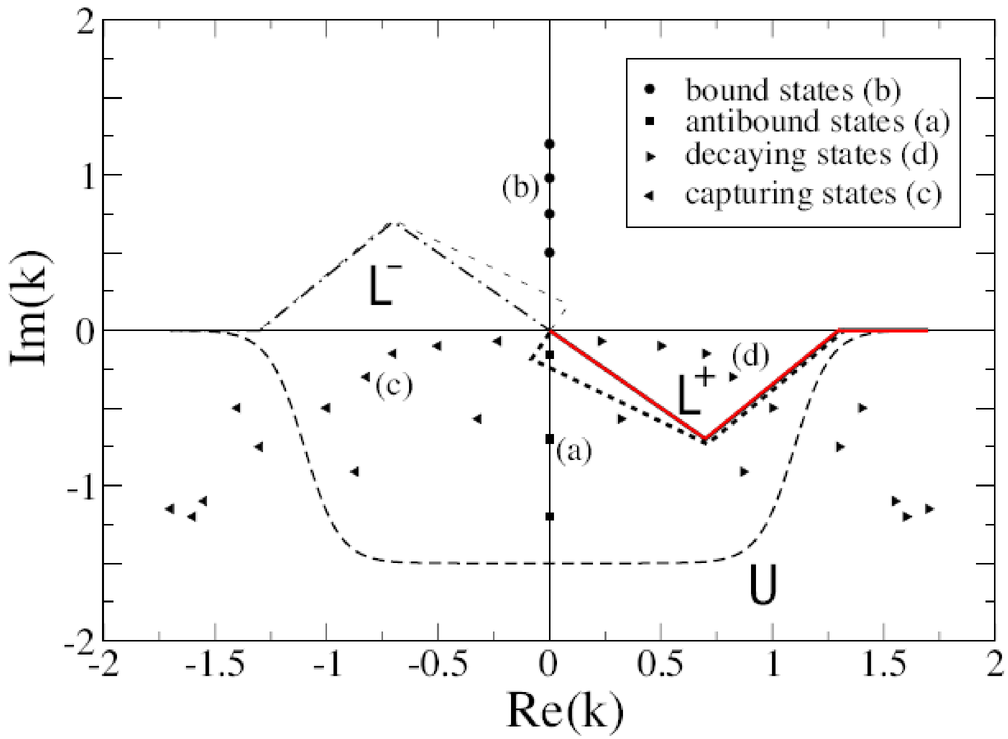
# The Gamow Shell Model (Open Quantum System)

N.Michel *et.al* 2002
PRL 89 042502



$$\left(-\frac{d^2}{dr^2}+v(r)+\frac{l(l+1)}{r^2}-k^2\right)u_l(k,r)=0$$

$$k=\sqrt{\frac{2mE}{\hbar^2}}$$

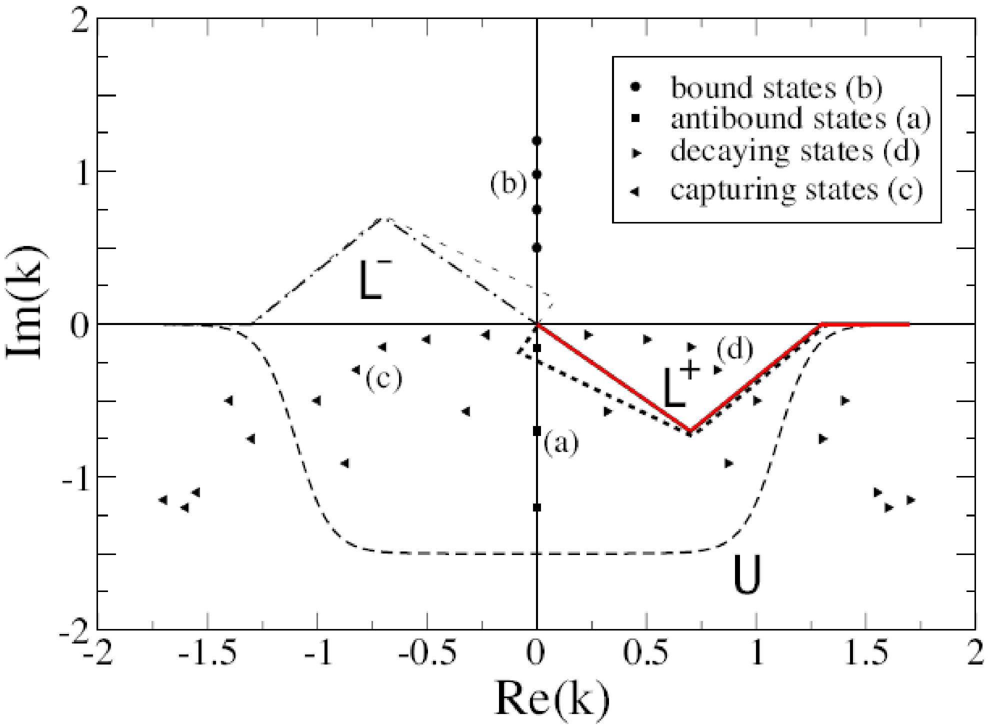



Poles of the S-matrix

$$u_l(k,r)\sim C_+H_l^+(k,r)\,,r\to\infty \quad \textit{bound states, resonances}$$

$$u_l(k,r)\sim C_+H_l^+(k,r)+C_-H_l^-(k,r)\,,r\to\infty \quad \textit{scattering states}$$

# Berggren's Completeness relation




T.Berggren (1968)
NP A109, 265

$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L^+} |u_k\rangle\langle\tilde{u}_k| dk = 1$$

**resonant states (bound, resonances…)**

**Non-resonant Continuum along the contour**

$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \sum_k |u_k\rangle\langle\tilde{u}_k| dk \cong 1$$

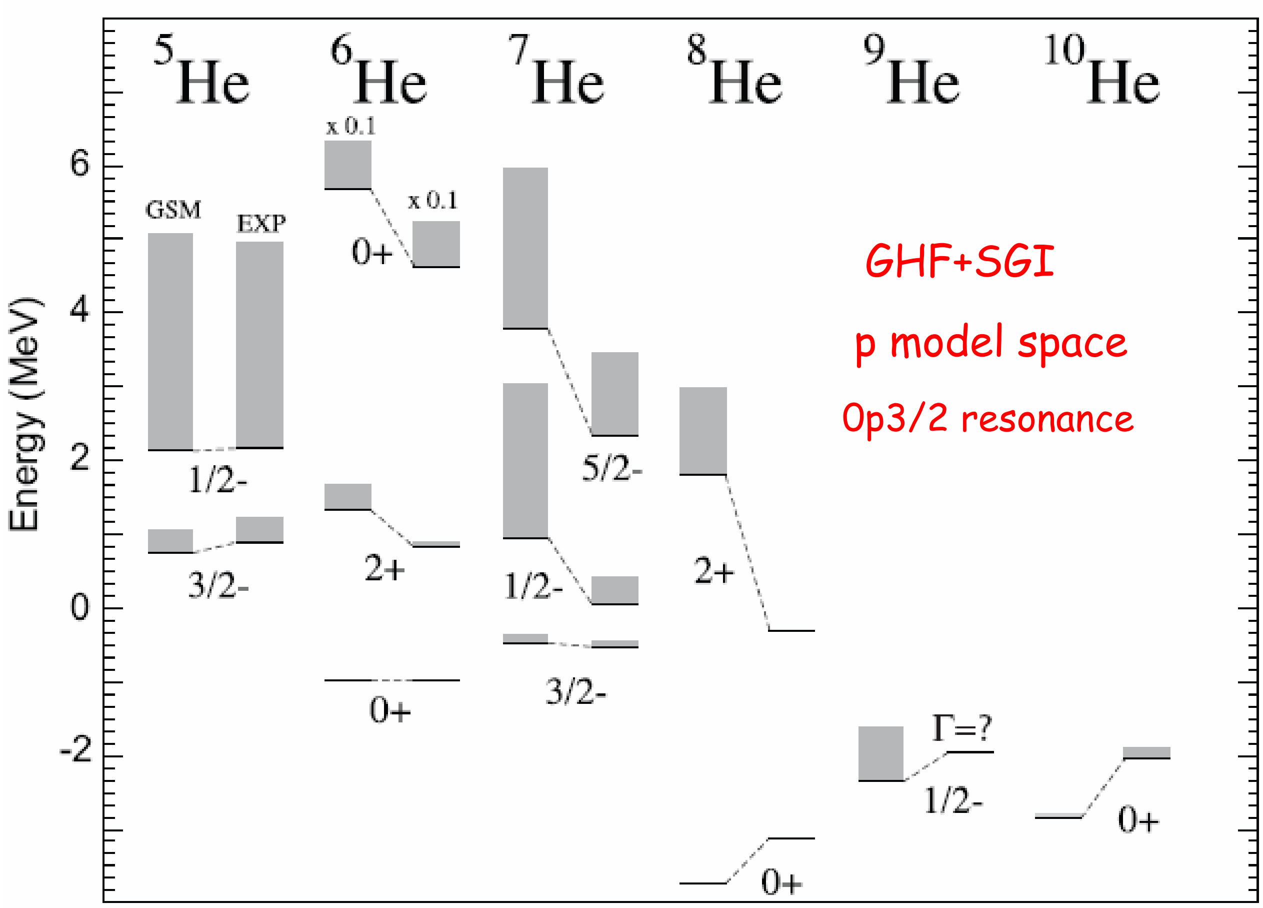
**Many-body discrete basis**

**Complex-Symmetric Hamiltonian matrix**

**Matrix elements calculated via complex scaling**

# GSM application for He chain

PRC 70, 064313 (2004)




GHF+SGI

p model space

0p3/2 resonance

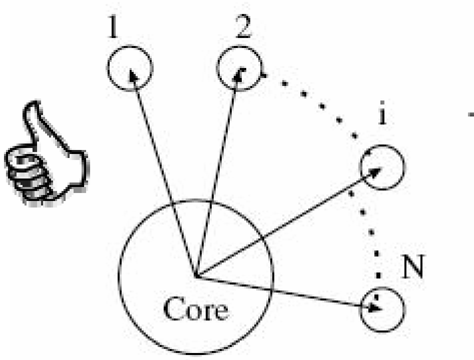
- ✓ **Optimal basis for each nucleus via the GHF method**
- ✓ **Borromean nature of $^{6,8}$He is manifested**
- ✓ **Helium anomaly is well reproduced**

# GSM HAMILTONIAN

<u>We want a Hamiltonian free from spurious CM motion</u>

Lawson method?

Jacobi coordinates?




Y.Suzuki and K.Ikeda
PRC 38,1 (1988)

$$H = \sum_{i=1}^{n} \left[ \frac{\mathbf{p_i}^2}{2\mu} + U_i \right] + \sum_{j>i=1}^{n} \left[ V_{ij} + \frac{1}{A_c} \mathbf{p_i p_j} \right]$$

"recoil" term coming from the expression of $H$ in the COSM coordinates. No spurious states

- pipj matrix elements $\langle ab|\mathbf{p}_i\mathbf{p}_j|cd\rangle = C\langle a||\mathbf{p}_i||c\rangle\langle b||\mathbf{p}_j||d\rangle$

- **complex scaling does not apply to this particular integral…**

# Recoil term treatment

✓Two methods which are equivalent from a numerical point of view

PRC 73 (2006) 064307

i) Transformation in momentum space

$$\frac{\hbar^2}{2\mu}k^2\psi_{nl}(k) + \int_{L_+} V_l(k,q)\psi_{nl}(q)q^2\,dq \;= E_{nl}\psi_{nl}(k)$$

✓disregard numerical derivatives

$$p_i \rightarrow k_i$$

Fourier transformation to return back to r-space

$$\phi_{nl}(r) = \sqrt{\frac{2}{\pi}}\sum_{i=1}^{N}\sqrt{\omega_i}k_i j_l(kr)\psi_{nl}(i)$$

ii) Expand $p_i$ in HO basis

$$\mathbf{p}_i = \sum_{\alpha<\gamma} |\alpha\rangle\langle\alpha|\mathbf{p}_i|\gamma\rangle\langle\gamma|$$

α,γ are oscillator shells
a,c are Gamow states

$$\langle a|\mathbf{p}_i|c\rangle = \sum_{\alpha<\gamma}\langle a|\alpha\rangle\langle\alpha|\mathbf{p}_i|\gamma\rangle\langle\gamma|c\rangle$$

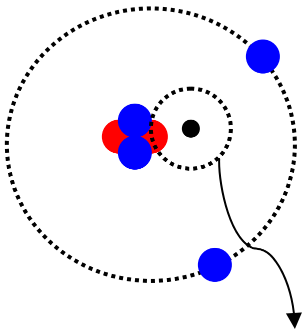
✓No complex scaling is involved

✓Gaussian fall-off of HO states provides convergence

✓Convergence is achieved with a truncation of about $N_{max}$ ~ 10 HO quanta

# EXPERIMENTAL RADII OF $^{6}He$, $^{8}He$, $^{11}Li$

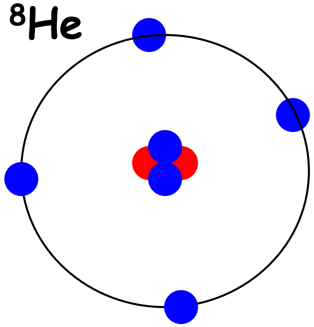
$^{6}He$



center of mass of the nucleus

charge radii determines the correlations between valence particles AND reflects the radial extent of the halo nucleus

$^{8}He$



$R_{charge}(^{6}He) > R_{charge}(^{8}He)$

Point proton charge radii

| | $^{4}He$ | $^{6}He$ | $^{8}He$ |
|---|---|---|---|
| L.B.Wang *et al* | **1.43fm** | **1.912fm** | |
| P.Mueller *et al* | **1.45fm** | **1.925fm** | **1.808fm** |

| | $^{9}Li$ | $^{11}Li$ |
|---|---|---|
| R.Sanchez *et al* | **2.217fm** | **2.467fm** |

| | $^{10}Be$ | $^{11}Be$ |
|---|---|---|
| W.Nortershauser *et al* | **2.357fm** | **2.460fm** |

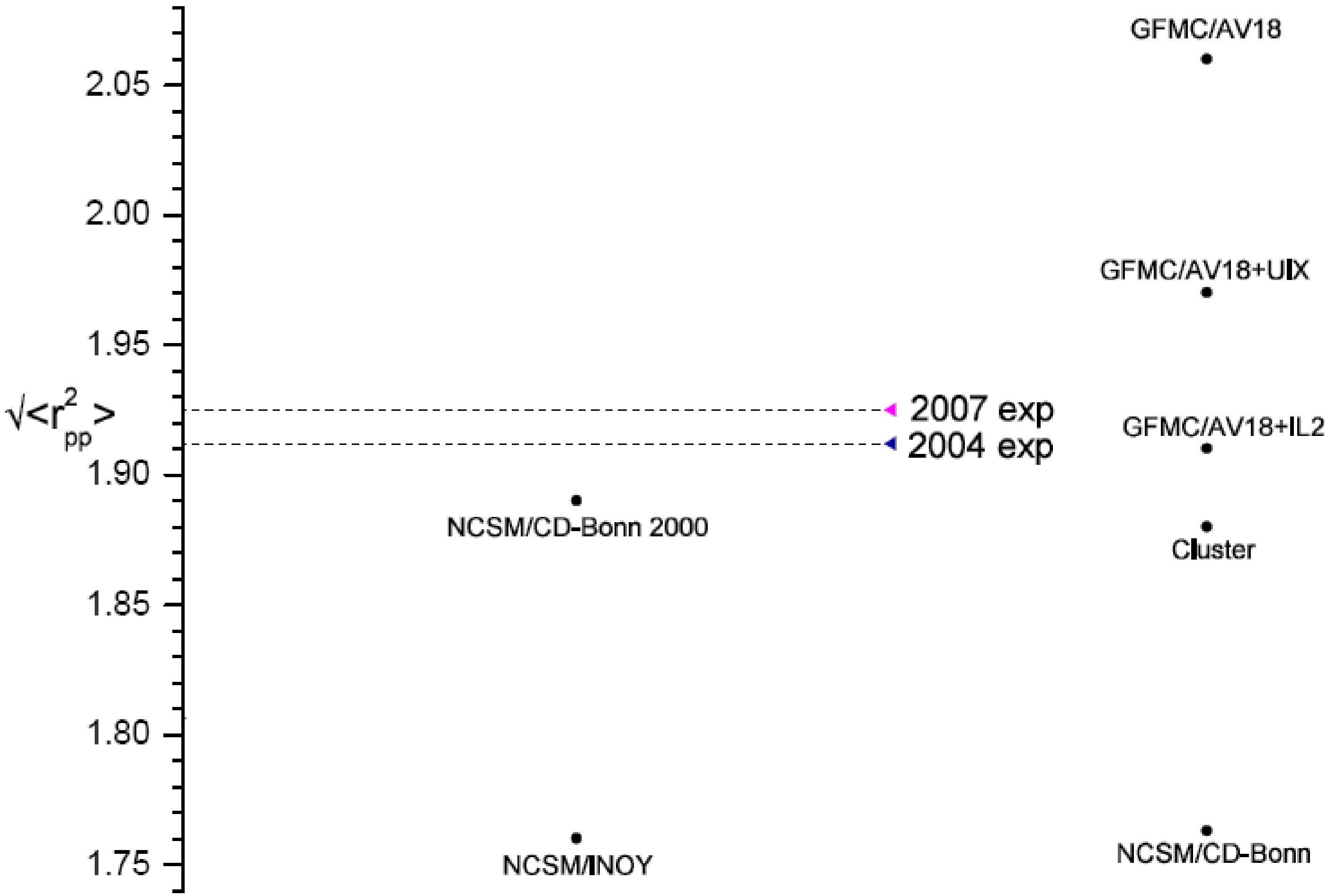
✓**"Swelling" of the core is not negligible**

$$\rho_{pp}(^{4}He) \neq \rho_{pp}(^{6}He) \neq \rho_{pp}(^{8}He)$$

Annu.Rev.Nucl.Part.Sci. **51**, 53 (2001)

L.B.Wang *et al*, PRL **93**, 142501 (2004)
P.Mueller *et al*, PRL **99**, 252501 (2007)
R.Sanchez *et al* PRL **96**, 033002 (2006)
W.Nortershauser *et al* nucl-ex/0809.2607v1 (2008)

# Comparison of $^{6}He$ radius data with nuclear theory models




Charge radii provide a benchmark test for nuclear structure theory!

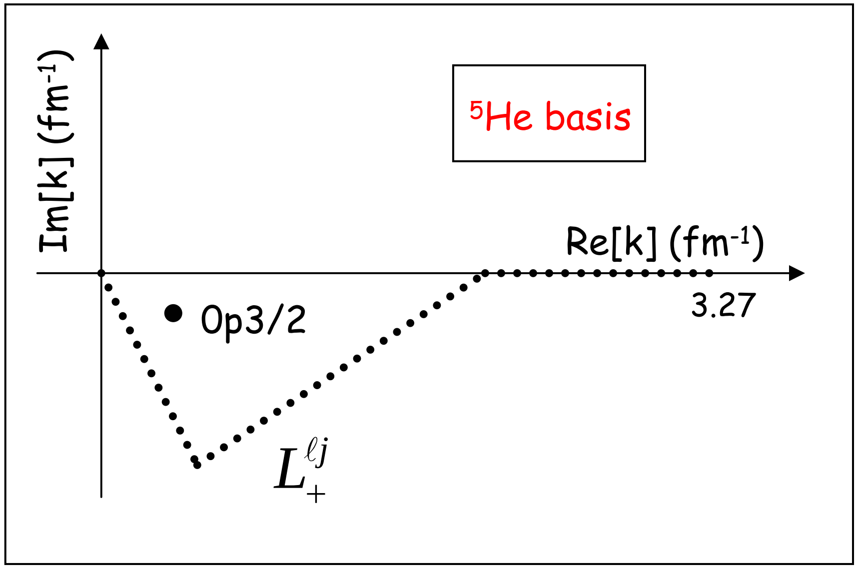
# GSM calculations for $^6$He nucleus

- WS basis parameterized to $^5$He s.p energies

<u>p-sd Valence space</u>

0p3/2 resonant state
plus
{ip3/2}, {ip1/2}, {is1/2}, {id5/2}, {id3/2}
non-resonant continua

With i=1,……$N_{sh}$ .
$N_{sh}$=60 with Gauss Legendre

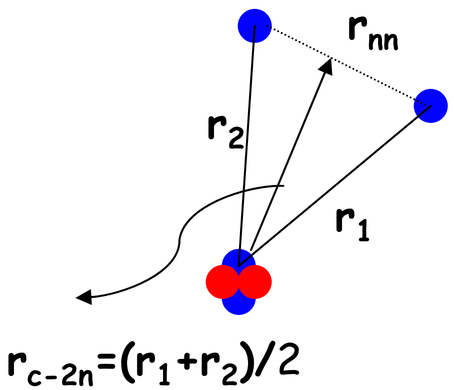



## Schematic two-body interactions employed

1. Separable Gaussian Interaction (**GI**) (PRC 71 044314)
2. Surface Delta Interaction (**SDI**) (PR 145, 830)
3. Surface Gaussian Interaction (**SGI**) (PRC 70, 064313)

- The parameter(s) of each force is(are) fitted on the g.s energy of $^6$He

# GSM calculations for $^6$He nucleus




**Expression of charge radius in these coordinates**

$$\left\langle r_p^2(Z,A)\right\rangle = \underbrace{\left\langle r_p^2(Z,A-2)\right\rangle}_{core} + \underbrace{\left(\frac{2}{A}\right)^2 \frac{1}{4}\left\langle \vec{r}_1^{\,2} + \vec{r}_2^{\,2} + 2\vec{r}_1\cdot\vec{r}_2\right\rangle}_{center\ of\ mass\ correction}$$

Generalization to n-valence particles is straightforward

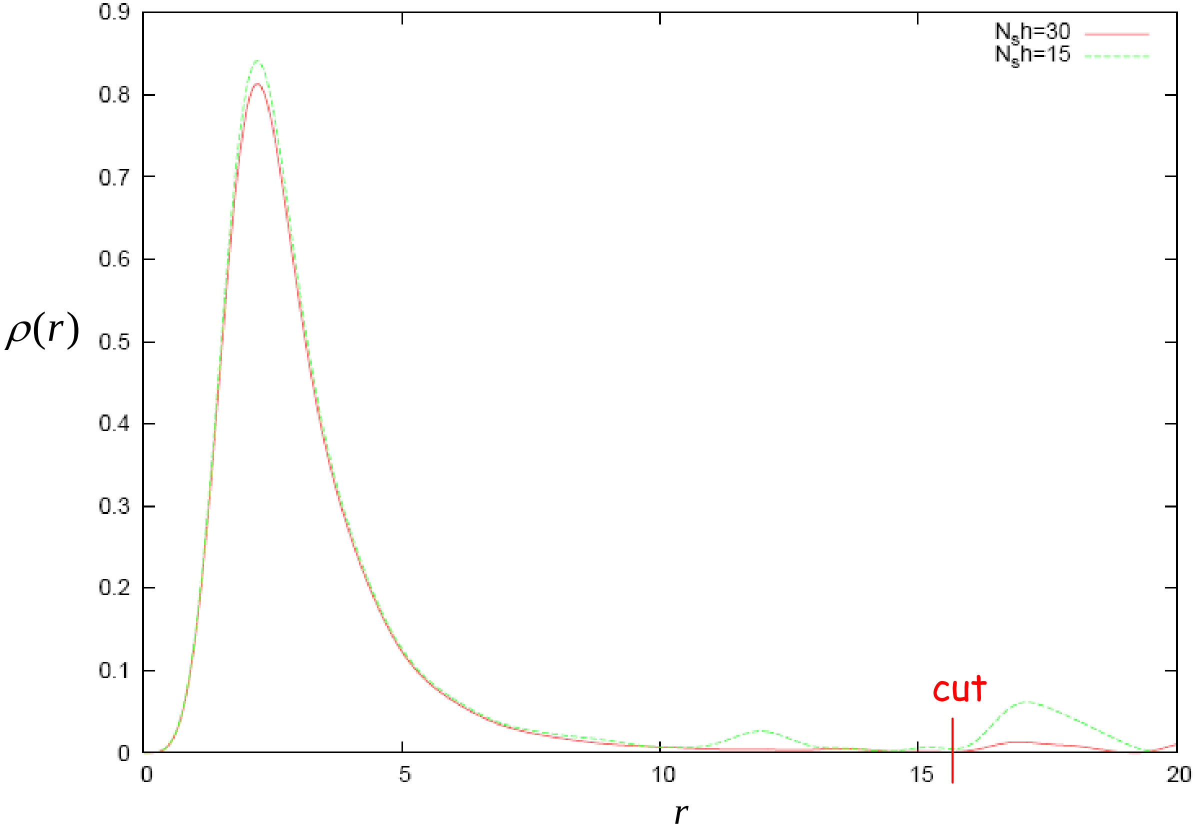
$$\int_0^\infty u_i(r)\, r^2 u_f(r)\, dr$$

→ **complex scaling cannot be applied!**

⇨ **Renormalization of the integral based on physical arguments (density)**

⇨ **In our calculations we carried out the radial integration until 25fm**

## Radial density of valence neutrons for the $^6He$




- ✓With an adequate number of points along the contour the fluctuations become minimal
- ✓We “cut” when for a given number of discretization points the fluctuations are smeared out

# Results and discussion

Angles estimated from the available B(E1) data and the average distances between neutrons.

| Interaction | GI | SGI | SDI |
|---|---|---|---|
| $\langle r_{pp}^2 \rangle^{1/2}$ [fm] | 1.912 | 1.924 | 1.920 |
| n-n angle $\theta_{nn}$ [degrees] | $90.3^o$ | $90.4^o$ | $89.77^o$ |

charge radii and angles for a p-sd model space employed

PRC 76, 051602

$$\langle \theta_{nn} \rangle = 83^{o}{}^{+20}_{-10}$$

$$\langle \theta_{nn} \rangle = 78^{o}{}^{+13}_{-18}$$

<u>Decomposition of the wavefunction</u>

| $(C_k)^2$ | GI | SGI | SDI |
|---|---|---|---|
| $(p_{3/2})^2_{res}$ | $(0.888, -0.789)$ | $(0.886, -0.776)$ | $(0.867, -0.747)$ |
| $(p_{3/2})^2_{cont}$ | $(-0.091, 0.027)$ | $(-0.096, 0.013)$ | $(-0.091, 0.007)$ |
| $(p_{3/2})^1_{res}(p_{3/2})^1_{cont}$ | $(0.125, 0.761)$ | $(0.130, 0.763)$ | $(0.115, 0.740)$ |
| $(s_{1/2})^2_{cont}$ | 0.005 | 0.007 | 0.004 |
| $(p_{1/2})^2_{cont}$ | 0.025 | 0.049 | 0.056 |
| $(d_{5/2})^2_{cont}$ | 0.0067 | 0.0156 | 0.033 |
| $(d_{3/2})^2_{cont}$ | 0.0010 | 0.006 | 0.0142 |

~91%

- The p3/2 occupancy is a crucial quantity for the correct determination of the charge radius in $^6$He

## Results and discussion

- ✓ Different interactions lead to different configuration mixing.

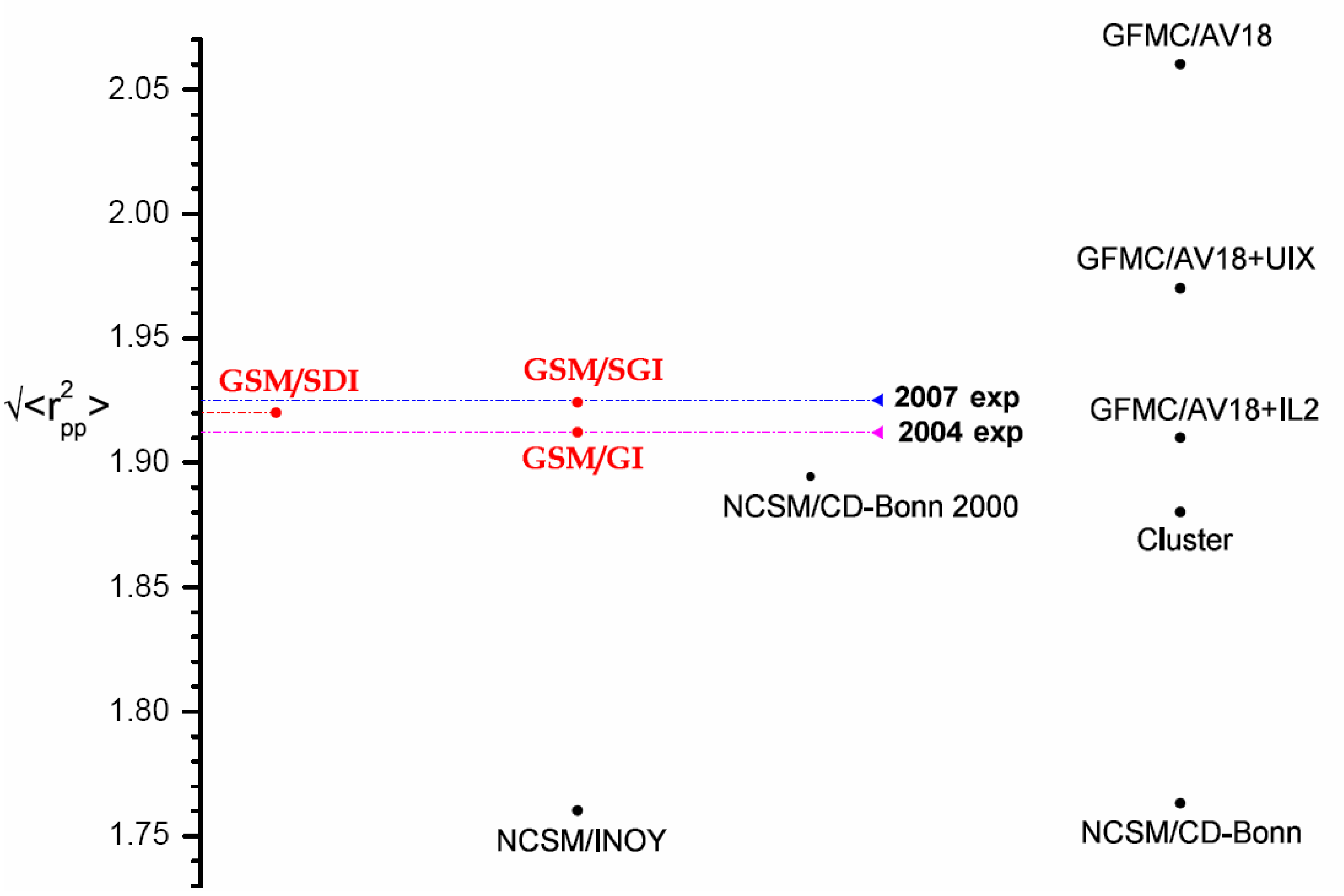
- ✓ $^{6}He$ charge radius ($R_{ch}$) is primarily related to the p3/2 occupation of the 2-body wavefunction.

- ✓ The recent measurements put a constraint in our GSM Hamiltonian which is related to the p3/2 occupation.

⬍

- ✓ We observe an overall weak sensitivity for both radii and the correlation angle.

## Comparison with other structure Models



## Conclusion and Future Plans

- The very precise measurements on $^{6}He$, $^{8}He$, $^{11}Li$ and $^{11}Be$ Halos charge radii give us the opportunity to constrain our GSM Hamiltonian.

- The GSM description is appropriate for modelling weakly bound nuclei with large radial extension.

- The next step: charge radii $^{8}He$, $^{11}Li$, $^{11}Be$ assuming an $^{4}He$ core. The rapid increase in the dimensionality of the space will be handled by the GSM+DMRG method.
  (J.Rotureau *et al* PRL 97 110603 (2006) and nucl-th/0810.0781.v1)

- The $2^{+}$ state of $^{6}He$ will be used to adjust the quadrupole strength V(J=2,T=1) of the interaction in $^{8}He$ and $^{11}Li$. For $^{11}Li$ the T=0 channel of the interaction will be fitted to the $^{6}Li$ nucleus.

- Develop effective interaction for GSM applications in the p and p-sd shells that will open a window for a detailed description of weakly bound systems. The effective GSM interaction depends on the valence space, but also in the position of the thresholds and the position of the S-matrix poles