Neutron Star Observations and the Equation of State

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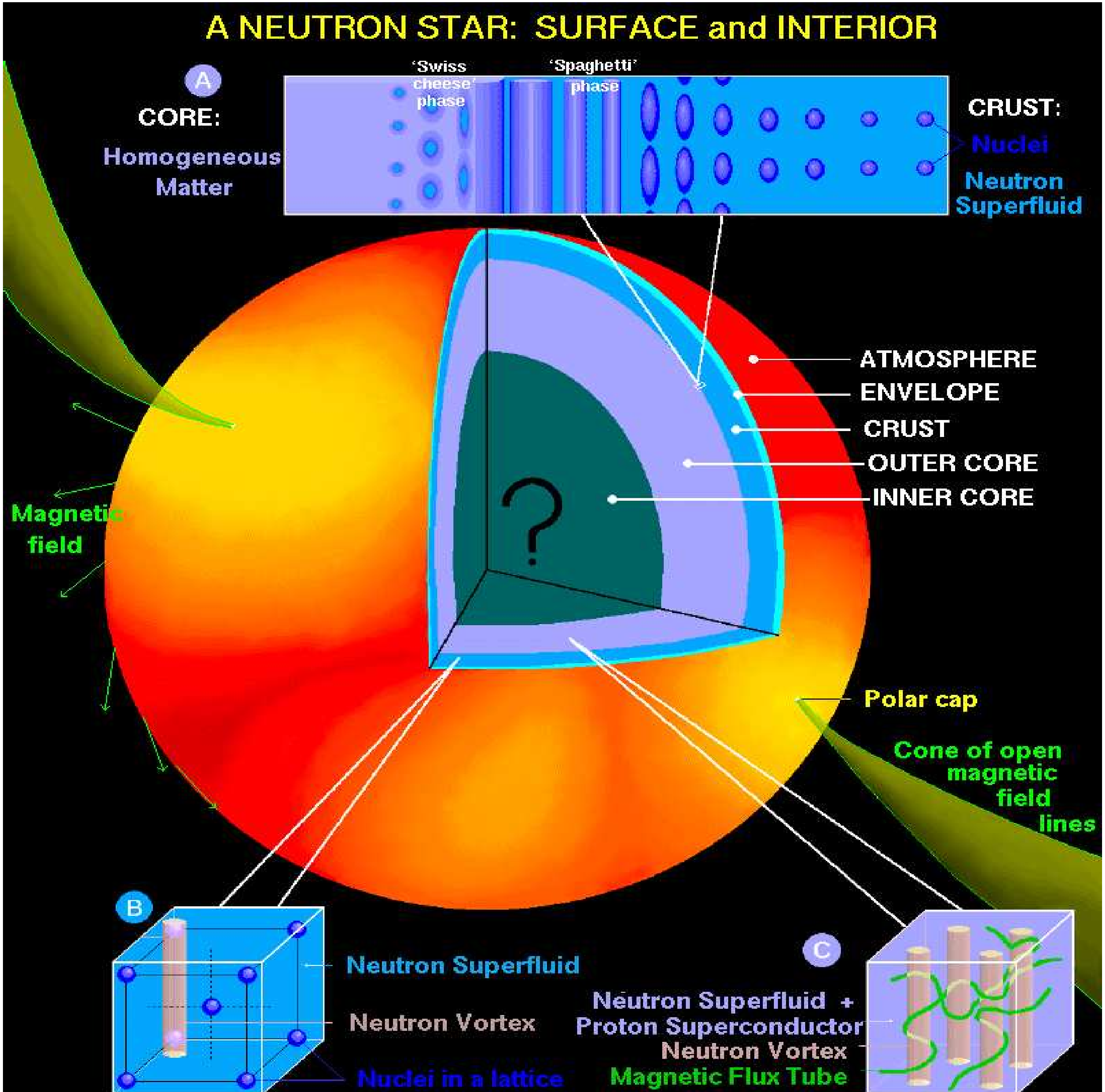
Outline

- **Neutron Star Structure**
    - Extreme Properties
    - Pulsar Constraints – Rotation and Mass
    - Pressure–Radius Correlation
    - Nuclear Symmetry Energy
    - Nuclear Structure Constraints
    - Inverting the TOV Equations
- **Radius Constraints**
    - Radiation Radius
    - Seismology
    - Moment of Inertia
    - Tidal Effects in Mergers
- **Neutron Star Cooling**
    - Binding Energy From Neutrinos
    - Crustal Cooling
    - Core Cooling and the URCA Process



Credit: Dany Page, UNAM

Neutron Star Structure

Tolman-Oppenheimer-Volkov equations of relativistic hydrostatic equilibrium:

$$
\begin{aligned}
\frac{dp}{dr} &= -\frac{G}{c^2}\frac{(m+4\pi pr^3)(\epsilon+p)}{r(r-2Gm/c^2)} \\
\frac{dmc^2}{dr} &= 4\pi\epsilon r^2
\end{aligned}
$$

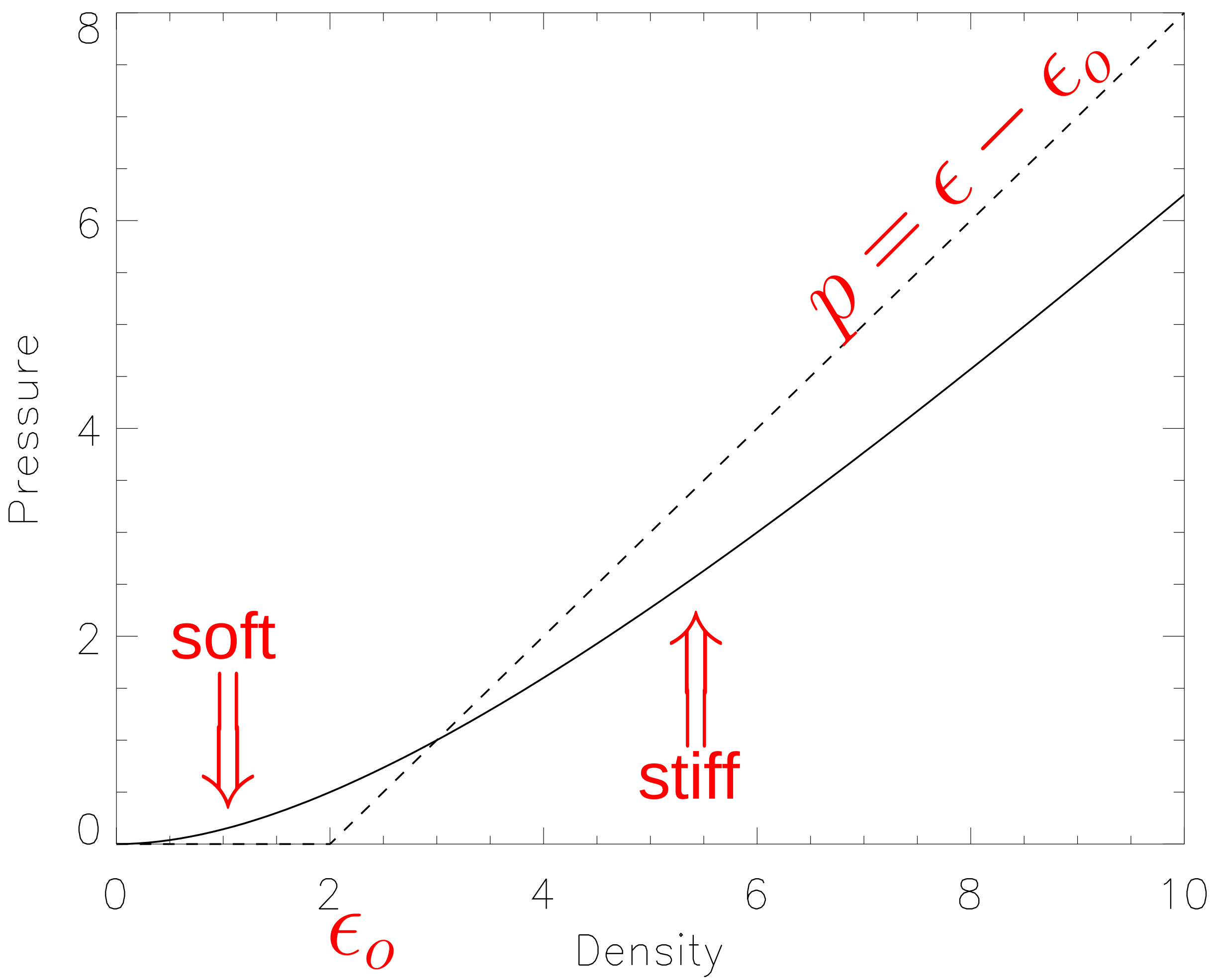
p is pressure, ϵ is mass-energy density

Useful analytic solutions exist:

- Uniform density $\epsilon =$ constant
- Tolman VII $\epsilon = \epsilon_c[1-(r/R)^2]$
- Buchdahl $\epsilon = \sqrt{pp_*} - 5p$

Extreme Properties of Neutron Stars

- The most compact configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff".



Maximally Compact Equation of State

Koranda, Stergioulas & Friedman (1997)

$$
\begin{aligned}
p(\epsilon) = 0, \qquad & \epsilon \leq \epsilon_o \\
p(\epsilon) = \epsilon - \epsilon_o, \qquad & \epsilon \geq \epsilon_o
\end{aligned}
$$

This EOS has a parameter ϵ_o, which corresponds to the surface energy density. The structure equations then contain only this one parameter, and can be rendered into dimensionless form using

$$y = m\epsilon_o^{1/2}, \qquad x = r\epsilon_o^{1/2}, \qquad q = p\epsilon_o^{-1}.$$

$$
\begin{aligned}
\frac{dy}{dx} &= 4\pi x^2(1+q) \\
\frac{dq}{dx} &= -\frac{(y+4\pi q x^3)(1+2q)}{x(x-2y)}
\end{aligned}
$$

The solution with the maximum central pressure and mass and the minimum radius:

$$
\begin{aligned}
q_{max} &= 2.026, \qquad y_{max} = 0.0851, \qquad x_{min}/y_{max} = 2.825 \\
p_{max} &= 307\left(\frac{\epsilon_o}{\epsilon_s}\right) \text{ MeV fm}^{-3}, \quad M_{max} = 4.2\left(\frac{\epsilon_s}{\epsilon_o}\right)^{1/2} \text{ M}_\odot, \quad R_{min} = 2.825\frac{GM_{max}}{c^2}.
\end{aligned}
$$

Moreover, the scaling extends to the axially-symmetric case, yielding

$$P_{min} \propto \left(\frac{M_{max}}{R_{min}^3}\right)^{1/2} \propto \epsilon_o^{-1/2}, \qquad P_{min} = 0.82\left(\frac{\epsilon_s}{\epsilon_o}\right)^{1/2} \text{ ms}$$

Maximum Mass, Minimum Period

Theoretical limits from GR and causality

- $M_{max} = 4.2(\epsilon_s/\epsilon_f)^{1/2}\ \mathrm{M}_\odot$ **Rhoades & Ruffini (1974), Hartle (1978)**
- $R_{min} = 2.9GM/c^2 = 4.3(M/\mathrm{M}_\odot)$ km

Lindblom (1984), Glendenning (1992), Koranda, Stergioulas & Friedman (1997)

- $\epsilon_c < 4.5 \times 10^{15}(\mathrm{M}_\odot/M_{largest})^2$ g cm^{-3} **Lattimer & Prakash (2005)**
- $P_{min} \simeq (0.74 \pm 0.03)(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}$ ms

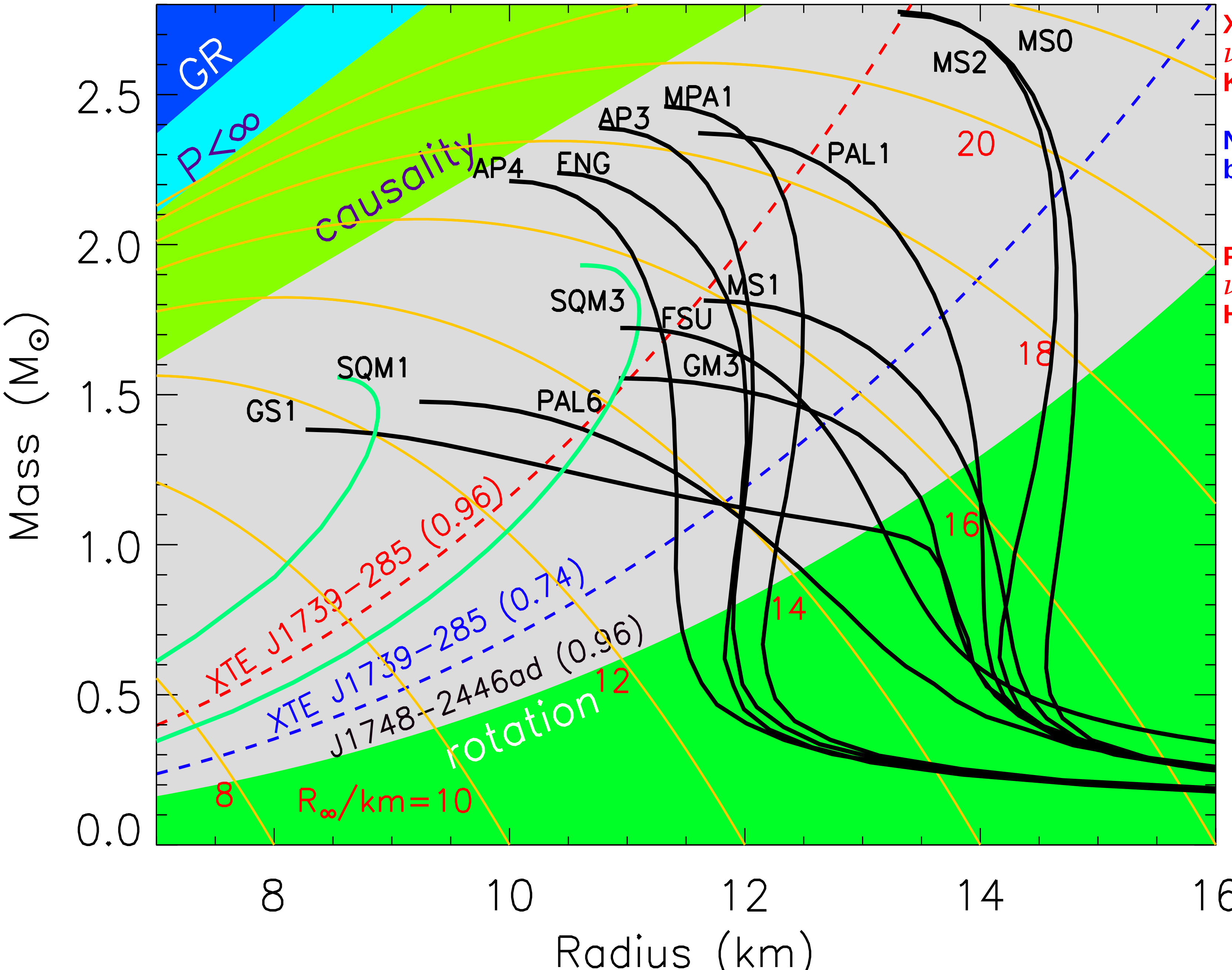
Koranda, Stergioulas & Friedman (1997)

- $P_{min} \simeq 0.96(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}$ ms (empirical)

Lattimer & Prakash (2004)

- $\epsilon_c > 0.91 \times 10^{15}(1\ \mathrm{ms}/P_{min})^2$ g cm^{-3} (empirical)
- $cJ/GM^2 \lesssim 0.5$ (empirical, neutron star)

Constraints from Pulsar Spins



XTE J1739-285
$\nu = 1122$ **Hz**
Kaaret et al. 2006

Not confirmed to be a rotation rate

PSR J1748-2446ad
$\nu = 716$ **Hz**
Hessels et al. 2006

Black hole? ⇒

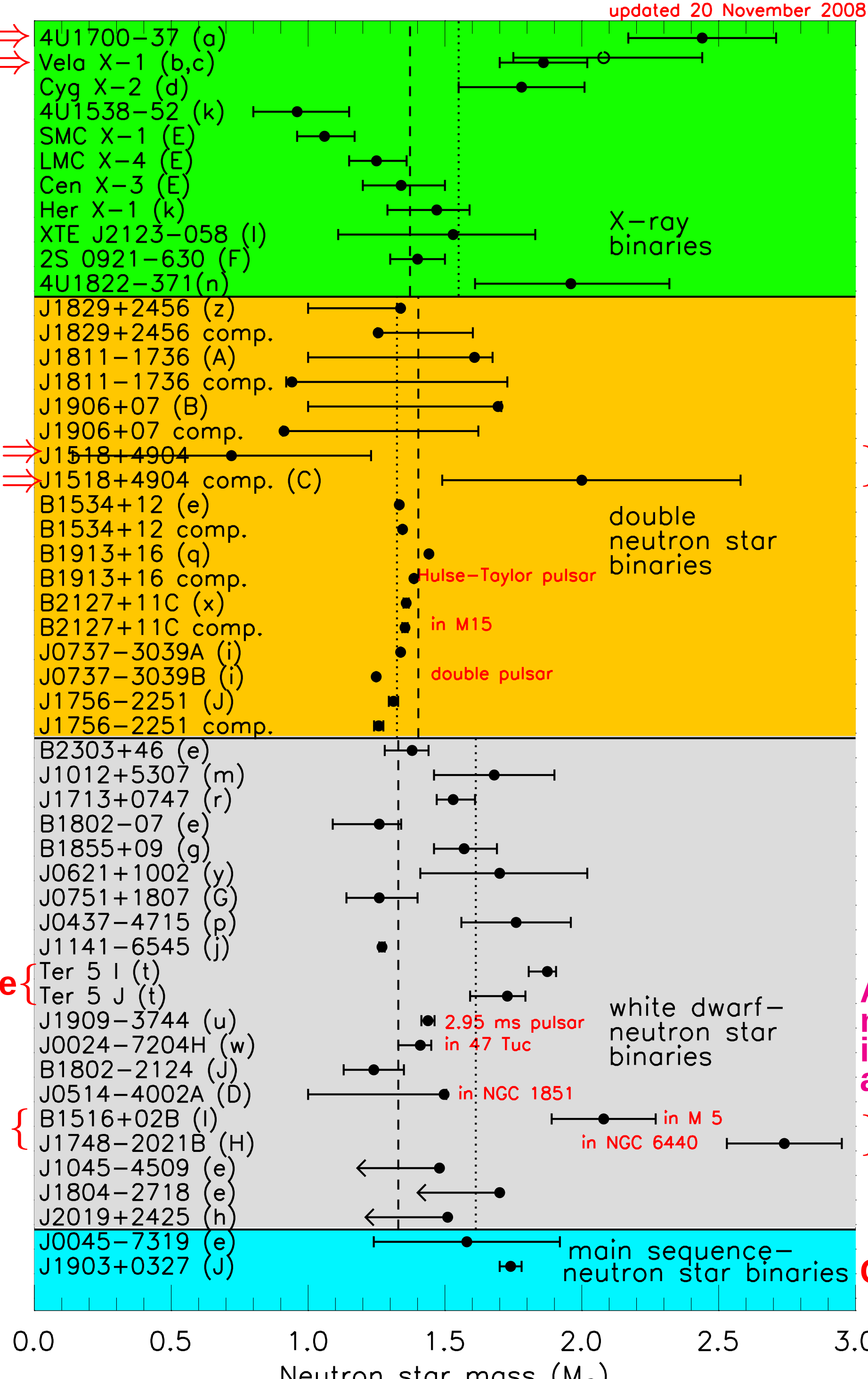
Firm lower mass limit? ⇒

$M \lesssim 1.17$ M$_\odot$, 95%confidence ⇒

$M > 1.55$ M$_\odot$, 95%confidence ⇒

$M > 1.68$ M$_\odot$, 95% confidence {

Freire et al. 2007 {



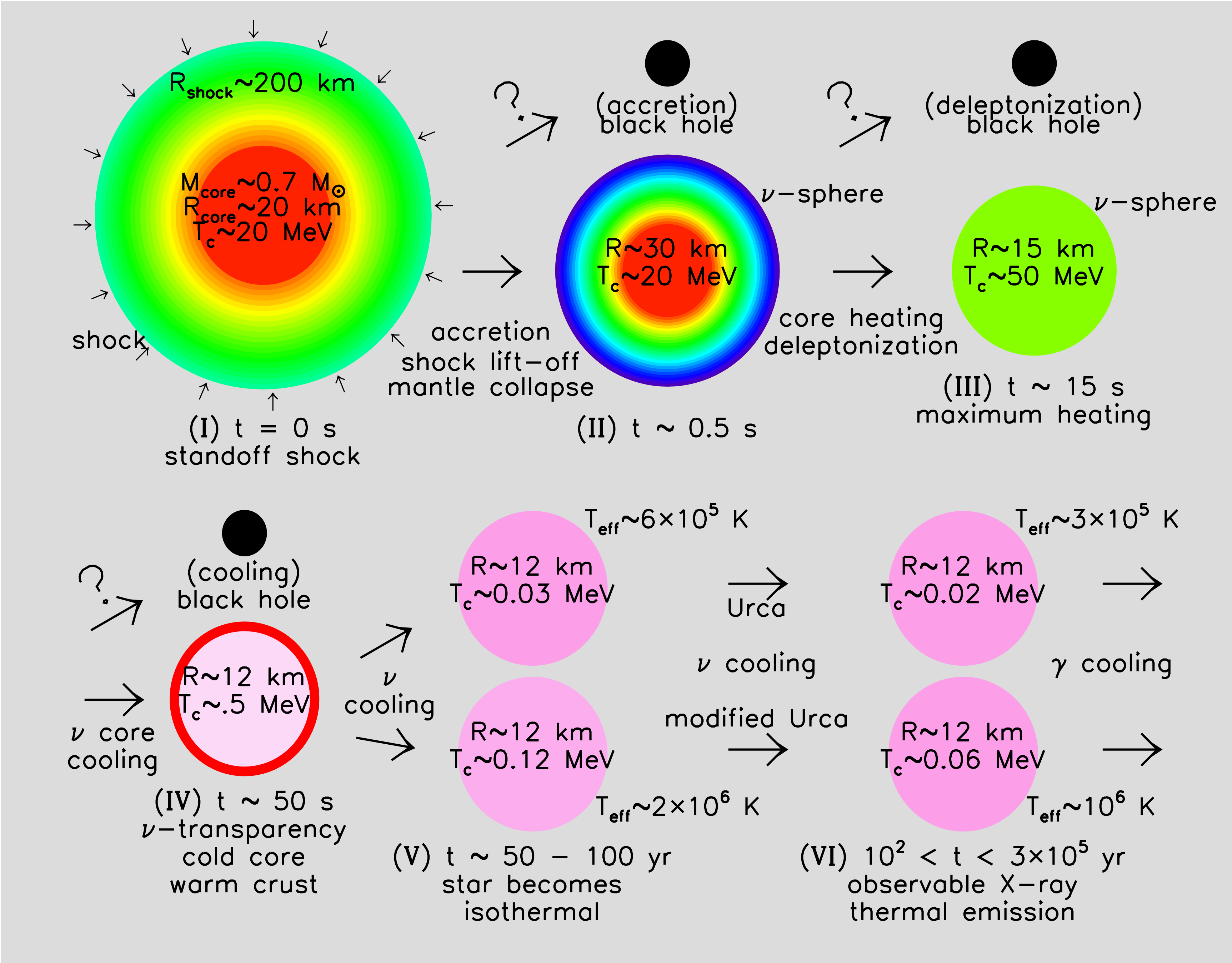
}Ferdman 2008, Janssen et al. 2008

Although simple average mass of w.d. companions is 0.27 M$_\odot$ larger, weighted average is 0.015 M$_\odot$ larger

} w.d. companion? statistics?

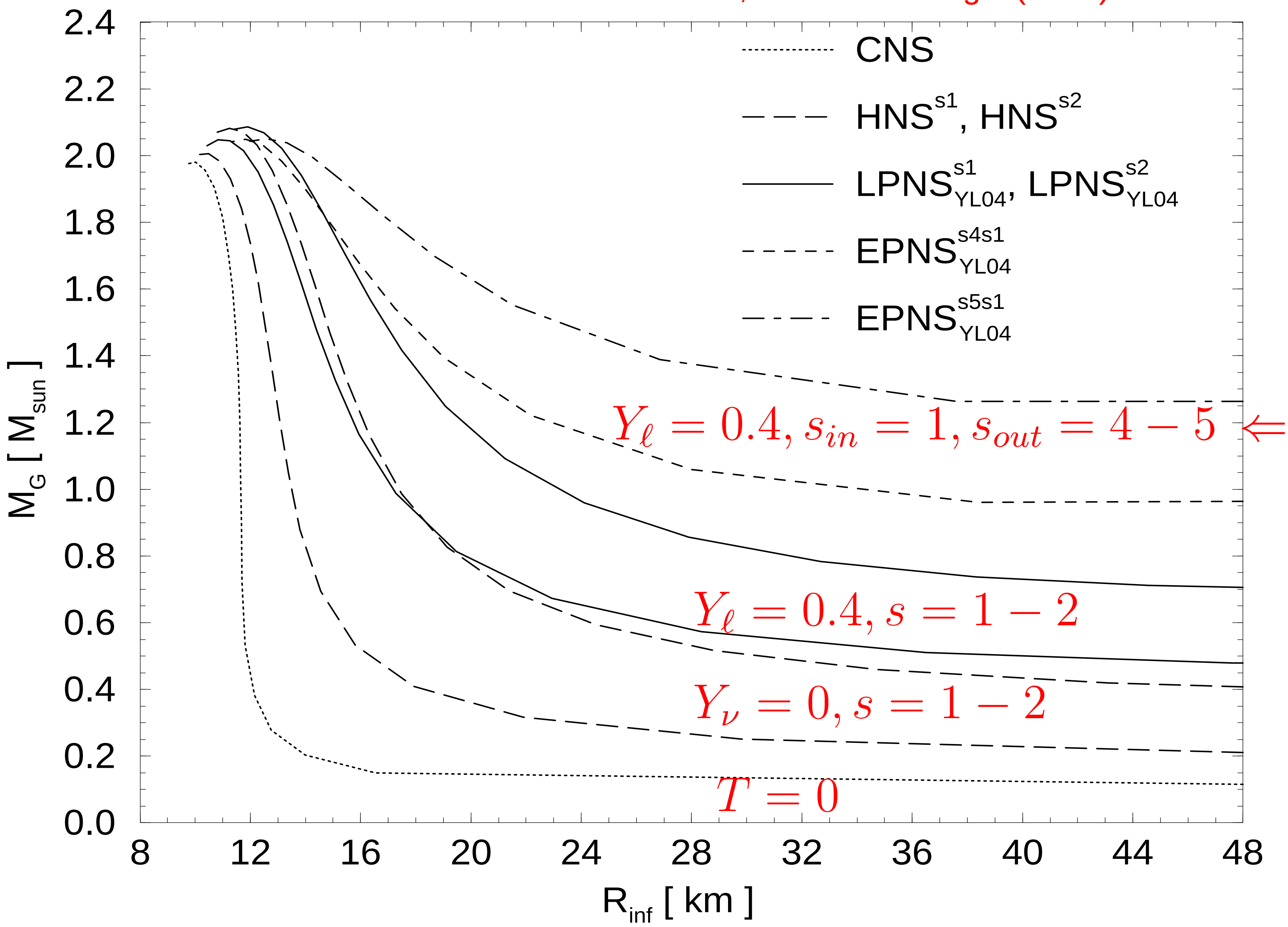
Champion et al. 2008

Proto-Neutron Stars



Effective Minimum Masses

Strobel, Schaab & Weigel (1999)



Neutron Star Matter Pressure and the Radius

$$p \simeq K\epsilon^{1+1/n}$$

$$n^{-1} = d\ln p/d\ln\epsilon - 1 \sim 1$$

$$R \propto K^{n/(3-n)}M^{(1-n)/(3-n)}$$

$$R \propto p_*^{1/2}\epsilon_*^{-1}M^0$$

$$(1 < \epsilon_*/\epsilon_0 < 2)$$

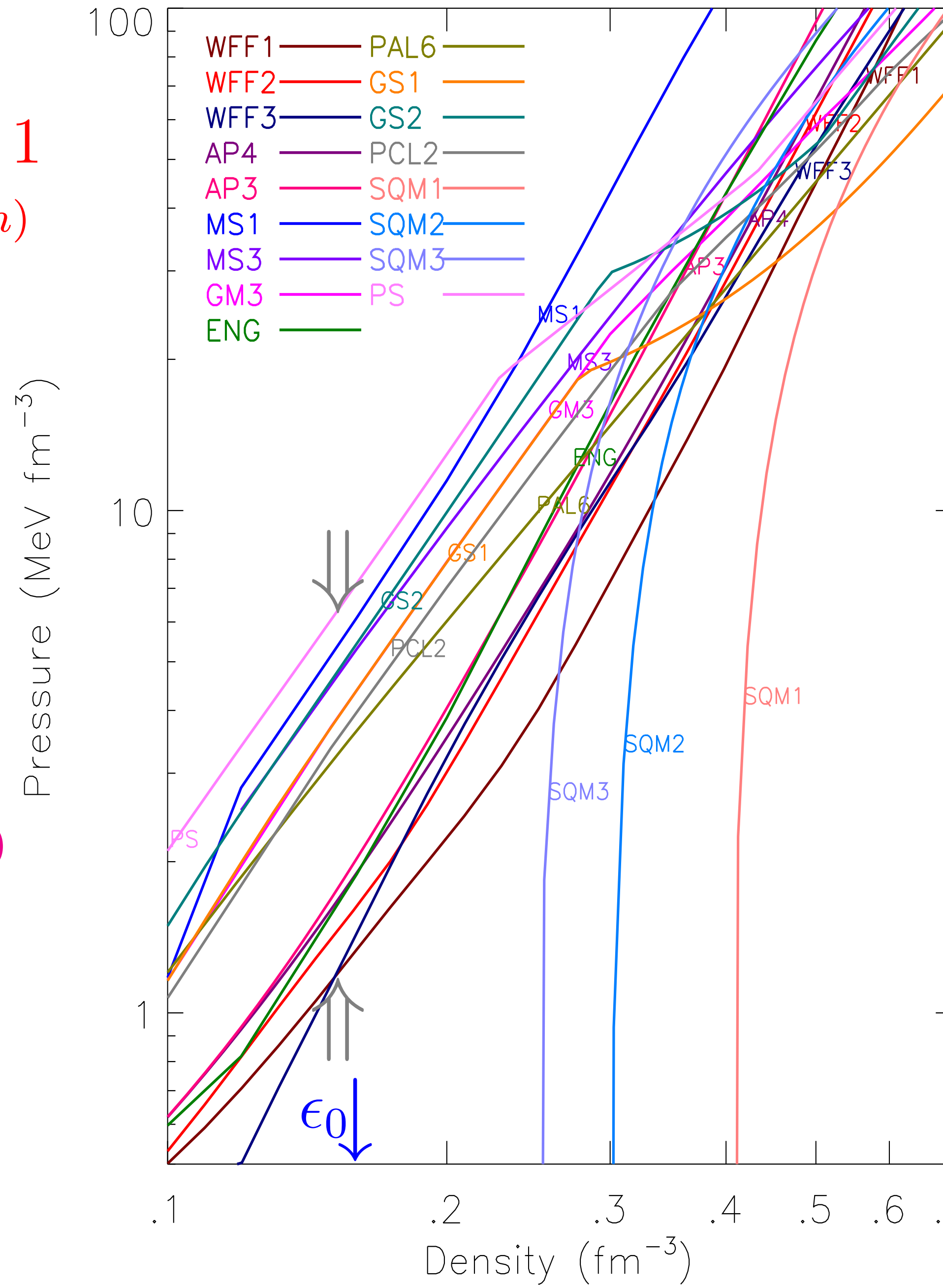
Wide variation:

$$1.2 < \frac{p(\epsilon_0)}{\mathrm{MeV\ fm^{-3}}} < 7$$

GR phenomenological result (Lattimer & Prakash 2001)

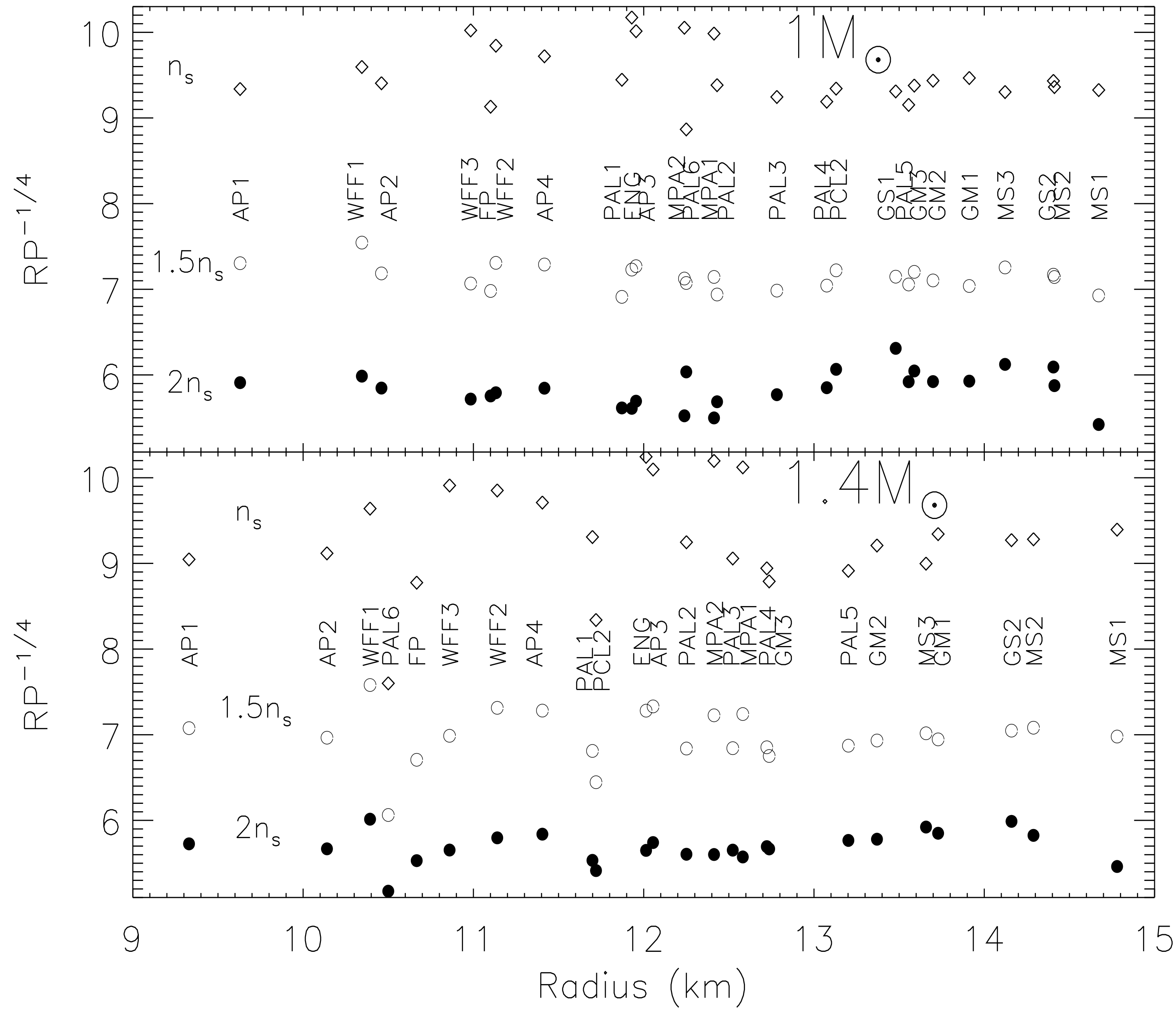
$$R \propto p_*^{1/4}\epsilon_*^{-1/2}$$

$$p_* = n^2\frac{dE_{sym}}{dn} = \frac{n^2 L}{3n_s}$$



The Radius – Pressure Correlation

$R \propto p^{1/4}$



Lattimer & Prakash (2001)

Nuclear Structure Considerations

Information about E_{sym} can be extracted from nuclear binding energies and models for nuclei. For example, consider the schematic liquid droplet model (Myers & Swiatecki):

$$E(A,Z) \simeq -a_v A + a_s A^{2/3} + \frac{S_v}{1+(S_s/S_v)A^{-1/3}} A + a_C Z^2 A^{-1/3}$$

Optimizing to energies of nuclei yields a strong correlation between S_v and S_s, but not highly significant individual values.

Blue: $\Delta E < 0.01$ MeV/b

Green: $\Delta E < 0.02$ MeV/b

Gray: $\Delta E < 0.03$ MeV/b

Circle: Moeller et al. (1995)

Crosses: Best fits

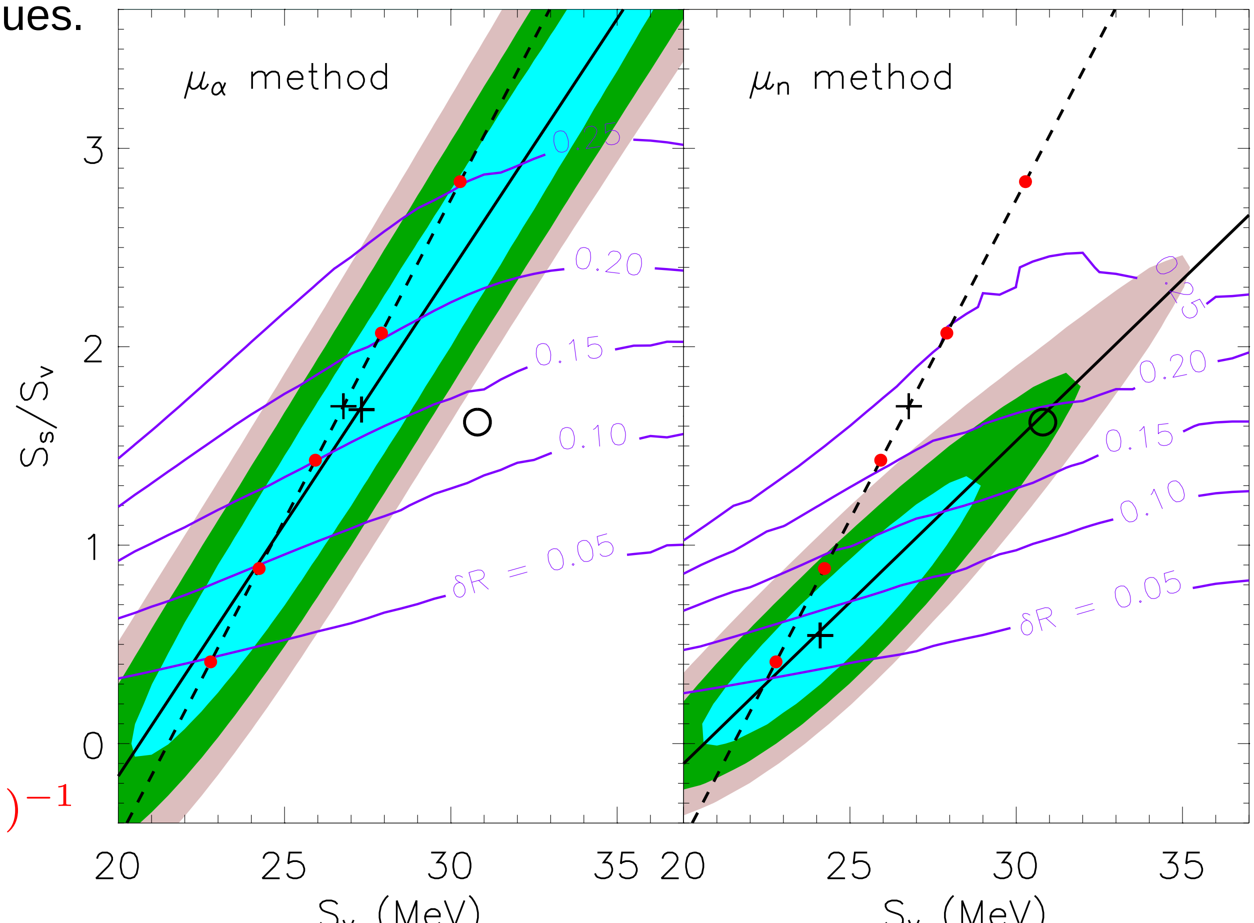
Dashed: Danielewicz (2004)

Solid: Steiner et al. (2005)

$a_C = \frac{3e^2}{5r_0} f$

$\mu_\alpha : f = 1 + \frac{A}{6Z}(1 + \frac{S_v}{S_s} A^{1/3})^{-1}$

$\mu_n : f = 1$



Schematic Dependence

Nuclear Hamiltonian:

$$H = H_B + \frac{Q}{2}n'^2\,, \quad H_B \simeq n\left[-B + \frac{K}{18}\left(1-\frac{n}{n_s}\right)^2\right] + E_{sym}(1-2x)^2$$

Lagrangian minimization of energy with respect to n (symmetric matter):

$$H_B - \mu_0 n = \frac{Q}{2}n'^2 = \frac{K}{18}n\left(1-\frac{n}{n_s}\right)^2\,, \qquad \mu_0 = -a_v$$

Liquid Droplet surface parameters: $a_s = 4\pi r_0^2\sigma_0, \qquad S_s = 4\pi r_0^2\sigma_\delta$

$$\sigma_0 = \int_{-\infty}^{+\infty}[H-\mu_0 n]dz = 2\int_0^{n_s}(H_B-\mu_0 n)\frac{dn}{n'} = \frac{4}{45}\sqrt{QKn_s^3}$$

$$t_{90-10} = \int_{0.1n_s}^{0.9n_s}\frac{dn}{n'} = 3\sqrt{\frac{Qn_s}{K}}\int_{0.1}^{0.9}\frac{du}{\sqrt{u}(1-u)} \simeq 9\sqrt{\frac{Qn_s}{K}}$$

$$\sigma_\delta = S_v\sqrt{\frac{Q}{2}}\int_0^{n_s} n\left(\frac{S_v}{E_{sym}}-1\right)(H_B-\mu_0 n)^{-1/2}dn$$

$$\frac{S_s}{S_v} = \frac{t_{90-10}}{r_0}\int_0^1\frac{\sqrt{u}}{1-u}\left(\frac{S_v}{E_{sym}}-1\right)du, \qquad \delta R = \sqrt{\frac{3}{5}}r_0\frac{S_s}{S_v}\frac{N-Z}{3Z}$$

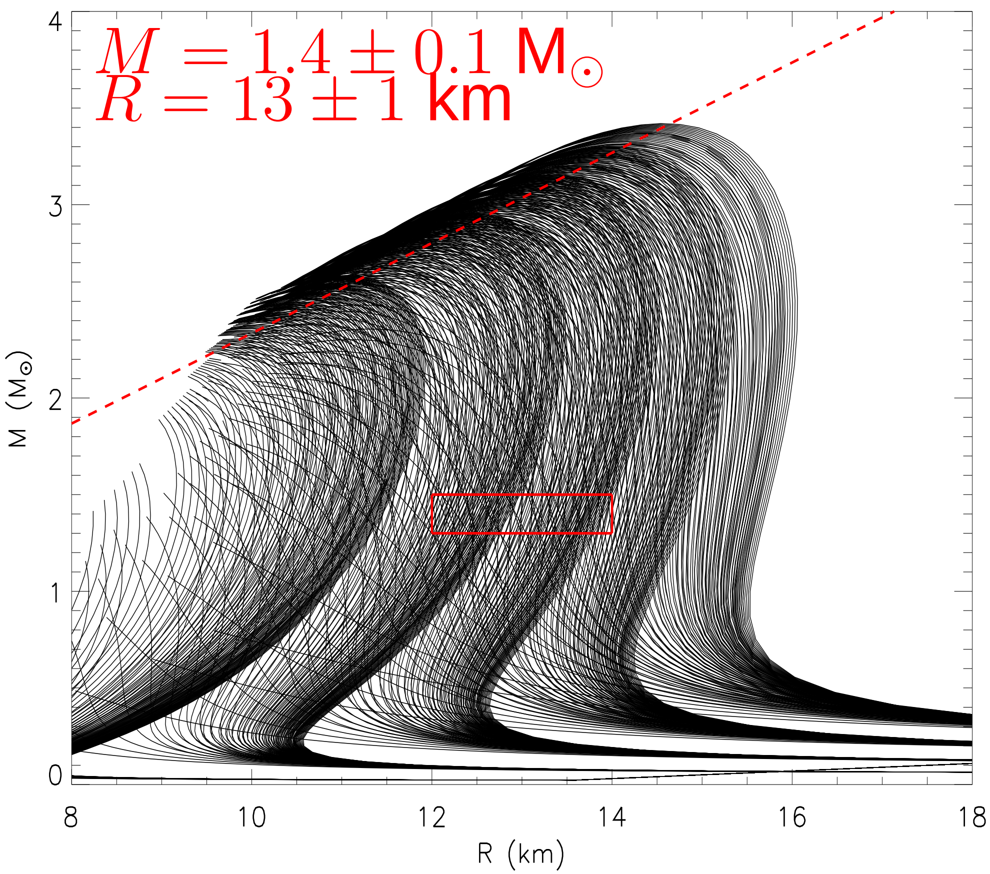
$$E_{sym} \simeq S_v\left(\frac{n}{n_s}\right)^p \Longrightarrow \int \to 0.28\ (p=\frac{1}{2})\,,\ 0.93\ (p=\frac{2}{3})\,,\ 2.0\ (p=1)$$

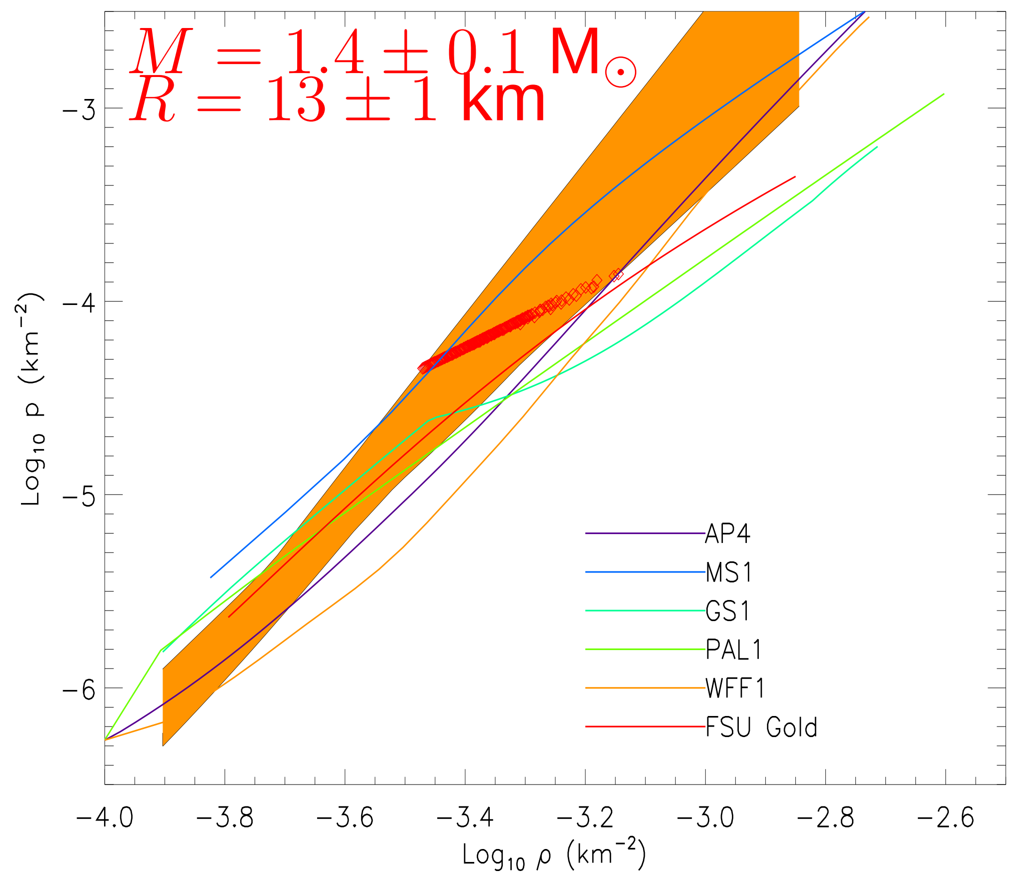
$$E_{sym} \simeq S_v + \frac{L}{3}\left(\frac{n}{n_s}-1\right) \Longrightarrow \int \to 2 - 2\sqrt{\frac{3S_v}{L}-1}\tan^{-1}\sqrt{\left(1+\frac{S_v}{3L}\right)^{-1}} \simeq \frac{2L}{3S_v}$$

TOV Inversion

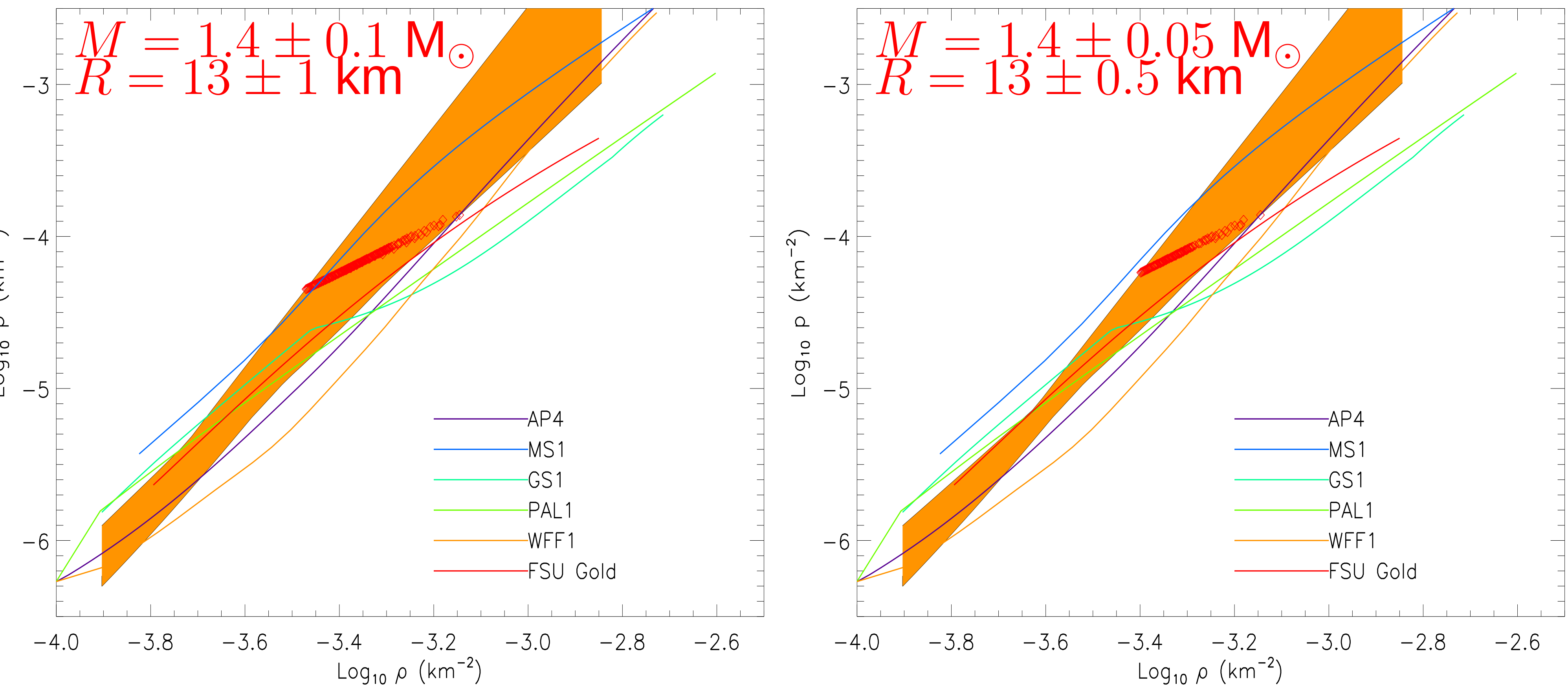
How would a simultaneous $M-R$ determination constrain the EOS? Each M-R curve specifies a unique $p-\rho$ relation.

- Generate physically reasonable $M-R$ curves and the $p-\rho$ relations that they specify.
- Generate physical $p-\rho$ relations and compute $M-R$ curves from them; select those $M-R$ curves passing within the error box.





TOV Inversion (cont.)



In this example, variations in the assumed subnuclear EOS are unrealistic. Realistic constraints on the EOS up to n_s will reduce the width of the allowed pressure-density regions.

Observational Constraints for Neutron Stars

- Maximum and Minimum Masses (binary pulsars)
- Minimum Rotational Period*
- Radiation Radius or Redshift*
- Neutron Star Thermal Evolution (URCA or not)*
- Crustal Cooling Timescale from X-ray Transients*
- X-ray Bursts from Accreting Neutron Stars*
- Seismology from Giant Flares in SGR's*
- Moment of Inertia*
- Proto-Neutron Star Neutrinos (Binding Energy, Opacities, Radii)*
- Pulse Shape Modulation*
- Gravitational Radiation* (Masses, Radii from tidal Love numbers)

* Significant dependence on symmetry energy

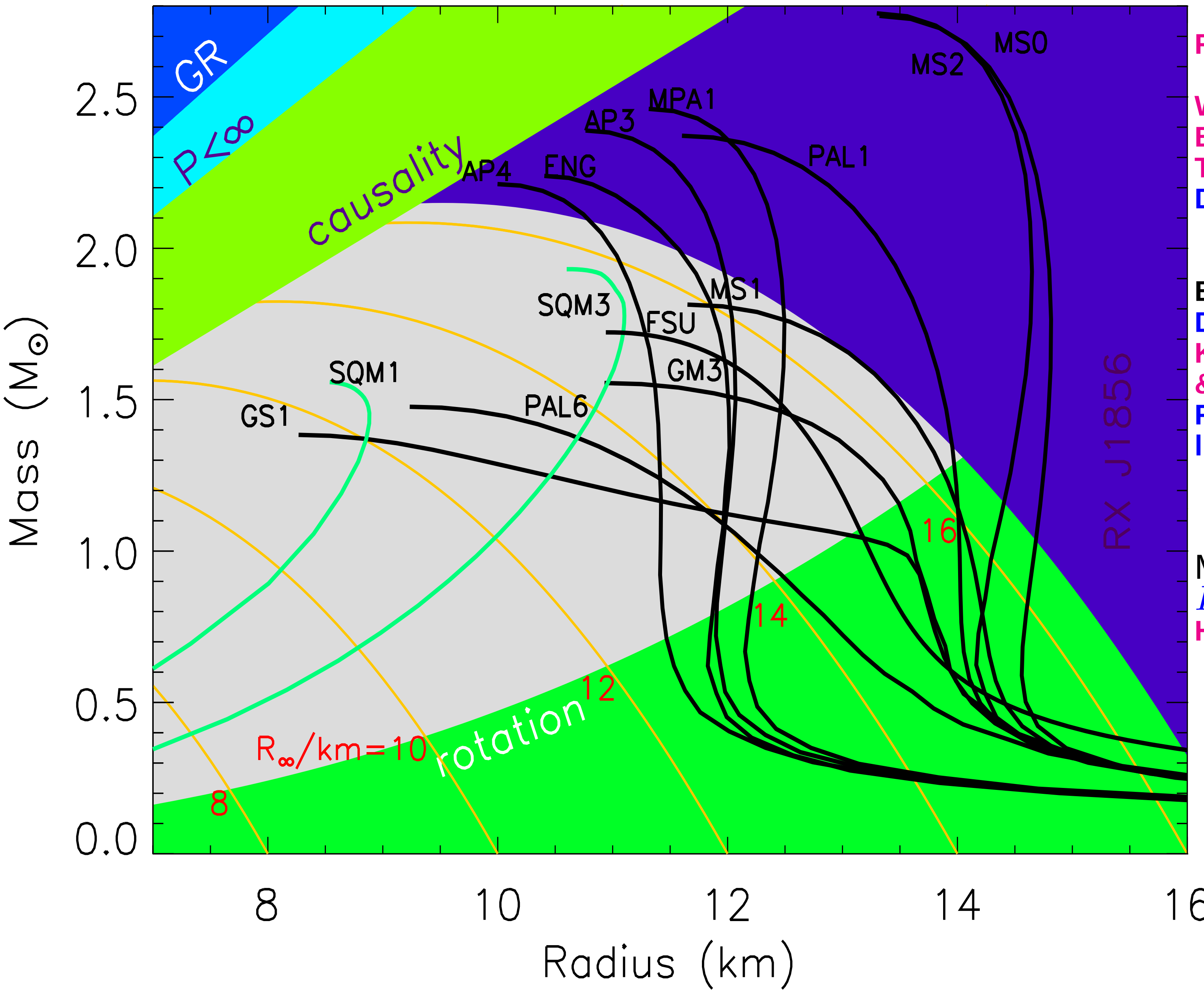
Radiation Radius

- Combination of flux and temperature measurements yields apparent angular diameter (pseudo-BB):

$$\frac{R_\infty}{d} = \frac{R}{d}\frac{1}{\sqrt{1-2GM/Rc^2}}$$

- Observational uncertainties include distance, interstellar H absorption (hard UV and X-rays), atmospheric composition
- Best chances for accurate radii are from
  - Nearby isolated neutron stars (parallax measurable)
  - Quiescent X-ray binaries in globular clusters (reliable distances, low B H-atmosperes)
  - X-ray pulsars in systems of known distance
    - CXOU J010043.1-721134 in the SMC: $R_\infty \geq 10.8$ km (Esposito & Mereghetti 2008)

Radiation Radius: Nearby Neutron Star



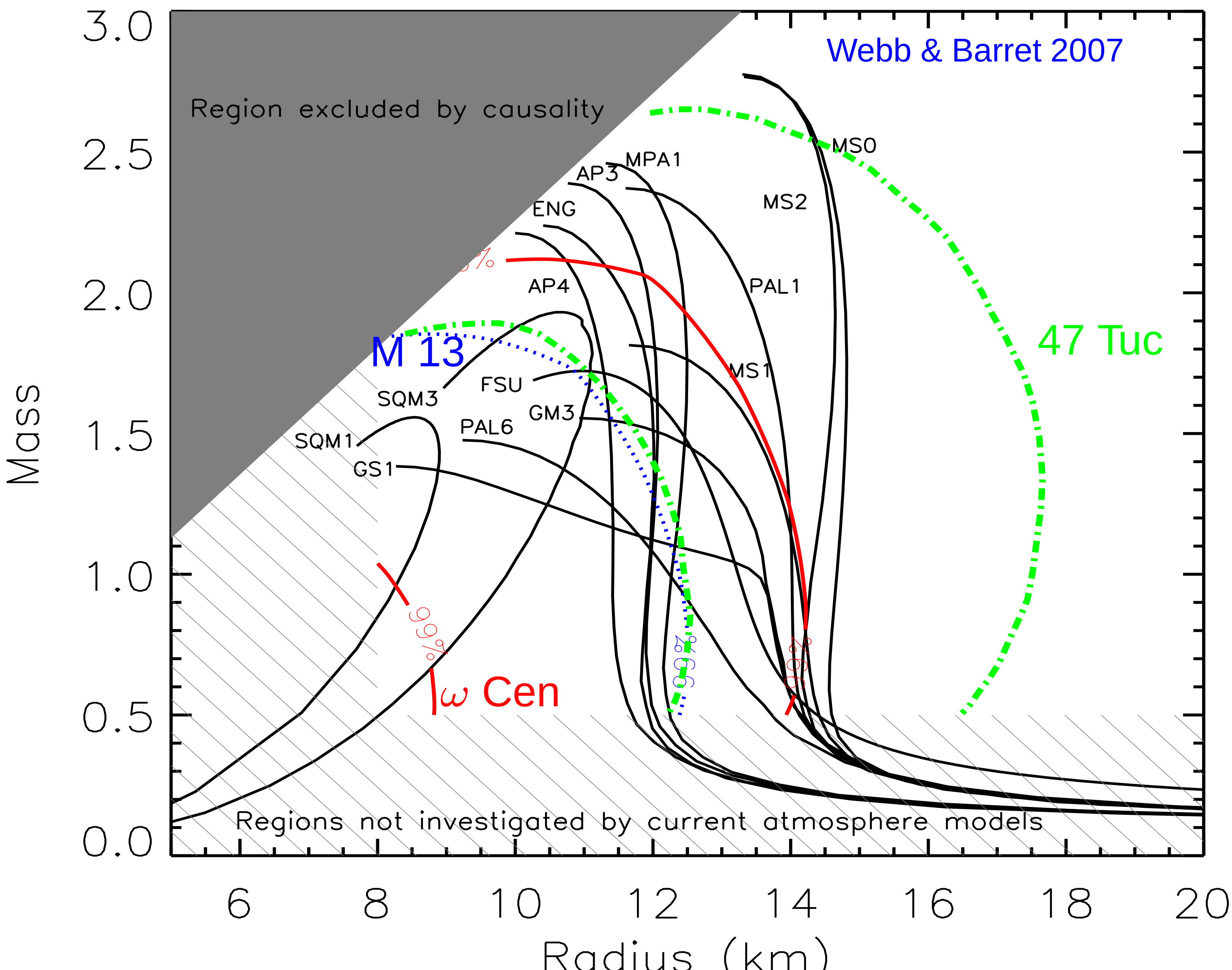
RX J1856-3754:

Walter & Lattimer 2002
Braje & Romani 2002
Truemper 2005
D=120 pc

BUT
D=140 pc
Kaplan, van Kerkwijk & Anderson 2002
Raises R_∞ limit to 19.5 km

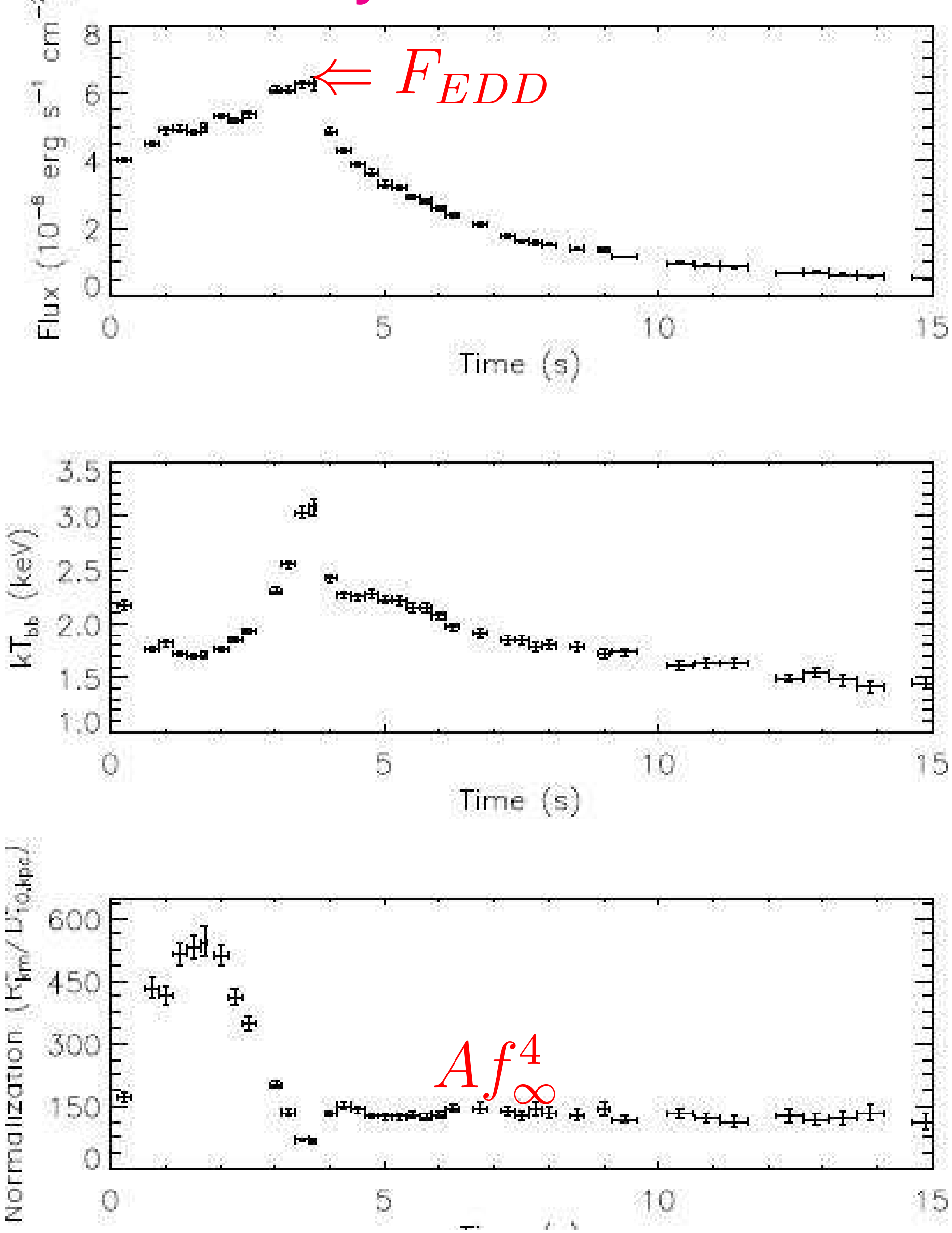
Magnetic H atmosphe
$R_\infty \approx 17$ **km**
Ho et al. 2007

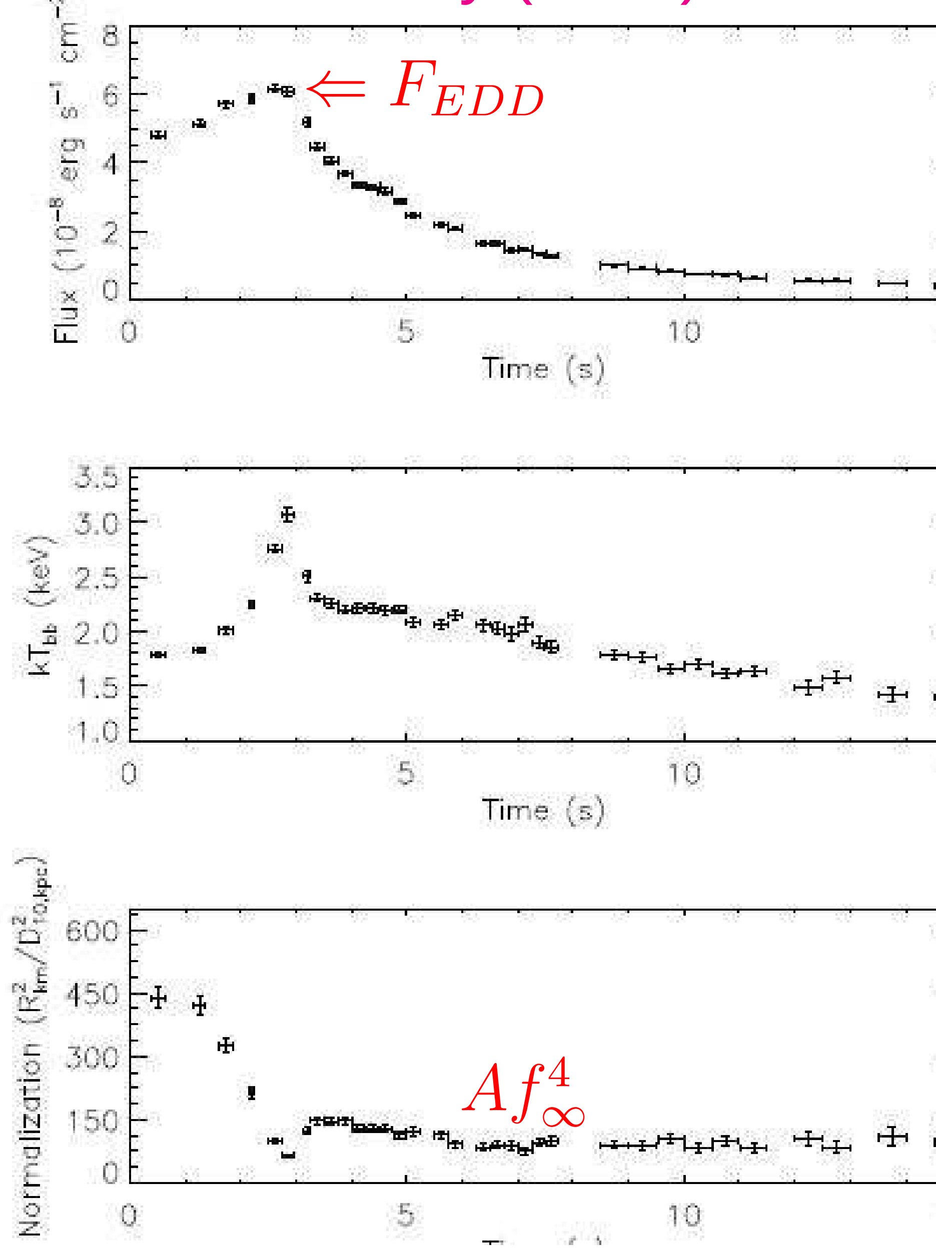
Radiation Radius: Globular Cluster Sources



Cooling Following An X-Ray Burst

Galloway, Muno, Hartman, Psaltis & Chakrabarty (2006)





Elementary Analysis

$$\alpha \equiv \frac{F_{EDD}\kappa D^2}{c^3} = \frac{GM}{c^2}\left(1-\frac{2GM}{Rc^2}\right)^{1/2}$$

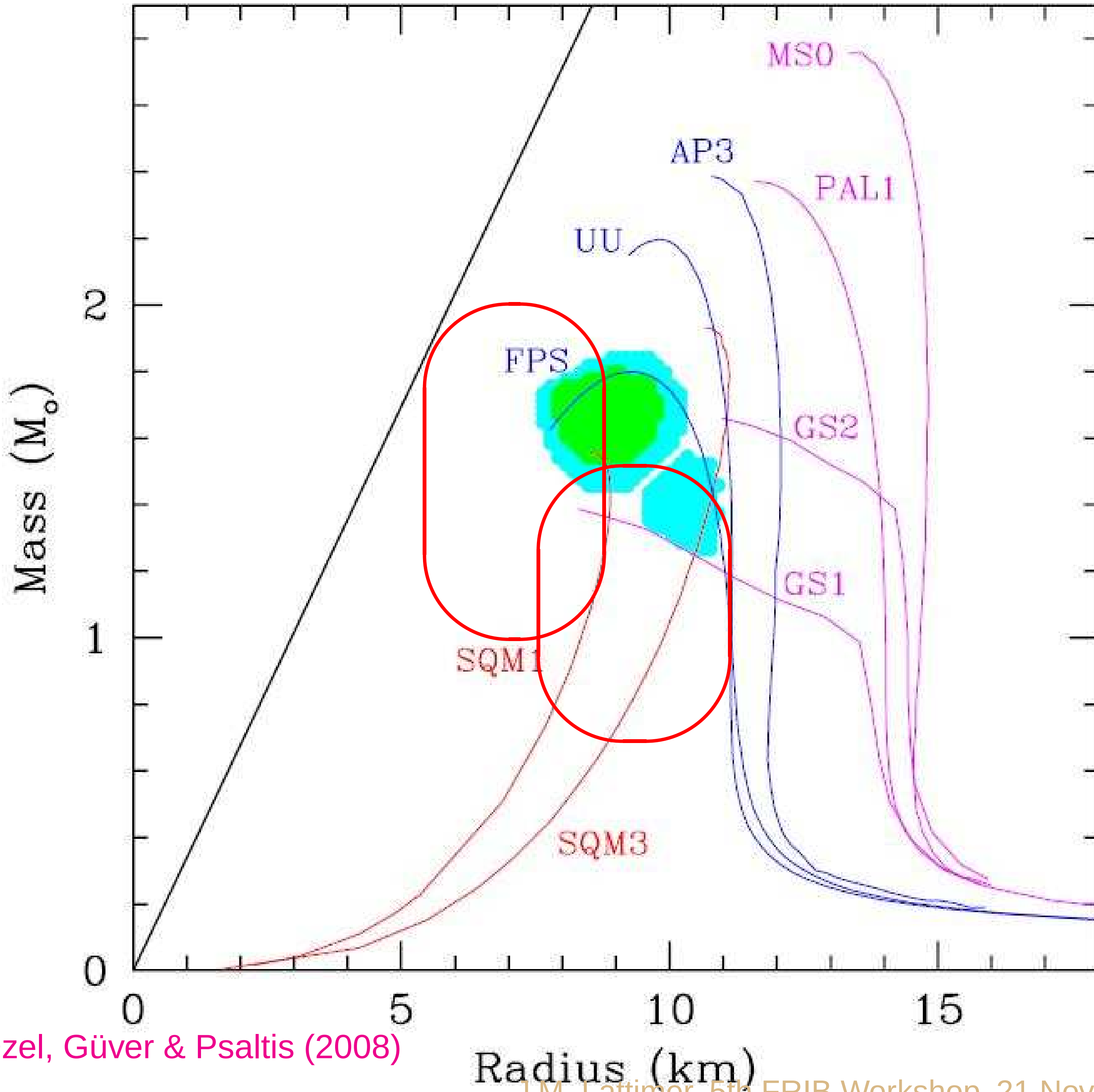
$$\beta \equiv A^{1/2} f_\infty^2 D = R\left(1-\frac{2GM}{Rc^2}\right)^{-1/2}$$

- $F_{EDD} = 6.25 \pm 0.20 \cdot 10^{-8}$ erg/cm^2/s
- $\kappa \simeq 0.2$ cm^2/g
- $D = 5.5 \pm 0.9$ kpc
- $A = 1.16 \pm 0.13$ km^2/kpc^2
- $f_\infty \simeq 1.4 \qquad \Rightarrow$
- $\alpha \simeq 1.35 \pm 0.31$ km
- $\beta \simeq 11.8 \pm 2.2$ km

$$\frac{GM}{Rc^2} = \frac{1}{4} \pm \frac{1}{4}\sqrt{1-8\frac{\alpha}{\beta}} = 0.323 \pm 0.036, \qquad 0.177 \pm 0.020$$

$$R = \sqrt{\alpha\beta\frac{Rc^2}{GM}} = 7.02 \pm 1.57 \text{ km}, \qquad 9.49 \pm 2.12 \text{ km}$$

$$M = \frac{c^2}{G}\alpha\left(1-\frac{2GM}{Rc^2}\right)^{-1/2} = 1.54 \pm 0.57\ \mathrm{M}_\odot, \qquad 1.14 \pm 0.42\ \mathrm{M}_\odot$$

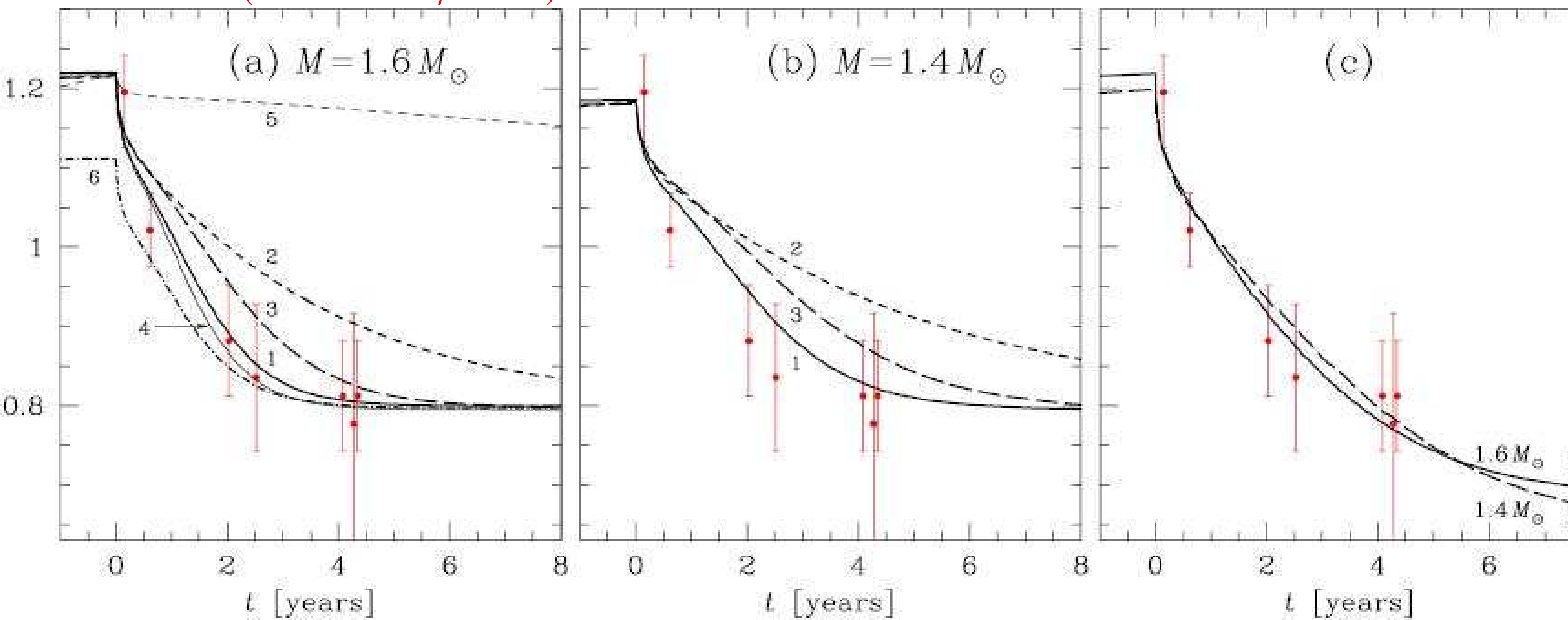


Özel, Güver & Psaltis (2008)

Crustal Heating in X-Ray Transients

Observations:
Cackett, Wijnands, Linares, Miller, Homan & Lewin (2006)

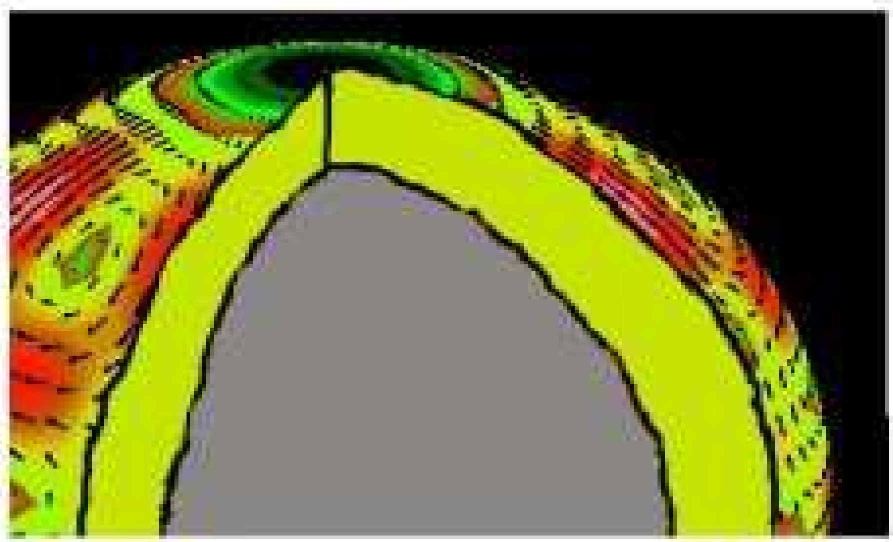
$$\tau \propto \frac{C_V \Delta^2}{\kappa(1-2GM/Rc^2)^{3/2}} \propto \frac{C_V(1-2GM/c^2)^{1/2}\ln^2 \mathcal{H} R^4}{\kappa M^2}$$

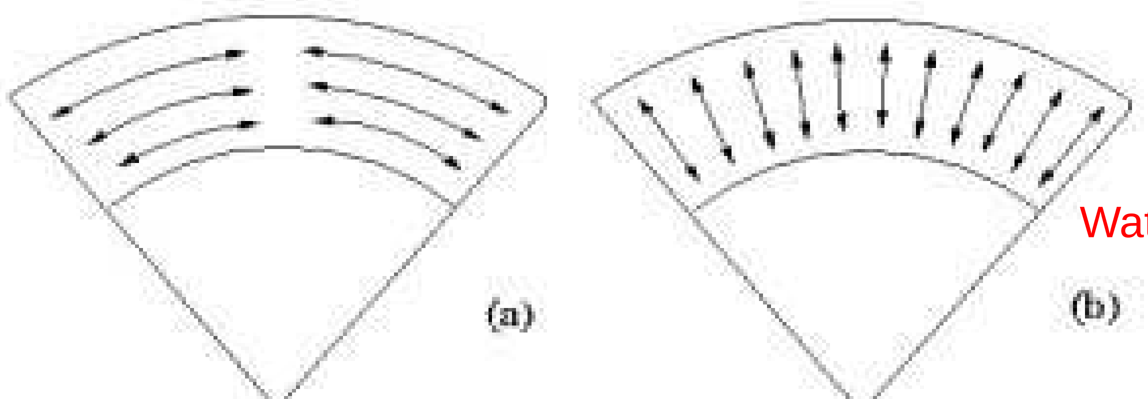


Shertnin, Yakovlev, Haensel & Potekhin (2007)

Giant Flares in Soft Gamma-Ray Repeaters (SGRs)

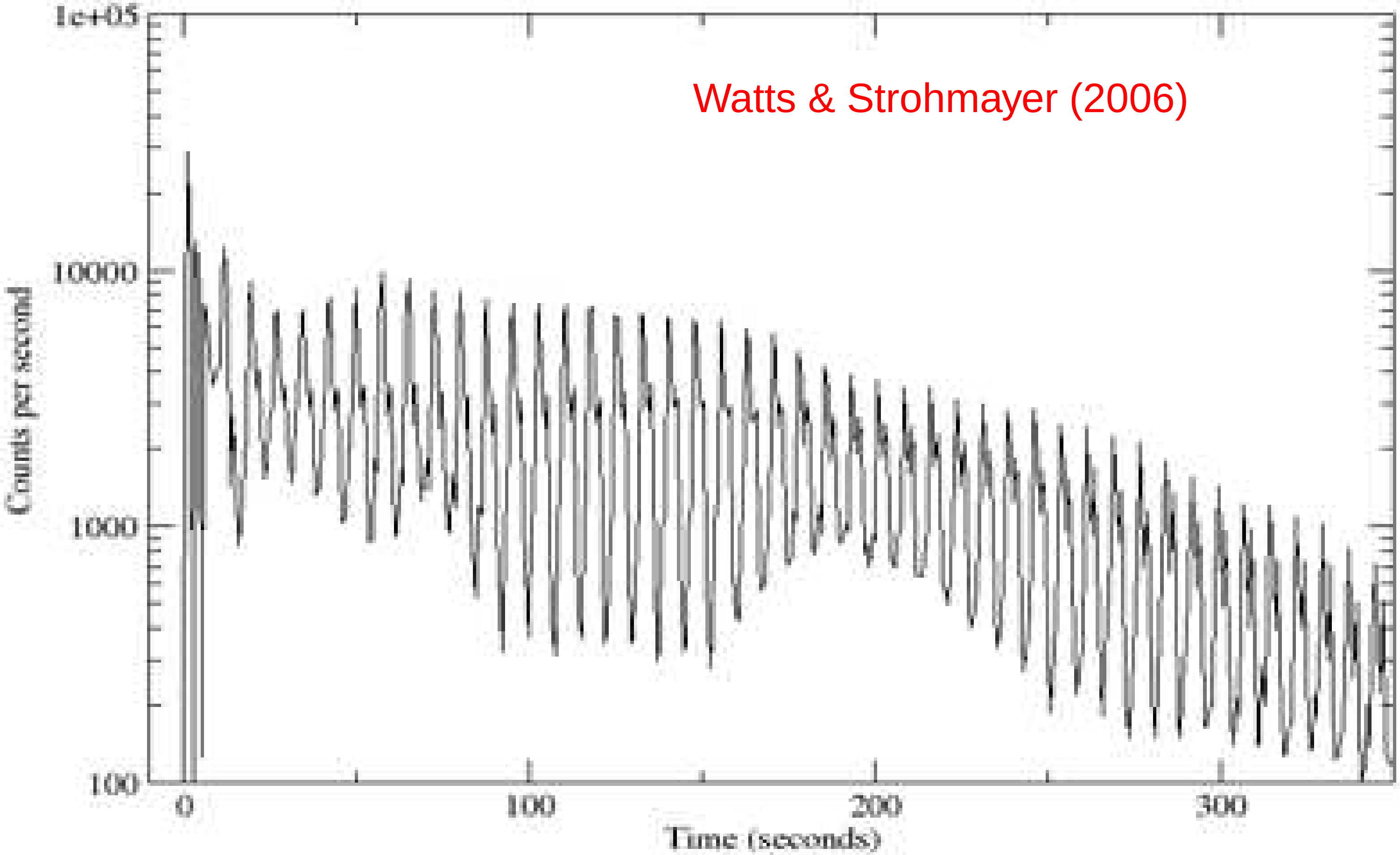
Quasi-periodic oscillations observed following giant flares in three soft gamma-ray repeaters (Israel et al. 2005; Strohmayer & Watts 2005, 6; Watts & Strohmayer 2006) which are believed to be highly magnetized neutron stars (magnetars).
Fields decay and twist, becoming periodically unstable. Eventually, the field lines snap and shift, launching starquakes and bursts of gamma-rays. Torsional shear modes are much easier to excite than radial modes.





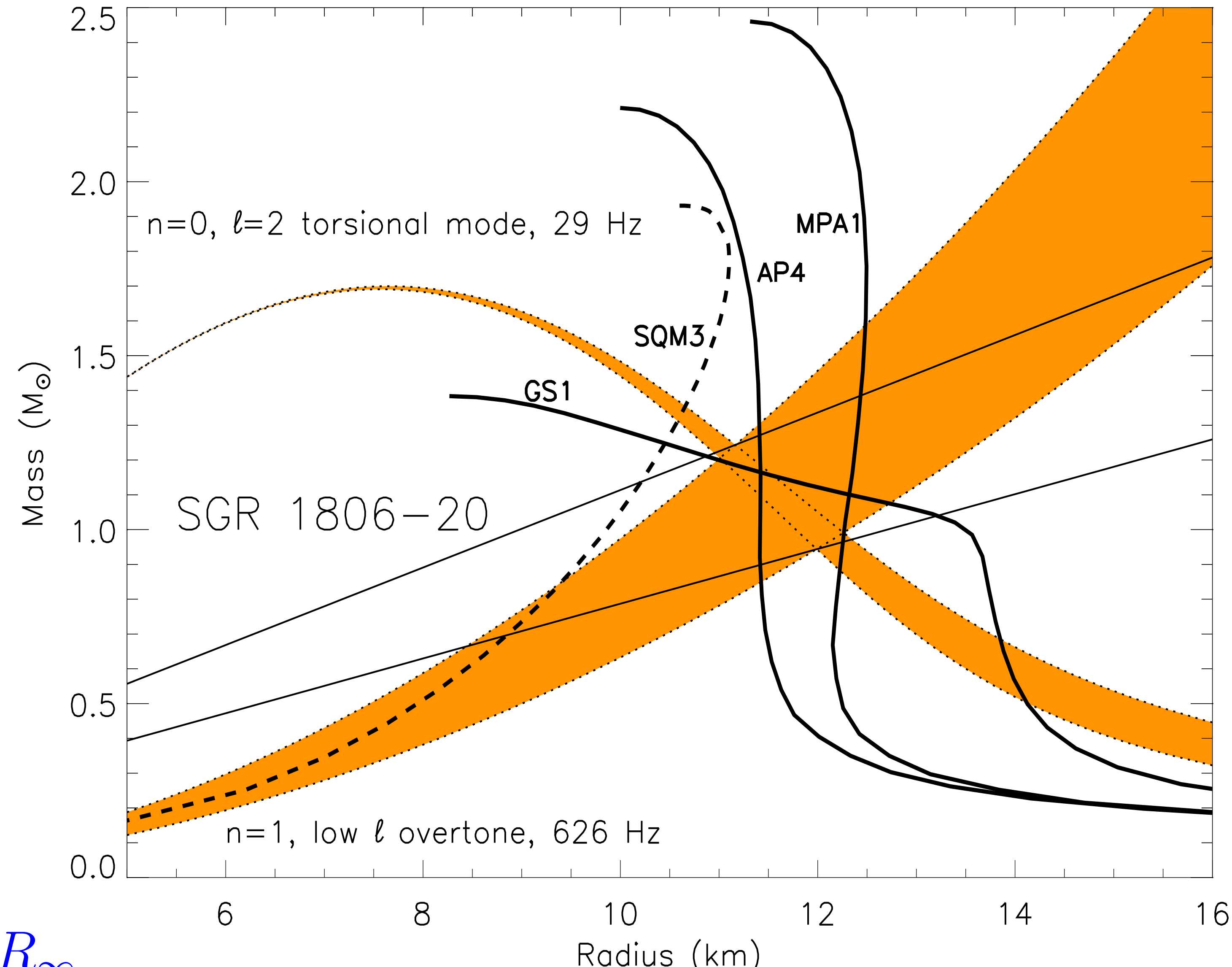
Watts & Reddy (2006)

Observations



Typical frequencies observed: 28-29 Hz, 50-150 Hz, 625 Hz
(SGRs 1806-20, 1900+14, 0526-66)
Frequencies of fundamental mode (28-29 Hz) agree well with expected torsional mode of neutron star crust (Duncan 1998)

Neutron Star Seismology



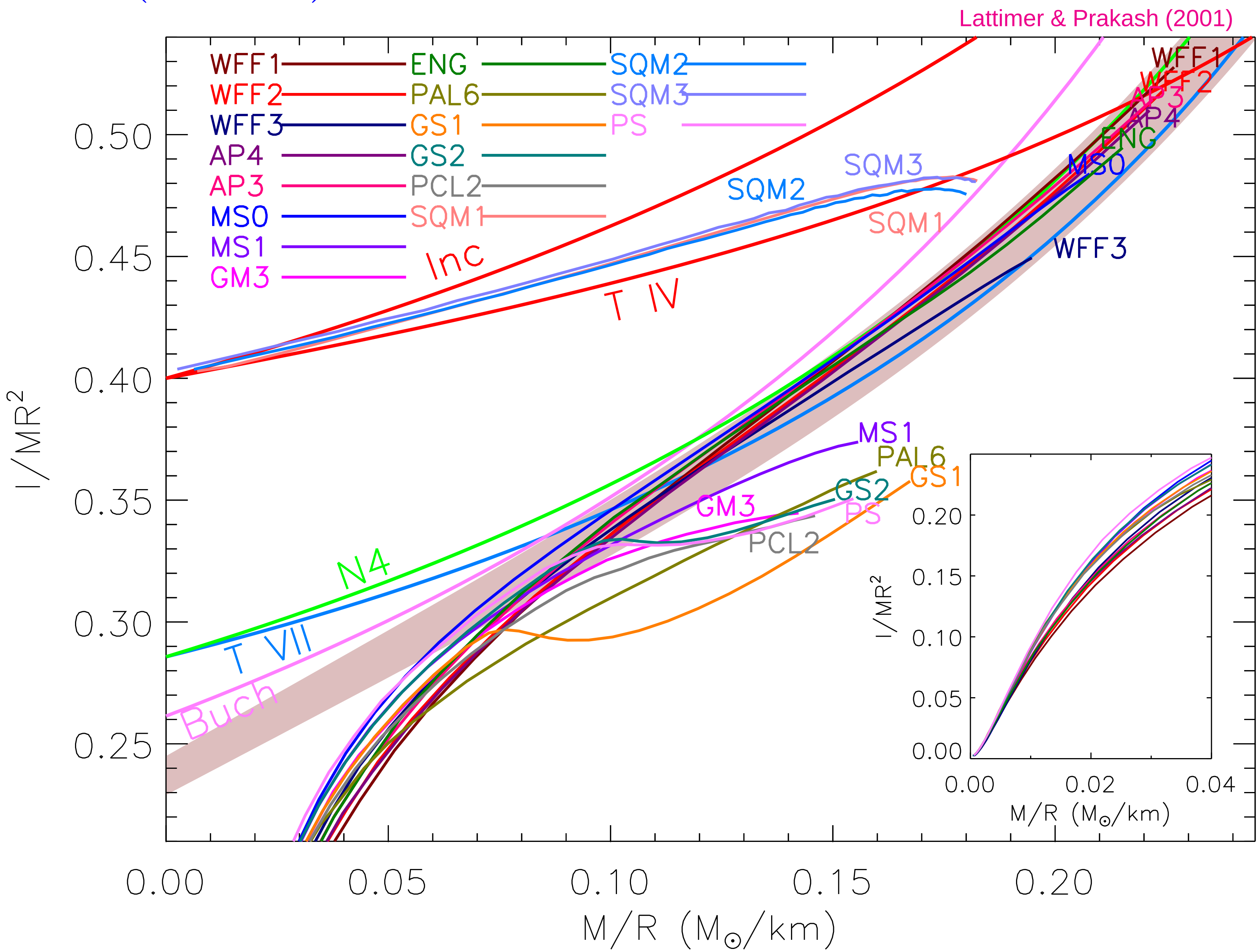
$$f_{n=0} \sim v_t / R_\infty$$
$$f_{n>0} \sim v_r(1-2\beta)/\Delta \sim (v_r M / R^2 \ln \mathcal{H})$$

Strohmayer & Watts (2005)
Samuelsson & Andersson (2006)
Lattimer & Prakash (2006)

Moment of Inertia

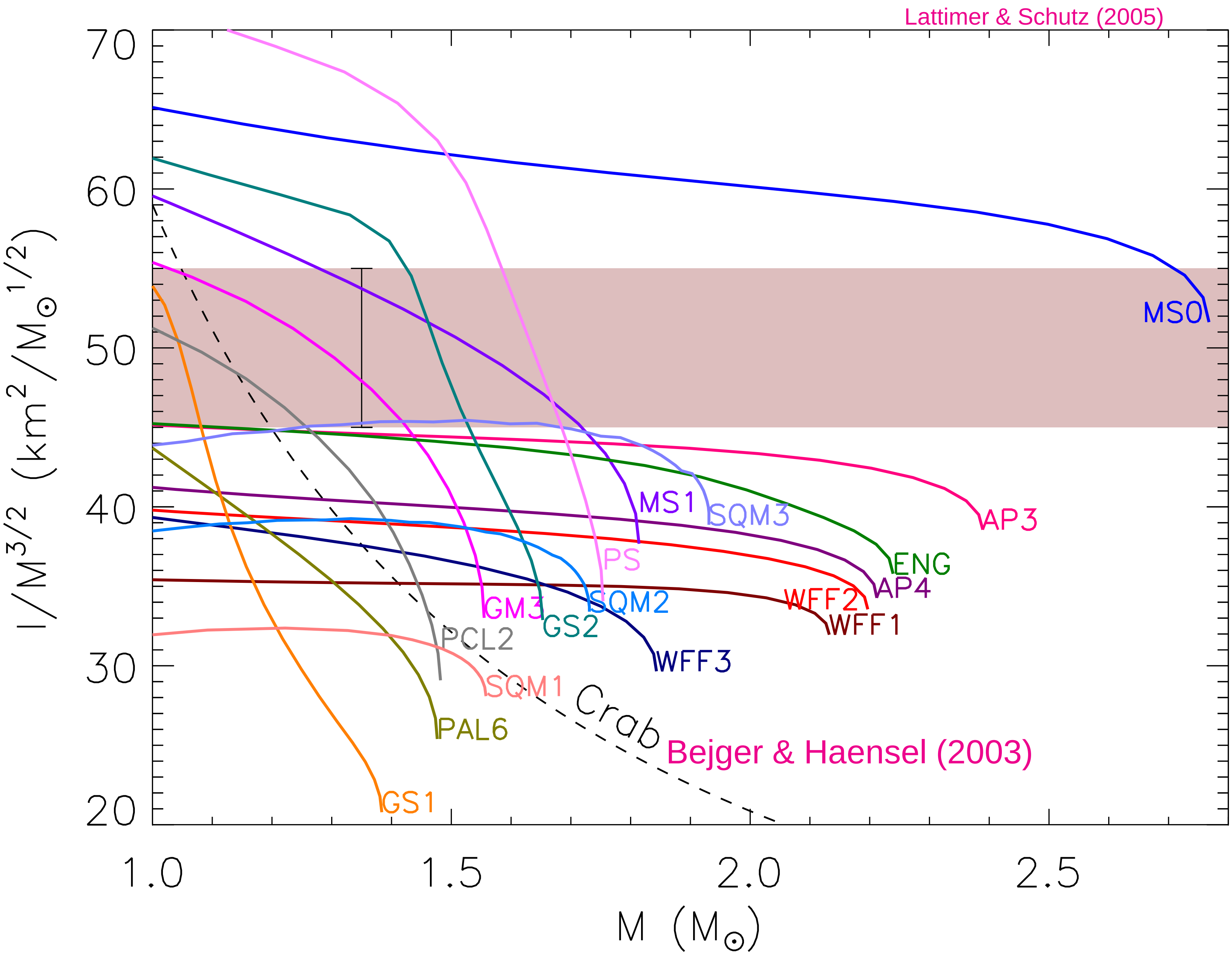
- Spin-orbit coupling of same magnitude as post-post-Newtonian effects (Barker & O'Connell 1975, Damour & Schaeffer 1988)
- Precession alters inclination angle and periastron advance
- More EOS sensitive than R: $I \propto MR^2$
- Requires extremely relativistic system to extract
- Double pulsar PSR J0737-3037 is a marginal candidate
- Even more relativistic systems should be found, based on dimness and nearness of PSR J0737-3037

$I(M,R)$

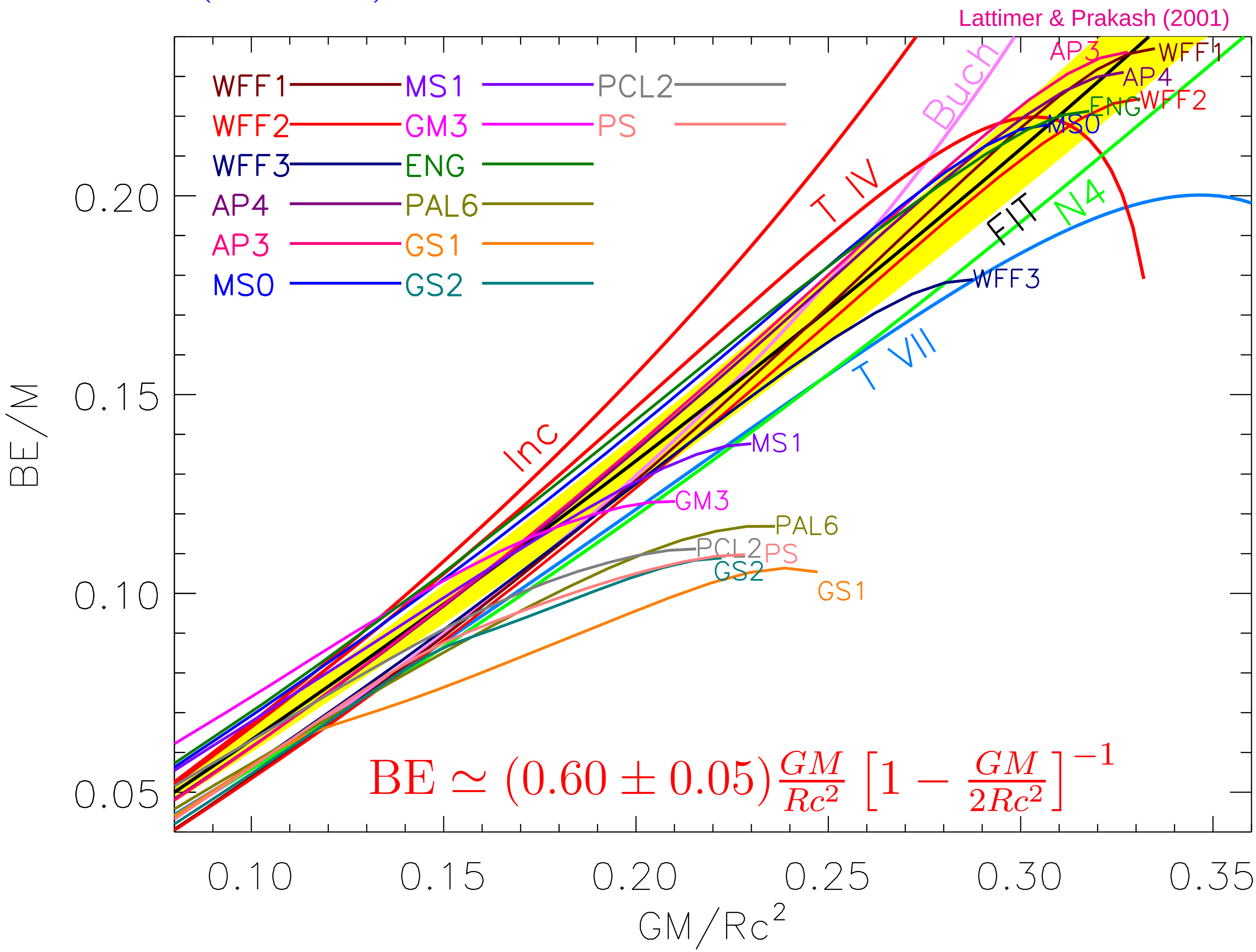


$$I \simeq (0.237 \pm 0.008) M R^2 \left[1 + 4.2 \frac{M \text{ km}}{R \text{ M}_\odot} + 90 \left(\frac{M \text{ km}}{R \text{ M}_\odot} \right)^4 \right]$$

EOS Constraint



$\mathrm{BE}(M, R)$

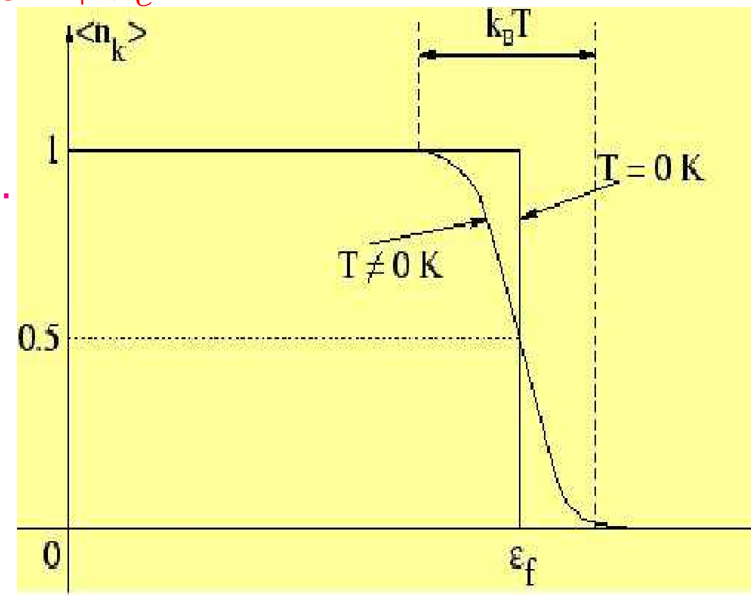


Lattimer & Prakash (2001)
WFF1
WFF2
WFF3
AP4
AP3
MS0
MS1
GM3
ENG
PAL6
GS1
GS2
PCL2
PS
Inc
T IV
Buch
FIT
N4
T VII
0.05
0.10
0.15
0.20
0.25
0.30
0.35
BE/M
GM/Rc^2
$\mathrm{BE} \simeq (0.60 \pm 0.05)\frac{GM}{Rc^2}\left[1 - \frac{GM}{2Rc^2}\right]^{-1}$

Neutron Star Cooling

Gamow & Schönberg proposed the direct Urca process: nucleons at the top of the Fermi sea beta decay.

$$n \rightarrow p + e^- + \nu_e \,, \qquad p \rightarrow n + e^+ + \bar{\nu}_e$$



Energy conservation guaranteed by beta equilibrium $\mu_n - \mu_p = \mu_e$

Momentum conservation requires $|k_{Fn}| \leq |k_{Fp}| + |k_{Fe}|$.

Charge neutrality requires $k_{Fp} = k_{Fe}$,

therefore $|k_{Fp}| \geq 2|k_{Fn}|$.

Degeneracy implies $n_i \propto k_{Fi}^3$, thus $x \geq x_{DU} = 1/9$.

With muons ($n > 2n_s$), $x_{DU} = \frac{2}{2+(1+2^{1/3})^3} \simeq 0.148$

If $x < x_{DU}$, bystander nucleons needed: modified Urca process is then dominant.

$$(n,p) + n \rightarrow (n,p) + p + e^- + \nu_e, \qquad (n,p) + p \rightarrow (n,p) + n + e^+ + \bar{\nu}_e$$

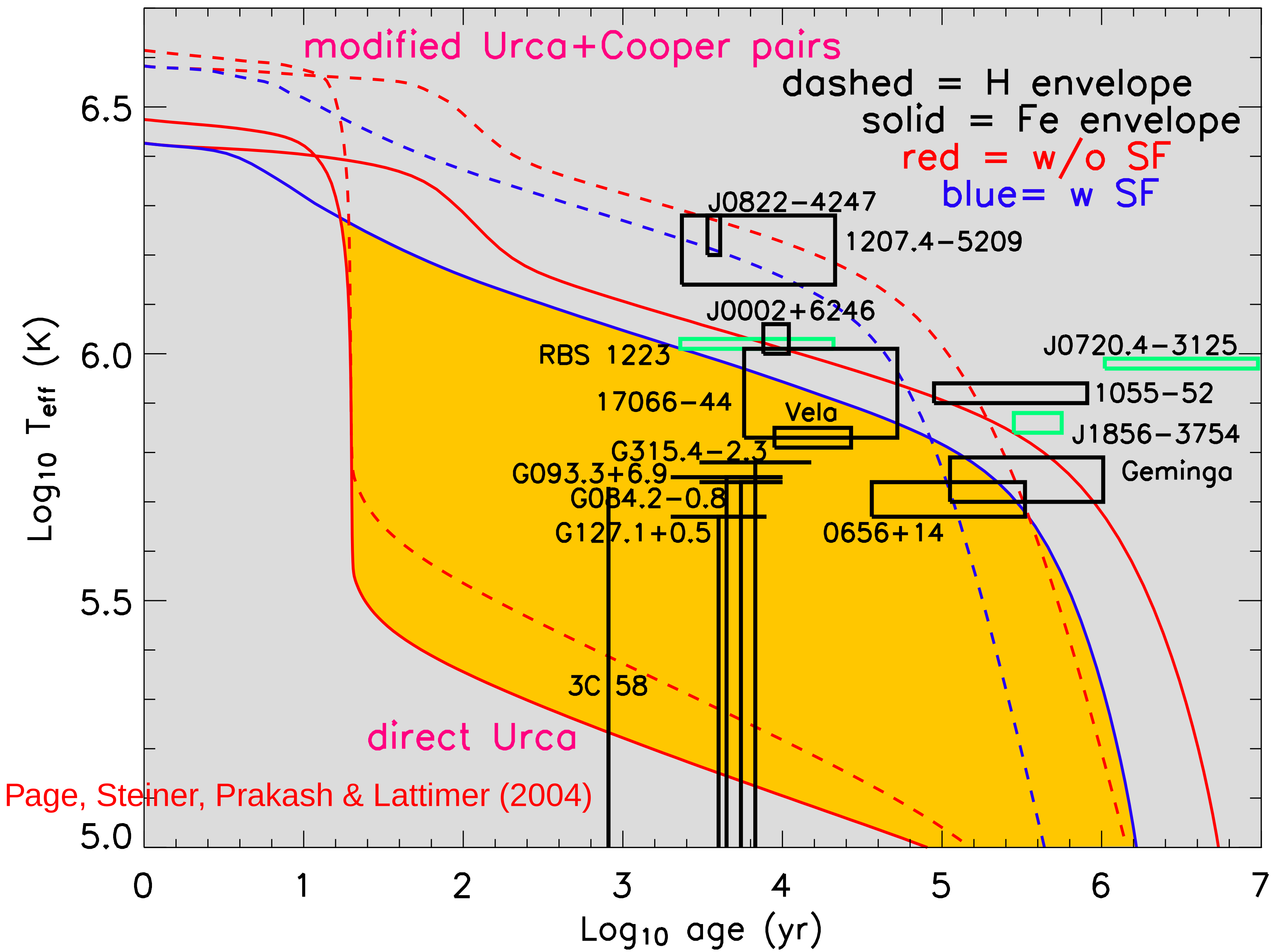
Neutrino emissivities:

$$\dot{\epsilon}_{MURCA} \simeq \left(\frac{T}{\mu_n}\right)^2 \dot{\epsilon}_{DURCA} \,.$$

Beta equilibrium composition:

$$x_\beta \simeq (3\pi^2 n)^{-1} \left(\frac{4E_{sym}}{\hbar c}\right)^3 \simeq 0.04 \left(\frac{n}{n_s}\right)^{0.5-2} \,.$$

Neutron Star Cooling



Conclusions

- **We thank Katsu for his many contributions to the study of hot, dense matter in supernovae and neutron stars.**
- **Neutron stars are a powerful laboratory to constrain dense matter physics, especially the symmetry energy and composition at supranuclear densities.**
- **Increasing evidence exists for massive neutron stars ($M \gtrsim 1.7$ $M_\odot$).**
- **Many kinds of observations are becoming available to measure neutron star radii, although no definitive measures yet exist.**