From Stopping to Viscosity in Nuclear Reactions

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Bulk Nuclear Properties
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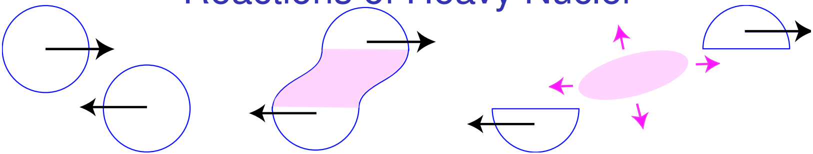
Reactions of Heavy Nuclei



Late in a reaction, matter describable in terms of a local temp T and velocity $\vec{v}$. Dissipation is responsible for equilibration.

$\Rightarrow$?Pace of the dissipation? ?Quantitative description of the dissipative transport??

Reactions well described in terms of the Boltzmann equation

$$\frac{\partial f}{\partial t} + \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} \frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{r}} \frac{\partial f}{\partial \mathbf{p}} = \int \mathrm{d}\mathbf{p}_2 \, \mathrm{d}\Omega' \, v_{12} \, \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega'} \left(\tilde{f}_1 \, \tilde{f}_2 \, f_1' \, f_2' - \tilde{f}_1' \, \tilde{f}_2' \, f_1 \, f_2 \right)$$

where f - nucleon Wigner function, $\tilde{f} = 1 - f$ - blocking factor, ϵ - single-nucleon energy related to the equation of state

Boltzmann valid at low-n/high-T. At high-n - phenomenological.

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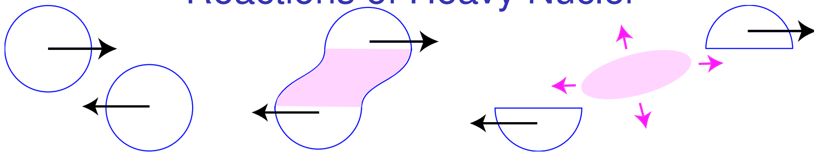

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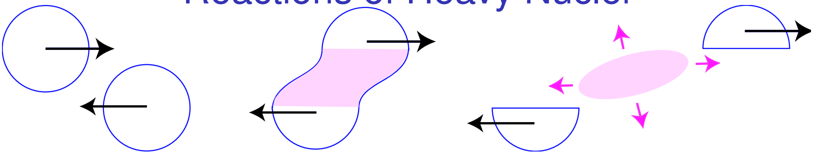

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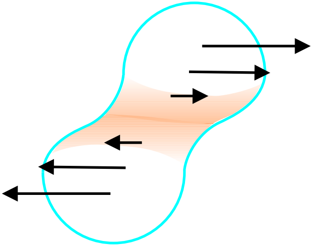
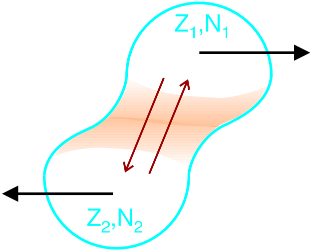
Dissipative Transport

Examples of the transport on the way towards equilibrium:



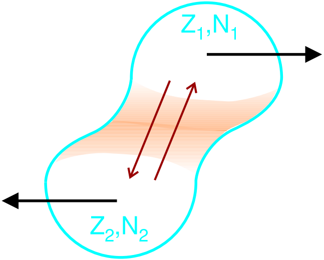
Transport of momentum



Transport of isospin, i.e. of neutron-proton imbalance

Transport of isospin of interest due to availability of exotic beams - with unusual neutron-proton content.

Transport Coefficients



For slow changes, the flux of a transported quantity is linear in gradients, with proportionality coefficients characteristic for the matter.

Complication: Different gradients present in a reaction - many coefficients?!

⇒ Curie (Pierre) Principle allows some sorting of relations.

Strategy: Identify nuclear reaction observables sensitive to a particular transport coefficient.

Manipulate the Boltzmann equation to reproduce data → deduce the coefficient; make sure that the sensitivity exclusive.

⇒ Need to know Boltzmann-eq. transport coefficients

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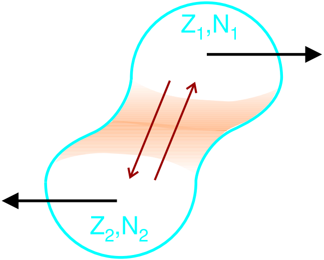

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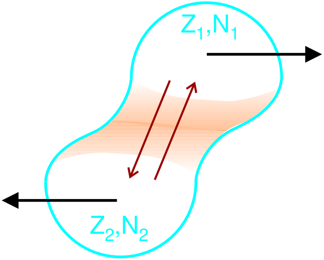

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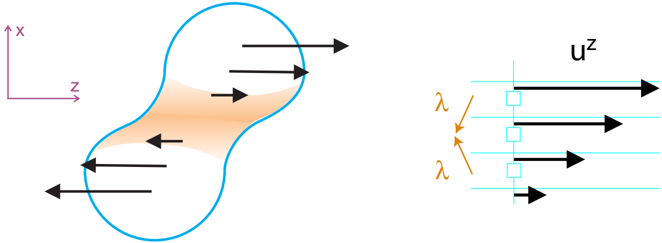
Shear Viscosity



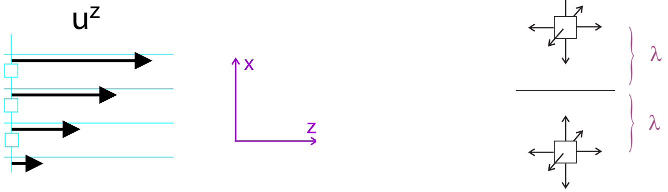
Flux of z-momentum in the x-direction, Π^{zx}, proportional to collective velocity gradient:

$$\Pi^{zx} = -\eta \frac{\partial u^z}{\partial x}$$

where η - shear viscosity coefficient.

Elementary estimate

Density $n = \text{const}$:



Net Momentum Flux Up = Flux Up - Flux Down

$$\Pi^{zx} = \frac{1}{6}\, n\, v_{kin}\, m\, u^z(x-\lambda) - \frac{1}{6}\, n\, v_{kin}\, m\, u^z(x+\lambda)$$

$$\simeq -\frac{1}{3}\, n\, v_{kin}\, m\, \lambda\, \frac{\partial u^z}{\partial x}$$

Thus, the shear viscosity coefficient:

$$\eta \simeq \frac{n\, m\, v_{kin}}{3}\, \lambda \sim \frac{0.16\,\text{fm}^{-3}\, 939\,\text{MeV}/c^2\, 0.3\, c}{3}\, 2\,\text{fm} \sim 30\,\text{MeV/fm}^2 c$$

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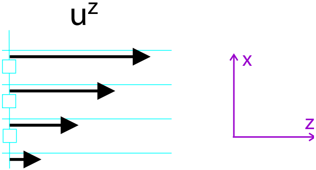

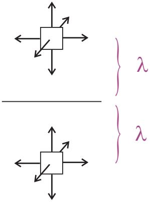

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Solving Boltzmann Equation

$$\frac{\partial f_j}{\partial t} + \frac{\partial \epsilon_j}{\partial \mathbf{p}} \frac{\partial f_j}{\partial \mathbf{r}} - \frac{\partial \epsilon_j}{\partial \mathbf{r}} \frac{\partial f_j}{\partial \mathbf{p}} = \mathcal{I}_j$$

where $j = 1, 2$ for neutrons and protons. Collision integrals $\mathcal{I}_j$ vanish, if the local equilibrium Wigner functions are substituted

$$f_i^{eq} = \frac{1}{\exp\left(\frac{\frac{(\mathbf{p}-m\,\mathbf{u})^2}{2m} - \mu_i}{T}\right) + 1}$$

However, the LHS does not vanish if there are gradients in the system. Thus, f_i^{eq} cannot be a precise solution and

$$f_i = f_i^{(0)} + f_i^{(1)} + f_i^{(2)} + \dots$$

where $f_i^{(0)} \equiv f^{eq}$ and $f^{(n)}$ is of n'th order in gradients.

f^{eq} produces no dissipative fluxes, while $f^{(1)}$ yields lowest-order fluxes linear in gradients & transport coefficients. $f^{(1)}$ obtained by substituting $f^{(0)}$ to the LHS and expanding $\mathcal{I}$ in $f^{(1)}$.

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NSCL

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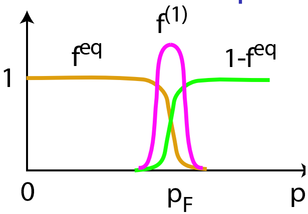
Shear Viscosity from Boltzmann Eq

From the structure of the Boltzmann equation

$$f_i^{(1)} = \phi_i \, f_i^{(0)} \, (1 - f_i^{(0)})$$

where ϕ is a smooth function



Following the Curie principle, anisotropy of symmetric momentum-flux tensor should be driven by the anisotropy of symmetric tensor of velocity gradient:

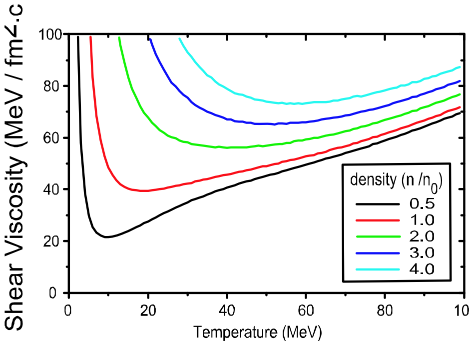
$$\phi_i = b_i \left(p_k \, p_\ell - \frac{p^2}{3} \, \delta_{k\ell} \right) \left(\nabla_k \, u_\ell + \nabla_\ell \, u_k - \frac{2}{3} \, \delta_{k\ell} \, \nabla_n \, u_n \right)$$

Upon substituting to the Boltzmann eq, the result on viscosity is

$$\eta = \frac{5T}{9} \frac{\left(\int \mathrm{d}\mathbf{p} \, f \, p^2 \right)^2}{\int \mathrm{d}\mathbf{p}_1 \, \mathrm{d}\mathbf{p}_2 \, \mathrm{d}\Omega \, f_1 \, f_2 \, (1 - f_1') \, (1 - f_2') \, v_{12} \, \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \, p_{12}^4 \, \sin^2 \theta}$$

Shi&PD PRC68(03)064604

Numerical Results



Free-space cross-sections used; density n in units of normal n_0

At low-T and high-n, divergence due to diverging mean-free-path.

Simple estimate gave $\eta \sim 30\,\mathrm{MeV}/\mathrm{fm}^2\,c$.

Validation in terms of data??

Calcs of in-medium cross-sections for Boltzmann eq.: Schnell PRC57(98)806 & Fuchs PRC64(01)024003 - effects of Pauli & eff mass on intermediate states; general suppression of cross-section, particulary of low-energy resonant behavior

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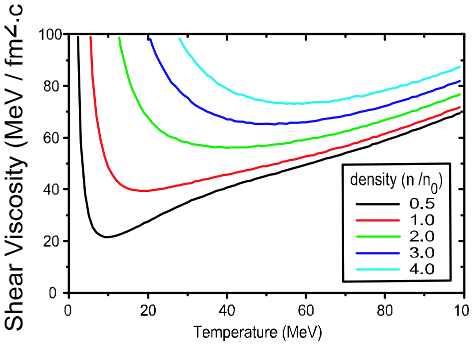

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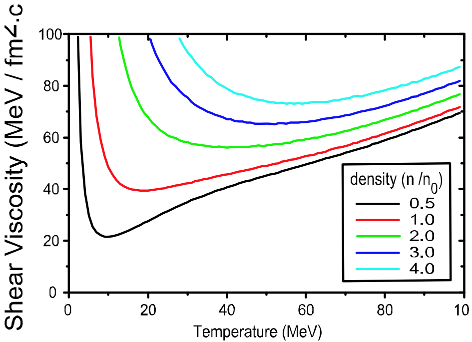

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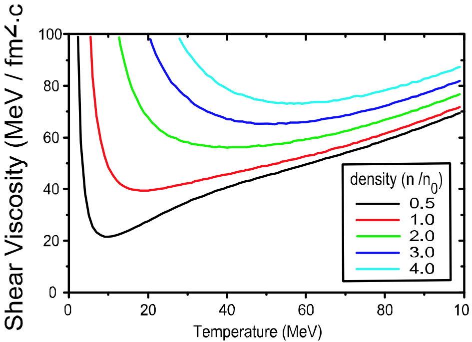

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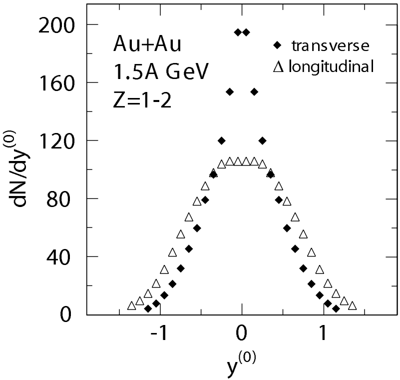
FOPI Measurements of Stopping

Central symmetric collisions of different nuclei from 0.09 to 1.93 GeV/nucleon. Generally, all ptcles with $Z < 10$, excluding pions, weighted with their charge.



Reisdorf *et al.* PRL92(04)232301

Rapidity distributions wider in the longitudinal than transverse direction: incomplete stopping

Ratio of rapidity widths:

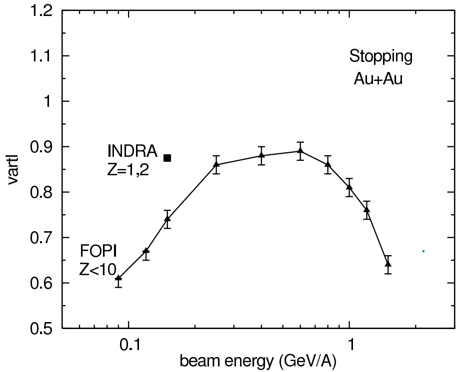
$$vartl = \frac{\Delta y_t}{\Delta y_l}$$

vartl Excitation Function

Stopping ↔ In-Medium Cross-Sections ↔ Viscosity

Boltzmann equation simulations by Brent Barker.

$vartl = \Delta y_t / \Delta y_l$. In the model $A \leq 3$, while FOPI data $Z < 10$: only $E_{lab} \gtrsim 400$ MeV/nucleon relevant; note the INDRA point.



symbols - data

free cross-sections?

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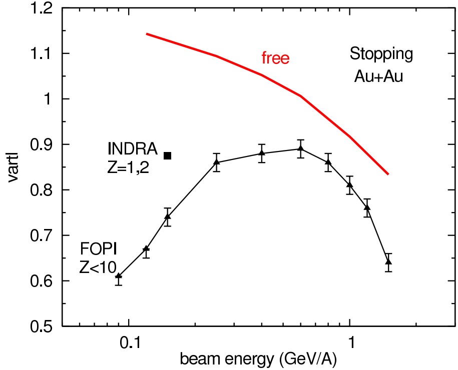

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free cross-sections yield far too much stopping

microscopic cross-sections?

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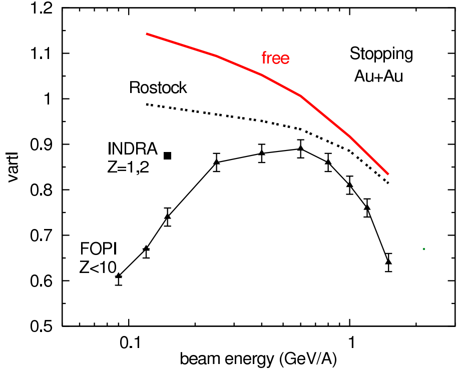

symbols - data

free cross-sections yield far too much stopping

Rostock cross-sections yield still too much stopping; Fuchs' similarly

Cross-Section Phenomenology

'Microscopic' cross-sections don't include effects of other collisions in the vicinity. Phenomenology: size of strong-int cross-section should not exceed interparticle distance.

$$\sigma \lesssim \sigma_0 = \nu \, n^{-2/3}$$

Practical realization:

$$\sigma = \sigma_0 \tanh \left(\frac{\sigma_{\text{free}}}{\sigma_0} \right)$$

with ν adjusted.

For $n \rightarrow 0$, $\sigma \rightarrow \sigma_{\text{free}}$.
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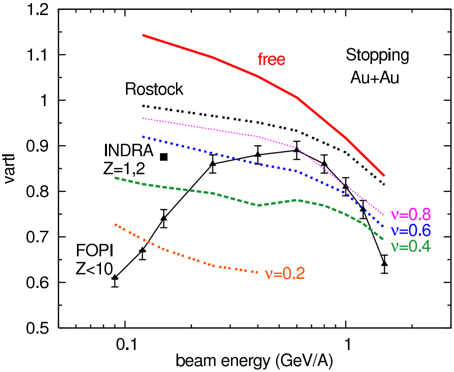
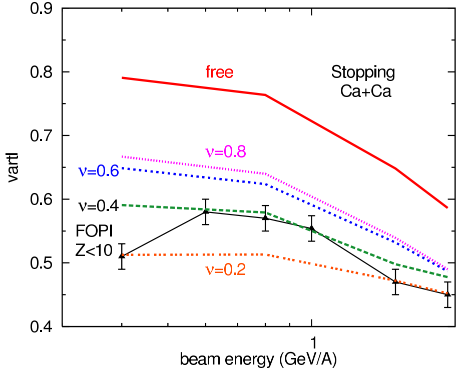


$\nu \sim 0.6$ best

Ca+Ca *vartl* Excitation Function

FOPI central Ca+Ca events - symbols
(Reisdorf *et al.* PRL92(04)232301)

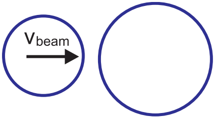


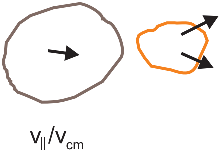
Again free cross-sections yield far too much stopping.

$\nu \sim$ 0.4 seems favored, but there may be an issue of the quality of central-event selection.

Stopping: Linear Momentum Transfer

Linear mo transfer:
Mass asymmetric reactions ($b \sim 0$) examined in lab frame





Velocity component along the beam of the most massive fragment determined and its average compared to the cm velocity.

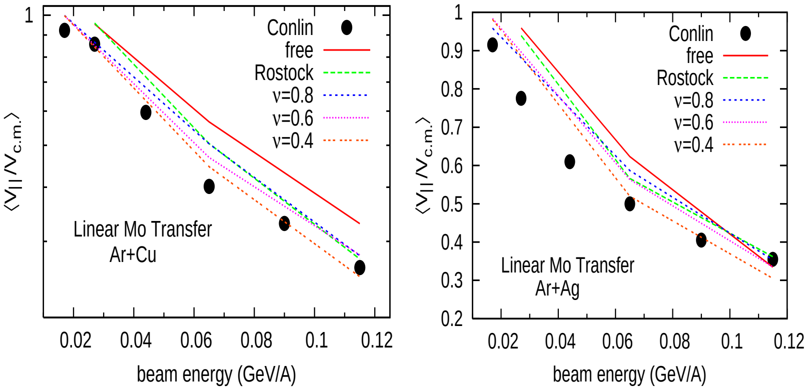
Limits:

Little stopping:	$\langle v_{\|\|} \rangle \sim 0$	small c.s.?
Large stopping, fusion:	$\langle v_{\|\|} \rangle \simeq v_{cm}$	large c.s.?

In-Medium Cross-Section Reduction?

Data: Conlin *et al.* PRC57(98)R1032 - symbols

Ar + Cu, Ag, Au High multiplicity events $\langle b \rangle \sim b_{max}/4$

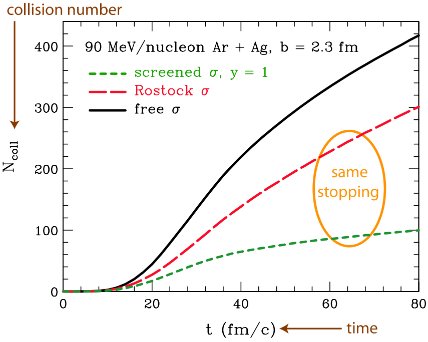


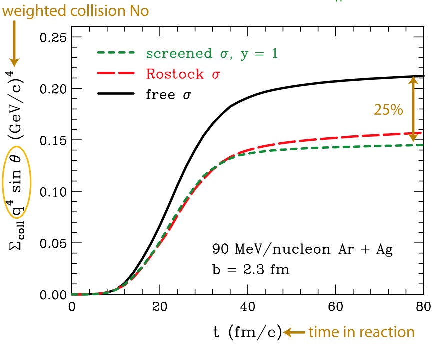
Free cross-sections overestimate stopping.

Low $\nu \sim 0.4$ favored.

Net In-Medium Cross-Sections??

Number of collisions:
Rostock σ yields the same $\langle v_{\parallel} \rangle / v_{cm}$ at all energies, as tempered σ with $\nu = 1$, but very different collision No.
What do these σs share that decides on the same $\langle v_{\parallel} \rangle$?





No of collisions with viscous weight $q^4 \sin^2 \theta'$ is nearly the same for the two σs, $\sim 3/4$ of the No for free σ. Found at all the energies in question.

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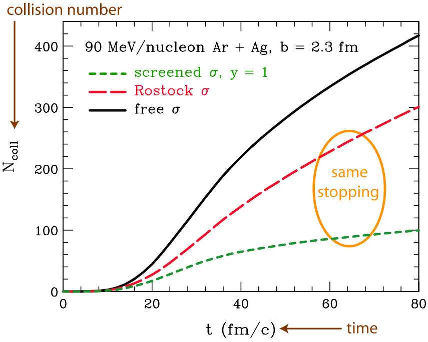

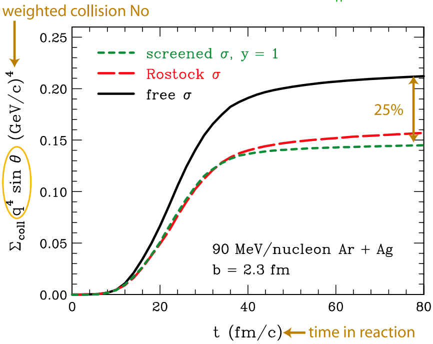

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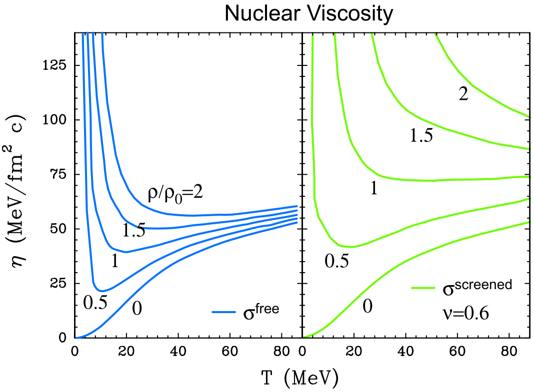
Viscosity from Transport Analysis of Reaction Data

Shi&PD PRC68(03)064604:

$$\eta = \frac{5T}{9} \frac{\left(\int d\mathbf{p}\, f\, p^2\right)^2}{\int d\mathbf{p}_1 \int d\mathbf{p}_2 \int d\Omega\, f_1\, f_2\, \tilde{f}_1'\, \tilde{f}_2'\, v_{12}\, \frac{d\sigma}{d\Omega}\, p_{12}^4\, \sin^2\theta}$$



Significant enhancement of the viscosity due to in-medium cross-section & effective-mass reduction.

At $n \sim n_0$,
$\eta \sim 75\,\mathrm{MeV/fm^2}\,c$

Conclusions

- Transport theory may be used for deducing macroscopic transport coefficients of nuclear matter.
- Free cross-sections yield more stopping in collisions than exhibited by data.
- Close correspondence results in simulations between reduced stopping and inferred shear viscosities.
- In-medium modifications appear to raise nuclear viscosity by nearly a factor of 2, compared to expectations based on free-space cross sections and dispersion relation.

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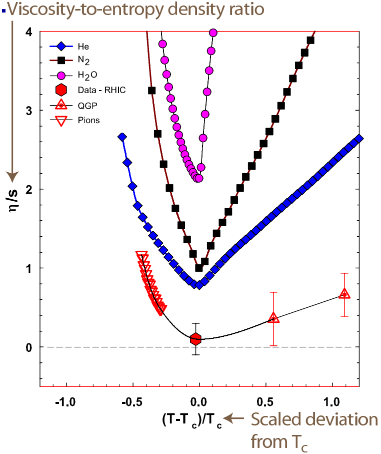
Viscosity vs EOS

QGP phase-transition search focused now on tricritical point

Empirical evidence: close to critical temperature viscosity normalized to entropy minimizes.



Compilation

Csernai *et al.* PRL97(06)152303

At RHIC η limited from above by the strength of collective flow (v_2).

Lower limit from strong-coupling limit of gauge theories: $\eta/s \geq 1/4\pi \sim 1/12$.

$\Rightarrow$Where is nuclear matter in medium-energy collisions?

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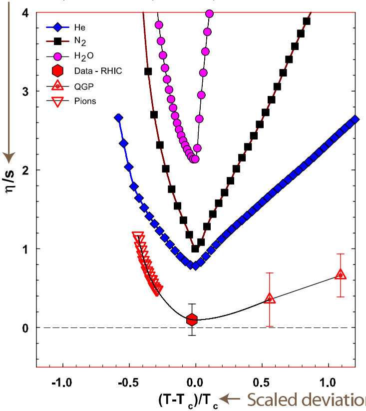
Viscosity-to-entropy density ratio



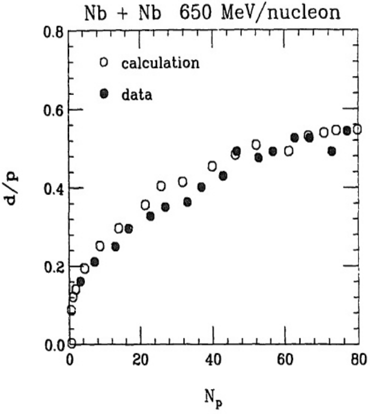
← Scaled deviation from T_c

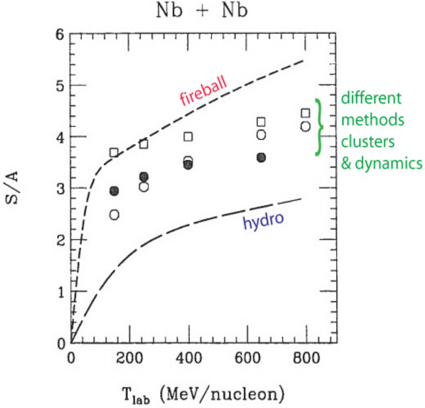
⇒Where is nuclear matter in medium-energy collisions?

Entropy from Cluster Yields

Simple formula: $S/A \simeq 3.9 - \log\left(N_d/N_p\right)$

Siemens&Kapusta PRL43(79)1486





Independent testing in transport theory

PD&Bertsch NPA533(81)712

Normalized Viscosity

In intermediate-energy reactions, $s/n \equiv S/A = $ (3-4.5), corresponding to $T = $ (40-70) MeV at $n = n_0$, yielding $\eta/s = $ (0.5-0.7)

Results fall in the ball-park of other but follow from nuclear data.

Lacey *et al* PRL98(07)092301