Nuclear many-body problem: new questions, new methods, and new solutions

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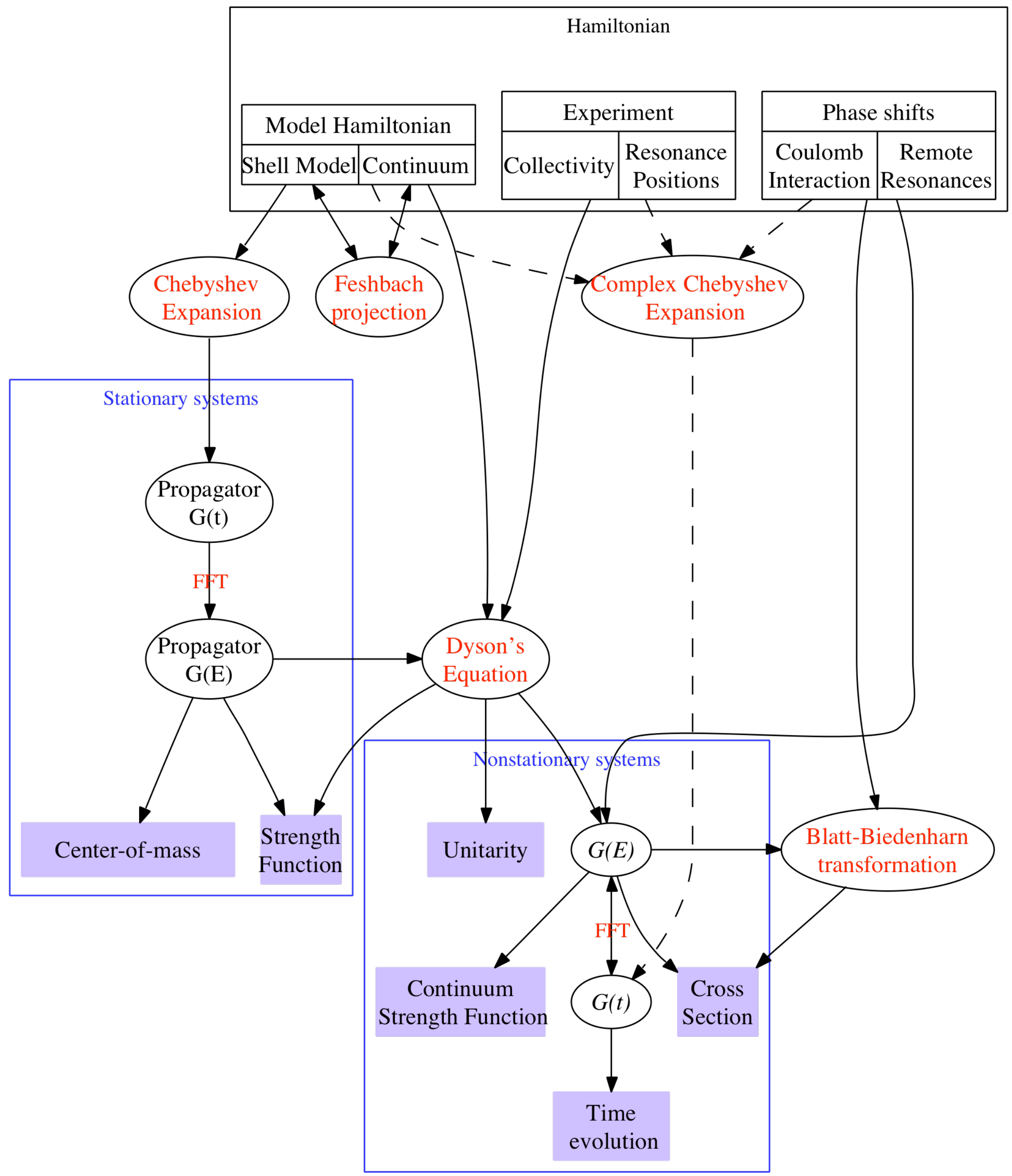
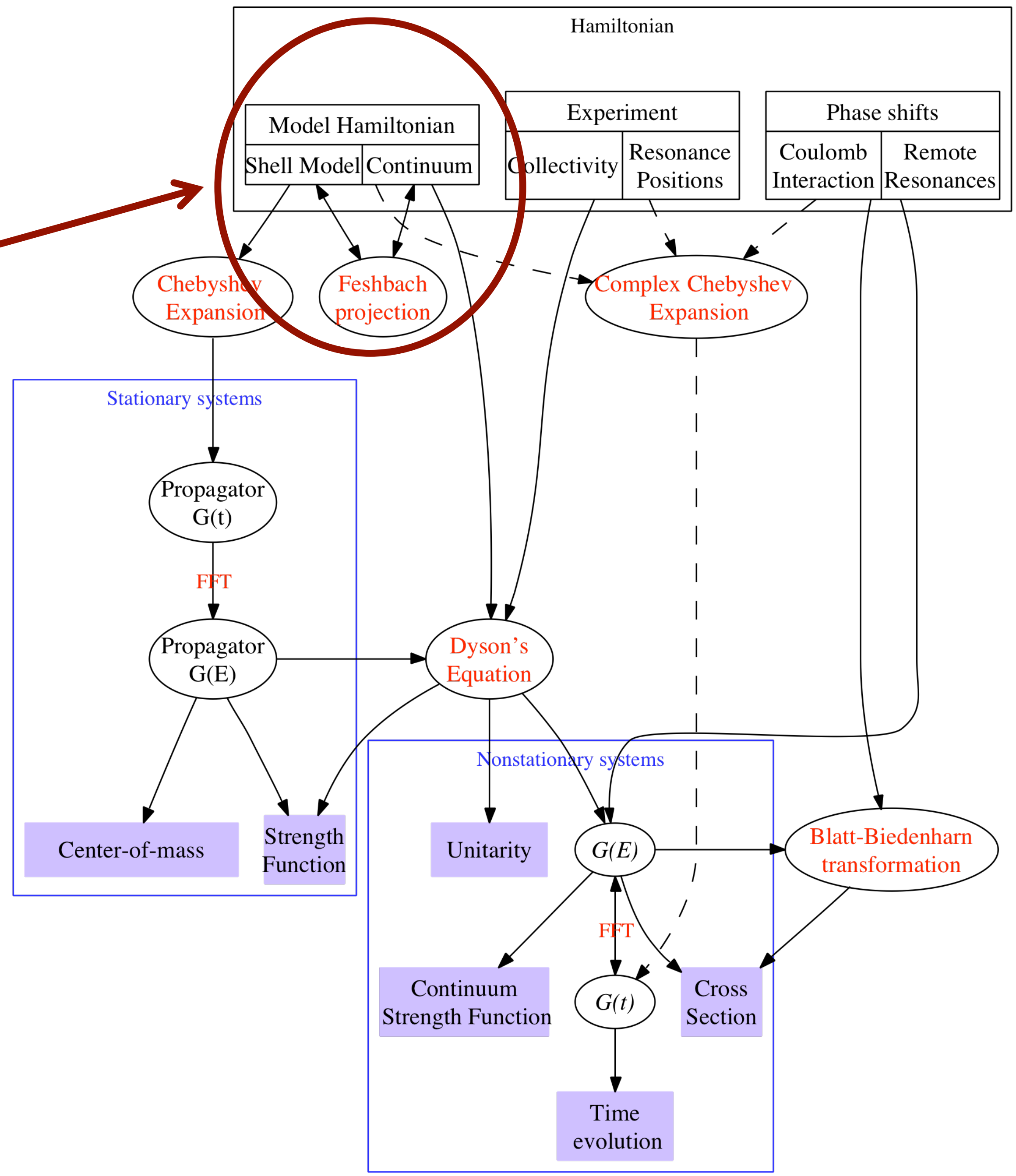
Nuclear structure and reactions

The features of time-dependent approach

- Reflects time-dependent physics of unstable systems
- Linearity of QM equations maintained
- No matrix diagonalization
- Stability for broad and narrow resonances
- Direct relation to observables
- Ability to work with experimental data
- New many-body numerical techniques


Hamiltonian
Model Hamiltonian
Shell Model
Continuum
Experiment
Collectivity
Resonance Positions
Phase shifts
Coulomb Interaction
Remote Resonances
Chebyshev Expansion
Feshbach projection
Complex Chebyshev Expansion
Stationary systems
Propagator G(t)
FFT
Propagator G(E)
Dyson's Equation
Center-of-mass
Strength Function
Nonstationary systems
Unitarity
G(E)
Blatt-Biedenharn transformation
FFT
Continuum Strength Function
G(t)
Cross Section
Time evolution

Initial Hamiltonian



Feshbach Formulations

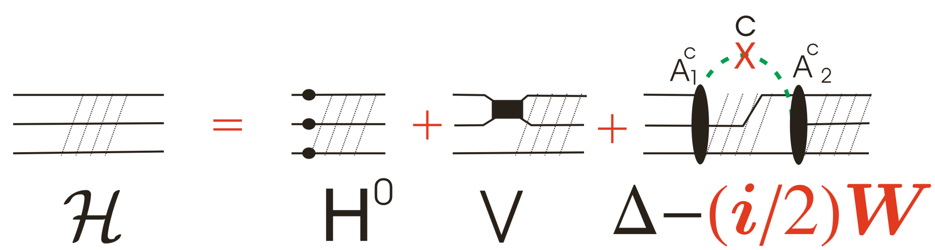
Hilbert space is separated into intrinsic P ($|1\rangle$) and external Q-subspaces ($|c;E\rangle$)

The Hamiltonian in P is: $$\mathcal{H}(E) = H + \Delta(E) - \frac{i}{2}W(E)$$

Channel-vector: $$|A^c(E)\rangle = P_{\mathcal{P}} H |c;E\rangle$$

Self-energy: $$\Delta(E) = \frac{1}{2\pi}\int dE' \sum_c \frac{|A^c(E')\rangle\langle A^c(E')|}{E-E'}$$

Irreversible decay to the excluded space: $$W(E) = \sum_{c(\mathrm{open})} |A^c(E)\rangle\langle A^c(E)|$$



[1] C. Mahaux and H. Weidenmüller, *Shell-model approach to nuclear reactions*, North-Holland Publishing, Amsterdam 1969

Channel Vectors and amplitudes

$$|A^c(E)\rangle = a^c(E)\,|c\rangle$$

Channel amplitude

Energy-independent channel vector: structure of spectator components

Perturbative limit in traditional Shell Model: $H|\alpha\rangle = E_\alpha|\alpha\rangle$

$$\Gamma_\alpha = \langle\alpha|W(E_\alpha)|\alpha\rangle \quad \Gamma_\alpha = \sum_c \Gamma_\alpha^c \quad \Gamma_\alpha^c = \gamma_c(E_\alpha)\,|\langle c|\alpha\rangle|^2$$

Single-particle decay width

$$\gamma_c(E) = |a^c(E)|^2$$

Spectroscopic factor or transition rate

$$C^2S = |\langle c|\alpha\rangle|^2$$

$$B(\mathrm{EM}) = |\langle c|\alpha\rangle|^2$$

Scattering matrix and reactions

$$\mathbf{T}_{cc'}(E) = \langle A^c(E)| \left(\frac{1}{E - \mathcal{H}(E)} \right) |A^{c'}(E)\rangle$$

$$\mathbf{S}_{cc'}(E) = \exp(i\xi_c)\ \{\delta_{cc'} - i\,\mathbf{T}_{cc'}(E)\}\exp(i\xi_{c'})$$

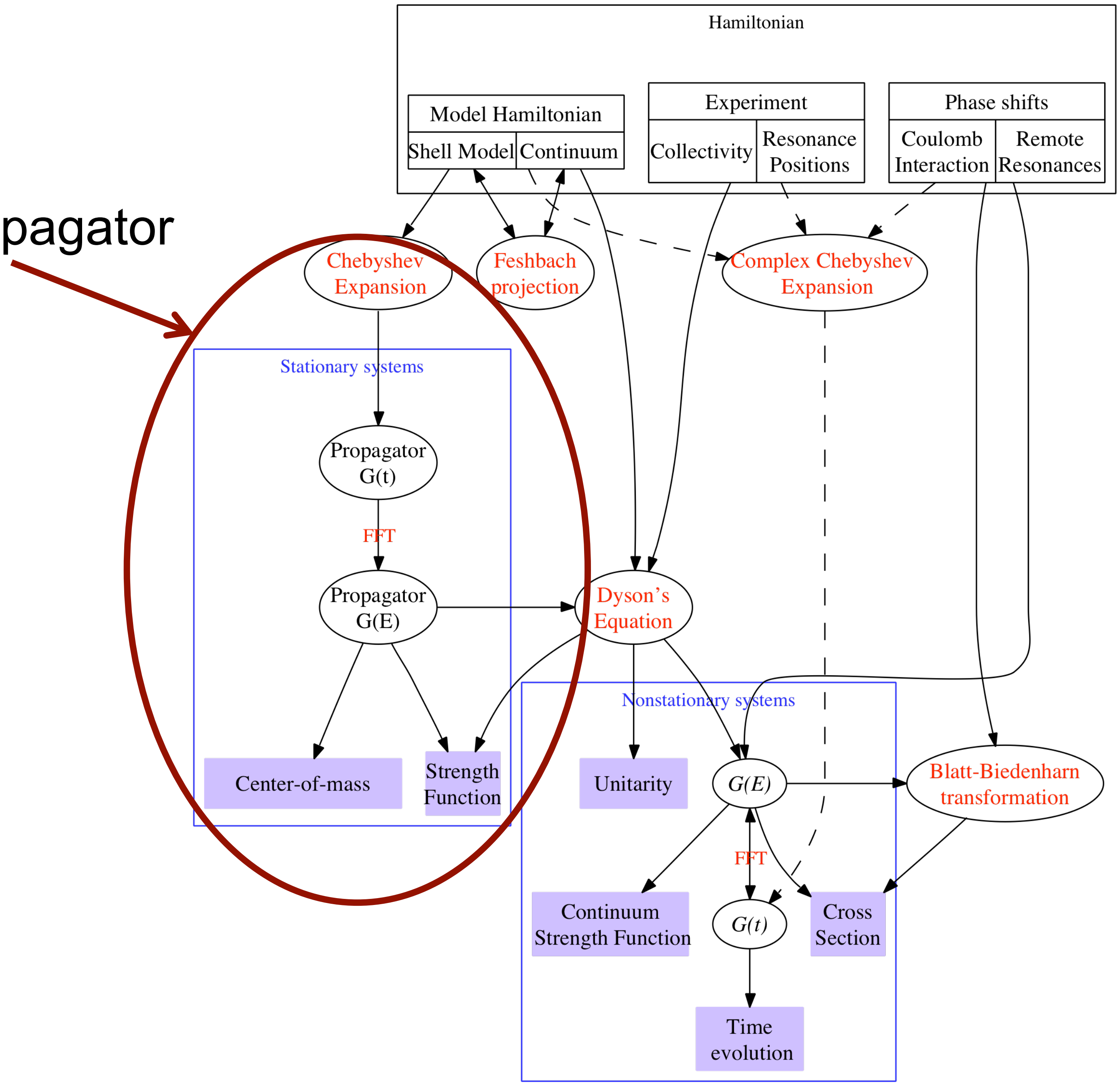
Cross section:

$$\sigma = \frac{\pi}{k'^2} \sum_{cc'} \frac{(2J+1)}{(2s'+1)(2I'+1)} |\mathbf{T}_{cc'}|^2$$

Additional topics:

- Angular (Blatt-Biedenharn) decomposition
- Coulomb cross sections, Coulomb phase shifts, and interference
- Phase shifts from remote resonances.

Stationary propagator



Calculation Details, Propagator-Strength Function

$$G(E) = \frac{1}{E - H} = -i \int_0^{\infty} dt\ \exp(iEt)\exp(-iHt)$$

- Scale Hamiltonian so that eigenvalues are in [-1 1]
- Expand Using evolution operator in Chebyshev polynomials

$$\exp(-iHt) = \sum_{n=0}^{\infty} (-i)^n (2 - \delta_{n0}) J_n(t) T_n(H)$$

- Chebyshev polynomial $T_n[\cos(\theta)] = \cos(n\theta)$
- Use iterative relation and matrix-vector multiplication to generate

$$|\lambda_n\rangle = T_n(H)|\lambda\rangle$$

$$|\lambda_0\rangle = |\lambda\rangle, \quad |\lambda_1\rangle = H|\lambda\rangle \quad |\lambda_{n+1}\rangle = 2H|\lambda_n\rangle - |\lambda_{n-1}\rangle$$

$$\langle\lambda'|T_{n+m}(H)|\lambda\rangle = 2\langle\lambda'_m|\lambda_n\rangle - \langle\lambda'|\lambda_{n-m}\rangle, \quad n \geqslant m$$

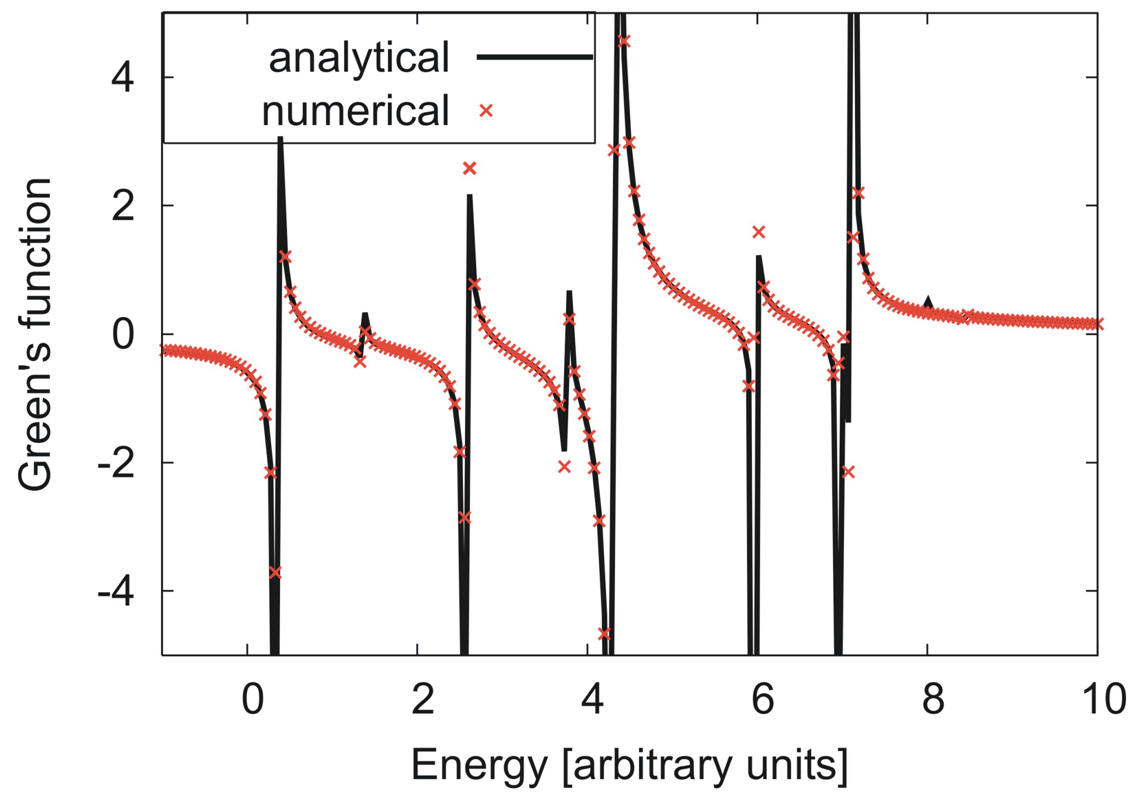
- Use FFT to find return to energy representation

T. Ikegami and S. Iwata, J. of Comp. Chem. **23** (2002) 310-318

Chebyshev expansion
Green's function calculation

Advantages of the method

- No need for full diagonalization or inversion at different E
- Only matrix-vector multiplications
- Numerical stability
- Controlled energy resolution



Center-of-mass problem
The strength-function example

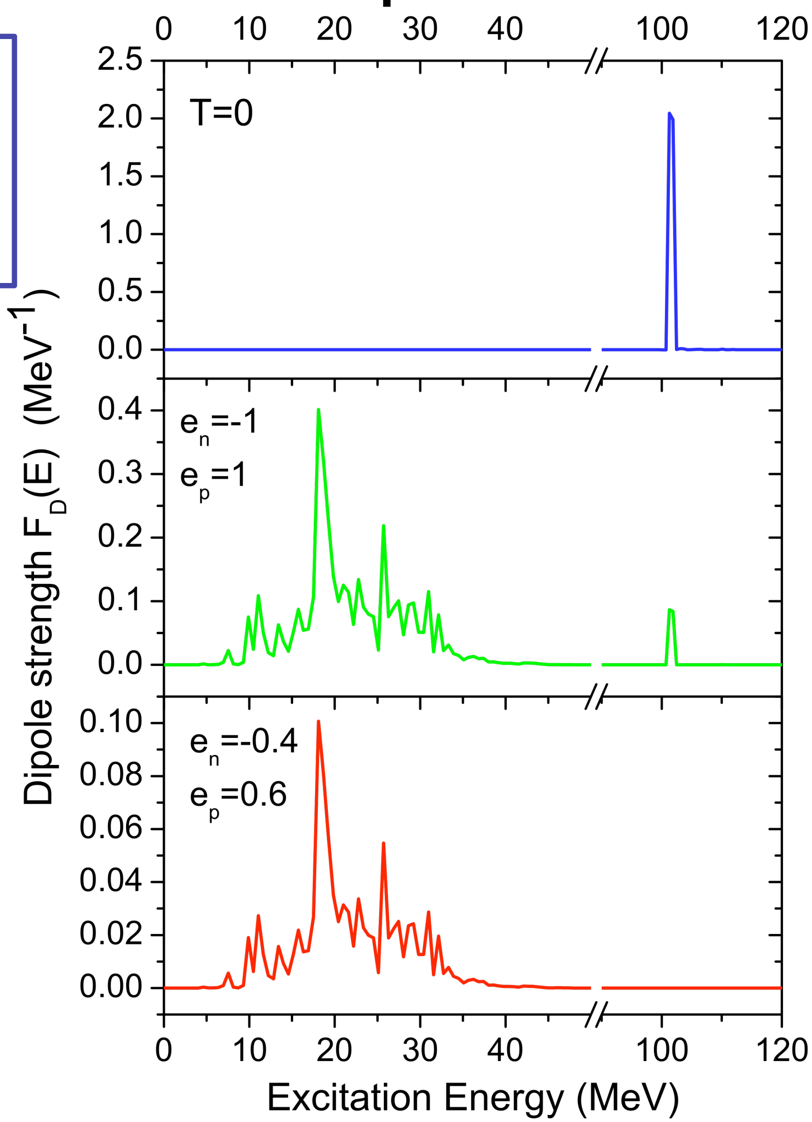
Figure: Strength function for E1 and CM excitation in ^{20}O example, spsdfp –shell model WBP interaction.

CM spurious states are moved to high energy

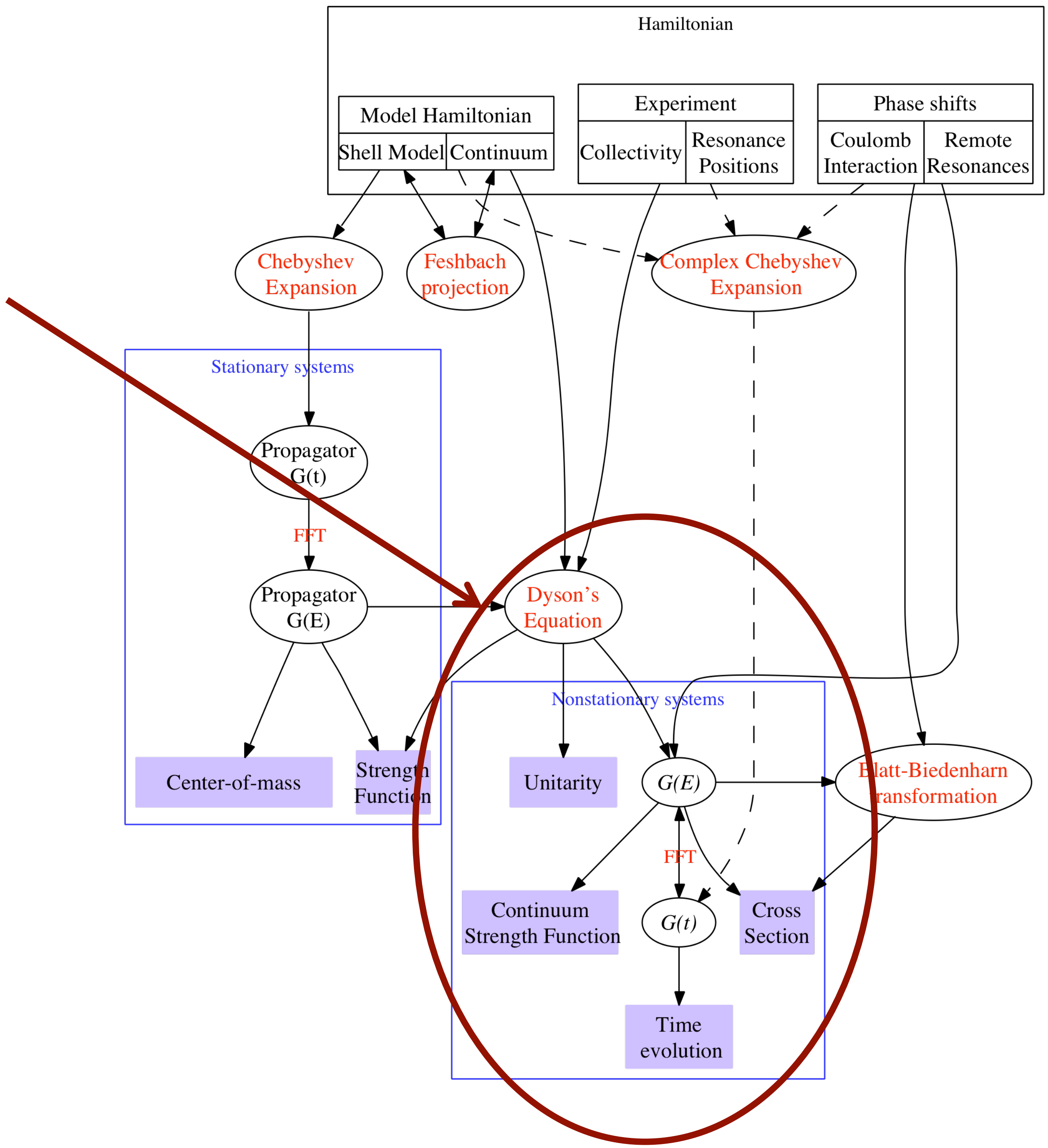
- Top plot-isoscalar dipole E1 T=0 excitation
- Center- E1 excitation with incorrect effective charges
- Bottom-E1 with e_p=0.6 and e_n=-0.4

$$F_\lambda(E) = \langle \lambda | \delta(E-H) | \lambda \rangle = -\frac{1}{\pi} \mathrm{Im} \langle \lambda | G(E) | \lambda \rangle$$

$$|D\rangle = D|0^+_{\mathrm{g.s.}}\rangle \qquad \vec{D} = \sum_a e_a \vec{r}_a$$



Full propagator



Dysion's equation, including other interaction terms

$$\mathcal{H}(E) = H + V(E) \quad V(E) = \sum_{ab} |a\rangle \mathbf{V}_{ab}(E) \langle b|$$

$$G(E) = \frac{1}{E - H} \qquad \mathcal{G}(E) = \frac{1}{E - \mathcal{H}(E)}$$

Propagators in channel space

$$\mathbf{G}_{ab} = \langle a|G(E)|b\rangle \qquad \mathbb{G}_{ab} = \langle a|\mathcal{G}(E)|b\rangle$$

Include non-Hermitian terms with Dyson's equation

$$\mathcal{G}(E) = G(E) + G(E)V(E)\mathcal{G}(E)$$

$$\mathbb{G} = \mathbf{G}\left[\mathbf{1} - \mathbf{V}\mathbf{G}\right]^{-1} = \left[\mathbf{1} - \mathbf{G}\mathbf{V}\right]^{-1}\mathbf{G}$$

Dyson's equation

- Work in channel space.
- Include any interaction confined to channel space, Herminian, non-Herminian or energy-dependent.
- T-matrix $\mathbf{T} = \mathbf{a}^\dagger \mathbb{G} \mathbf{a}$ where channel matrix $\mathbf{a}_{cc'} = \delta_{cc'} a^c(E)$

Examples:

- Non-Hermitian decay components $\mathbf{W} = \mathbf{a}\mathbf{a}^\dagger$, show unitarity.
- Non-Hermitian components : time evolution of decaying states
- Hermitian terms and GR collectivities $V = \kappa |D\rangle\langle D|$
- Position of resonances
- Self energy, full inclusion of continuum effects $\mathbf{V} = \mathbf{\Delta} - i\mathbf{W}/2$

Unitarity and flux conservation

Take: $\mathbf{W} = \mathbf{a}\mathbf{a}^\dagger$

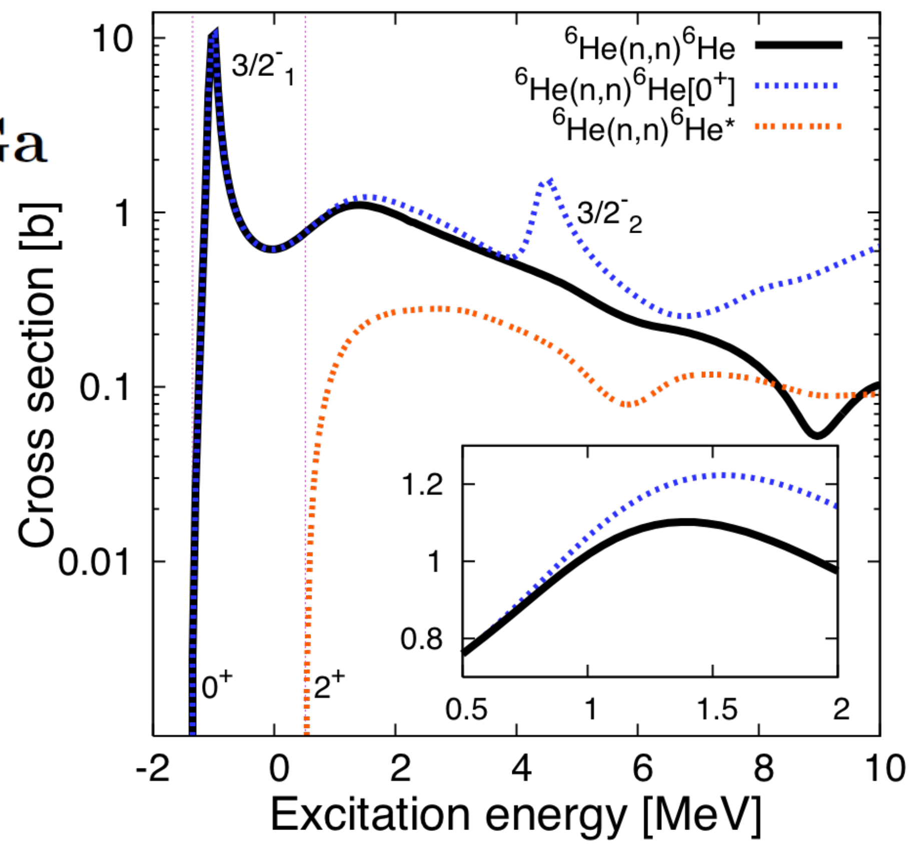
Exact relation:

$$\mathbf{S} = \frac{\mathbf{1} - i/2\,\mathbf{K}}{\mathbf{1} + i/2\,\mathbf{K}} \qquad \mathbf{K} = \mathbf{a}^\dagger \mathbf{G} \mathbf{a}$$

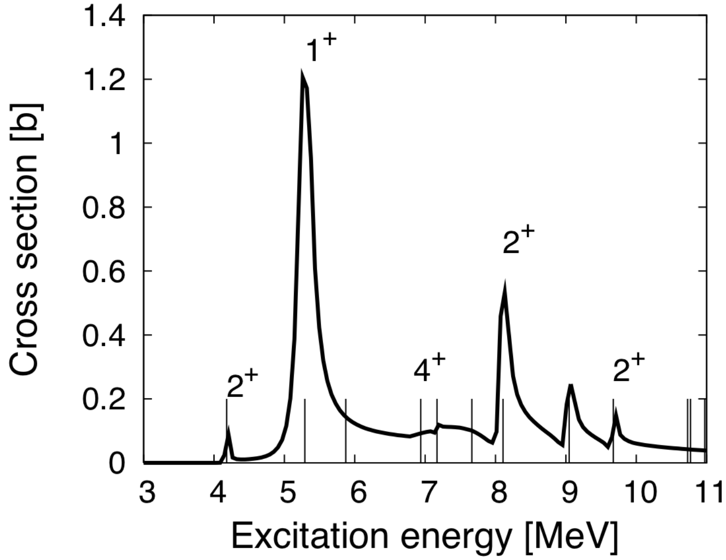
$$\mathbf{S}\mathbf{S}^\dagger = \mathbf{S}^\dagger \mathbf{S} = \mathbf{1}$$

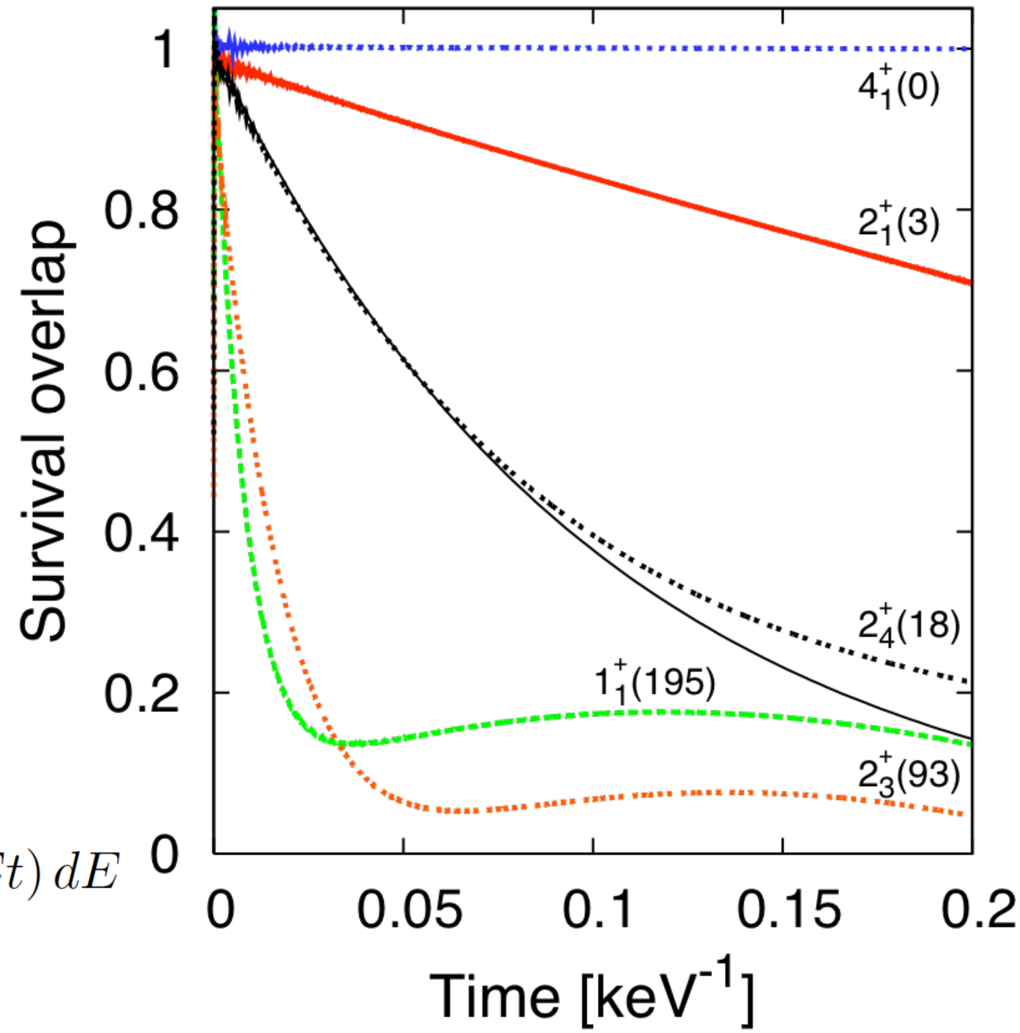
•Cross section has a cusp when inelastic channels open
•The cross section is reduced due to loss of flux
•The p-wave discontinuity $E^{3/2}$

Figure: ^{6}He(n,n) cross section
•Solid curve-full cross section
•Dashed (blue) only g.s. channel
•Dotted (red) inelastic channel



Time evolution of decaying states





Time evolution of several SM states in ^{24}O. The absolute value of the survival overlap is shown $|\langle\alpha|\mathcal{U}(t)|\alpha\rangle|$

$$\mathcal{U}(t) = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} \mathcal{G}(E)\ \exp\left(-iEt\right) dE$$

For an isolated narrow resonance

$$|\langle\alpha|\exp(-i\mathcal{E}_\alpha t)|\alpha\rangle| = \exp(-\Gamma_\alpha t/2)$$

Dipole collectivity

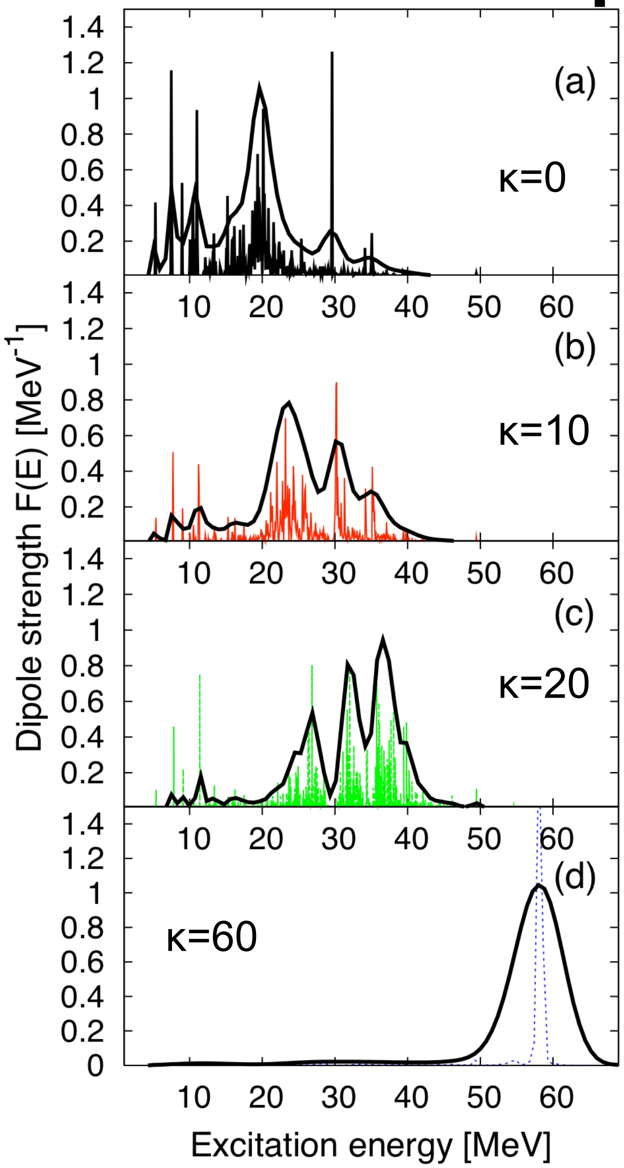


Figure: Strength function of the isovector dipole operator in ^{22}O. WBP SM Hamiltonian plus interaction term:

$$V = \kappa|D\rangle\langle D|$$

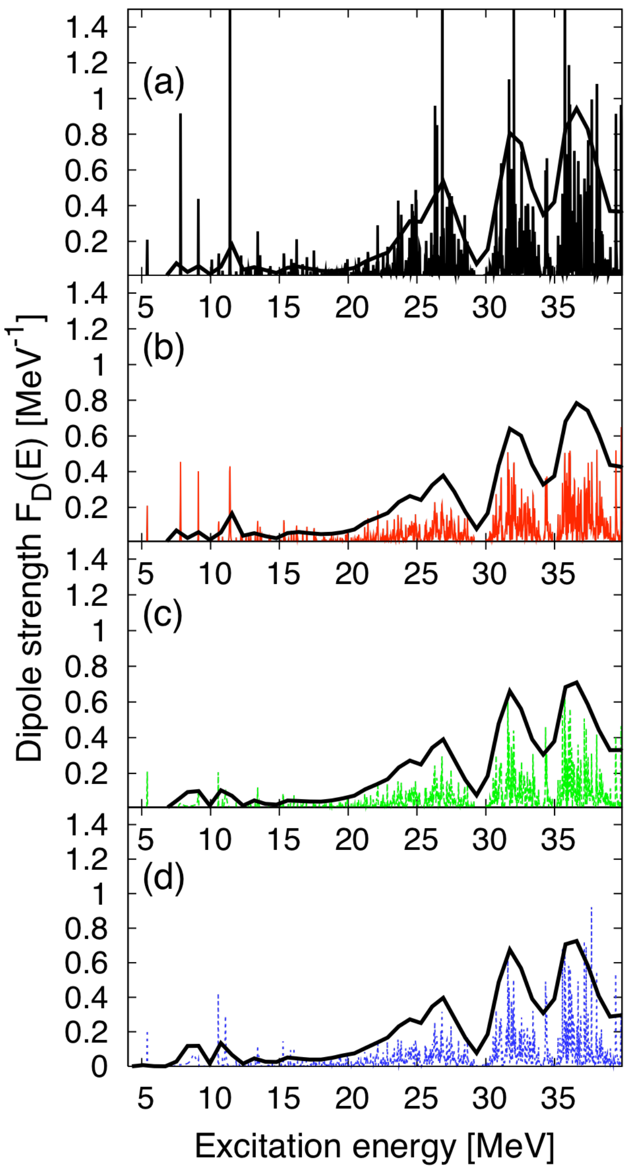
$$|D\rangle = D|0^+_{\mathrm{g.s.}}\rangle$$

$$\kappa = 10,\ 20,\ \mathrm{and}\ 60$$

Strength Function

for system with dipole collectivity and neutron decay

$$F_\lambda(E) = -\frac{1}{\pi} \operatorname{Im} \langle \lambda | \mathcal{G}(E) | \lambda \rangle$$



Dipole strength in ^{22}O. WBP Shell Model, enhanced dipole collectivity k=20, neutron decay l=1 Woods-Saxon potential for reactions.

(a) No decay
(b) Realistic decay
(c) Enhanced by a factor of 3 continuum coupling
(d) Enhanced 10 times continuum coupling

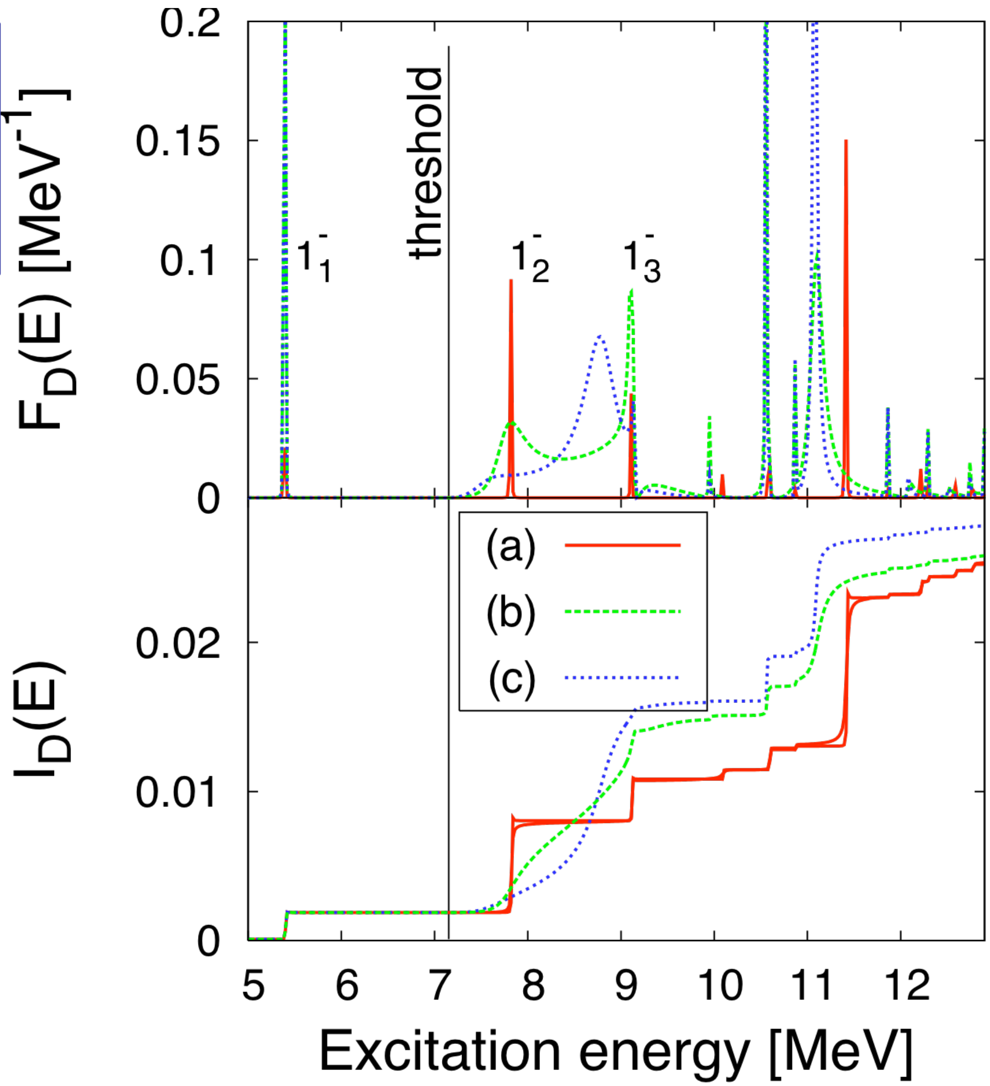
Strength function and decay in ^{22}0

Upper panel: Isovector dipole strength in ^{22}O low-energy region.
Lower panel: Integrated strength

$$I_\lambda(E) = \int_{-\infty}^{E} F_\lambda(E')dE'$$

In the limit of weak decay

$$I_D(E) = \sum_{\alpha}^{E_\alpha < E} B(\mathrm{E1}; \alpha \to 0^+_{\mathrm{g.s.}})$$



Manipulation with resonance positioning

For SM eigenstate

$$H|\alpha\rangle = E_\alpha|\alpha\rangle$$

Include term in full Hamiltonian

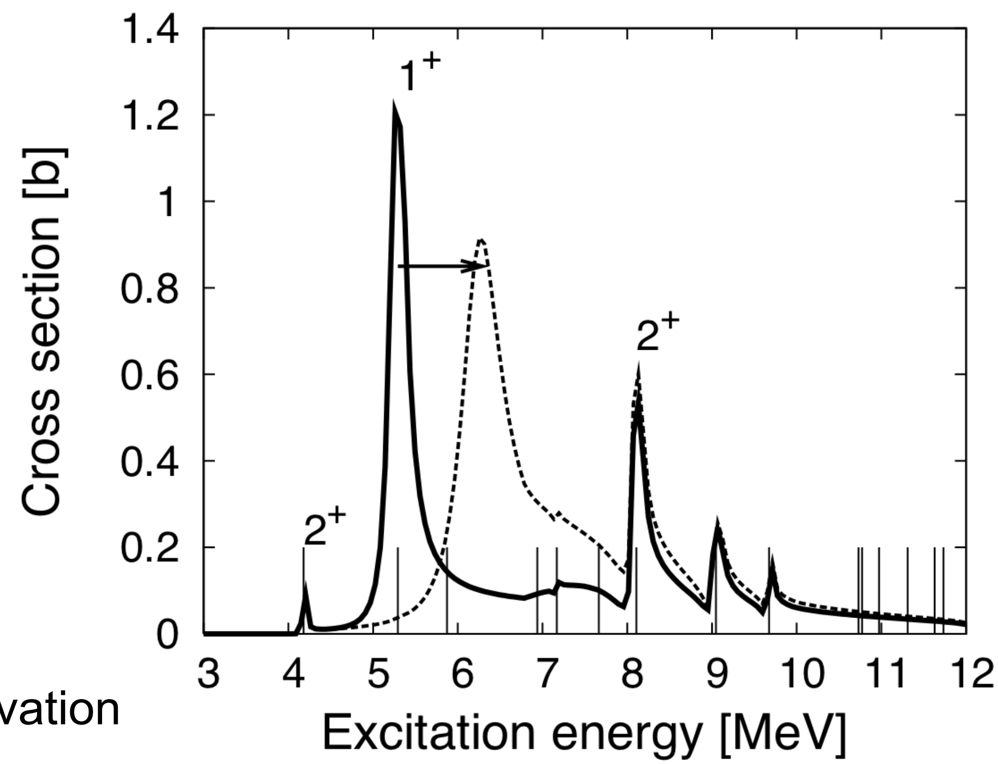
$$V = \sum_\alpha |\alpha\rangle V_\alpha \langle\alpha|$$

The position of the resonance will shift

Advantages

- Factorized form, Dyson's equation
- Fast work in channel space
- Practical method to analyze observation

Figure: the l=2 cross section $^{23}O(n,n)^{23}O$. Solid line: USD interaction + neutron decay (WS potential). Dashed line 1^+ state moved up by 1 MeV (from 5.29 to 6.29)



The role of self-energy

Energy-dependent contribution from virtual excitation to continuum, the self-energy.

In channel space

$$\mathbf{\Delta}_{cc'} = \delta_{cc'}\Delta_c(E)$$

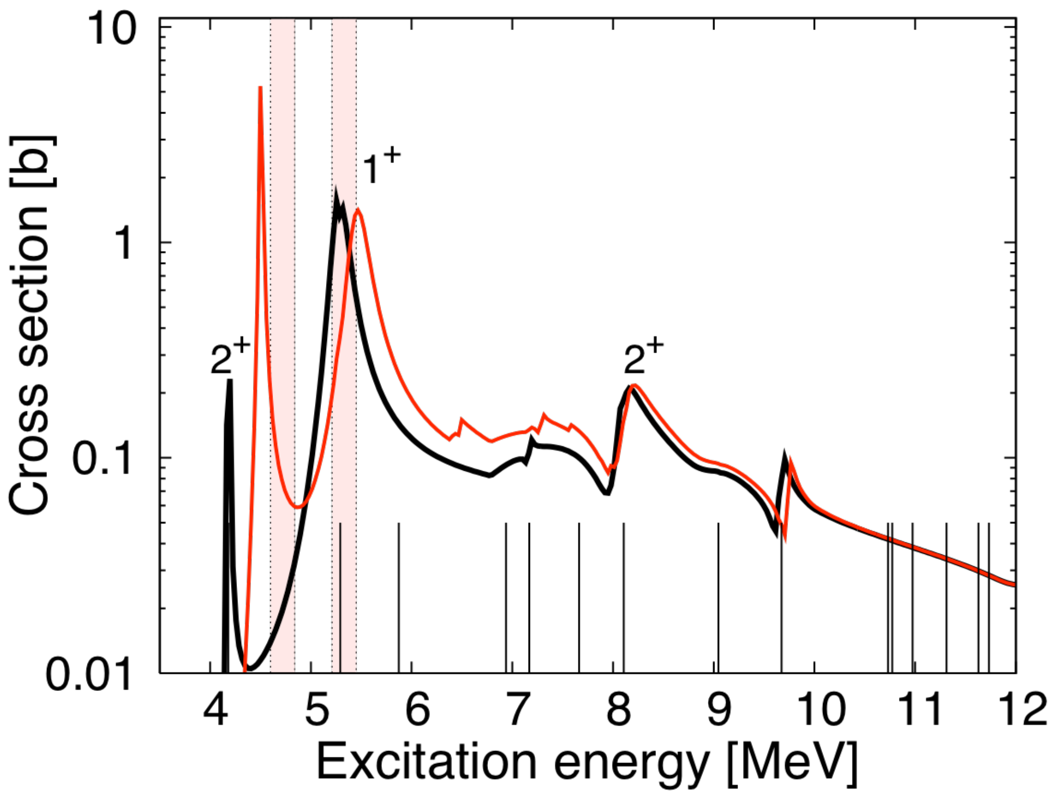
$$\Delta_c(E) = \frac{1}{2\pi}⨍ dE' \frac{|a^c(E')|^2}{E - E'}$$

Near-threshold form

$$\Delta_c(\epsilon) = \frac{\varkappa^2}{2}\Theta(-\epsilon)\epsilon^l\sqrt{-\epsilon}$$

Experimental data from:
C. Hoffman, et.al. Phys. Lett. **B672**, 17 (2009)

Figure: $^{23}O(n,n)^{23}O$ Effect of self-energy term (red curve). Shaded areas show experimental values with uncertainties.



Correcting USD interaction

Figure: Theory predictions for states in ^{24}O

Theoretical Models:

OBE05 -A. Obertelli, et.al.
Phys. Rev. C **71,** 024304 (2005).

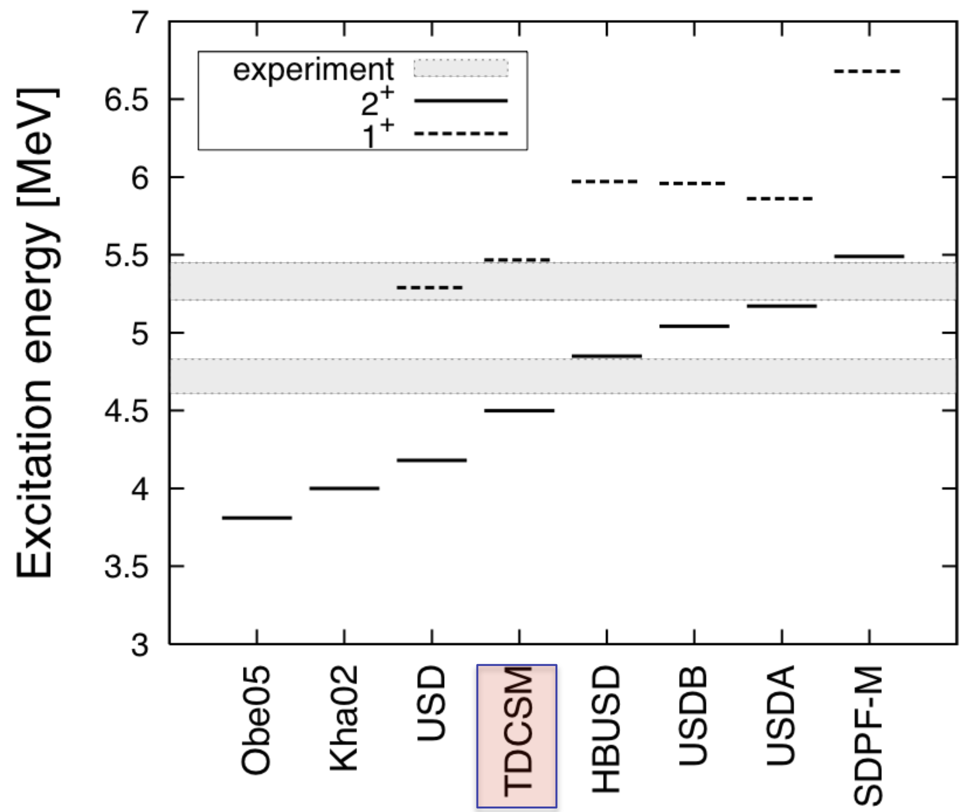
Khau02- E. Khan, et.al.
Phys. Rev. C **66**, 024309 (2002).

USD, USDA, USDB- B.A. Brown, et.al.
Phys. Rev. C **74**, 034315 (2006).

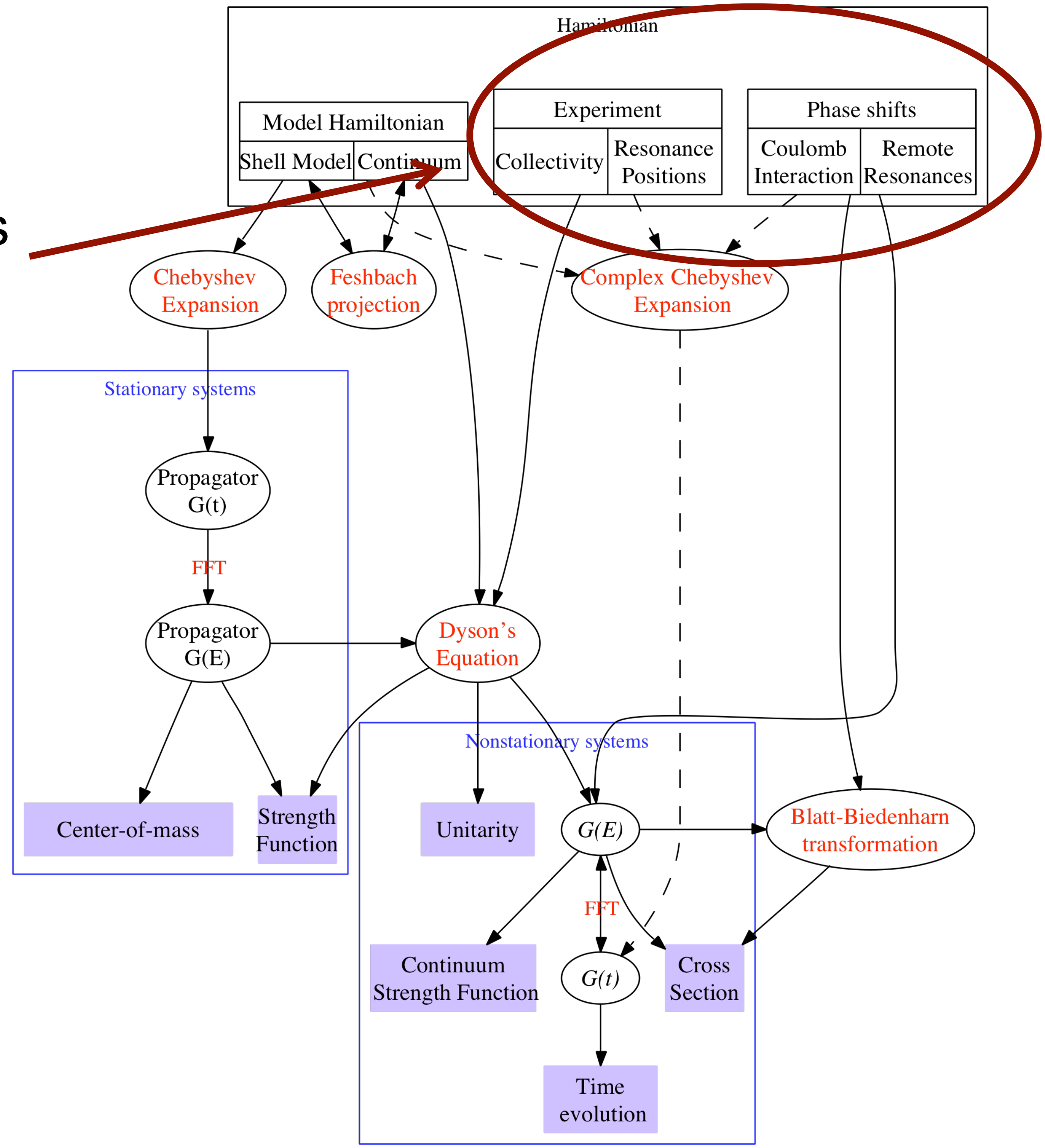
HBUSD- B.A. Brown, et.al.
Prog. Part. Nucl. Phys. **47**, 517 (2001).

SDPF-M – Y.Utsuno, et.al.
Phys. Rev. C **60,** 054315 (1999).

TDCSM – This work

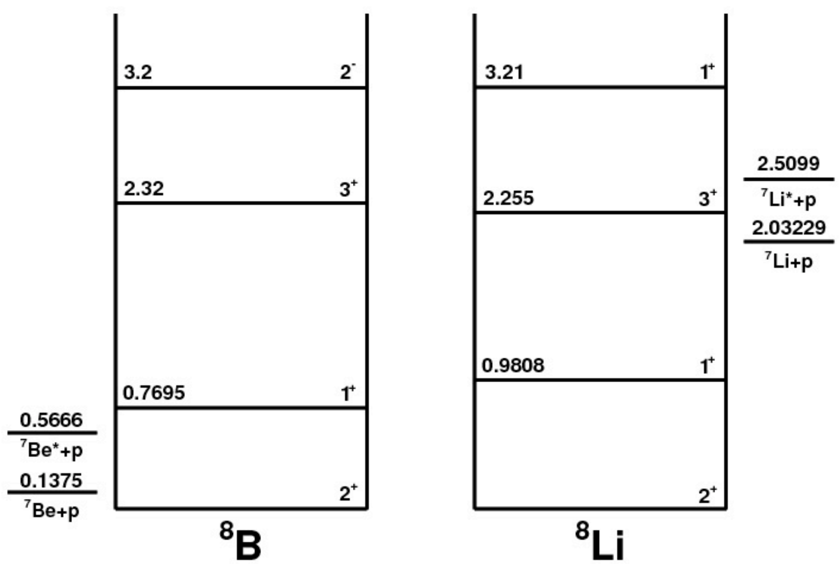


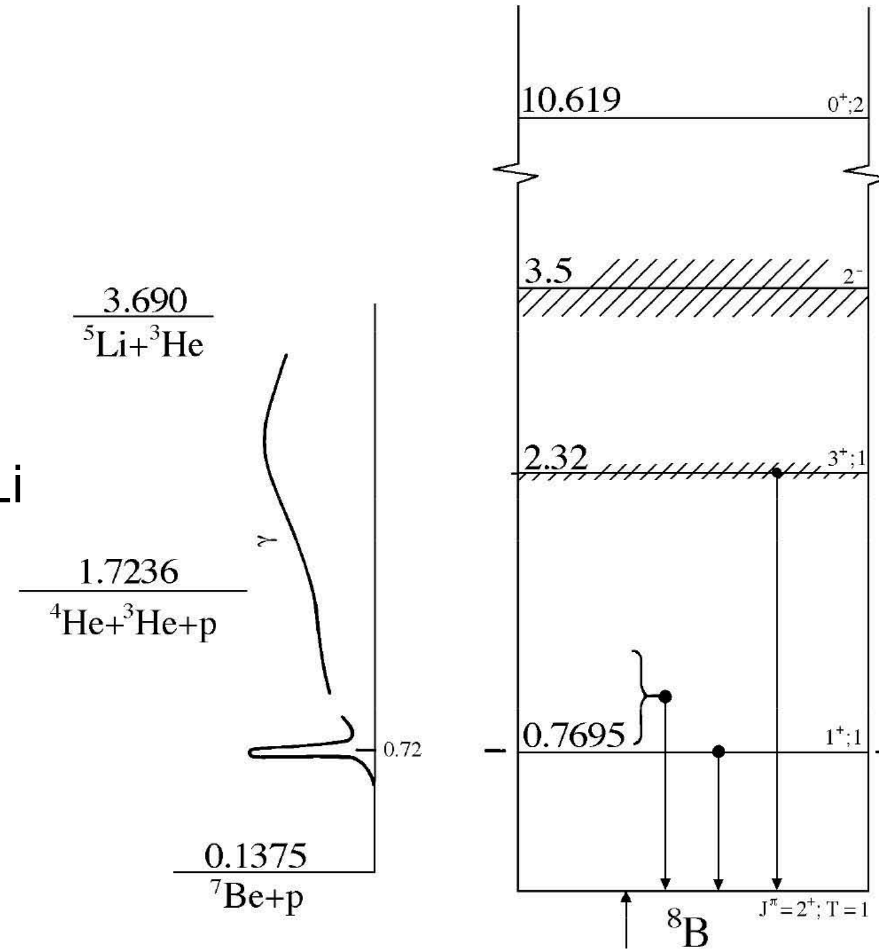
Additional components in Hamiltonian



States in ^{8}B

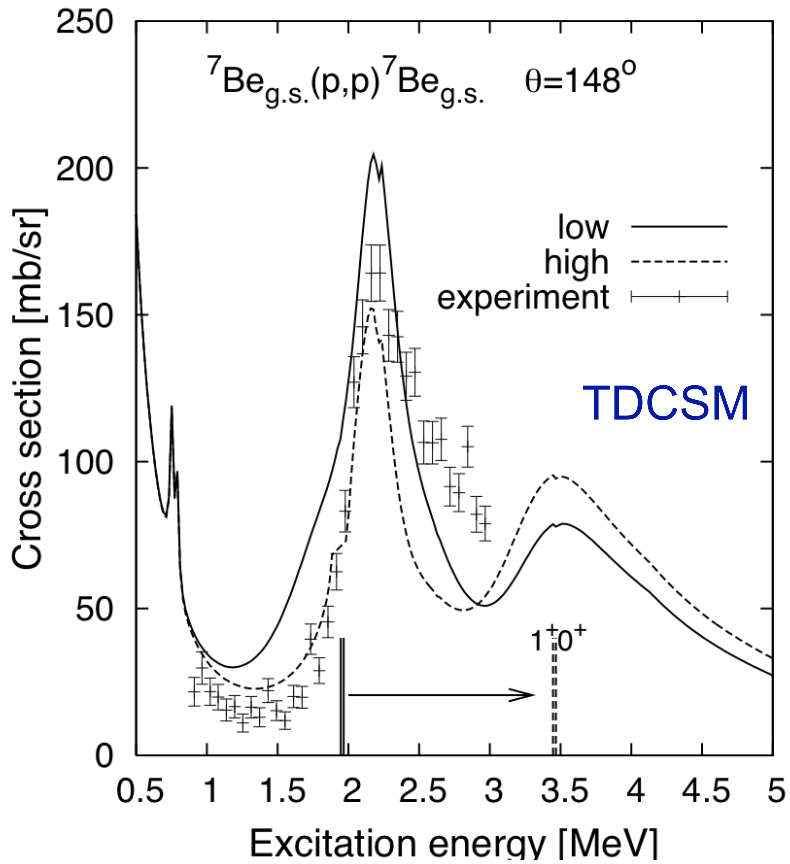
- Ab-initio and no core theoretical models predict low-lying 2^+, 0^+, and 1^+ states
- Recoil-Corrected CSM suggests low-lying states
- Traditional SM mixed results
- These states were not seen in ^{8}B and in ^{8}Li



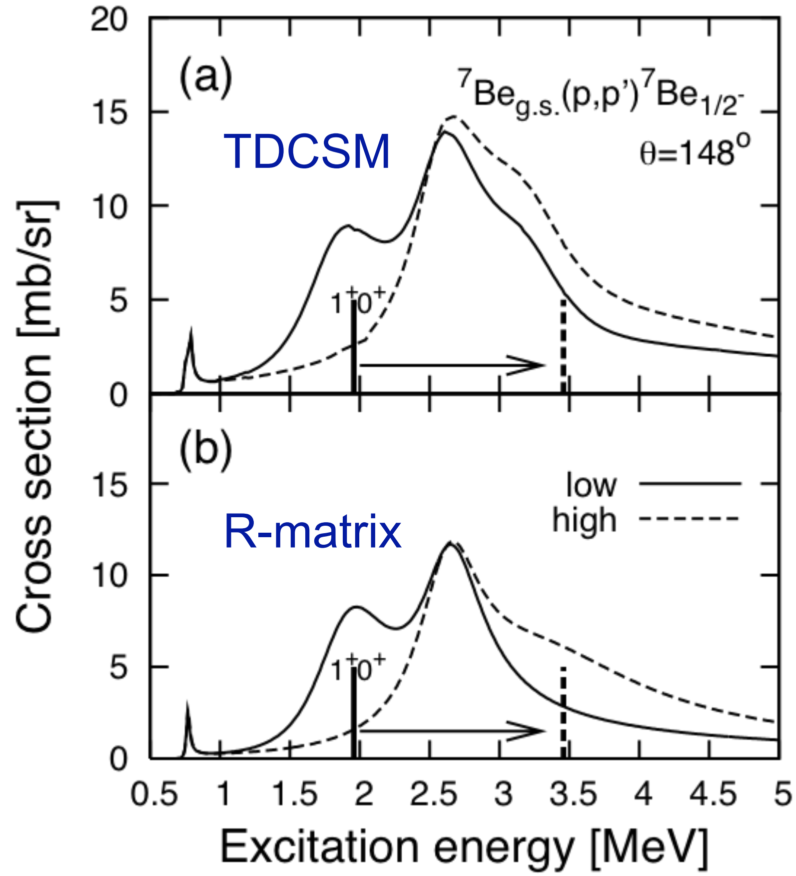


Experimental observation of 2^+, 0^+, and 1^+ states can be done in inelastic reaction

Elastic cross section



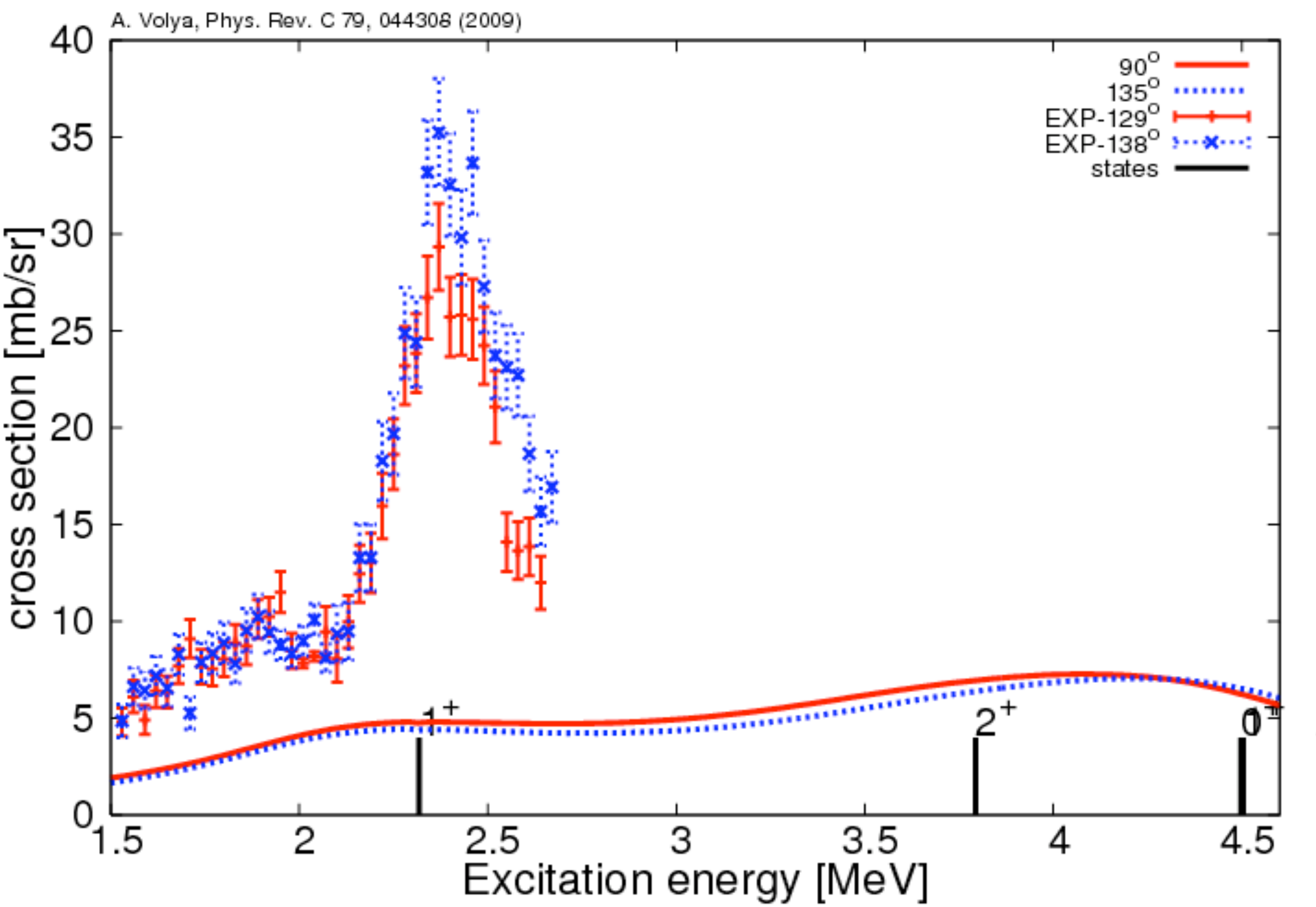
Inelastic cross section



TDCSM: WBP interaction +WS potential, threshold energy adjustment.
R-Matrix: WBP spectroscopic factors, R_c=4.5 fm, only 1^+ 1^+ 0^+ 3^+ and 2^+ l=1 channels
Experimental data from: G.Rogachev, et.al. Phys. Rev. C **64,** 061601(R) (2001).

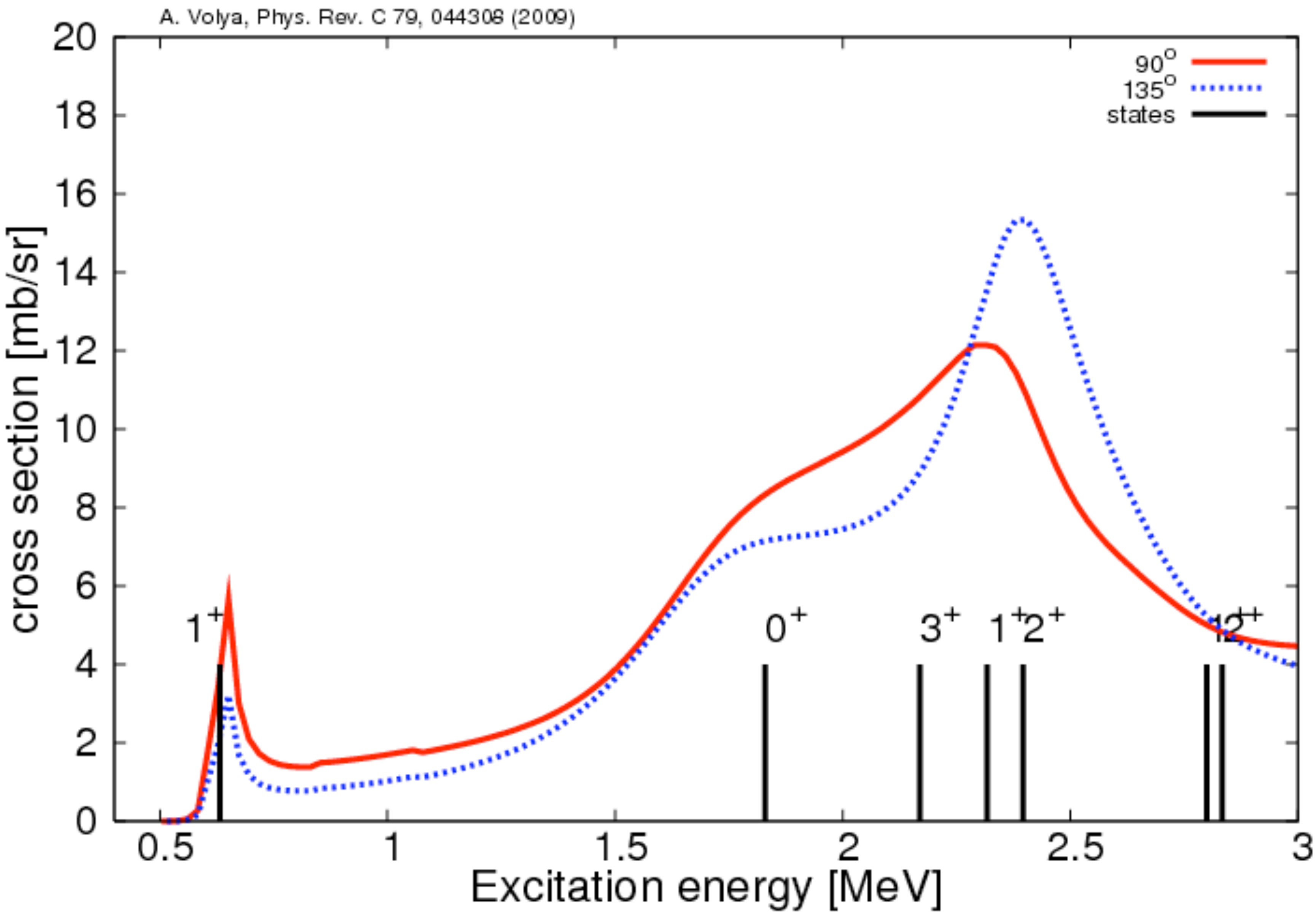
Resonances and their positions inelastic $^7Be(p,p')^7Be$ reaction in TDCSM

CKI+WS Hamiltonian



See animation at www.volya.net

Position of the 2+ and its role in $^7Be(p,p)^7Be$

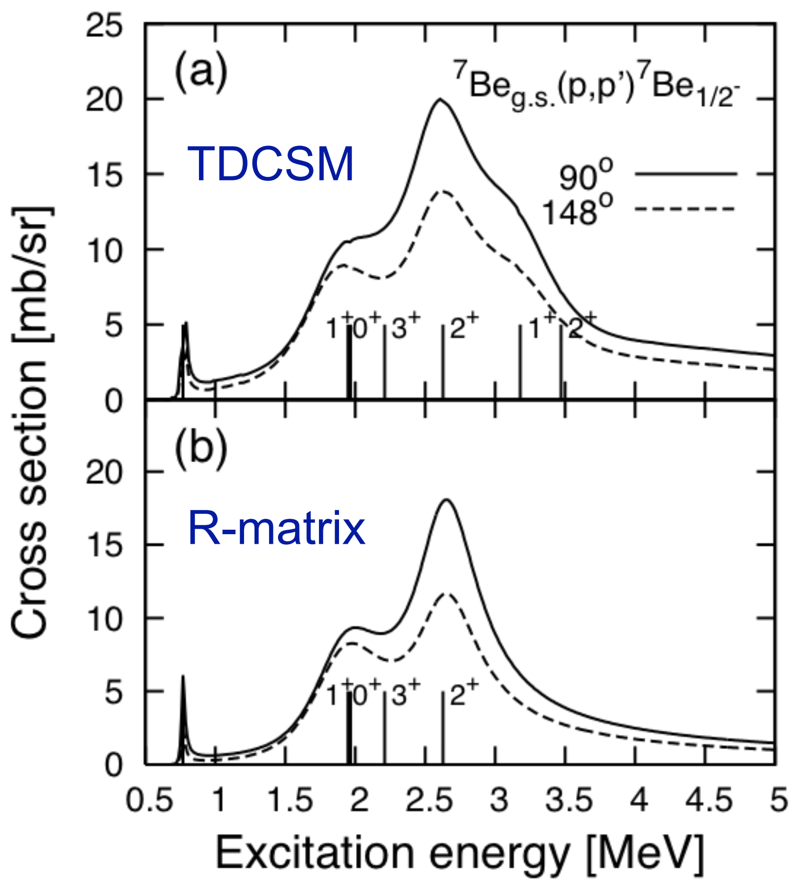


See animation at www.volya.net

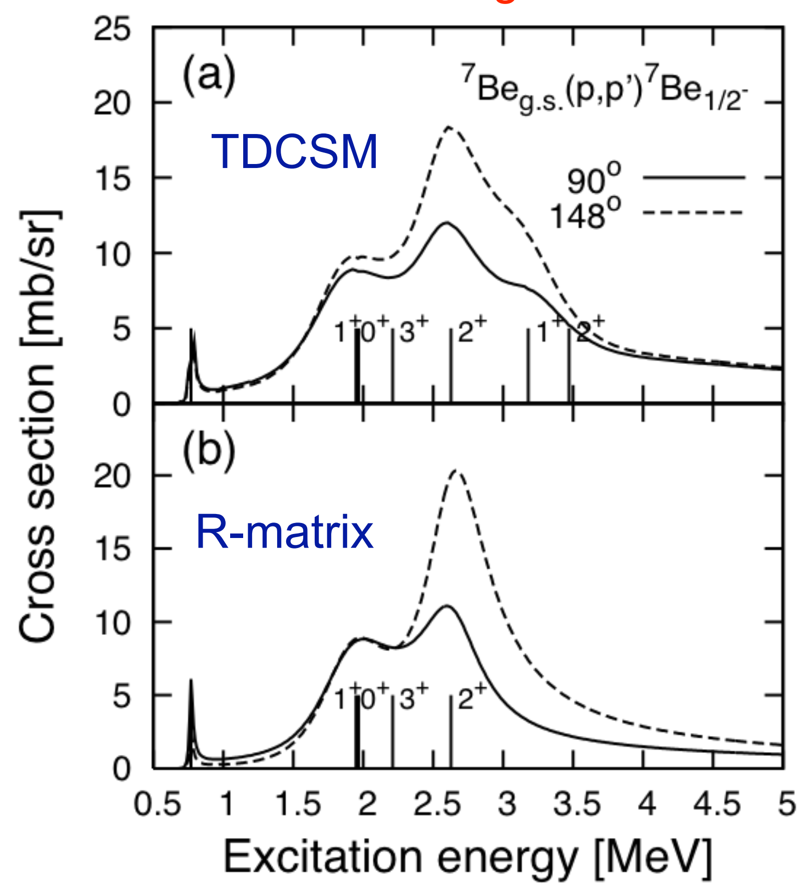
From cross section to many-body structure $^7Be(p,p)^7Be$

J^π	E(MeV)	$p_{3/2}$(g.s.)	$p_{1/2}$(gs)	$p_{3/2}$	$p_{1/2}$
1_1^+	0.7693	-0.563	0.303	0.867	-0.138
1_2^+	1.947	0.597	0.826	0.284	0.240
0_1^+	1.967	0.693	0	0	-0.918
3_1^+	2.2098	0.612	0	0	0
2_2^+	2.628	0.149	0.326	-0.632	0

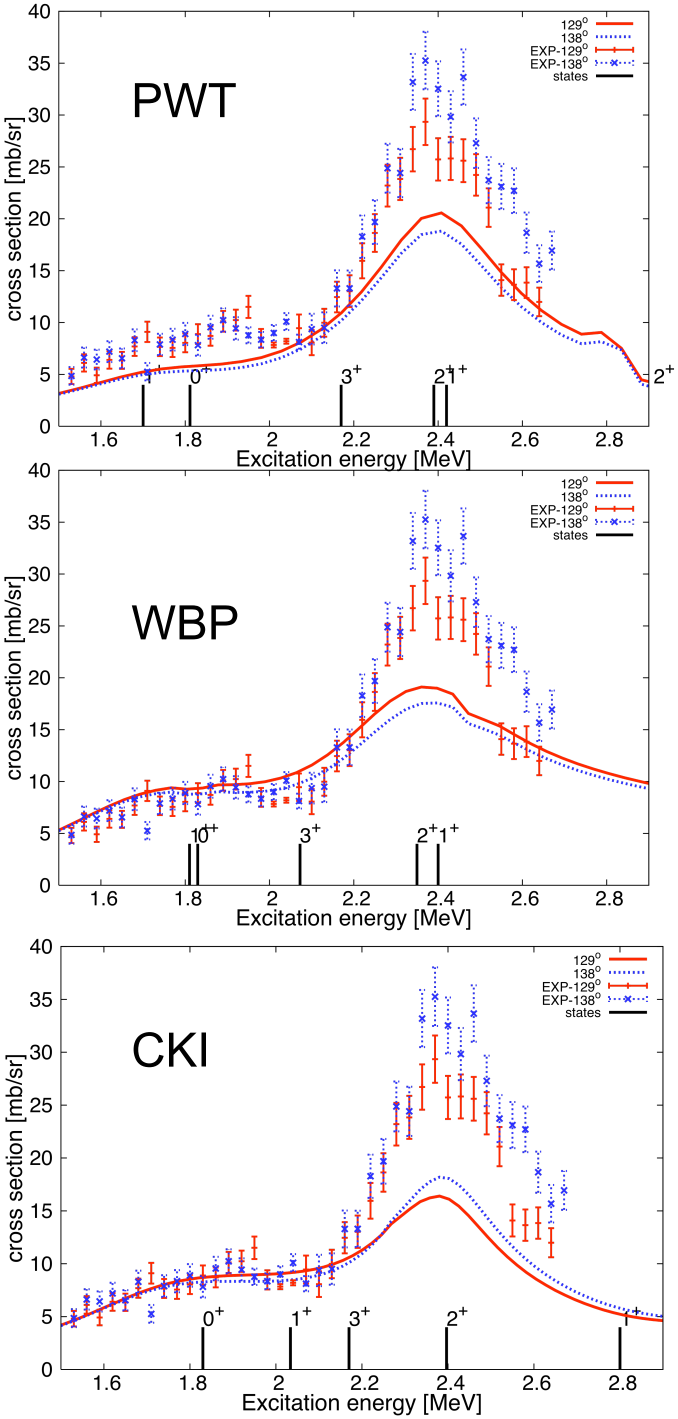
identical energies, identical widths, identical spectroscopic factors but different cross section

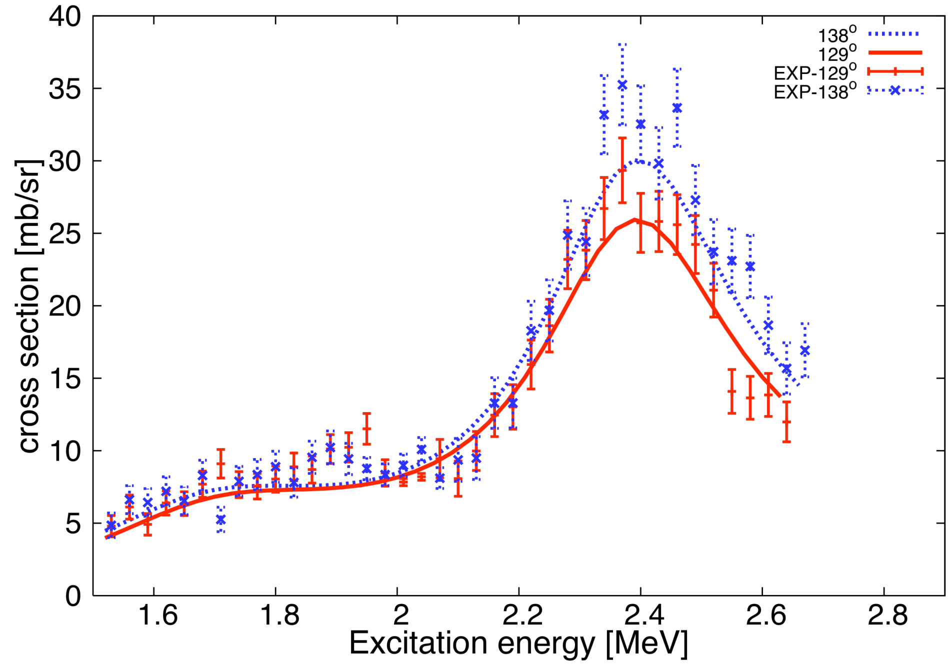


Phase changed



R-matrix fit and TDCSM for $^{7}Be(p,p)^{7}Be$

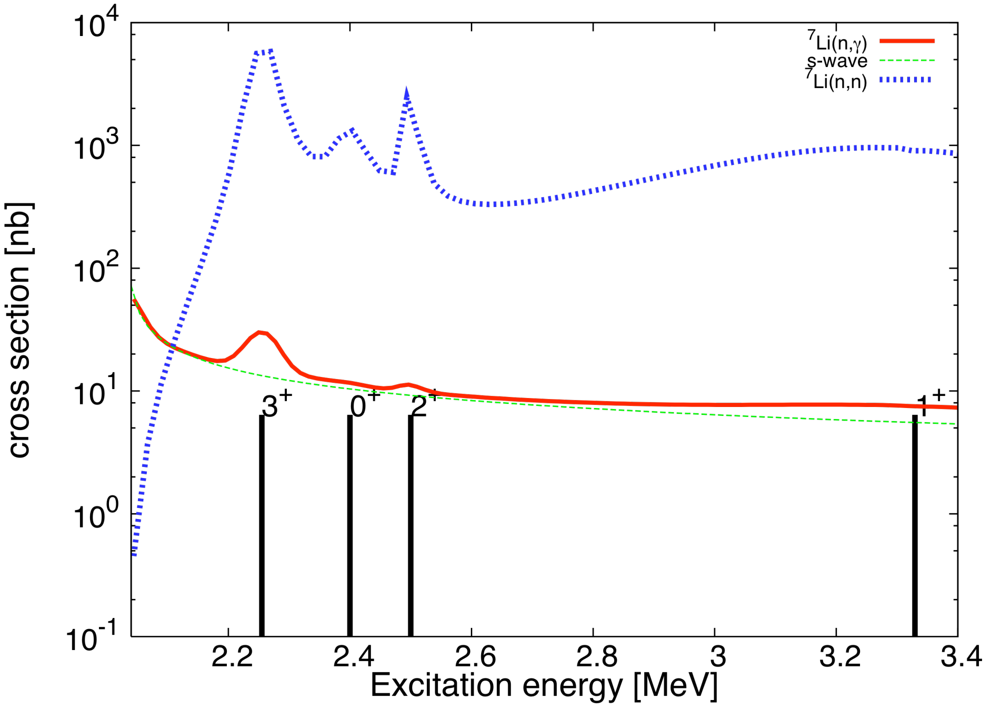


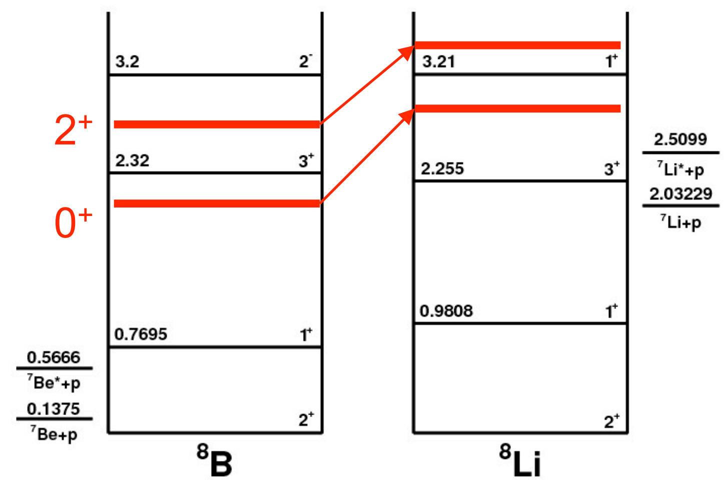


Chanel Amplitudes from TDCM and final best fit

	J^{π}	$p_{1/2}$, I=3/2	$p_{3/2}$, I=3/2	$p_{1/2}$, I=1/2	$p_{3/2}$, I=1/2
FIT	2^+	-0.293	0.293		0.534
CKI	2^+	-0.168	0.164		0.521
FIT	1^+	-0.821	-0.612	0.375	0.175
CKI	1^+	-0.840	-0.617	0.332	0.178

Studies of the mirror nucleus ^{8}Li



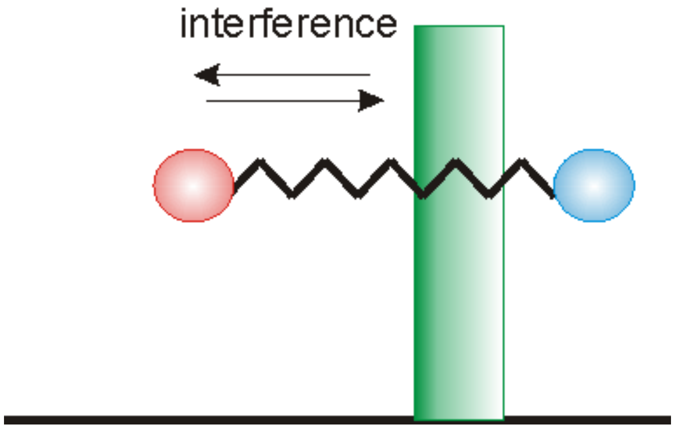


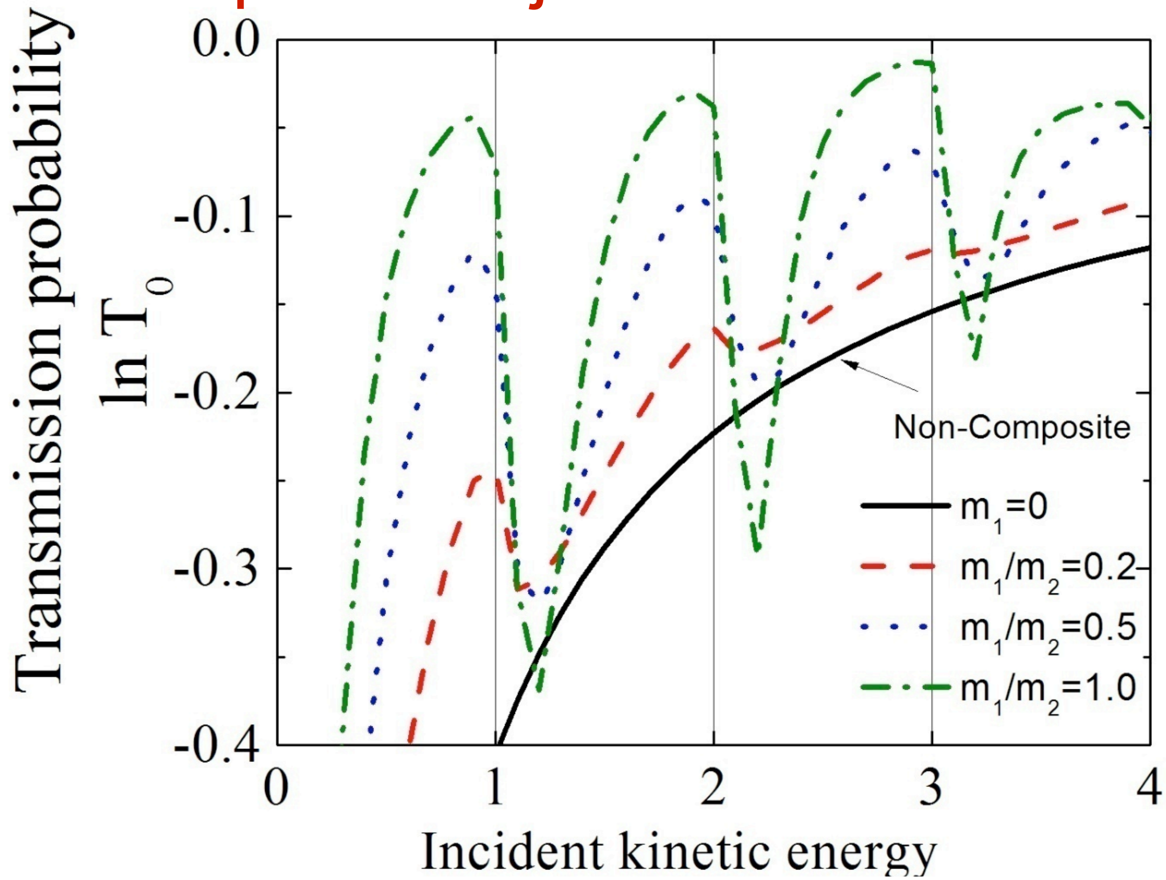
From reactions to structure

looking to the future

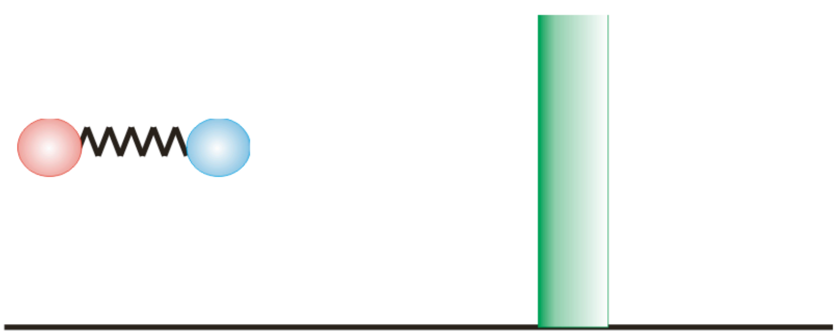
- Dynamics for channels
- Coupled channels
- Folded potential
- Virtual excitation dynamics on complex plane

Resonant tunneling of composite objects





N. Ahsan and A.Volya, in
CHANGING FACETS OF NUCLEAR STRUCTURE
World Scientific (2008)



Variable amplitude technique

Factorization $\Psi(R,r) = \sum \chi_n(R)\psi_n(r)$

Folded potential $$v_{nm}(R) = \frac{2M}{\hbar^2}\int_{\infty}^{\infty} V(R,r)\psi_n^*(r)\,\psi_m(r)dr$$

Center-of-mass $$\left(\frac{d^2}{dR^2} + k_n^2\right)\chi_n(R) - \sum_{m=0}^{\infty} v_{nm}(R)\chi_m(R) = 0$$

Reflection $$C_{nm}^- = \frac{1}{2ik_n}\sum_p \int dR' e^{ik_nR'} v_{np}(R')\chi_{pm}(R')$$

Transmission $$C_{nm}^+ = \delta_{nm} + \frac{1}{2ik_n}\sum_p \int dR' e^{-ik_nR'} v_{np}(R')\chi_{pm}(R')$$

Differential equations for amplitudes

Truncation of potential leads to the following

$$C^{-}_{nm}(R) = e^{ik_n R}\left[2ik_m U_{nm}(R) - \delta_{nm}\right] e^{ik_m R}$$

$$\frac{dU_{nm}}{dR} = \delta_{nm} - i(k_n + k_m)U_{nm} - \sum_{pq} U_{np} v_{pq} U_{qm}$$

$$C^{-}_{nm}(\infty) = 0$$

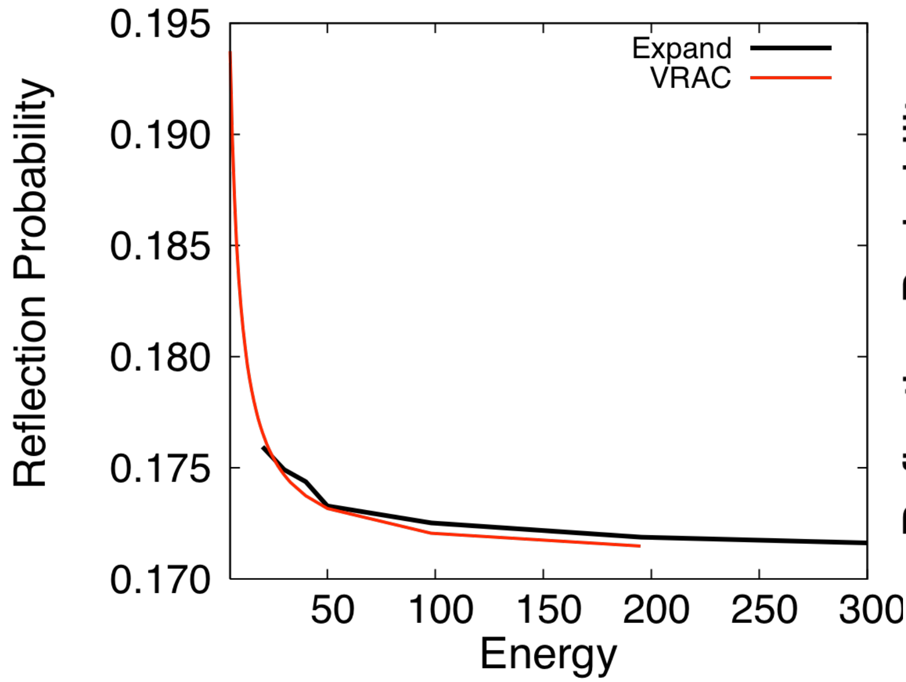
Y. Tikochinsky, *Ann. Phys.* ***103,*** *185 (1977)*.

M. Razavy, *Quantum theory of tunneling (World Scientific, River Edge, NJ ; Singapore, 2003), p. 549.*

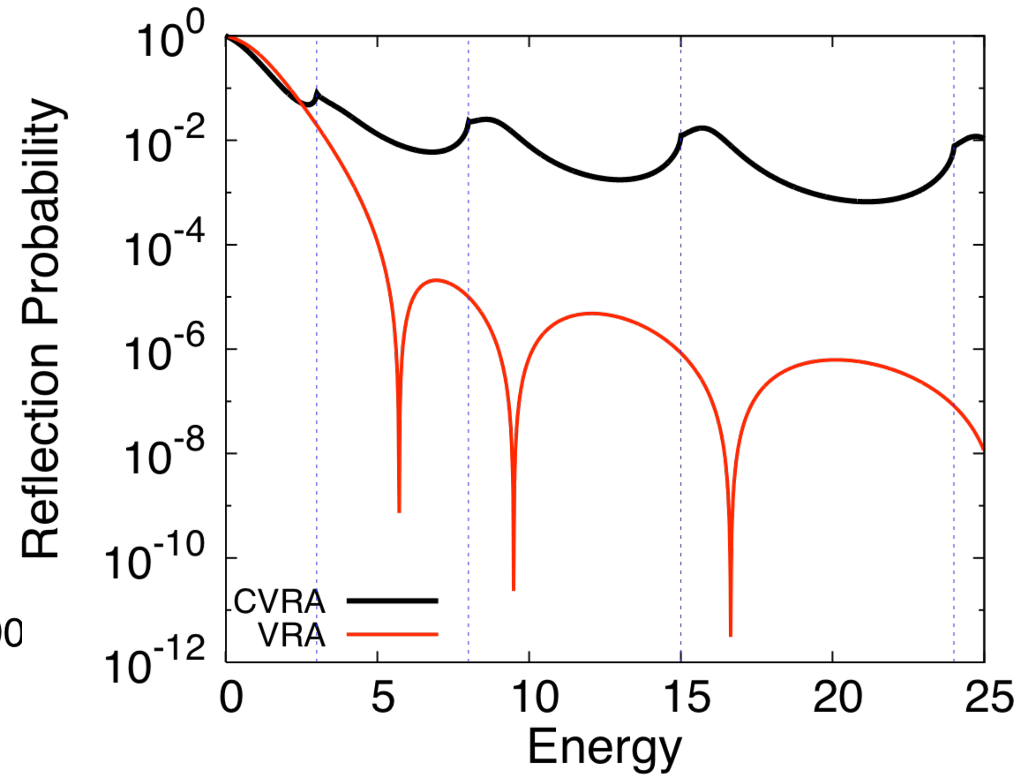
Role of virtual channels, convergence

Intrinsic binding: Infinite square well External Potential : delta function

Convergence



Role of virtual channels

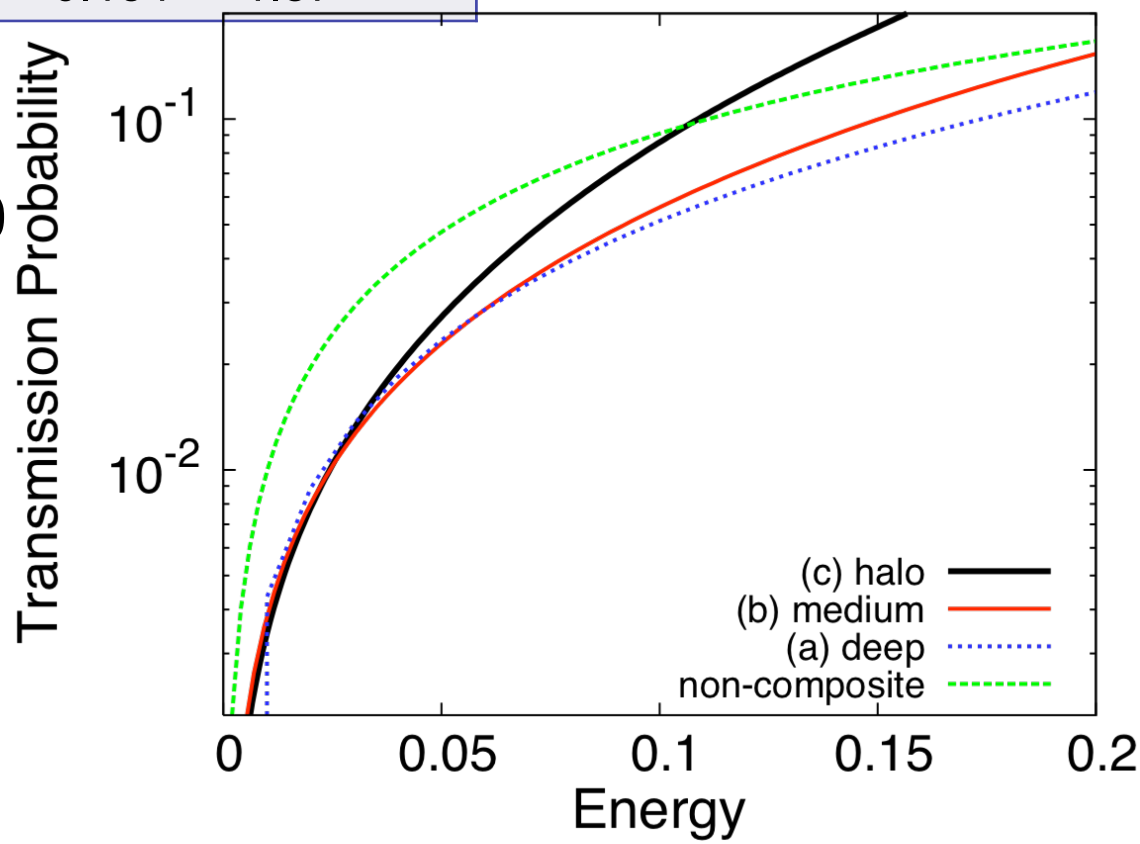


Two-nucleon system with continuum

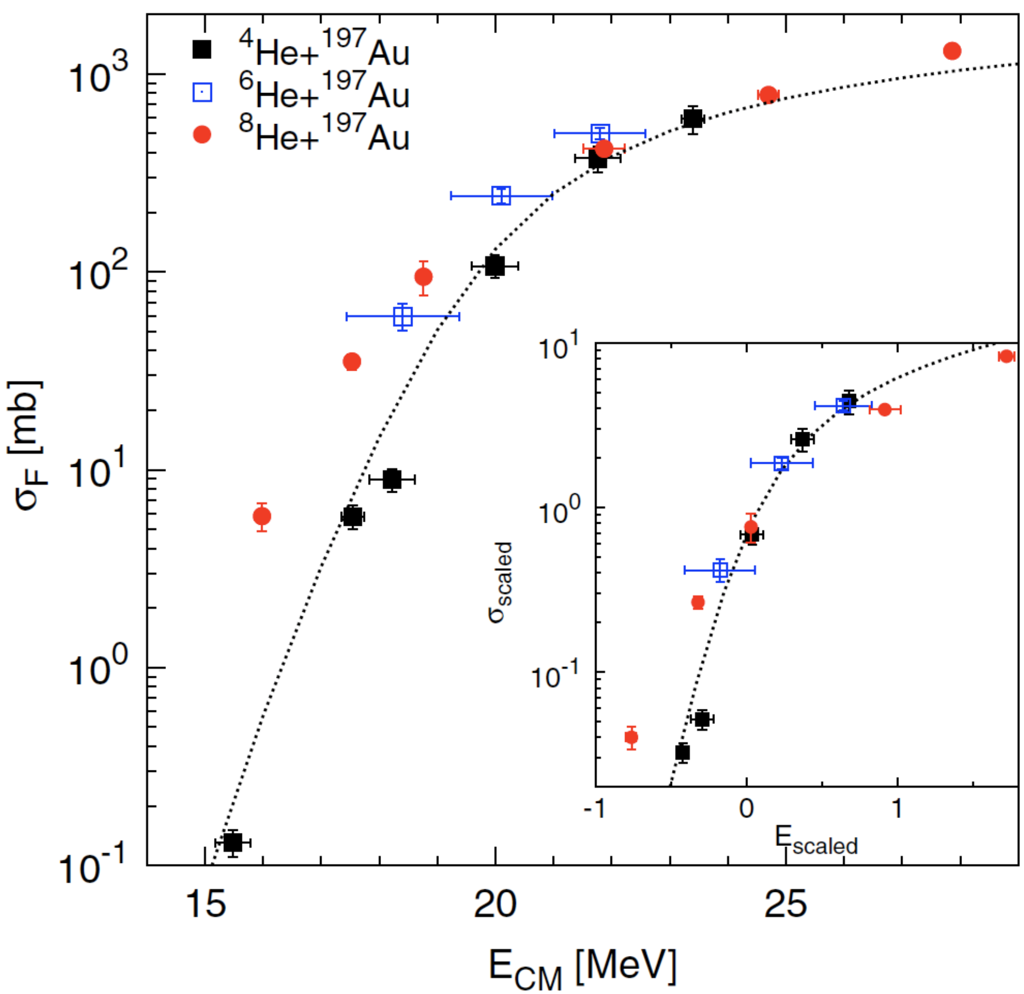
finite square well binding

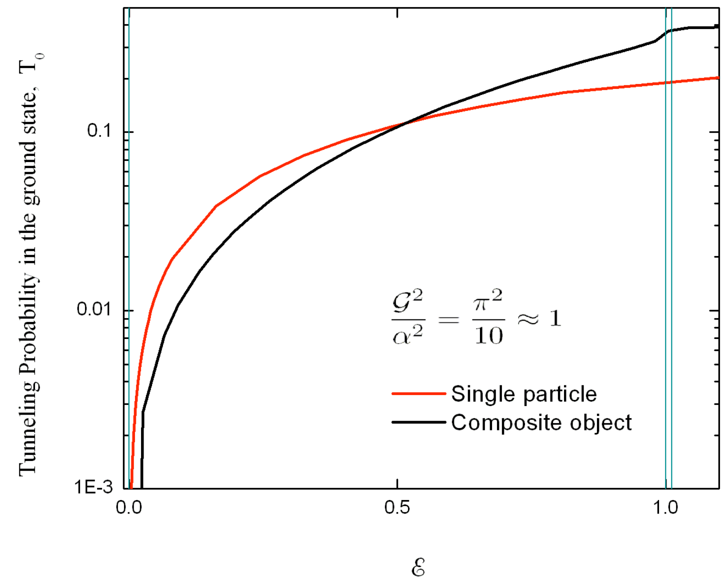
Model	Depth	size	energy	WF RMS
(a) Deep	2	1.0	-1.209	0.81
(b) Medium	1	1.0	-0.455	1.16
(c) Halo	0.5	1.0	-0.154	1.87

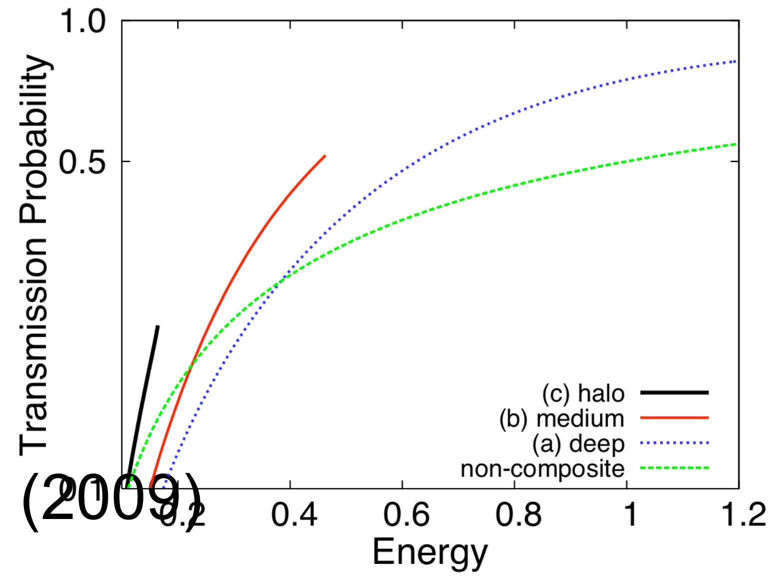
Treat arbitrary potential
Quantization in a box 30
Discrete points 12k
Virtual channels 120



Enhanced tunneling probability for composite objects







A. Lemasson, et.al. PRL 103, 232701 (2009)

Summary:

- TDCSM new approach to many-body physics on the verge of stability
  - Direct relation to physics of unstable systems
  - Overcoming technical difficulties
  - New numerical methods
  - Treatment of complicated interaction terms
- Practical applications
  - Adjustments of interactions
  - Position of resonances
  - Direct experimental test of theoretical assumptions
- Future methods

Recent publication:

A. Volya, Phys. Rev. C 79, 044308 (2009).

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