Coupled-Channels Density-Matrix Approach to Nuclear Reaction Dynamics

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Outline

- Introduction
  - ✓ Quantum decoherence in a broad context
  - ✓ Coherent coupled channels model
  - ✓ Quantum decoherence in nuclear collisions
- Coupled-channels density-matrix approach
  - ✓ Picture and formulation
  - ✓ Applications
- Summary

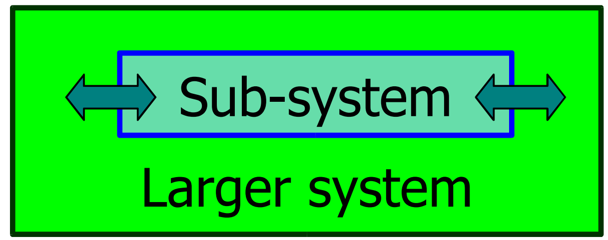
The Quantum to Classical transition - from coherent superposition to irreversibility

W.H. Zurek, Rev. Mod. Phys. **75** (2003) 715; Phys. Today **44** (1991) 36
M. Schlosshauer, Decoherence and the quantum to classical transition, Springer (2007)
P. Ball, Nature **453** (2008) 22

Idealized isolated system

Superposition of basis states

Described by coherent Q.M.



Sub-system "entangled" with environment

Loss of coherence in smaller system

⇒ Irreversible outcome (classical)

Can be described by density matrix (Q.M.)

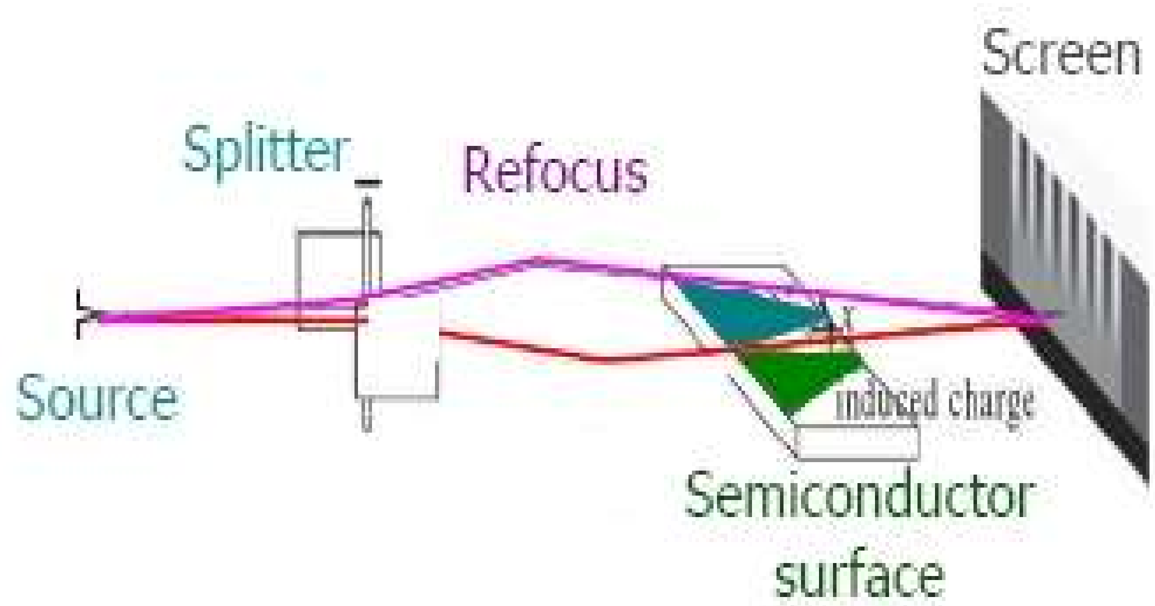
- Quantum decoherence – "dynamical dislocalization of Q.M. superpositions"

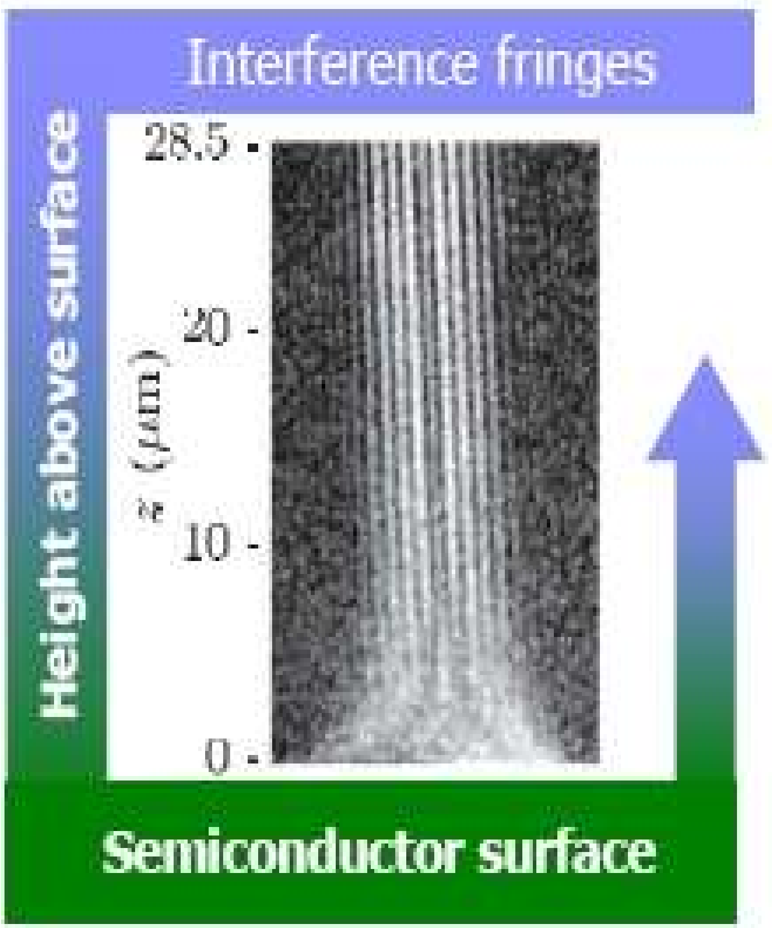
(H.D. Zeh arXiv:quant-ph/0512078 v2) coherence shared with (lost in) environment

Quantum Decoherence

Example: Electron entanglement with a surface

Double-slit type experiment with single electrons

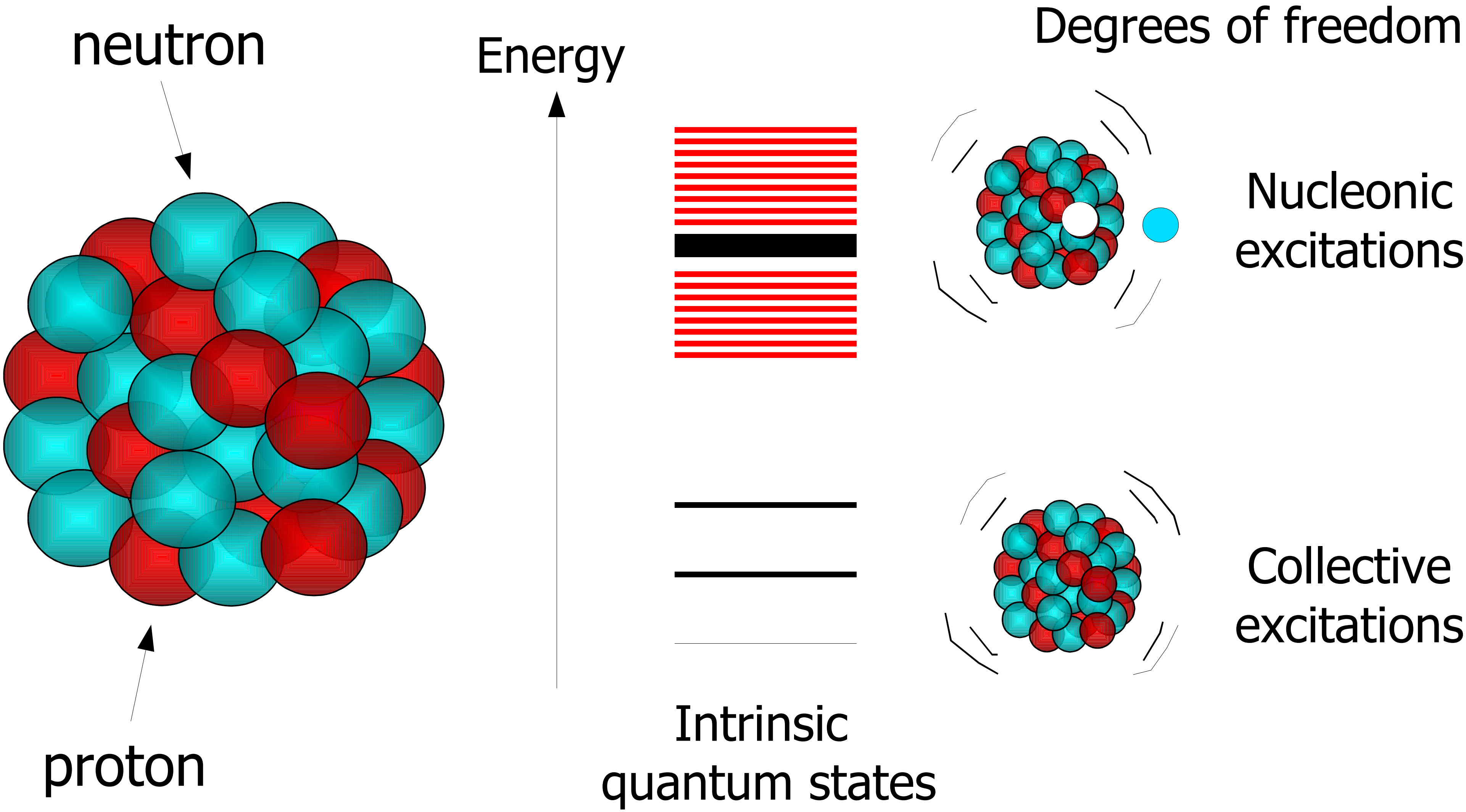




P. Sonnentag et al., PRL **98** (2007) 200402

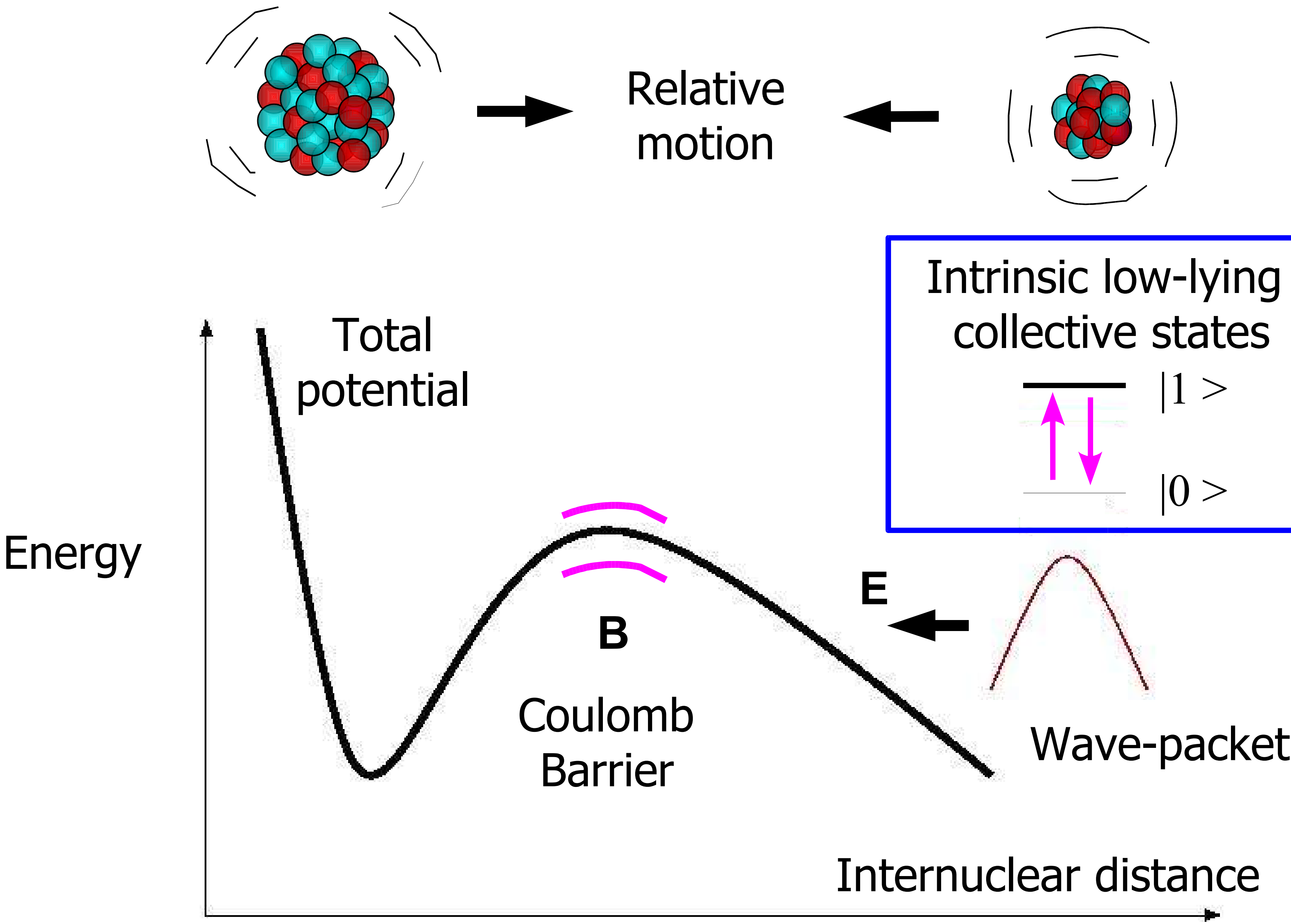
Can a quantitative model be developed for nuclear collisions ?

Composite Atomic Nucleus



How do these excitations affect the nuclear collision dynamics ?

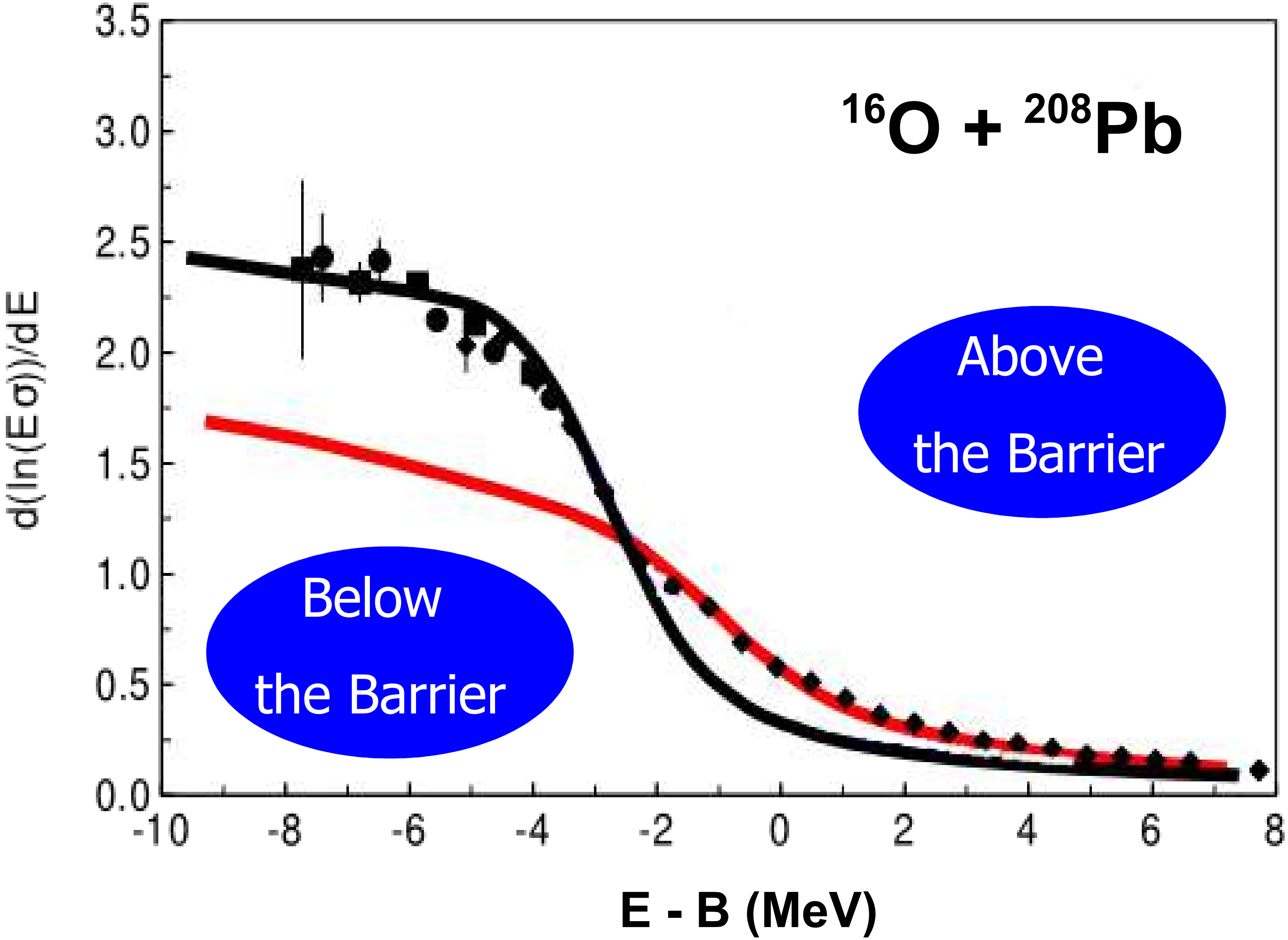
Low-Energy Collision Dynamics: Coherent Quantum Description



Failure of the Coherent Quantum Description

Coupling Assisted Quantum Tunnelling: Nuclear Fusion

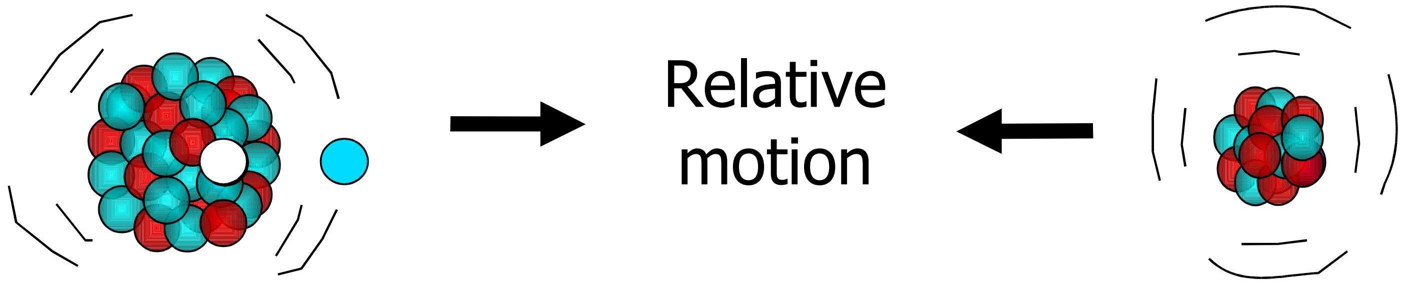
Measure of Fusion Probability

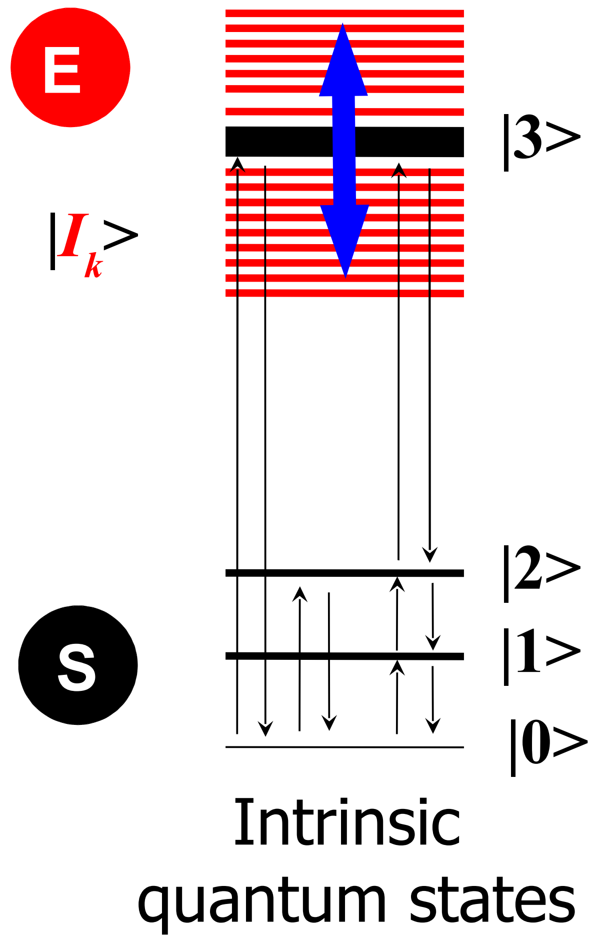


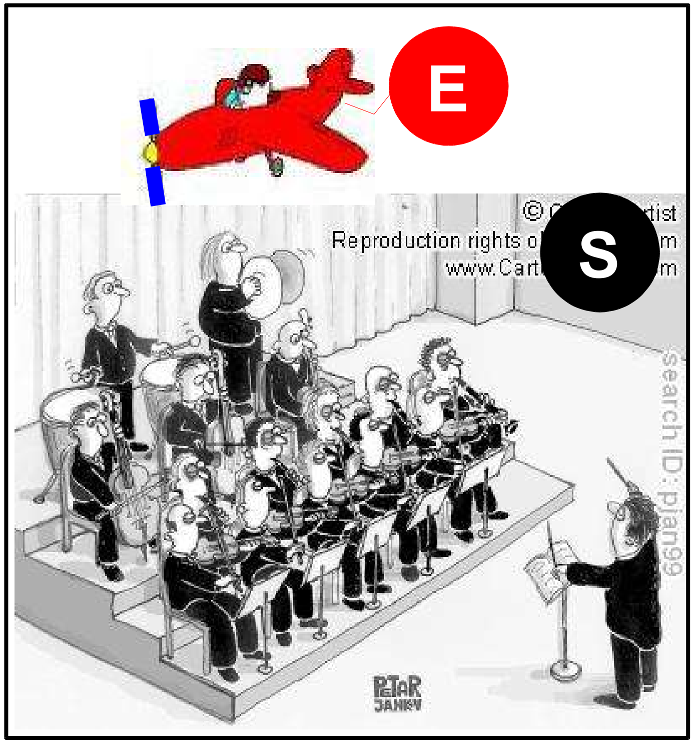
Dasgupta, Hinde, AD-T, Bouriquet, Low, Milburn & Newton, PRL 99 (2007) 192701

Quantum Decoherence in Nuclear Collisions

AD-T, Hinde, Dasgupta, Milburn & Tostevin, PRC 78 (2008) 064604



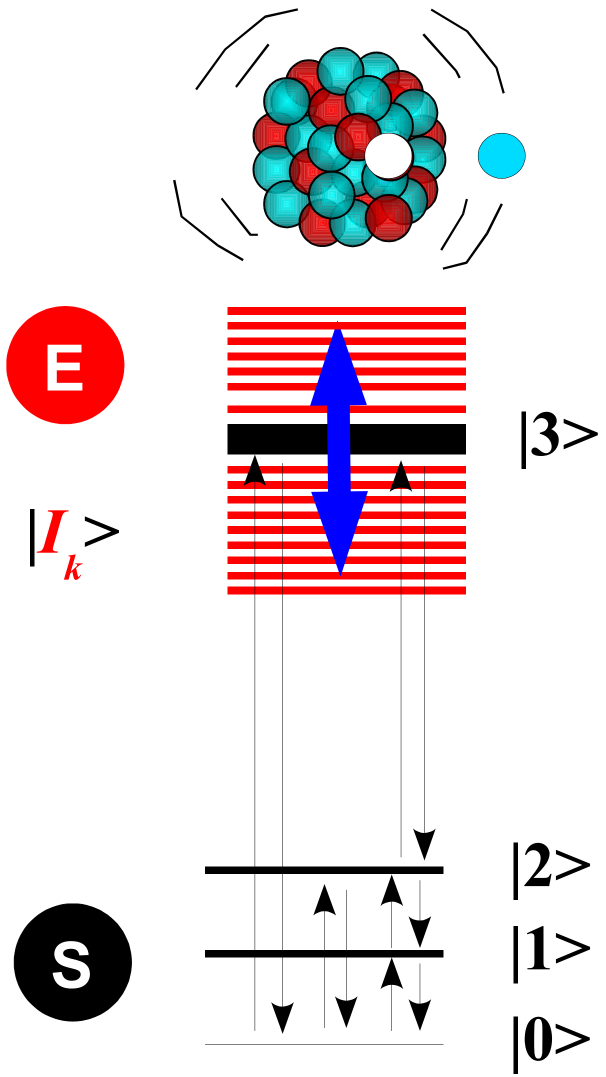




Suppresses
quantum tunnelling
(sub-barrier fusion)

Quantum Decoherence in Nuclear Collisions

AD-T, Hinde, Dasgupta, Milburn & Tostevin, PRC 78 (2008) 064604



Intrinsic quantum states

$$\partial\hat{\rho}/\partial t = [\hat{\mathcal{L}}_H + \hat{\mathcal{L}}_D]\hat{\rho} \quad , \quad \hat{\rho}(0) = \hat{\rho}_0$$ Master equation

$$\hat{\mathcal{L}}_H \hat{\rho} = -i[\hat{H}, \hat{\rho}]/\hbar$$ Schrödinger description

$$\hat{\mathcal{L}}_D \hat{\rho} = \sum_{\mathbf{k}} \left(\hat{C}_{\mathbf{k}} \hat{\rho} \hat{C}^\dagger_{\mathbf{k}} - \tfrac{1}{2} [\hat{C}^\dagger_{\mathbf{k}} \hat{C}_{\mathbf{k}}, \hat{\rho}]_+ \right)$$

$$\hat{C}_{Ij} = \sqrt{\Gamma_{Ij}} |I\rangle\langle j|$$

Decoherence & Absorption

Example : 1- dimensional model

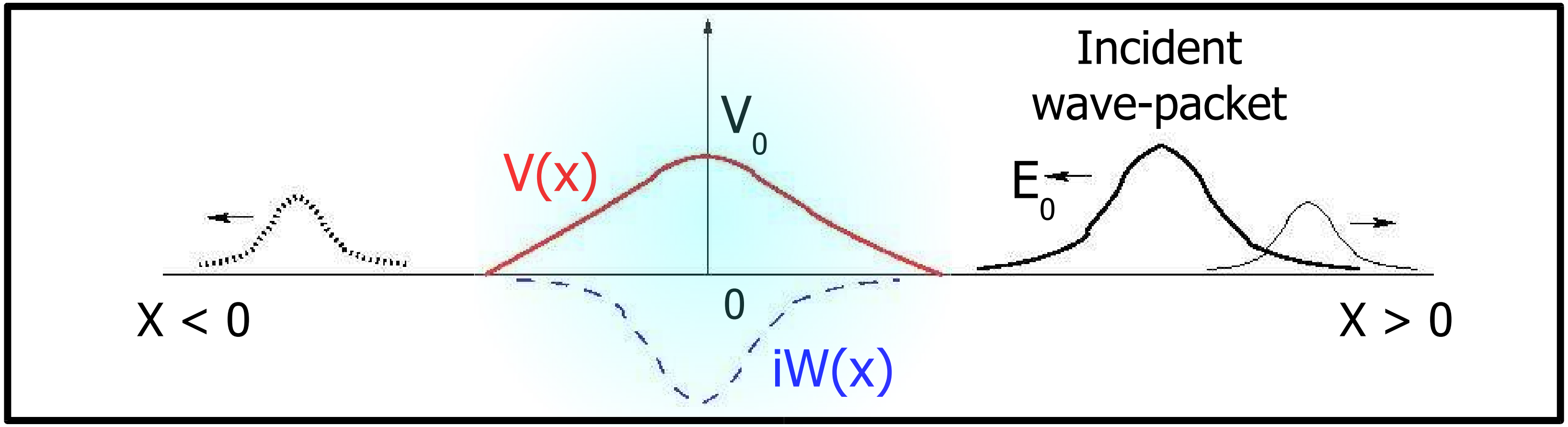
$$\hat{\rho}(t) = \sum_{ij,rs} |r)|i\rangle \, \rho^{rs}_{ij}(t) \, \langle j|(s| \quad , \quad \rho^{rs}_{ij}(0) = \rho^{rs}_{00}(0) = g_0(r) g_0^*(s)$$

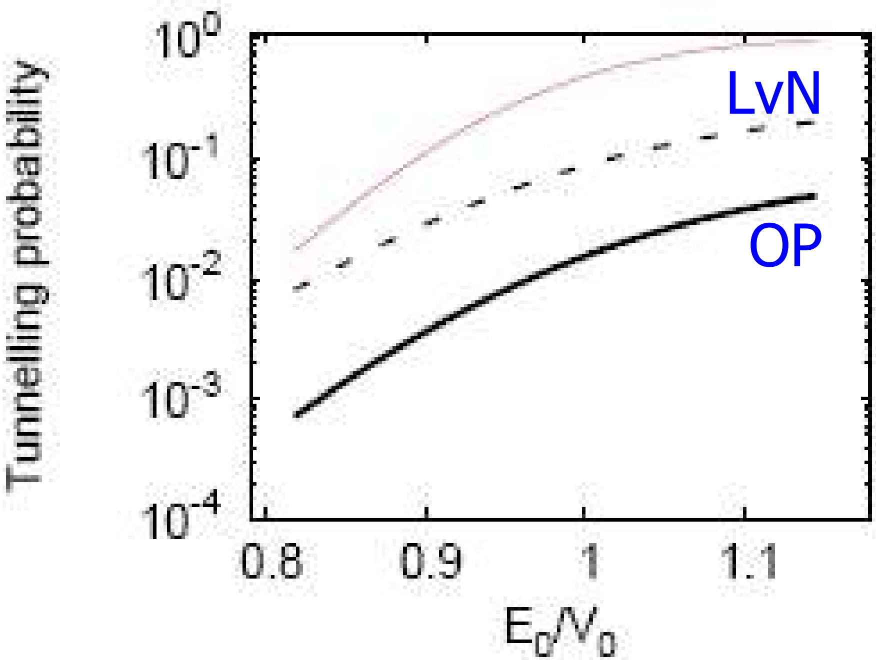
$|i\rangle, i = 1, \ldots N$ Intrinsic (energy) basis

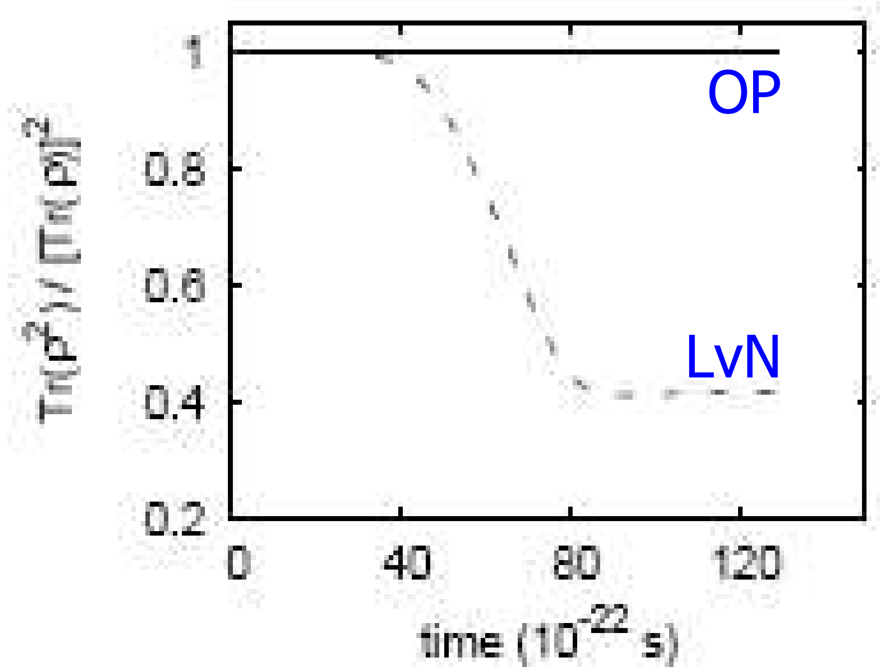
$|r), r = 1, \ldots M$ Coordinate (grid) basis

Quantum Decoherence in Nuclear Collisions

Absence of Decoherence in the Optical Potential Model

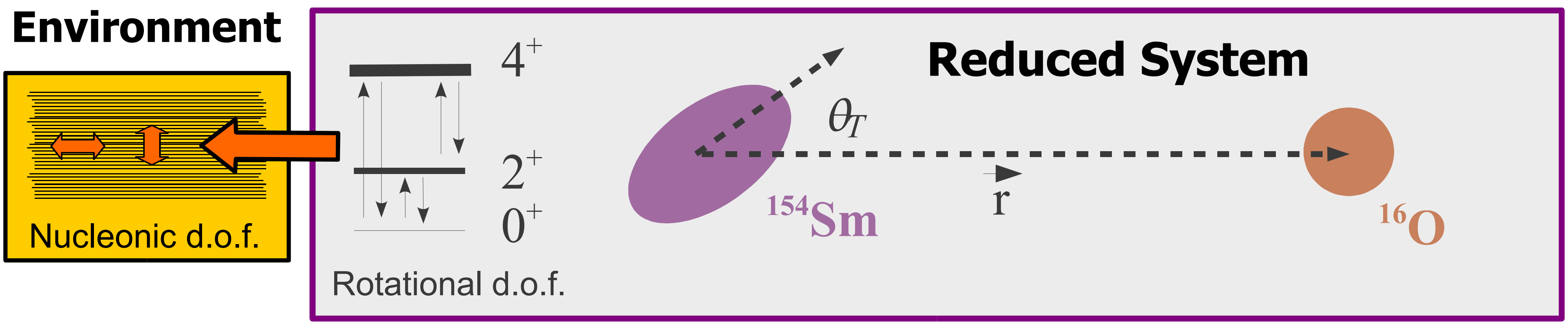






Decoherence significantly affects quantum tunnelling, and thus scattering as well

Coupled-Channels Density-Matrix Approach



$$|\chi\rangle = \sum_{LJM} \psi_{k_0}(r)\,|0\,L;JM\rangle \;\Rightarrow\; \hat{\rho}_0 = |\chi\rangle\langle\chi|$$

$$\hat{\rho}_0 = \sum_{\alpha,\alpha',rs} |r)|\alpha\rangle\, \rho^{rs}_{\alpha\alpha'}(t=0)\, \langle\alpha'|(s|,$$

where $\alpha \equiv (IL;JM)$, $|\alpha\rangle$ and $|r)$ are the coupled angular momentum basis and the discrete grid-basis describing the internuclear separations, respectively.

$$\begin{aligned}\rho^{rs}_{\alpha\alpha'}(t=0) &= N^2 \exp\left[-\frac{(r-r_0)^2}{2\sigma^2}\right] e^{ik_0\,r} \\ &\times \exp\left[-\frac{(s-r_0)^2}{2\sigma^2}\right] e^{-ik_0\,s}\; \delta_{I\,0}\; \delta_{I'\,0},\end{aligned}$$

where N is determined from the normalization condition $\sum_{r\alpha} \rho^{rr}_{\alpha\alpha} = 1$.

Coupled-Channels Density-Matrix Approach

Equations of Motion

S

$$
\begin{aligned}
i\hbar\,\dot{\rho}^{rs}_{\alpha\alpha'} \;=\; & \sum_t \left(T^{rt}\,\rho^{ts}_{\alpha\alpha'} - \rho^{rt}_{\alpha\alpha'}\,T^{ts} \right) \\
& + \left[U_\alpha(r) - U_{\alpha'}(s) \right] \rho^{rs}_{\alpha\alpha'} \\
& + \sum_\beta \left[V_{\alpha\beta}(r)\,\rho^{rs}_{\beta\alpha'} - \rho^{rs}_{\alpha\beta}\,V_{\beta\alpha'}(s) \right] \\
& + \left(\varepsilon_\alpha - \varepsilon_{\alpha'} \right) \rho^{rs}_{\alpha\alpha'} \\
& + i\hbar \left\{ \delta_{\alpha\alpha'} \sum_\mu \sqrt{\Gamma^{rr}_{\alpha\mu}}\,\rho^{rs}_{\mu\mu}\,\sqrt{\Gamma^{ss}_{\alpha\mu}} \right. \\
& \left. - \frac{1}{2} \sum_\mu \left(\Gamma^{rr}_{\mu\alpha} + \Gamma^{ss}_{\mu\alpha'} \right) \rho^{rs}_{\alpha\alpha'} \right\}
\end{aligned}
$$

E

$$
\begin{aligned}
\dot{\rho}^{rs}_{\bar{\alpha}\bar{\alpha}'} \;=\; & \delta_{\bar{\alpha}\bar{\alpha}'} \sum_\mu \sqrt{\Gamma^{rr}_{\bar{\alpha}\mu}}\,\rho^{rs}_{\mu\mu}\,\sqrt{\Gamma^{ss}_{\bar{\alpha}\mu}} \\
& - \frac{1}{2} \sum_\mu \left(\Gamma^{rr}_{\mu\bar{\alpha}} + \Gamma^{ss}_{\mu\bar{\alpha}'} \right) \rho^{rs}_{\bar{\alpha}\bar{\alpha}'}
\end{aligned}
$$

Expectation value of an observable: $\langle \hat{\mathcal{O}}(t) \rangle = \mathrm{Tr}[\hat{\mathcal{O}}\,\hat{\rho}(t)]$

Coupled-Channels Density-Matrix Approach

Asymptotic Observables

The probability for producing the target in state (I, M_I) with the relative coordinate in the direction $\hat{r}'$:

$$\frac{dW}{d\Omega}(I, M_I) = \sum_q C^{JM}_{LmIM_I} Y_{Lm}(\hat{r}') \mathcal{S}_{\gamma\lambda}(t_f) \times C^{J'M'}_{L'm'IM_I} Y^*_{L'm'}(\hat{r}'),$$

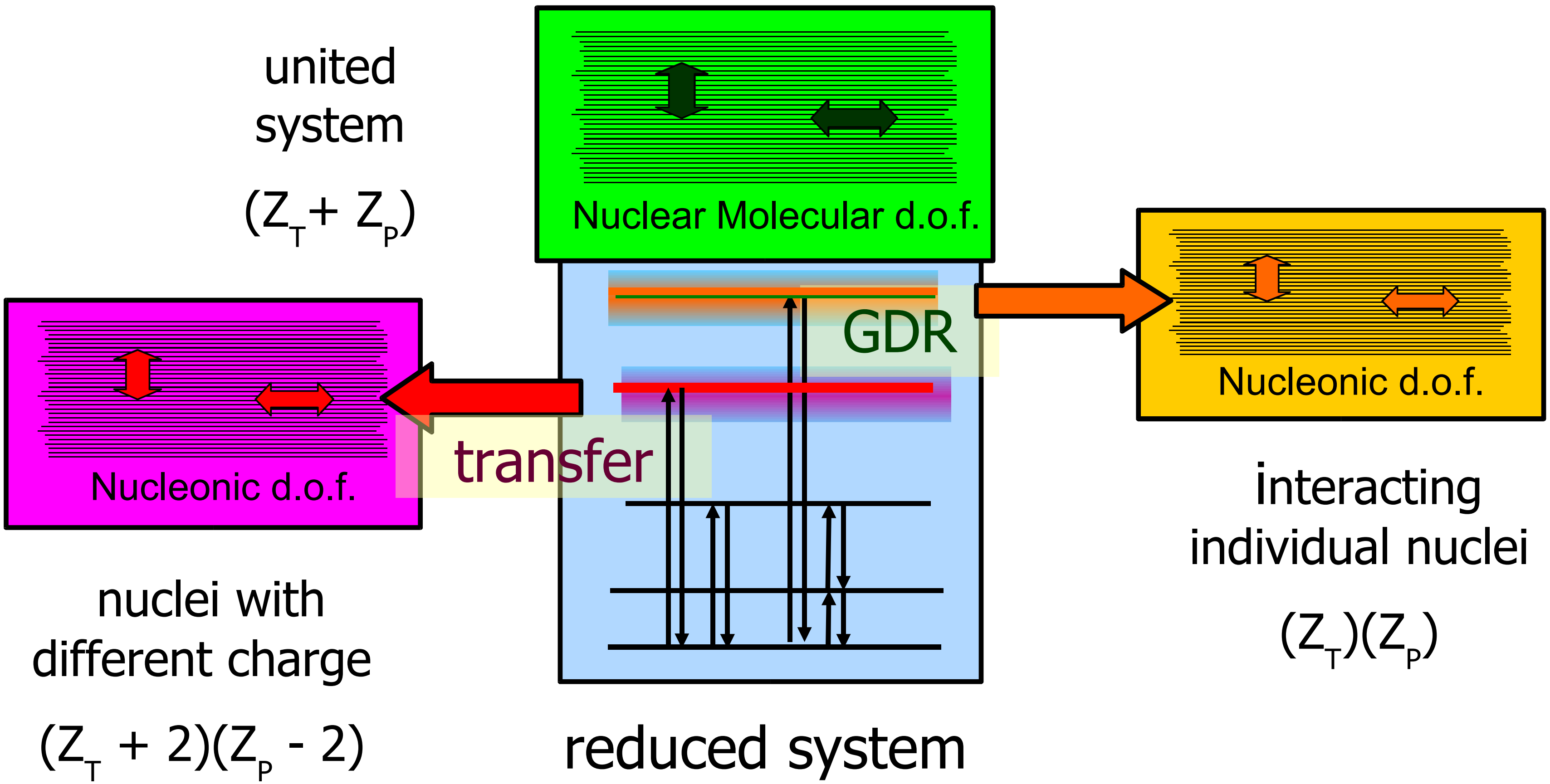
where $q \equiv (L, m, J, M, L', m', J', M')$, $\gamma \equiv (IL; JM)$, $\lambda \equiv (IL'; J'M')$, and $\mathcal{S}_{\gamma\lambda}(t_f) = \sum_{r'} \rho^{r'r'}_{\gamma\lambda}(t_f)$.

Integrating over all directions $\hat{r}'$ of solid angles, and summing over all M_I, the total probability for producing the target in state I (population) is obtained:

$$W(I) = \sum_{M_I} \sum_{LmJM} (C^{JM}_{LmIM_I})^2 \mathcal{S}_{\gamma\gamma}(t_f)$$

Coupled-Channels Density-Matrix Approach

Different Environments



Track decoherence and absorption through different mechanisms

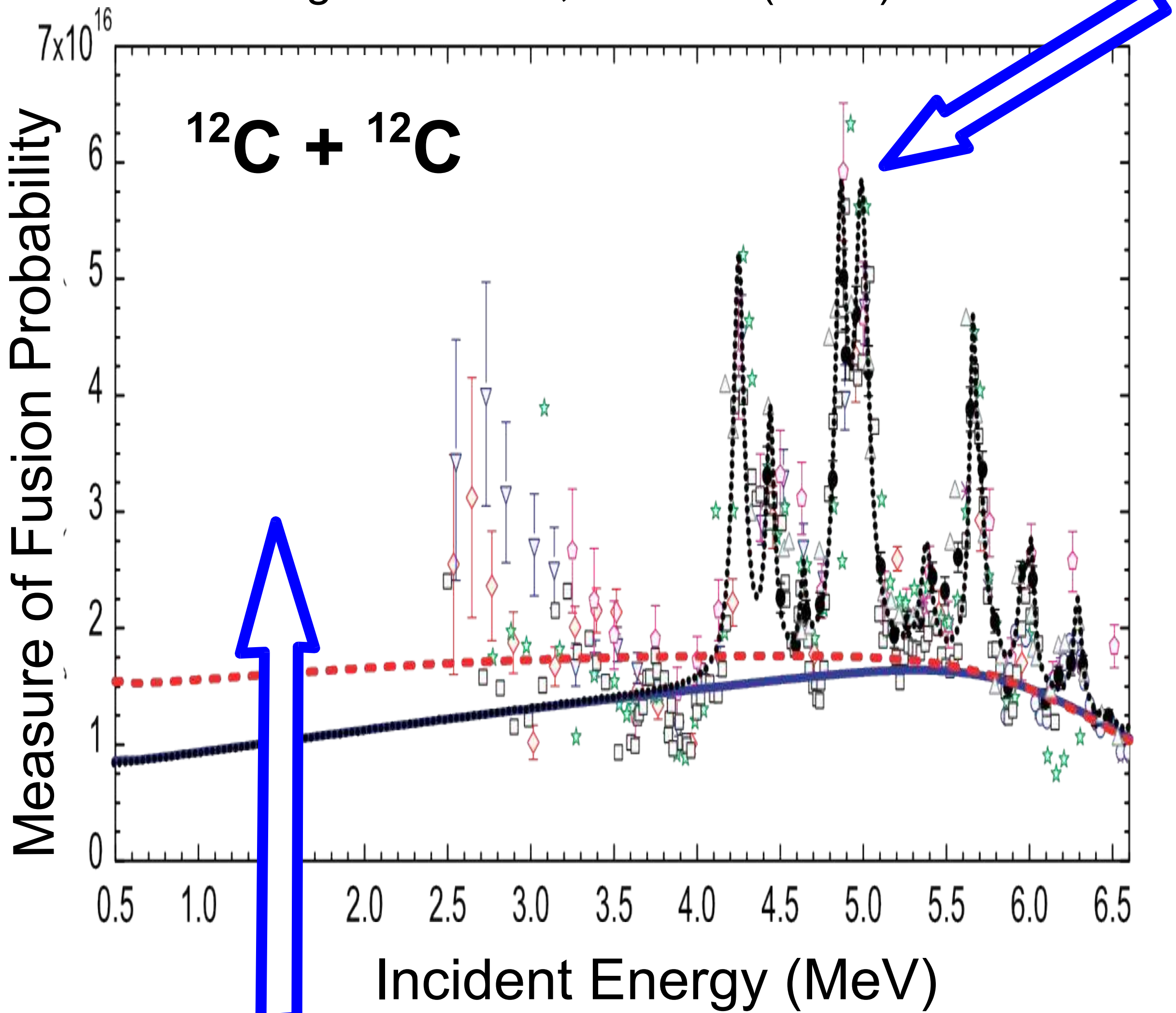
Environments are specific to particular degrees of freedom

Application: Understanding fusion of astrophysically-important collisions at low energies

AD-T, Gasques & Wiescher, PLB 652 (2007) 255

E.F. Aguilera et al., PRC **73** (2006) 064601

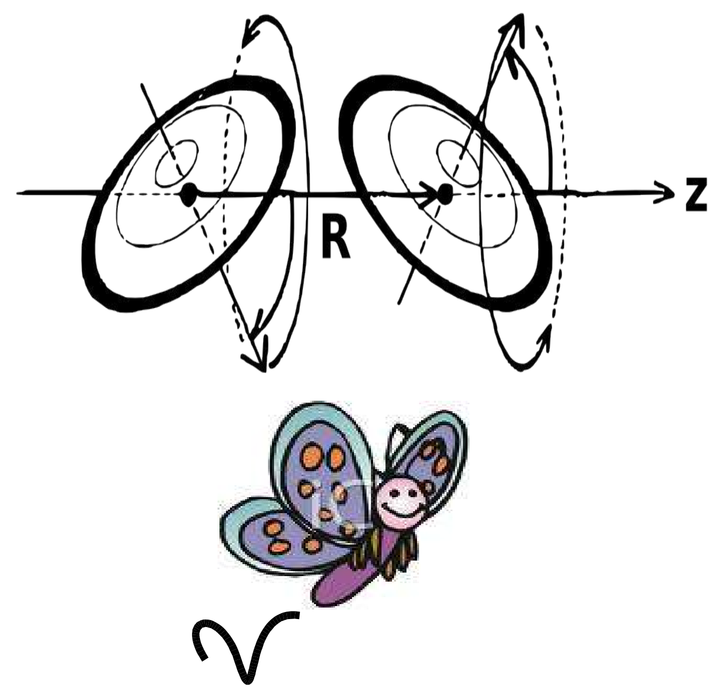
Origin of the resonances ?



Fusion probability at astrophysical energies ?

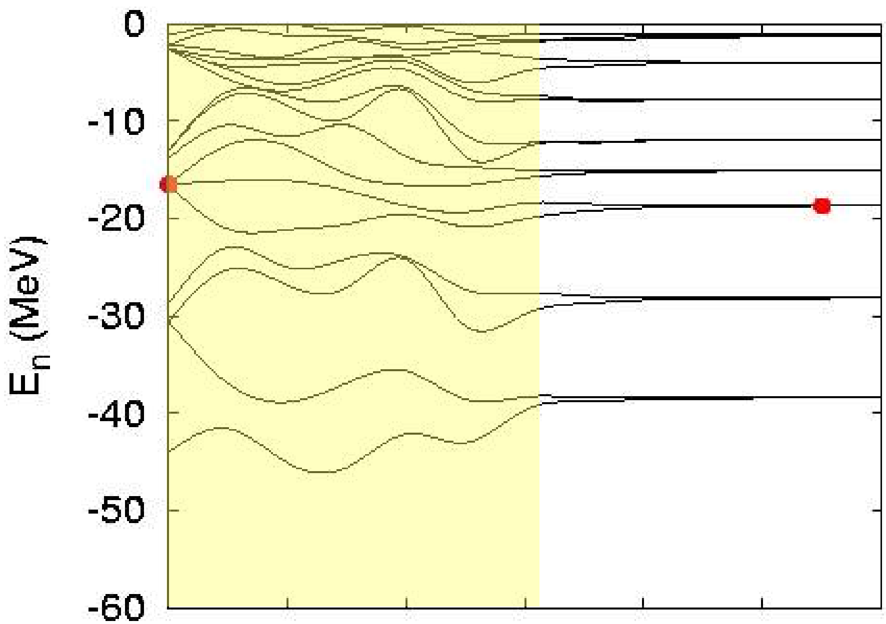
Complex excitation modes

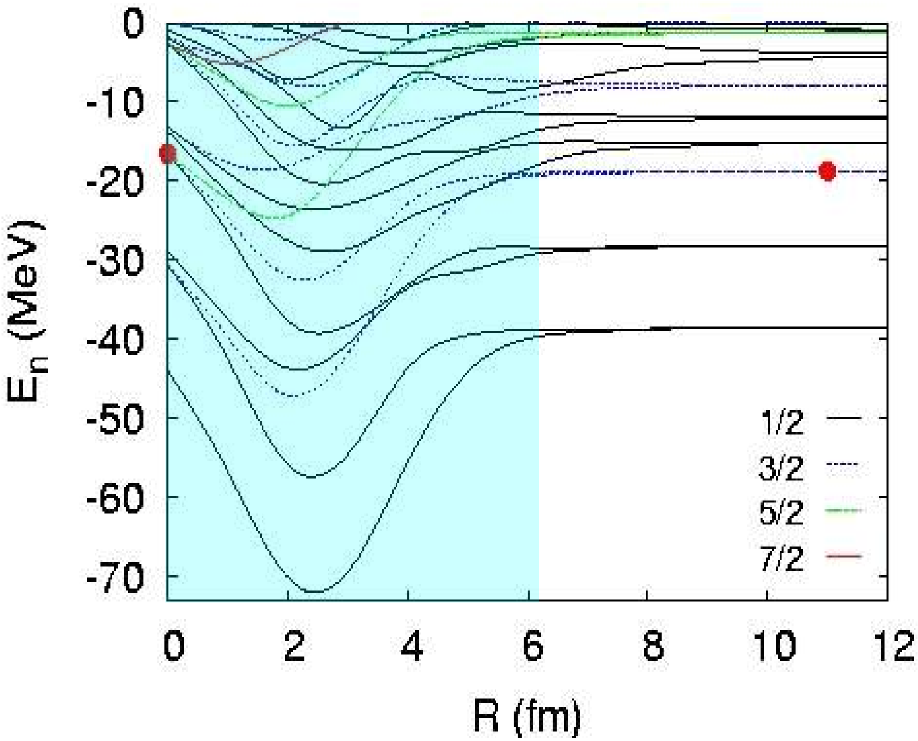
in dinuclear system

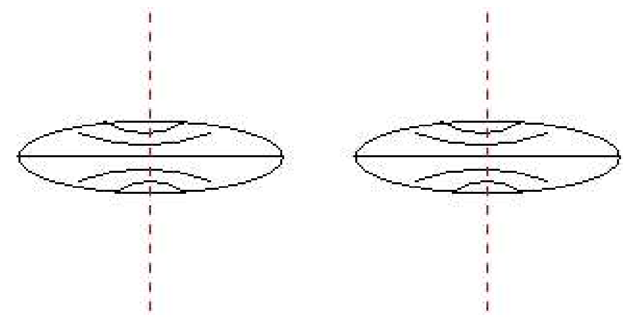


AD-T, PRL 101 (2008) 122501

Neutron molecular shell structure of two interacting deformed ^{12}C

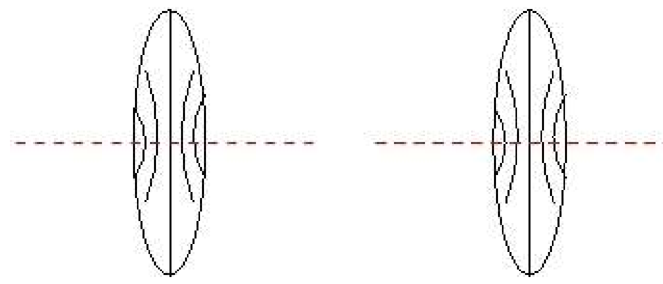






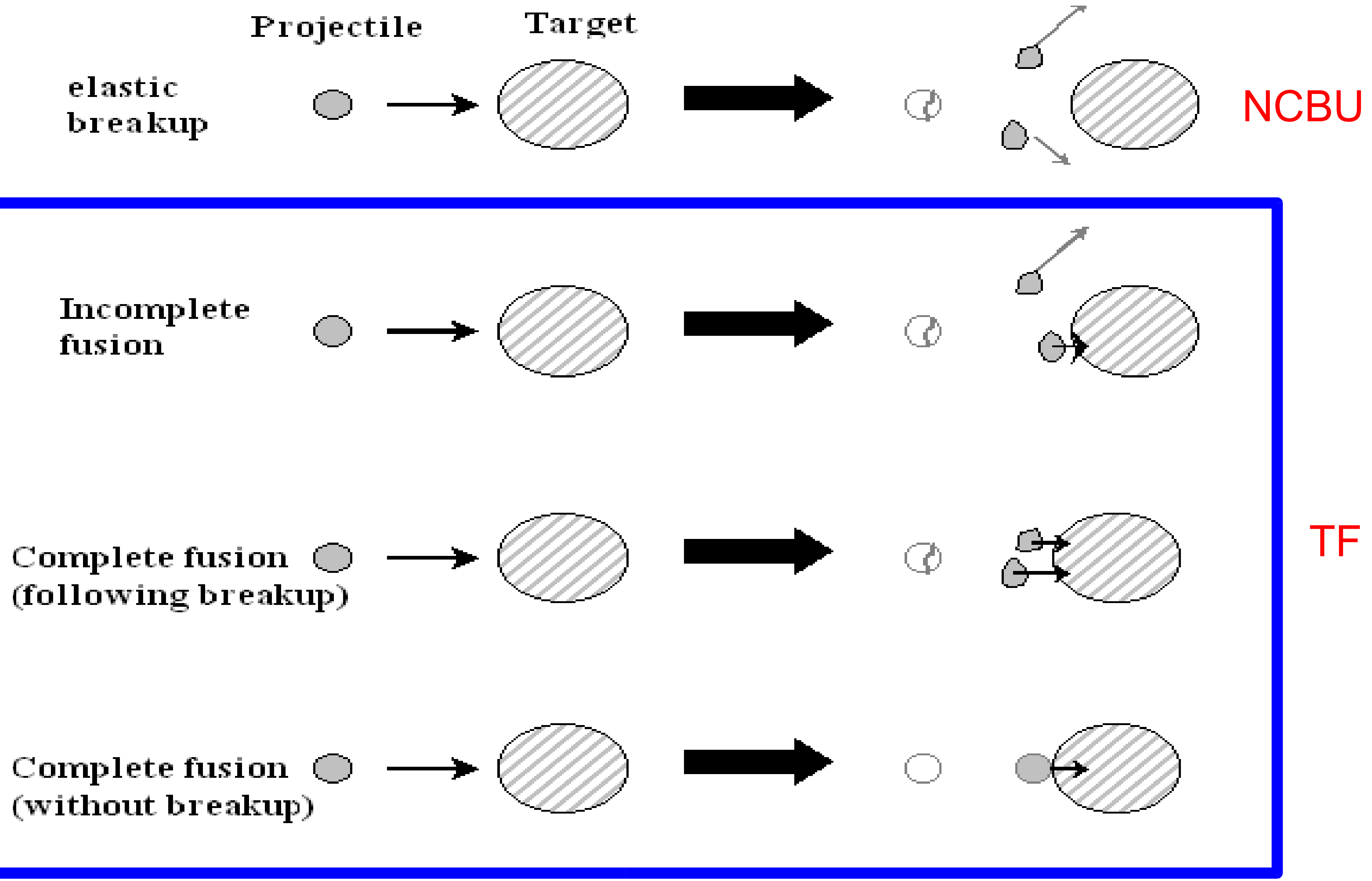
$$V = \sum_{s=1}^{2} e^{-i\mathbf{R}_s \hat{k}}\, \hat{U}(\Omega_s)\, V_s\, \hat{U}^{-1}(\Omega_s)\, e^{i\mathbf{R}_s \hat{k}}$$

$$V_s \approx \sum_{\nu\mu}^{N} |s\nu\rangle\, V^s_{\nu\mu}\, \langle s\mu|$$



AD-T, PRL 101 (2008) 122501

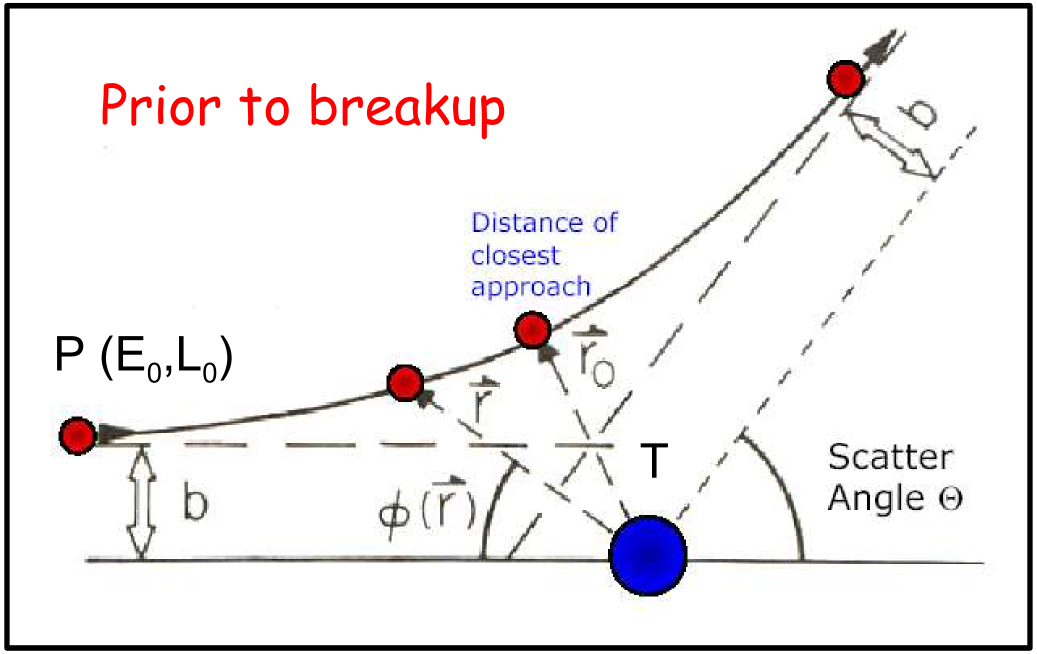
Application: Unified quantum description of reaction processes of neutron-rich, weakly-bound nuclei



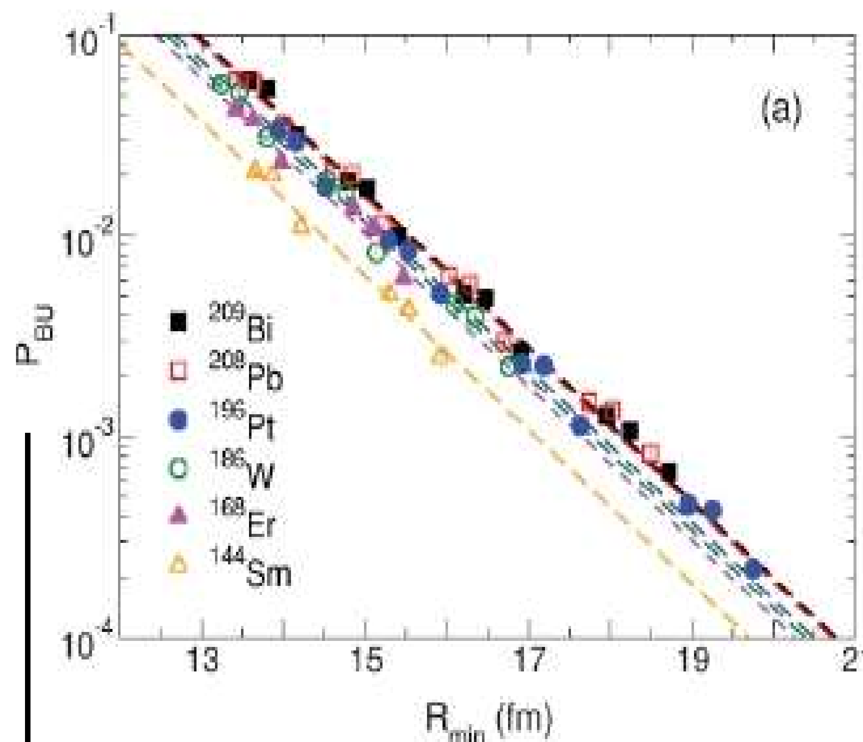
Classical dynamical model

AD-T, Hinde, Tostevin, Dasgupta & Gasques, PRL 98 (2007) 152701

Quantitative calculations of CF, ICF and NCBU yields above the barrier



PHYSICAL REVIEW C 81, 024601 (2010)



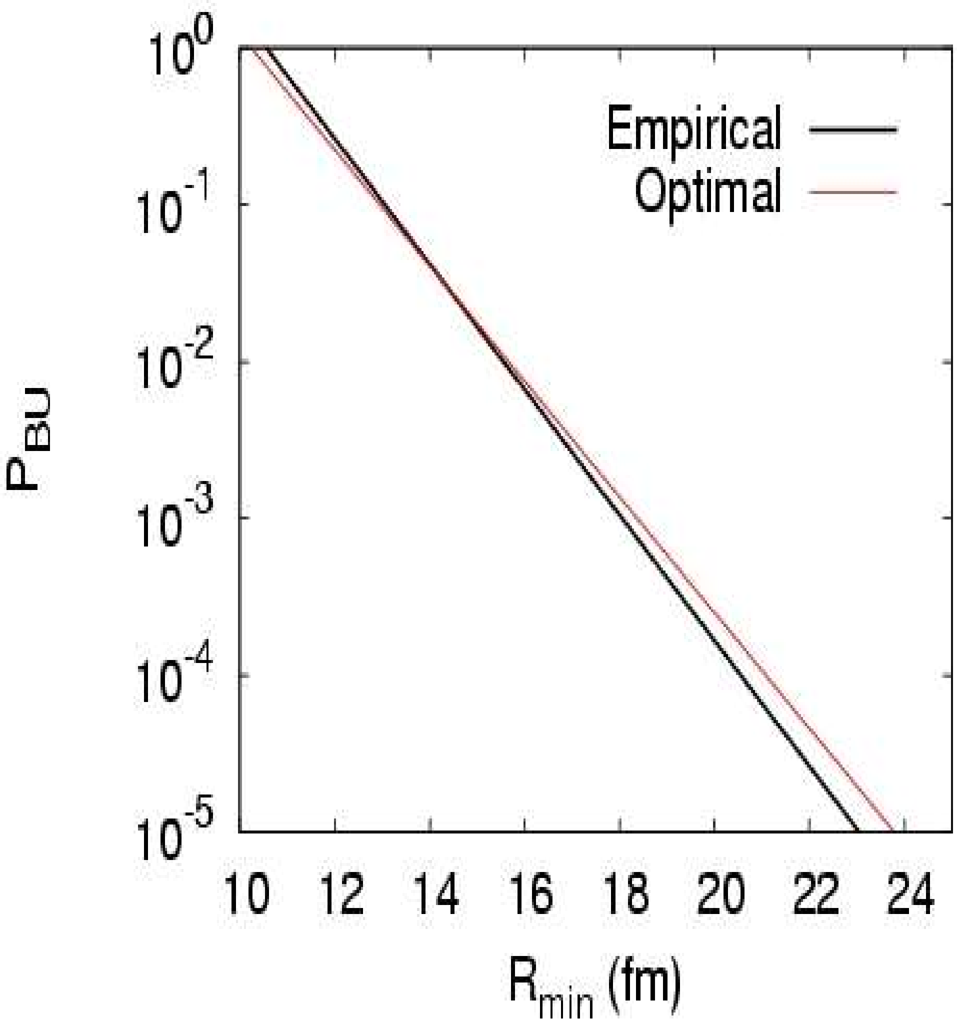
Main Ingredient :

$P^L_{BU}(R)dR$ *probability of breakup on the interval* $R + dR$

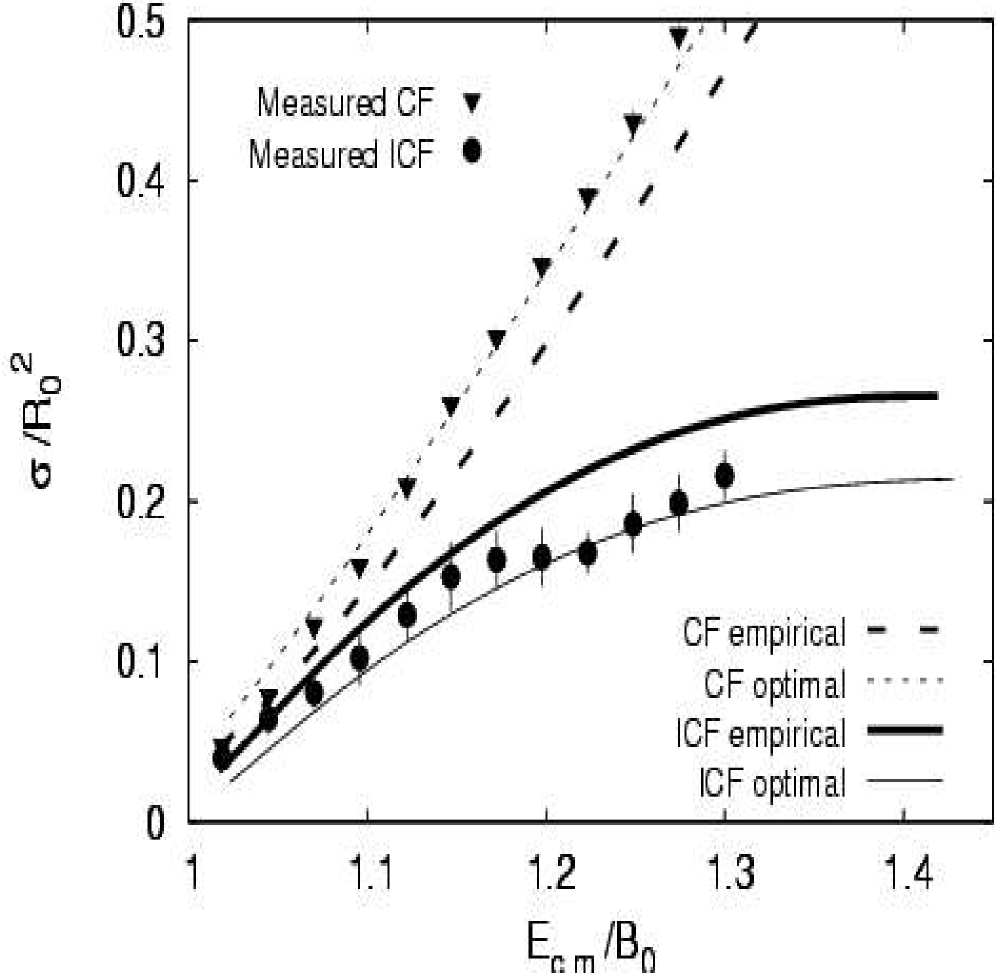
$$P_{BU}(R_{min}) = 2\int_{R_{min}}^{\infty} P^L_{BU}(R)dR = A exp(-\alpha R_{min})$$

Fusion excitation functions: “ ^{8}Be ” + ^{208}Pb

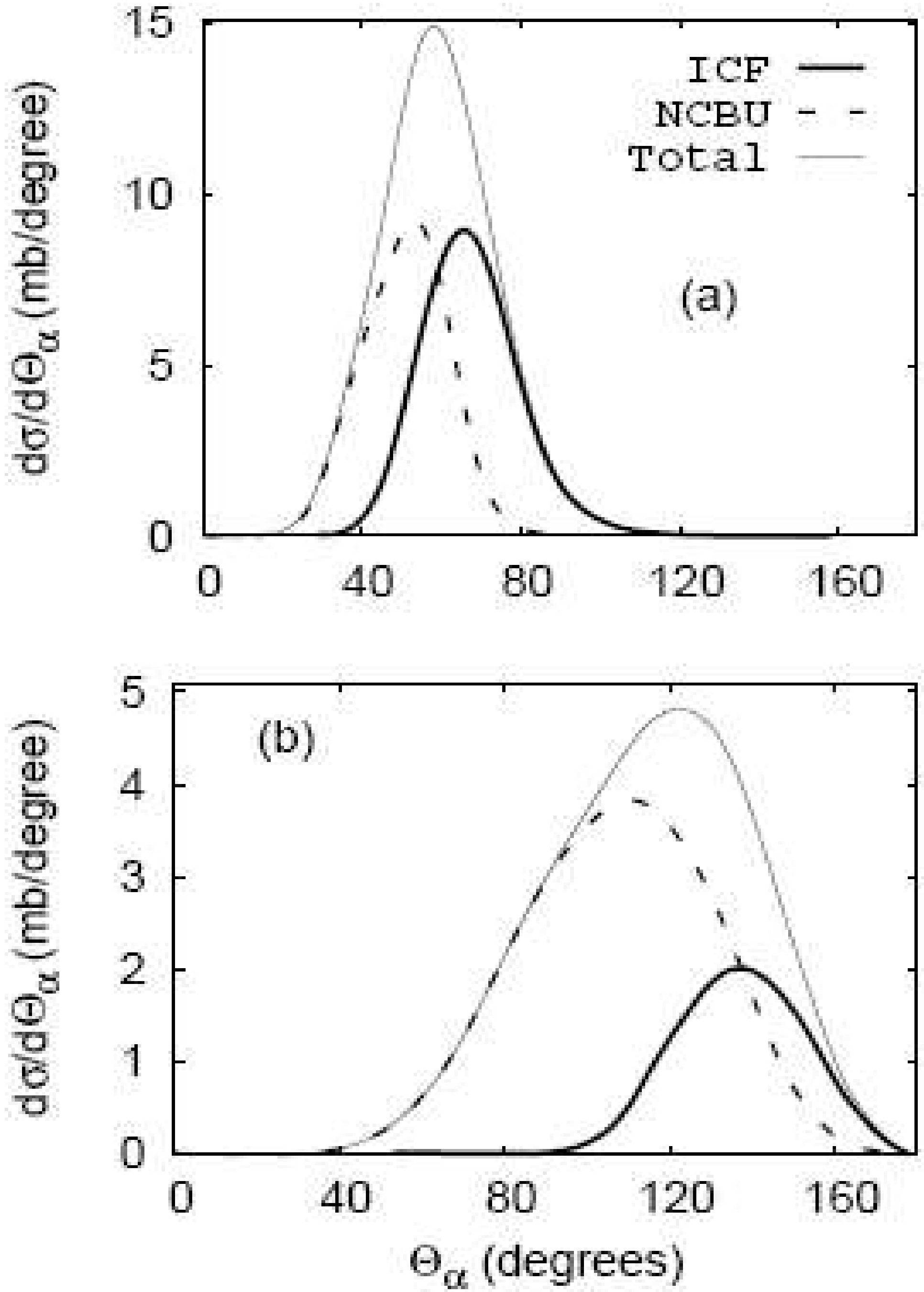
Breakup function



CF & ICF excitation functions



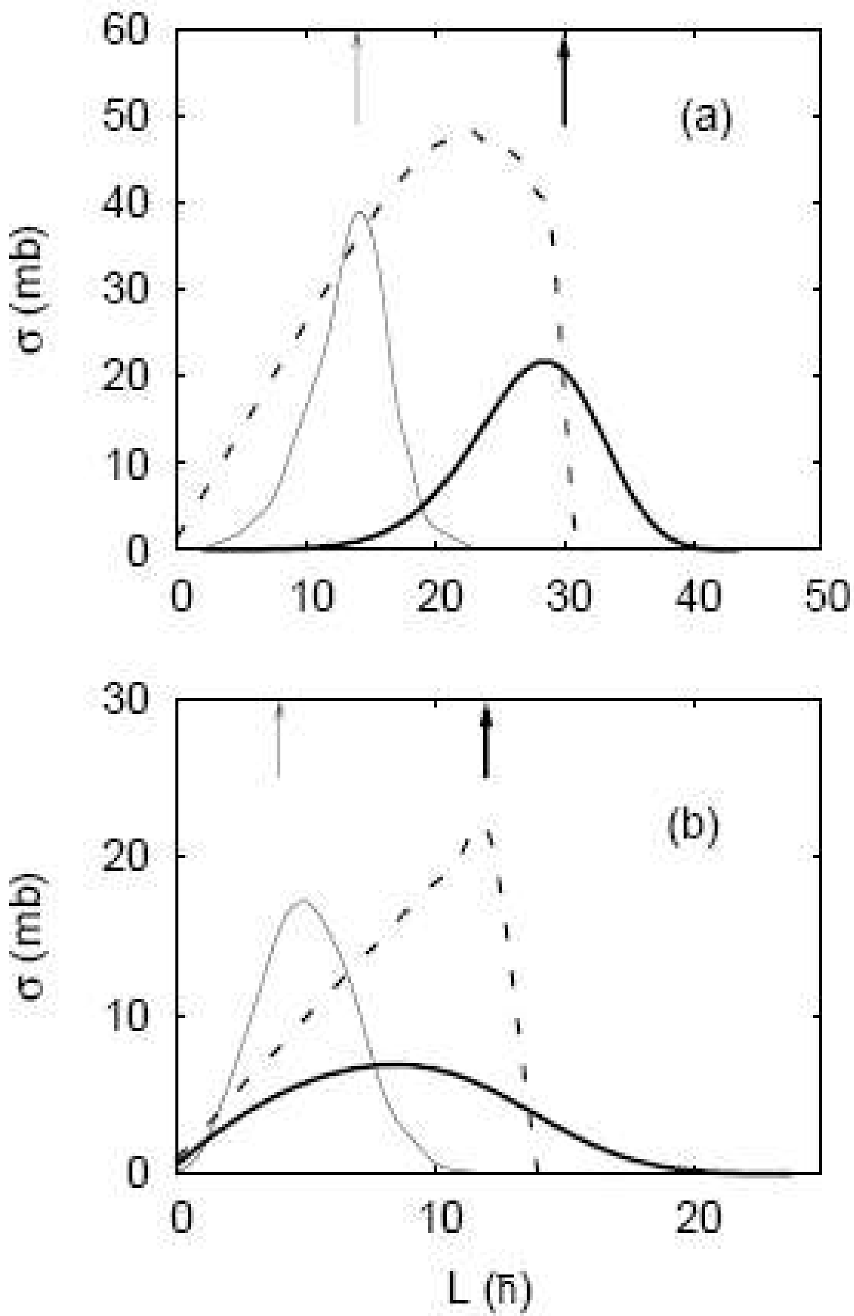
Direct alpha-production yields: “ ^{8}Be ” + ^{208}Pb



$E_{cm}/B_0 = 1.57$

$E_{cm}/B_0 = 1.08$

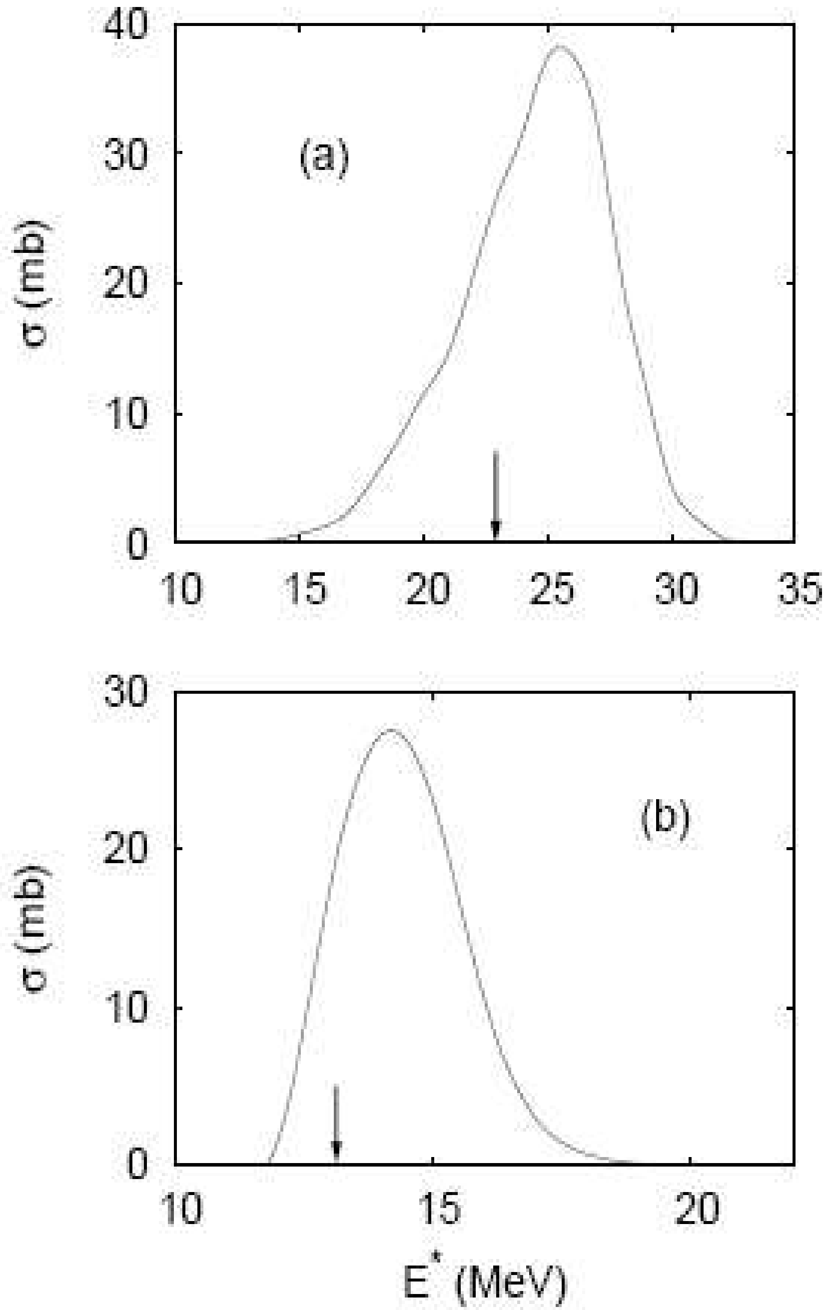
Spin distribution of fusion products: ^{216}Rn & ^{212}Po



$E_{cm}/B_0 = 1.57$

$E_{cm}/B_0 = 1.08$

Excitation energy distribution of ICF product ^{212}Po



$E_{cm}/B_0 = 1.57$

$E_{cm}/B_0 = 1.08$

Summary

- **Innovative quantum dynamical approach**, which will quantify the **importance of quantum decoherence** in various areas of reaction theory of stable and exotic nuclei
- **Quantum decoherence should always be explicitly included** when modelling low-energy nuclear collision dynamics within a truncated model space of reaction channels
- Links with decoherence in **other quantum systems**

Workshop on Decoherence in Quantum Dynamical Systems

To be held at ECT* Trento, IT, April 26th – 30th, 2010

Registration: until April 9

http://www.nucleartheory.net/Decoherence/

Breakup probability function

Let us define two probabilities: (i) the probability of breakup between R and $R + dR$, $\rho(R)dR$ [being $\rho(R)$ a density of probability], and (ii) the probability of the weakly-bound projectile's survival from ∞ to R, $S(R)$. The survival probability at $R + dR$, $S(R + dR)$, can be written as follows

$$S(R + dR) = S(R)\,[1 - \rho(R)dR]. \qquad \text{(A.1)}$$

Expression (A.1) suggests the following differential equation for the survival probability $S(R)$,

$$\frac{dS(R)}{dR} = -S(R)\,\rho(R), \qquad \text{(A.2)}$$

whose solution is $[S(\infty) = 1]$:

$$S(R) = \exp(-\int_{\infty}^{R} \rho(R)dR). \qquad \text{(A.3)}$$

From (A.3), the breakup probability at R, $B(R) = 1 - S(R)$. If $\int_{\infty}^{R} \rho(R)dR \ll 1$, $B(R)$ can be written as

$$B(R) \approx \int_{\infty}^{R} \rho(R)dR. \qquad \text{(A.4)}$$

From (A.4), identifying $\rho(R)$ with $\mathcal{P}_{BU}^{L}(R)$, we obtain expression (1) for the breakup probability integrated along a given classical orbit.

Measure of Coherence

For a *pure* state described by the state vector $|\chi\rangle$:

$\hat{\rho} = |\chi\rangle\langle\chi|$, and $\mathrm{Tr}(\hat{\rho}) = \langle\chi|\chi\rangle$.

$\hat{\rho}^2 = |\chi\rangle\langle\chi|\chi\rangle\langle\chi|$, and $\mathrm{Tr}(\hat{\rho}^2) = \langle\chi|\chi\rangle\,\langle\chi|\chi\rangle = [\mathrm{Tr}(\hat{\rho})]^2$.

Hence, $\mathrm{Tr}(\hat{\rho}^2)/[\mathrm{Tr}(\hat{\rho})]^2 = 1$, for nonzero values of $\mathrm{Tr}(\hat{\rho})$.

For a *mixed* state, there is no single state vector describing the system:

$\mathrm{Tr}(\hat{\rho}^2)/[\mathrm{Tr}(\hat{\rho})]^2 < 1$.

The transition from a pure state to a mixed state is caused by decoherence.