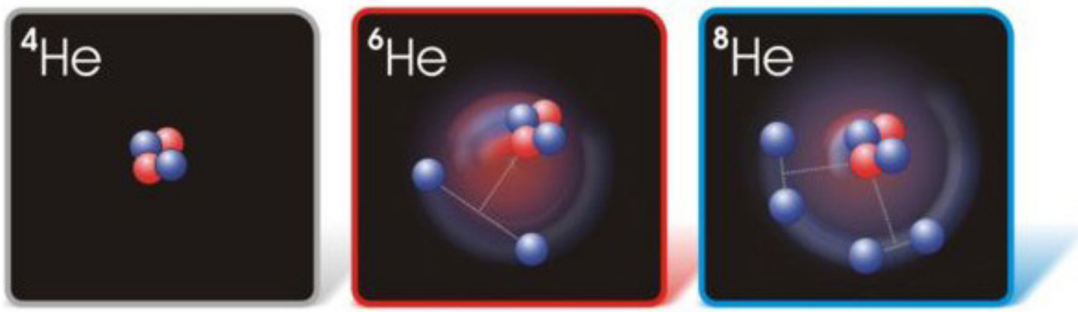
Nuclei, Neutron Matter and Cold Atoms

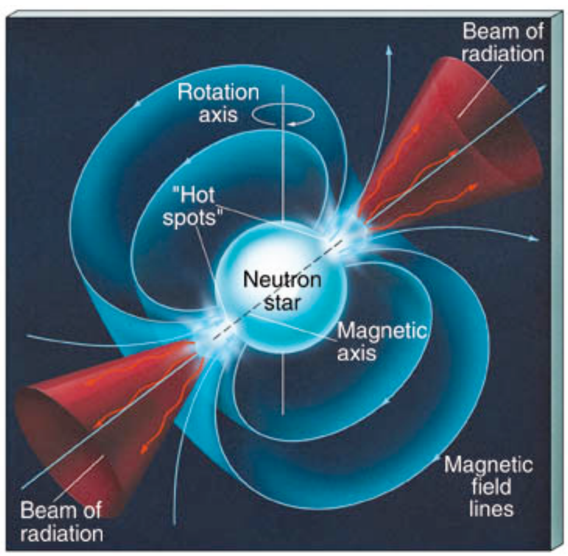
J. Carlson - LANL

Light Nuclei



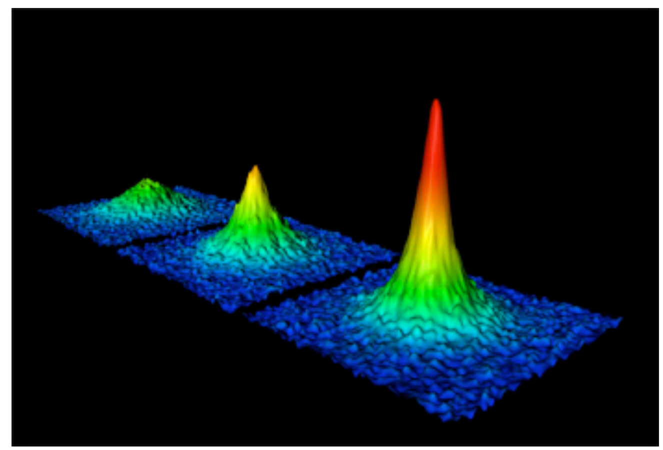
ANL

Neutron Star



U. Arizona

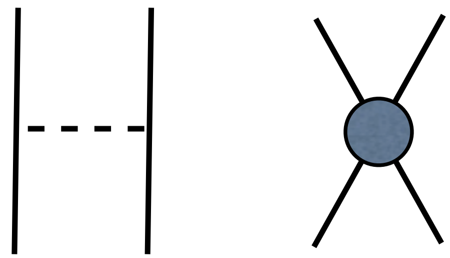
Cold Atom Fermi Condensates



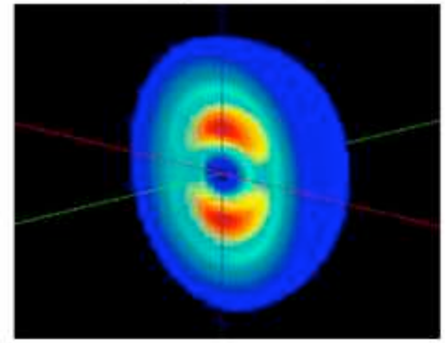
C. Regal et al. PRL 2004

```
mpi=(mpi0+2.*mpic)/3.
mu0=mpi0/hc
muc=mpic/hc
mu=mpi/hc
fsq=.075
cpi=2.1
rws=.5
aiws=5.
x=mu*r
x0=mu0*r
xc=muc*r
if (r.le.small) then
  tpi=3*cpi**2*r/mu**3
  ypi0=(mpi0/mpic)**2*(mpi0/3)*cpi*r/mu0
  tpi0=3*cpi*ypi0/mu0**2
  ypic=(mpic/3)*cpi*r/muc
  tpic=3*cpi*ypic/muc**2
else
  rcut=1-exp(-cpi*r*r)
  ypi=exp(-x)*rcut/x
  tpi=(1+(3+3/x)/x)*ypi*rcut
  ypi0=(mpi0/mpic)**2*(mpi0/3)*exp(-x0)*rcut/x0
  tpi0=(1+(3+3/x0)/x0)*ypi0*rcut
  ypic=(mpic/3)*exp(-xc)*rcut/xc
  tpic=(1+(3+3/xc)/xc)*ypic*rcut
end if
ypi0=fsq*ypi0
ypic=fsq*ypic
tpi0=fsq*tpi0
tpic=fsq*tpic
tpi2=tpi*tpi
ws=1/(1+exp((r-rws)*aiws))
ws0=1/(1+exp(-rws*aiws))
wsp=ws*(1+aiws*exp(-rws*aiws)*ws0*r)
wsx=ws*x
wsx2=wsx*x
dypi00=(mpi0/mpic)**2*(mpi0/3)*cpi/mu0
dypic0=(mpic/3)*cpi/muc
ypi0p=ypi0-fsq*dypi00*ws*r/ws0
ypicp=ypic-fsq*dypic0*ws*r/ws0
ypi=(ypi0+2*ypic)/3
tpi=(tpi0+2*tpic)/3
p11pp=  -7.62701*tpi2+1815.4920*wsp+1847.8059*wsx2+ypi0p
p11np=  -7.62701*tpi2+1813.5315*wsp+1847.8059*wsx2-ypi0p+2*ypicp
p11nn=  -7.62701*tpi2+1811.5710*wsp+1847.8059*wsx2+ypi0p
pt1pp=   1.07985*tpi2 -190.0949*wsx -811.2040*wsx2+tpi0
pt1np=   1.07985*tpi2 -190.0949*wsx -811.2040*wsx2-tpi0+2*tpic
pt1nn=   1.07985*tpi2 -190.0949*wsx -811.2040*wsx2+tpi0
pls1=    -.62697*tpi2 -570.5571*wsp +819.1222*wsx2
pl211=    .06709*tpi2 +342.0669*wsp -615.2339*wsx2
pls21=    .74129*tpi2   +9.3418*wsp -376.4384*wsx2
p10=    -8.62770*tpi2+2605.2682*wsp +441.9733*wsx2-ypi0p-2*ypicp
pt0=    1.485601*tpi2-1126.8359*wsx +370.1324*wsx2-tpi0-2*tpic
pls0=     .10180*tpi2  +86.0658*wsp -356.5175*wsx2
pl210=   -.13201*tpi2 +253.4350*wsp   -1.0076*wsx2
pls20=    .07357*tpi2 -217.5791*wsp  +18.3935*wsx2
p01pp= -11.27028*tpi2+3346.6874*wsp-3*ypi0p
p01np= -10.66788*tpi2+3126.5542*wsp-3*(-ypi0p+2*ypicp)
p01nn= -11.27028*tpi2+3342.7664*wsp-3*ypi0p
pl201=    .12472*tpi2  +16.7780*wsp
p00=    -2.09971*tpi2+1204.4301*wsp-3*(-ypi0p-2*ypicp)
pl200=   -.31452*tpi2 +217.4559*wsp
p11=(p11pp+p11nn+p11np)/3
p11cd=(.5*(p11pp+p11nn)-p11np)/6
p11cs=(p11pp-p11nn)/4
pt1=(pt1pp+pt1nn+pt1np)/3
pt1cd=(.5*(pt1pp+pt1nn)-pt1np)/6
pt1cs=(pt1pp-pt1nn)/4
p01=(p01pp+p01nn+p01np)/3
p01cd=(.5*(p01pp+p01nn)-p01np)/6
p01cs=(p01pp-p01nn)/4
```

A Nuclear Hamiltonian:



Deuteron

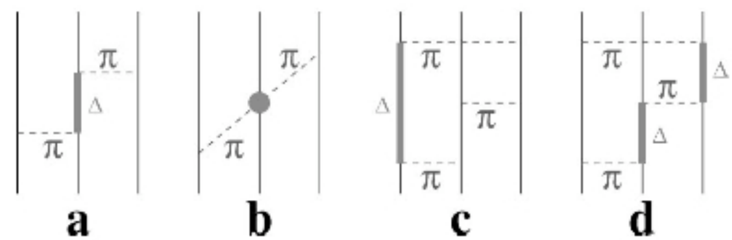


Pion Exchange

Shorter-range interactions

spin-orbit, etc.

Also need a three-body force



Monte Carlo Methods

$$\Psi \; = \; \exp[-H\tau] \; \Psi_0$$

Initial Guess or `Source'
Shell Model plus Correlations

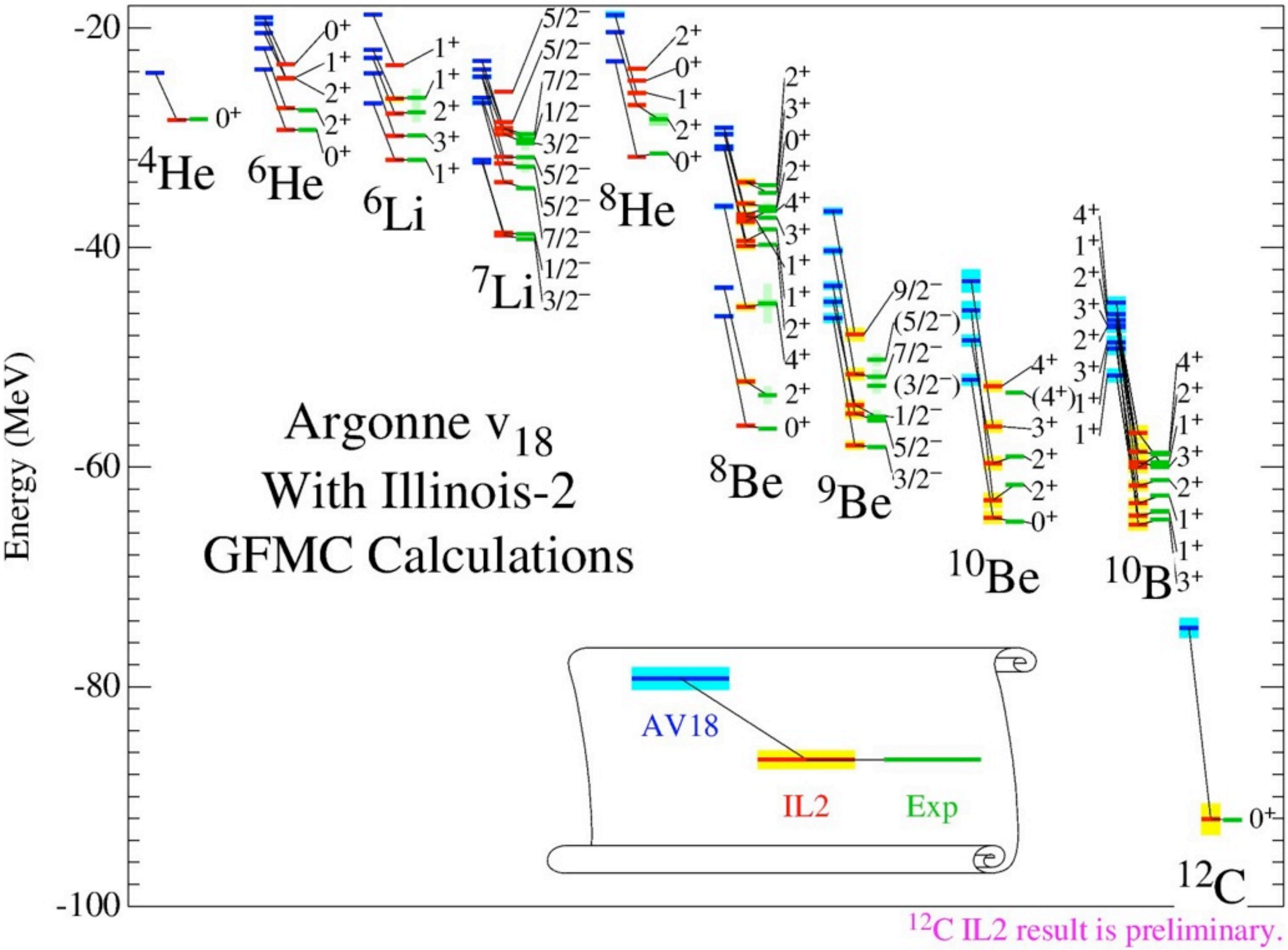
Two-Body solutions plus 3-nucleon interaction

- Monte Carlo for spatial integrals
- Explicit Sums for spin/isospin
- In general approximate solutions

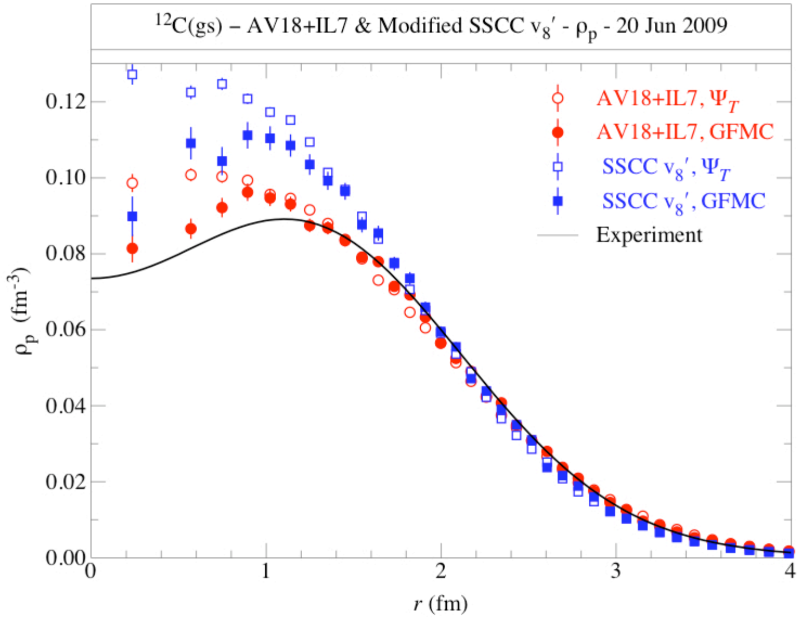
Alpha Particle,... Trivial

^{12}C uses 100K cores (or more) for Ground/Hoyle states

Light Nuclear Spectra

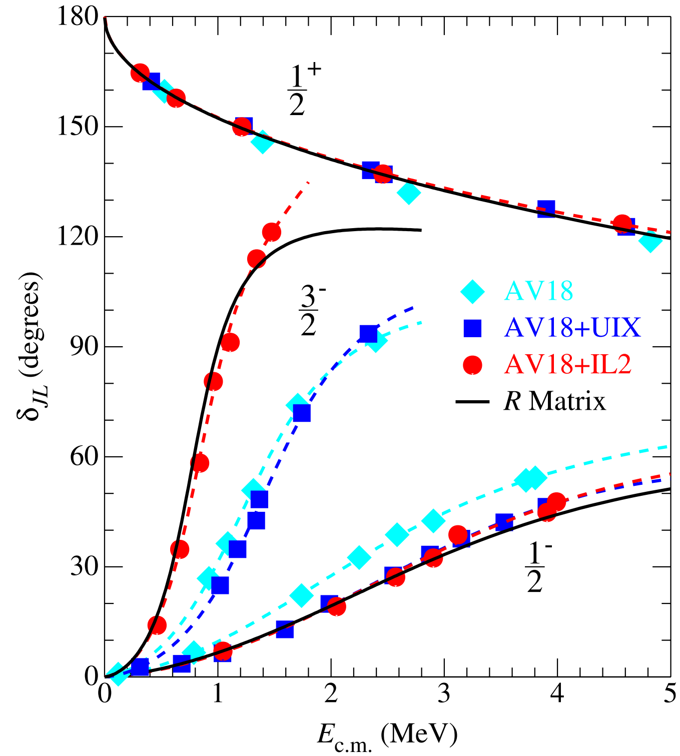


Recent Calculations



^{12}C Form Factor

n-alpha scattering

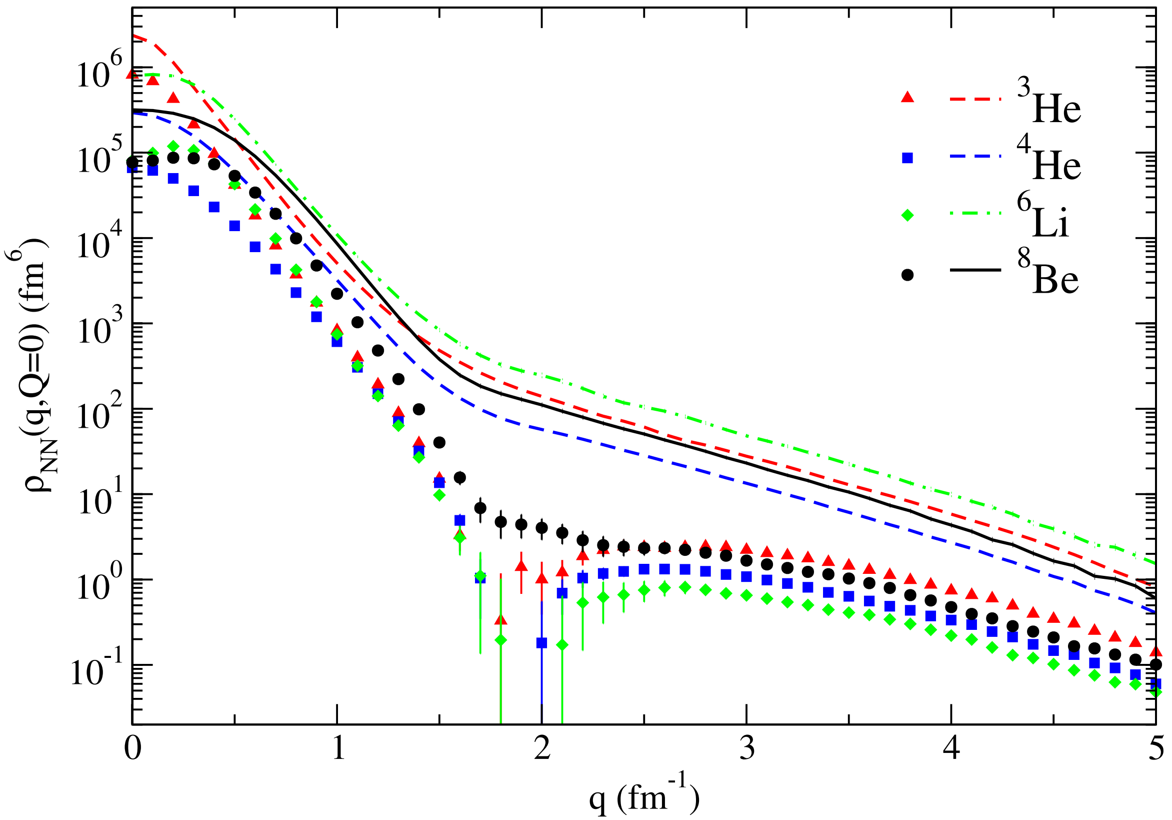


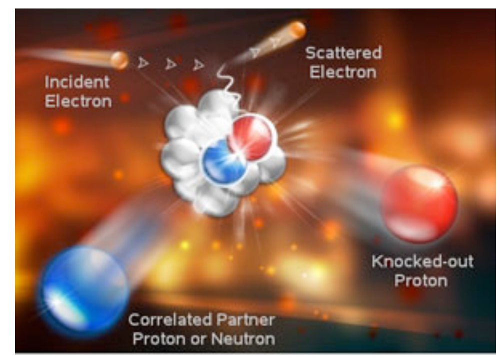
Mostly ‘Postdictions’

Nollett, et al, PRL 2007

Higher Momentum States in the Nucleus
Back-to-Back np vs. pp momentum distribution

$$n_{N,N'}(k,P=0) = \langle 0|\, a^\dagger_N(\mathbf{k}) a^\dagger_{N'}(-\mathbf{k})\; a_N(\mathbf{k}) a_{N'}(-\mathbf{k})\; |0\rangle$$

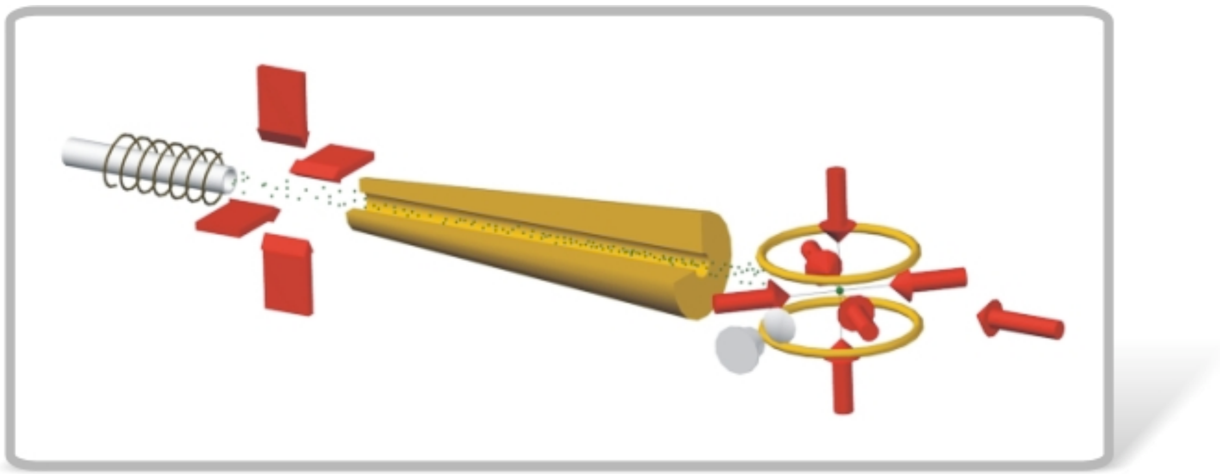


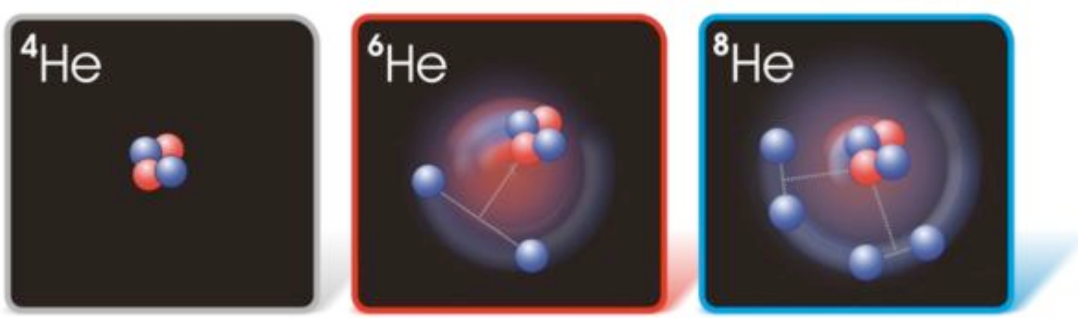


Schiavilla, et al, PRL 2007
see Alvioli, et al PRL 2008

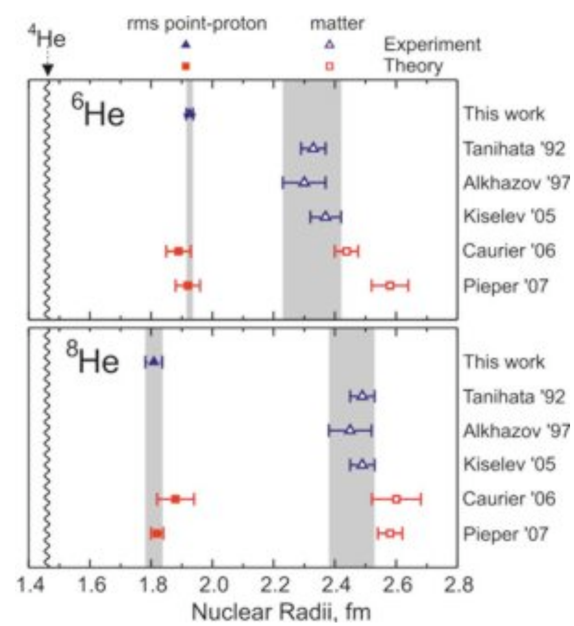
Piasetsky et al PRL 2006; Shneor et al 2007

Neutron Rich Matter: Helium Charge Radii

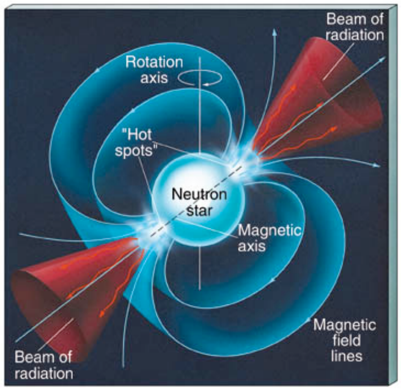




Mueller, et al, PRL 2007
Norterhauser, et al, PRL 2009



Neutron/Neutron-Star Matter



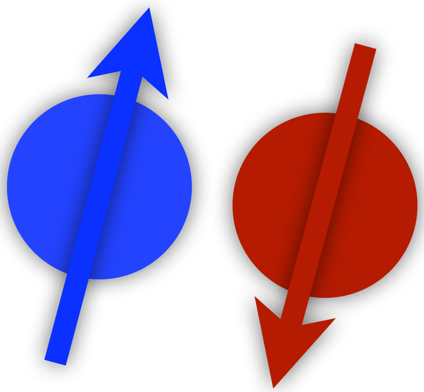
Low Density: Exterior of Nuclei
Neutron Star Crust

Simple well-understood interaction
Intriguing physics: strongly paired fermions
Many experimental tests

High Density: Bulk of Neutron Star

Interaction Poorly Known
Required to understand cold, dense matter
Observational Tests

Unitary Regime and
Low-Density Neutron Matter



$$H = \sum_i T_i + \sum_{i<j} V_0 \, \delta(r_{ij})$$

One Parameter: V_0

Cold Atom Experiments:

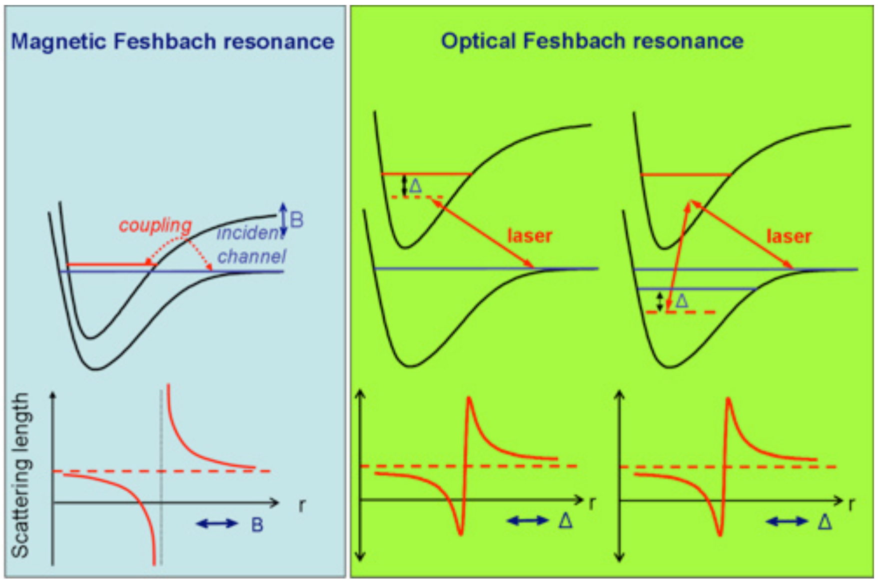
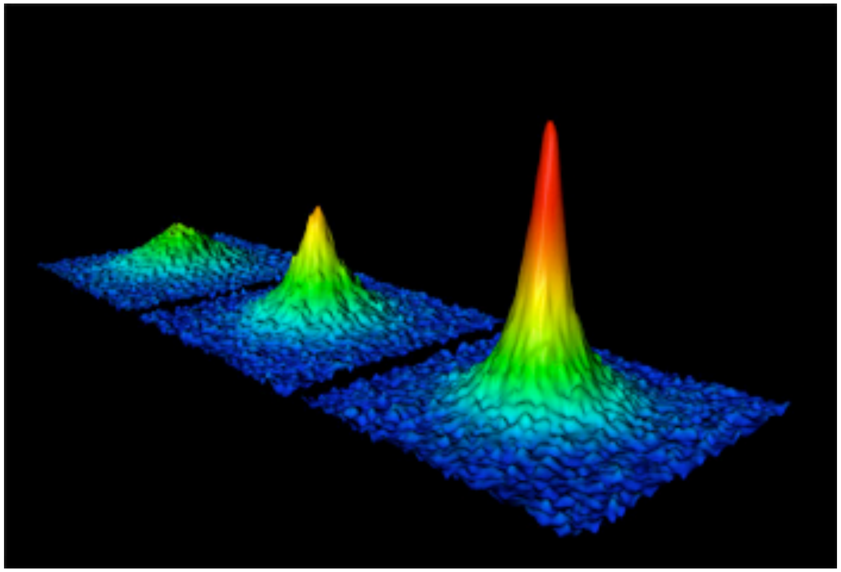


Diagram from Innsbruck



First Fermi Condensates 2004

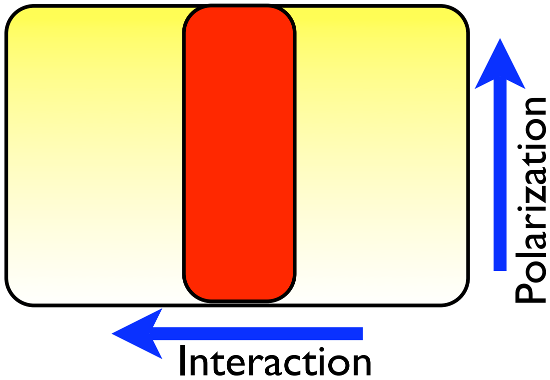
Fermions: ^{6}Li, ^{40}K

Density ~ 1/ μm^3

Temperature ~ 200 nK ~ 0.1 Ef

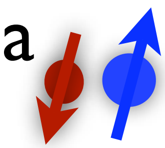
Experimental Results in Cold Atoms

- (nearly) Free Fermions
- (nearly) Free Bosons
- ‘Universality’ and the BCS-BEC transition
- Polarons
- Efimov States
- Superfluid Fermions (s-, p-, d-wave,... pairing)
- Exotic Polarized Superfluids (FFLO, breached pair,...)
- PseudoGap States
- Itinerant Ferromagnetism
- `Perfect’ Fluids
- Reduced Dimensionality
- More than pairing (3-,4-body condensates, ...)
- Bose, Fermi Hubbard Models,



Unitarity

Unitarity = limit of 0 pair binding

a  = ∞

All quantities multiples of Fermi Gas at same ρ

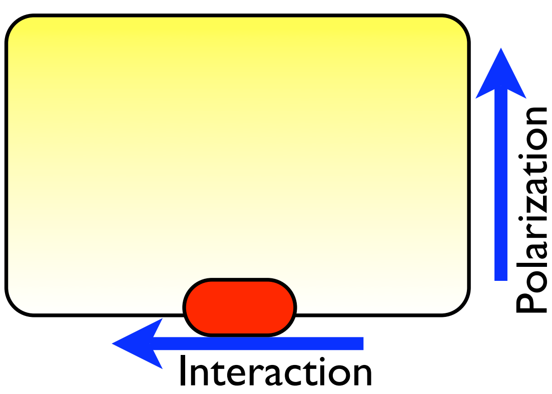
At zero polarization, expect strong pairing

$$E = \xi \; E_{FG} = \xi \; \frac{3}{5} \; \frac{\hbar^2 k_F^2}{2m}$$

$$\Delta \; = \; \delta \; \frac{\hbar^2 k_F^2}{2m}$$

$$T_c \; = \; t \; \frac{\hbar^2 k_F^2}{2m}$$

Values of ξ, δ, t are independent of ρ



Unitarity: #  = # 

JC, Chang, Pandharipande, Schmidt, PRL 2003

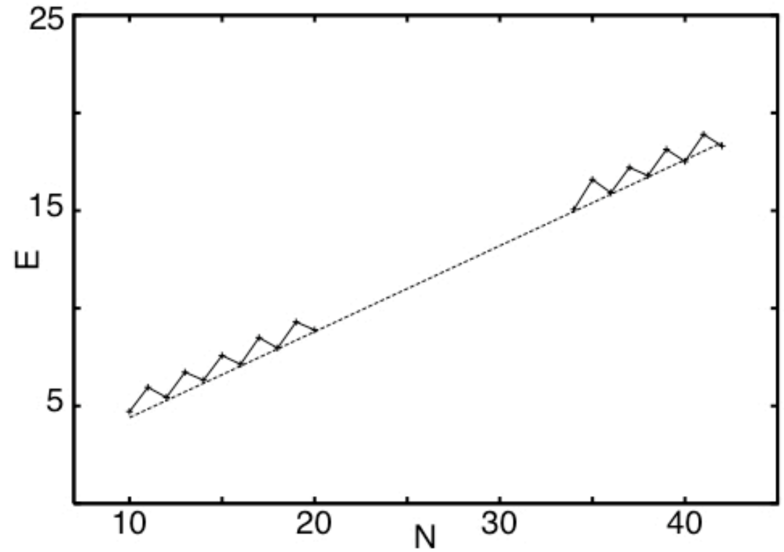


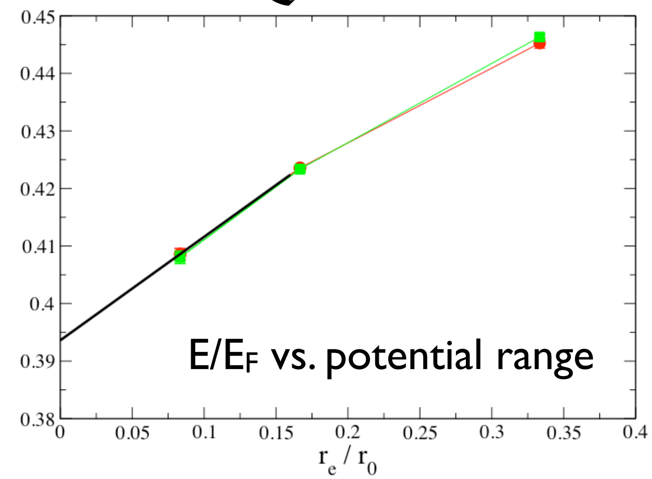
FIG. 3: The $E(N)$ in units of E_{FG}

$$E = \xi\ E_{FG} = \xi\ \frac{3}{5}\ \frac{\hbar^2 k_F^2}{2m}$$

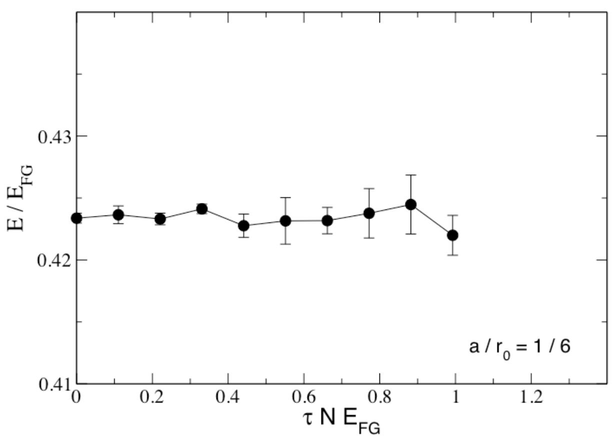
$$\Psi_0 = \begin{pmatrix} \phi(r_{11'}) & \phi(r_{12'}) & \ldots & \phi(r_{1(n+d)'}) & \psi_{1\uparrow}(\mathbf{r}_1) & \psi_{2\uparrow}(\mathbf{r}_1) \\ \phi(r_{21'}) & \phi(r_{22'}) & \ldots & \phi(r_{2(n+d)'}) & \psi_{1\uparrow}(\mathbf{r}_2) & \psi_{2\uparrow}(\mathbf{r}_2) \\ \vdots & \vdots & \vdots\vdots\vdots & \vdots & \vdots & \vdots \\ \phi(r_{(n+u)1'}) & \phi(r_{(n+u)2'}) & \ldots & \phi(r_{(n+u)(n+d)'}) & \psi_{1\uparrow}(\mathbf{r}_{n+u}) & \psi_{2\uparrow}(\mathbf{r}_{n+u}) \\ \psi_{1\downarrow}(\mathbf{r}_{1'}) & \psi_{1\downarrow}(\mathbf{r}_{2'}) & \ldots & \psi_{1\downarrow}(\mathbf{r}_{(n+d)'}) & 0 & 0 \\ \psi_{2\downarrow}(\mathbf{r}_{1'}) & \psi_{2\downarrow}(\mathbf{r}_{2'}) & \ldots & \psi_{2\downarrow}(\mathbf{r}_{(n+d)'}) & 0 & 0 \\ \psi_{3\downarrow}(\mathbf{r}_{1'}) & \psi_{3\downarrow}(\mathbf{r}_{2'}) & \ldots & \psi_{3\downarrow}(\mathbf{r}_{(n+d)'}) & 0 & 0 \end{pmatrix}$$

Calculations at Unitarity: #  = # 

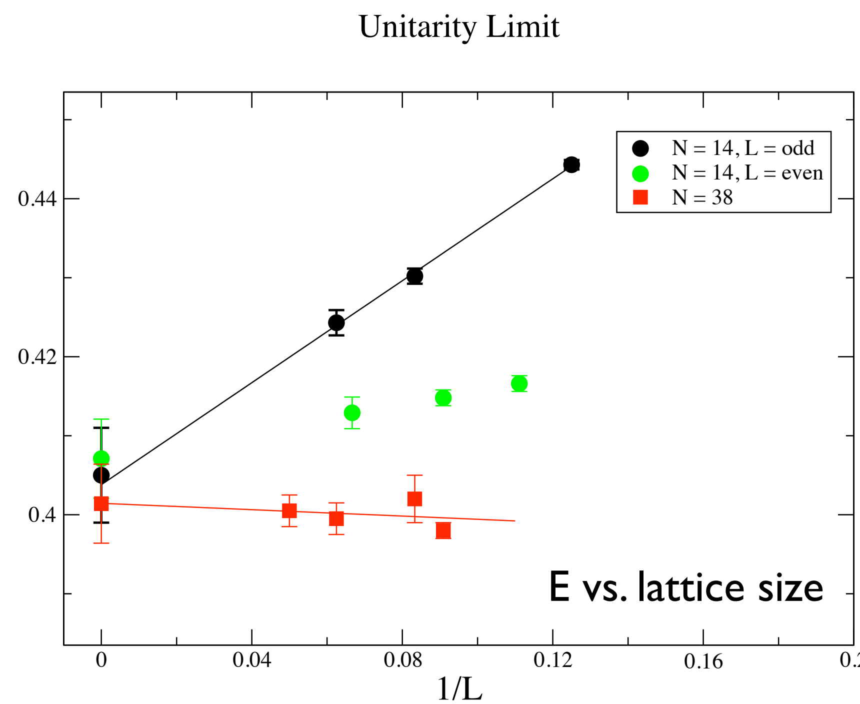
QMC



Transient Estimation



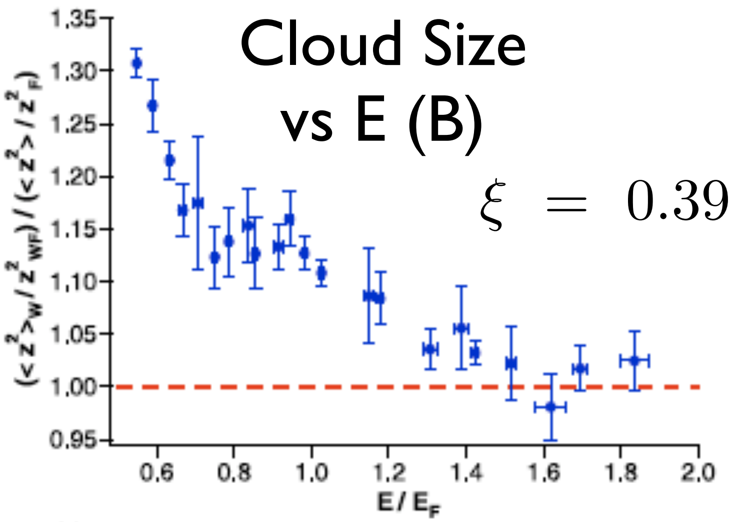
AFMC



$$\xi = 0.40(01)$$

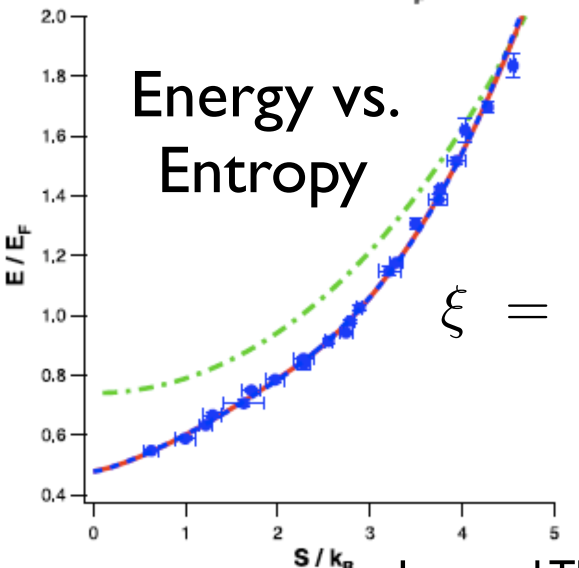
Carlson and Reddy, PRL 2005, 2007

Experiments at Unitarity: #  = # 
Cloud Size and Sound Velocity



Cloud Size vs E (B)

$$\xi = 0.39(02)$$



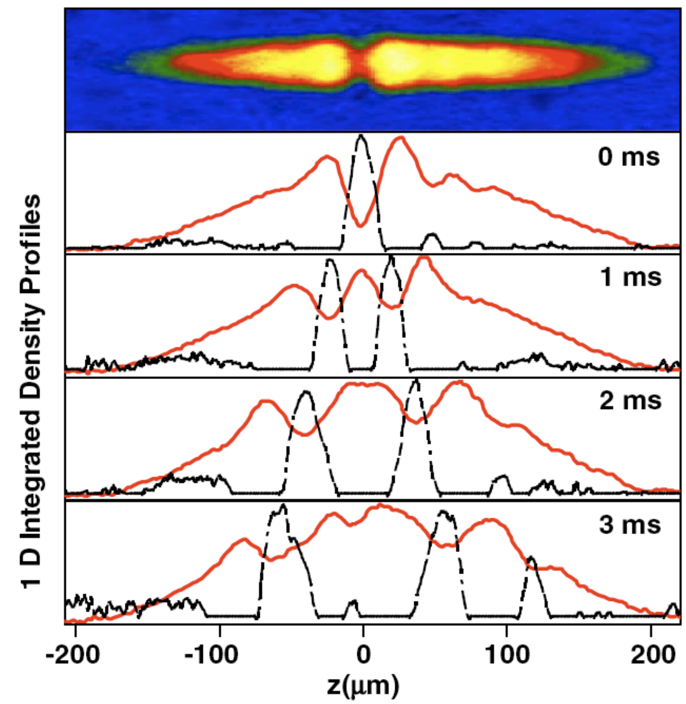
Energy vs. Entropy

$$\xi = 0.41(02)$$

Luo and Thomas, JLTP, 2009

Sound Propagation

Joseph, et al., PRL 2007

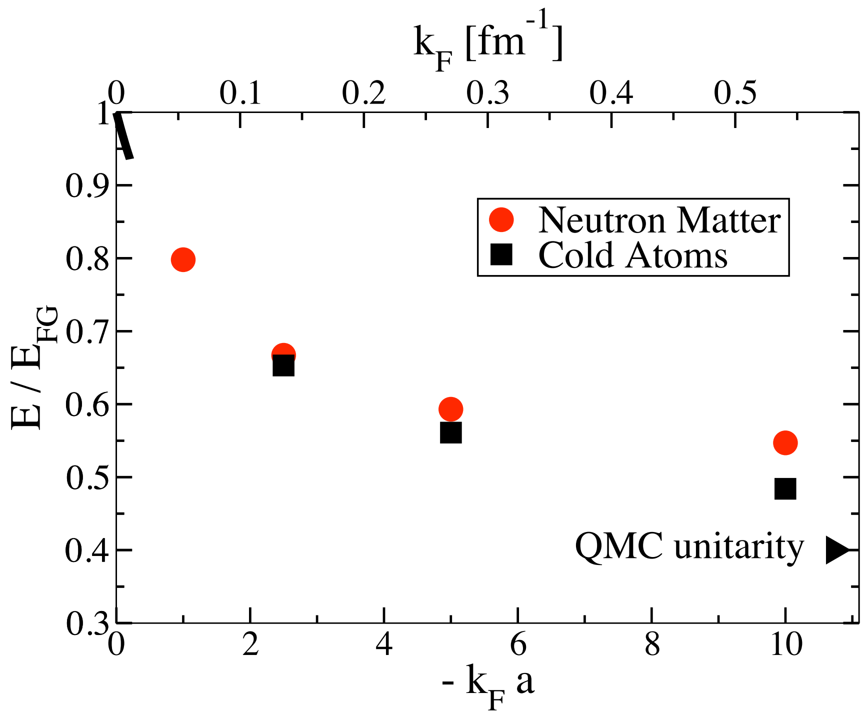


$$\frac{c_0}{v_f} = \frac{\xi^{1/4}}{\sqrt{(5)}}$$

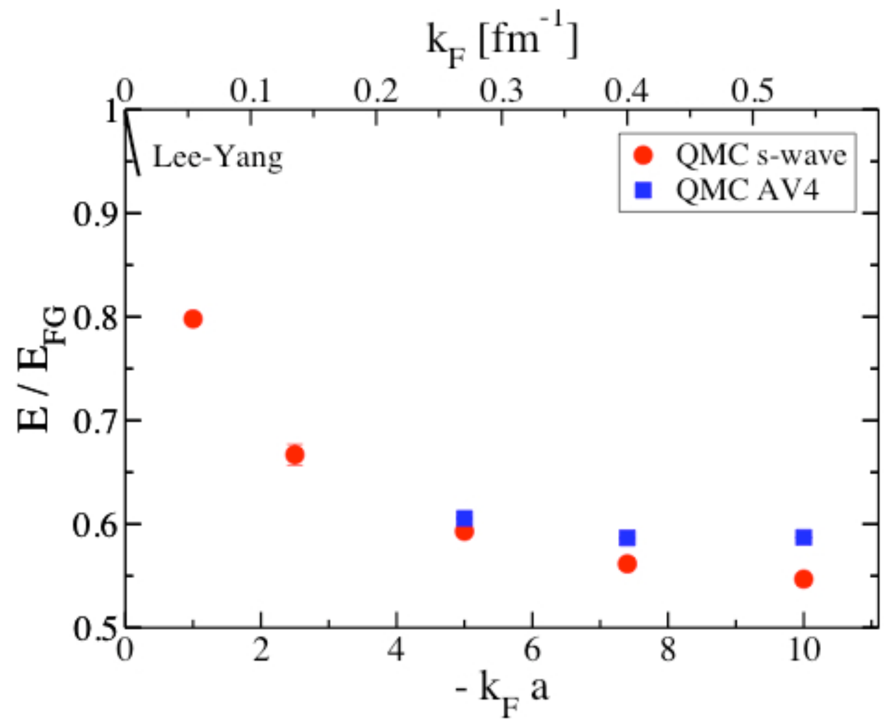
scaling verified as ρ varied by 30!

$$\xi = 0.435(15)$$

Relation to Neutron Matter: Equation of State



Neutrons vs. Cold Atoms



adding p-wave interactions

Gezerlis and JC, PRC 2008, 2010

Pairing Gap at Unitarity - Cold Atom Calculations

Add one  to fully-paired system

Energy cost for an unpaired particle: $\mu + \Delta$

Computational Cost Large:

$E(N+1) - 1/2(E(N)+E(N+2))$

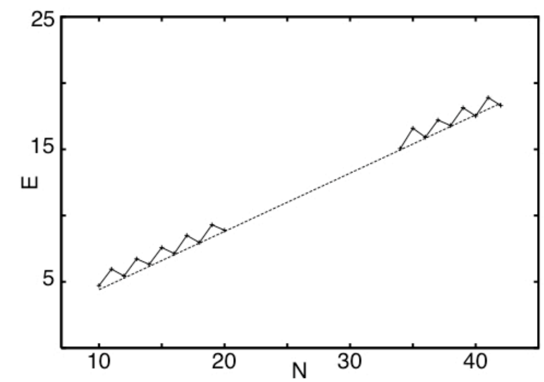
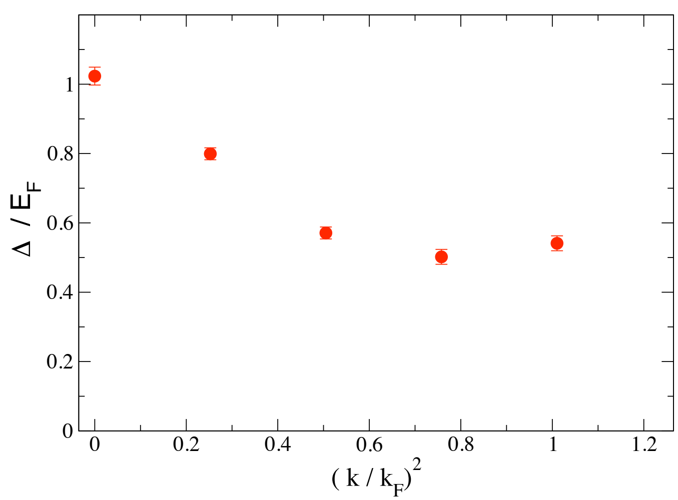


FIG. 3: The $E(N)$ in units of E_{FG}

Quasiparticle Dispersion

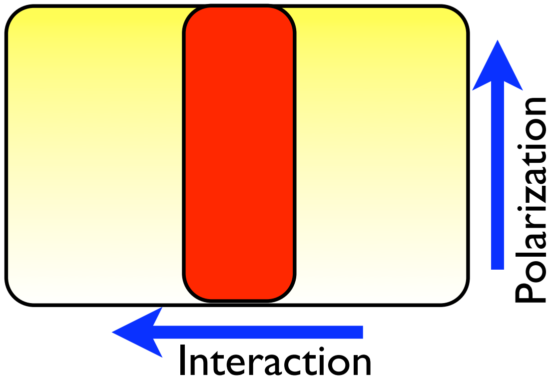


$$\Delta = \delta \frac{\hbar^2 k_F^2}{2m}$$

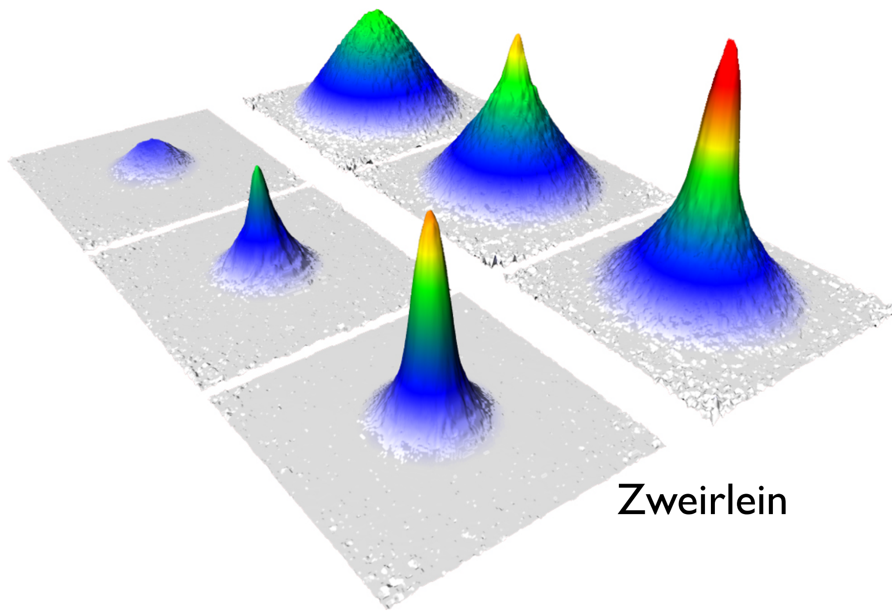
$$\delta = 0.50\ (03)$$

$$(k_{min}/k_f)^2 = 0.80(10)$$

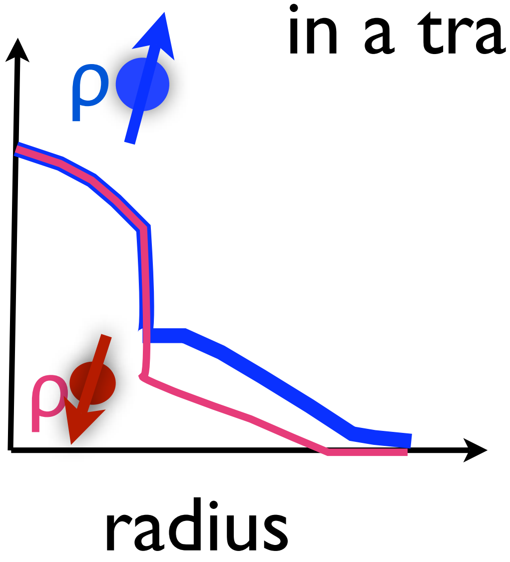
JC and Reddy, PRL 2005

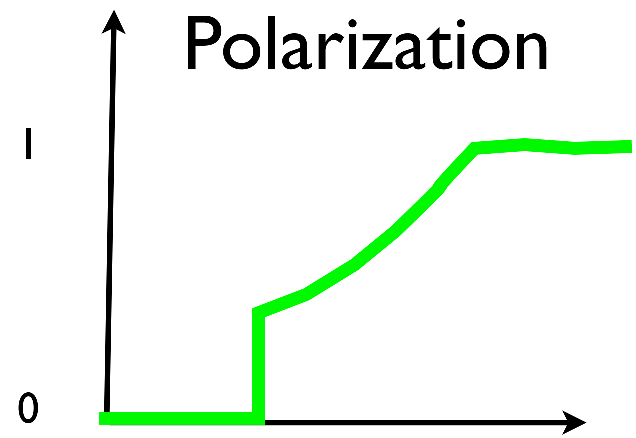


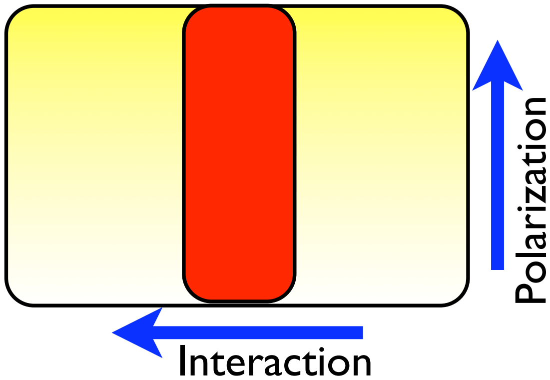
Pairing Gap at Unitarity - Experiment



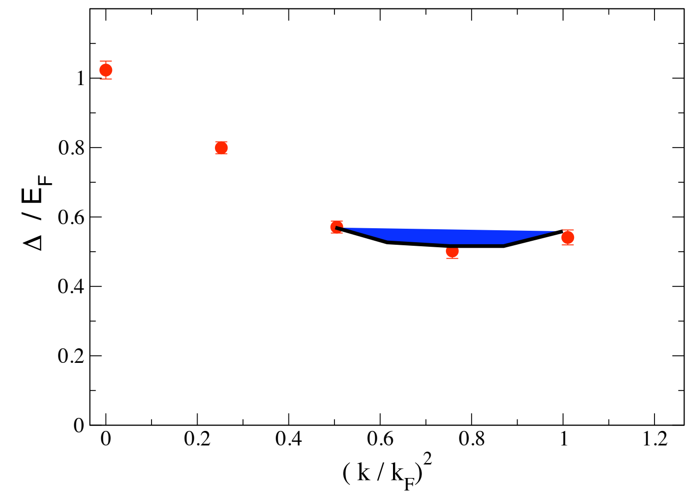
Spin up, down densities in a trap



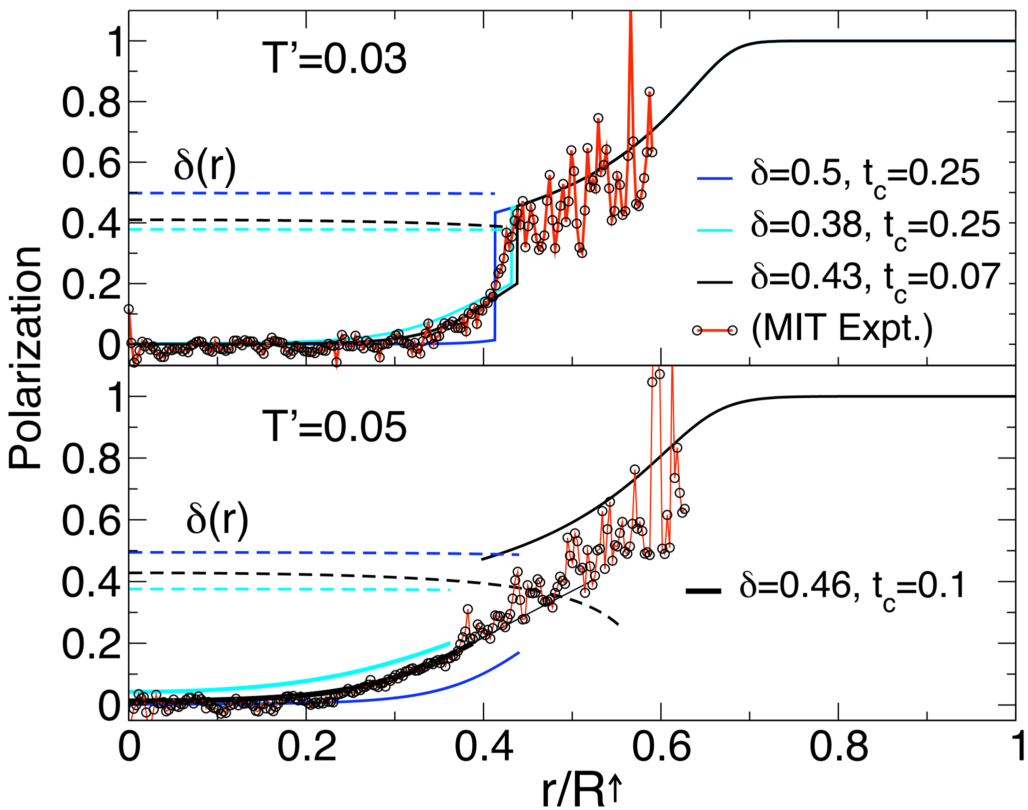




Pairing Gap at Unitarity - Experiment



Polarization in a trap



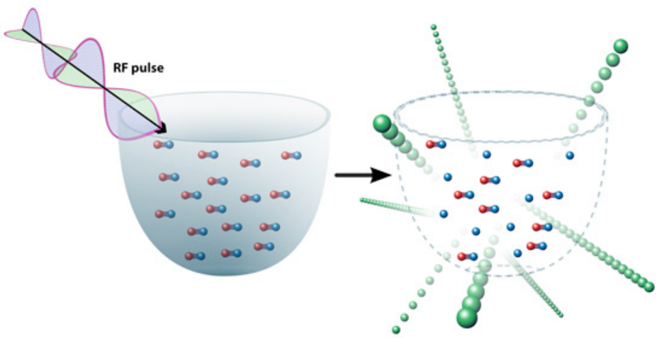
Largest Δ/E_f

$$\Delta = \delta \frac{\hbar^2 k_F^2}{2m}$$

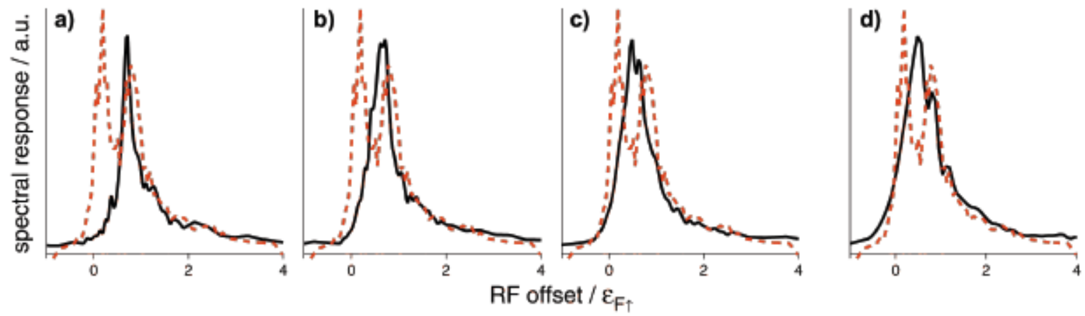
$$\delta = 0.45(05)$$

JC and Reddy, PRL 2007
analyzing MIT data

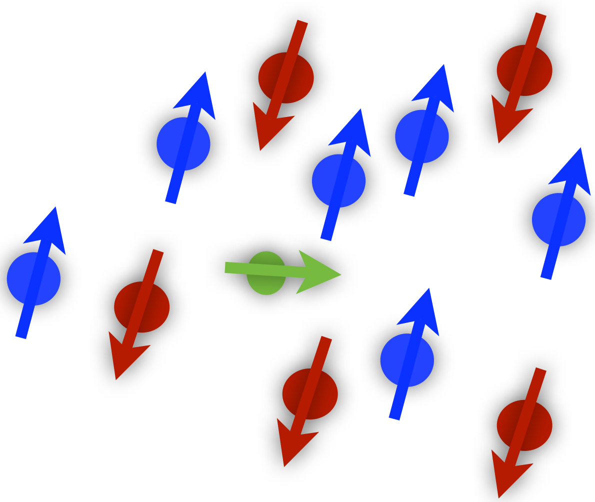
Pairing Gap at Unitarity - Experiment
RF response

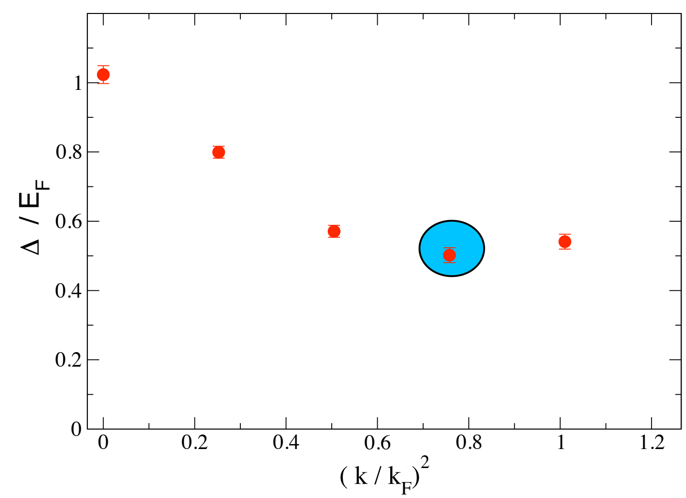


Credit: Greg Kuebler, JILA



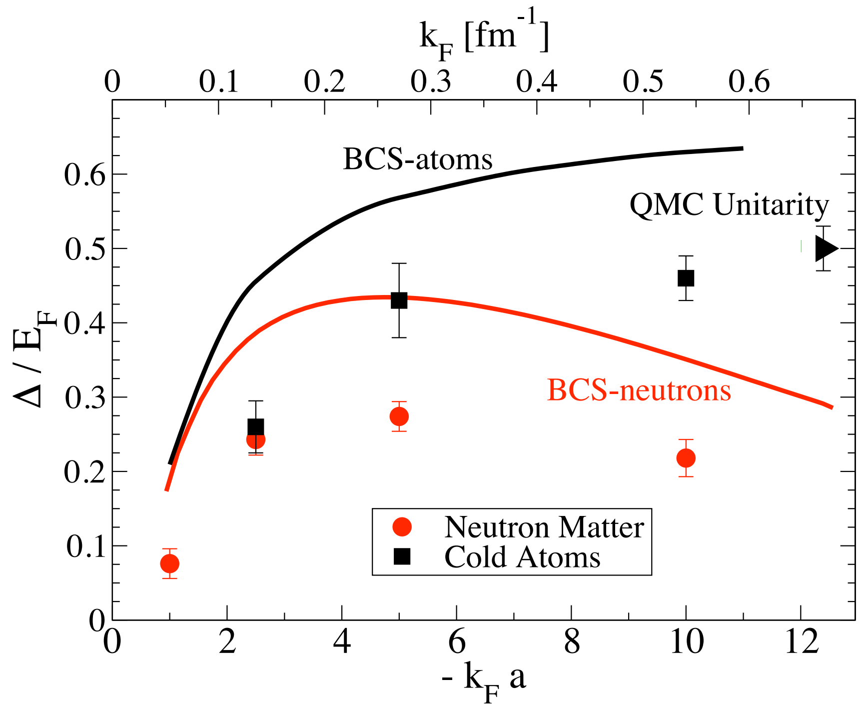
Shift of response of paired vs. unpaired atoms

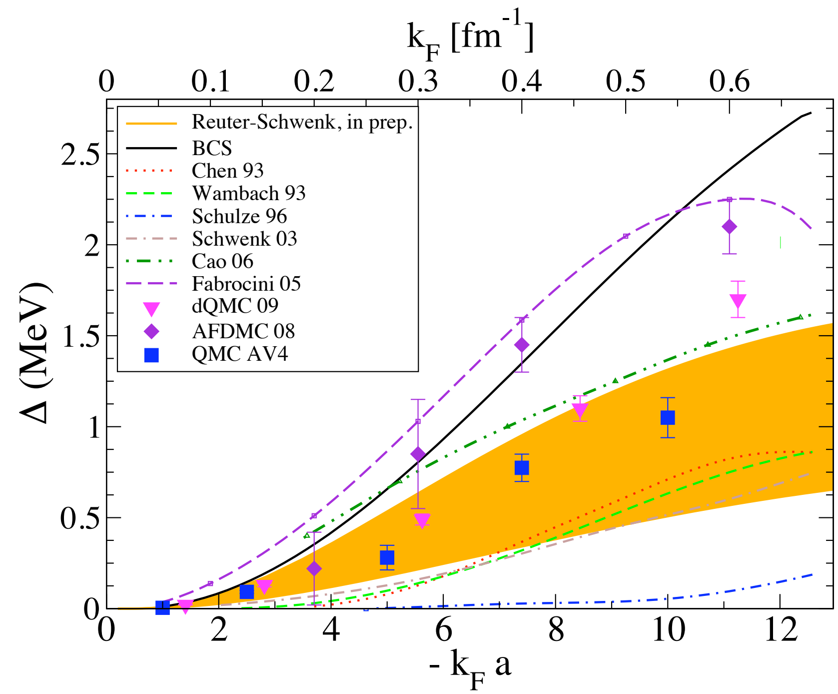




Shin, Ketterle, ... 2008

Pairing Gap: Cold Atoms and Neutron Matter





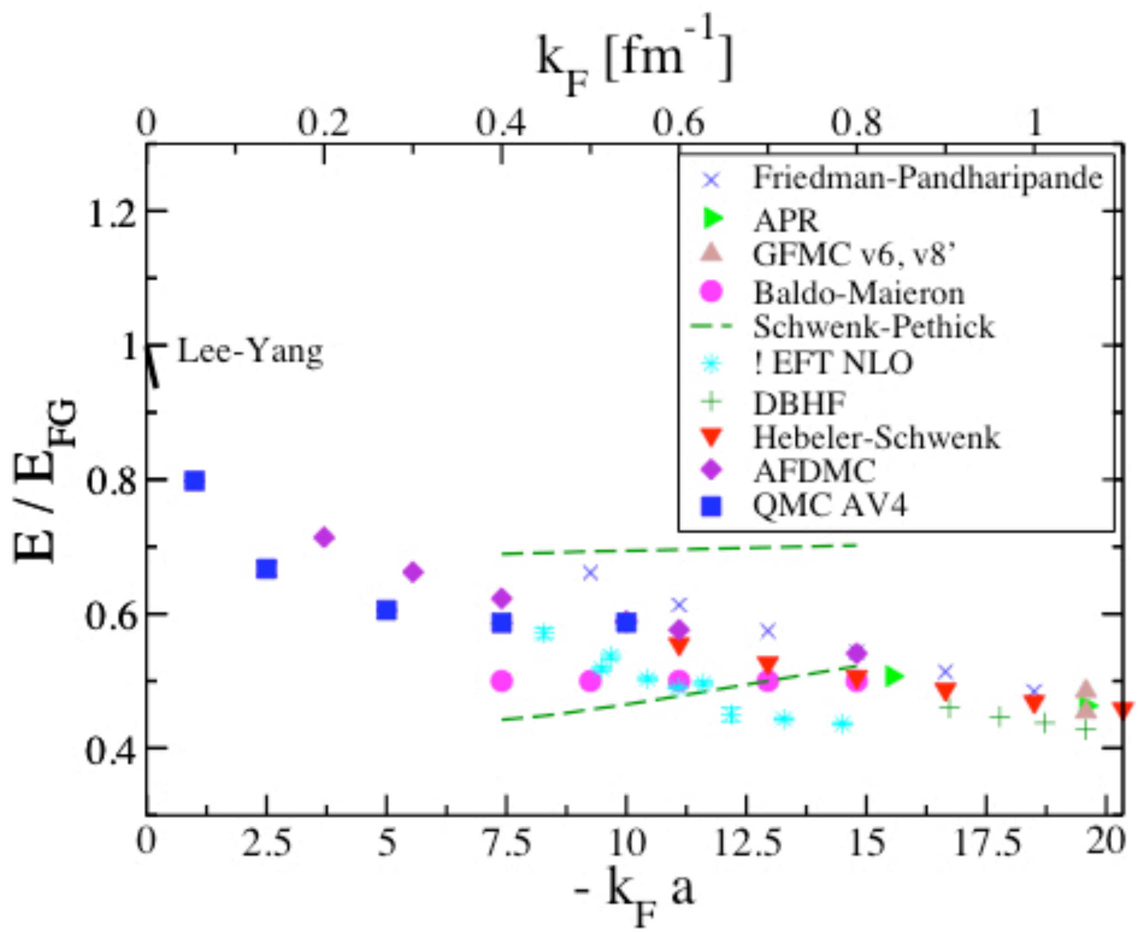
Gezerlis and Carlson, PRC 2008

At small |kF a| consistent with
Gorkov polarization suppression of BCS

$$\Delta/\Delta_{BCS} = 1/(4e)^{(1/3)} \approx 0.45$$

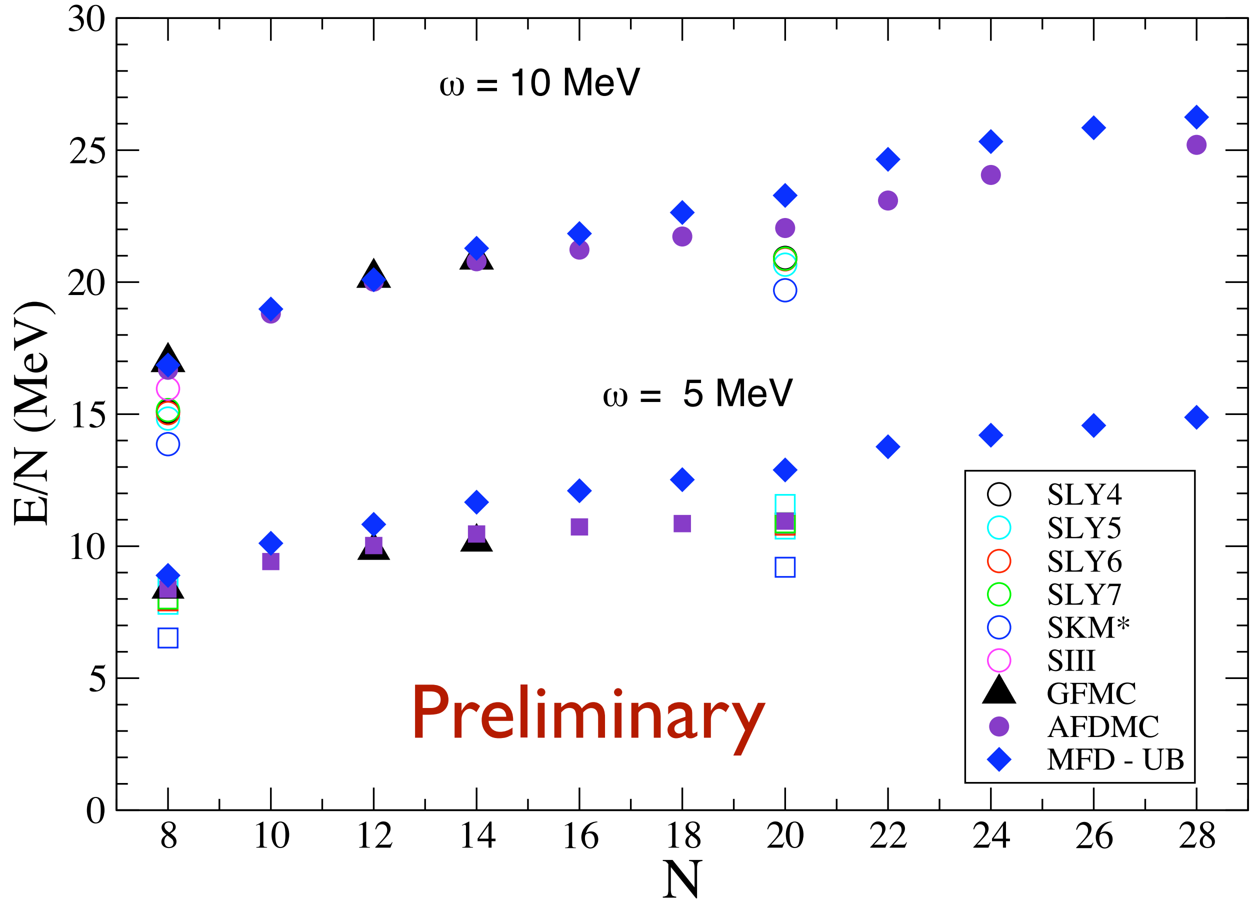
At large densities consistent with unitary results

Neutron Matter EOS strongly constrained at low-moderate densities



Beyond Bulk Matter

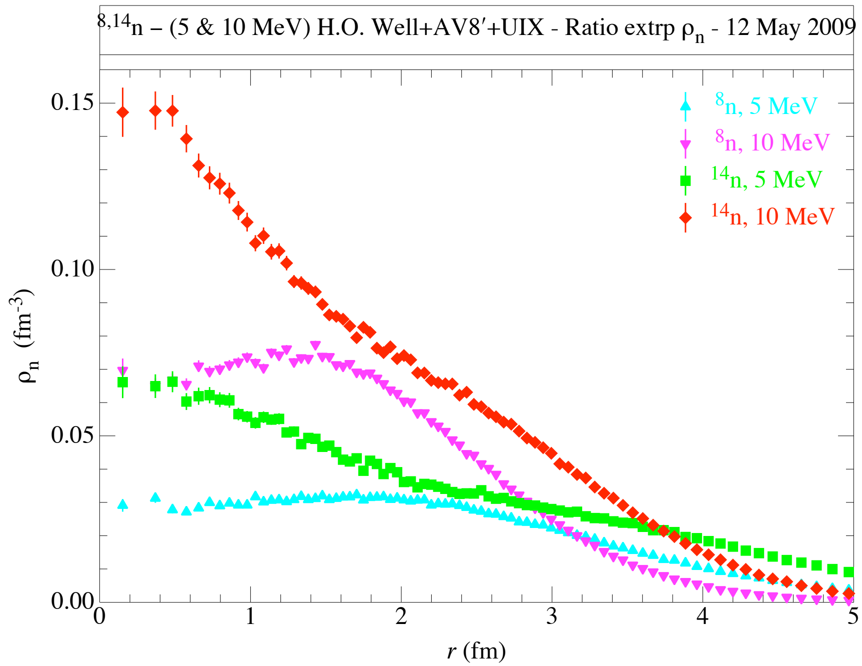
Neutron Drops in an External Well (HO)

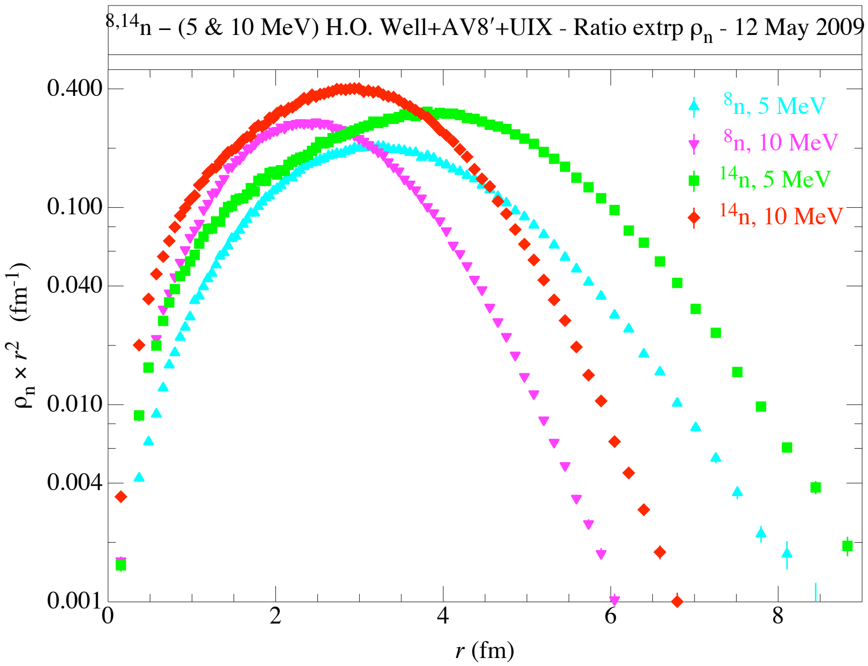


Implies significantly more repulsive isovector gradient terms

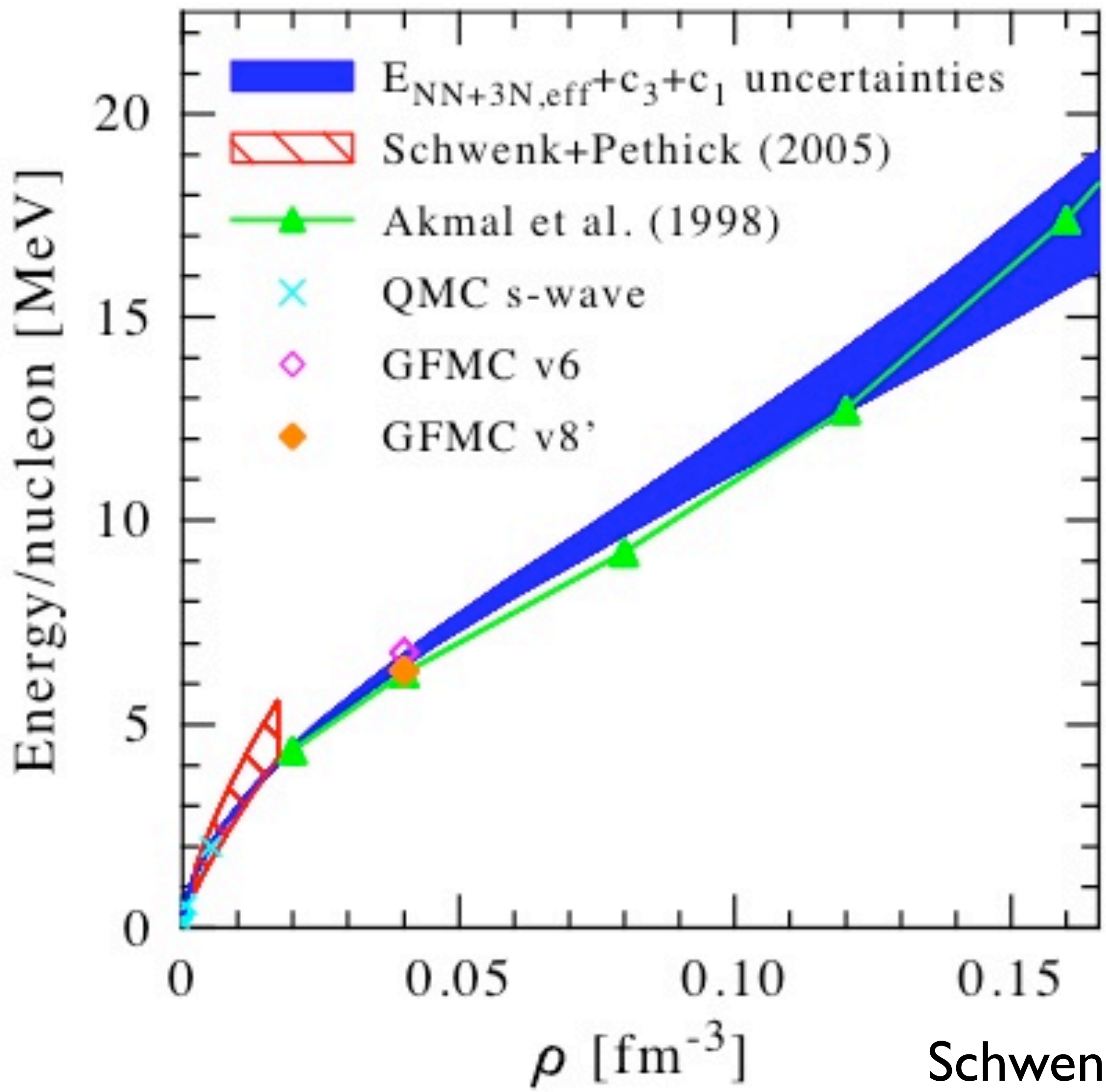
Carlson, Pieper, Gandolfi, preliminary

Neutron Drop Densities





Neutron Matter at Intermediate Densities
Equation of State

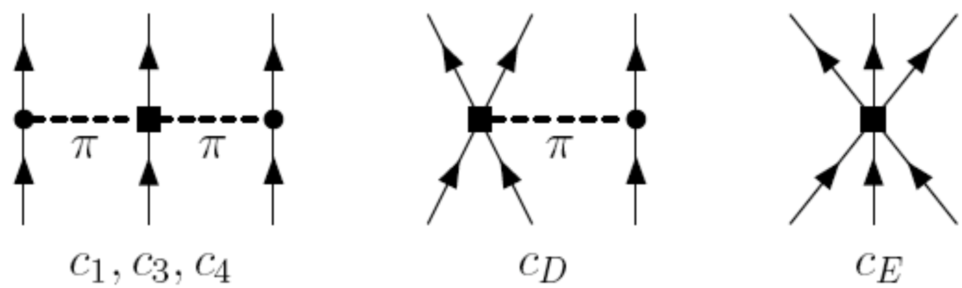


Schwenk, 2010

Low-Momentum 2 Nucleon Forces

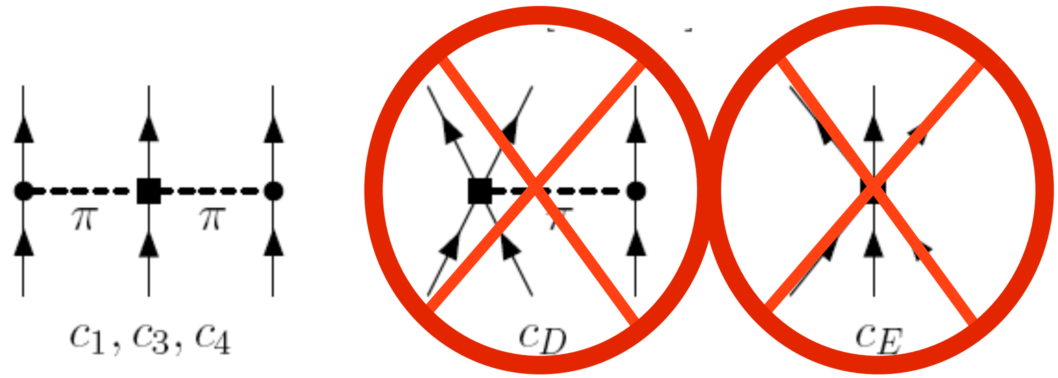
2N interaction yields identical results in truncated space

Chiral 3-N forces



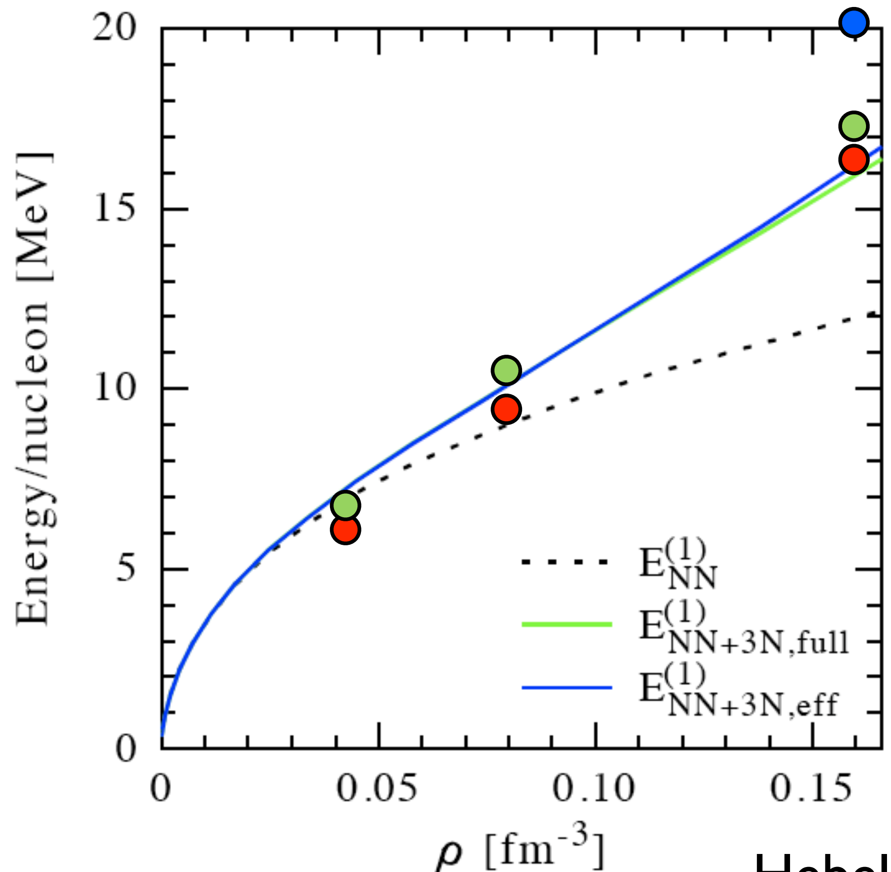
2 Pion Exchange similar to Urbana, Illinois models
Others typically fit to A=3,4......

Chiral 3-N forces



Only 2-Pion Exchange contributes in neutron matter

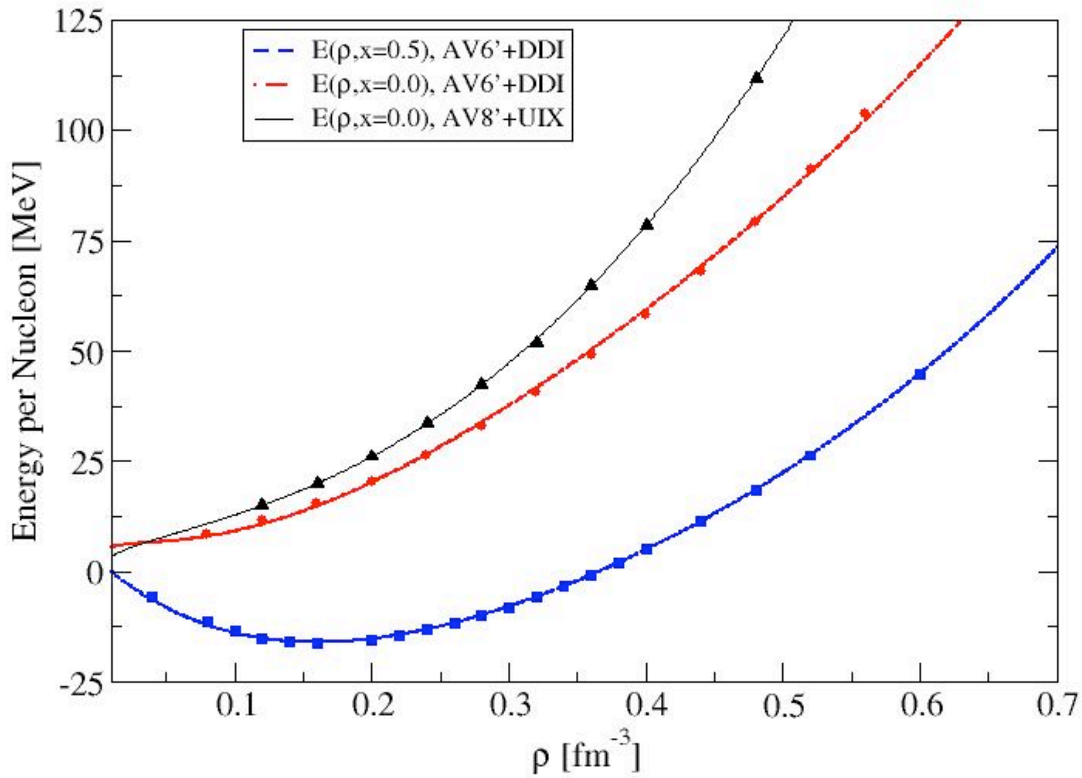
Neutron Matter at Intermediate Densities



Hebeler and Schwenk, 2010

- GFMC- AV8': JC, Morales, Pandharipande
- AFDMC-AV8': Gandolfi, et al, PRC 2009
- AFDMC-AV8'+UIX: Gandolfi, et al

Also introduced density-dependent TNI



Gandolfi, Illarionov, Fantoni,
Miller, Pederivak, Schmidt : arxiv 0909.3487

$$S = S_0 + \frac{L}{3}\left(\frac{\rho - \rho_0}{\rho_0}\right) + \ldots$$

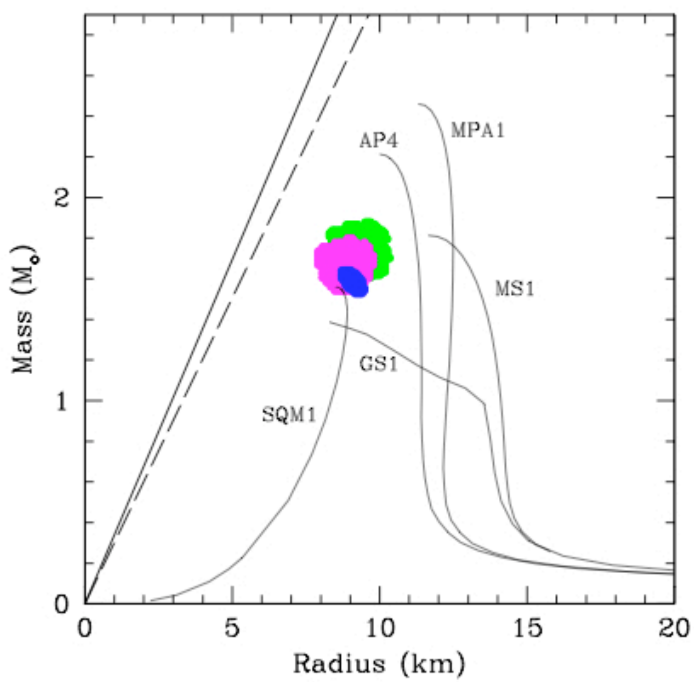
Gives reasonable symmetry energy: S0=31.3 MeV; L ~ 70
Compare with Tsang, et al (PRL 2009) and references therein: 30-34

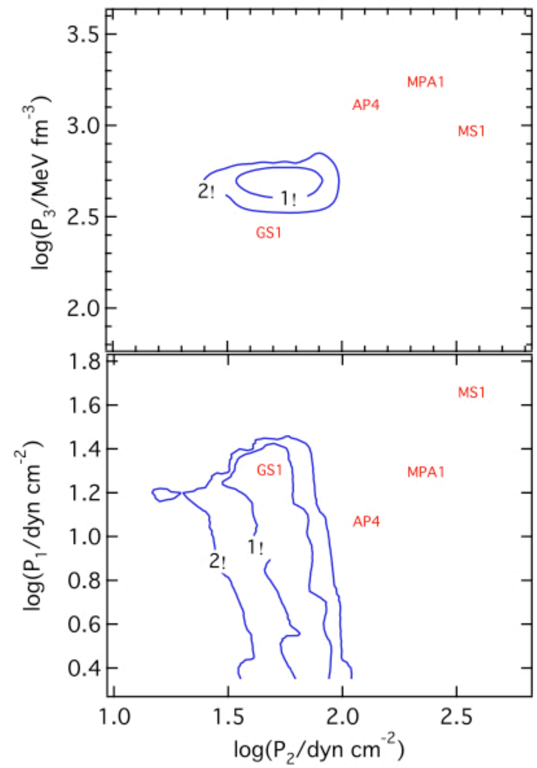
Astrophysical Constraints on Neutron Star Matter

Apparent surface area during cooling phase of burst:

$$A = \frac{R^2}{D^2 f_c^4}\left(1 - \frac{2GM}{Rc^2}\right)^{-1}$$

Eddington Luminosity $$F_{Edd} = \frac{GMc}{k_{es}D^2}(1 - \frac{2GM}{Rc^2})^{1/2}$$





Ozel, Baym, Guyver arXiv:1002.3153

FIG. 2: The pressure of cold matter at (top) 7.4 and 3.7 ns and (bottom) 1.85 and 3.7 ns.

Really Need:

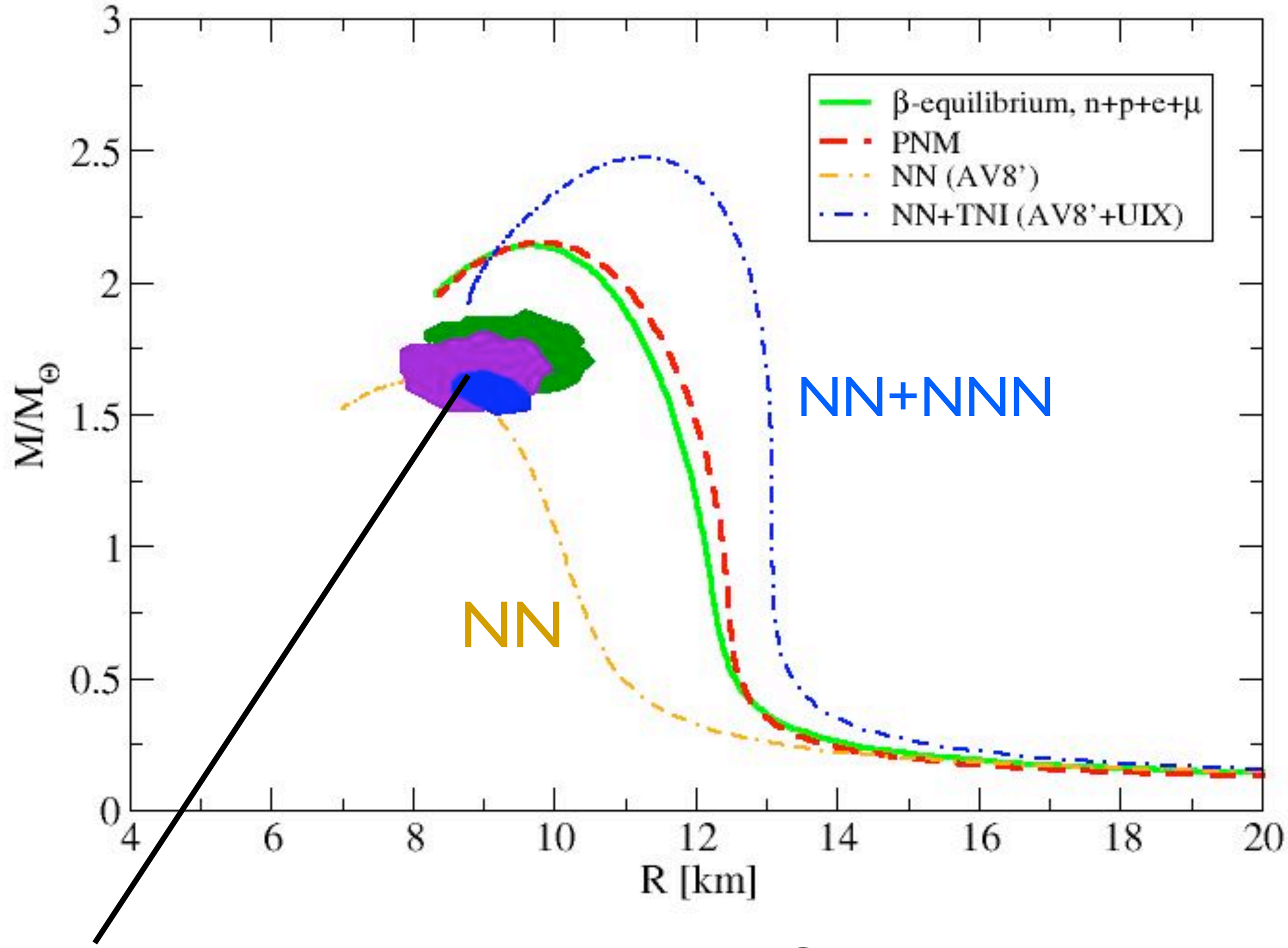
3-neutron interactions
Hyperon-Nucleon interactions
Hyperon-Hyperon interactions

from Lattice QCD?

Mass/Radius for range of neutron stars

Could know neutron star matter EOS
better than nuclear matter

Microscopic Constraints from Observations



Observations:
Ozel, Baym, Guyver

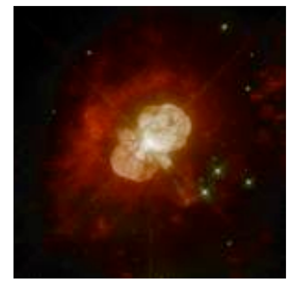
Calculations

Gandolfi, Illarionov, Fantoni,
Miller, Pederivak, Schmidt : arxiv 0909.3487

Future Challenges in Theory/Computation

Neutron Matter:

- Few Protons (Neutron Star Matter)
- Data from Expt (PREX, FRIB,...)
- Generalized Static Response
- Drops in Various External Fields
- Matter in the Crust



Nuclei / Nuclear Matter:

- Low-Energy Reactions w/ GFMC
- AFDMC with `realistic' interactions
- Pairing in Finite Nuclei
- Larger Nuclei, Matter, ...
- Neutrino Response
- Finite Temperature