QCD Factorization and Nucleon Structure

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UC RIVERSIDE | 2023 National Nuclear Physics Summer School

QCD in 1973

- Asymptotic freedom: papers (Gross-Wilczek, Politzer) published in PRL side by side on June 25, 1973

Ultraviolet Behavior of Non-Abelian Gauge Theories*

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(Received 27 April 1973)

It is shown that a wide class of non-Abelian gauge theories have, up to calculable logarithmic corrections, free-field–theory asymptotic behavior. It is suggested that Bjorken scaling may be obtained from strong-interaction dynamics based on non-Abelian gauge symmetry.

Non-Abelian gauge theories have received much attention recently as a means of constructing unified and renormalizable theories of the weak and electromagnetic interactions.[1] In this note we report on an investigation of the ultraviolet (UV) asymptotic behavior of such theories. We have found that they possess the remarkable feature, perhaps unique among renormalizable theories, of asymptotically approaching free-field theory. Such asymptotically free theories will exhibit, for matrix elements of currents between on-mass-shell states, Bjorken scaling. We therefore suggest that one should look to a non-Abelian gauge theory of the strong interactions to provide the explanation for Bjorken scaling, which has so far eluded field-theoretic understanding.

The UV behavior of renormalizable field theories can be discussed using the renormalization-group equations,[2,3] which for a theory involving one field (say $g\varphi^4$) are

$$[m\partial/\partial m+\beta(g)\,\partial/\partial g-n\gamma(g)]\Gamma_{\text{asy}}^{(n)}(g;P_1,\ldots,P_n)=0. \tag{1}$$

$\Gamma_{\text{asy}}^{(n)}$ is the asymptotic part of the one-particle–irreducible renormalized n-particle Green's function, $\beta(g)$ and $\gamma(g)$ are finite functions of the renormalized coupling constant g, and m is either the renormalized mass or, in the case of massless particles, the Euclidean momentum at which the theory is renormalized.[4] If we set $P_i=\lambda q_i^0$, where q_i^0 are (nonexceptional) Euclidean momenta, then (1) determines the λ dependence of $\Gamma^{(n)}$:

$$\Gamma^{(n)}(g;P_i)=\lambda^D\Gamma^{(n)}(\bar{g}(g,t);q_i)\exp\left[-n\int_0^t\gamma(\bar{g}(g,t'))\,dt'\right], \tag{2}$$

where $t=\ln\lambda$, D is the dimension (in mass units) of $\Gamma^{(n)}$, and $\bar{g}$, the invariant coupling constant, is the solution of

$$d\bar{g}/dt=\beta(\bar{g}),\quad \bar{g}(g,0)=g. \tag{3}$$

The UV behavior of $\Gamma^{(n)}$ $(\lambda\to+\infty)$ is determined by the large-t behavior of $\bar{g}$ which in turn is controlled by the zeros of β: $\beta(g_f)=0$. These fixed points of the renormalization-group equations are said to be UV stable [infrared (IR) stable] if $\bar{g}\to g_f$ as $t\to+\infty$ $(-\infty)$ for $\bar{g}(0)$ near g_f. If the physical coupling constant is in the domain of attraction of a UV-stable fixed point, then

$$\Gamma^{(n)}(g;P_i)\underset{\lambda\to\infty}{\approx}\lambda^{D-n\gamma(g_f)}\Gamma^{(n)}(g_f;q_i)\exp\left\{-n\int_0^\infty[\gamma(\bar{g}(g,t))-\gamma(g_f)]\,dt\right\}, \tag{4}$$

so that $\gamma(g_f)$ is the anomalous dimension of the field. As Wilson has stressed, the UV behavior is determined by the theory at the fixed point $(g=g_f)$.[5]

In general, the dimensions of operators at a fixed point are not canonical, i.e., $\gamma(g_f)\neq 0$. If we wish to explain Bjorken scaling, we must assume the existence of a tower of operators with canonical dimensions. Recently, it has been argued for all but gauge theories, that this can only occur if the fixed point is at the origin, $g_f=0$, so that the theory is asymptotically free.[6,7] In that case the anomalous dimensions of all operators vanish, one obtains naive scaling up to finite and calculable powers of $\ln\lambda$, and the structure of operator products at short distances is that of free-field theory.[7] Therefore, the existence of such a fixed point, for a theory of the strong interactions, might explain Bjorken scaling and the success of naive light-cone or parton-model relations. Unfortunately, it appears that the fixed point at the origin, which is common to all theories, is not UV stable.[8,9] The only exception would seem to be non-Abelian gauge theories, which hitherto have not been explored in this re-

[14]Y. Nambu and G. Jona-Lasino, Phys. Rev. 122, 345 (1961); S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).
[15]K. Symanzik (to be published) has recently suggested that one consider a $\lambda\varphi^4$ theory with a negative λ to achieve UV stability at $\lambda=0$. However, one can show, using the renormalization-group equations, that in such theory the ground-state energy is unbounded from below (S. Coleman, private communication).
[16]W. A. Bardeen, H. Fritzsch, and M. Gell-Mann, CERN Report No. CERN-TH-1538, 1972 (to be published).
[17]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 28, 1494 (1972); S. Weinberg, Phys. Rev. D 5, 1962 (1972).
[18]For a review of this program, see S. L. Adler, in Proceedings of the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Illinois, 1972 (to be published).

Reliable Perturbative Results for Strong Interactions?*

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(Received 3 May 1973)

An explicit calculation shows perturbation theory to be arbitrarily good for the deep Euclidean Green's functions of any Yang-Mills theory and of many Yang-Mills theories with fermions. Under the hypothesis that spontaneous symmetry breakdown is of dynamical origin, these symmetric Green's functions are the asymptotic forms of the physically significant spontaneously broken solution, whose coupling could be strong.

Renormalization-group techniques hold great promise for studying short-distance and strong-coupling problems in field theory.[1,2] Symanzik[2] has emphasized the role that perturbation theory might play in approximating the otherwise unknown functions that occur in these discussions. But specific models in four dimensions that had been investigated yielded (in this context) disappointing results.[3] This note reports an intriguing contrary finding for any generalized Yang-Mills theory and theories including a wide class of fermion representations. For these one–coupling-constant theories (or generalizations involving product groups) the coefficient function in the Callan-Symanzik equations commonly called $\beta(g)$ is negative near $g=0$.

The constrast with quantum electrodynamics (QED) might be illuminating. Renormalization of QED must be carried out at off-mass-shell points because of infrared divergences. For small e^2, we expect perturbation theory to be good in some neighborhood of the normalization point. But what about the inevitable logarithms of momenta that grow as we approach the mass shell or as some momenta go to infinity? In QED, the mass-shell divergences do not occur in observable predictions, when we take due account of the experimental situation. The renormalization-group technique[4] provides a somewhat opaque analysis of this situation. Loosely speaking,[5] the effective coupling of soft photons goes to zero, compensating for the fact that there are more and more of them. But the large-p^2 divergence represents a real breakdown of perturbation theory. It is commonly said that for momenta such that $e^2\ln(p^2/m^2)\sim 1$, higher orders become comparable, and hence a calculation to any finite order is meaningless in this domain. The renormalization group technique shows that the effective coupling grows with momenta.

The behavior in the two momentum regimes is reversed in a Yang-Mills theory. The effective coupling goes to zero for large momenta, but as p^2's approach zero, higher-order corrections become comparable. Thus perturbation theory tells *nothing* about the mass-shell structure of the symmetric theory. Even for arbitrarily small g^2, there is no sense in which the interacting theory is a small perturbation on a free multiplet of massless vector mesons. The truly catastrophic infrared problem makes a symmetric particle interpretation impossible. Thus, though one can well approximate asymptotic Green's functions, to what particle states do they refer?

Consider theories defined by the Lagrangian

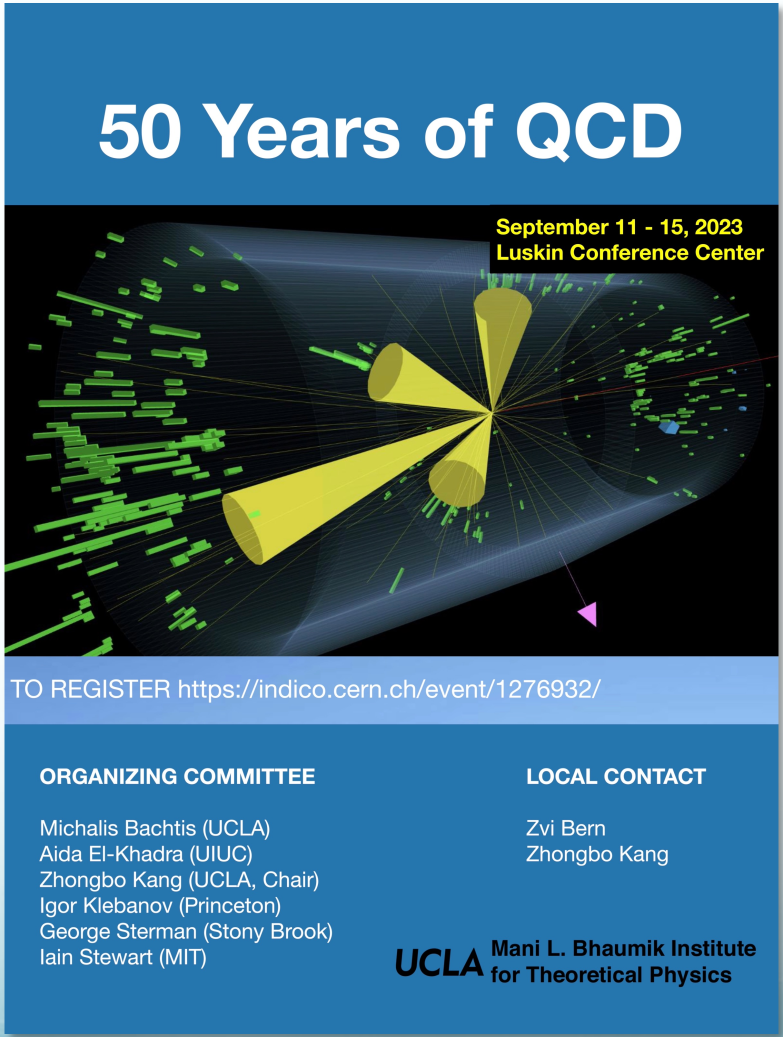
$$\mathcal{L}=-\tfrac{1}{4}F_{\mu\nu}{}^aF^{a\mu\nu}+i\bar{\psi}_i\gamma\cdot D_{ij}\psi_j, \tag{1}$$

where

$$F_{\mu\nu}{}^a=\partial_\mu A_\nu{}^a-\partial_\nu A_\mu{}^a+gf^{abc}A_\mu{}^bA_\nu{}^c,$$

50 Years of QCD

- A dedicated conference will be held at UCLA during Sept. 11 – 15, 2023

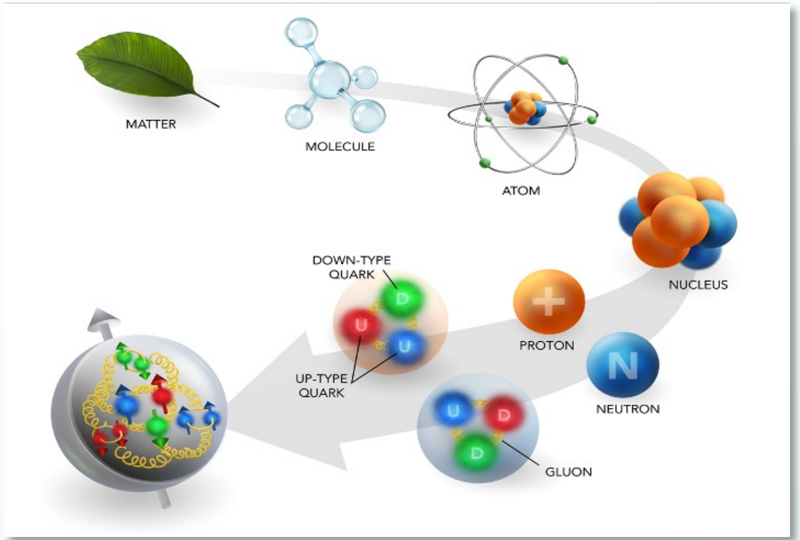


Outline

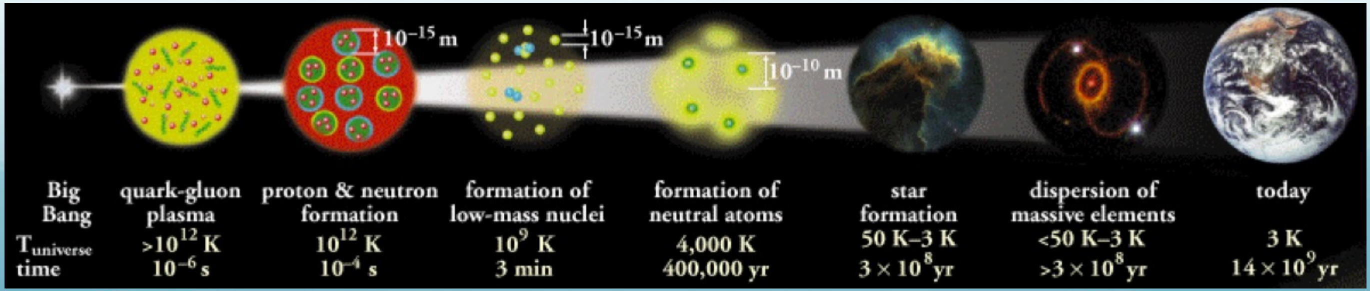
- Lecture 1: QCD factorization
  - QCD basics, collinear factorization
- Lecture 2: Nucleon structure
  - TMDs, SCET, and phenomenology

The structure of matter

- The exploration on the structure of matter has a really long history
  - Dalton 1803 (atom)
  - Rutherford 1911 (nucleus)
  - Chadwick 1932 (neutron)
  - Gell-Mann and Zweig 1964 (quark model)
  - Feynman 1969 (parton model), ...



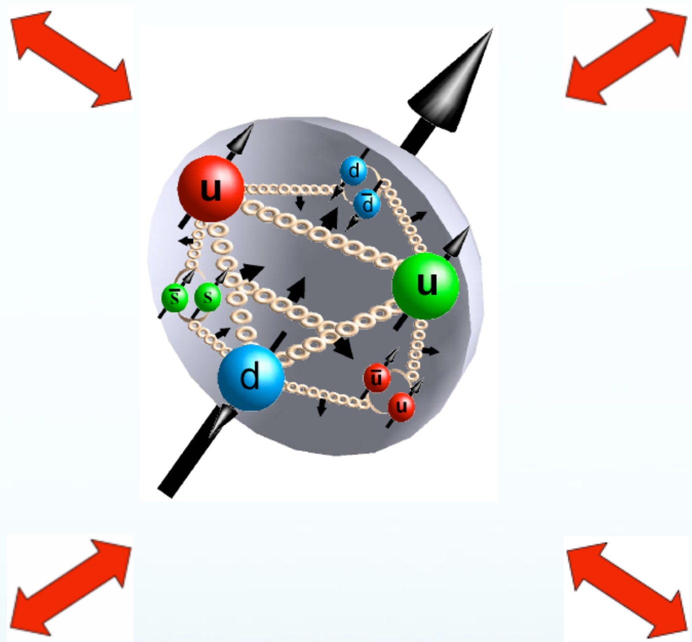
- Central goal of nuclear science
  - **To discover, explore, and understand all forms of nuclear matter and the associated dynamics**



Exploring the nucleon: fundamental importance in science

Know what we are made of:
Most abundant particles around us
Building blocks of all elements

Fundamental properties:
Proton mass, spin, magnetic moment, understand them in terms of the internal degrees of freedom

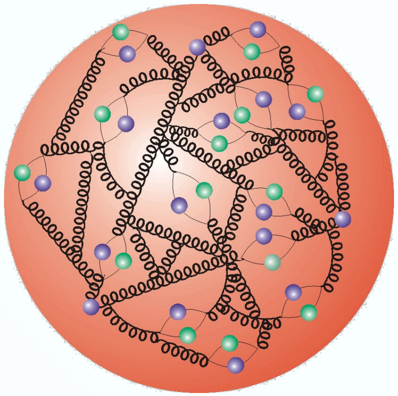


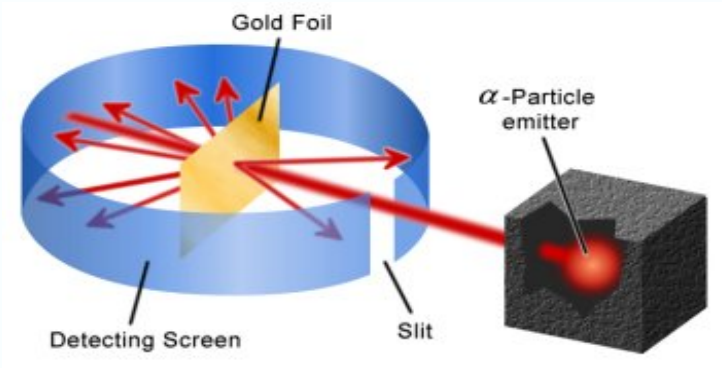
Tool for discovery:
Colliding high energy nucleons
New Physics beyond SM
LHC, Tevatron, RHIC, HERA, ...

Exploring QCD and strong interaction:
Confinement, Lattice QCD, Asymptotic freedom, perturbative QCD, ...

Hadron structure

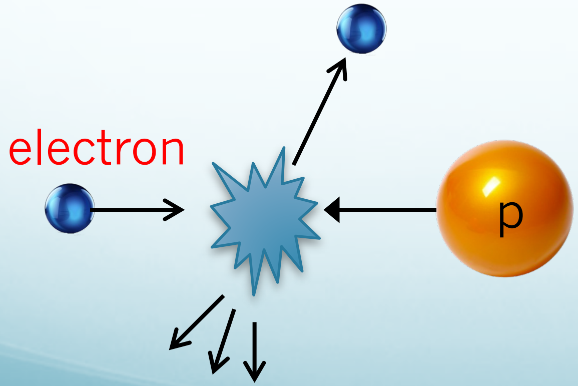
- Nucleon: quantum many-body system of quarks and gluons



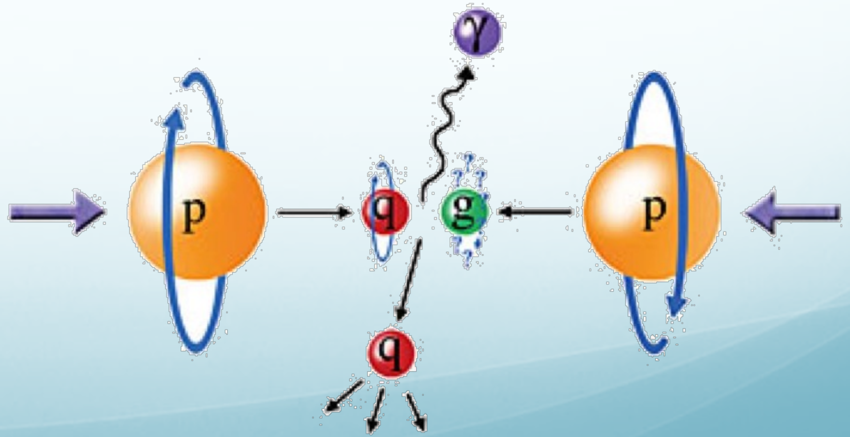


Rutherfold's experiment

- High energy scattering: to extract information on the nucleon structure, we send in a probe and measure the outcome of the collisions



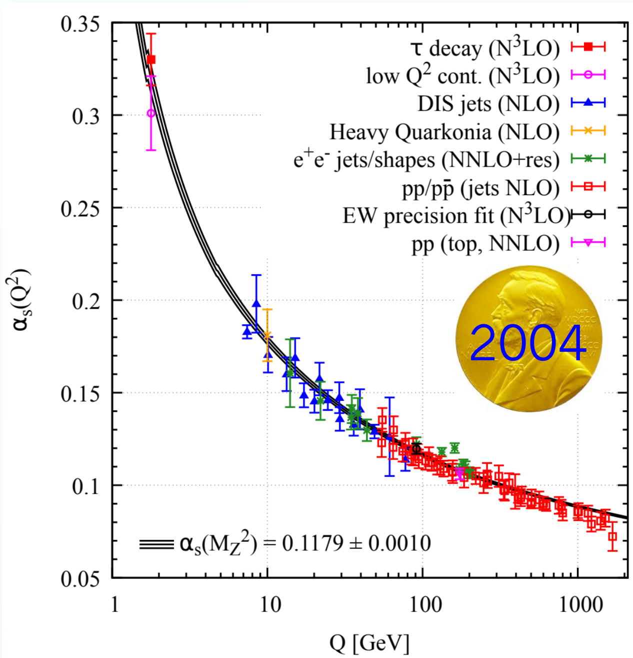
Deep Inelastic Scattering (DIS)



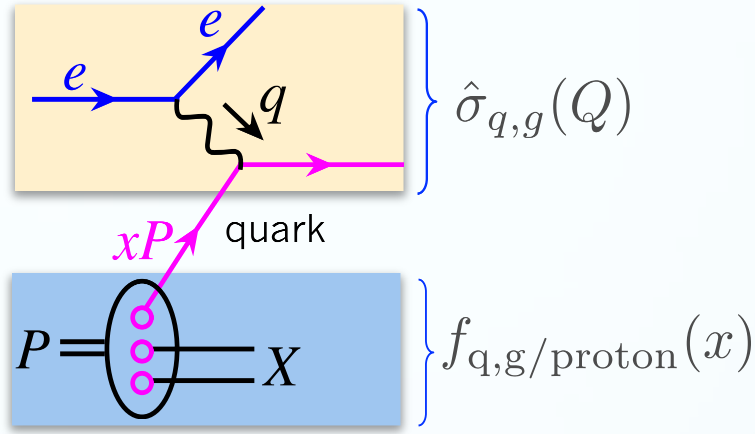
Proton-Proton collisions

How to trace back?

- Perturbative QCD paradigm: Asymptotic freedom + QCD factorization

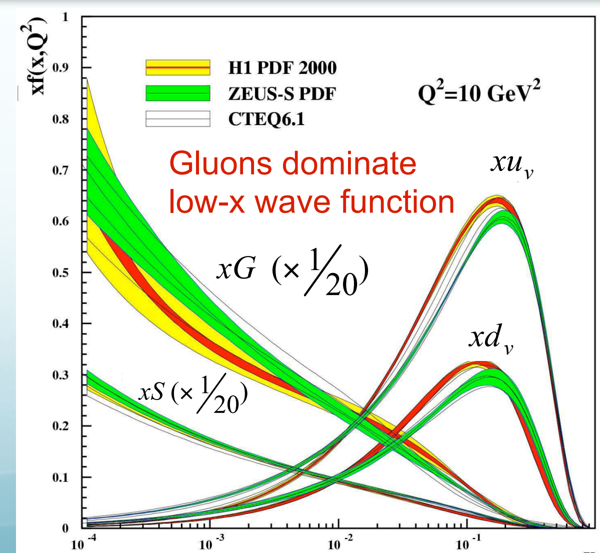


Parton distribution function



$$\sigma_{\mathrm{proton}}(Q) = f_{\mathrm{q,g/proton}}(x) \otimes \hat{\sigma}_{q,g}(Q)$$

measured　　extracted　　calculable



asymptotic freedom

↓

perturbation theory for quarks and gluons

↓

only hadrons are observed in experiments (confinement)

Lecture 1: QCD factorization

- QCD basics, collinear factorization, DGLAP, PDFs, FFs

QED: the fundamental theory of electo-magnetic interaction

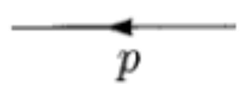
- QED Lagrangian:

$$\boxed{\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + e\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$

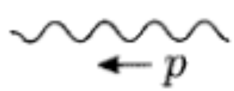
- Feynman rule: photon has no charge, thus does not self-interact

Dirac propagator:



$$= \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}$$

Photon propagator:



$$= \frac{-ig_{\mu\nu}}{p^2 + i\epsilon}$$

QED vertex:

$$= iQe\gamma^\mu$$

($Q = -1$ for an electron)

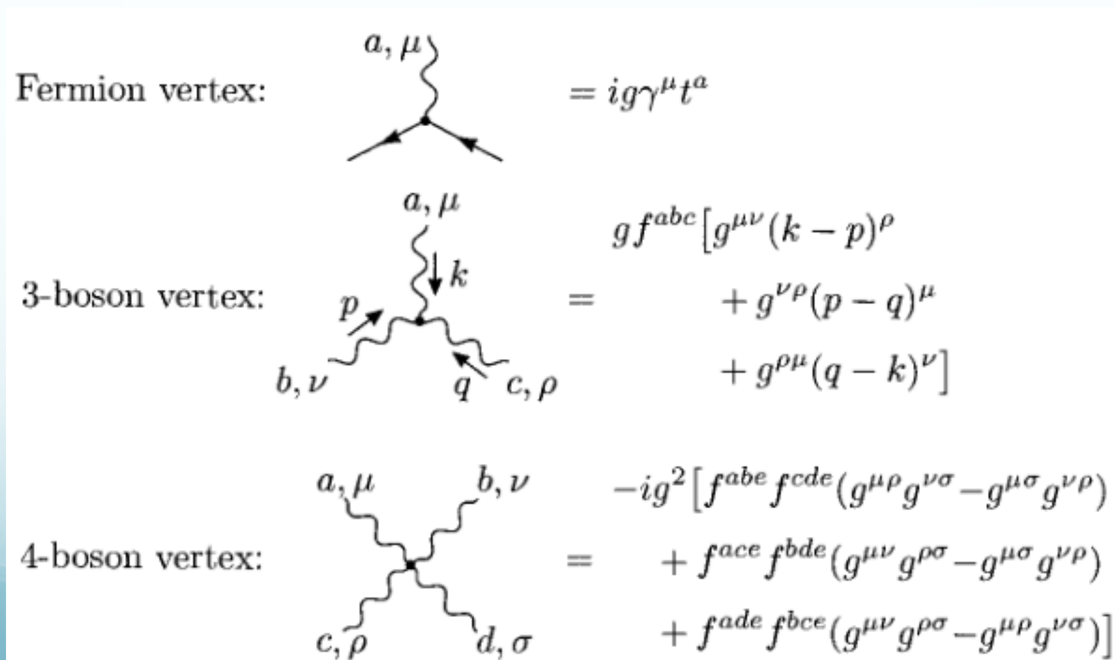
QCD: the fundamental theory of the strong interaction

- QCD describes the interaction between quarks and gluons

$$\boxed{\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + g\bar{\psi}\gamma^\mu t_a \psi G^a_\mu - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a}$$

$$G^a_{\mu\nu} \equiv \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g f_{abc} G^b_\mu G^c_\nu$$

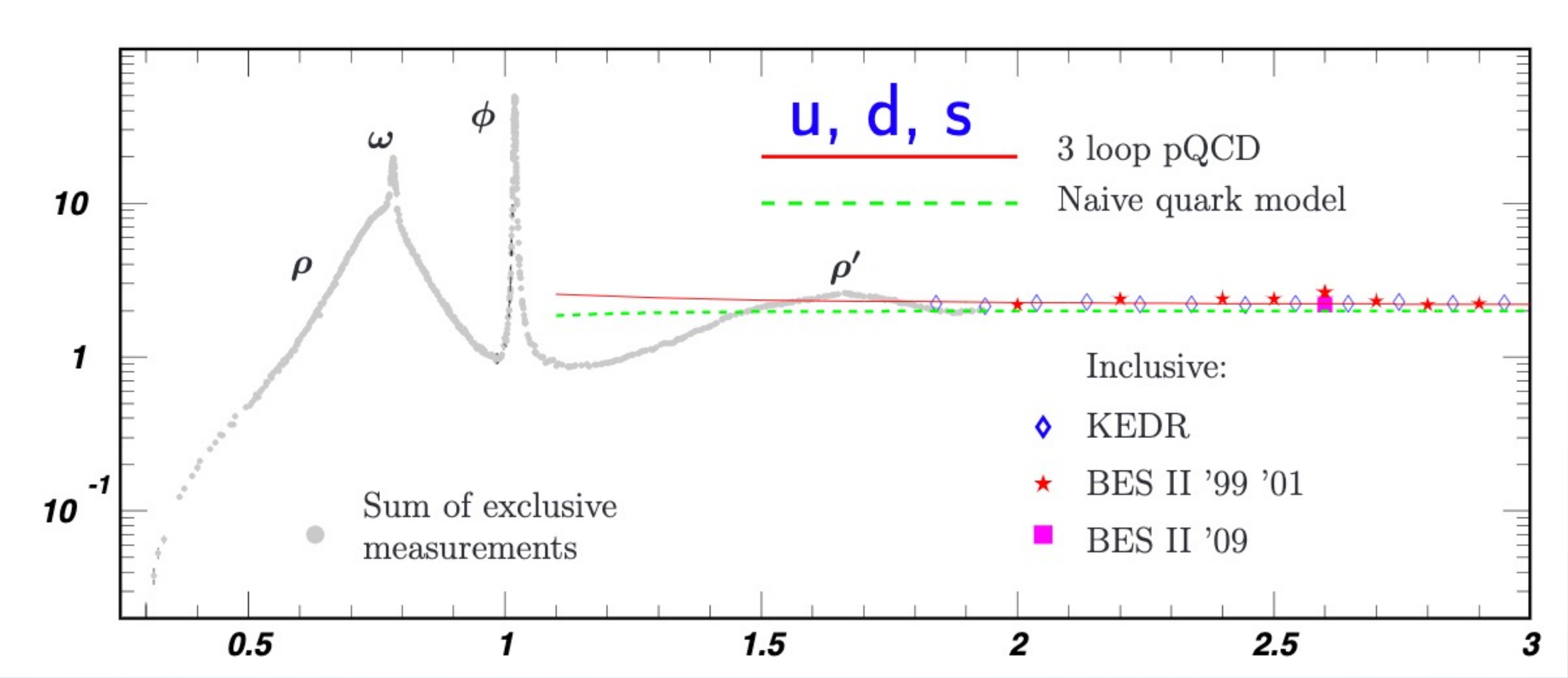
- Feynman rules: gluon carries color, thus can self-interact



Experimental verification of the color

- The color does exist: color of quarks Nc = 3 (low energy R=2/3 v.s. 2)

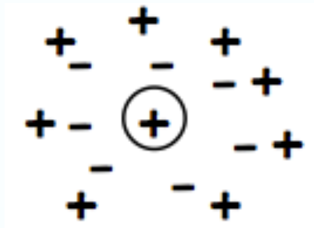
$$R_{e^+e^-} = \frac{\sigma(e^+e^- \to q\bar{q})}{e^+e^- \to \mu^+\mu^-} = N_c \sum_q e_q^2$$



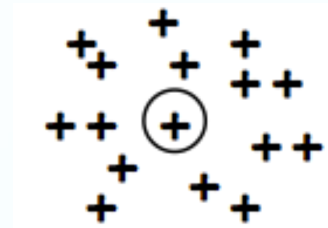
$\sqrt{S}$ [GeV]

Understanding QCD: running coupling (asymptotic freedom)

- Gluon carries color charges
  - Strong coupling α_s depends on the distance (i.e., energy)



Screening: $\alpha_{em}(r) \uparrow \;\; \text{as} \;\; r \downarrow$



Anti-screening: $\alpha_s(r) \downarrow \;\; \text{as} \;\; r \downarrow$

Asymptotic Freedom ⇔ antiscreening

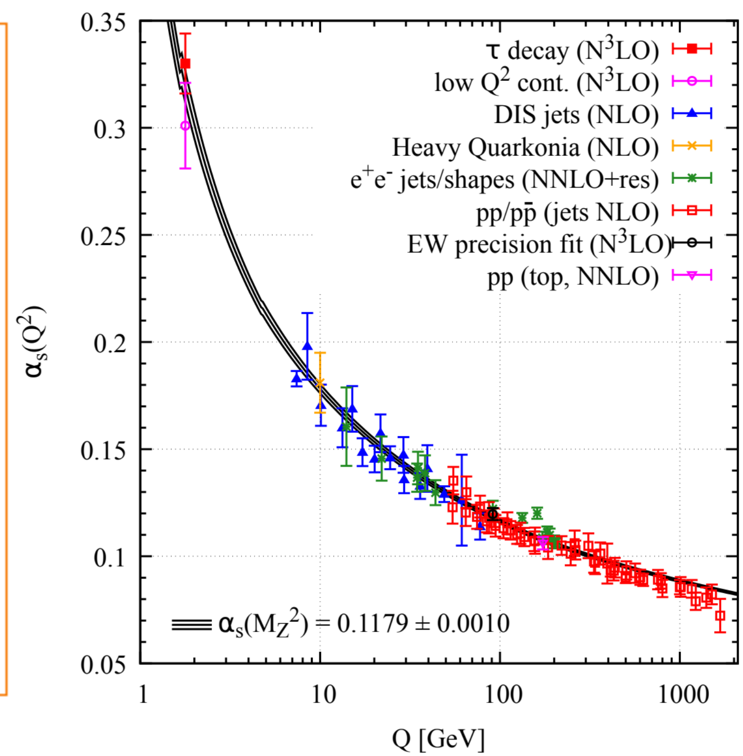
$$\text{QCD: } \frac{\partial \alpha_s(Q^2)}{\partial \ln Q^2} = \beta(\alpha_s) < 0$$

Compare

$$\text{QED: } \frac{\partial \alpha_{EM}(Q^2)}{\partial \ln Q^2} = \beta(\alpha_{EM}) > 0$$

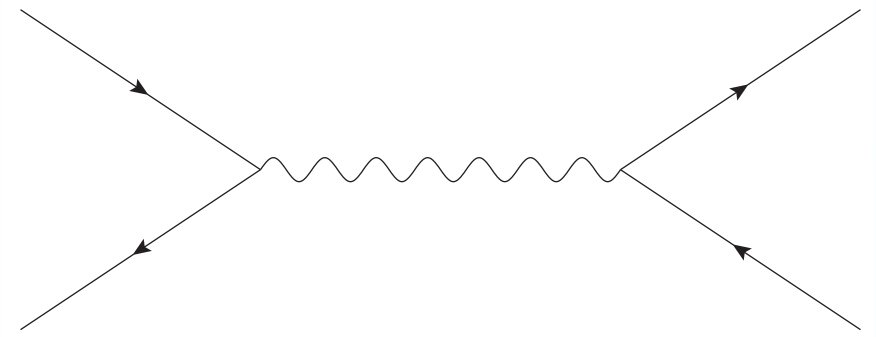
D.Gross, F.Willczek, Phys.Rev.Lett 30,(1973)
H.Politzer, Phys.Rev.Lett 30, (1973)

2004 Nobel Prize in Physics

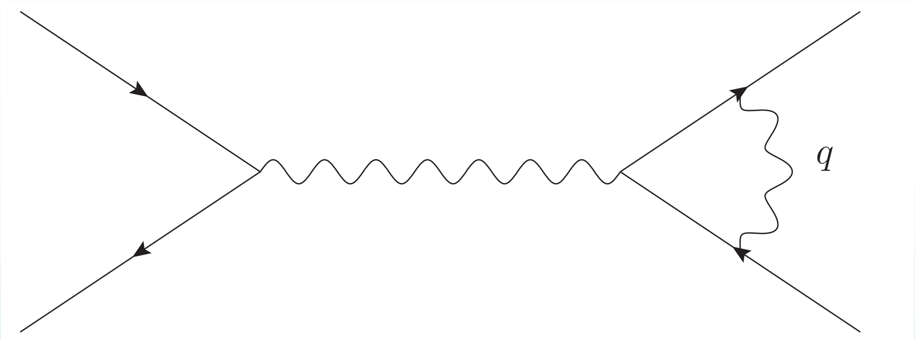


Why does the coupling constant run?

- Leading order calculation is simple: tree diagrams – always finite



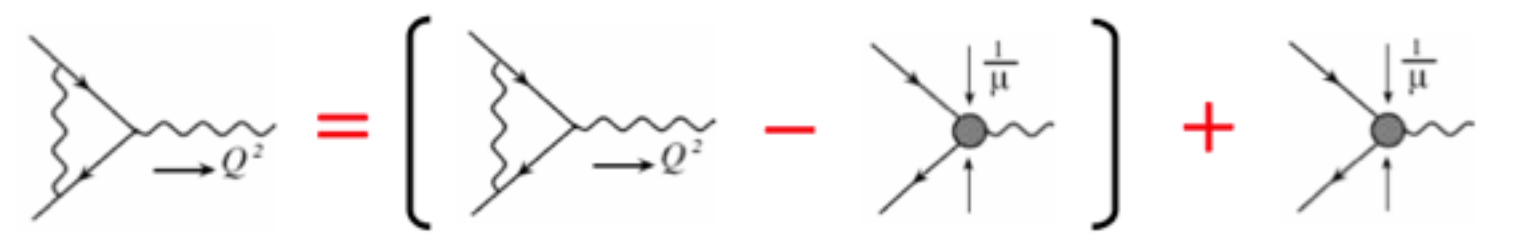
- Study a higher order Feynman diagram: one-loop, the diagram is divergent as q $\rightarrow \infty$



- Make sense of the result: redefine the coupling constant to be physical

Renormalization (Redefine the coupling constant)

- Renormalization
  - UV divergence due to “high momentum” states
  - Experiments cannot resolve the details of these states

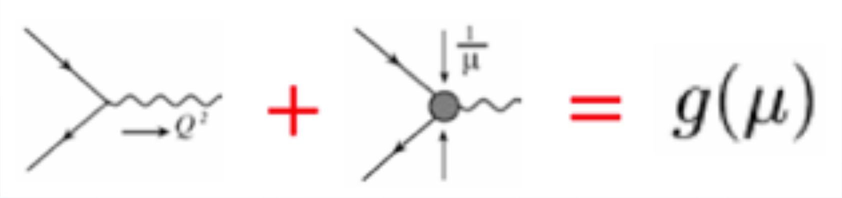


Low momentum state

High momentum state

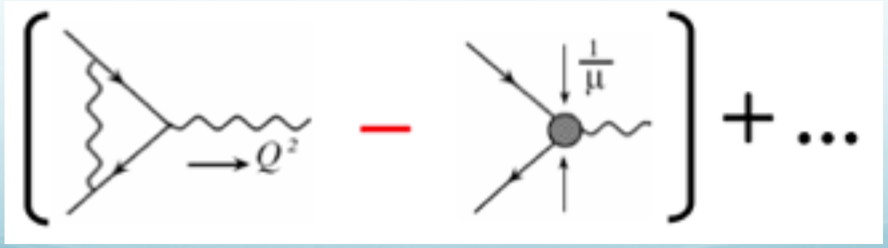
- Combine the “high momentum” states with leading order

LO:



Renormalized coupling

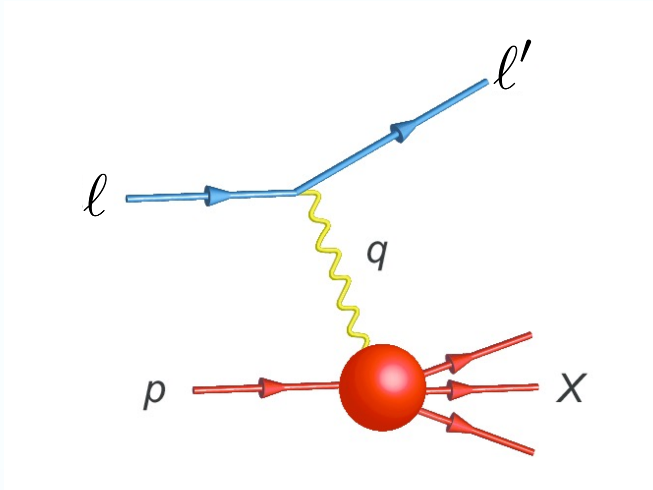
NLO:



No UV divergence!

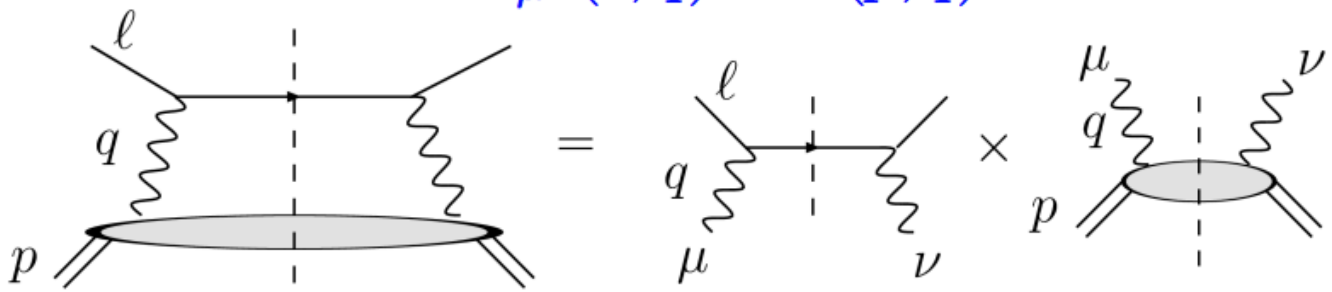
Simple study of Deep Inelastic Scattering (DIS): parton model

- DIS has been used a lot in extracting hadron structure



- Leptonic and hadronic tensor

$$d\sigma \propto L_{\mu\nu}(\ell, q) W^{\mu\nu}(p, q)$$



– Electron is elementary: $L_{\mu\nu}$ can be calculated perturbatively

Structure functions

- Hadronic tensor: Lorentz decomposition + parity invariance (for photon case) + time-reversal invariance + gauge invariance

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1\left(x_B, Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) F_2\left(x_B, Q^2\right)$$

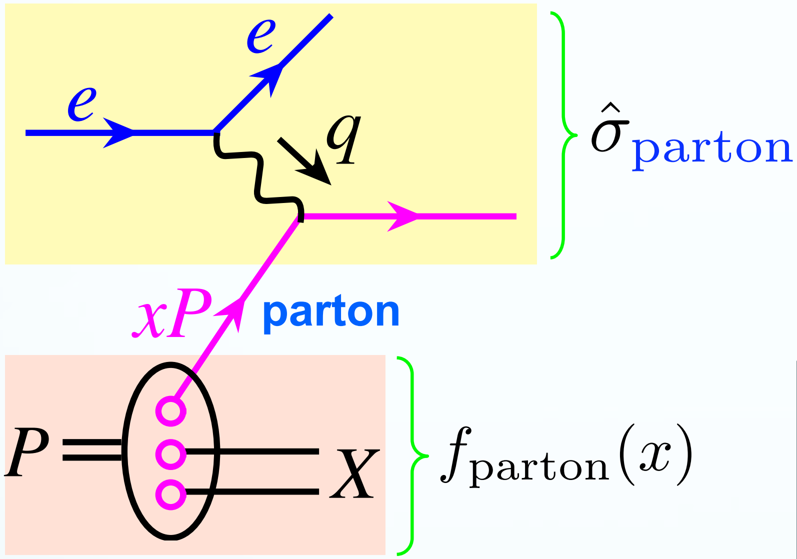
- All the information about hadron structure is contained in the structure functions

$$L_{\mu\nu} = 2\left(\ell_\mu \ell'_\nu + \ell'_\mu \ell_\nu - \ell\cdot\ell' g_{\mu\nu}\right)$$

$$\frac{d\sigma}{dx_B dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{x_B Q^4}\left[(1-y)\, F_2(x_B, Q^2) + y^2 x_B F_1(x_B, Q^2)\right]$$

The paradigm of perturbative QCD

- The common wisdom: to trace back what's inside the proton from the outcome of the collisions, we rely on QCD factorization



Parton Distribution Functions (PDFs):
Probability density for finding a parton in a proton with momentum fraction x

$$\sigma_{\text{proton}}(Q) = f_{\text{parton}}(x) \otimes \hat{\sigma}_{\text{parton}}(Q)$$

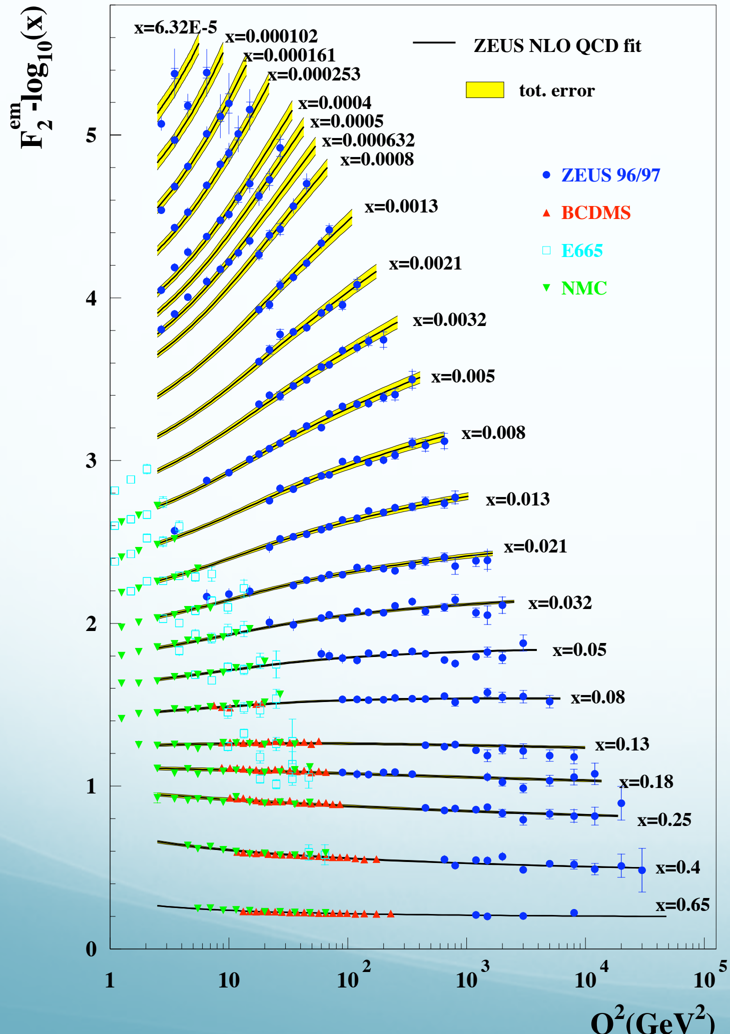
Universal (measured) **calculable**

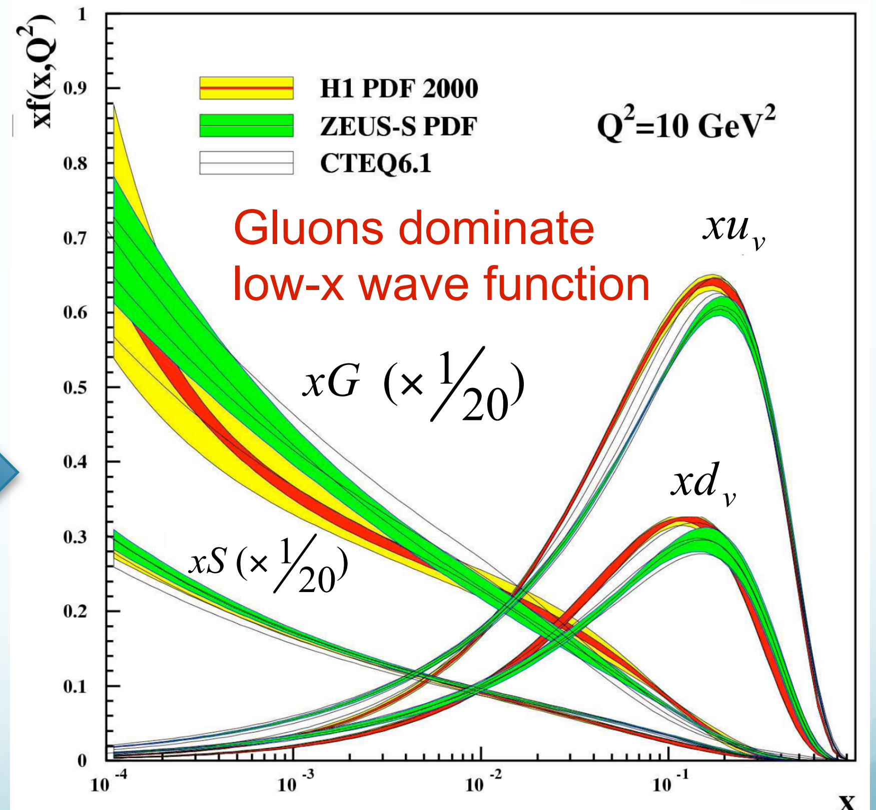
- Hadron structure: encoded in PDFs
- QCD dynamics at short-distance: partonic cross section, perturbatively calculable

Universality of PDFs: extraction from DIS

$$\sigma_{\mathrm{proton}}(Q) = f_{\mathrm{parton}}(x) \otimes \hat{\sigma}_{\mathrm{parton}}(Q)$$

Universal (measured) **calculable**

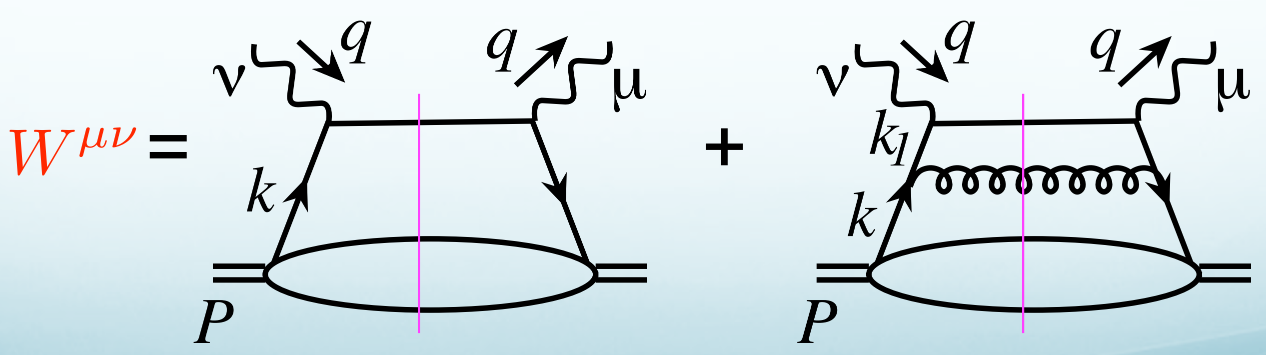




What about higher order?

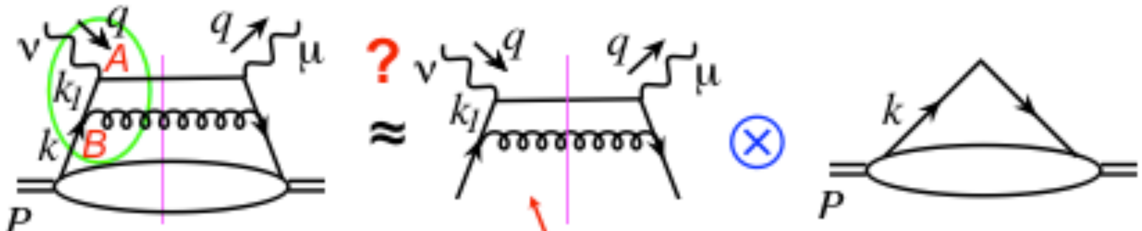
- pQCD calculations: understand and make sense of all kinds of divergences
  - Ultraviolet (UV) divergence $k \to \infty$: renormalization (redefine coupling constant)
  - Collinear divergence $k//P$: redefine the PDFs and FFs
  - Soft divergence $k \to 0$: usually cancel between real and virtual diagrams for collinear PDFs/FFs; do not cancel for TMDs, leads to new evolution equations

- Going beyond the leading order of the DIS, we face another divergence



QCD dynamics beyond tree level

- Going beyond leading order calculation



Collinear divergence!!! (from $k_1^2 \sim 0$)

$$\Rightarrow \int d^4k_1 \frac{i}{k_1^2 + i\epsilon} \frac{-i}{k_1^2 - i\epsilon} \Rightarrow \infty$$

$$k_1^2 = (k + k_g)^2 = 2EE_g(1 - \cos\theta)$$

- $k_1^2 \sim 0$ intermediate quark is on-shell

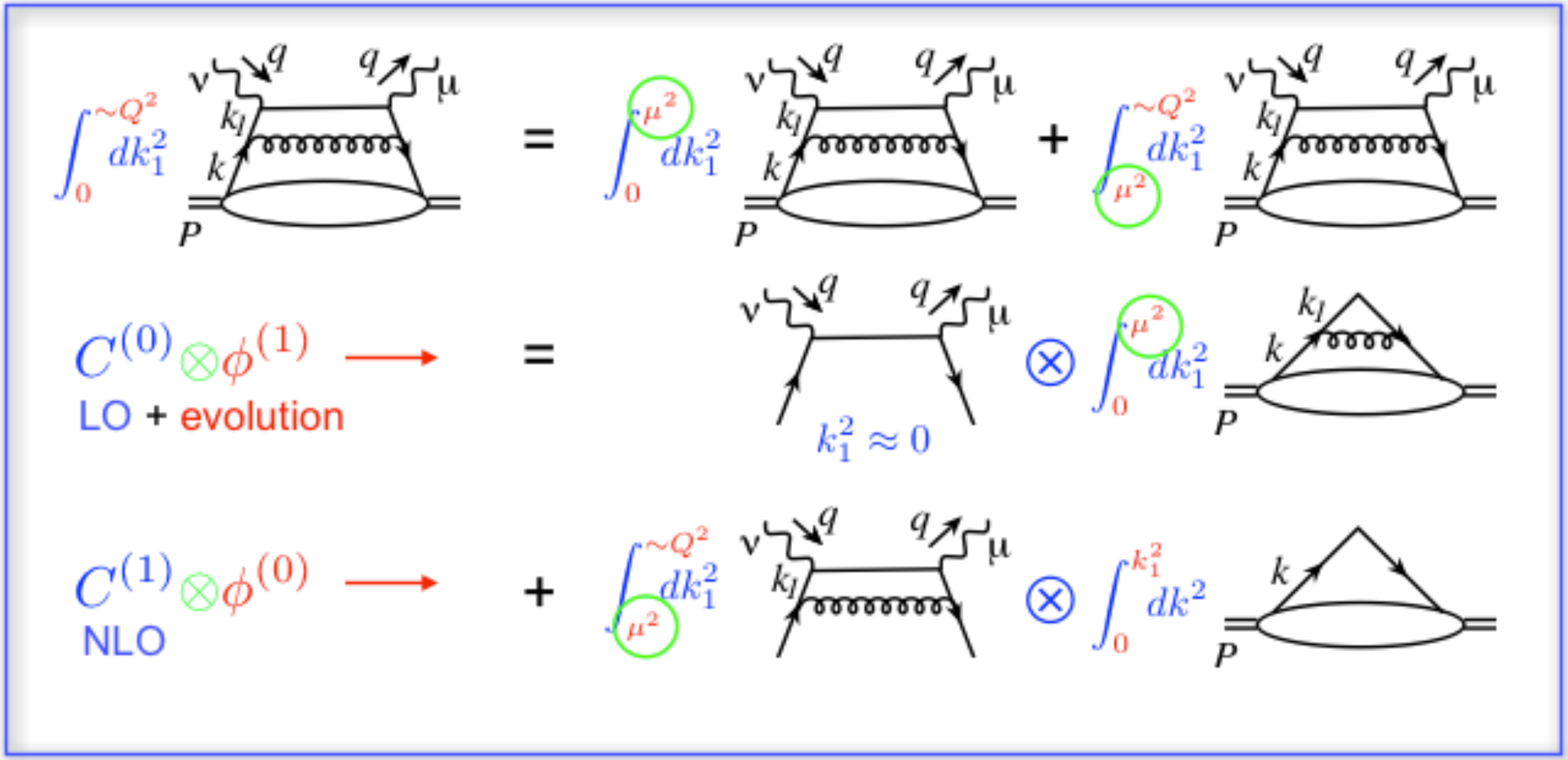
  $t_{AB} \rightarrow \infty$

- gluon radiation takes place long before the photon-quark interaction
  $\Longrightarrow$ a part of PDF

Partonic diagram has both long- and short-distance physics

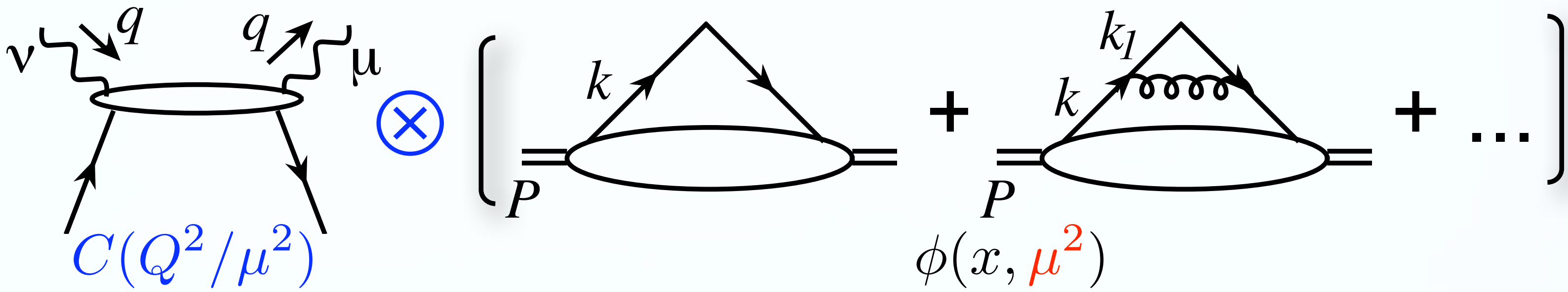
QCD factorization: beyond parton model

- Systematic remove all the long-distance physics into PDFs

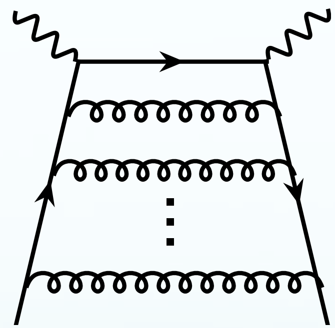


Scale-dependence of PDFs

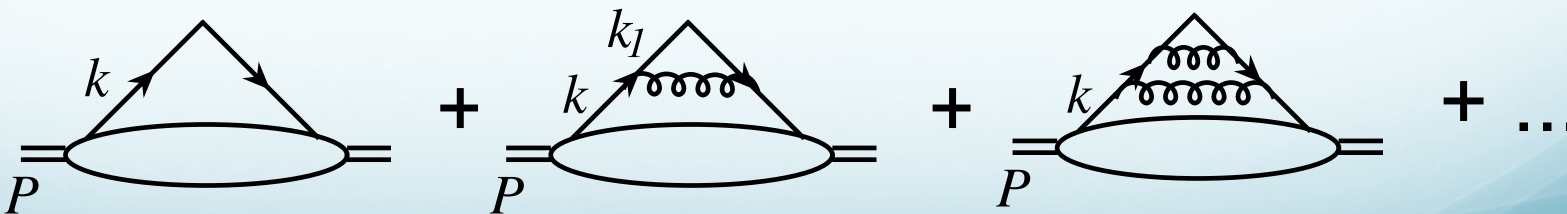
- Logarithmic contributions into parton distributions



- Going to even higher orders: QCD resummation of single logs



$\phi(x,\mu^2)$ =

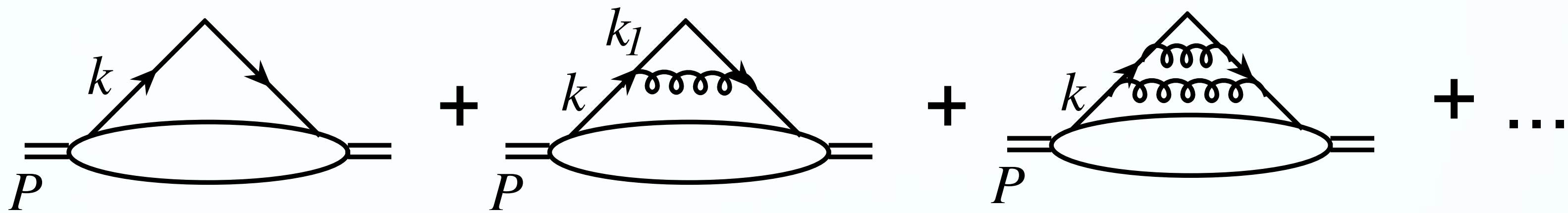


$$\alpha_s \ln \frac{\mu^2}{\Lambda^2} \qquad \left(\alpha_s \ln \frac{\mu^2}{\Lambda^2}\right)^2$$

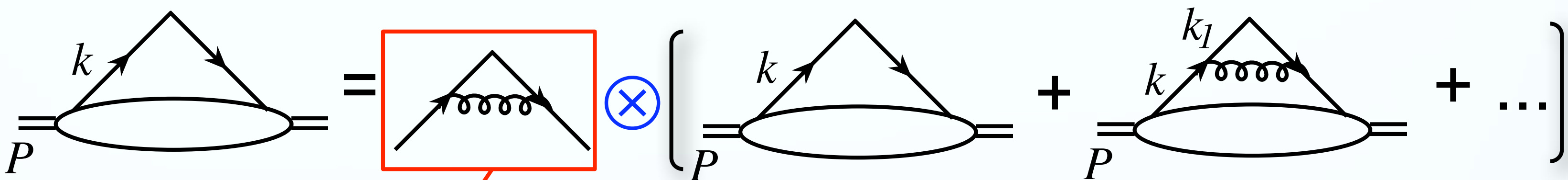
DGLAP evolution = resummation of single logs

- Evolution = Resum all the gluon radiation

$\phi(x,\mu^2) =$



$\phi(x,\mu^2)$ -



DGLAP Equation

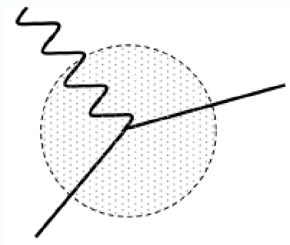
Evolution kernel
splitting function

$$\frac{\partial}{\partial \ln \mu^2}\phi_i(x,\mu^2) = \sum_j \boxed{P_{ij}\left(\frac{x}{x'}\right)} \otimes \phi_j(x',\mu^2)$$

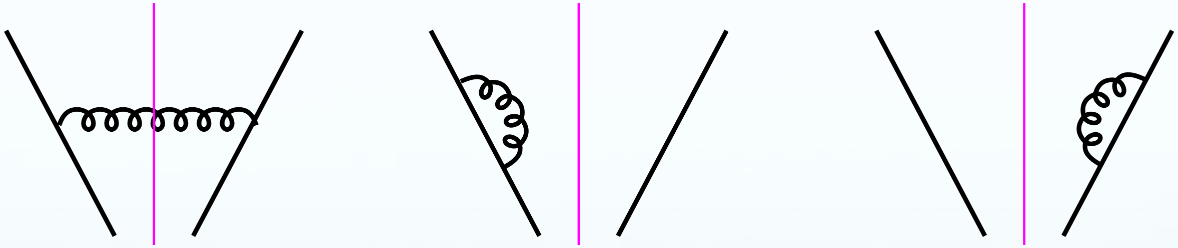
- By solving the evolution equation, one resums all the single logarithms of type $\left(\alpha_s \ln \frac{\mu^2}{\Lambda^2}\right)^n$

Evolutions of PDFs

- Perturbative change:



- Feynman diagrams for unpolarized PDFs



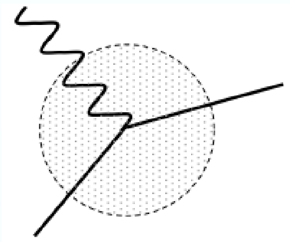
$$\frac{q(x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[P_{qq}(z)\, q(\xi,\mu_F)\right]$$

$$P_{qq}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$

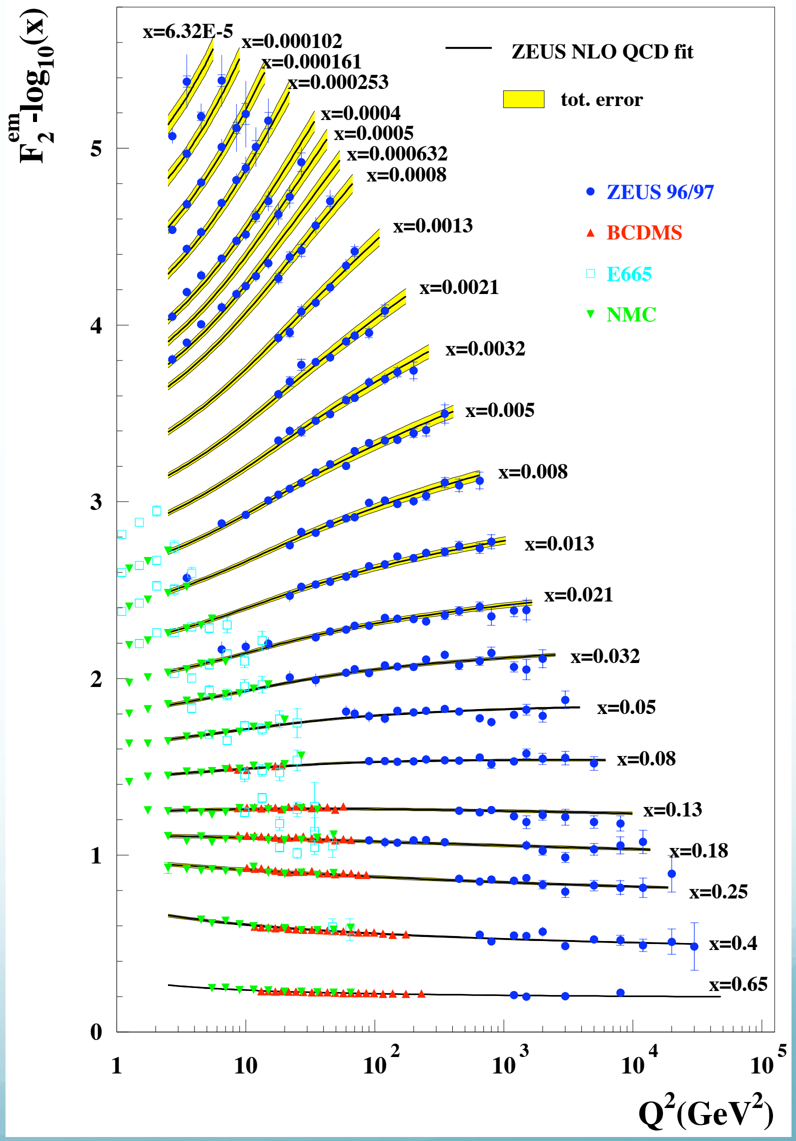
PDFs also depends on the scale of the probe

- Increase the energy scale, one sees parton picture differently

Q_0^2



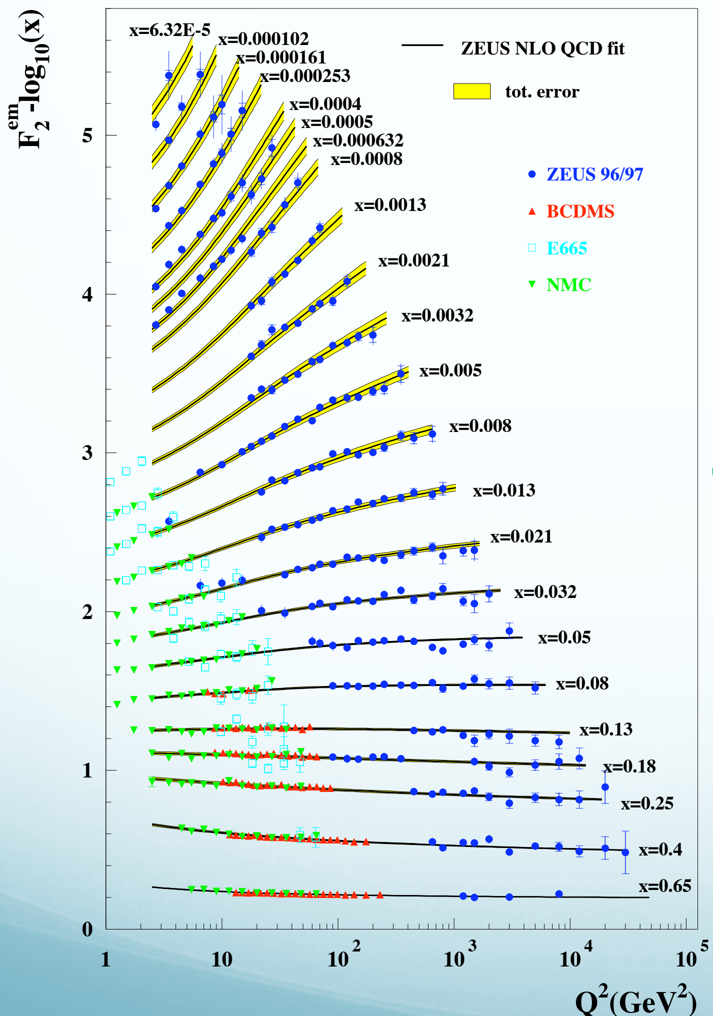
$Q^2 > Q_0^2$



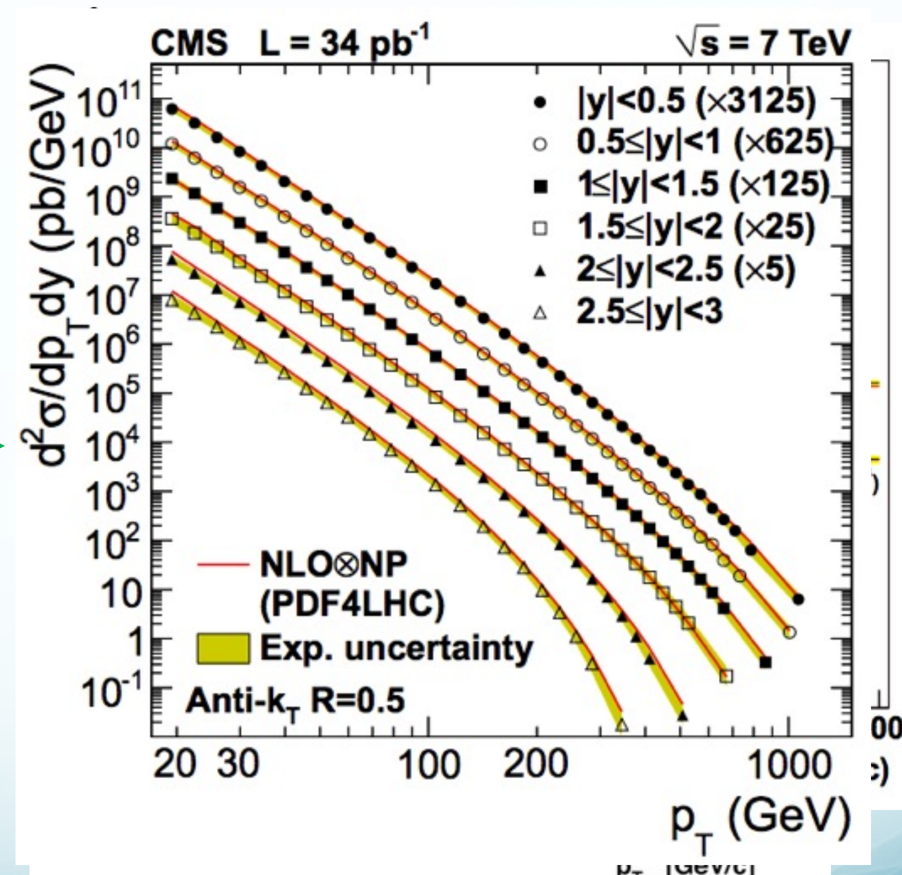
Success of QCD collinear factorization

- PDFs are universal and evolve via DGLAP equations

DIS: e+p

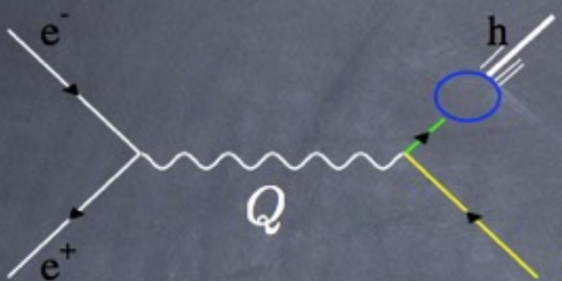


TevaRHICLHC p+p(bar) @ 200 GeV 7 TeV



Same idea for hadron production

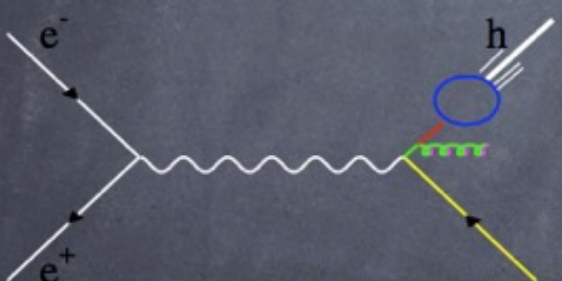
- Fragmentation function: another probability
  - Going to NLO, needs also to absorb the collinear divergence, and thus the scale dependence of the fragmentation function



$$z \equiv \frac{E_h}{E_q} = \frac{2E_h}{Q}$$

$$\frac{d\sigma}{dz}(e^+e^- \to h\,X) = \sum_q \sigma(e^+e^- \to q\bar{q})\,\left[D_q^h(z) + D_{\bar{q}}^h(z)\right]$$

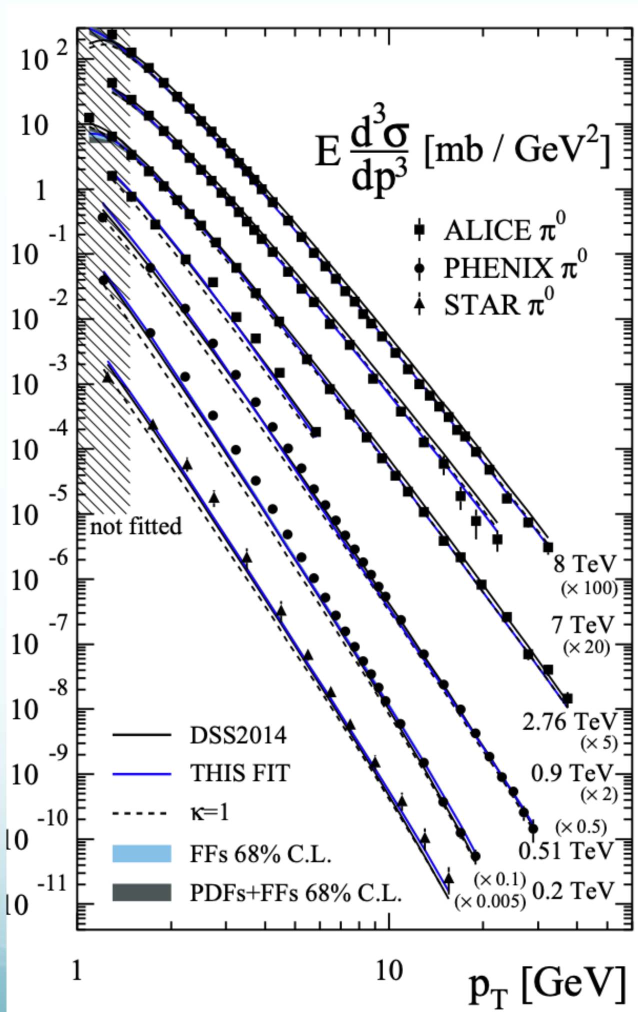
$$\sum_h \int_0^1 z\,D_q^h(z)\,dz = 1 \qquad \sum_q \int_0^1 \left[D_q^h(z) + D_{\bar{q}}^h(z)\right]\,dz = n_h$$

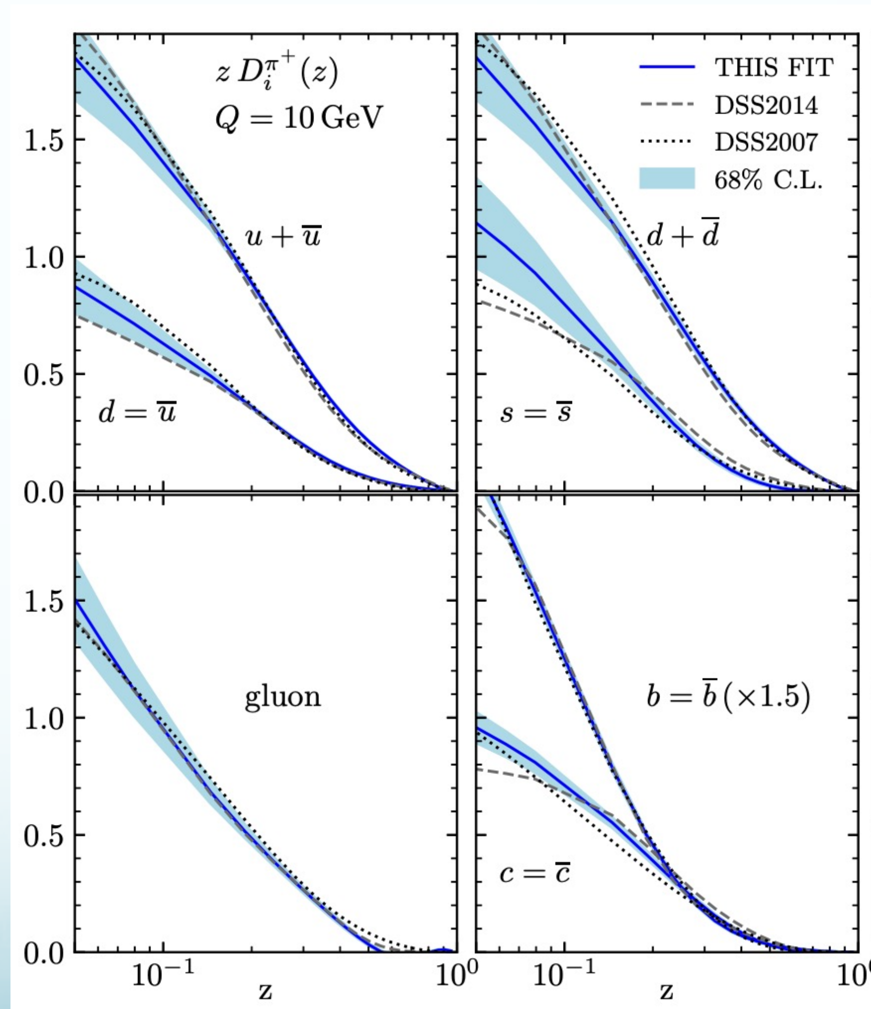


$$D_q^h(z) \longrightarrow D_q^h(z, Q^2)$$

It also works well

- DSS parameterizations for the fragmentation function





Summary

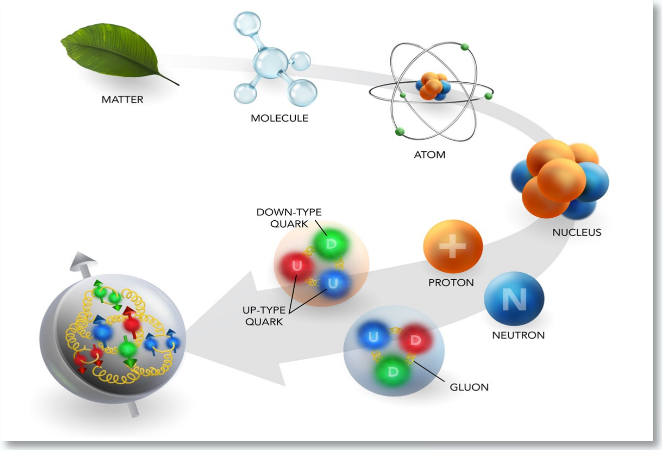
- Asymptotic freedom: allow one to calculate partonic cross sections
- Parton distribution functions and fragmentation functions
- Renormalization scale and factorization scale

Lecture 2: Nucleon structure

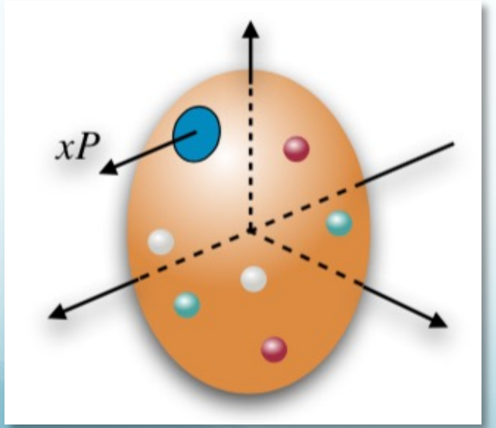
- TMD factorization, jets, SCET

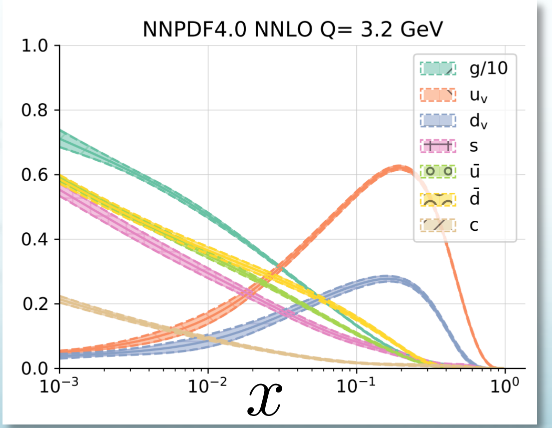
EIC era: Imaging of proton and nucleus

- Unraveling the mysteries of relativistic hadronic bound states



- Beyond 1D: collinear PDFs provide 1D structure – longitudinal motion

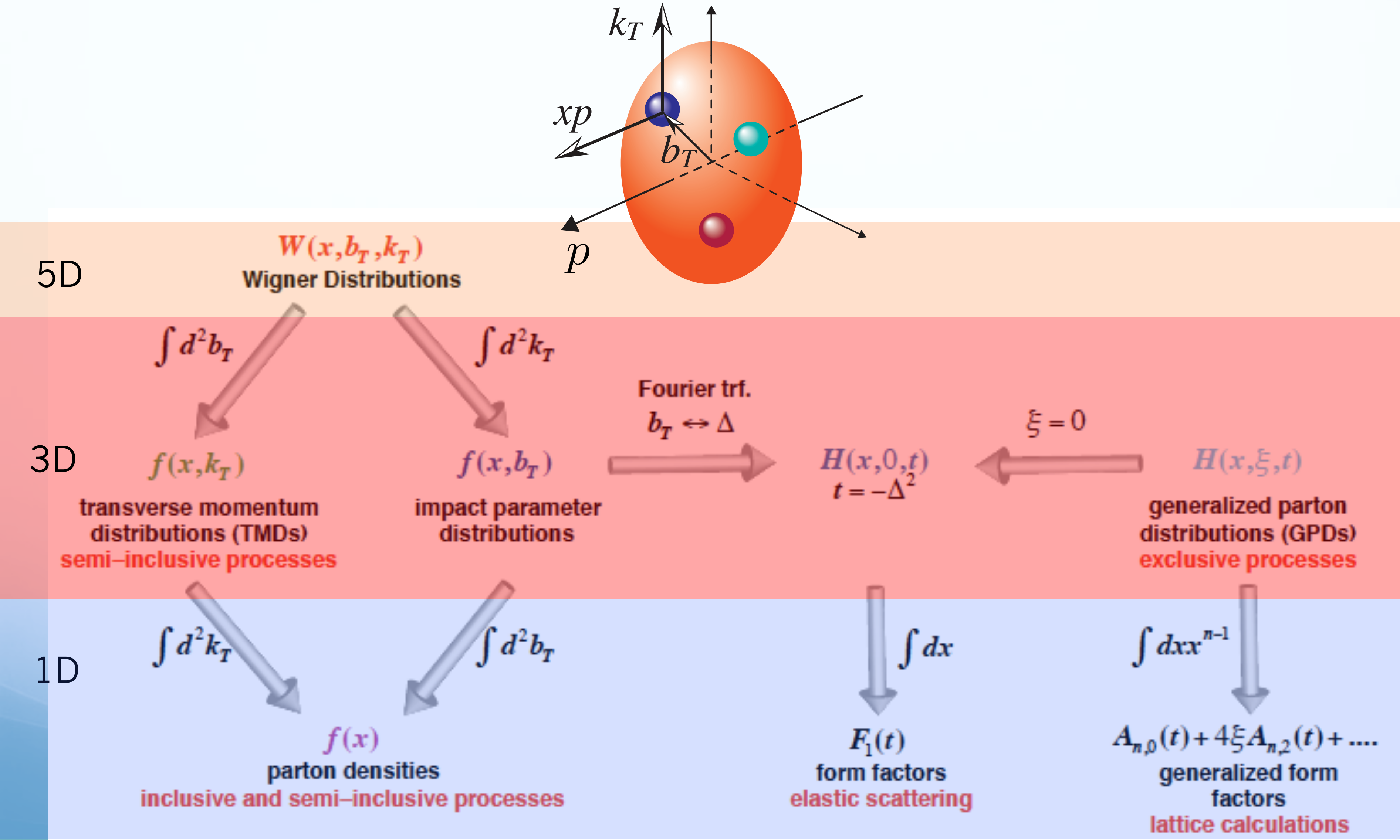




Proton Movie

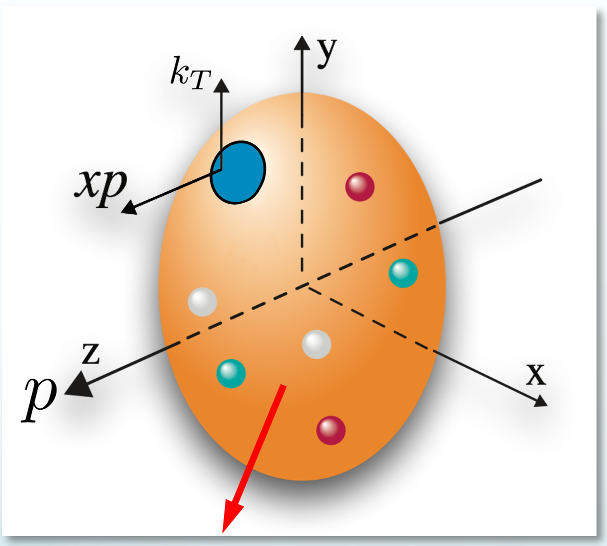
Unified view: internal landscape

- Wigner distributions: a quantum version of phase-space distribution

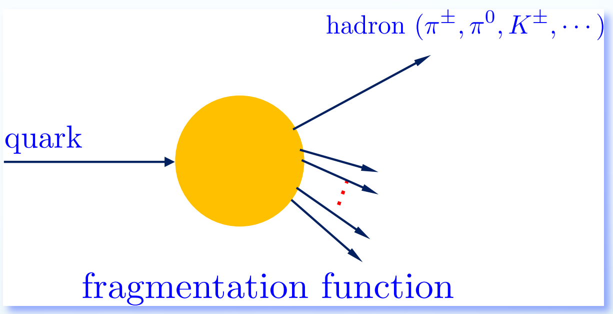


Transverse Momentum Dependent distributions (TMDs)

- 3D imaging in momentum space
  - Both longitudinal and transverse motion
  - What are the quantum correlations between the motion of the quarks/gluons, their spin and the spin of the proton? (TMD PDFs)
  - Similarly precision information on hadronization (TMD FFs)



Proton spin



TMDs with polarization

TMD PDFs

Leading Twist TMDs

Nucleon Spin — Quark Spin

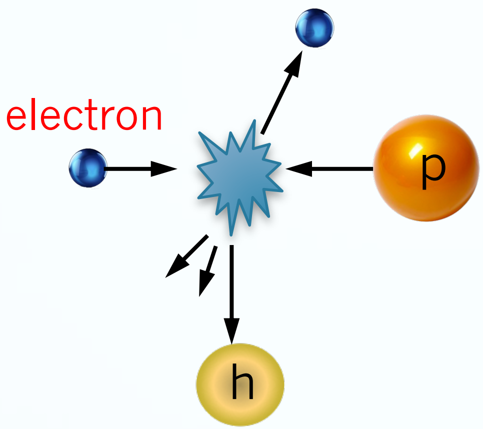
Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	f_1 =		$h_1^{\perp}$ = − Boer-Mulders
L		g_{1L} = − Helicity	$h_{1L}^{\perp}$ = −
T	$f_{1T}^{\perp}$ = − Sivers	g_{1T} = − Transversal Helicity	h_1 = − Transversity $h_{1T}^{\perp}$ = −

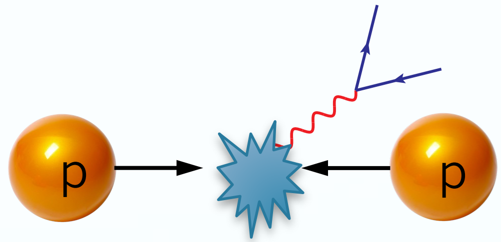
TMD FFs

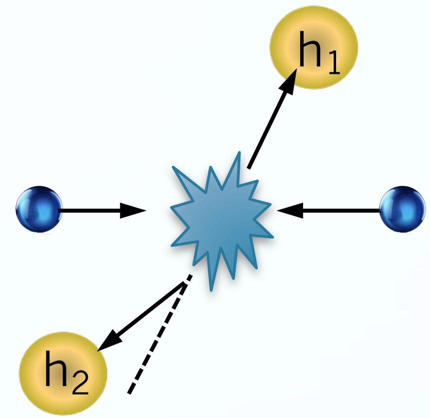
	Quark Polarization: U	L	T
Pion	D_1		$H_1^{\perp}$ **Collins**

Processes to extract TMDs

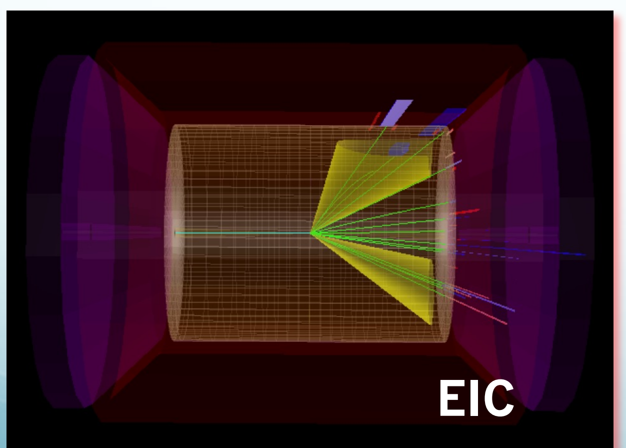
- Standard processes: SIDIS, Drell-Yan, dihadron in e^+e^-

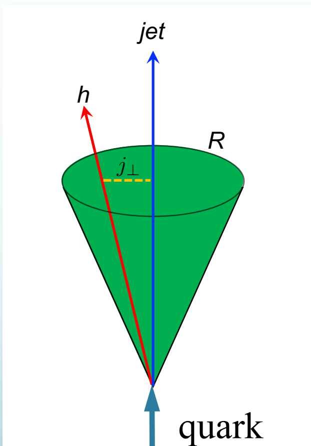


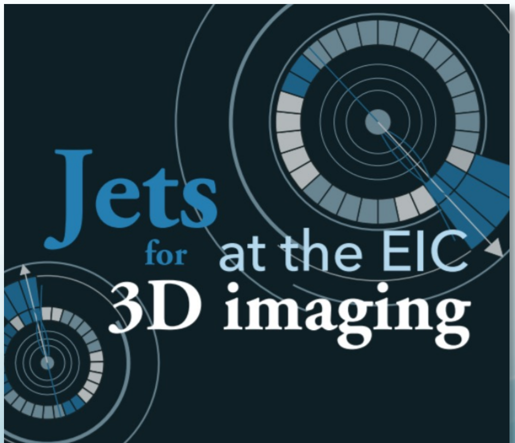




- New opportunities: jets and jet-like observables

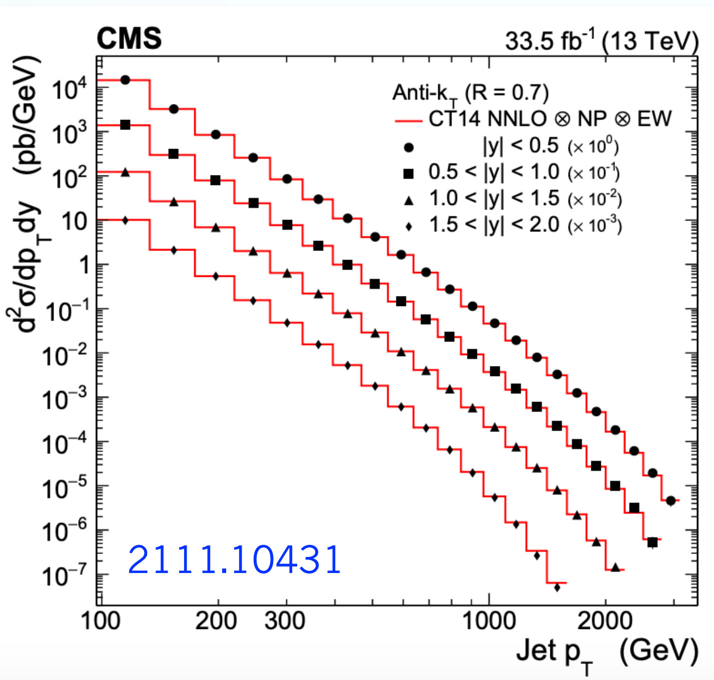


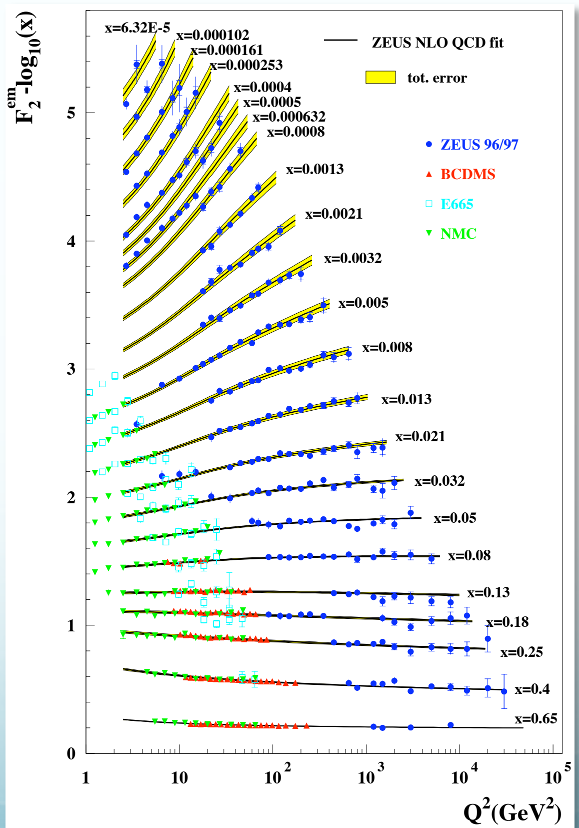


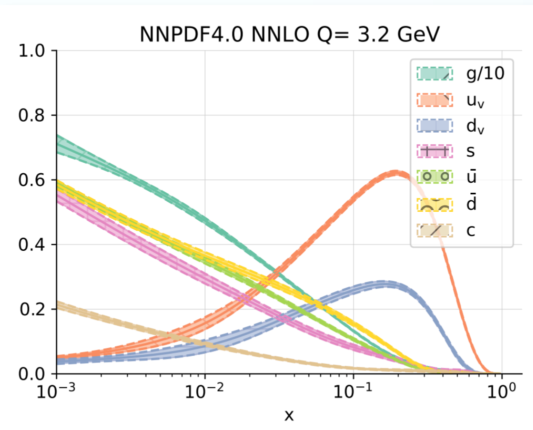


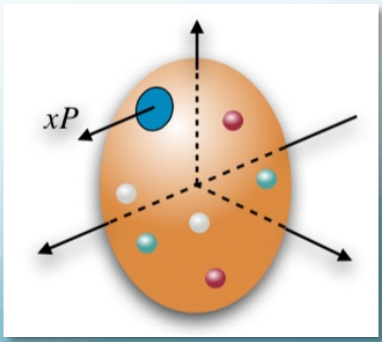
Two kinds of QCD factorization

- There are different types of QCD factorization, one should use them accordingly to include world data in different processes
  - **Collinear factorization**: process with ONE single hard scale
    - Single-inclusive hadron/jet in p+p collisions: pT is the hard scale
    - Inclusive deep inelastic scattering (DIS): Q is the hard scale



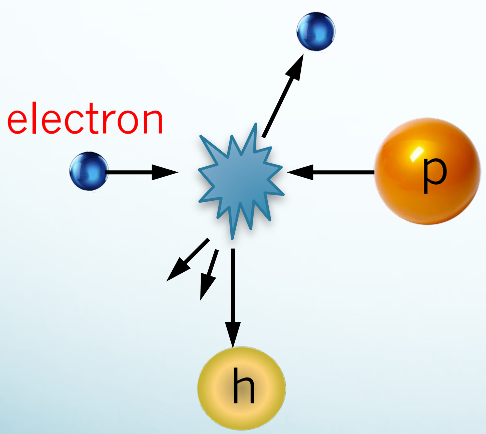


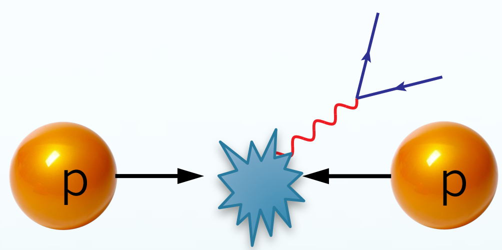


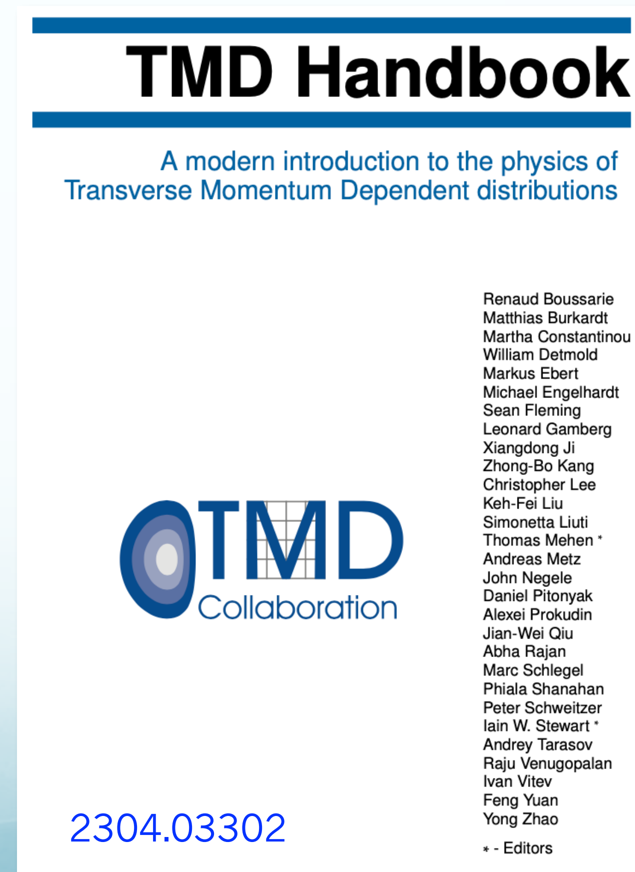


Another type: TMD factorization

- Transverse-momentum dependent (TMD) factorization, used for more differential processes
  - **TMD factorization**: process with TWO momentum scales (Q, qT) with qT << Q
    - Semi-inclusive hadron production in e+p collisions (SIDIS): qT, Q
    - Drell-Yan production in p+p collisions: qT, Q






TMD Handbook

A modern introduction to the physics of
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Alexei Prokudin
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Marc Schlegel
Phiala Shanahan
Peter Schweitzer
Iain W. Stewart *
Andrey Tarasov
Raju Venugopalan
Ivan Vitev
Feng Yuan
Yong Zhao

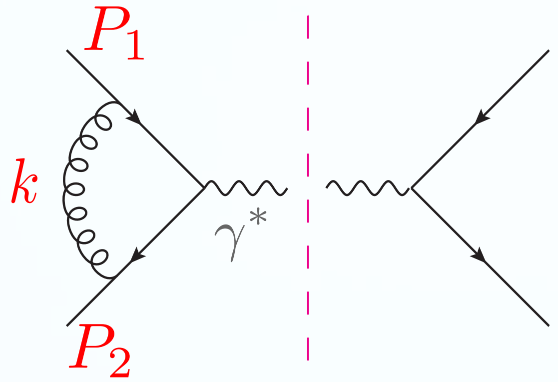
* - Editors

TMD Collaboration

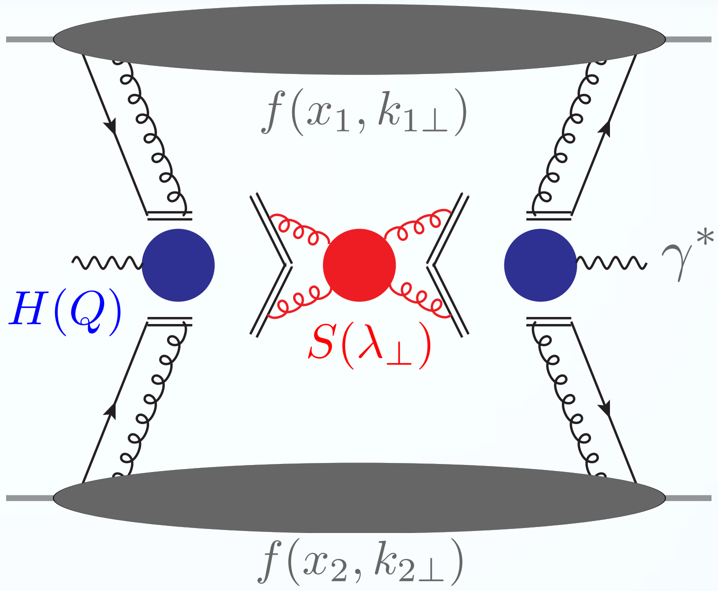
2304.03302

TMD factorization in a nut-shell

- Drell-Yan: $p + p \to [\gamma^* \to \ell^+ \ell^-] + X$



Factorization of regions:
(1) k//P$_1$, (2) k//P$_2$, (3) k soft, (4) k hard



- Factorized form and mimic "parton model"

$$\frac{d\sigma}{dQ^2 dy d^2 q_\perp} \propto \int d^2 k_{1\perp}\, d^2 k_{2\perp}\, d^2 \lambda_\perp H(Q) f(x_1, k_{1\perp}) f(x_2, k_{2\perp}) S(\lambda_\perp) \delta^2(k_{1\perp} + k_{2\perp} + \lambda_\perp - q_\perp)$$

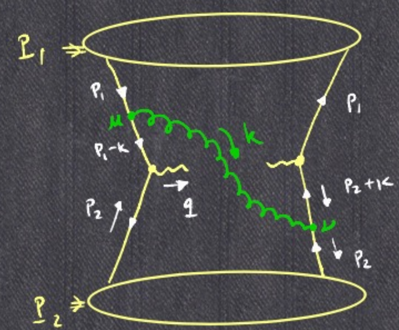
$$= \int \frac{d^2 b}{(2\pi)^2} e^{i q_\perp \cdot b} H(Q) f(x_1, b) f(x_2, b) S(b)$$

$$F(x, b) = f(x, b) \sqrt{S(b)}$$

$$\boxed{= \int \frac{d^2 b}{(2\pi)^2} e^{i q_\perp \cdot b} H(Q) F(x_1, b) F(x_2, b)}$$

mimic "parton model"

Illustration I



$p_1^+ = \frac{1}{\sqrt{2}}(p_1^0 + p_1^z)$ large

$p_2^- = \frac{1}{\sqrt{2}}(p_2^0 - p_2^z)$ large

$p_1^+ \sim p_2^-$

$p_1 = (p_1^+, p_1^-, p_{1\perp}) \sim p_1^+(1, \lambda^2, \lambda)$ $\qquad \lambda \sim \frac{p_{1\perp}}{p_1^+}$

$p_2 = (p_2^+, p_2^-, p_{2\perp}) \sim p_2^-(\lambda^2, 1, \lambda)$

$\bar{n}^\mu = (\bar{n}^+, \bar{n}^-, \bar{n}_\perp) = (1, 0, 0)$

$n^\mu = (n^+, n^-, n_\perp) = (0, 1, 0)$

$$p_1^\mu = p_1^+ \bar{n}^\mu + p_1^- n^\mu + p_{1\perp}^\mu$$

$$p_2^\mu = p_2^+ \bar{n}^\mu + p_2^- n^\mu + p_{2\perp}^\mu$$

if k is soft $\quad k = (k^+, k^-, k_\perp) \sim p^+(\lambda, \lambda, \lambda)$

$$I = \frac{i(\not{p}_1 - \not{k})}{(p_1 - k)^2 + i\epsilon} (-ig\gamma^\mu)\, \bar{\psi}(p_1)$$

$$\begin{aligned}
\not{p}_1 = p_1^\mu \gamma_\mu &= [p_1^+ \bar{n}^\mu + p_1^- n^\mu + p_{1\perp}^\mu]\gamma_\mu \\
&= \underset{O(1)}{p_1^+ \not{\bar{n}}} + \underset{O(\lambda^2)}{p_1^- \not{n}} + \underset{O(\lambda)}{\not{p}_{1\perp}} \\
&\approx p_1^+ \not{\bar{n}}
\end{aligned}$$

$$\not{k} = k^\mu \gamma_\mu = O(\lambda)$$

$$(p_1 - k)^2 = \underset{0}{p_1^2} + \underset{0}{k^2} - 2p_1 \cdot k = -2\overset{1}{p_1^+}\overset{\lambda}{k^-} - 2\overset{\lambda^2}{p_1^-}\overset{\lambda}{k^+} + 2\overset{\lambda}{\vec{p}_{1\perp}} \cdot \overset{\lambda}{\vec{k}_\perp}$$

$$\approx -2p_1^+ k^-$$

Illustration II

$$I = \frac{g\, p_1^+ \bar{\not n}\, \gamma^\mu}{-2p_1^+ k^- + i\epsilon}\, \bar\psi(p_1)$$

$$\Downarrow \quad \bar{\not n}\,\gamma^\mu = 2\bar n^\mu - \gamma^\mu \bar{\not n}$$

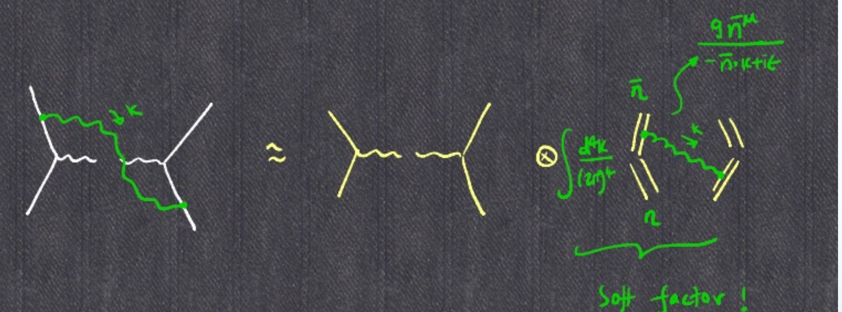
$$= \frac{g(2\bar n^\mu - \gamma^\mu \bar{\not n})}{-2k^- + i\epsilon}\, \bar\psi(p_1)$$

$$\Downarrow \quad p_1^+ \bar{\not n}\, \bar\psi(p_1) \to 0$$

[recall for now $\not p_1 \bar u(p_1) = 0$]

$$= g\, \frac{\bar n^\mu}{-k^- + i\epsilon}\, \bar\psi(p_1)$$

$$= g\, \frac{\bar n^\mu}{-\bar n\cdot k + i\epsilon}\, \bar\psi(p_1) \qquad \bar n \cdot k = k^-$$



Similarly for collinear contribution

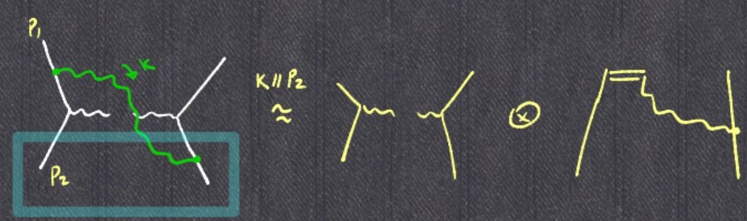


Illustration III

- Please check this hand-writing note
- It tells how you know how many independent correlator you could have and how you define them

TMD evolves

- Just like collinear PDFs, TMDs also depend on the scale of the probe = evolution

Collinear PDFs

$F(x, Q)$

✓ DGLAP evolution

✓ Resum $\left[\alpha_s \ln(Q^2/\mu^2)\right]^n$

✓ Kernel: purely perturbative

TMDs

$F(x, k_\perp; Q)$

✓ Collins-Soper/rapidity evolution equation

✓ Resum $\left[\alpha_s \ln^2(Q^2/k_\perp^2)\right]^n$

✓ Kernel: can be non-perturbative when $k_\perp \sim \Lambda_{\rm QCD}$

$$F(x, Q_i) \downarrow R^{\rm coll}(x, Q_i, Q_f) \downarrow F(x, Q_f)$$

$$F(x, k_\perp, Q_i) \downarrow R^{\rm TMD}(x, k_\perp, Q_i, Q_f) \downarrow F(x, k_\perp, Q_f)$$

TMD evolution in a nutshell

$$F(x, k_\perp; Q) = \frac{1}{(2\pi)^2} \int d^2 b e^{ik_\perp \cdot b} F(x, b; Q) = \frac{1}{2\pi} \int_0^\infty db\, b J_0(k_\perp b) F(x, b; Q)$$

$$F(x, b; Q) \approx C \otimes F(x, c/b^*) \times \exp\left\{ -\int_{c/b^*}^{Q} \frac{d\mu}{\mu} \left(A \ln \frac{Q^2}{\mu^2} + B \right) \right\} \times \exp\left(-S_{\text{non-pert}}(b, Q) \right)$$

longitudinal/collinear part

transverse part

- ✓ Non-perturbative: fitted from data
- ✓ The key ingredient – ln(Q) piece is spin-independent

The presence of non-perturbative evolution kernel makes TMD global analysis much more involved

TMD Handbook

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arXiv:2304.03302

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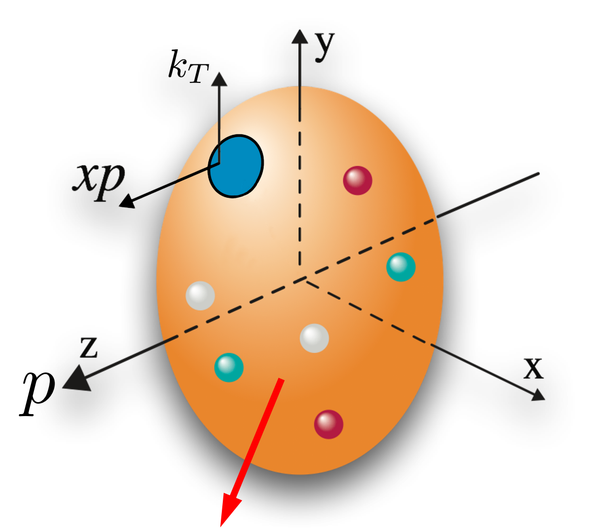


Proton spin

TMD evolution

- TMD evolution
  - Usual renormalization scale + a rapidity scale

$$F(x,b,\mu,\zeta) = \exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_\mu(\mu',\zeta_0)\right] \exp\left[\frac{1}{2}\gamma_\zeta(\mu,b)\ln\left(\frac{\zeta}{\zeta_0}\right)\right] F(x,b,\mu_0,\zeta_0)$$

Collins-Soper kernel

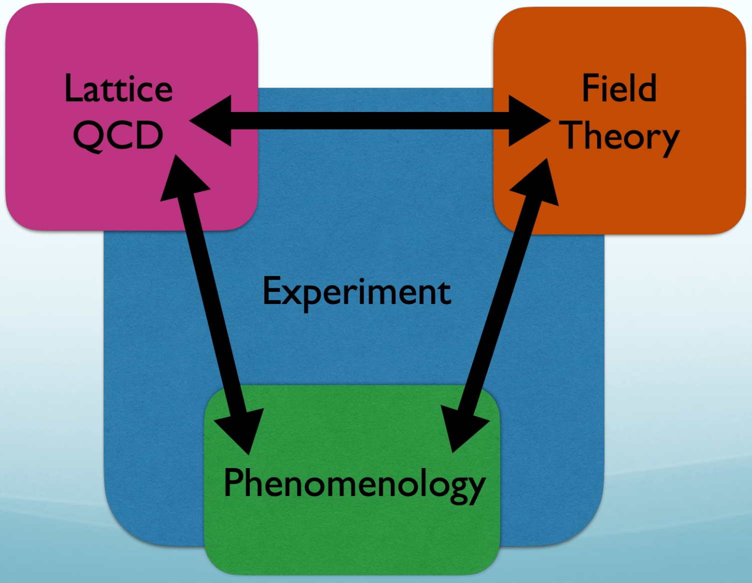
$$\mu = \sqrt{\zeta} = Q$$

$$\mu_0 = \sqrt{\zeta_0} = Q_0$$

Sum large logarithms: $\ln(Q^2 b_T^2) \sim \ln \frac{Q^2}{q_T^2}$

Perturbative γ_i^q : Leading Log (LL) → Next-to-leading log (NLL) → NNLL → N^{3}LL → N^{4}LL

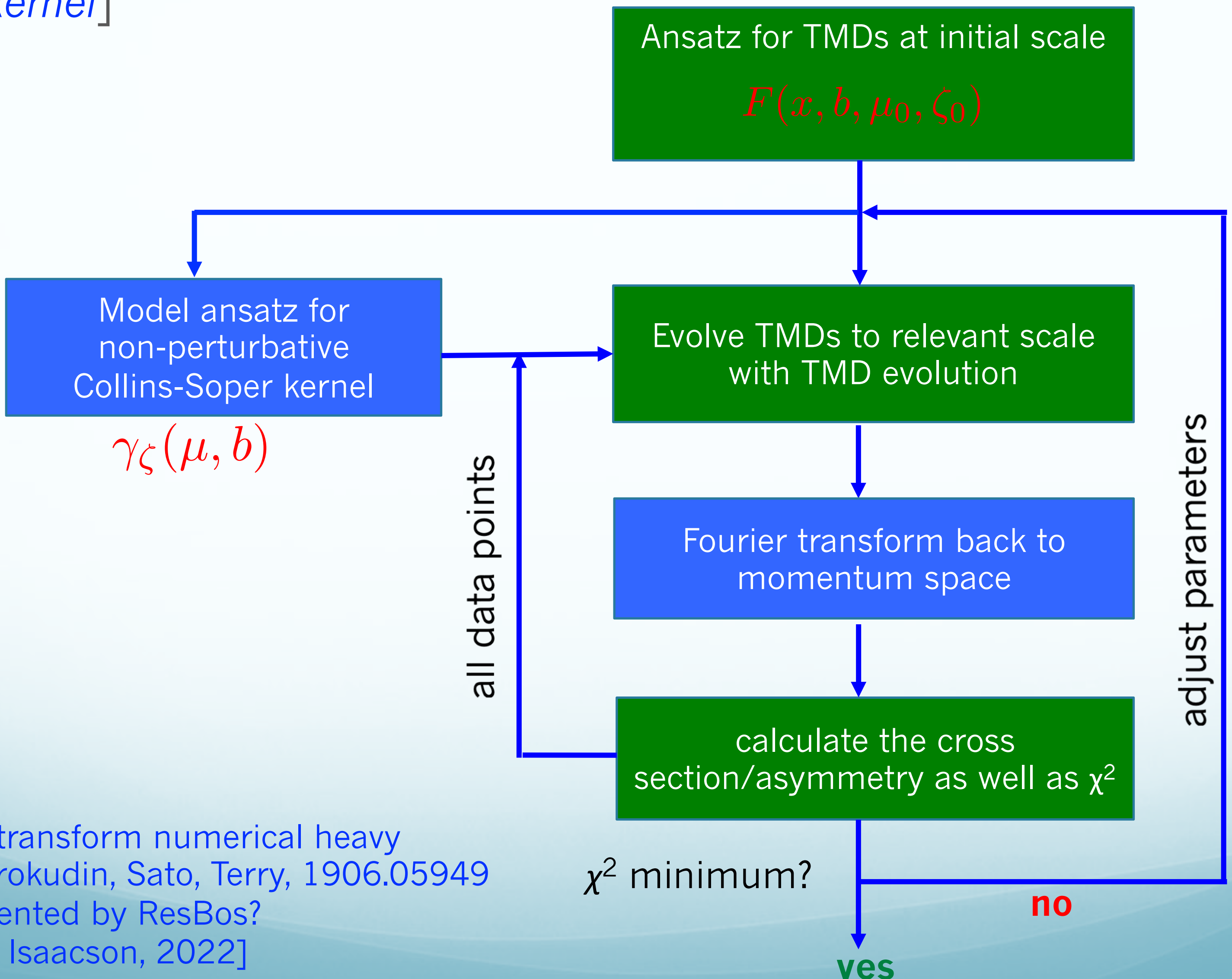
Nonperturbative γ_ζ^q : fit to data using models, or calculate with Lattice QCD



Courtesy of I. Stewart

TMD global analysis

- Outline of a TMD global analysis [*lattice input on initial TMD and CS kernel*]



Fourier transform numerical heavy
Kang, Prokudin, Sato, Terry, 1906.05949
Implemented by ResBos?
[Joshua Isaacson, 2022]

Unpolarized cross sections

- Good fit achieved in the “TMD region” (N^{3}LL)

$$q_T/Q \lesssim 0.25$$

Non-perturbative structure of semi-inclusive deep-inelastic and Drell-Yan scattering at small transverse momentum

Ignazio Scimemi[a] and Alexey Vladimirov[b]

JHEP (2020)

Unpolarized Transverse Momentum Distributions from a global fit of Drell-Yan and Semi-Inclusive Deep-Inelastic Scattering data

The **MAP** Collaboration*

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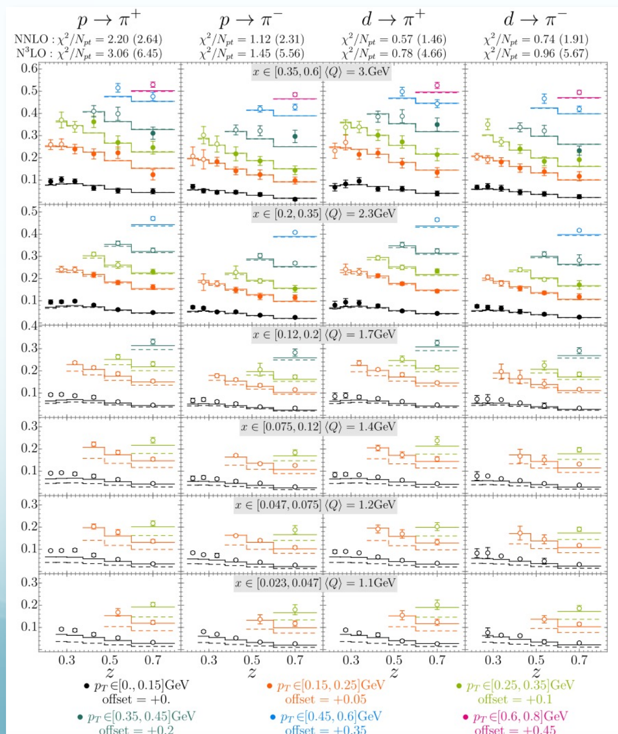
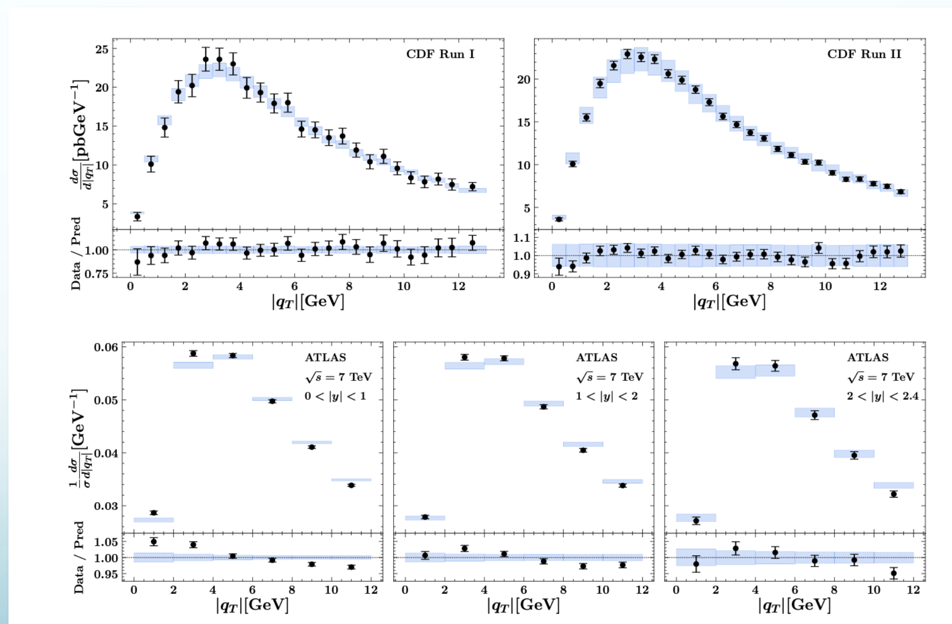
2206.07598

SIDIS

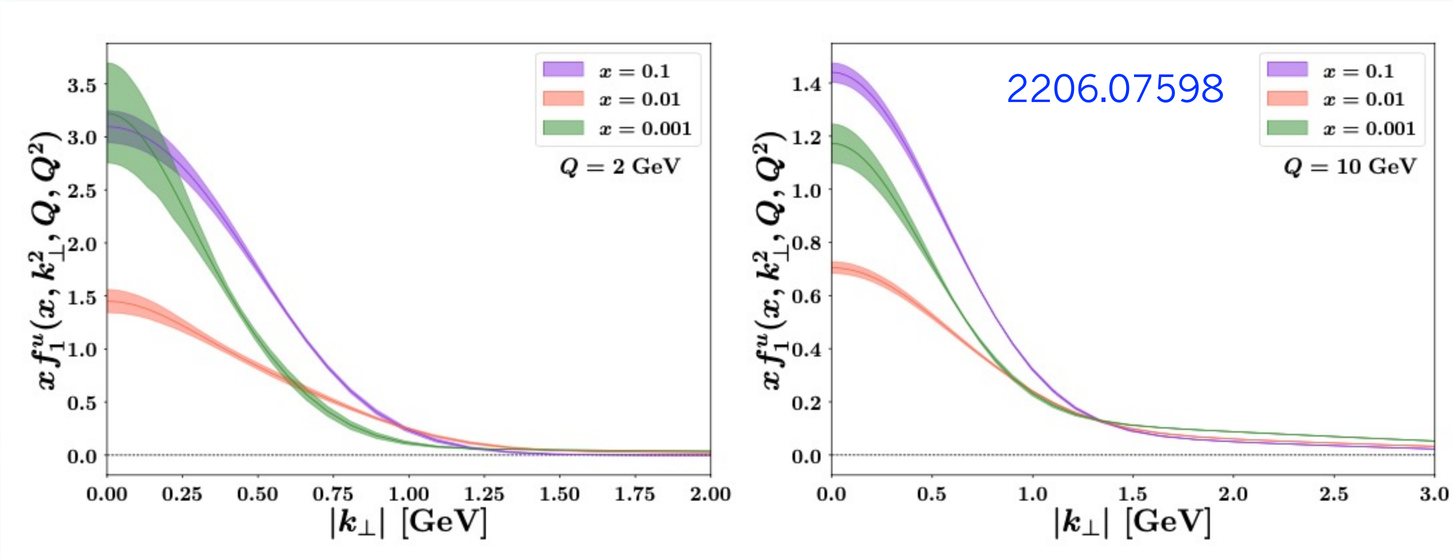


Drell-Yan



Unpolarized TMDs

- Extracted unpolarized TMD PDFs and FFs



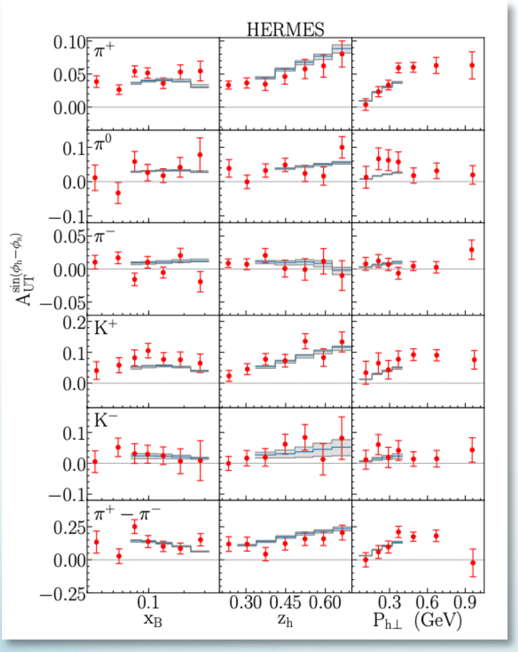
- Useful tools 2103.09741

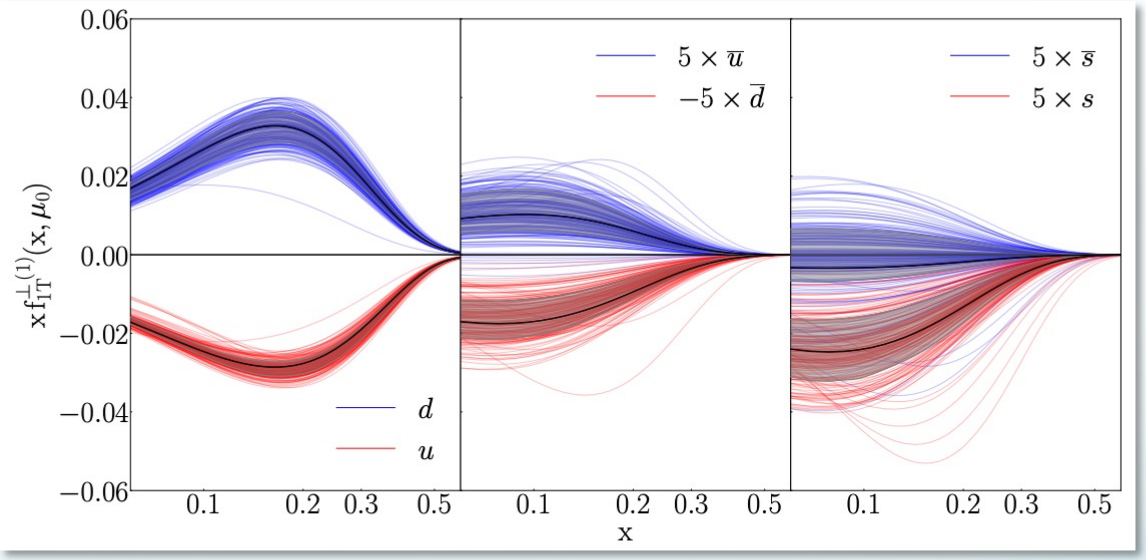
TMDlib2 and TMDplotter:
a platform for 3D hadron structure studies

Progress from standard processes

- Take an example: Sivers function
  - Data: Jlab 12, HERMES, COMPASS, RHIC W boson

$$f_{q/P^{\uparrow}}(x,\mathbf{k}_{\perp},\vec{S}) \equiv f_{q/P}(x,k_{\perp}) - \frac{1}{M} f_{1T}^{\perp q}(x,k_{\perp})\, \vec{S}\cdot(\hat{p}\times\mathbf{k}_{\perp})$$

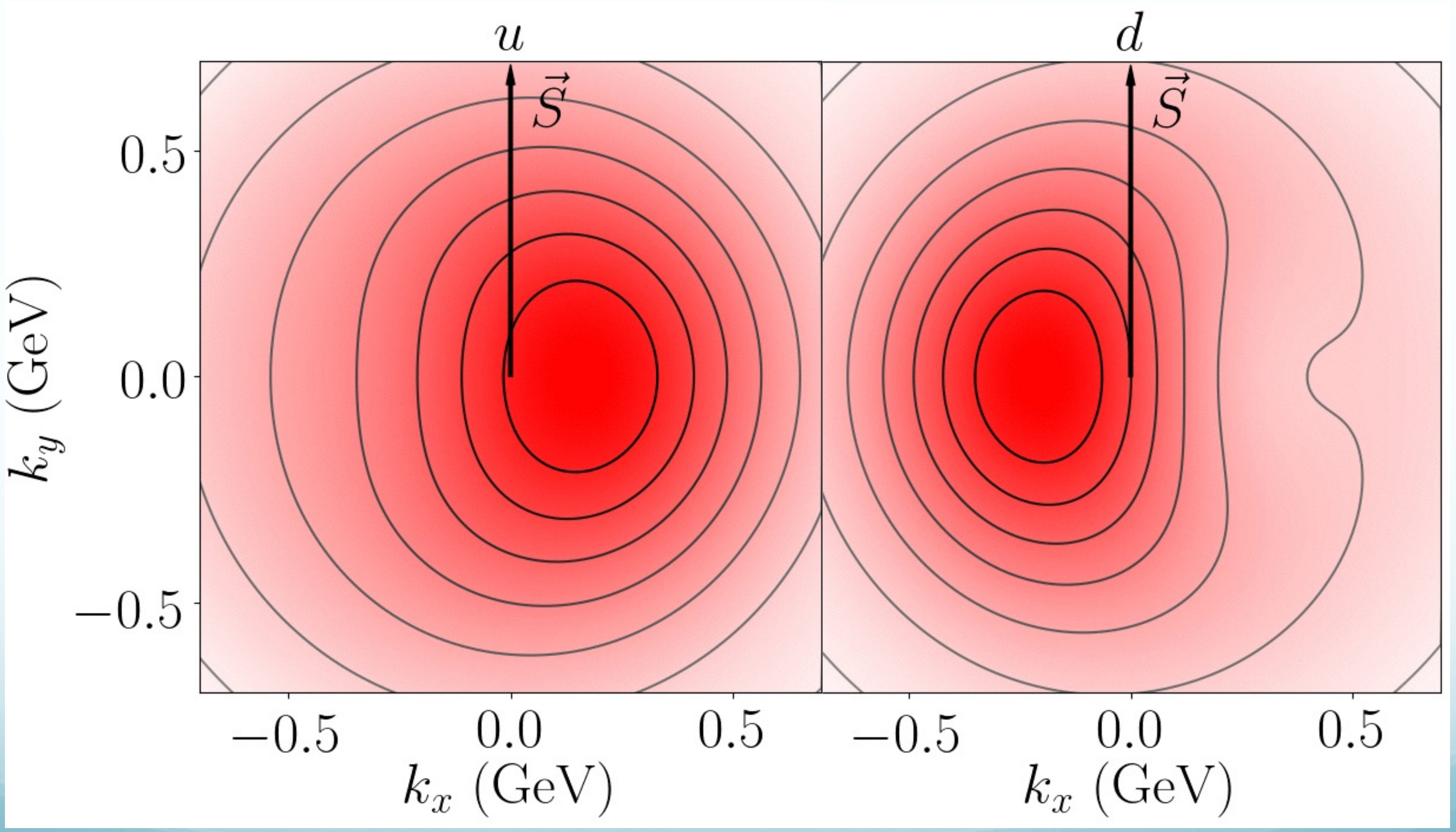




Echevarria, Kang, Terry, JHEP (2021)

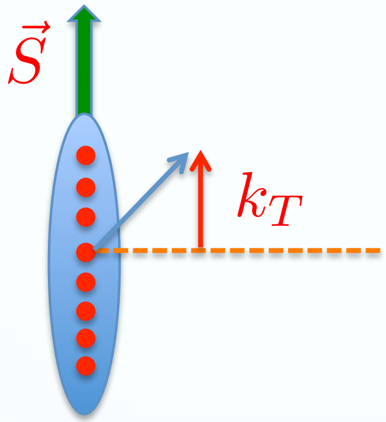
Distorted distribution

- Sivers correlation leads to a distortion in u/d quark distribution
  - Left or right shift



Fundamental feature of Sivers function

- Sivers function: check its definition

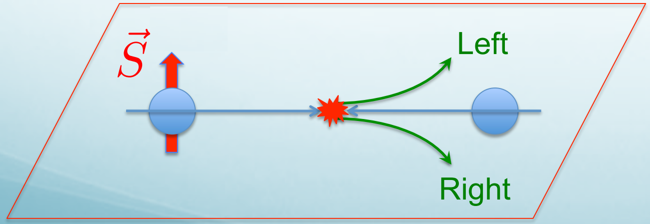


$$\vec{S} \cdot (\hat{p} \times \mathbf{k}_T)$$

Sivers function

Sivers effect

left-right asymmetry



Look closer

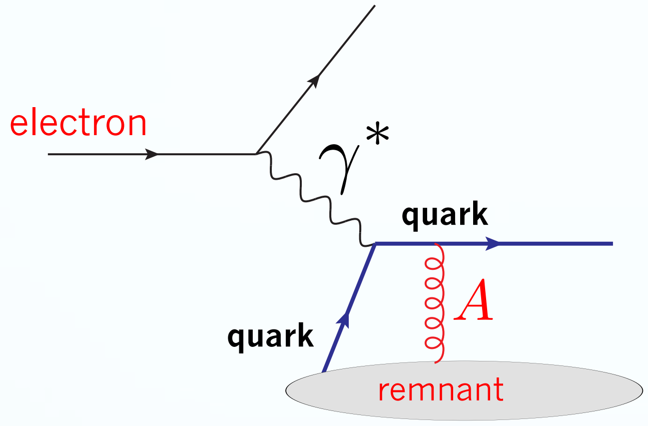
✓ **Naïve time-reversal-odd**: recall momentum and spin change sign under T

✓ **Forbidden?**: such a correlation is forbidden in QCD, unless there is a phase

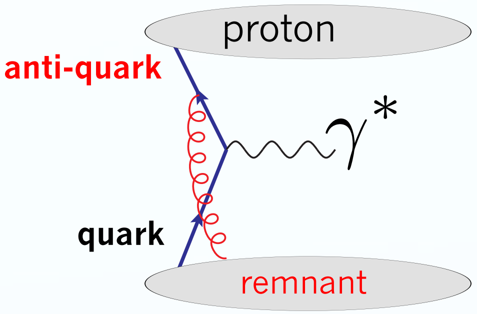
Recall: $\hat{T}\psi(t) \rightarrow \psi^*(-t)$

Quantum mechanical phase

- Quark passes through a color gauge field, generated by the remnant of the proton, it will accumulate a phase



DIS: after the interaction
final state



Drell-Yan: before the interaction
initial state

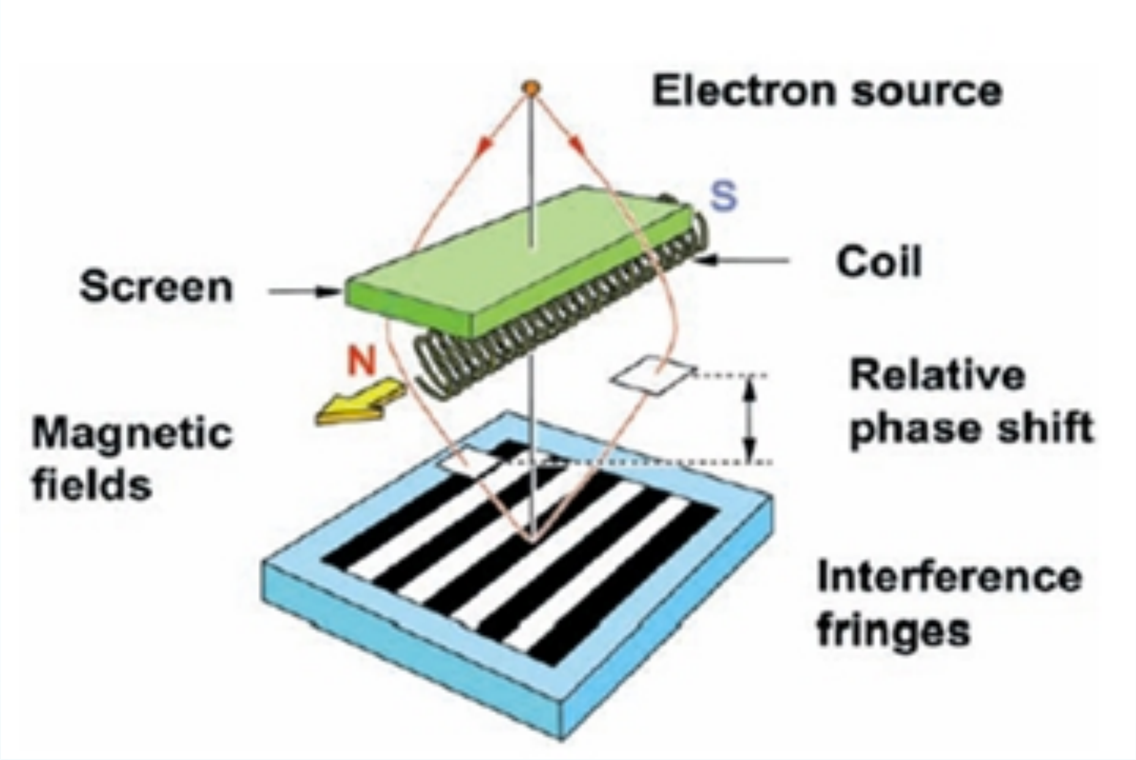
$$e^{i\phi} \qquad \phi = g_s \int_{\text{path}} dr \cdot A$$

$$\text{Sivers function}|_{\text{DIS}} = - \text{Sivers function}|_{\text{DY}}$$

Collins 02, Boer-Mulders-Pijlman 03, Collins-Metz 04, Kang, Qiu, PRL 09, ...

Sivers effect: QCD version of Aharonov-Bohm effect

- Pure quantum effect: different paths lead to interference

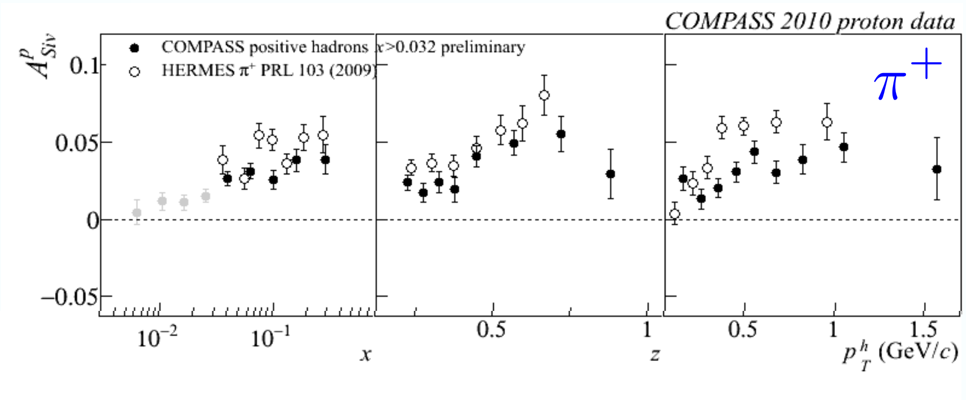


- Physics today, September 2009

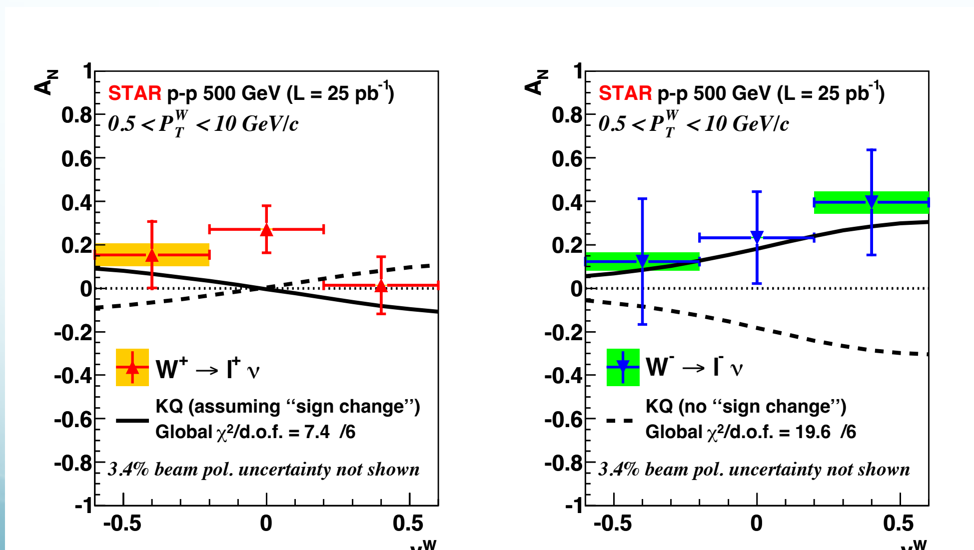
$$\Psi = \Psi_1\, e^{i\phi_1} + \Psi_2\, e^{i\phi_2} \qquad \phi_i = e\int_{\text{path i}} d\vec{r}\cdot\vec{A}$$

Sivers asymmetry from SIDIS and W

- Sivers asymmetry has been measured in DIS process

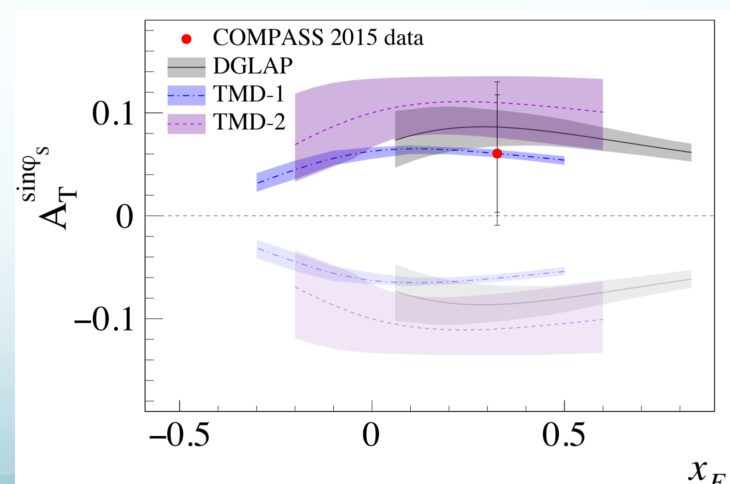


- Predictions comparison with DY/W



KQ = **Kang**, Qiu

Echevarria, **Kang**, et.al., 2014
COMPASS, PRL, 2017

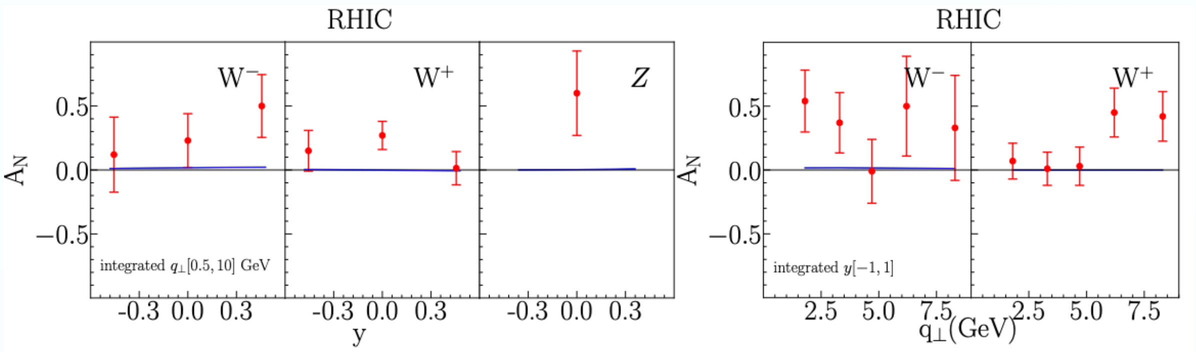


STAR, PRL, 2016

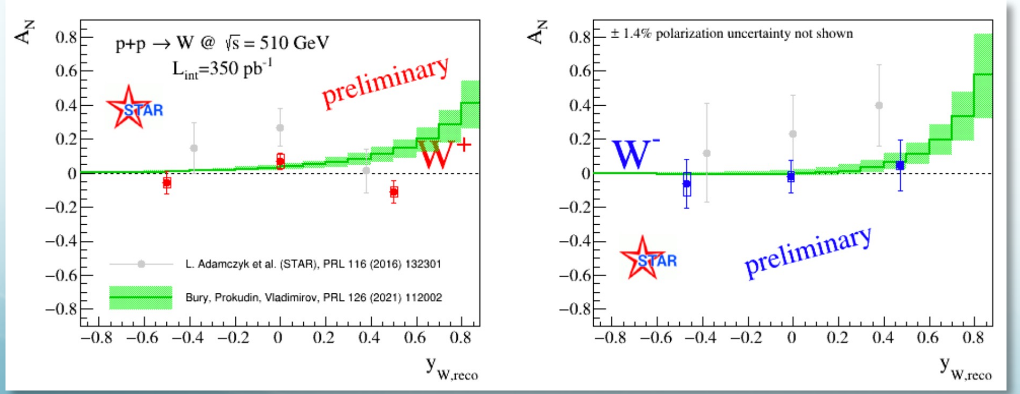
RHIC data: update

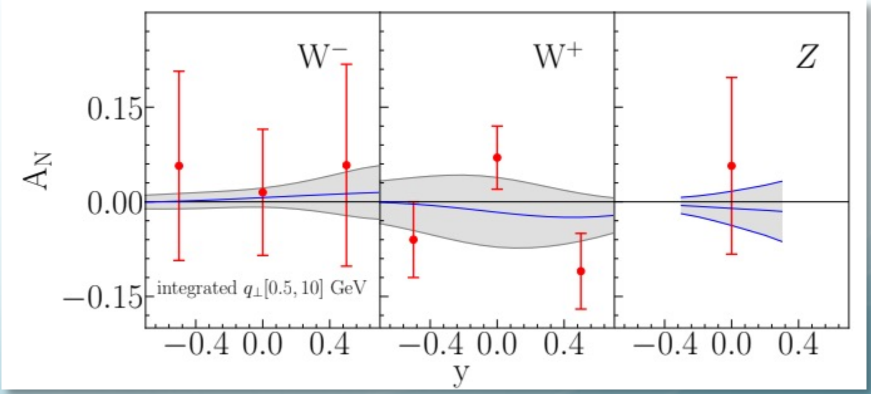
- With TMD evolution, tension in fitting (2016) RHIC W/Z data (very large asymmetry), but consistent with 2021 preliminary data
  - Emphasizing the importance of the precision measurement

OLD data



NEW data



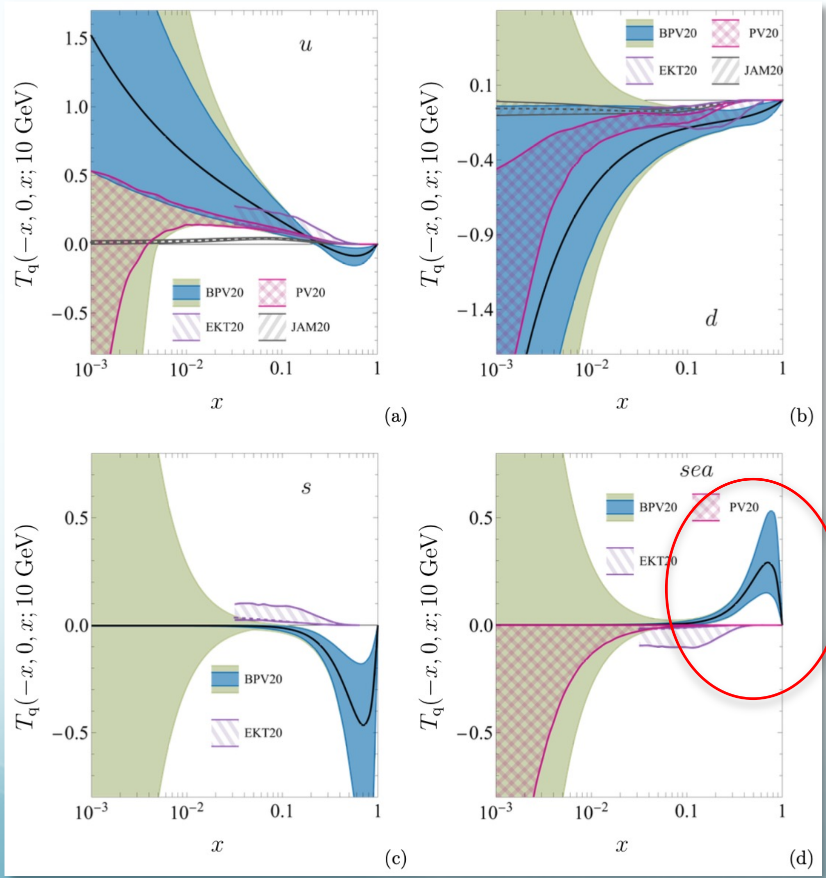


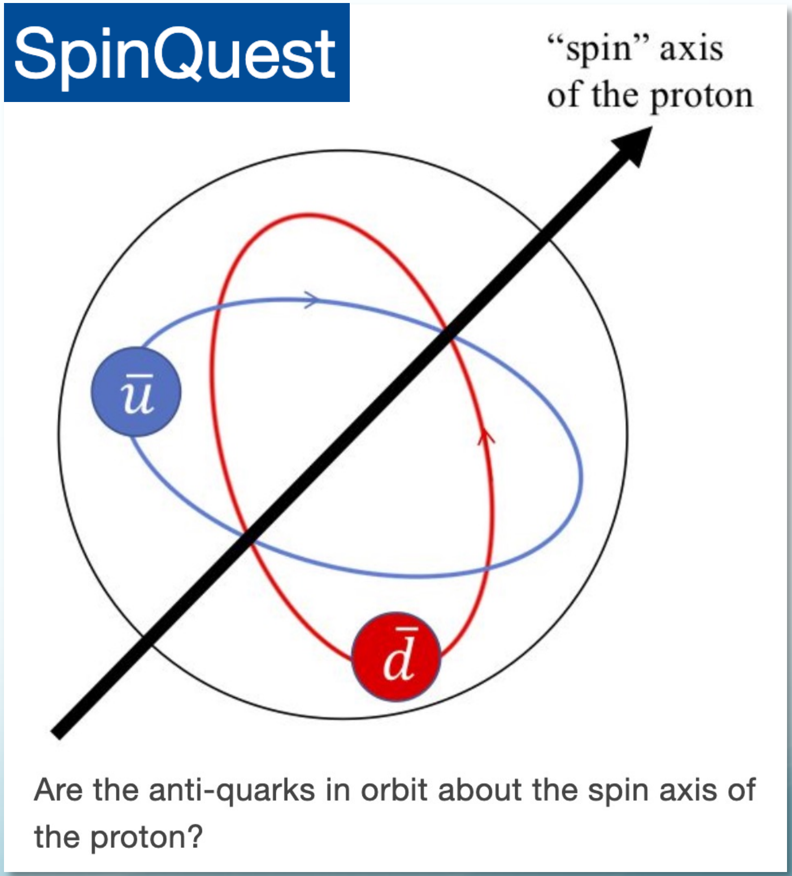
Bury, Vladimirov, Prokudin

Echevarria, Kang, Terry

RHIC data: update

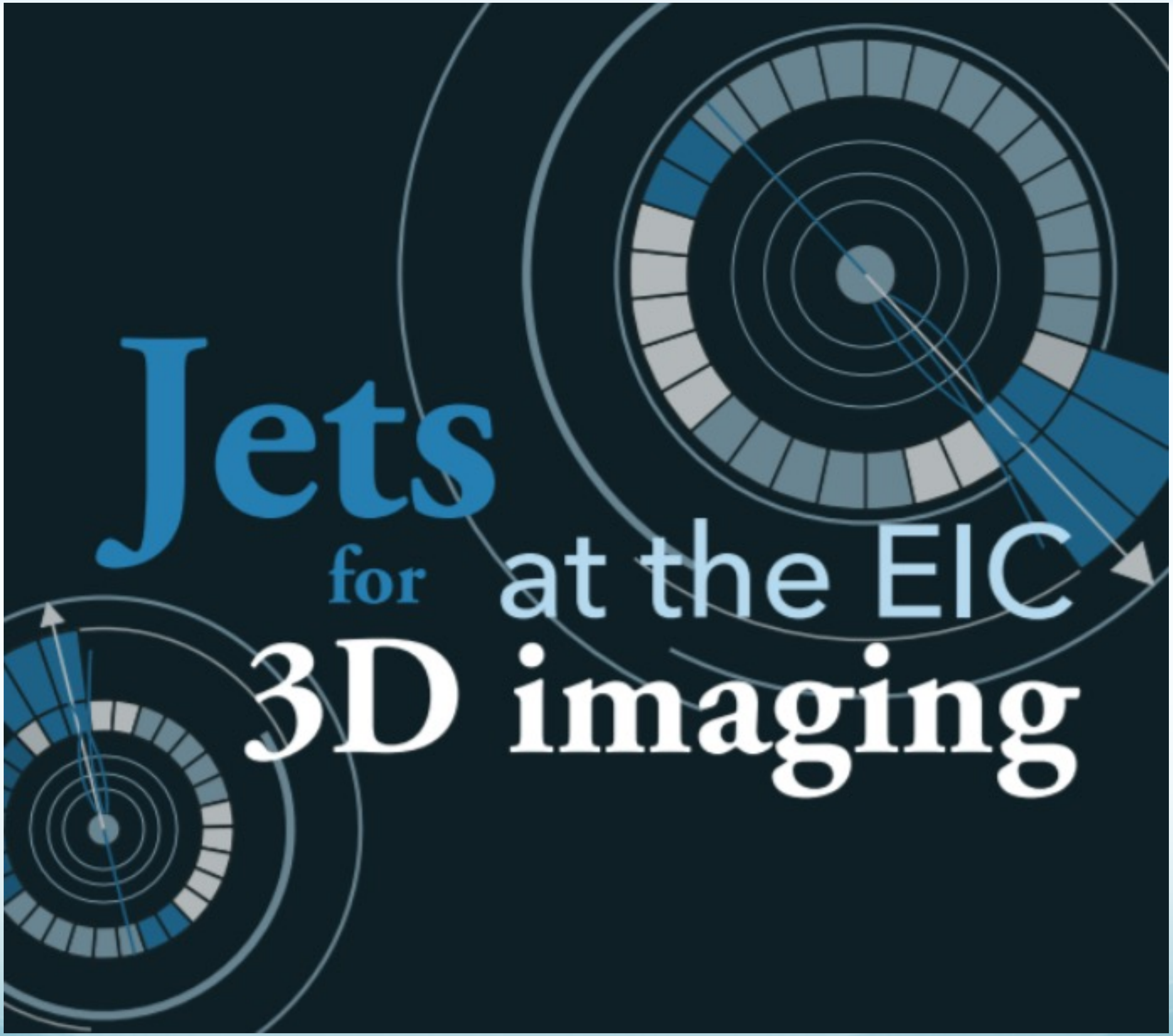
- Major difference: sea quark in relatively large x region
- This is precisely the main goal of SpinQuest/E1039 Drell-Yan experiment at Fermilab is to determine sea quark Sivers functions





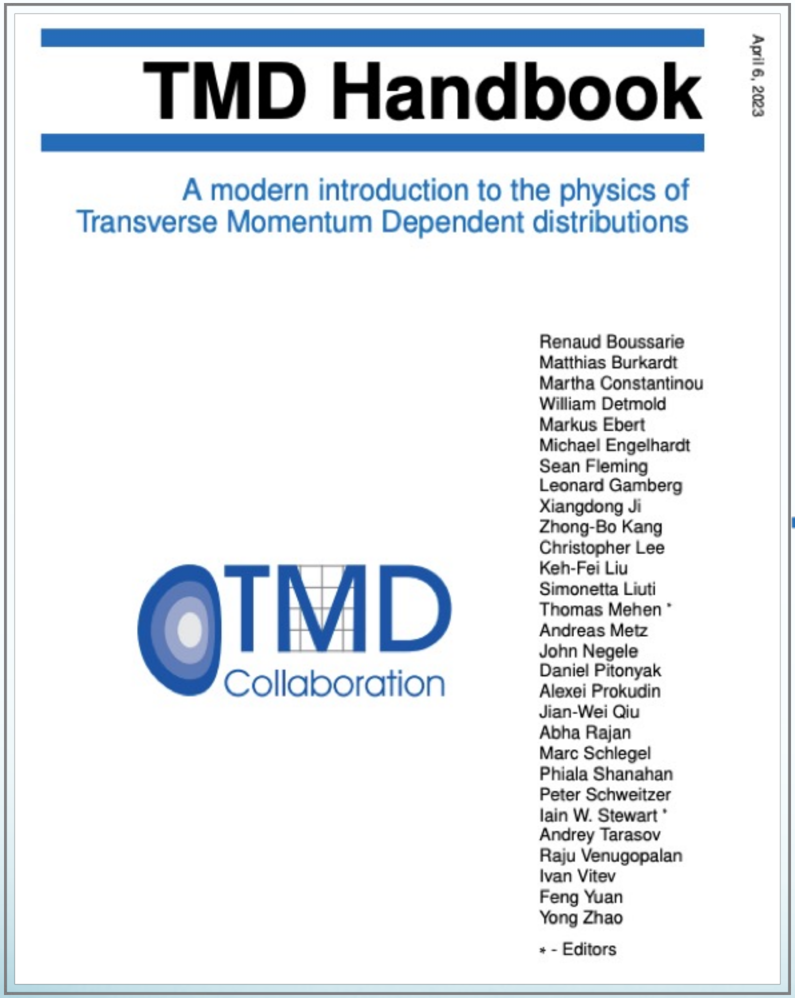
Courtesy of A. Prokudin

Jets for 3D imaging



Designed by M. Arratia

Jet fragmentation function



TMD Handbook

A modern introduction to the physics of Transverse Momentum Dependent distributions

April 6, 2023

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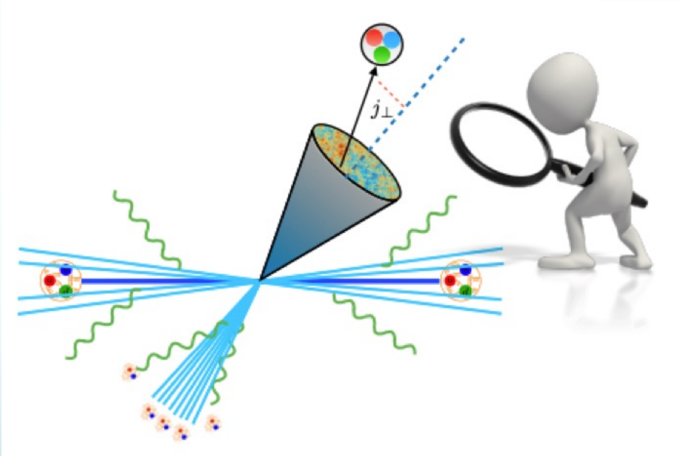
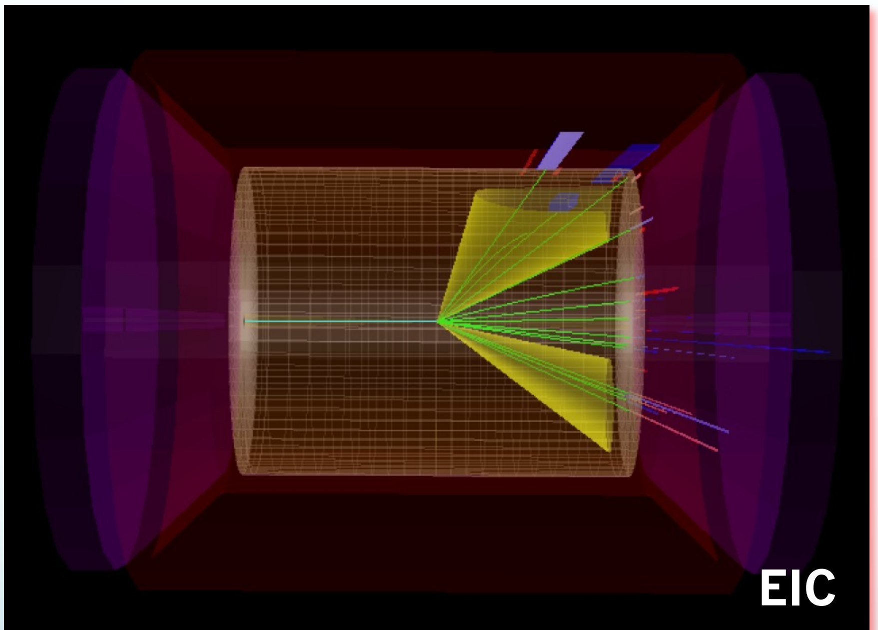
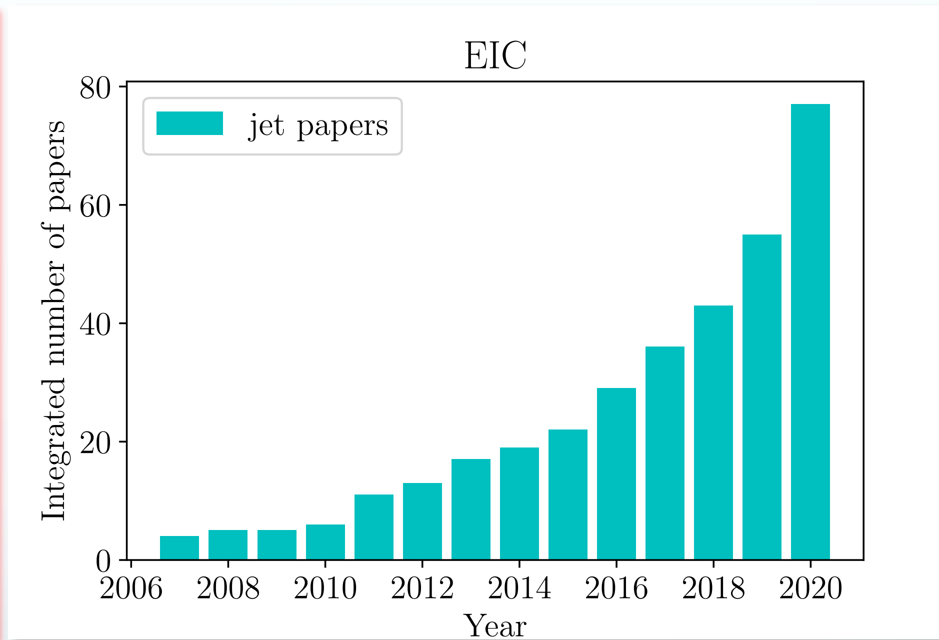


Jets are useful tools for TMD physics

- Active study at the EIC
  - EIC jet papers grow exponentially

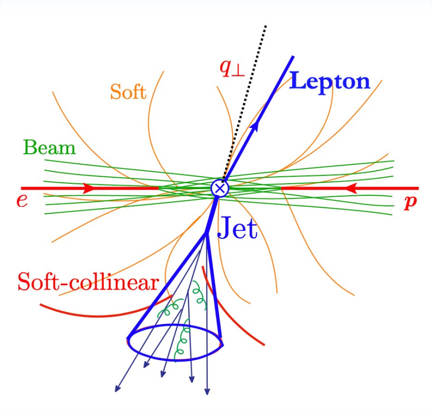




Back-to-back lepton-jet production

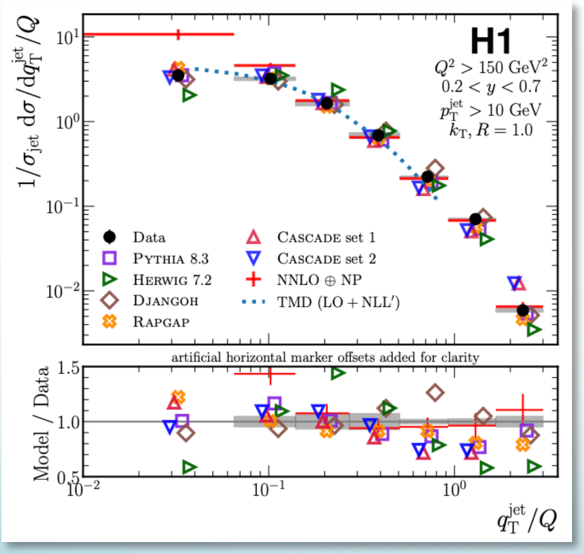
- Probing TMD PDFs

Liu, Ringer, Vogelsang, Yuan, 18, 20, …
Arratia, Kang, Prokudin, Ringer, 2007.07281
Kang, Lee, Shao, Zhao, 2106.15624

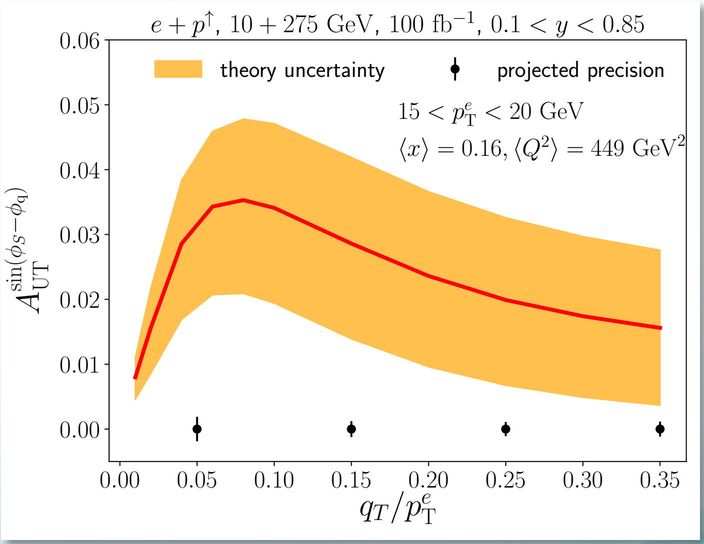


  - Unpolarized scattering: recent HERA measurement

HERA, arXiv:2108.12376, PRL 22



Arratia, Kang, Prokudin, Ringer, PRD 20

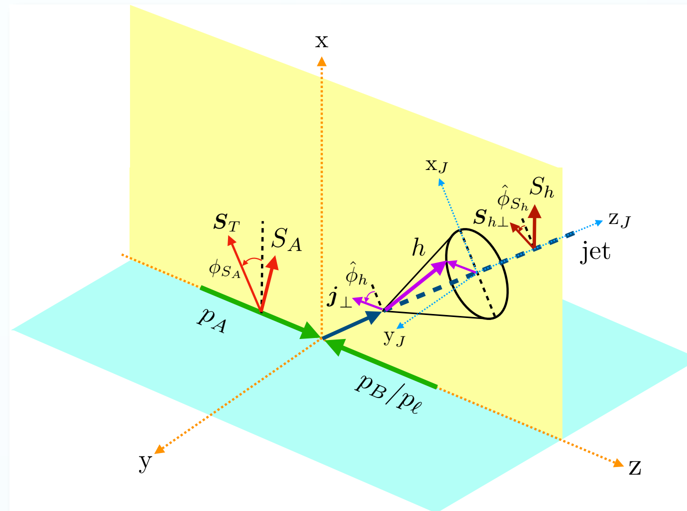


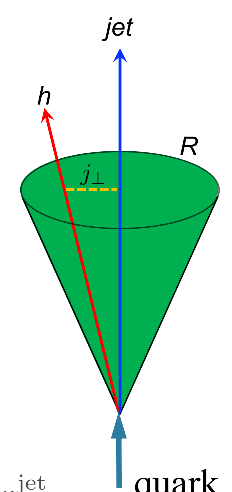
  - Transversely polarized proton scattering: Sivers function (right plot)

Jet substructure: polarized jet fragmentation function

- One can further measure distribution of hadrons inside the jet
  - Two axes: imbalance controls TMD PDFs, while the hadron transverse momentum w.r.t jet axis controls TMD FFs

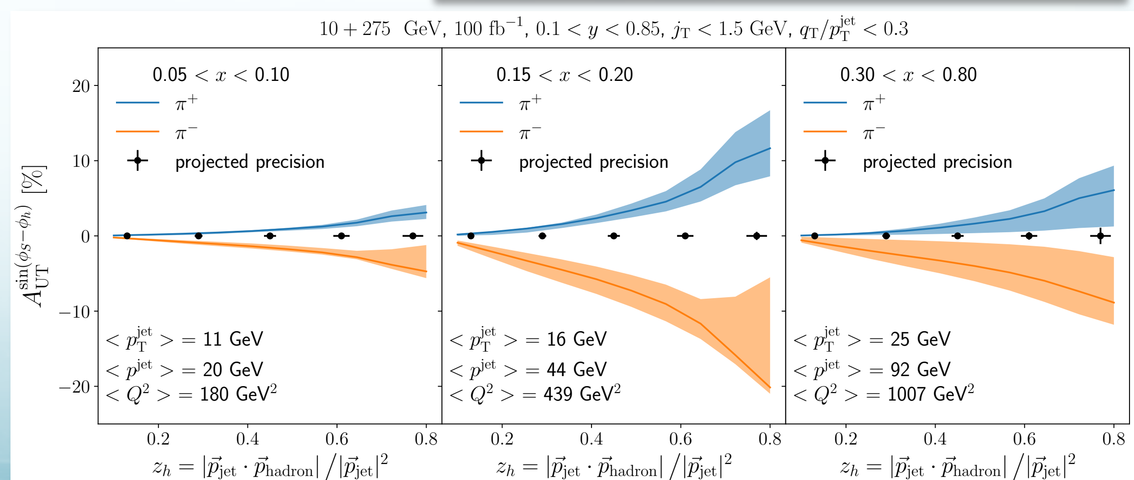
Kang, Lee, Zhao, PLB 20
Arratia, Kang, Ringer, Prokudin, 20
Kang, Lee, Shao, Zhao, JHEP 21
Kang, Terry, Vossen, Xu, Zhang, PRD 22





$z_h = p_T^h/p_T^{jet}$

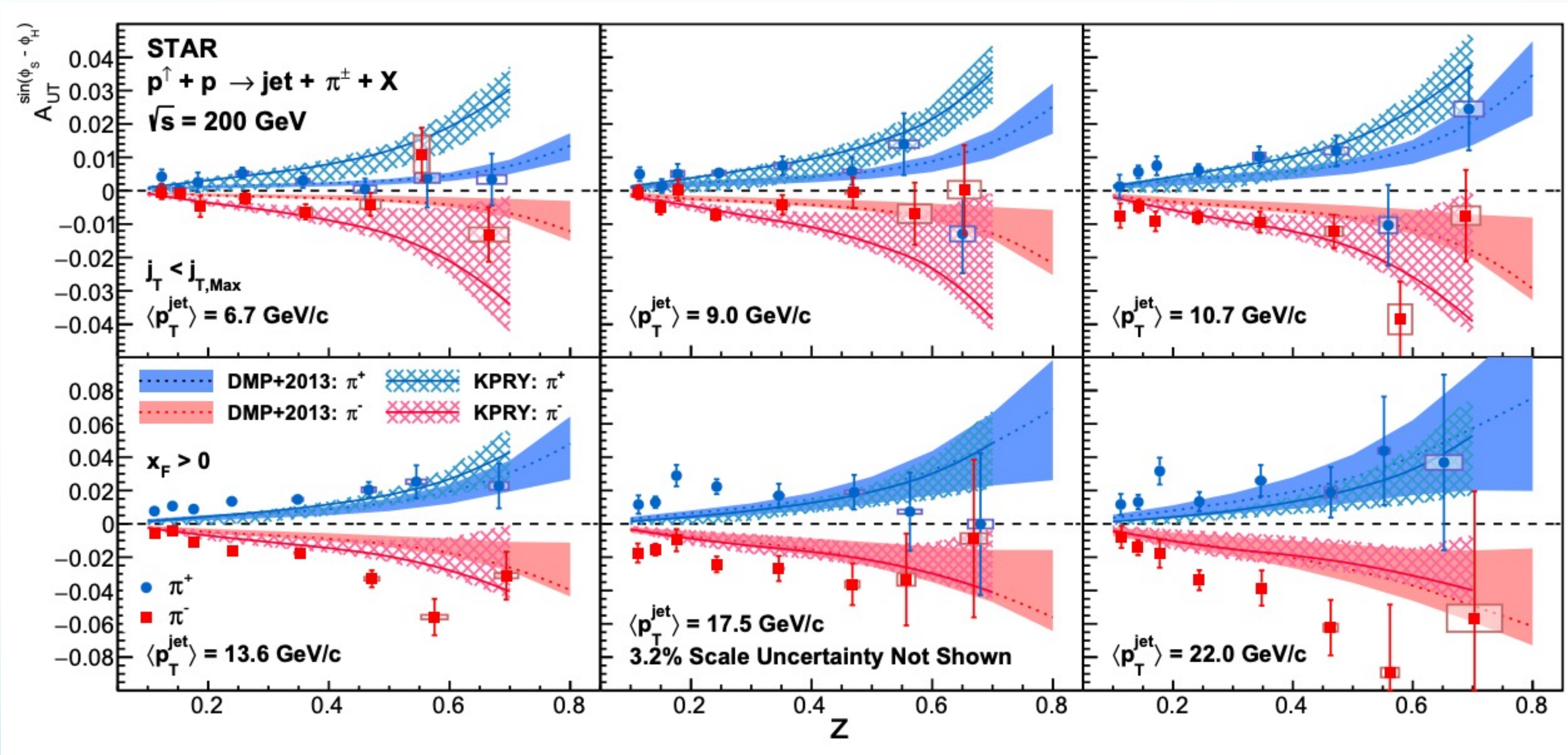
$j_\perp$: hadron transverse momentum with respect to the jet



Collins effect inside the jet

- STAR measurements: Collins asymmetry for hadron inside the jet in transversely polarized p+p collisions

STAR, 2205.11800

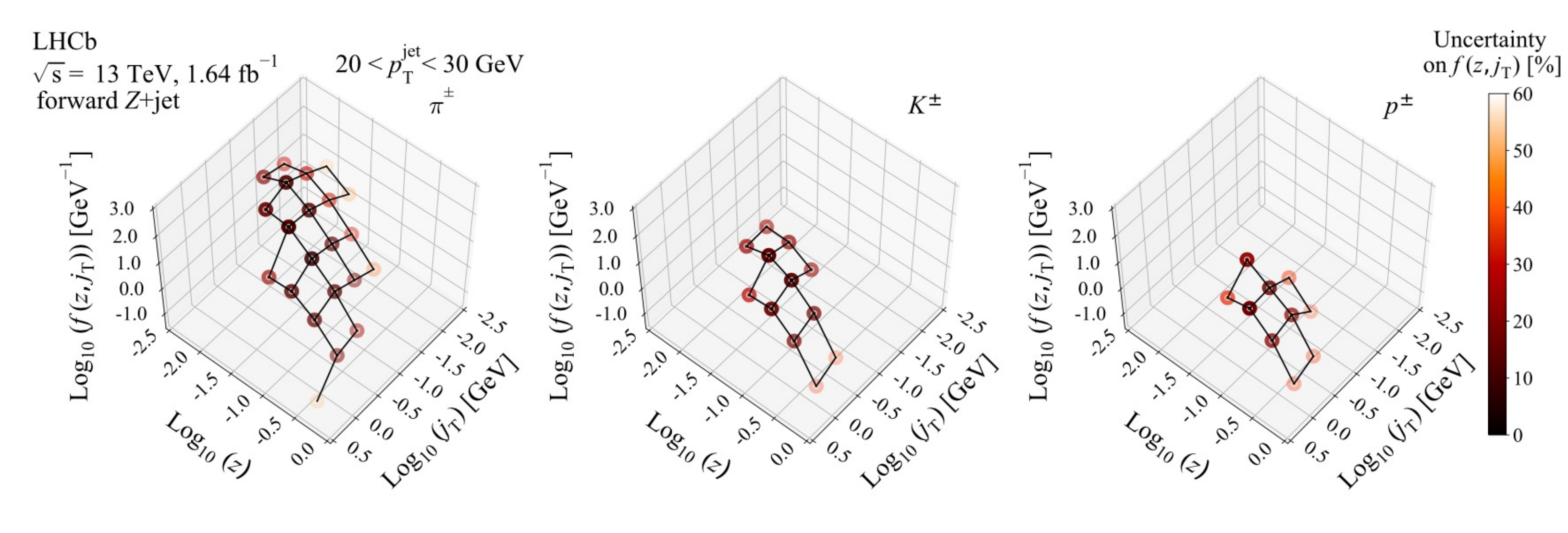


Kang, Prokudin, Ringer, Yuan, 1707.00913, PLB

Kang, Kyle, Zhao, arXiv:2005.02398, PLB

Jet fragmentation function at LHC

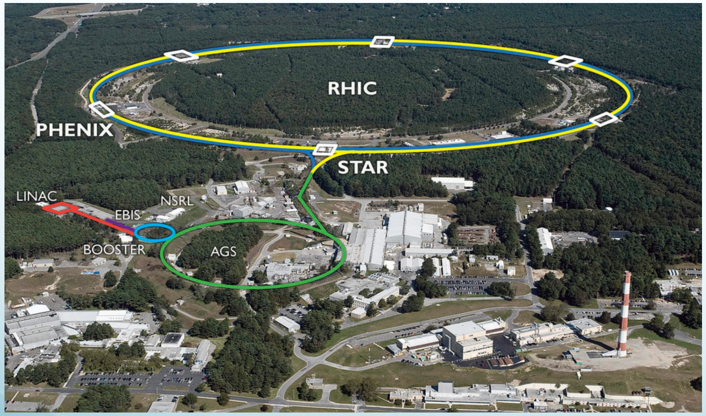
- Example: recent LHCb measurement Z-tagged jet production

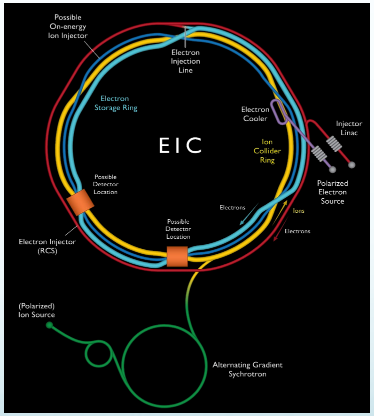


LHCb, 2208.11691

Summary

- Great progress has been made for TMD structure of the proton
- Current: HERMES, COMPASS, Jlab 12, HERA, RHIC spin, and LHC provide great experimental measurements for TMD physics
- Novel new opportunities: use jet/substructure for TMDs (synergy with high-energy QCD community)
- Looking forward to the bright future at the EIC





Thank you!