Nuclear astrophysics I

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My world line

UNIVERSITY of GUELPH

National Technical University of Athens

BEng
1998-2003

UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

Msc + started PhD
2003-2006

Los Alamos NATIONAL LABORATORY

finished PhD
2006-2009

UNIVERSITY of WASHINGTON

postdoc
2009-2012

TECHNISCHE UNIVERSITÄT DARMSTADT

Herzberg fellow
2012-2013

UNIVERSITY of GUELPH

(all kinds of)
professor
2013-

Nuclear astrophysics

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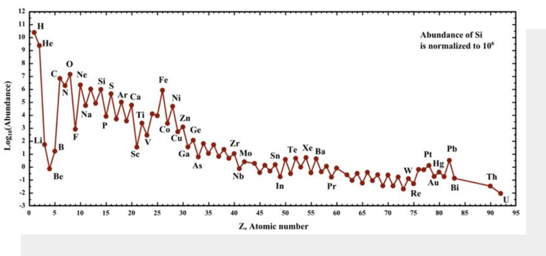
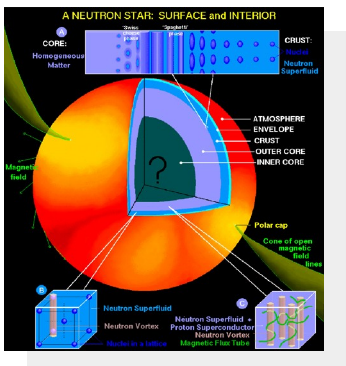
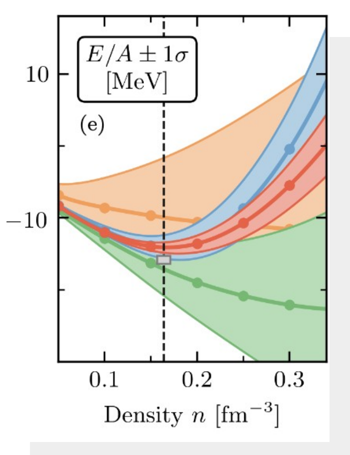


Element formation

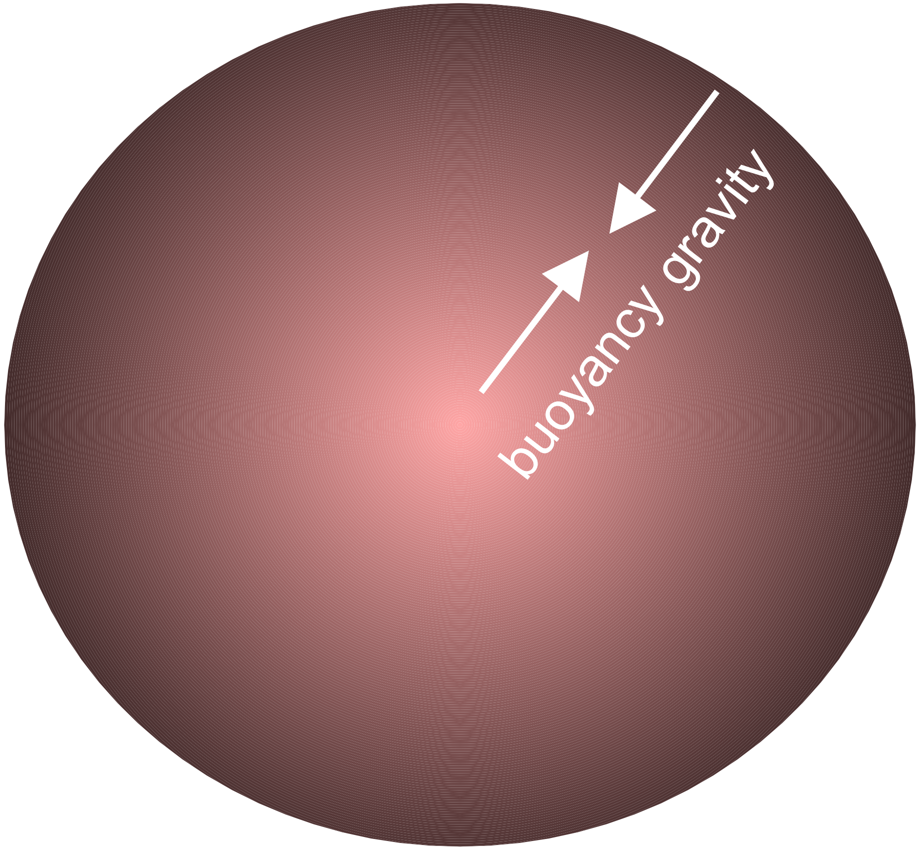


Neutron stars



Exotic matter

Stars



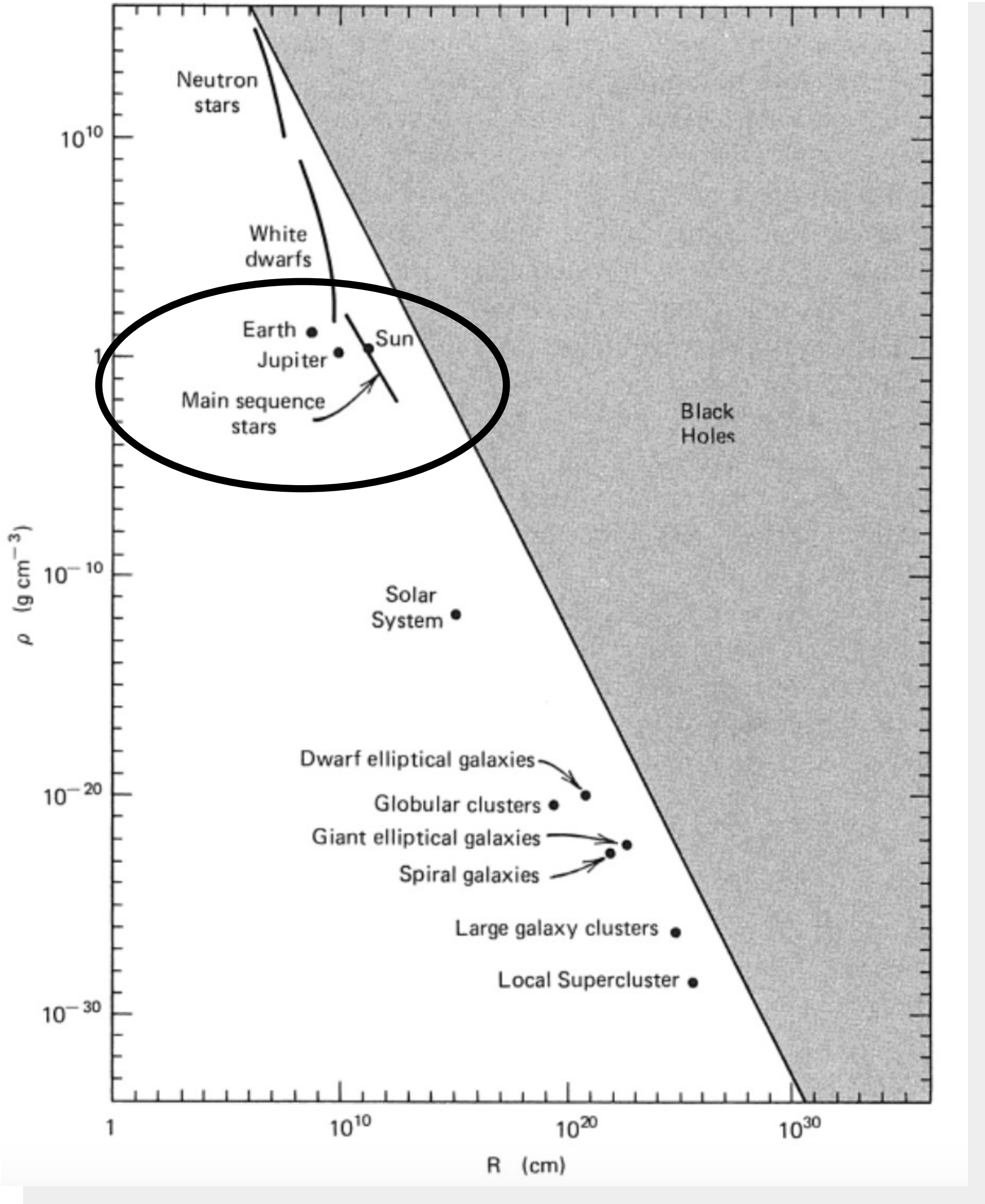
buoyancy aka upthrust aka pressure-gradient

Origin of the pressure

- **Normal stars**: thermal pressure (via nuclear energy generation) counteracts gravity

- **Compact stars**: mostly degeneracy pressure (since nuclear fuel basically spent) counteracts gravity (if at all)

Traditional astrophysics



Shapiro-Teukolsky, p. 3

Stars

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What is the “main sequence”?

Let’s turn to the Hertzsprung-Russell diagram, via a short digression on visualization in science

Hertzsprung-Russell diagram

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In practice, the Hertzsprung–Russell diagram of a cluster is a thick curve, not strictly one-dimensional. This is because the cluster stars did not all begin at precisely the same time with precisely the same chemical composition. There are also observational problems: a star's color and spectrum do not give a precise value for the effective temperature, and it is often difficult to distinguish binary stars from single stars. Even so, one can clearly see in the data that there is a one-dimensional curve of luminosity versus effective temperature, not just points everywhere in the plot.

The Hertzsprung–Russell diagram for a cluster commonly contains a *main sequence*, consisting of stars like the Sun that are still burning hydrogen at their cores. On the main sequence L increases smoothly with T_{eff}, with the most massive stars the hottest and most luminous. (In Section 1.6 we will show how to estimate the shape of the main sequence curve by applying dimensional analysis to Eqs. (1.3.1)–(1.3.4).) As the cluster evolves, the Hertzsprung–Russell diagram develops a red giant branch, consisting of stars that have converted most of the hydrogen at their cores to helium, and are burning hydrogen only in a shell around the inert helium core. On this branch, the effective temperature *decreases* (and radius increases) with increasing luminosity, accounting for the red color of very luminous red giant stars such as Betelgeuse and Antares. The heavier stars on the main sequence have larger L and therefore evolve more quickly, so as time passes more and more of the upper part of the main sequence bends over into the red giant branch. Observations of this main sequence

Weinberg, Astro, p. 19

Hertzsprung-Russell diagram

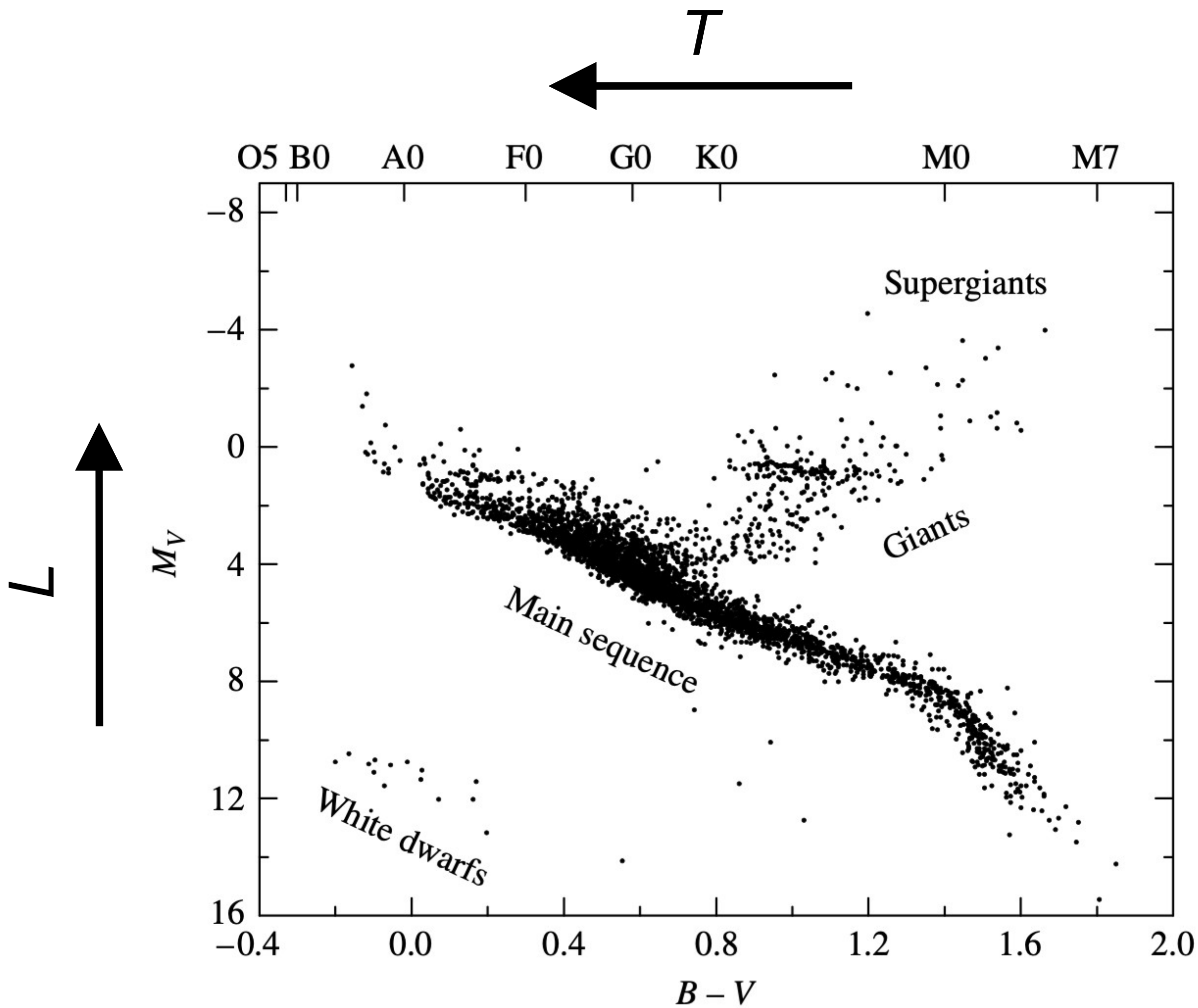

FIGURE 8.13 An observer's H–R diagram. The data are from the Hipparcos catalog. More than 3700 stars are included here with parallax measurements determined to better than 20%. (Data courtesy of the European Space Agency.)

Carroll-Ostlie, p. 223

Hertzsprung-Russell diagram

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In practice, the Hertzsprung–Russell diagram of a cluster is a thick curve, not strictly one-dimensional. This is because the cluster stars did not all begin at precisely the same time with precisely the same chemical composition. There are also observational problems: a star's color and spectrum do not give a precise value for the effective temperature, and it is often difficult to distinguish binary stars from single stars. Even so, one can clearly see in the data that there is a one-dimensional curve of luminosity versus effective temperature, not just points everywhere in the plot.

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Weinberg, Astro, p. 19

Traditional astrophysics

How does a star stay there?

Reminder: **thermal pressure** (via nuclear energy generation) counteracts gravity

Hydrostatic equilibrium

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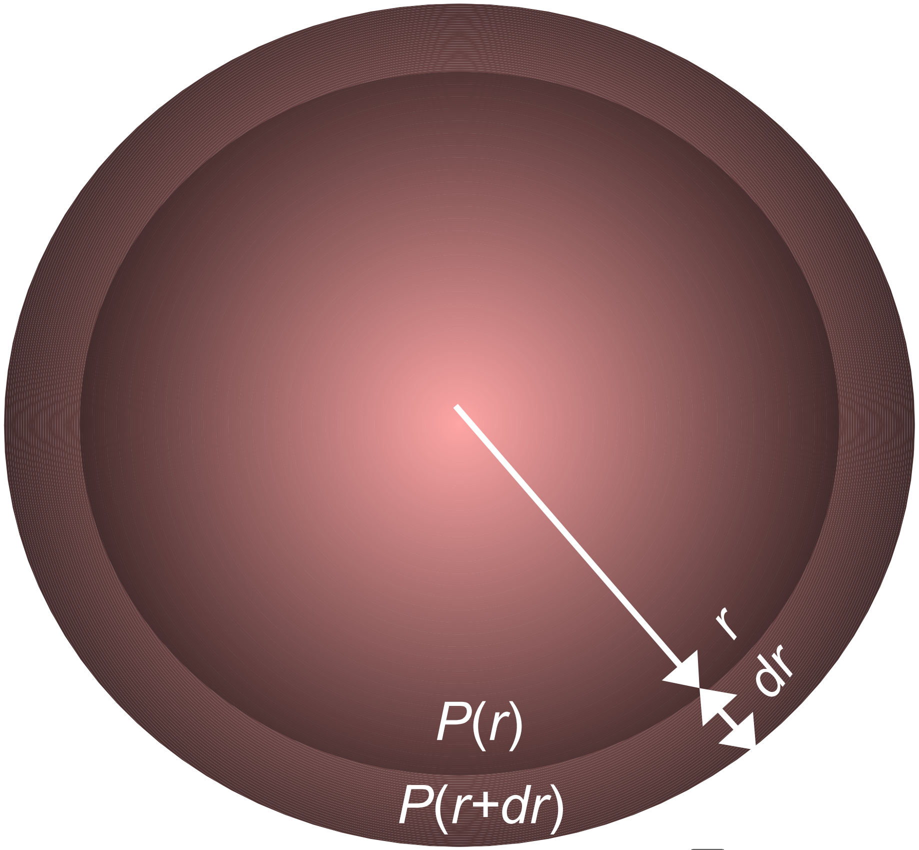

Distinguish between the mass within *r* :

$$m(r) = \int_0^r 4\pi {r'}^2 \rho(r') dr'$$

and that of the thin shell: $4\pi r^2 \rho(r) dr$

Shell experiences gravitation:

$$F_{\mathrm{gravitational}} = -G \frac{4\pi r^2 \rho(r) dr m(r)}{r^2} = -4\pi G \rho(r) m(r) dr$$

and buoyancy:

$$F_{\mathrm{buoyant}} = 4\pi r^2 \left[P(r) - P(r+dr)\right] = -4\pi r^2 P'(r) dr$$

Hydrostatic equilibrium

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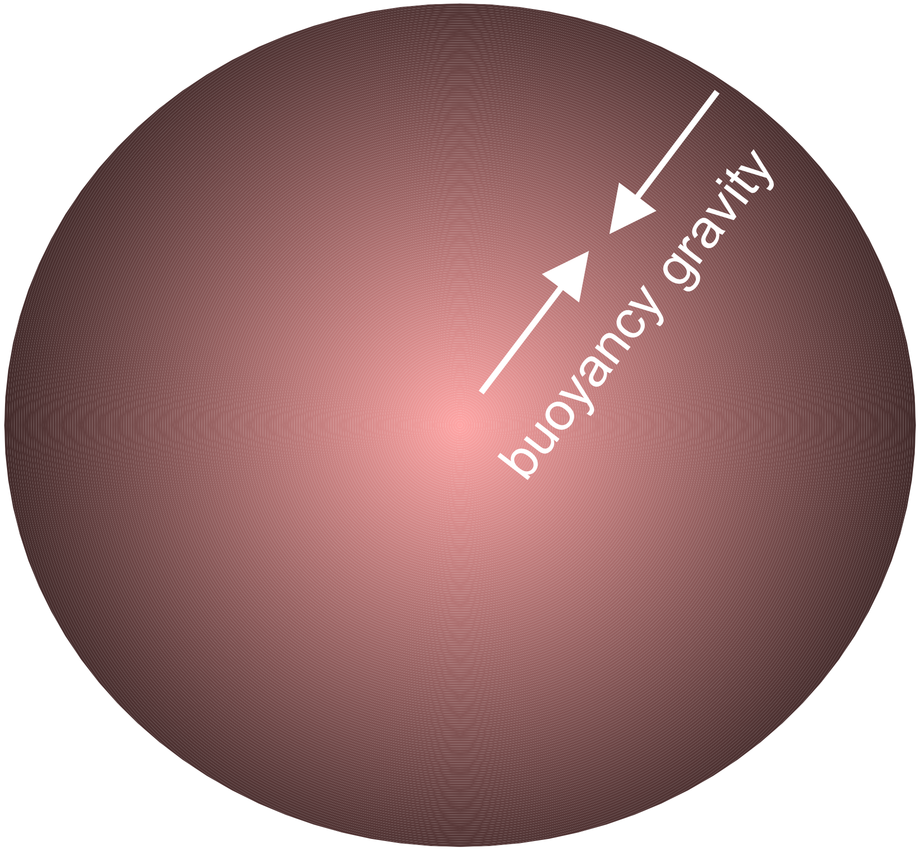

At equilibrium the sum of the two forces vanishes, so:

$$\frac{dP(r)}{dr} = -\frac{Gm(r)\rho(r)}{r^2}$$

To be solved together with:

$$m(r) = \int_0^r 4\pi r'^2 \rho(r') dr'$$

or, equivalently:

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r)$$

Hydrostatic equilibrium

Our two equations:

$$\frac{dP(r)}{dr} = -\frac{Gm(r)\rho(r)}{r^2}$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r)$$

seem to involve three quantities $P(r)$, $m(r)$, and $\rho(r)$.

To make further progress, use *equation-of-state* (EOS):

$$P = P(\rho, T)$$

Traditional astrophysics

How does a star stay there?

Reminder: thermal pressure (via **nuclear energy generation**) counteracts gravity

Coulomb barrier

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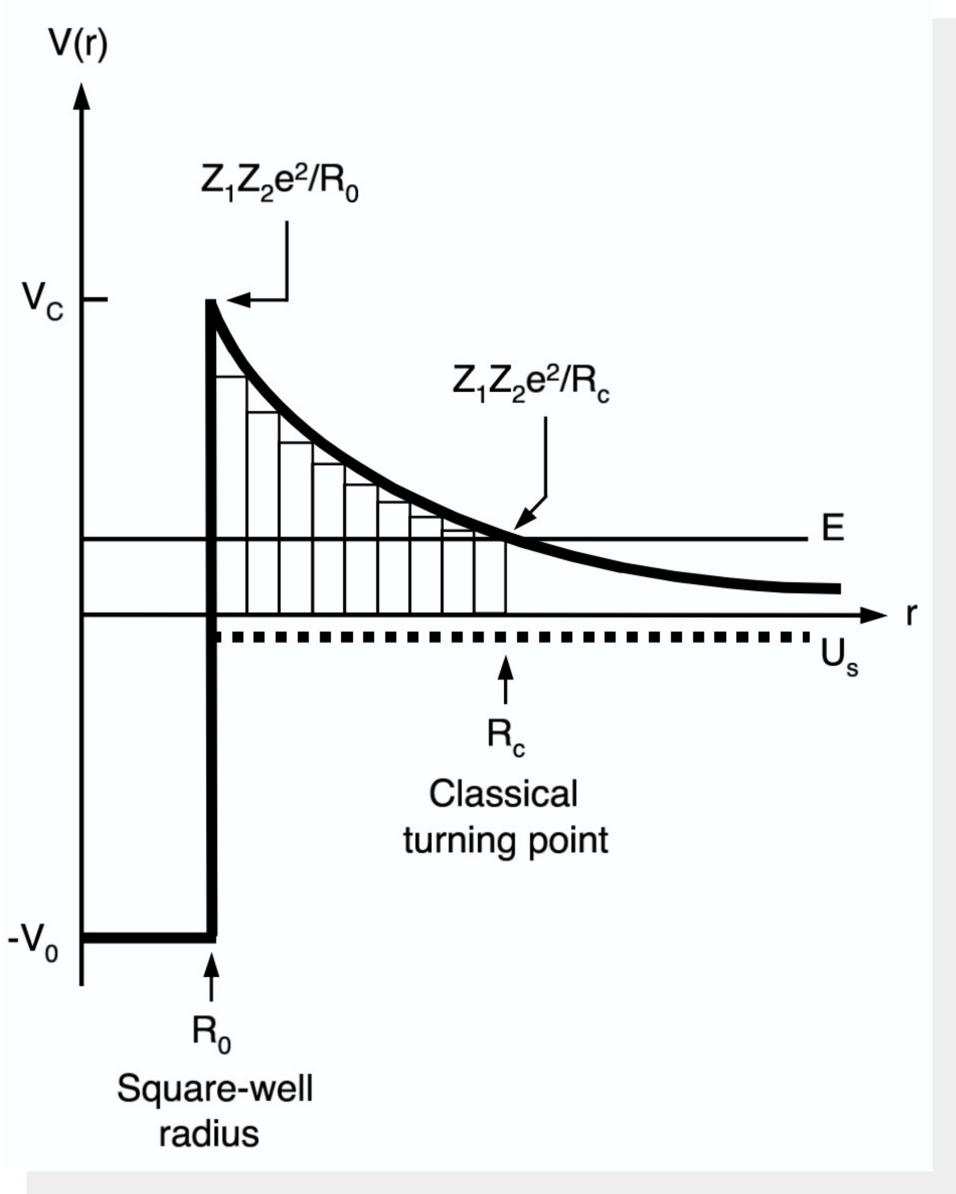

Using the WKB approximation, you can show that the reaction is suppressed by a factor

$$B(E) = \exp\left[-\pi Z_1 Z_2 e^2 \sqrt{\frac{2\mu}{\hbar^2 E}}\right]$$

C. Iliadis, p. 107

Coulomb barrier

Using the WKB approximation, you can show that the reaction is suppressed by a factor:

$$B(E) = \exp\left[-\pi Z_1 Z_2 e^2 \sqrt{\frac{2\mu}{\hbar^2 E}}\right]$$

Rate of nuclear reactions per mass:

$$\epsilon(\rho, T) = \int_0^\infty dE\, f(E, \rho, T) \exp(-E/k_\mathrm{B} T) B(E)$$

Dominated by peak of the exponential:

$$B_\mathrm{T} = \exp\left(-\frac{E_\mathrm{T}}{k_\mathrm{B} T} - \frac{C}{\sqrt{E_\mathrm{T}}}\right) = \exp\left(-3\left(\frac{\pi Z_1 Z_2 e^2 \sqrt{\mu}}{\hbar \sqrt{2 k_\mathrm{B} T}}\right)^{2/3}\right)$$

Reminder: **thermal pressure** (via **nuclear energy generation**)

Proton-proton chain

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$$\text{I}: \quad {}^1\text{H} + {}^1\text{H} \to {}^2\text{H} + e^+ + \nu_e + 1.18\ \text{MeV}$$
$$\text{II}: \quad {}^1\text{H} + {}^2\text{H} \to {}^3\text{He} + \gamma + 5.49\ \text{MeV}$$
$$\text{III}: \quad {}^3\text{He} + {}^3\text{He} \to {}^4\text{He} + {}^1\text{H} + {}^1\text{H} + 12.85\ \text{MeV}$$

$$\text{III}': \quad {}^3\text{He} + {}^4\text{He} \to {}^7\text{Be} + \gamma$$

followed by either

$$\text{IV}: \quad {}^7\text{Be} + e^- \to {}^7\text{Li} + \nu_e$$
$$\text{V}: \quad {}^7\text{Li} + {}^1\text{H} \to {}^4\text{He} + {}^4\text{He}$$

or else

$$\text{IV}': \quad {}^7\text{Be} + {}^1\text{H} \to {}^8\text{B} + \gamma$$
$$\text{V}': \quad {}^8\text{B} \to {}^8\text{Be} + e^+ + \nu_e$$
$$\text{VI}': \quad {}^8\text{Be} \to {}^4\text{He} + {}^4\text{He}.$$

Weinberg, Astro, p. 35

Proton-proton chain

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$$\begin{aligned}
\text{I}: &\quad {}^1\text{H} + {}^1\text{H} \to {}^2\text{H} + e^+ + \nu_e + 1.18\ \text{MeV} \\
\text{II}: &\quad {}^1\text{H} + {}^2\text{H} \to {}^3\text{He} + \gamma + 5.49\ \text{MeV} \\
\text{III}: &\quad {}^3\text{He} + {}^3\text{He} \to {}^4\text{He} + {}^1\text{H} + {}^1\text{H} + 12.85\ \text{MeV}
\end{aligned}$$

The idea now is to plug in the appropriate proton numbers and reduced masses, thereby finding the corresponding Coulomb barrier suppression.
(Of course, one must also include, e.g., weak suppression.)

Since two ^{3}He nuclei are destroyed in III, the rates per volume should obey the relation:

$$\Gamma \equiv \Gamma(\text{I}) = \Gamma(\text{II}) = 2\Gamma(\text{III}),$$

assuming time-independence, i.e., stable abundances

CNO cycle

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$$\begin{aligned}
\text{i}: &\quad {}^{1}\mathrm{H} + {}^{12}\mathrm{C} \rightarrow {}^{13}\mathrm{N} + \gamma + 1.95\ \mathrm{MeV} \\
\text{ii}: &\quad {}^{13}\mathrm{N} \rightarrow {}^{13}\mathrm{C} + \mathrm{e}^{+} + \nu_{\mathrm{e}} + 1.50\ \mathrm{MeV} \\
\text{iii}: &\quad {}^{1}\mathrm{H} + {}^{13}\mathrm{C} \rightarrow {}^{14}\mathrm{N} + \gamma + 7.54\ \mathrm{MeV} \\
\text{iv}: &\quad {}^{1}\mathrm{H} + {}^{14}\mathrm{N} \rightarrow {}^{15}\mathrm{O} + \gamma + 7.35\ \mathrm{MeV} \\
\text{v}: &\quad {}^{15}\mathrm{O} \rightarrow {}^{15}\mathrm{N} + \mathrm{e}^{+} + \nu_{\mathrm{e}} + 1.73\ \mathrm{MeV} \\
\text{vi}: &\quad {}^{1}\mathrm{H} + {}^{15}\mathrm{N} \rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He} + 4.96\ \mathrm{MeV}
\end{aligned}$$

You can play the same game of establishing relations between the rates per volume:

$$\Gamma(\mathrm{i}) = \Gamma(\mathrm{ii}) = \Gamma(\mathrm{iii}) = \Gamma(\mathrm{iv}) = \Gamma(\mathrm{v}) = \Gamma(\mathrm{vi}) \equiv \Gamma.$$

This multiplied by the sum of the energies listed above can give the total rate of energy production per volume

Liquid-drop formula

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Binding-energy systematics

$$\frac{E_B}{N+Z} = a_V - \frac{a_S}{(N+Z)^{1/3}} - a_C \frac{Z(Z-1)}{(N+Z)^{4/3}} - a_{sym} \frac{(N-Z)^2}{(N+Z)^2} - \lambda \frac{a_p}{(N+Z)^{3/2}}$$

volume surface Coulomb symmetry

pairing $\lambda = \begin{cases} +1, & \text{odd } N\text{-odd } Z \\ 0, & \text{odd } N\text{-even } Z\text{, or even } N\text{-odd } Z \\ -1, & \text{even } N\text{-even } Z \end{cases}$

Liquid-drop formula

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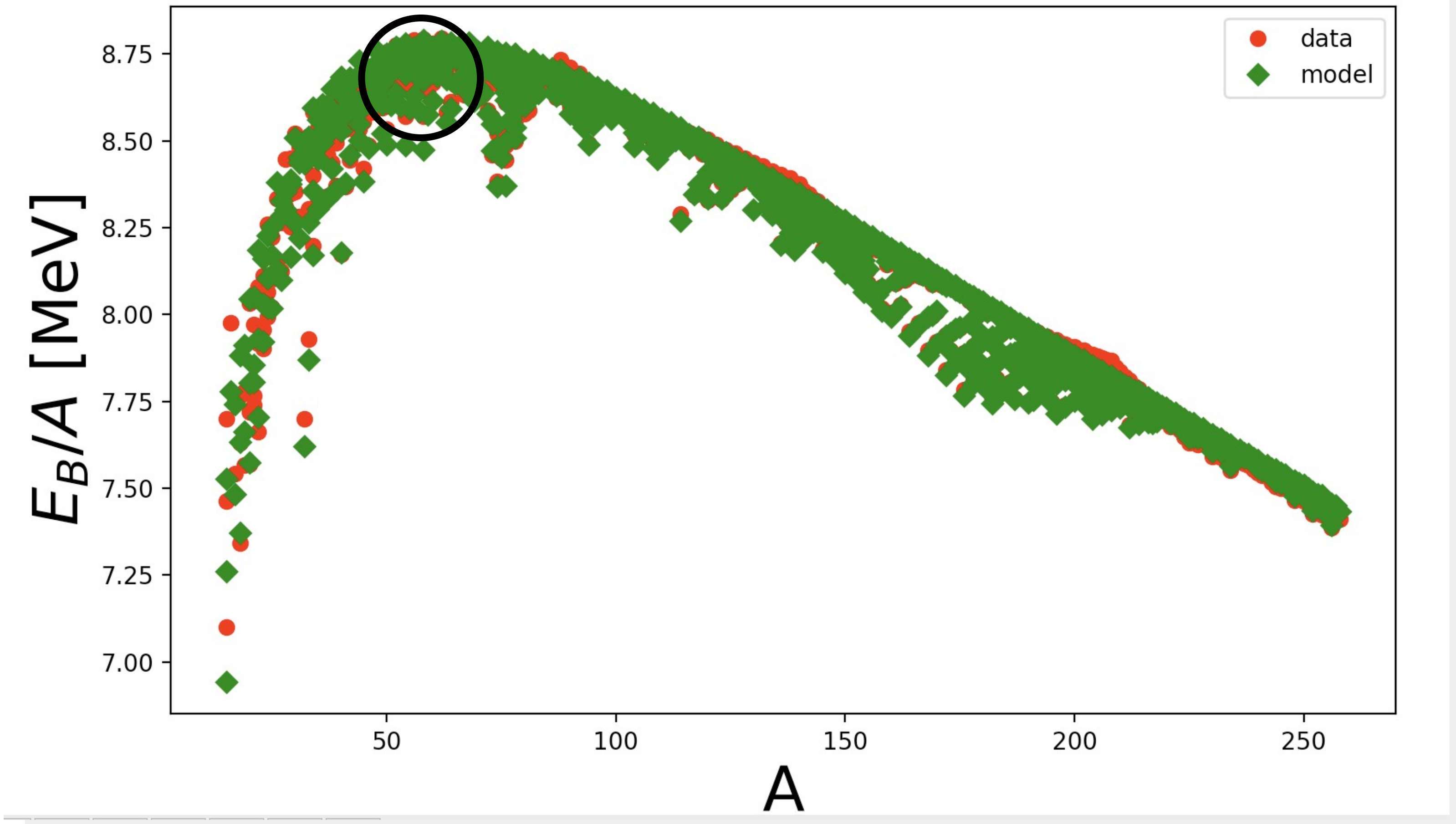
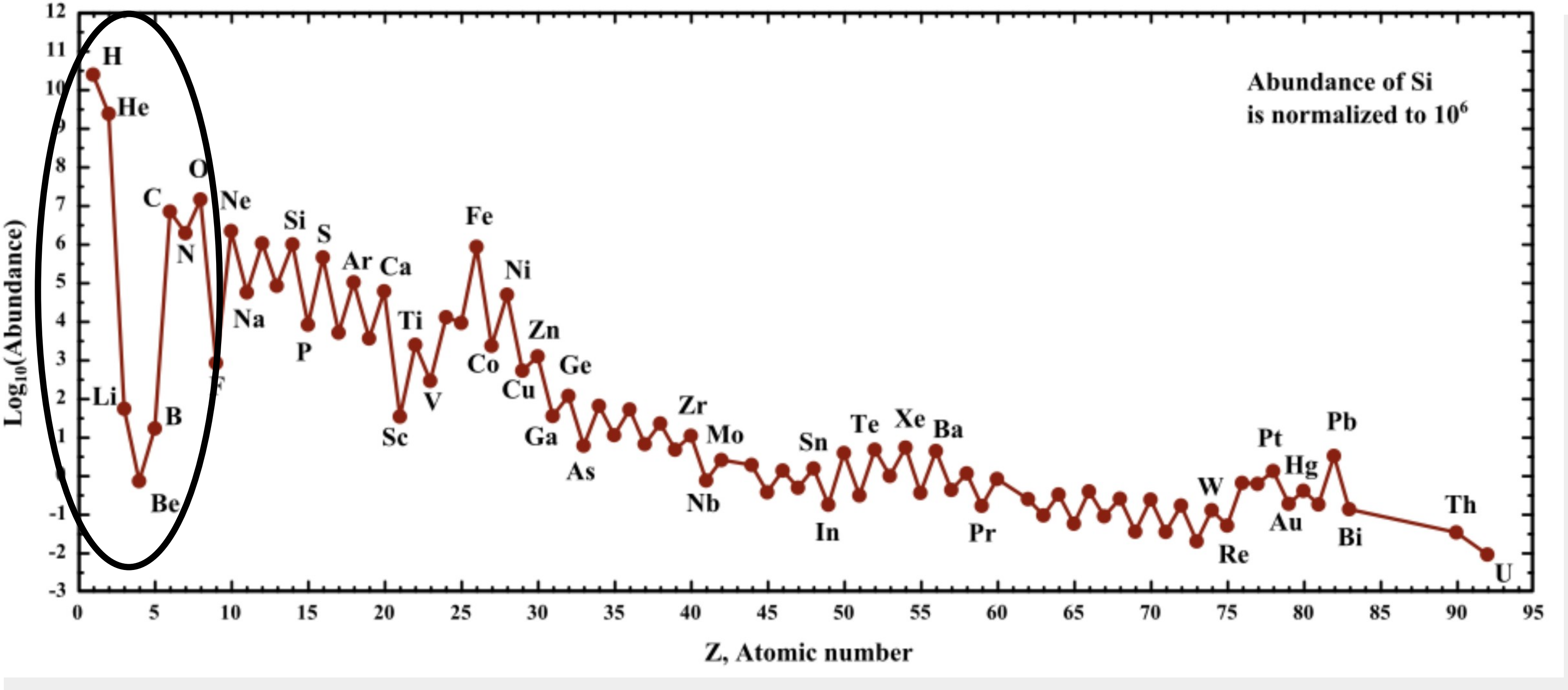
For nuclei lighter than this peak, fusion releases energy (just like in the proton-proton chain and the CNO cycle)

Of course, the energy released via a given reaction also depends on the abundances of a given nuclide in nature

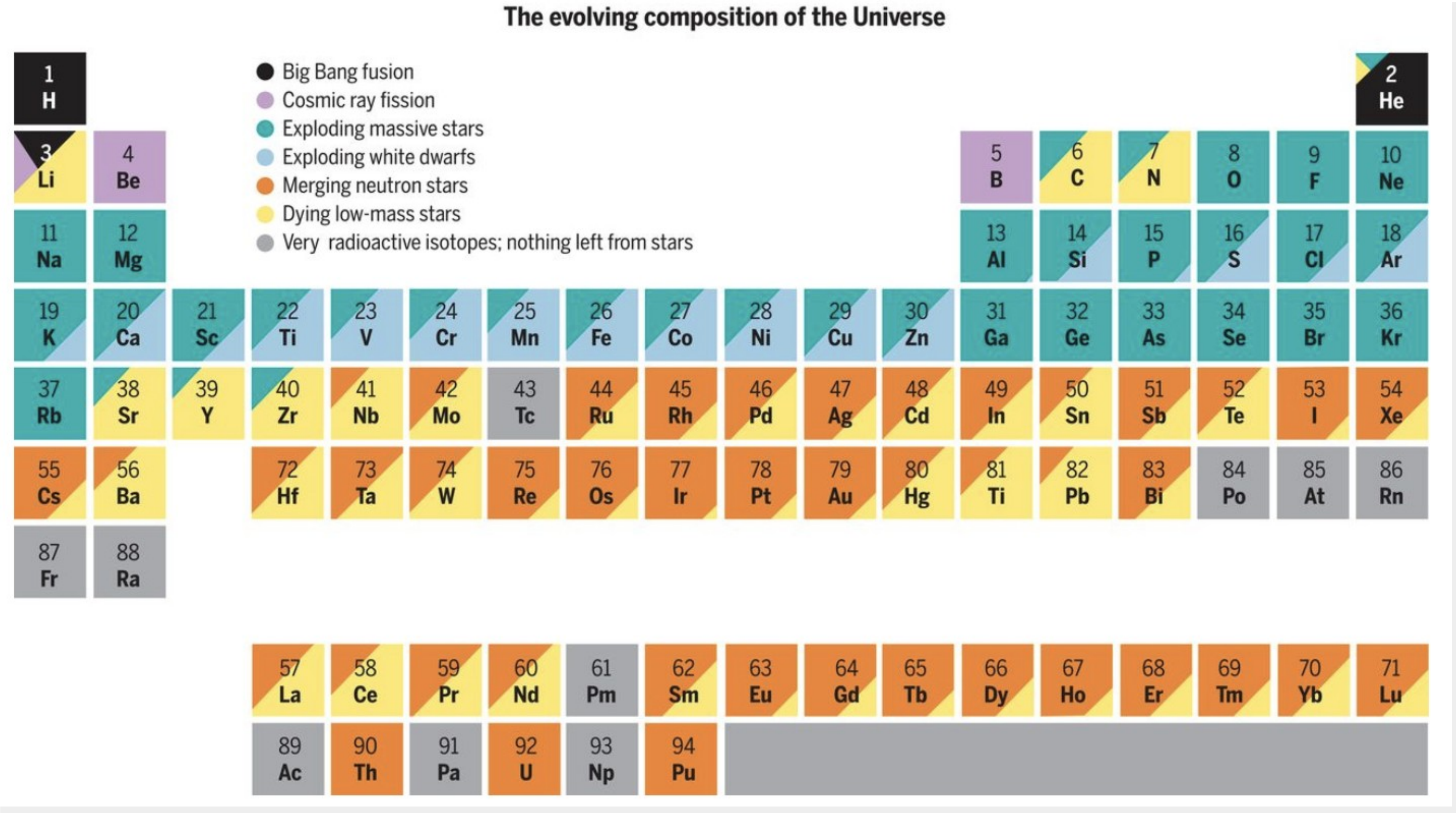


Abundances



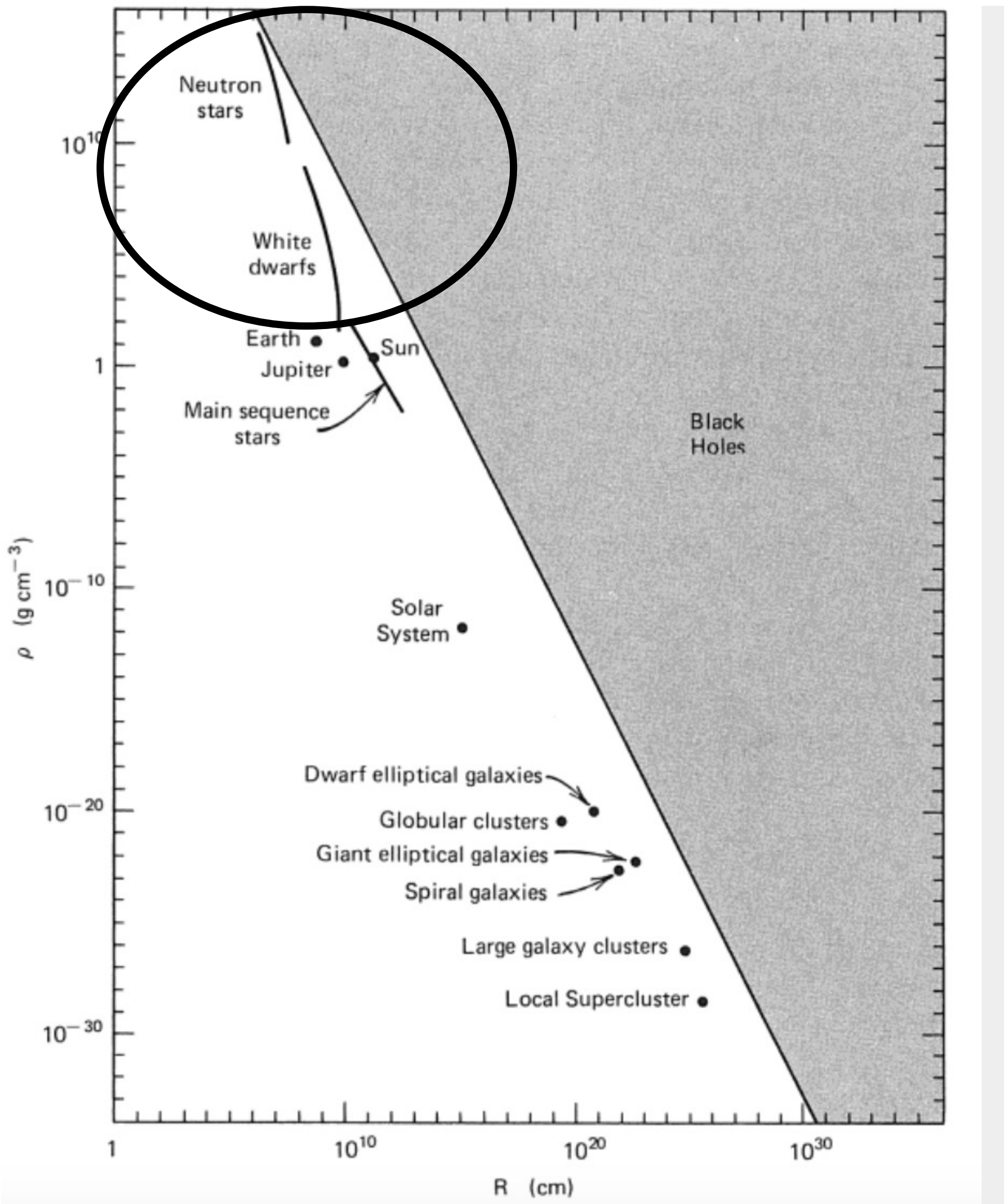
K. Lodders, arXiv:1010.2746

Sites of nucleosynthesis



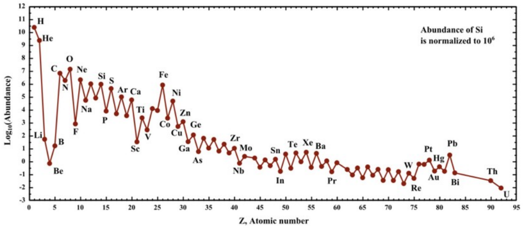
J. A. Johnson, *Science* **363**, 474 (2019)

Neutron astrophysics

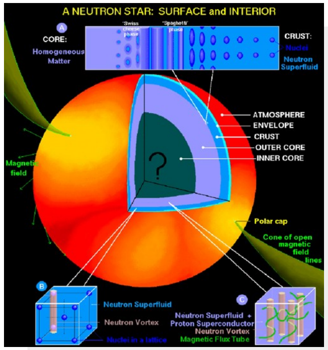


Shapiro-Teukolsky, p. 3

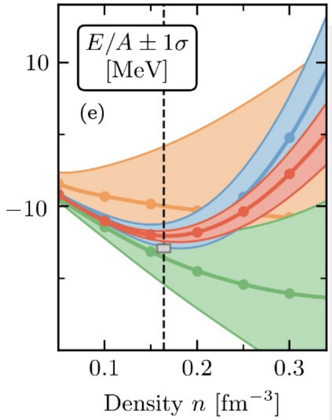
Nuclear astrophysics



Element formation



Neutron stars



Exotic matter

Stars

- **Normal stars**: thermal pressure (via nuclear energy generation) to counteract gravity

- **Compact stars**: mostly degeneracy pressure (since nuclear fuel basically spent) counteracts gravity (if at all)
  - *White dwarfs*: electron degeneracy pressure
  - *Neutron stars*: neutron degeneracy pressure (and repulsion)
  - *Black holes*: the struggle is real

MASS ↓

Neutron stars: key properties

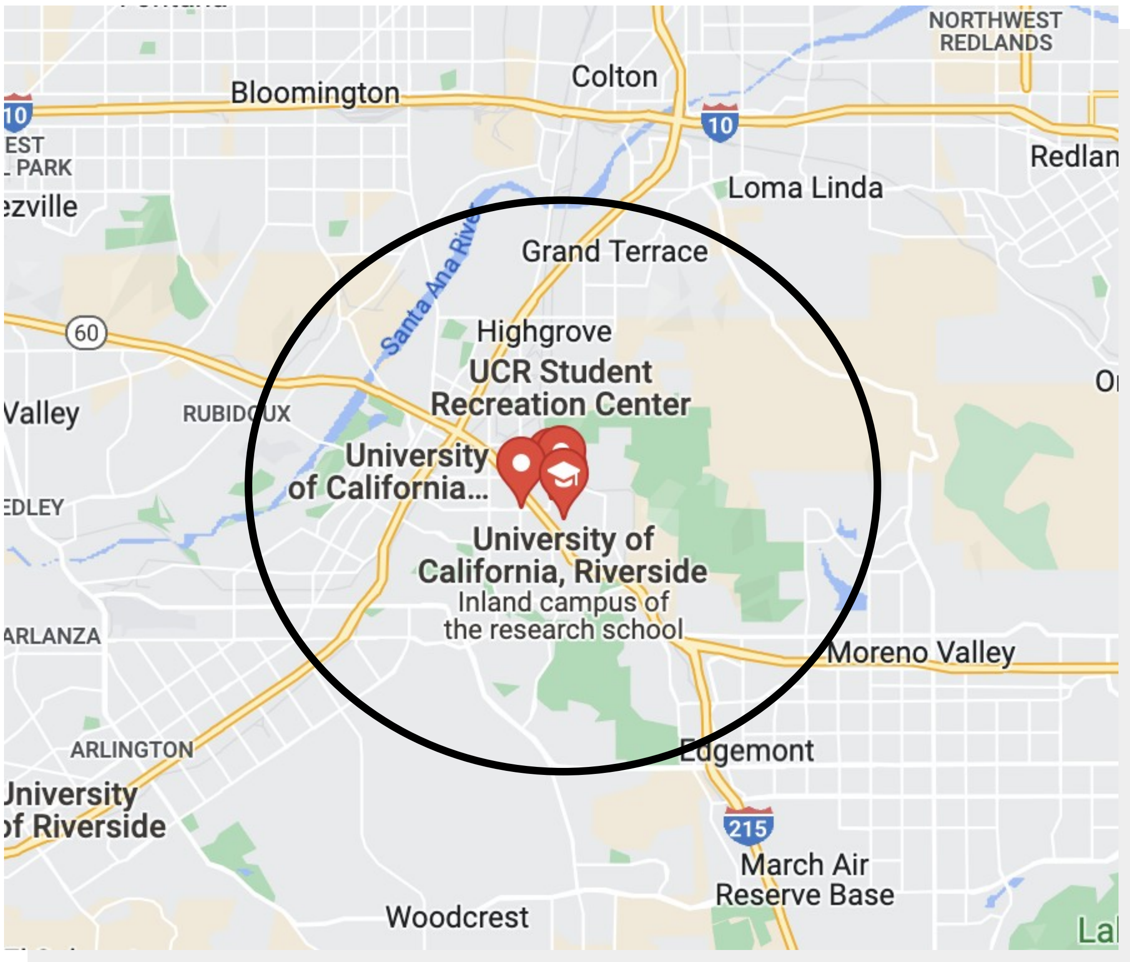


Image credit: Google maps

- Ultra-dense: 1.2-2 solar masses within a radius of 10 km
- Magnetic fields of 10^4 to 10^{11} T

Neutron stars: mass measurements

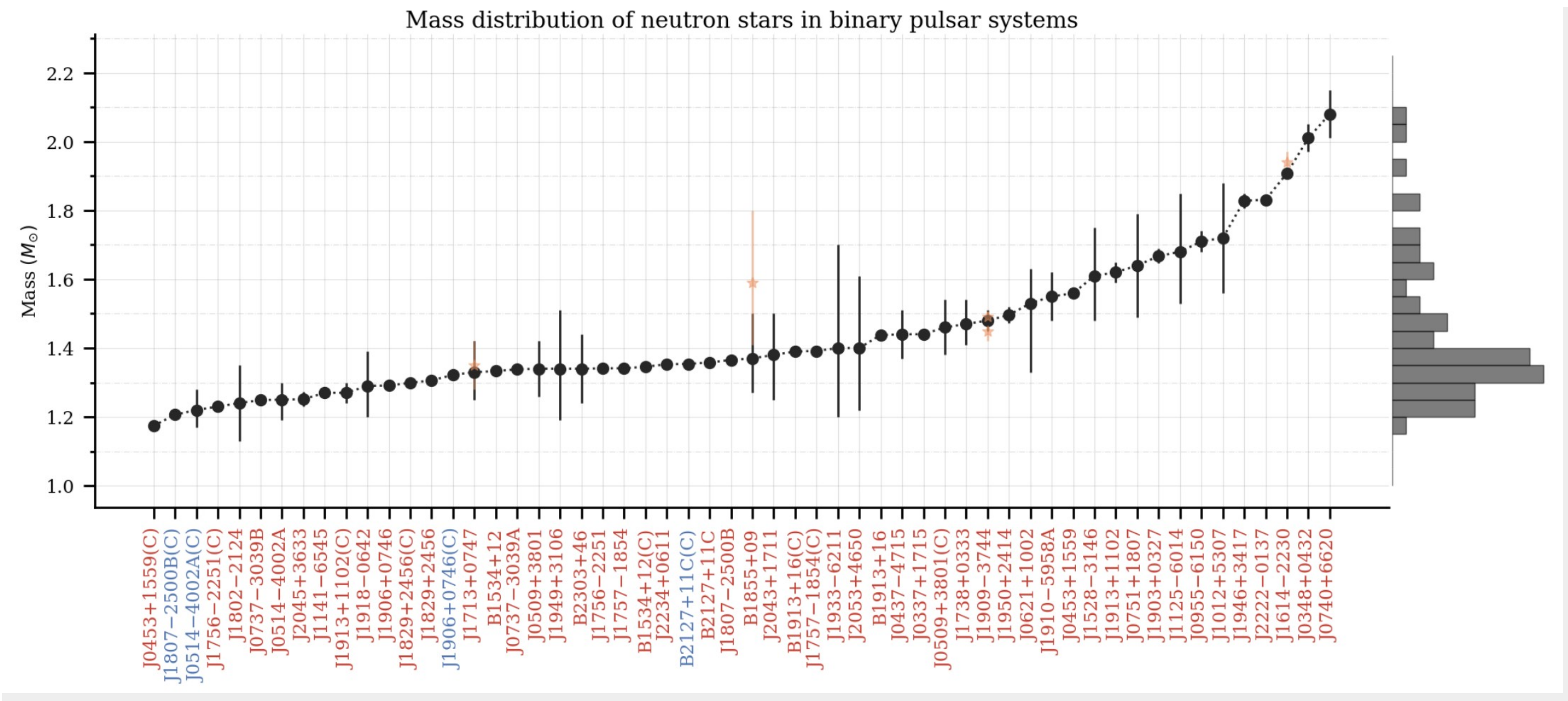


Image credit: Paulo Freire

Neutron stars: key properties

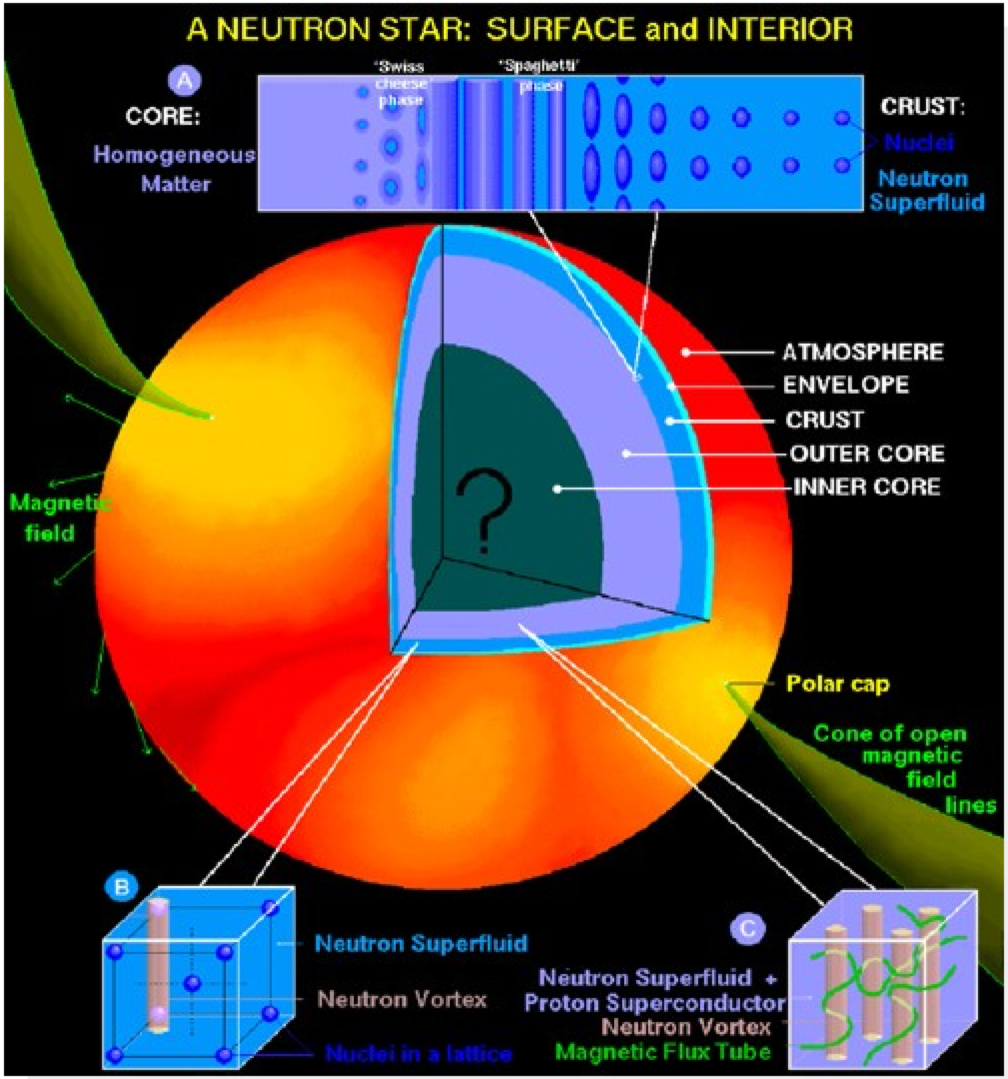


Image credit: Dany Page

- Ultra-dense: 1.2-2 solar masses within a radius of 10 km
- Magnetic fields of 10^4 to 10^{11} T
- Terrestrial-like (outer layers) down to exotic (core) behavior
- Observationally probed, i.e., not accessible in the lab (see, however, LIGO)
- We'll talk about describing the exotic matter later. For now, assuming an equation of state, how does one arrive at the composition?

Neutron-star composition

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Transition density ($kg\ m^{-3}$)	Composition	Degeneracy pressure
	iron nuclei, nonrelativistic free electrons	electron
$\approx 1 \times 10^{9}$	electrons become relativistic	
	iron nuclei, relativistic free electrons	electron
$\approx 1 \times 10^{12}$	neutronization	
	neutron-rich nuclei, relativistic free electrons	electron
$\approx 4 \times 10^{14}$	neutron drip	
	neutron-rich nuclei, free neutrons, relativistic free electrons	electron
$\approx 4 \times 10^{15}$	neutron degeneracy pressure dominates	
	neutron-rich nuclei, superfluid free neutrons, relativistic free electrons	neutron
$\approx 2 \times 10^{17}$	nuclei dissolve	
	superfluid free neutrons, superconducting free protons, relativistic free electrons	neutron
$\approx 4 \times 10^{17}$	pion production	
	superfluid free neutrons, superconducting free protons, relativistic free electrons, other elementary particles (pions, ...?)	neutron

Carroll-Ostlie, p. 582

Relativistic hydrostatic equilibrium

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Static, spherically symmetric metric

$$g_{\mu\nu} = \begin{pmatrix} A(r) & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2 \sin^2\theta & 0 \\ 0 & 0 & 0 & -B(r) \end{pmatrix}$$

Einstein field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Ricci tensor   Energy-momentum tensor

Inside the source (for isotropic fluid without shear forces):
Tolman-Oppenheimer-Volkoff (TOV) equation(s)

$$\frac{dP(r)}{dr} = -\frac{Gm(r)\rho(r)}{r^2} \left[1 + \frac{P(r)}{\rho(r)c^2}\right] \left[1 + \frac{4\pi r^3 P(r)}{m(r)c^2}\right] \left[1 - \frac{2Gm(r)}{c^2 r}\right]^{-1}$$

Relativistic hydrostatic equilibrium

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Thus, our task is a messier version of what we had before:

$$\frac{dP(r)}{dr} = -\frac{Gm(r)\rho(r)}{r^2}\left[1+\frac{P(r)}{\rho(r)c^2}\right]\left[1+\frac{4\pi r^3 P(r)}{m(r)c^2}\right]\left[1-\frac{2Gm(r)}{c^2 r}\right]^{-1}$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r)$$

(Incidentally, you can take the Newtonian limit via $c^2 \to \infty$.)

Again, this seems to involve three quantities *P*(*r*), *m*(*r*), and *ρ*(*r*).

To make further progress, use *equation-of-state* (EOS), *P* = *P*(*ρ*)

together with $\frac{dP(r)}{dr} = \left(\frac{dP(\rho)}{d\rho}\right)\left(\frac{d\rho}{dr}\right)$

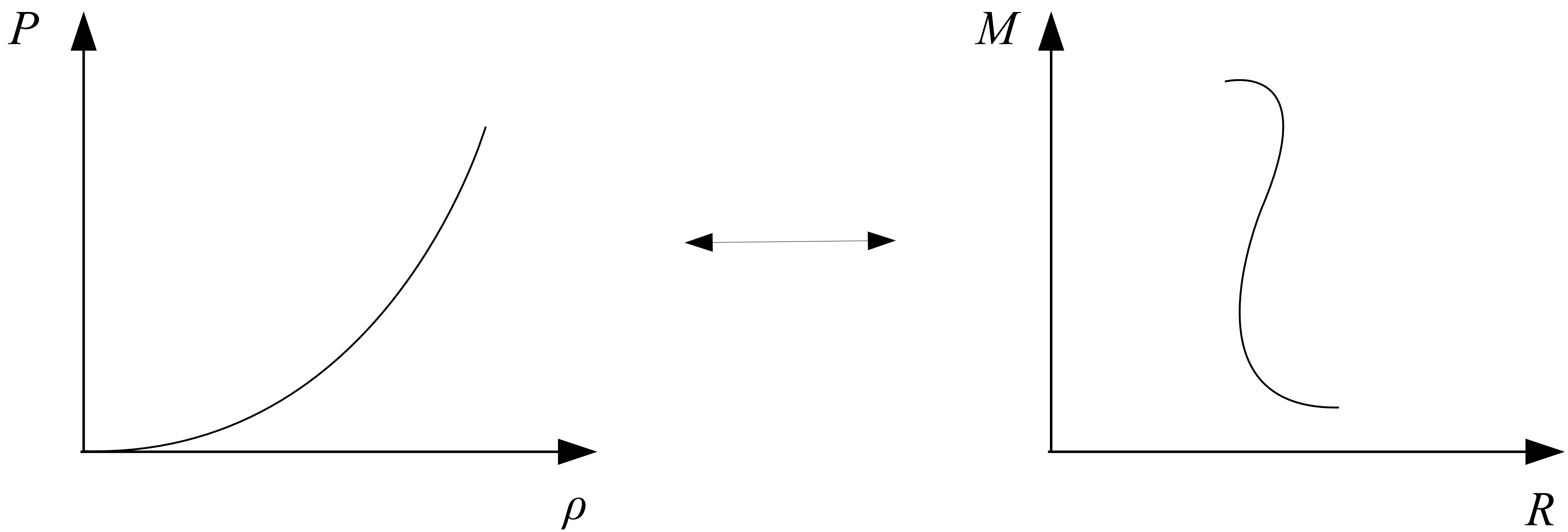
Relativistic hydrostatic equilibrium

Putting it all together: $$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dP(r)}{dr} = -\frac{Gm(r)\rho(r)}{r^2}\left[1+\frac{P(r)}{\rho(r)c^2}\right]\left[1+\frac{4\pi r^3 P(r)}{m(r)c^2}\right]\left[1-\frac{2Gm(r)}{c^2 r}\right]^{-1}$$

Pressure vs density

Mass vs radius



Neutron astrophysics

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How does a neutron star stay there?

Reminder: thermal pressure not an option. Instead, we need **degeneracy pressure**.

Acknowledgments

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