



BERKELEY LAB

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Low-Energy Nuclear Structure

Lecture 2

Heather Crawford

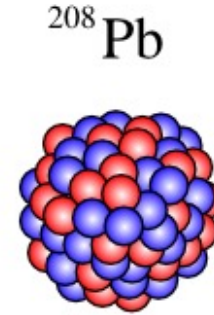
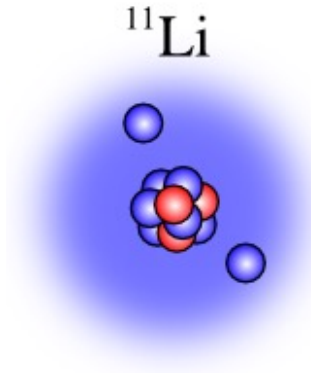
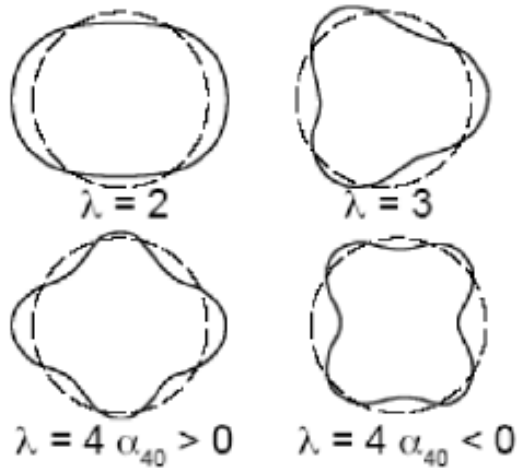
Nuclear Science Division, Lawrence Berkeley National Laboratory

2023 National Nuclear Physics Summer School (NNPSS)

Ground State Properties

Nuclear radii and nuclear shapes

A fundamental property of the ground state is the shape and size of the nucleus – the nuclear radius provides insight into nuclear extent (matter and charge).



The nuclear shape can deviate from spherical, but most frequently maintains axial symmetry – e.g. quadrupole deformation.

Nuclear radii definitions

$$\langle r_c^2 \rangle = \frac{\int_0^R \rho(r) r^2 dr}{\int_0^R \rho(r) dr}$$

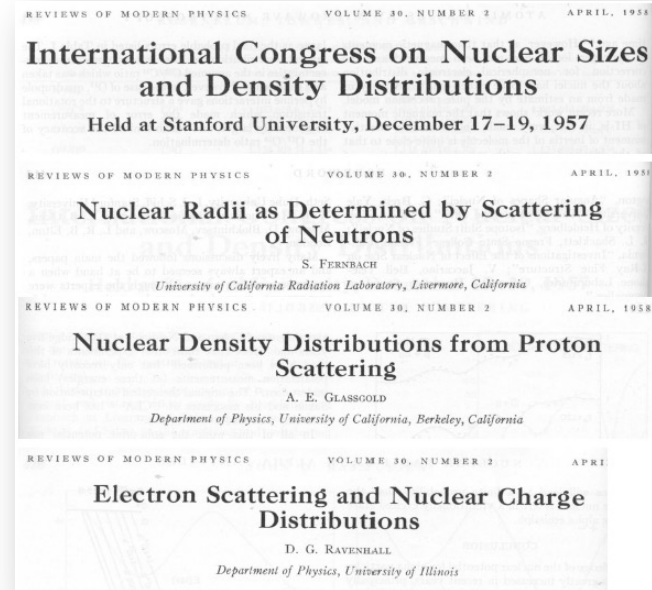
$$\langle r_m^2 \rangle^{1/2}$$

Consider RMS radii
(matter and
neutron)

$$\langle r_n^2 \rangle^{1/2}$$

Nuclear quadrupole deformation

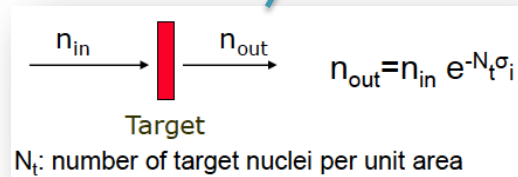
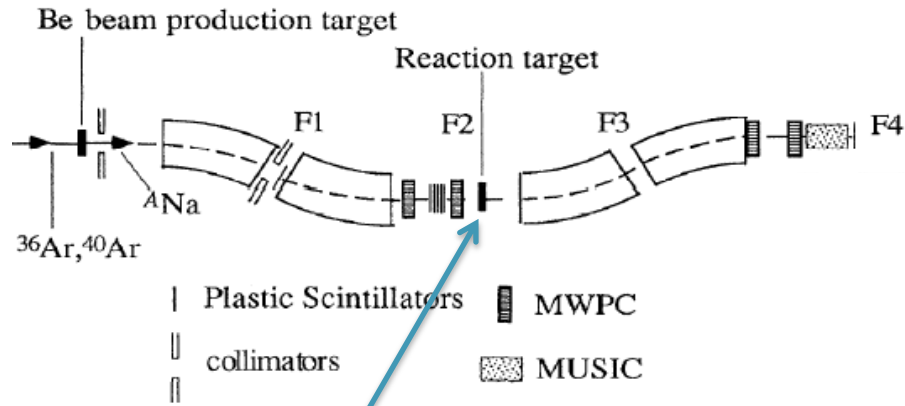
$$R = R_0 \left[1 + \sum_{\mu=-2}^2 a_{2\mu} Y_{2\mu}(\theta, \phi) \right]$$



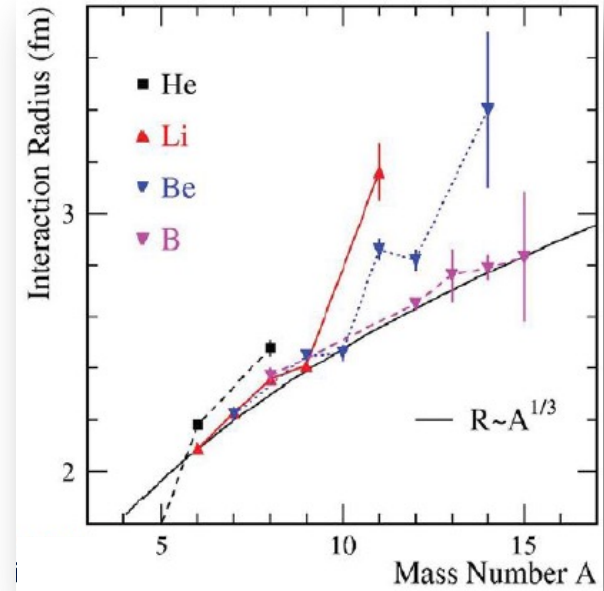
Near stability we know:

$$R = r_0 A^{1/3}$$

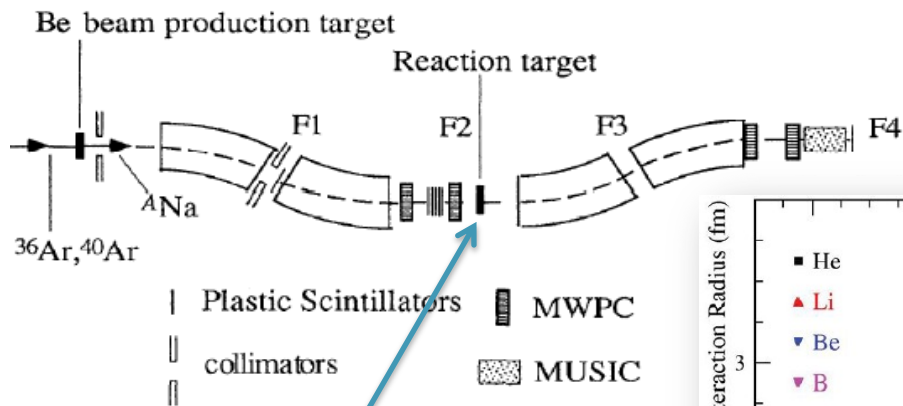
Matter radii: total interaction cross-sections



Theoretical model: $\sigma_i = \pi[R_i(p) + R_i(t)]^2$



Matter radii: total interaction cross-sections



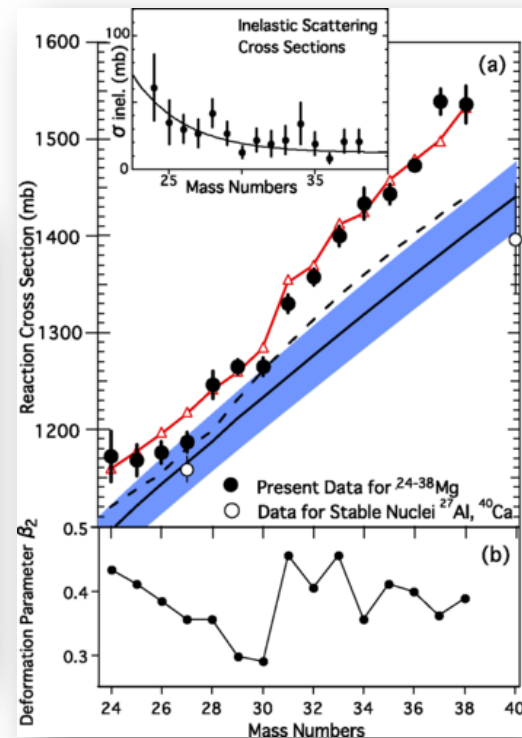
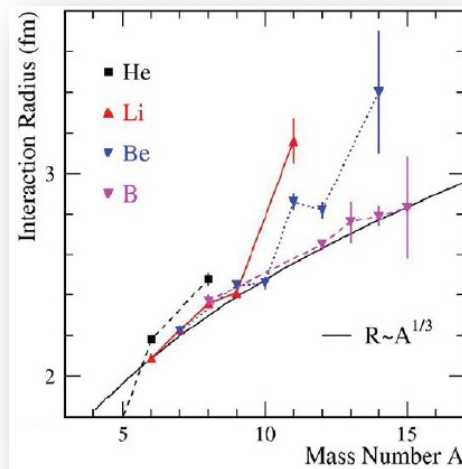
$n_{\text{in}} \rightarrow n_{\text{out}}$

Target

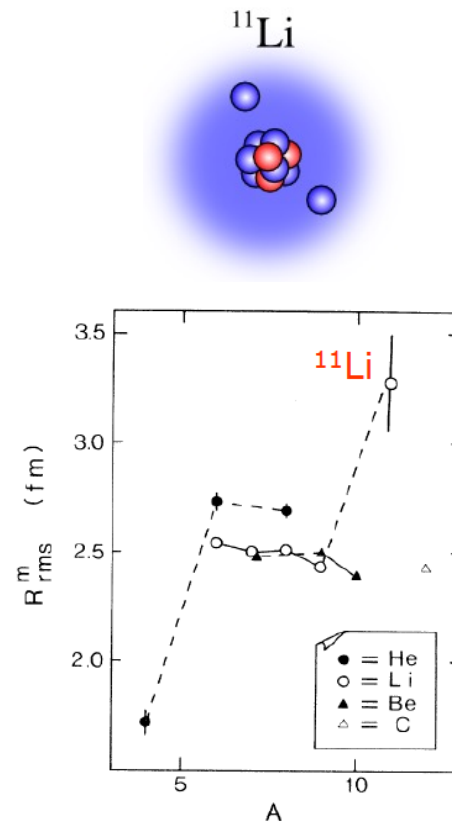
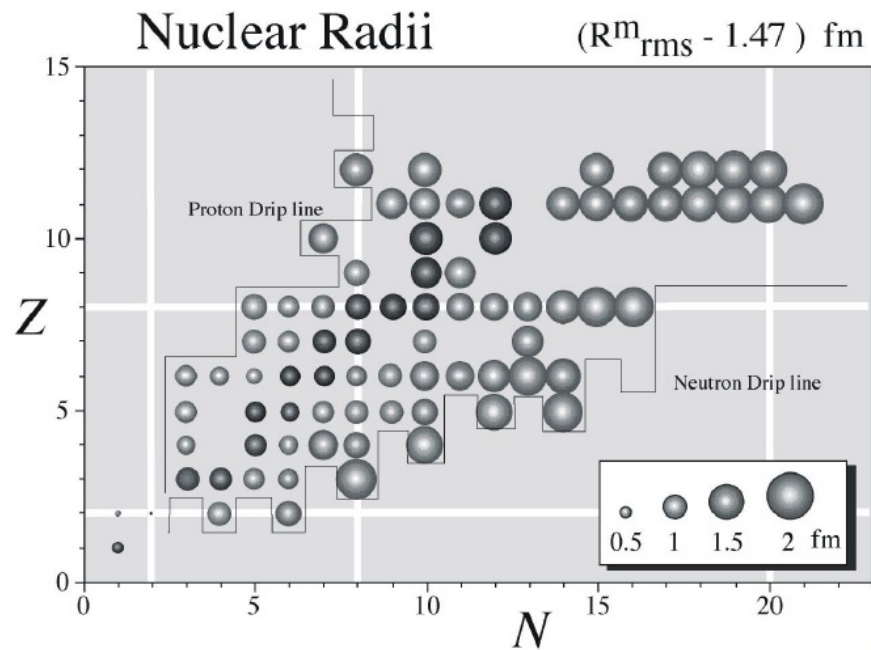
$n_{\text{out}} = n_{\text{in}} e^{-N_t \sigma_i}$

N_t : number of target nuclei per unit area

Theoretical model: $\sigma_i = \pi[R_i(p) + R_i(t)]^2$



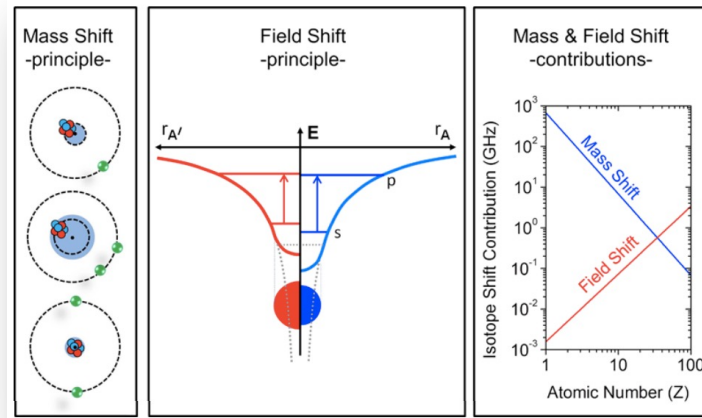
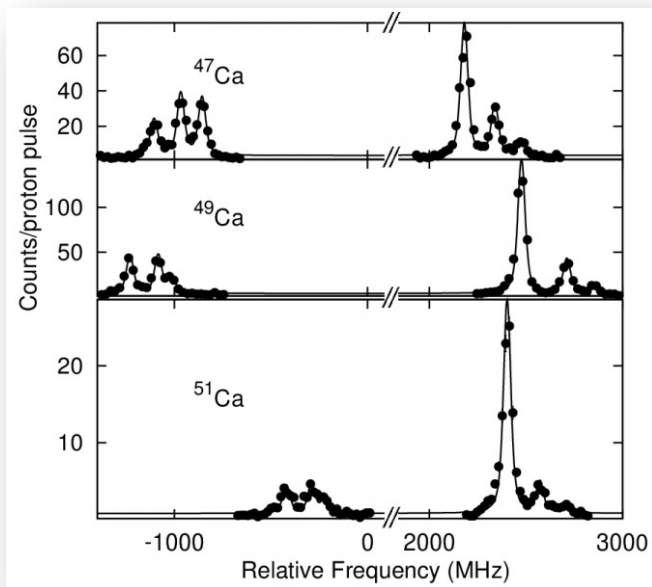
Skins and halos



I. Tanihata, Phys. Rev. Lett. 55, 2676 (1985).

Laser spectroscopy for charge radii → isotope shifts

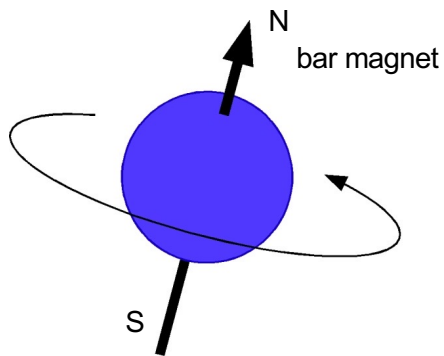
$$\delta\nu_{IS}^{AA'} = \delta\nu_{MS}^{AA'} + F \delta\langle r_c^2 \rangle^{AA'}$$



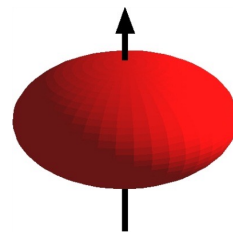
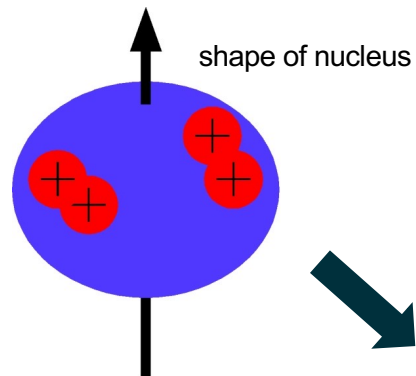
The finite size and mass of the atomic nucleus has a distinct influence on the optical spectrum, which can be probed with high precision using laser spectroscopy.

Ground state nuclear moments

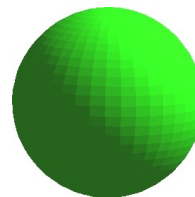
Magnetic moment - μ



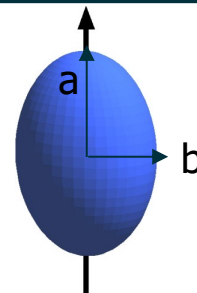
Quadrupole moment - Q



Oblate: $Q < 0$
frisbee



Spherical: $Q = 0$
baseball



Prolate: $Q > 0$
football

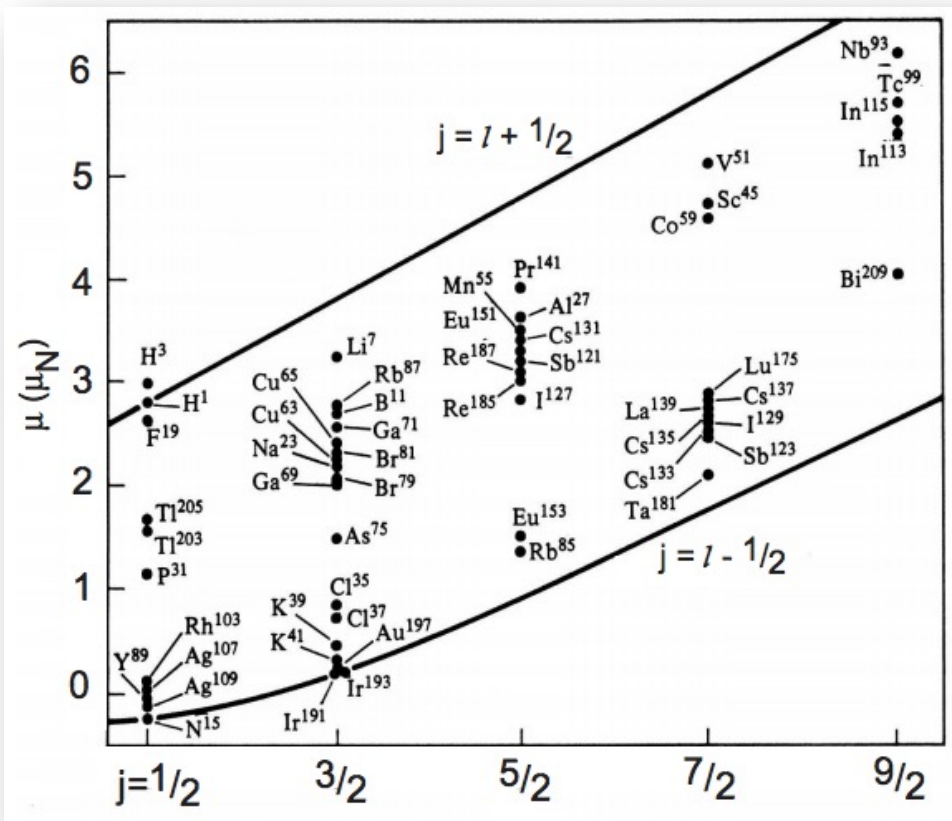
Magnetic moments

$$\mu = \int \psi_{J,M}^* (\vec{\mu})_z \psi_{J,M}$$

$$\vec{\mu} = \sum_{k=1}^A g_L^{(k)} \vec{L}^{(k)} + \sum_{k=1}^A g_S^{(k)} \vec{S}^{(k)}$$



$$\mu_{s.p.} = j \left[g_l \pm \frac{g_s - g_l}{2l + 1} \right] \text{ for } j = l \pm \frac{1}{2}$$



Hyperfine structure

Hyperfine structure refers to the splitting of a single electronic level for nuclei with $I > 0$

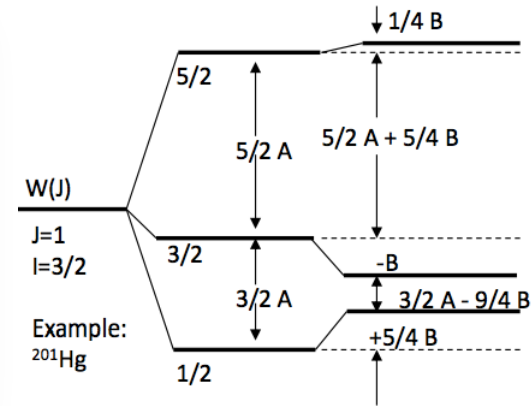
$$\Delta E_{mag} = |g_I| \cdot \mu_N \cdot B + \frac{1}{2} Q \cdot V_{zz}$$

Derived properties of nuclei:

- **Spin** (orbital+intrinsic angular momentum), **parity** (I^π)
- Nuclear ***g*-factor** and **magnetic dipole moment** (g_I and μ_I)
- Electric quadrupole moment (Q)
- **Charge radius** ($\langle r^2 \rangle$)

Give information on:

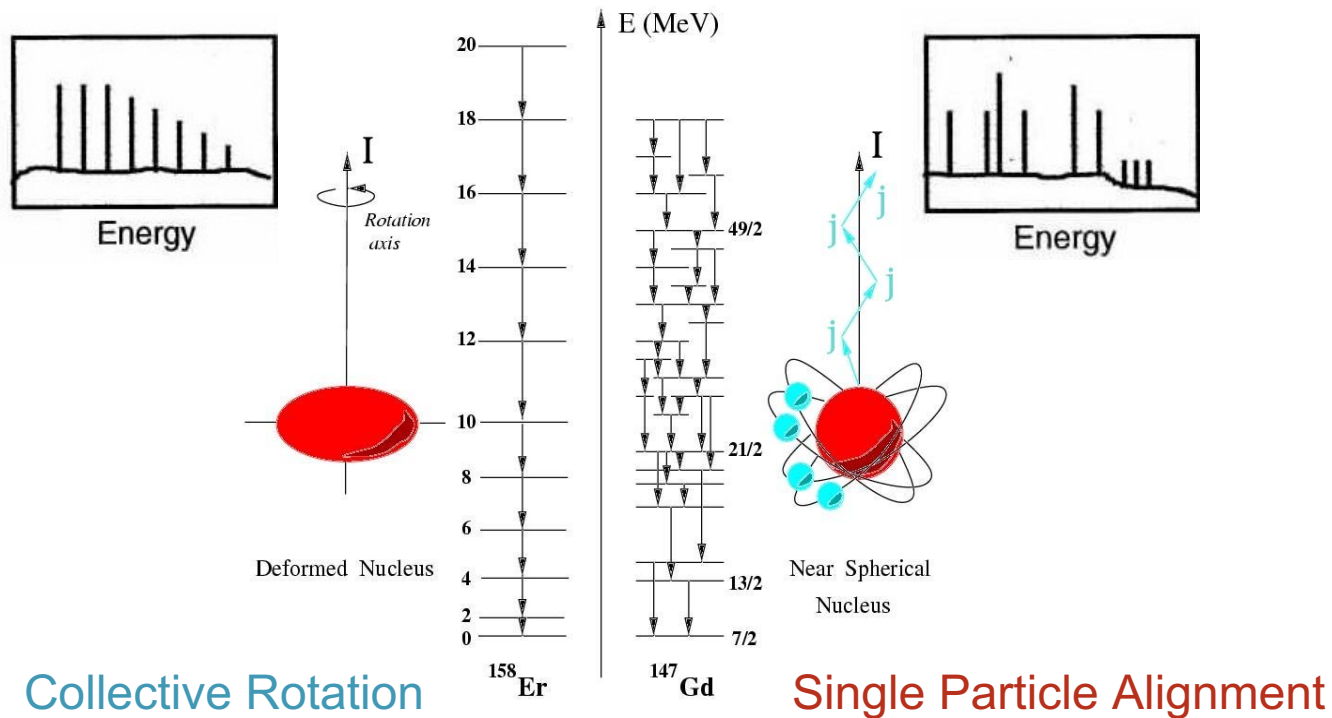
- Configuration of neutrons and protons in nucleus
- Size and form of nucleus



Excitation Spectra

Level schemes – collective vs. single particle

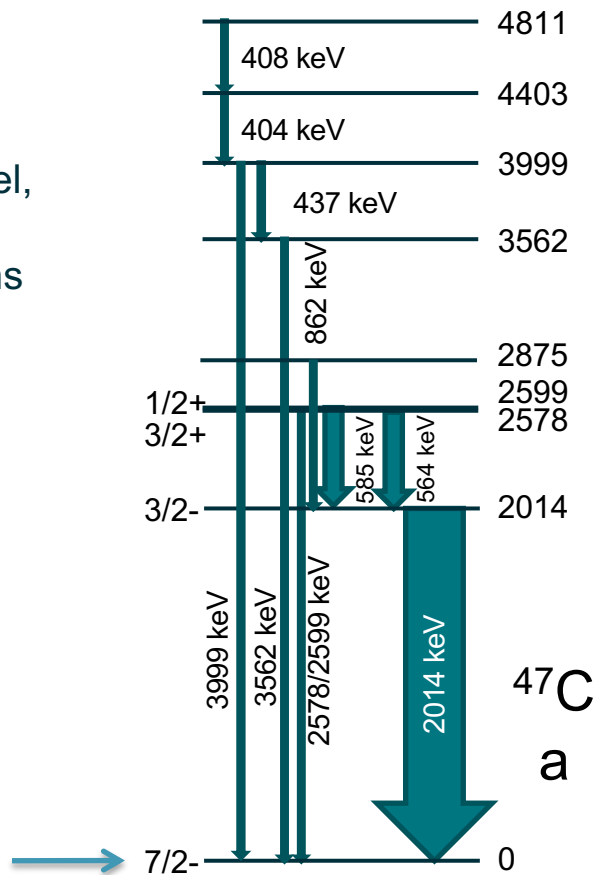
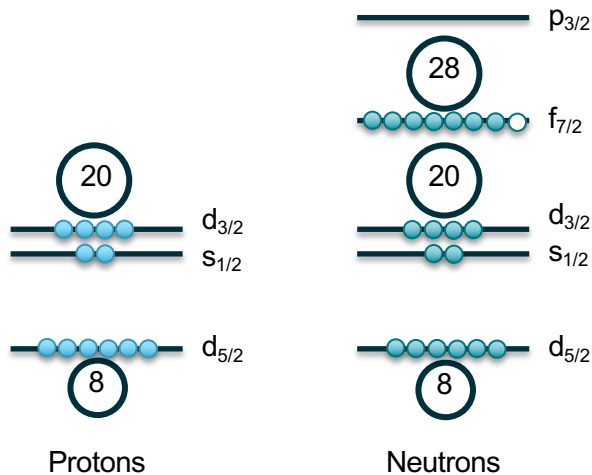
Level Schemes Contain Structural Information



Nuclear excitations

Single particle excitations

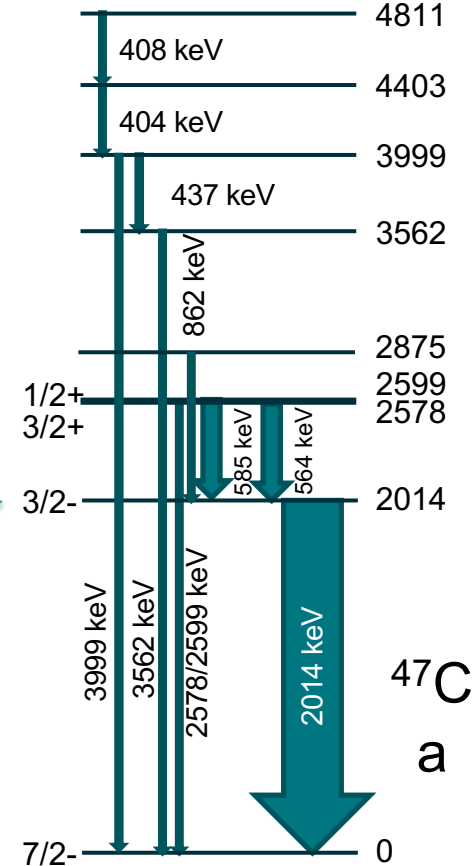
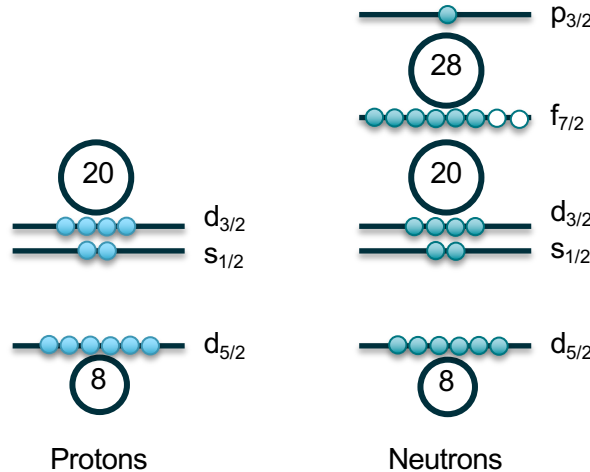
- Within an independent particle model, a subset of nuclear excitations correspond to different configurations of protons and neutrons



Nuclear excitations

Single particle excitations

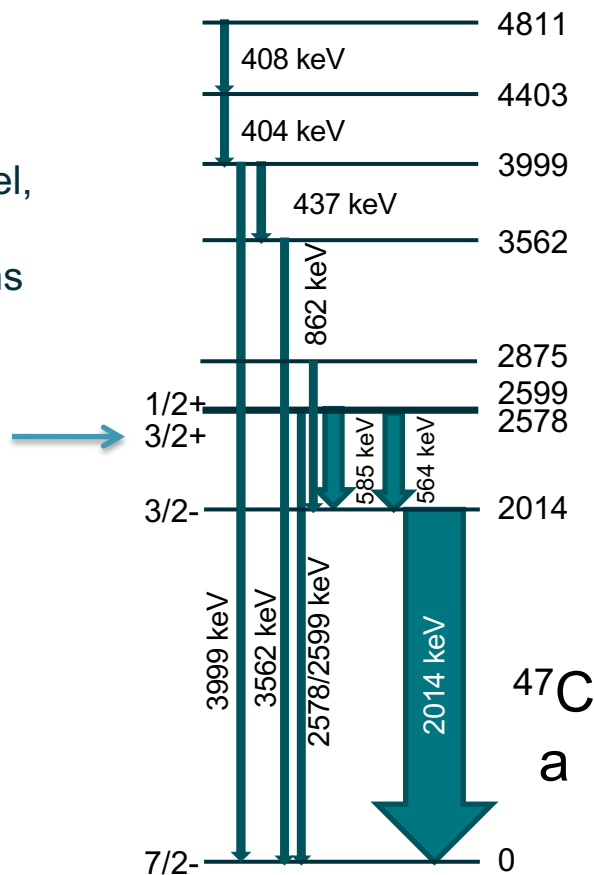
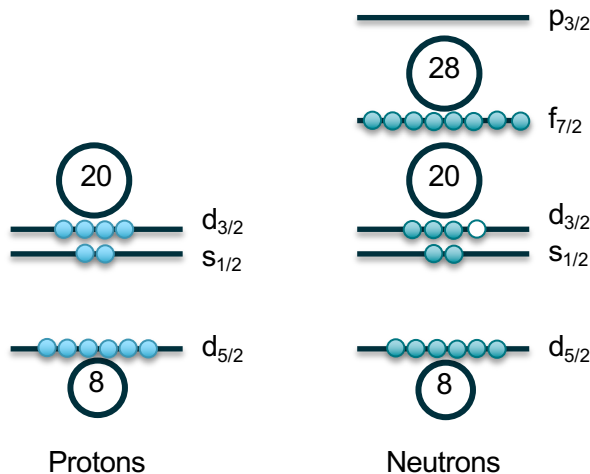
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Nuclear excitations

Single particle excitations

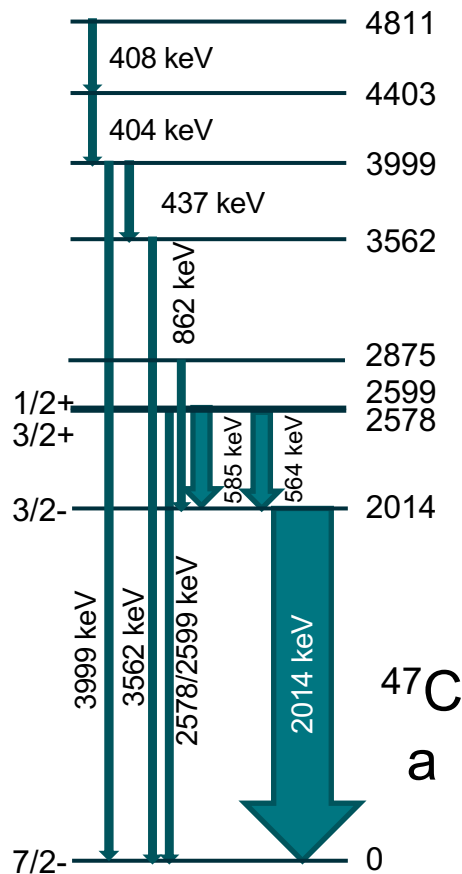
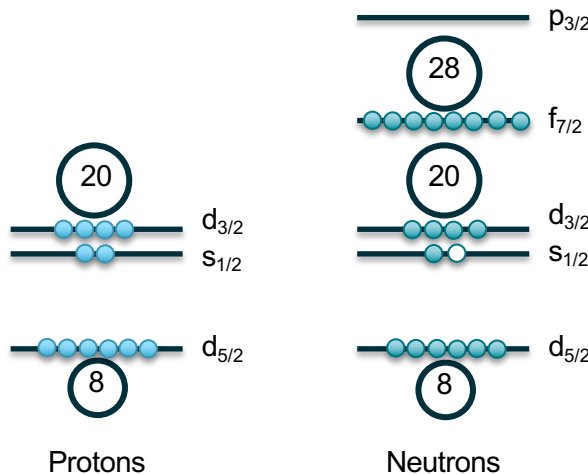
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Nuclear excitations

Single particle excitations

- Within an independent particle model, a subset of nuclear excitations correspond to different configurations of protons and neutrons



Nuclear excitations

Collective excitations

- Many nucleons outside a closed shell contribute coherently to excitations
- Vibrations and rotations (for non-spherical nuclei) have excitation energies comparable to single-particle energy excitations

Nuclear vibration

Treat nuclear vibrations as time-dependent deformation

$$R(\theta, \phi) = R_0 \left(1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu}^* Y_{\lambda\mu}(\theta, \phi) \right)$$
$$H_{\text{vib}} = \frac{1}{2} \sum_{\lambda\mu} (B_{\lambda} |\dot{\alpha}_{\lambda\mu}|^2 + C_{\lambda} |\alpha_{\lambda\mu}|^2)$$

n=3 ————— 0,2,3,4,6+

n=2 ————— 0,2,4+

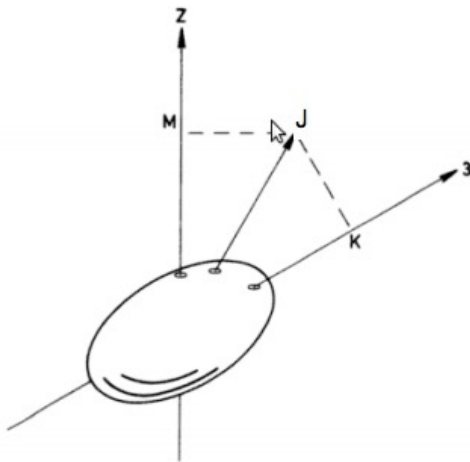
n=1 ————— 2+

————— 0+

Give rise to characteristic excitation spectra – vibration phonons couple as angular momenta

i.e. Quadrupole vibrations

Nuclear rotation



From A. Bohr and B. R. Mottelson.
Nuclear structure, volume 2

$$E_{\text{rot}}(J) = \hbar^2/2I \times J(J+1)$$

I = Moment of inertia

Deformed nuclei can also undergo collective rotational motion; nuclear rotation is parameterized in the same way as classical rotors

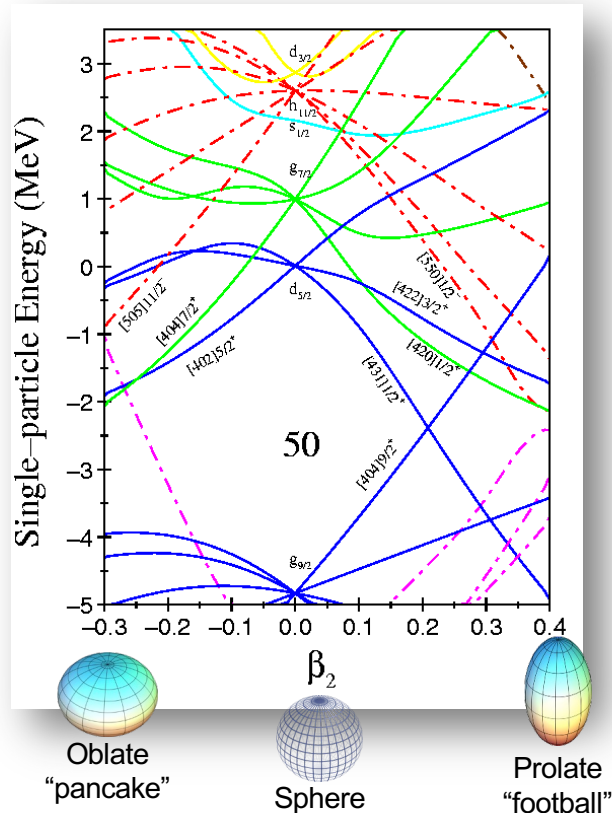
6+

4+

2+

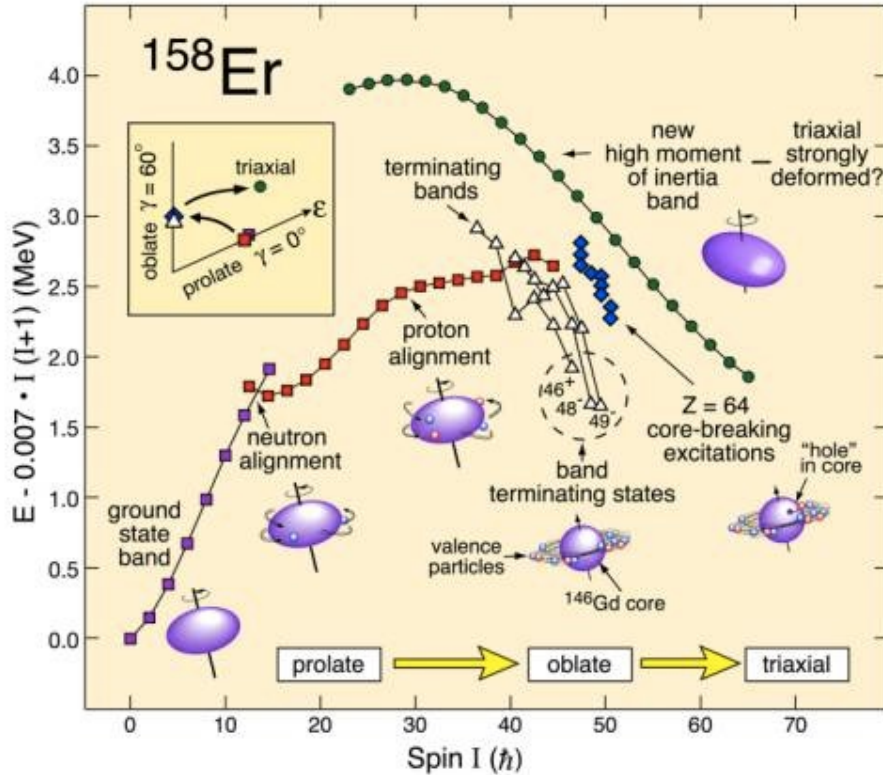
0+

Deformation and the Nilsson model



- Nuclear rotation is a collective excitation, but interfaces to single-particle structure
- Nilsson model is a shell-model description in a deformed basis, which provides a good description in well-deformed nuclei

Moment of inertia in nuclei

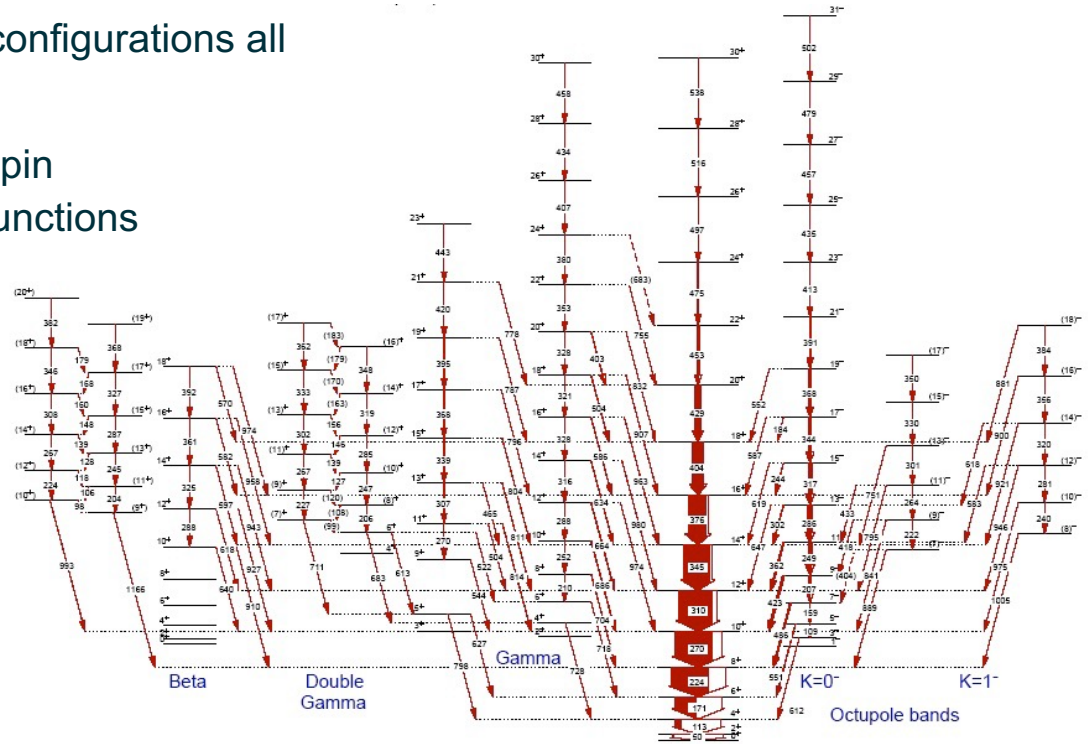


- Rigid body estimate for the moment of inertia is consistently larger than experimental data
- Irrotational flow value (like a liquid drop...) is too small
- Data puts the nuclear moment of inertia between these two limits; moment of inertia dynamic

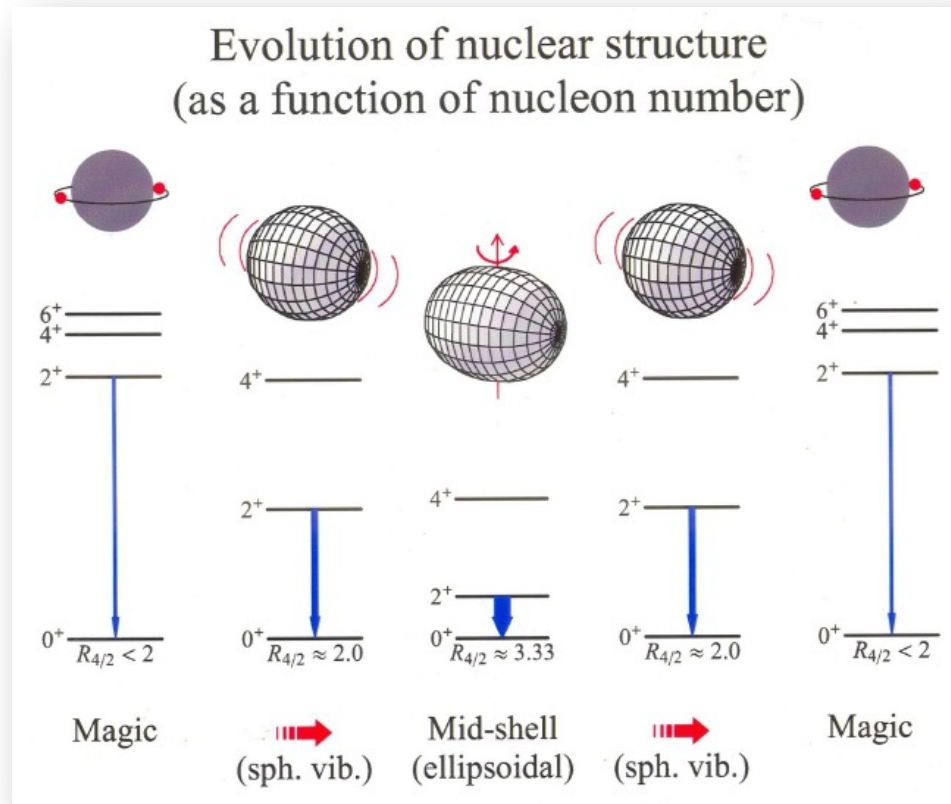
Excitations in the real world

Nuclei are not limited to a single type of excitation – vibration, rotation and single-particle configurations all coexist at similar excitation energies.

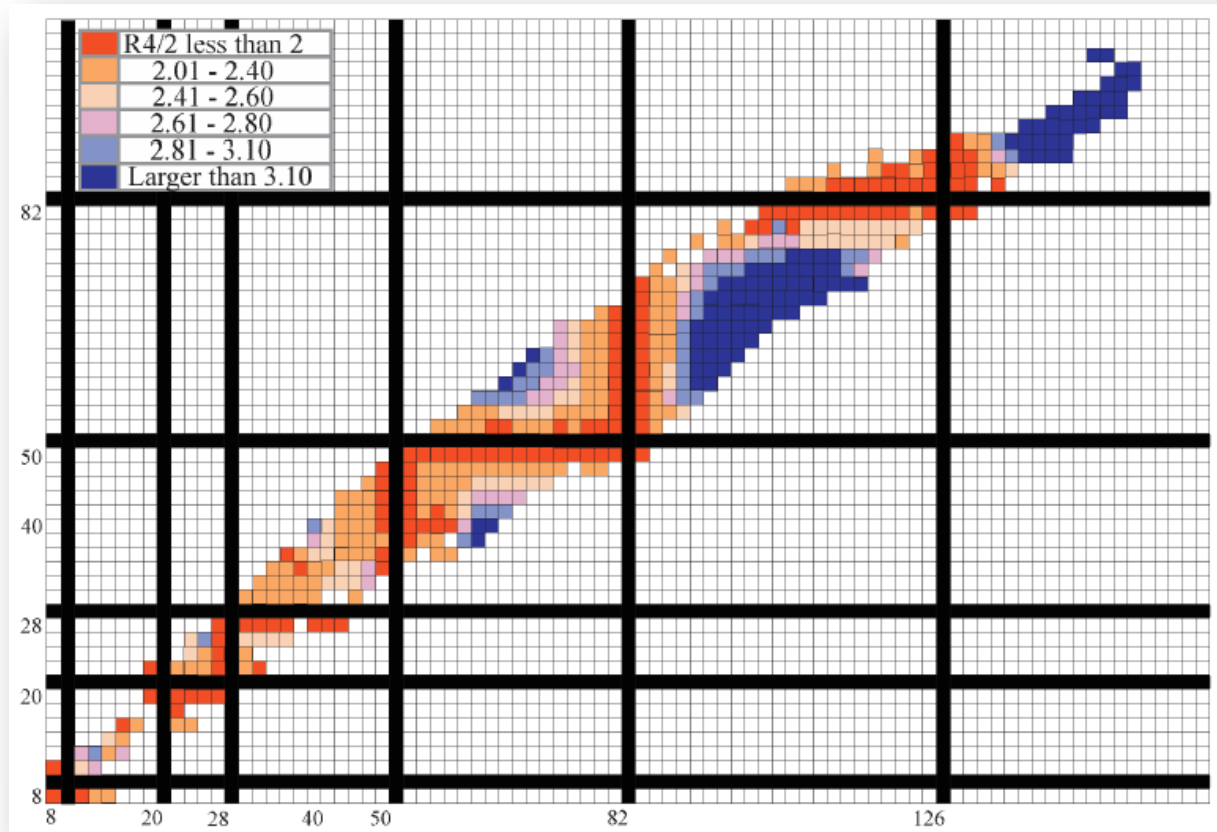
States near in energy with the same spin interact and interfere – nuclear wavefunctions are complex superpositions of ‘pure’ configurations.



Simple patterns still tell us about structure



$R_{4/2}$ – A powerful ratio



Question!

In ^{42}Si , a gamma-ray from 2^+ to 0^+ is observed at 742 keV, and a gamma-ray from the 4^+ state to the 2^+ state is observed at 2032 keV. What can we say about the excitation?

- (A) Nothing
- (B) It's pretty rotational – deformed
- (C) It seems vibrational
- (D) It's unbound



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$$E(4+)/E(2+) = (1431 + 742)\text{keV} / 742 \text{ keV} \\ = 2.9$$



Gamma Rays!

How can we build level schemes?

Nuclear excited state decay

- Excited states in nuclei can decay in a number of ways:
 - β^+ , β^- , electron capture (EC) -- $^{177}\text{Lu}^m$
 - Particle emission -- $^{53}\text{Co}^m$, $^{211}\text{Po}^m$
 - Fission $^{239}\text{Pu}^m$
 - Internal conversion
 - Gamma-ray emission

Dominant Excited State Decay

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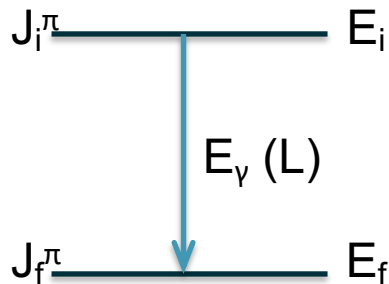
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Nuclear properties from gamma-ray studies

- Coincidence relation --> Level schemes
- Angular distribution/correlation --> Multipolarity, spin
- Doppler shifts --> excited state lifetimes
- Linear polarization --> E/M, parity
- Intensity of transitions --> $B(E2)$

Selection Rules

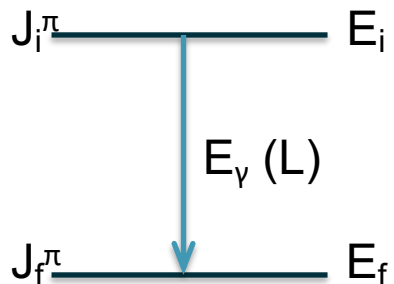


$$E_\gamma = E_i - E_f$$
$$|J_i - J_f| \leq L \leq J_i + J_f$$

$$\Delta\pi(EL) = (-1)^L$$

$$\Delta\pi(ML) = (-1)^{L+1}$$

Selection Rules

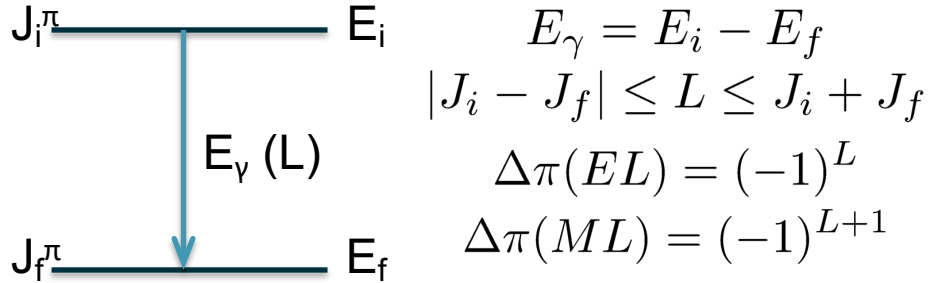


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The transition probability for a state decaying by transition of multipole order L is:

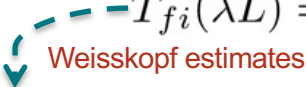

$$T_{fi}(\lambda L) = \frac{8\pi(L+1)}{\hbar L((2L+1)!!)^2} \left(\frac{E_\gamma}{\hbar c} \right)^{2L+1} B(\lambda L : J_i \rightarrow J_f)$$

Selection Rules



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 Weisskopf estimates
 

Reduced matrix element – i.e.

$T(E1) = 1.03 \times 10^{24} A^{2/3} E_\gamma^3$
 $T(E2) = 7.28 \times 10^7 A^{4/3} E_\gamma^5$
 $T(M1) = 3.15 \times 10^{13} E_\gamma^3$
 $T(M2) = 2.24 \times 10^7 A^{4/3} E_\gamma^5$
 \dots

$$B(E2 : J_i \rightarrow J_f) = \frac{1}{2J_i + 1} \langle \psi_f || E2 || \psi_i \rangle^2$$

Lifetimes and Gamma Decay

- The bulk of electromagnetic (gamma) transitions have lifetimes of $10^{-15} - 10^{-13}$ s
 - Explains why excited states primarily undergo gamma decay (compare to beta-decay lifetimes » ms, or alpha decay » s)

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Recoil-distance (plunger) method

The lifetime of excited states in the range of 10-100s of ps can be measured by populating the state via Coulomb excitation or knock-out reactions, and observing the Doppler-shift of the decay gamma-ray.

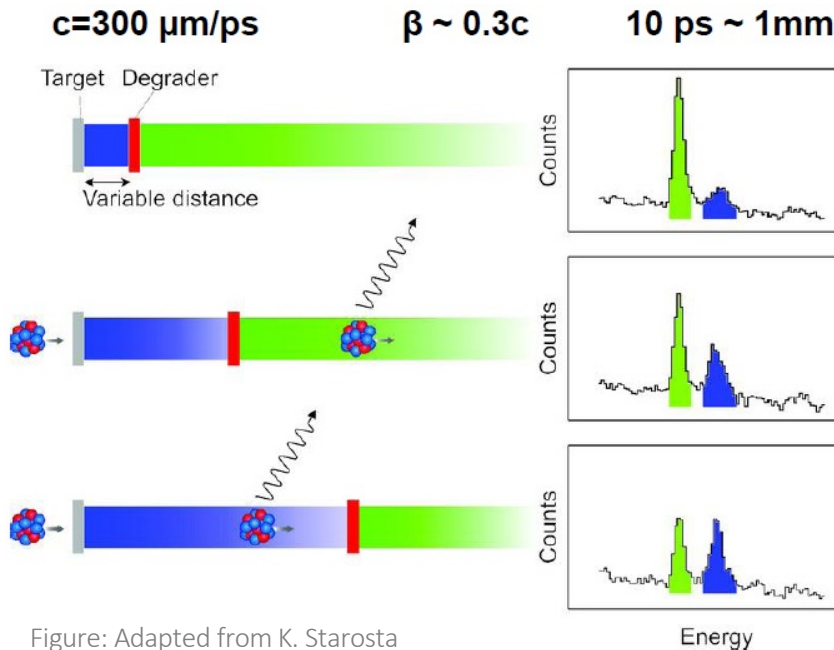


Figure: Adapted from K. Starosta

Question!

What would you expect to be the dominant character of the gamma-ray transition linking the second 0^+ excited state at 1.06 MeV in ^{32}Mg with the ground state (0^+)?

- (A) E1
- (B) M2
- (C) No gamma transition
- (D) M1



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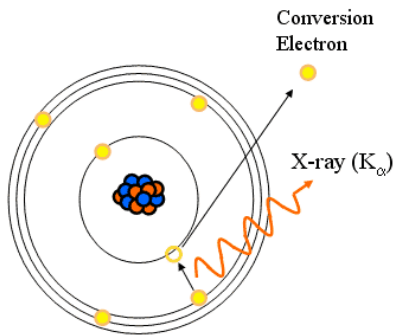
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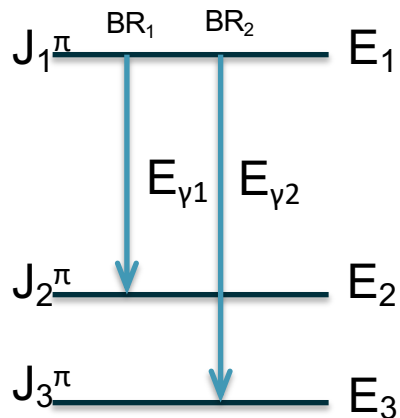


Gamma rays must carry at least one \hbar of angular momentum – cannot link two 0^+ states
When gamma transition is not possible, internal conversion is an alternative electromagnetic transition.



Properties of Gamma Decay

- Energies \rightarrow spacing between nuclear levels
- Lifetimes \rightarrow information about transition probabilities, links to nuclear matrix elements (structure!)
- Intensities \rightarrow experiment dependent – generally relates to transition probabilities (branching ratios)

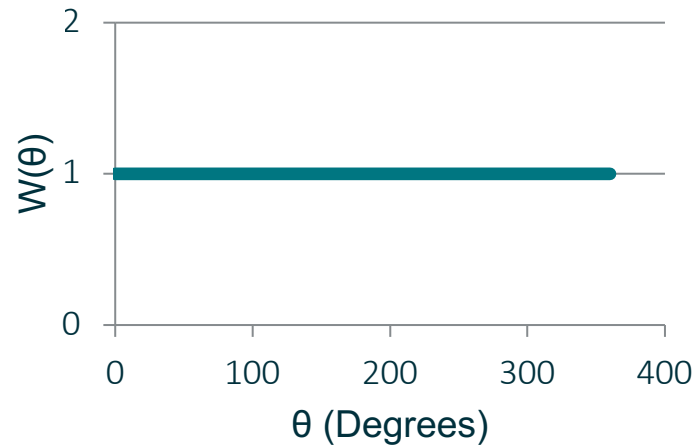
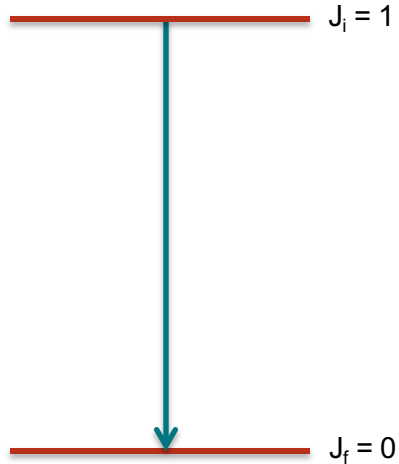


Properties of Gamma Decay

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-
- Knowledge of J_i and J_f limit the multipolarity (L) of gamma-ray transitions
 - To measure multipole order (L) we can measure angular distributions
 - To determine E vs. M we need to measure polarization of the transition

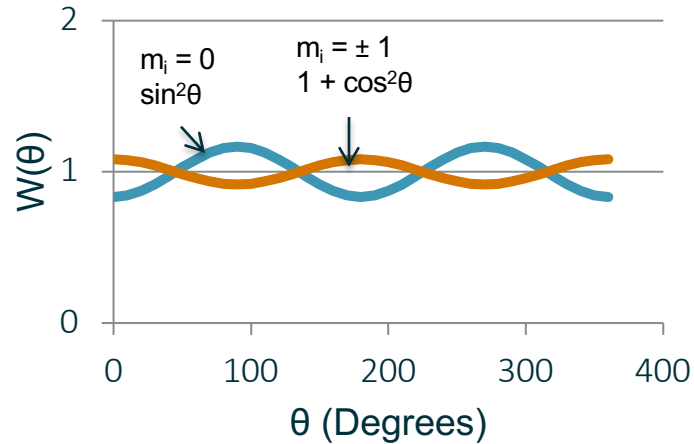
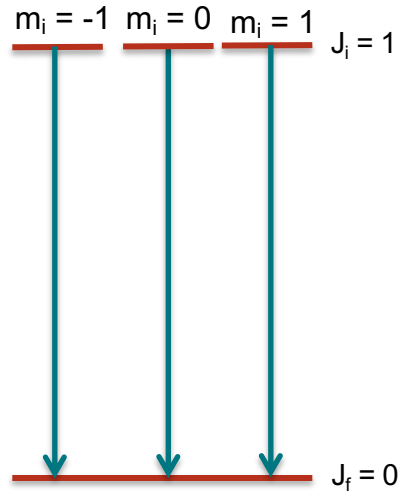
Gamma-Ray Angular Distributions

- Angular distribution of a gamma-ray depends on the values of m_i and m_f



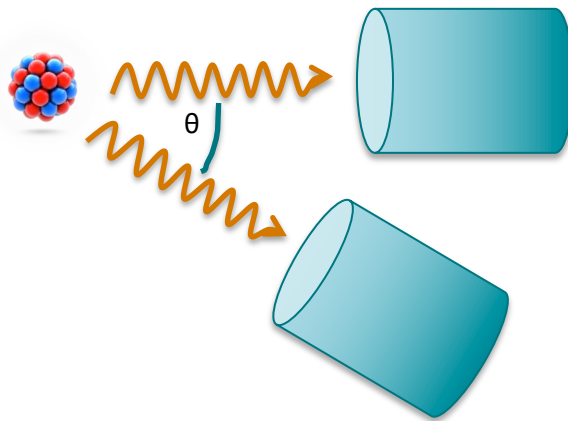
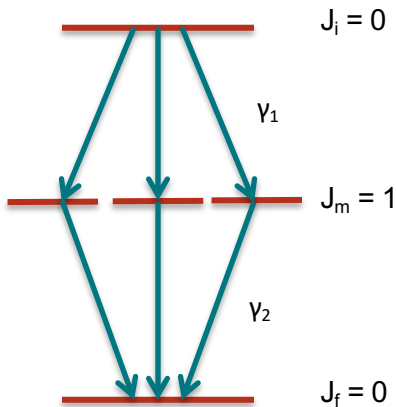
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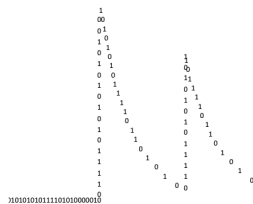
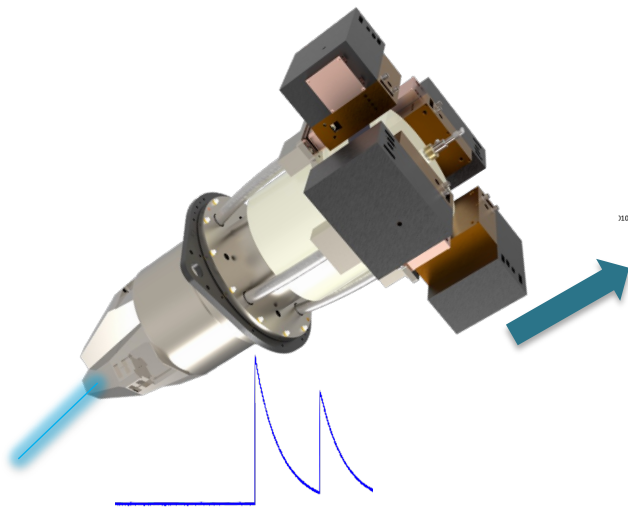
Gamma-Ray Angular Correlations

- Observation of a previous radiation selects an unequal mixture of populations $p(m_i)$



- First gamma defines z-axis -- $\theta_1 = 0$
 - $p(m_m) = 0$ for $m_m = 0$
- Distribution of γ_2 relative to γ_1 is $m = \pm 1 \rightarrow m = 0$
 - $W(\theta) \rightarrow 1 + \cos^2\theta$

Gamma Ray Energy Tracking Array

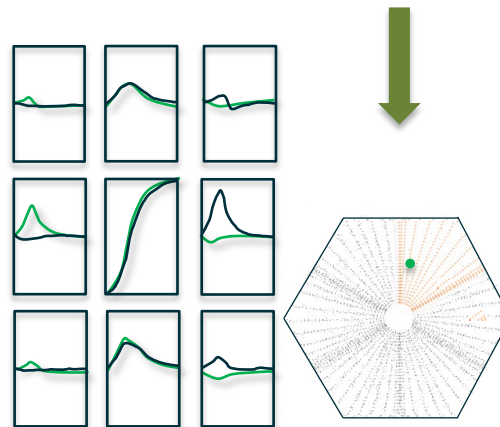


- Continuous 100MHz digitization of **40** preamplifier signals **per crystal**

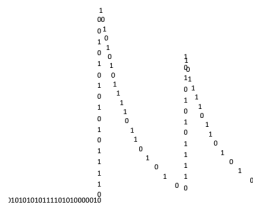
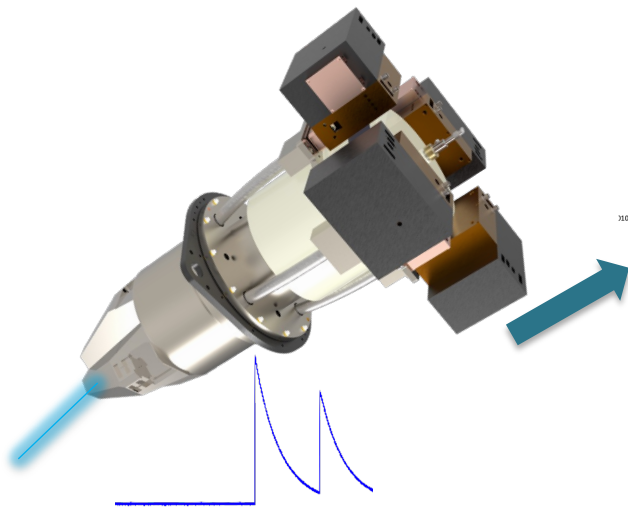


$E, t, \begin{bmatrix} 1 \\ 00 \\ 01 \\ 10 \\ 10 \\ 01 \\ 00 \\ 10 \\ 01 \\ 10 \\ 10 \\ 01 \\ 10 \\ 01 \\ 10 \\ 00 \\ 0000 \end{bmatrix} \times 40$

- FPGA-based energy filters, event selection in response to physics triggers



Gamma Ray Energy Tracking Array



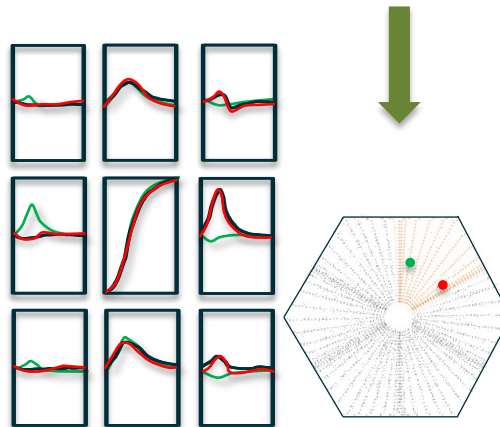
- Continuous 100MHz digitization of **40** preamplifier signals **per crystal**



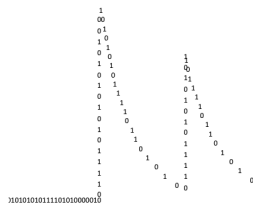
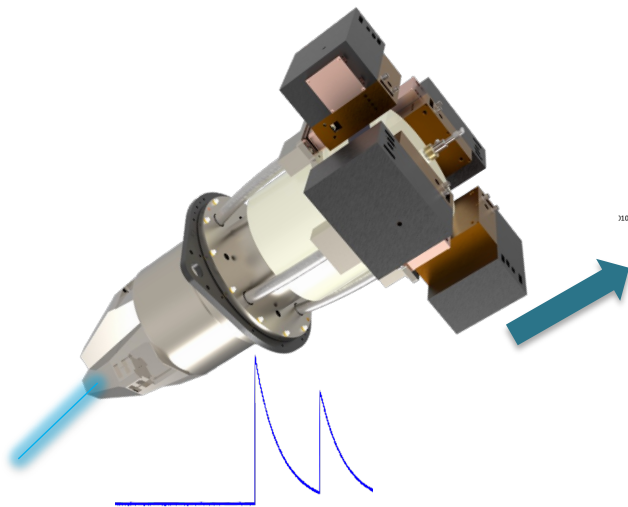
E, t, x 40

	1
00	
01	
10	
1	
0	1
0	
1	1
0	
1	
0	1
1	
0	1
1	
0	1
1	1
0	1
1	0
0	
1	
1	
1	
1	
0	
00010	

- FPGA-based energy filters, event selection in response to physics triggers



Gamma Ray Energy Tracking Array

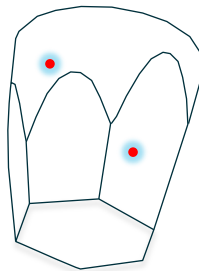


- Continuous 100MHz digitization of **40** preamplifier signals **per crystal**

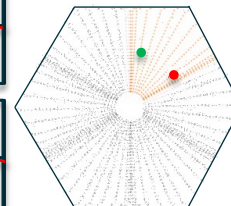
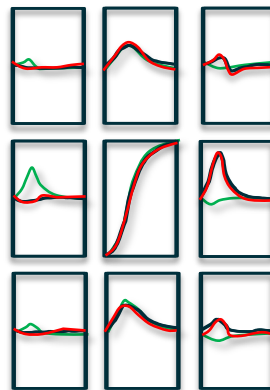


$E, t, \begin{bmatrix} 1 \\ 00 \\ 01 \\ 10 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0000 \end{bmatrix} \times 40$

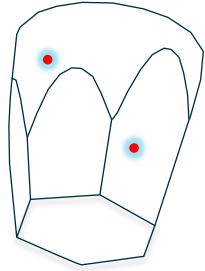
- FPGA-based energy filters, event selection in response to physics triggers



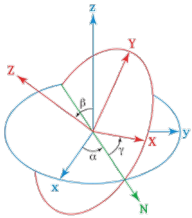
$$\left[E, t, (x, y, z)_{\text{crystal}} \right]_n$$



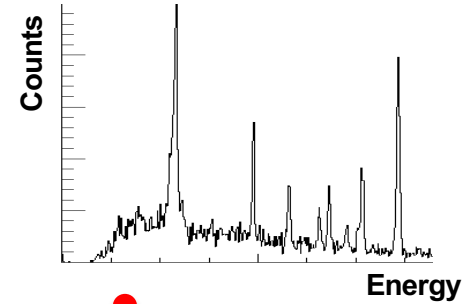
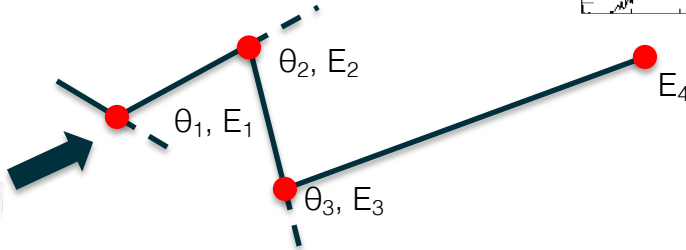
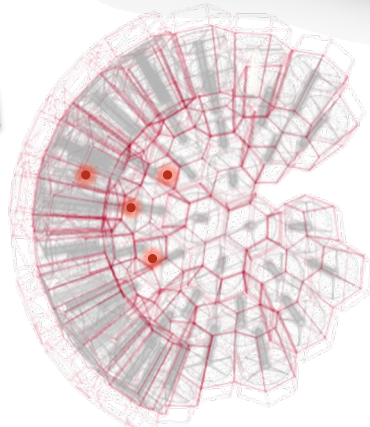
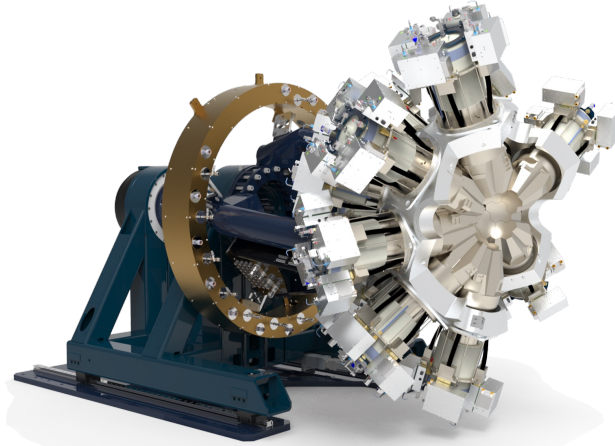
Gamma Ray Energy Tracking Array



$$\left[E, t, (x, y, z)_{\text{crystal}} \right]_n$$



- Detectors located absolutely in space to +/- 0.4 mm

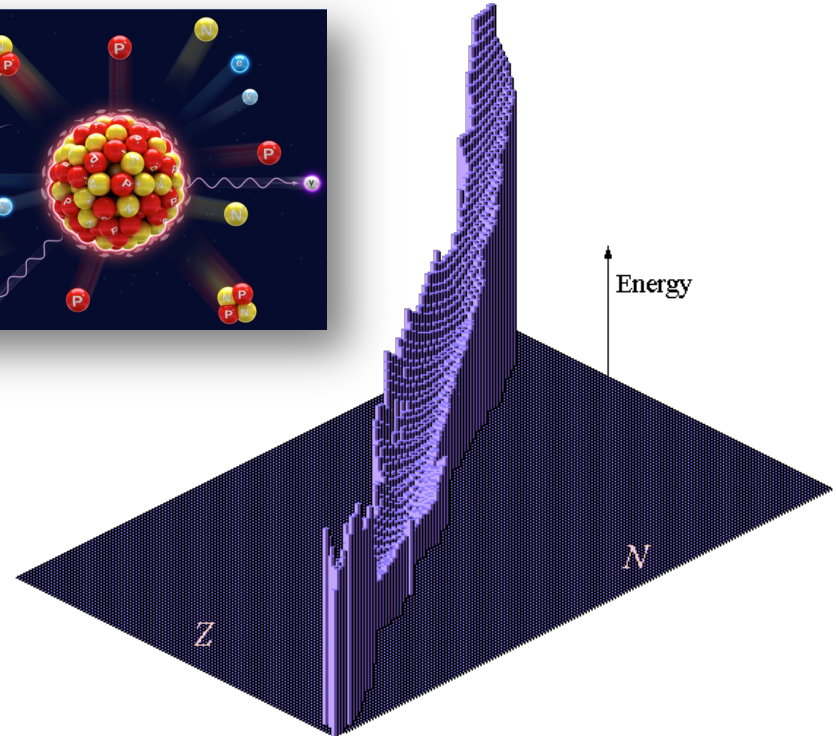
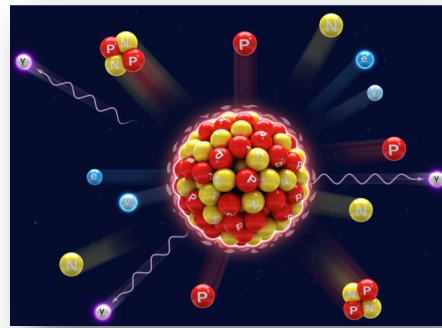


Decay Spectroscopy

Nuclear ground-state decay

Nuclei decay toward stability (and a lower energy state) via one of four basic decay modes:

- Alpha decay ($\rightarrow Z-2, N-2$)
- Beta(-) decay ($\rightarrow Z+1, N-1$)
- Beta(+) decay ($\rightarrow Z-1, N+1$)
- Fission ($\rightarrow 2 \text{ fragments} + n$)
- 1p & 2p radioactivity

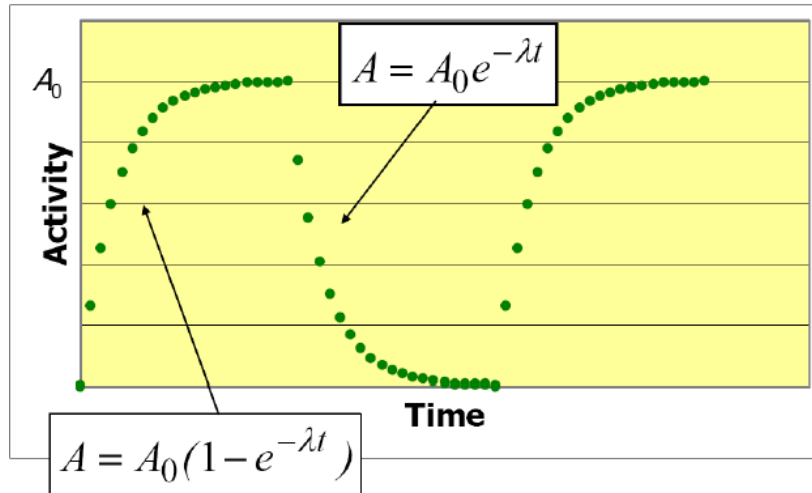


Decay observables

- Nuclear decay measurements allow access to a number of observables
 - Half-life information for decaying state
 - Energies for emitted particles (spectroscopic information in daughter nucleus)
 - Gamma-rays de-exciting daughter states populated in decay
 - Excited state spins and parities based on selection rules for primary decay and subsequent gamma decay

Decay half-lives

All radioactive decay modes obeys Poisson statistics and are described by straight-forward differential equations.



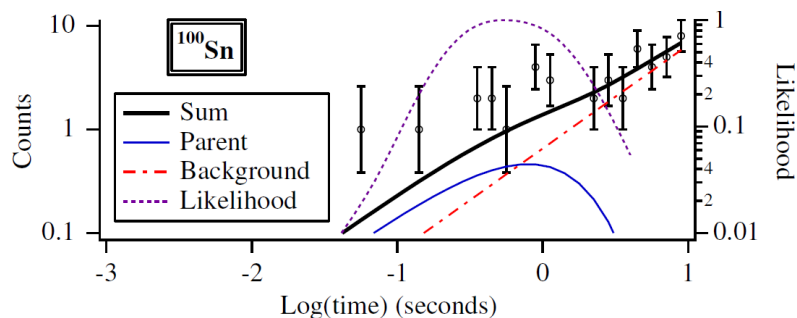
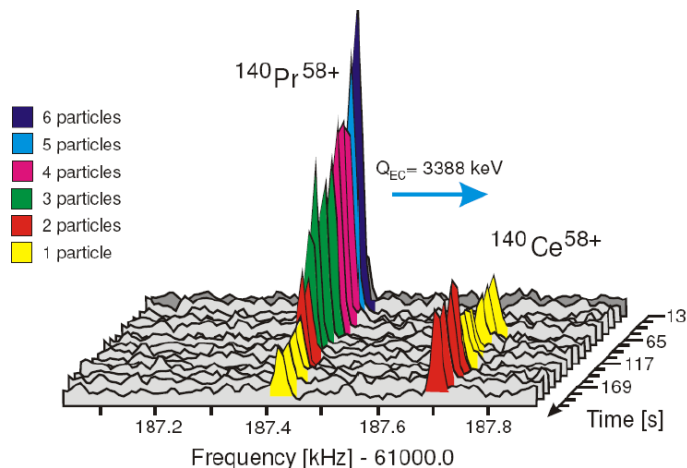
$$A = -dN/dt = \lambda N$$

$$t_{1/2} = \ln(2) / \lambda$$

Decay half-lives

Even with the most limited statistics, half-lives can be extracted

The equations of exponential decay are well known and can be applied using statistical techniques such as maximum likelihood to obtain half-lives from tens of observed decays



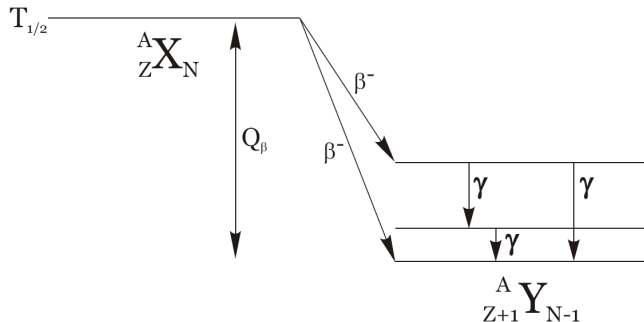
Decay observables

- Nuclear decay measurements allow access to a number of observables
 - Half-life information for decaying state
 - Energies for emitted particles (spectroscopic information in daughter nucleus)
 - Gamma-rays de-exciting daughter states populated in decay
 - Excited state spins and parities based on selection rules for primary decay and subsequent gamma decay
 - Consider β -decay (and other decays) as a tool to populate excited states in daughter nuclei, with a unique selectivity

Forbiddenness	ΔJ	$\Delta\pi$
Superallowed	$0^+ \rightarrow 0^+$	no
Allowed	0, 1	no
First forbidden	0, 1, 2	yes
Second forbidden	1, 2, 3	no
Third forbidden	2, 3, 4	yes

Implantation β decay spectroscopy

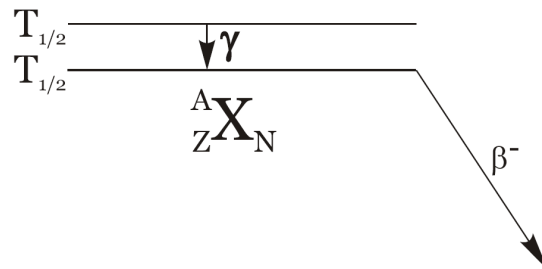
β -Delayed Gamma Spectroscopy



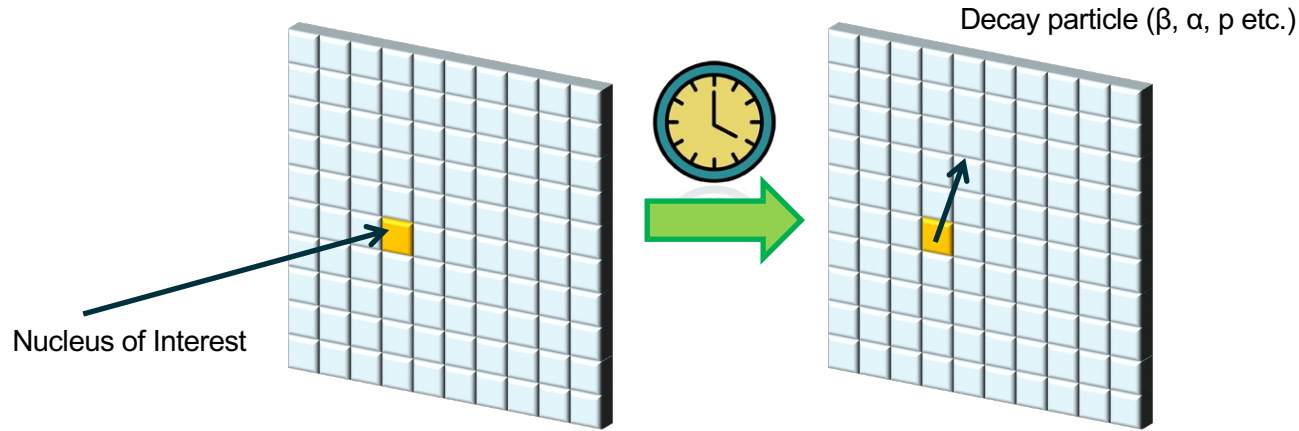
- gamma rays following decay events provide information on low-level structure of daughter nuclei

Isomeric Decay

- depending on the production mechanism, nuclei may be produced in long-lived excited states (isomeric states)
- a TAC for implantation-gamma provides the possibility for isomer lifetime determination, if you look for gammas following an implantation

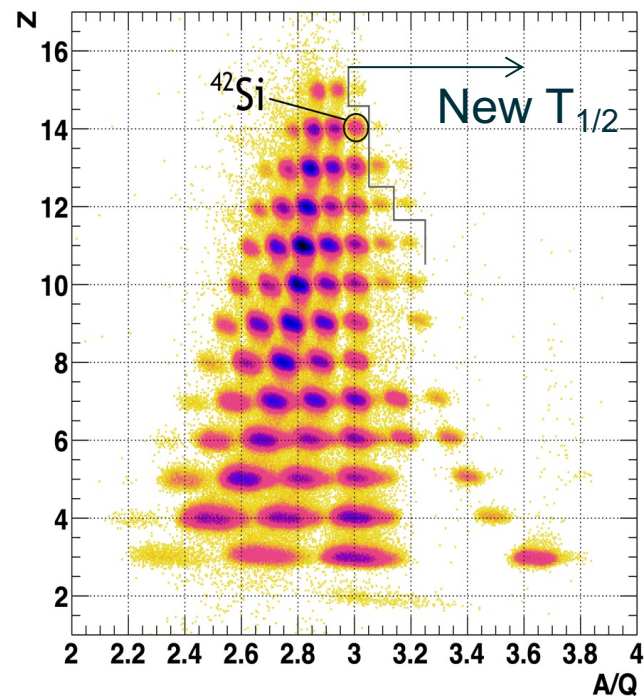
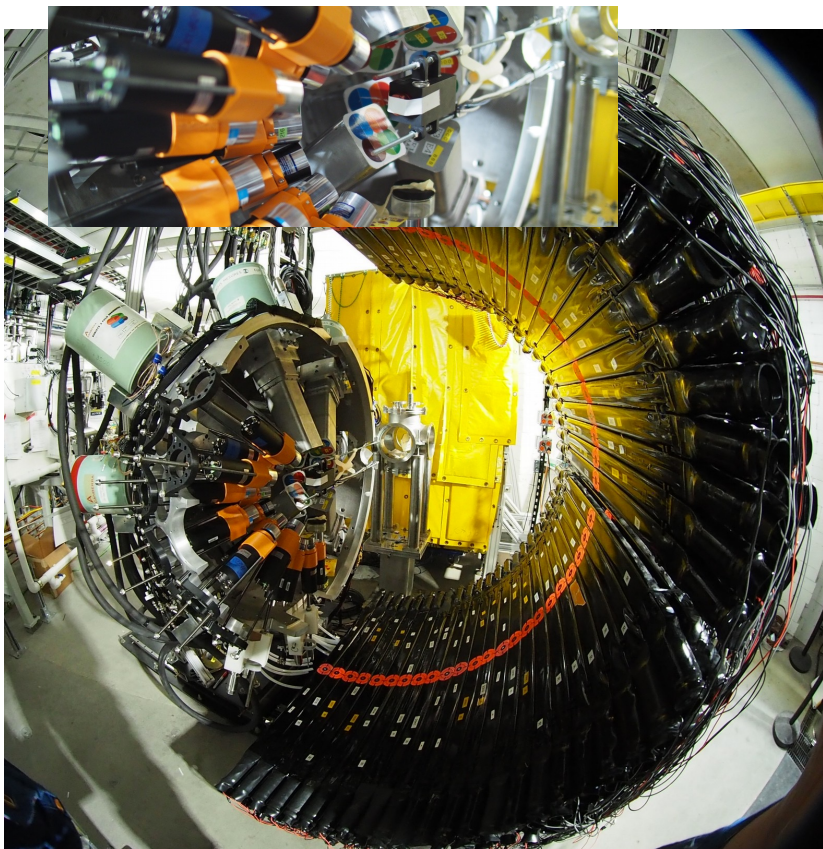


Implant-decay correlation technique



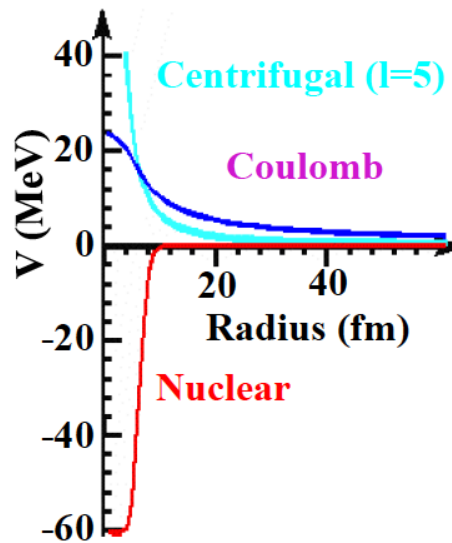
The use of highly-segmented detectors allows temporal and spatial correlations between implanted nuclei, and their subsequent decays → detect the implant and the decay to obtain half-lives and information on levels in the daughter relative to the parent ground state

β -decay spectroscopy at FRIB

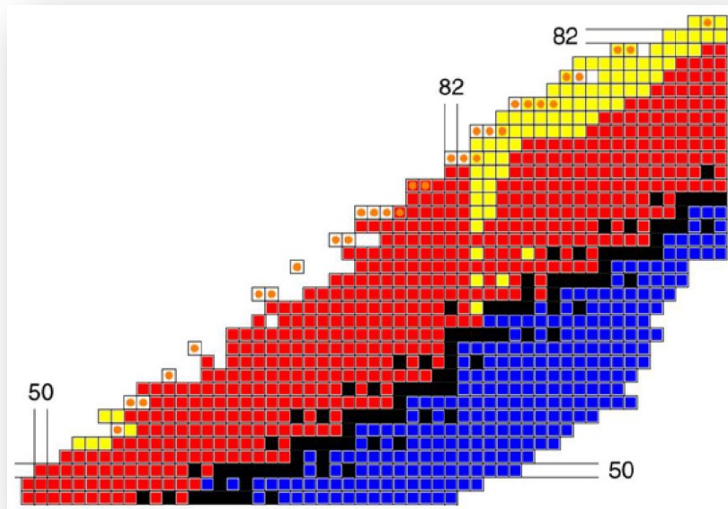


The FRIB Decay Station initiator (FDSi) is being led by ORNL and UTK, and includes (in addition to HPGe), fast timing scintillators, neutron-detection and possibilities for TAS measurements

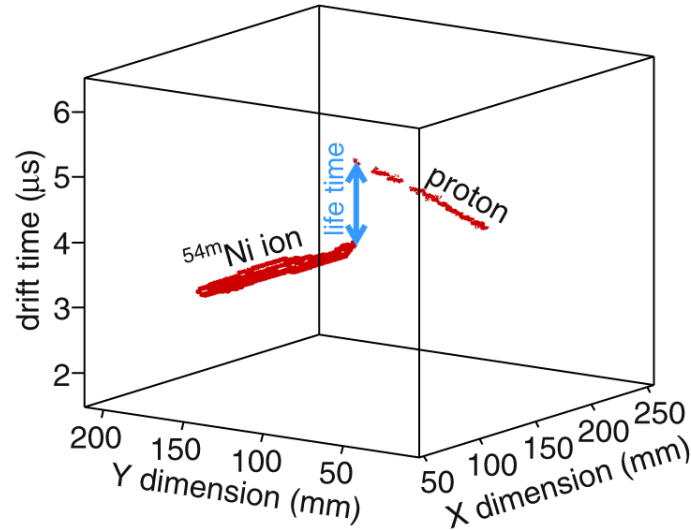
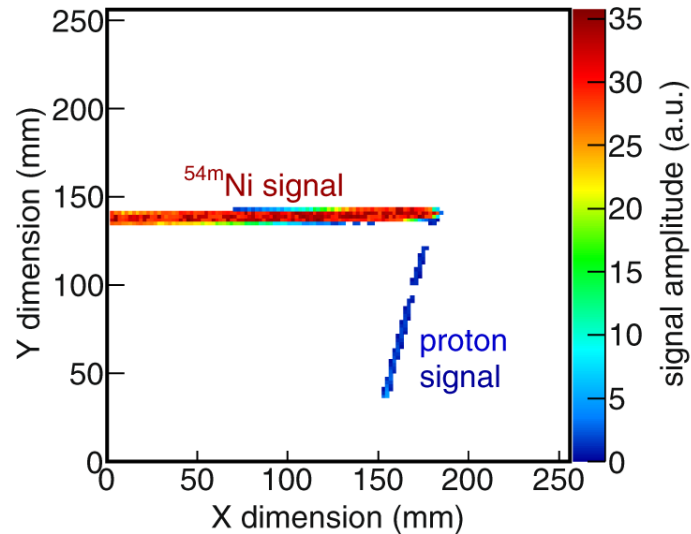
Proton decay



- Even when the Q value for proton removal becomes positive, proton emission is hindered due to the Coulomb (and centrifugal) barriers \rightarrow radioactivity



Proton emission branches in ^{54m}Ni



- A recent experiment with the ACTAR TPC measured proton decay from isomeric states in ^{54}Ni
- Data were reproduced reasonably well with shell-model calculations for the initial and final state wavefunctions and a barrier penetration model for the proton emission

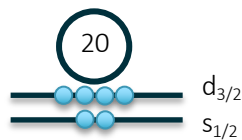
Probing Wavefunctions

Direct Reactions

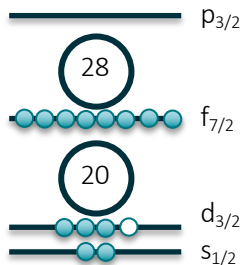
Beyond excitation energies and spins?

Can we probe the details of the wavefunction 'directly'?

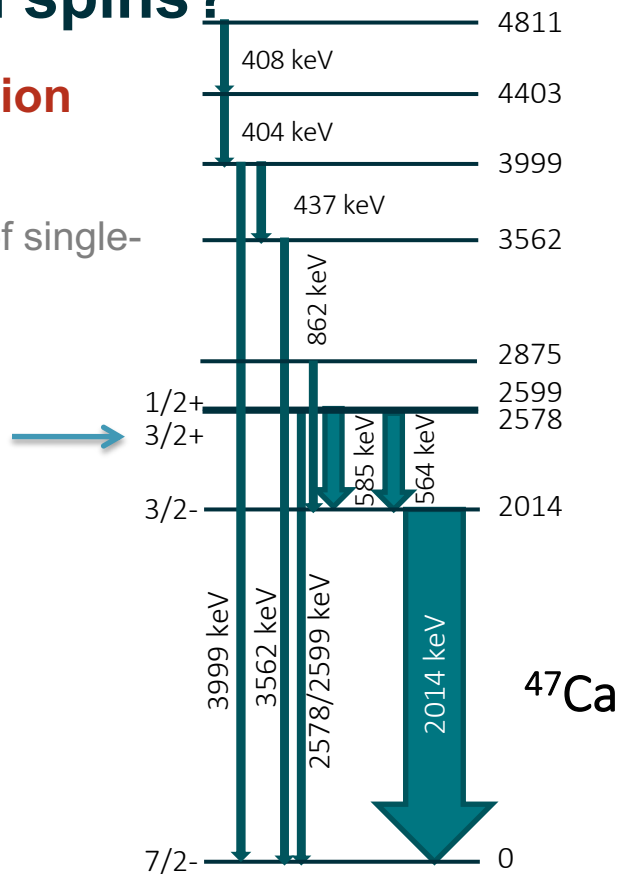
Is there a way to tell where the particles are in terms of single-particle states (even within a specific model)?



Protons

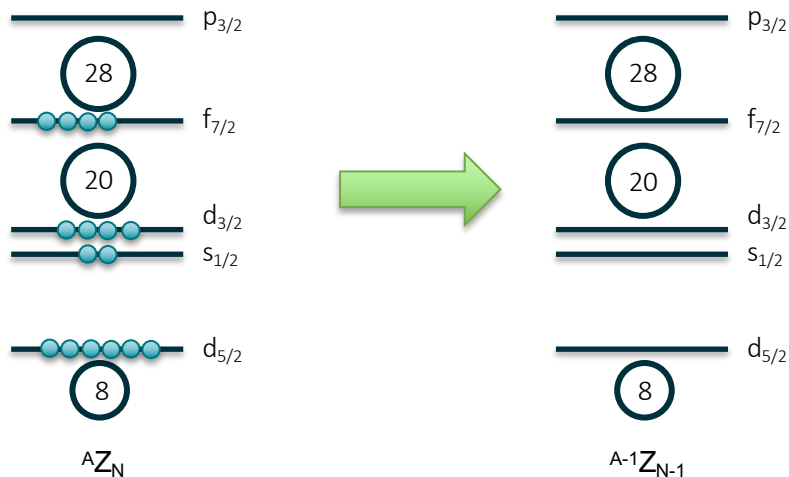


Neutrons



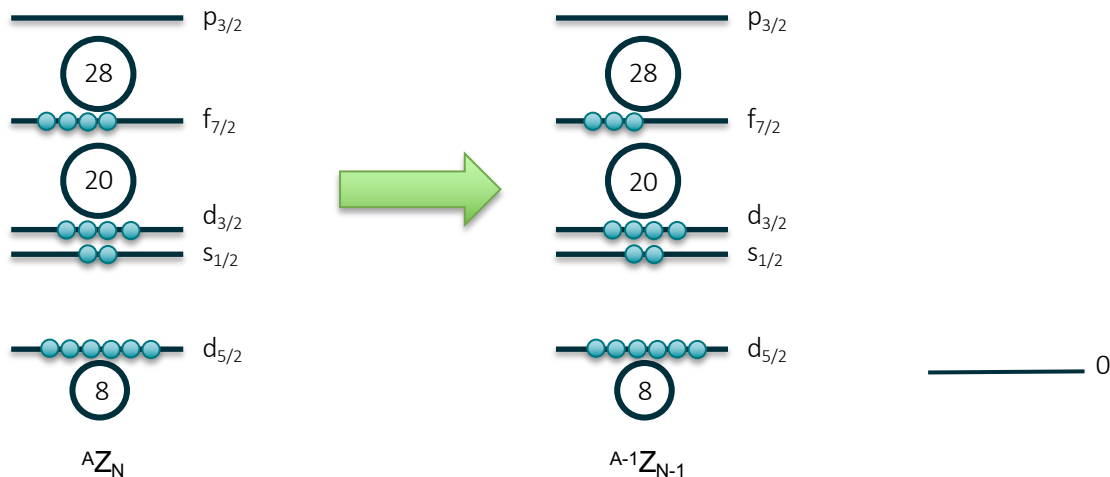
Direct nucleon removal (or addition)

- Information regarding the ‘occupancy’ of single-particle states can be investigated within a model framework
- Two energy regimes --> low-energy transfer experiments and intermediate energy knockout



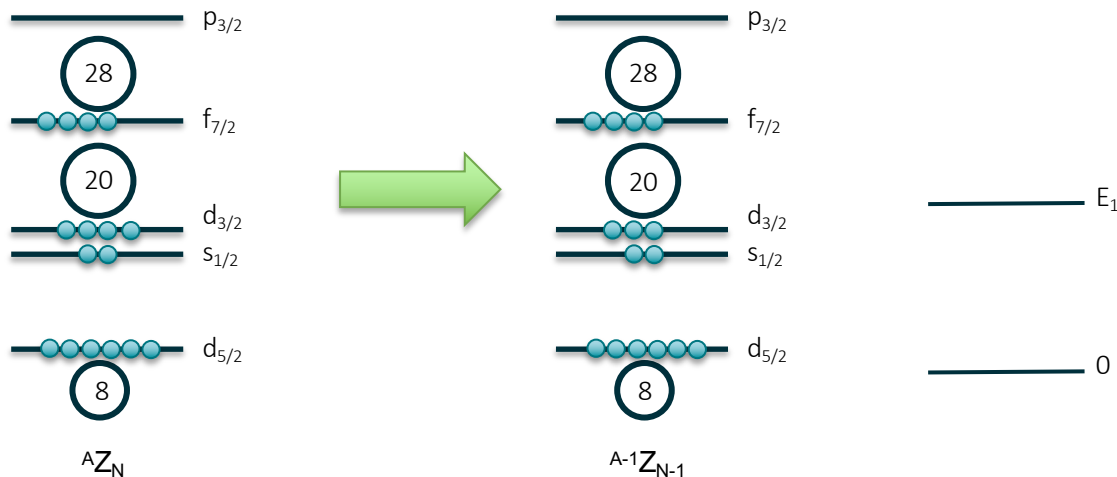
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Direct nucleon removal (or addition)

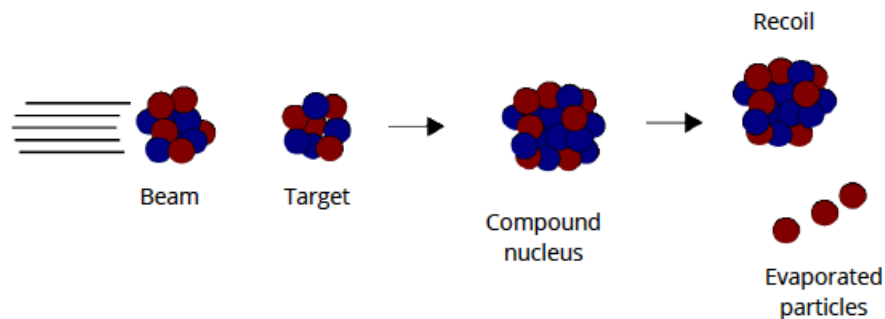
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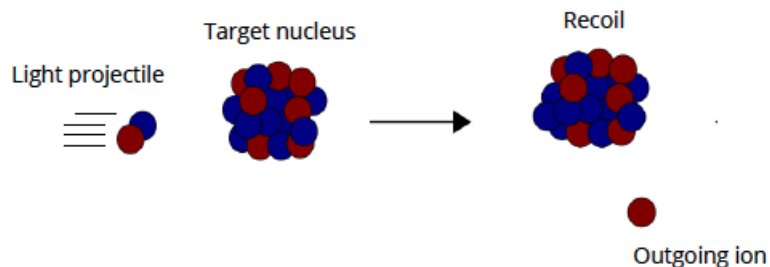
Selectivity of the reaction mechanism

- Knockout / nucleon removal
- Fusion – evaporation
- Transfer
- Deep inelastic
- Scattering (elastic / inelastic)
- Capture

Fusion evaporation vs. direct transfer

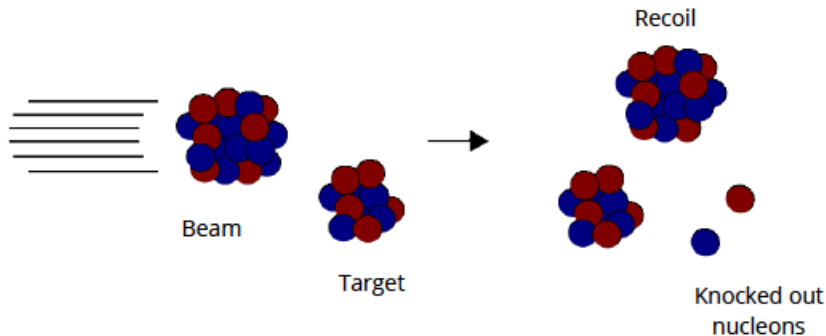


- $A + b = C \rightarrow D + X$
 - $^{12}\text{C}(^{18}\text{O}, 3n)^{27}\text{Si}^*$
- Compound system has NO memory of its formation
- Evaporated particle energies give excitation energies of final states

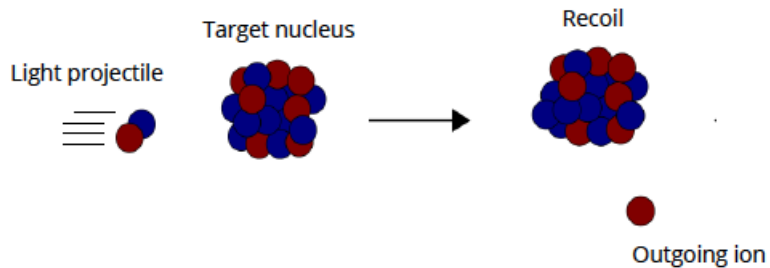


- Two-body $A(b,c)D$
 - $^{16}\text{O}(d,p)^{17}\text{O}^*$
- Outgoing particle DO retain knowledge of transferred particles

Knockout reaction vs. direct transfer

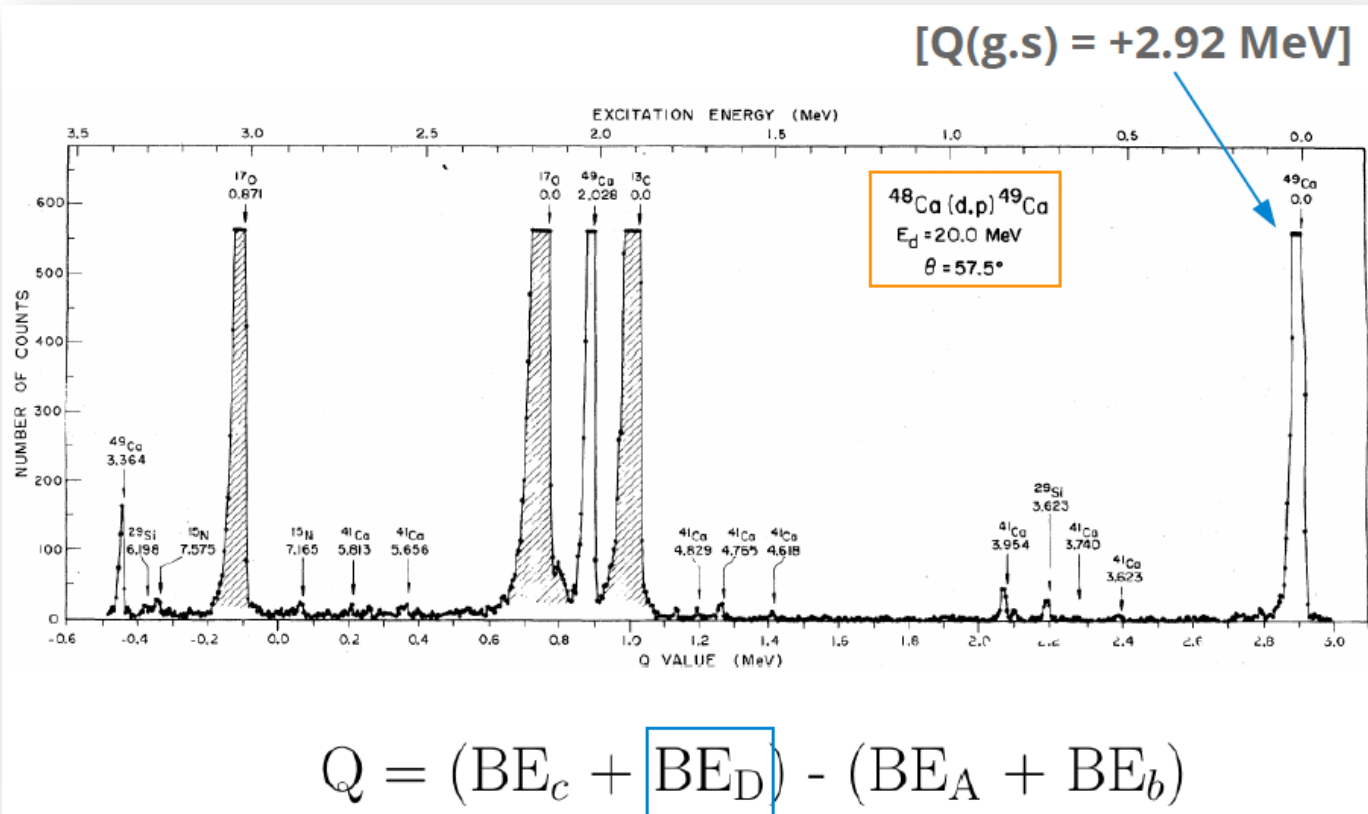


- $A + b = c - X_n - X_p$
 - ${}^9\text{Be}({}^{44}\text{S}, -1p1n){}^{42}\text{P}^*$
- Momentum distribution of recoil reflects orbital momentum transfer



- Two-body $A(b,c)D$
 - ${}^{16}\text{O}(d,p){}^{17}\text{O}^*$
- Outgoing particle DO retain knowledge of transferred particles

Transfer reactions: measured quantities



Transfer reactions: extracted quantities

Sensitivity of the differential cross sections to orbital angular momenta (l) of transferred nucleon(s)

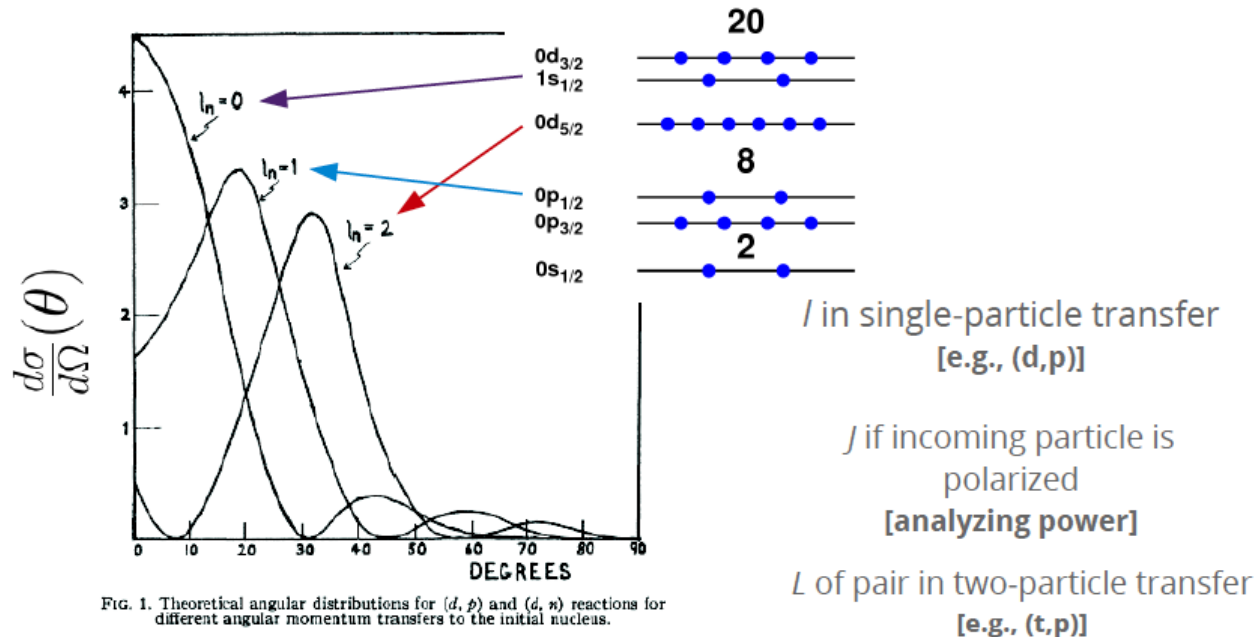
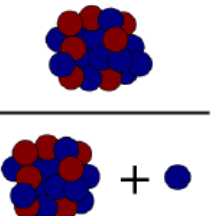


FIG. 1. Theoretical angular distributions for (d, p) and (d, n) reactions for different angular momentum transfers to the initial nucleus.

Transfer reaction: extracted quantities

Experimental spectroscopic factor
 [Relative values are typically reliable (<25%)]
 [absolute values can be tricky (>30%)!]

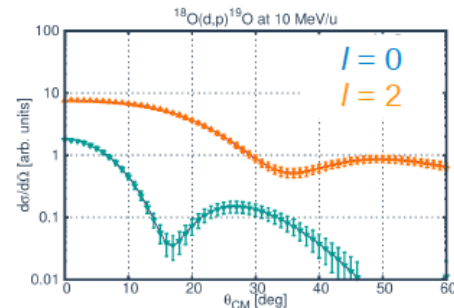
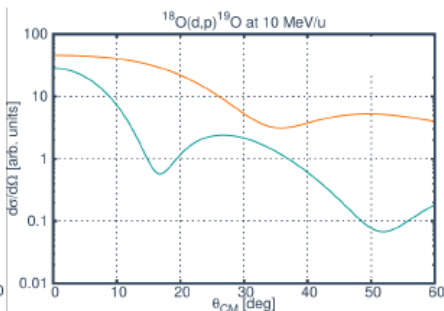
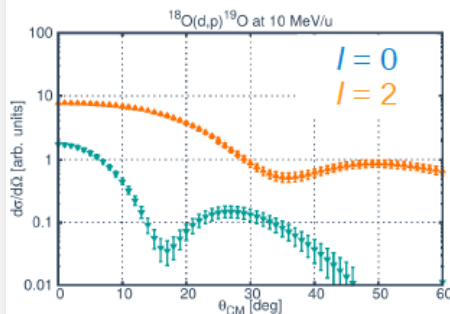
$$S_{lj} = \frac{\text{Final state}}{\text{Initial state} + \text{nucleon}}$$


$$\left. \frac{d\sigma}{d\Omega} \right|_{\text{Measured}} = g S_{lj} \left. \frac{d\sigma}{d\Omega} \right|_{\text{DWBA}}$$

Statistical factor \rightarrow g

Calculated cross section for "pure" single-particle like state \rightarrow $\left. \frac{d\sigma}{d\Omega} \right|_{\text{DWBA}}$

Amount of overlap between initial and final states
Spectroscopic Factor \rightarrow S_{lj}



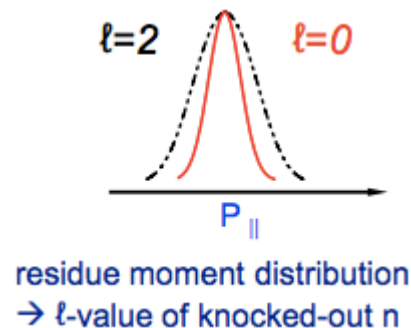
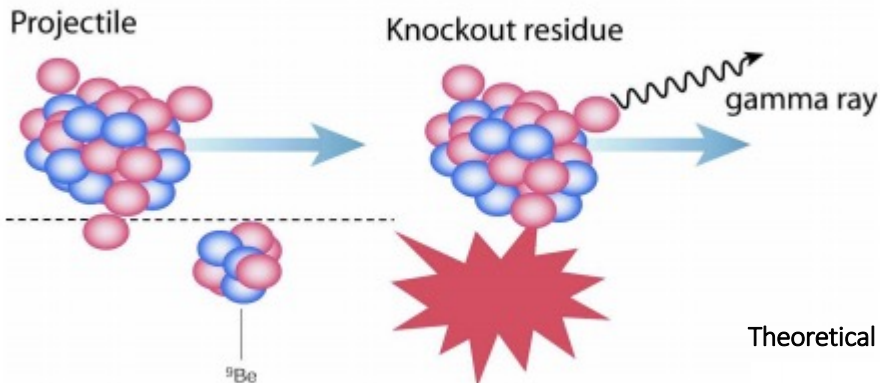
Nucleon knockout reactions

Intermediate energy beams (> 50 MeV/nucleon)

- Sudden approximation + eikonal approach for reaction theory

Spectroscopic strengths \rightarrow exclusive cross-sections

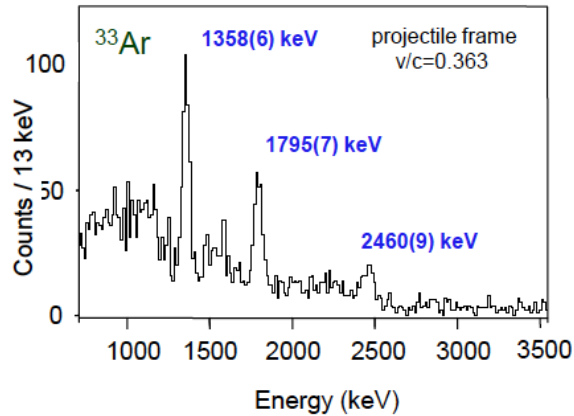
- Populated states in A-1 residue provide detailed measure of beam structure



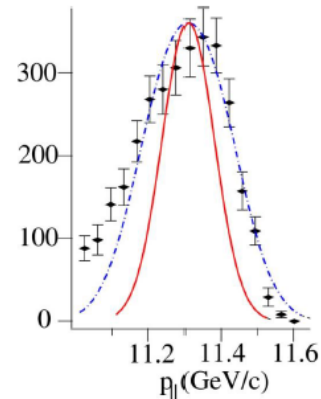
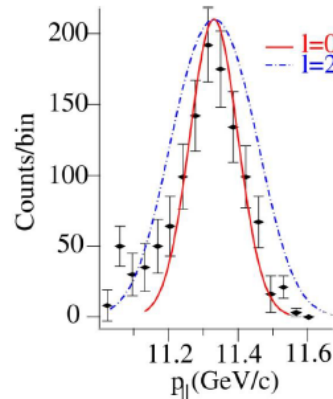
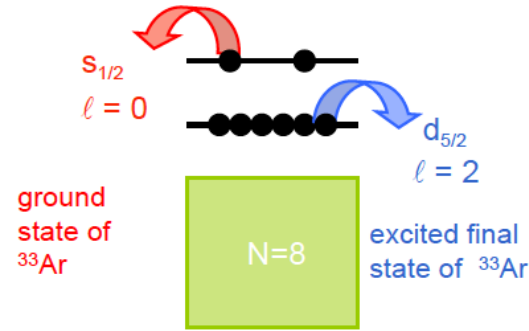
Theoretical cross-section

$$\sigma(j^\pi) = \left(\frac{A}{A-1} \right)^N \underbrace{C^2 S(j^\pi)}_{\text{Structure theory}} \underbrace{\sigma_{sp}(j, S_N + E_x[j^\pi])}_{\text{Reaction theory}}$$

Neutron knockout – $^9\text{Be}(^{34}\text{Ar}, ^{33}\text{Ar})\text{X}$



	BR (%)	σ_{exp} (mb)	C^2S_{exp}
$1/2^+$	30.2(46)	4.7(9)	0.38(6)
$3/2^+$	20.2(44)	3.2(8)	0.36(9)
$5/2^+$	31.7(31)	4.9(7)	0.56(8)
$(5/2^+)$	17.9(30)	2.8(6)	$>0.34(7)$

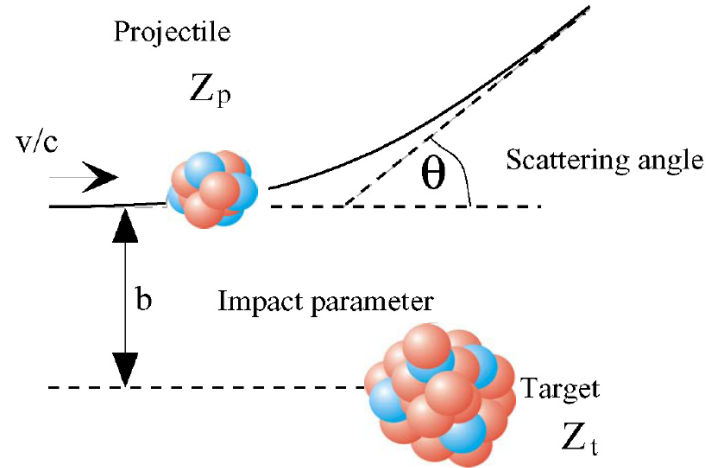


A. Gade et al., PRC 69, 034311 (2004).

Collectivity: B(E2) from excitation probability

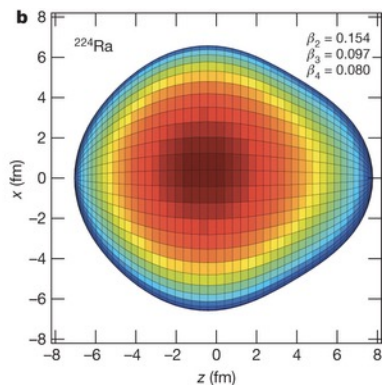
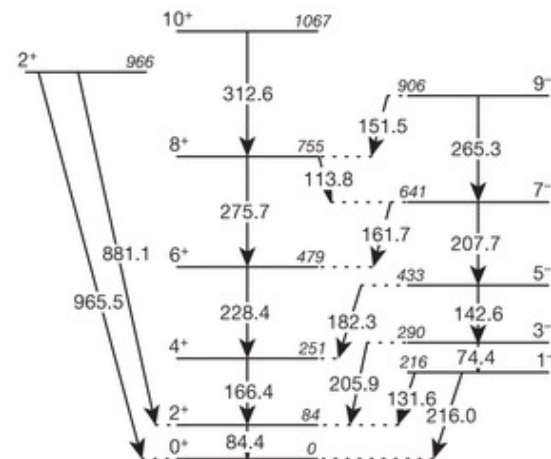
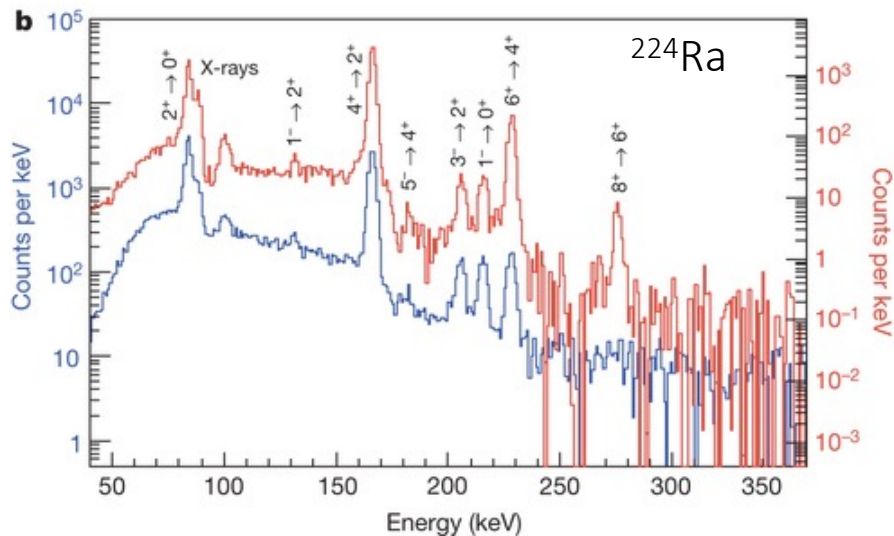
Coulomb excitation:

- purely Coulomb interaction causes excitation of the nucleus of interest
- well described interaction, and cross-section relates to transition matrix element, i.e. B(E2) for $0^+ \rightarrow 2^+$ in even-even nuclei.



$$\sigma_{\pi\lambda} \approx \left(\frac{Z_{\text{pro}} e^2}{\hbar c} \right)^2 \frac{\pi}{e^2 b_{\text{min}}^{2\lambda-2}} B(\pi\lambda, 0 \rightarrow \lambda) \begin{cases} 1/(\lambda - 1) & \text{for } \lambda \geq 2 \\ 2 \ln(b_a/b_{\text{min}}) & \text{for } \lambda = 1 \end{cases}$$

Pear shaped nuclei and atomic EDM



$$\langle I' || E\lambda || I \rangle = \sqrt{(2I' + 1)(2\lambda + 1) / 16\pi} (I' 0 \lambda 0 | I 0) Q_\lambda$$

L. P. Gaffney *et al.*, Nature **497**, 199 (2013).

Summary

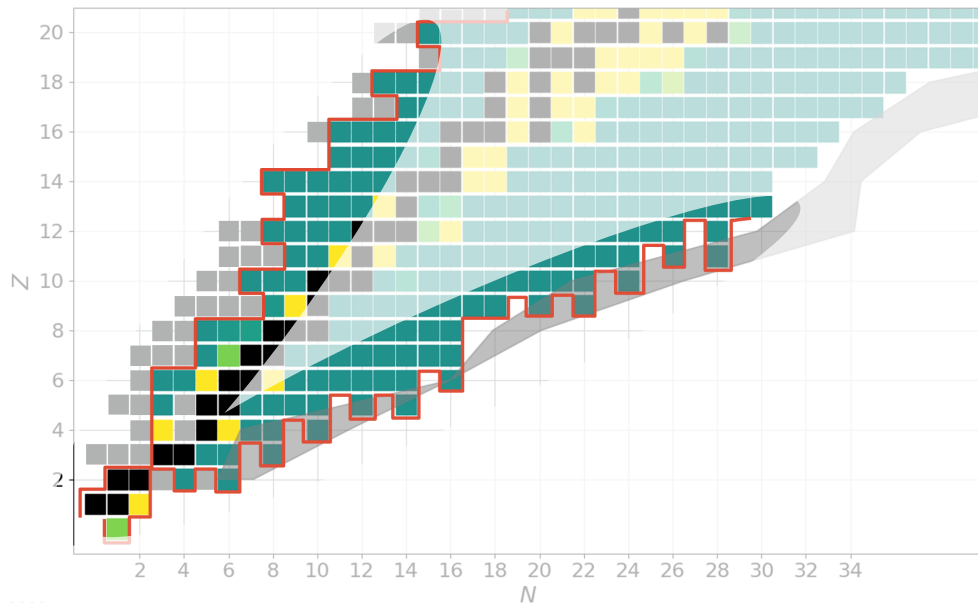
- Nuclear excitation spectra (energies, spins and parities of excited states) are fundamental experimental observables
- Patterns of excitation provide insight into symmetries and collective properties of nucleus
 - Vibrational spectra
 - Rotational spectra
 - Single-particle excitations
- Nuclear decay provides access to excitation spectra, as well as fundamental observable such as half-life

Thank You

Locating the driplines

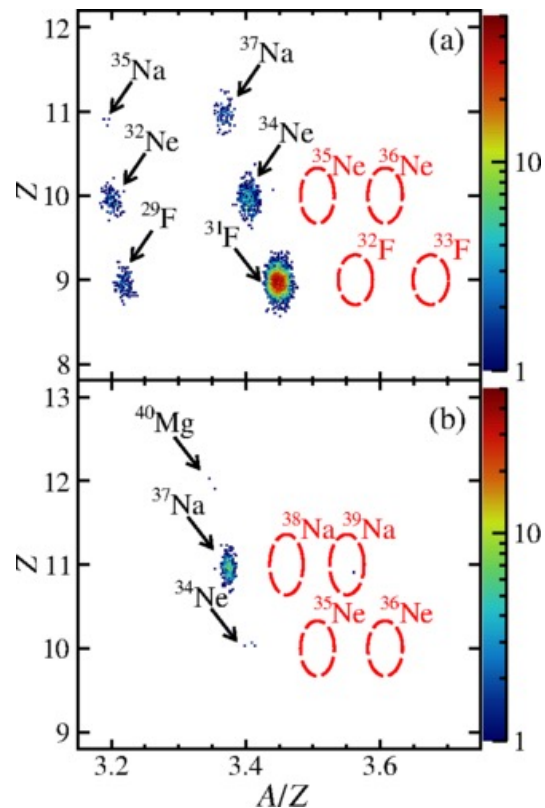
The limits of existence are defined by the proton and neutron driplines

S_n (or S_p) become positive \rightarrow neutron/proton are not bound;
emission does not require energy input

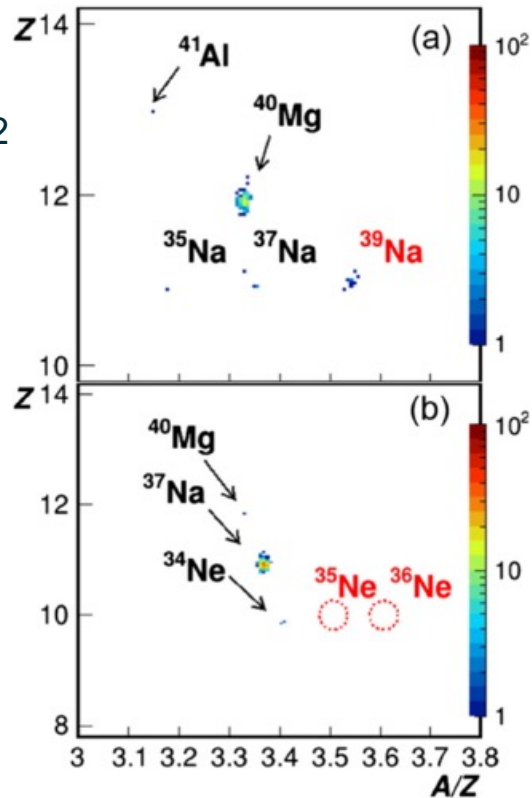


Mapping the Neutron Dripline up to Ne

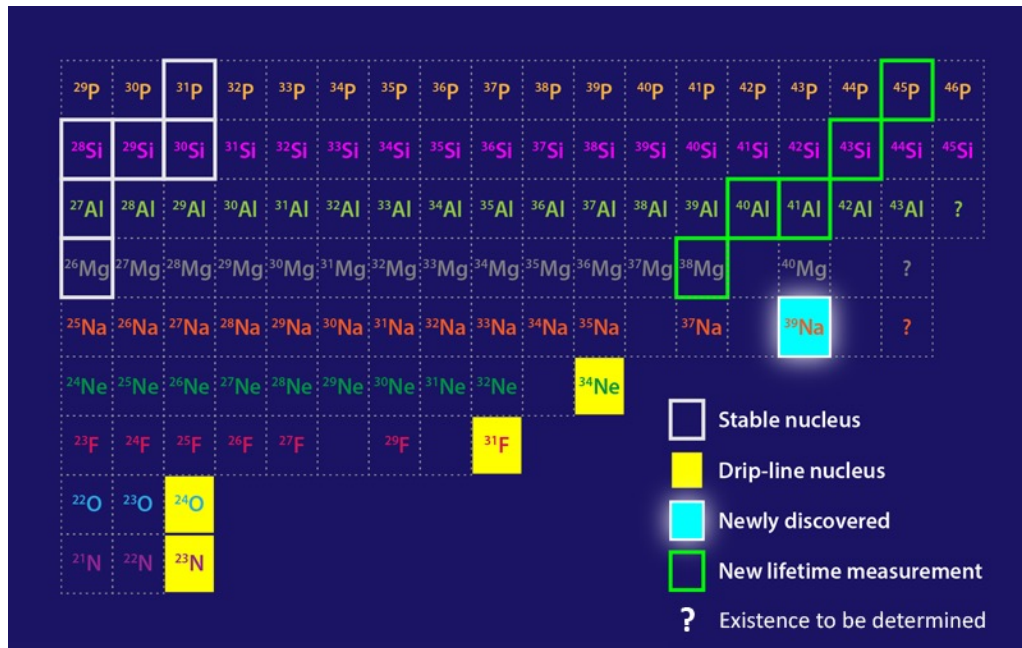
2019



2022

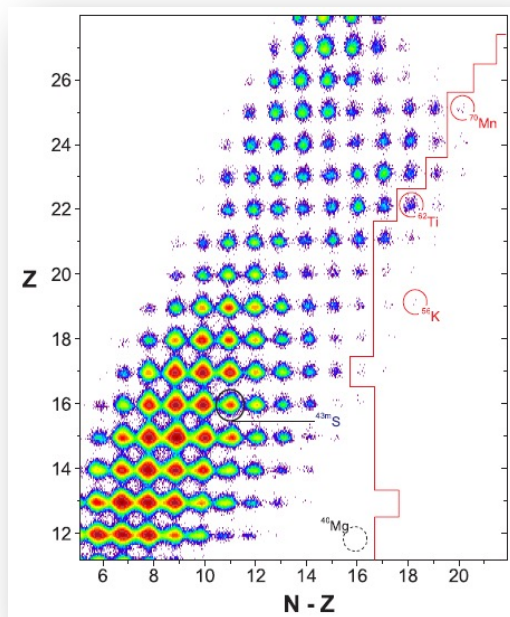


Mapping the Neutron Dripline up to Ne



Fragmentation cross-sections

A change in the trend of fragmentation cross-sections indicates a change in the binding – enhanced binding suggestive of a change in nuclear structure?



O. Tarasov et al., PRL 102, 142501 (2009)

