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Low-Energy Nuclear Structure
Lecture 1: The “Basics”

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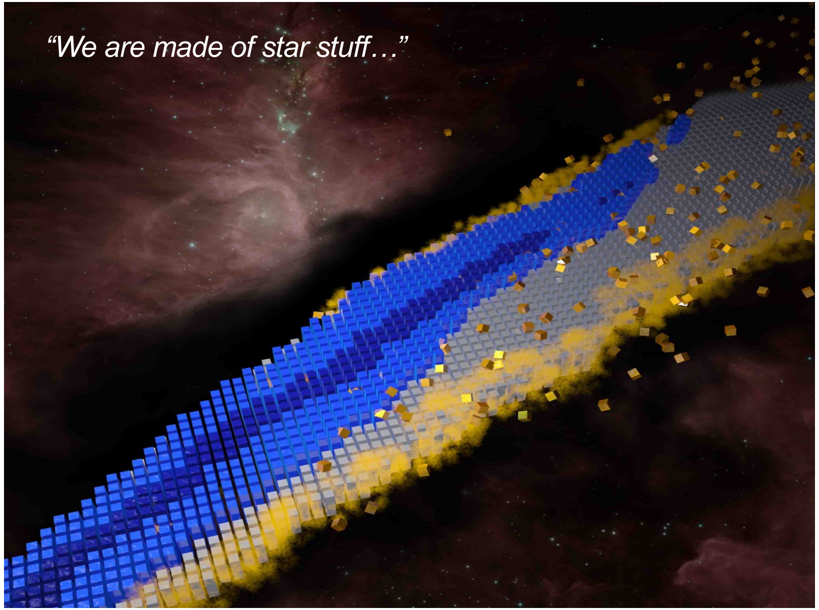
Why study nuclear structure and reactions?

- What are the limits of nuclear existence, and what features arise near and beyond these limits?
- What is the nature of the nuclear force that binds protons and neutrons into stable nuclei and rare isotopes, and how do the rich phenomena of nuclear structure and reactions emerge?
- How do single-nucleon, cluster, and collective degrees of freedom coexist and evolve with increasing proton-neutron imbalance and excitation energies?
- What is the nature of dense matter and neutron stars?



Why study nuclear structure?

Studying exotic nuclei extends the range over which theories can be tested.

Goal: Establish the physical properties of exotic nuclei and their interactions (reactions) to constrain theory and improve predictive power

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Goal: Establish the physical properties of exotic nuclei and their interactions (reactions) to constrain theory and improve predictive power ⇔ Theory

Physical properties

Mass, decay lifetime, production cross-section, electric and magnetic moments, excited state properties (moments, energies, lifetimes), etc…

Note ⇨ the observables in an experiment may or may not require interpretation to relate to physical properties

Observables

Ultimately, we would like to understand the wavefunction of nuclear states. But these are not observable quantities.

Observables:

Half-life, mass, decay modes, electric/magnetic moments, cross-sections, momentum distributions, transition probabilities, ….

Challenges for radioactive nuclei

The observables we're interested in for stable nuclei are the same ones we're interested in for exotic systems.

Most techniques translate as well…but radioactive nuclei add some experimental challenges.

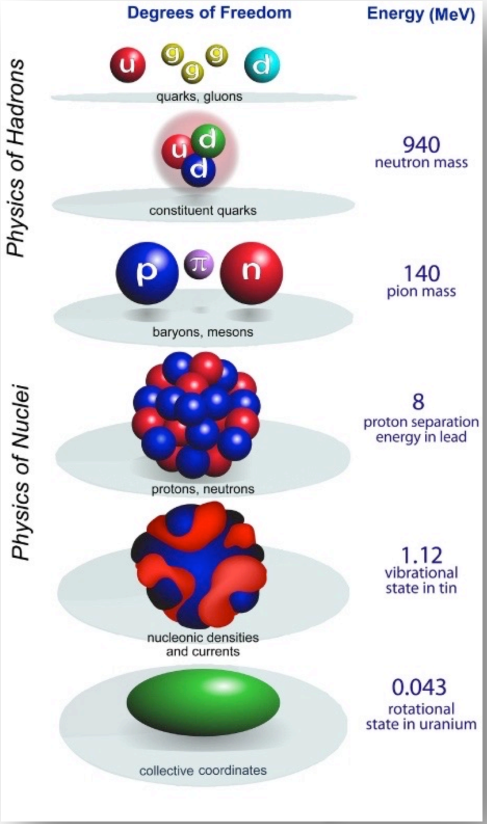
Biggest challenges:

Half-life → how do you study an isotope that lives for a fraction of a second (ms timescale for beta-unstable nuclei)

Production → how do you study nuclei you only see once a week, or less?

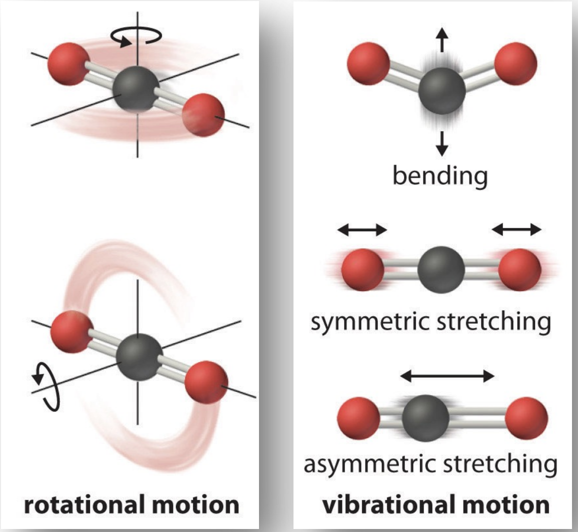
Scales: Energy and size

Nuclear structure physics



R J Furnstahl and K Hebeler, Rep. Prog. Phys. **76**, 126301 (2013).
doi:10.1088/0034-4885/76/12/126301

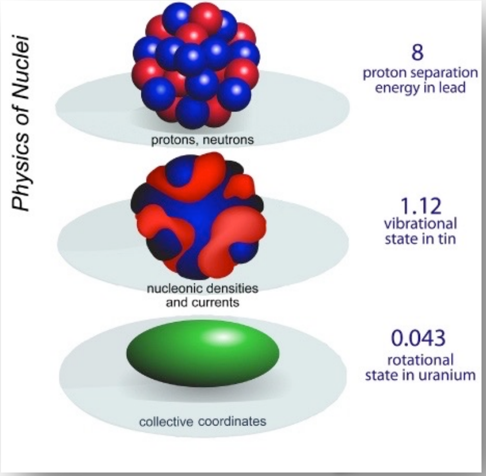
Scales: Energy and size



Molecular excitations:
$E_{rot} << E_{vib} << E_{electric}$
(μeV << meV << eV)

Molecular excitations are separable – wavefunctions can be treated as product of terms

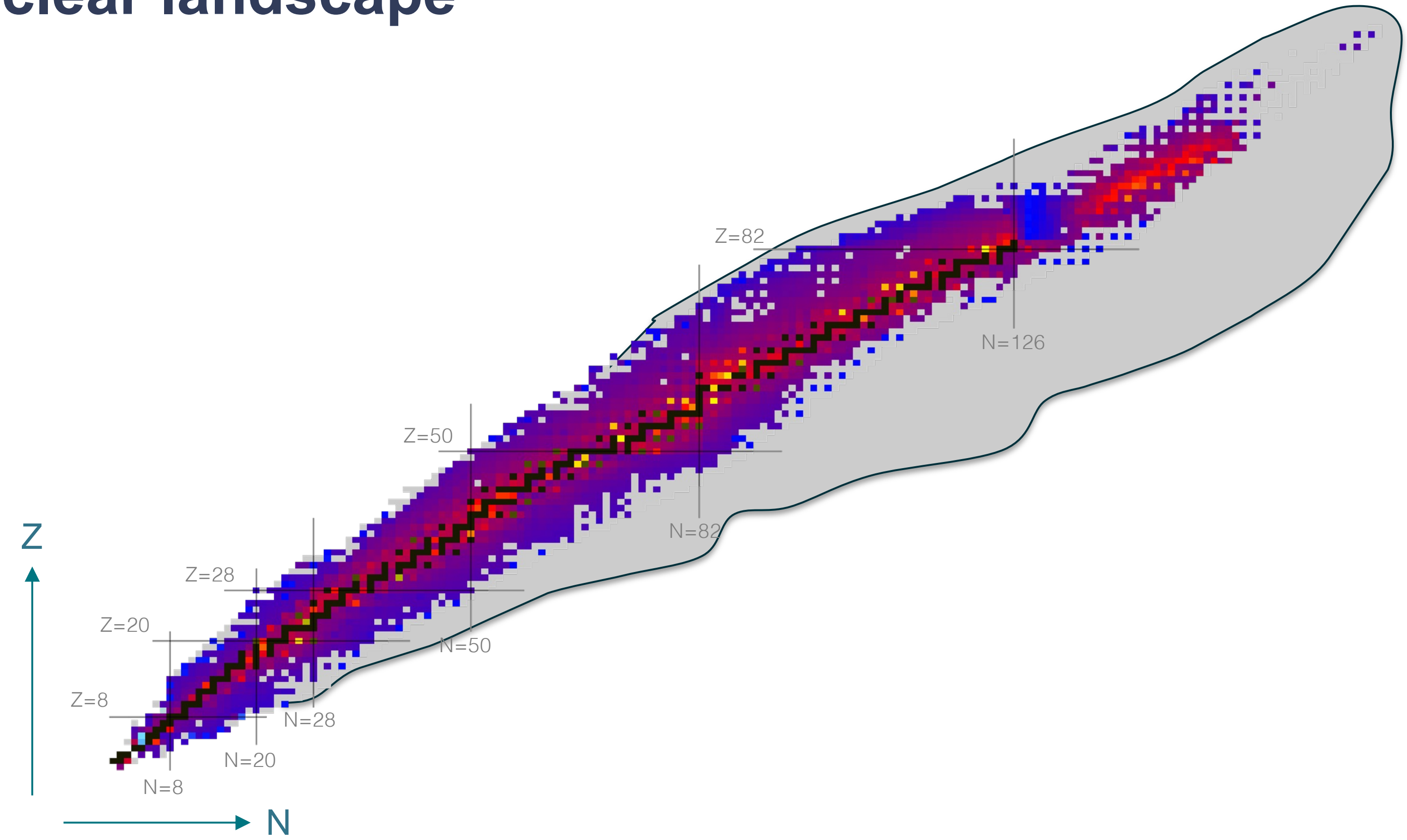
Nuclear structure physics



Nuclear excitations:
$E_{rot} \sim E_{vib} \sim E_{single\ particle}$
(all MeV)

Collective and single-particle excitations are all of a similar energy scale and interact

Nuclear landscape



Nuclear landscape basics

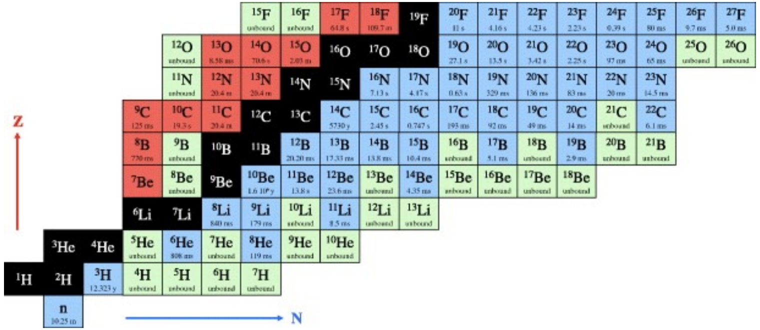
A given nucleus is a bound system of N neutrons and Z protons, with a total mass number A = N + Z

Shorthand: ${}^{A}_{Z}X_{N}$

Isotopes: same Z, but different N
${}^{9}C$, ${}^{10}C$, ${}^{11}C$, ${}^{12}C$, …

Isotones: Same N, but different Z
${}^{12}C$, ${}^{11}B$, ${}^{10}Be$, ${}^{9}Li$, ${}^{8}He$, …

Isobars: Nuclei with the same mass number A
${}^{12}N$, ${}^{12}C$, ${}^{12}B$, ${}^{12}Be$, …



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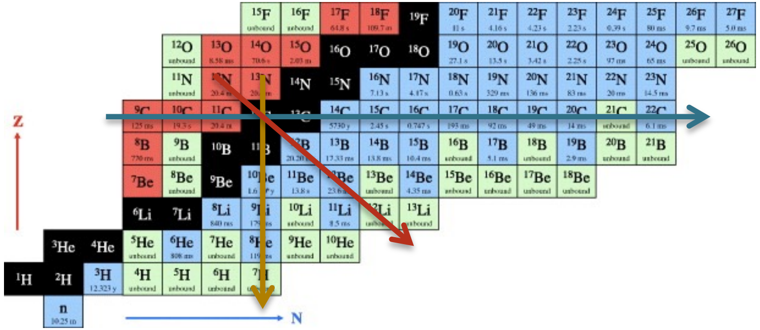
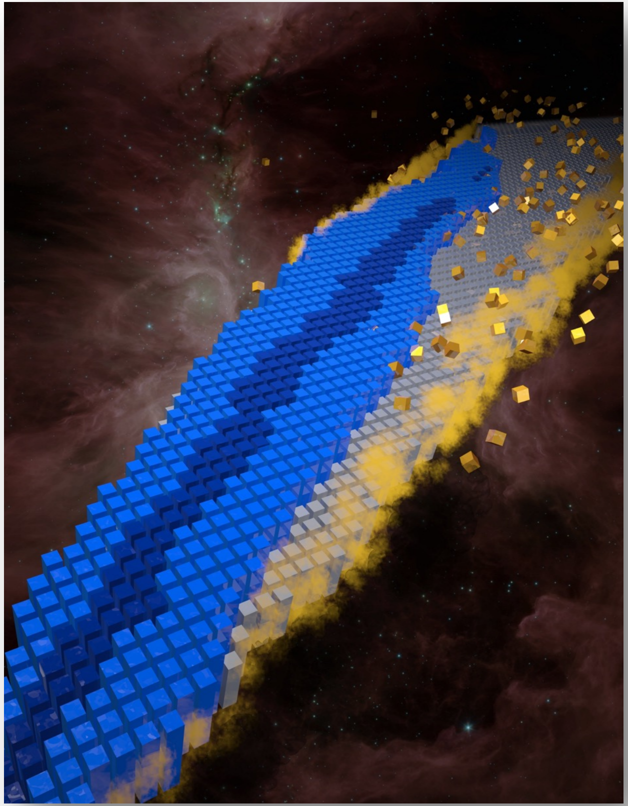
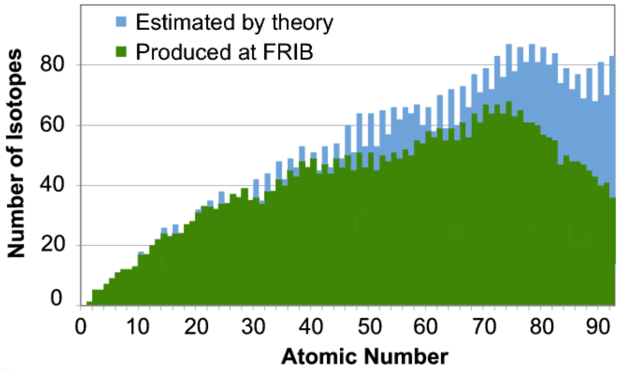


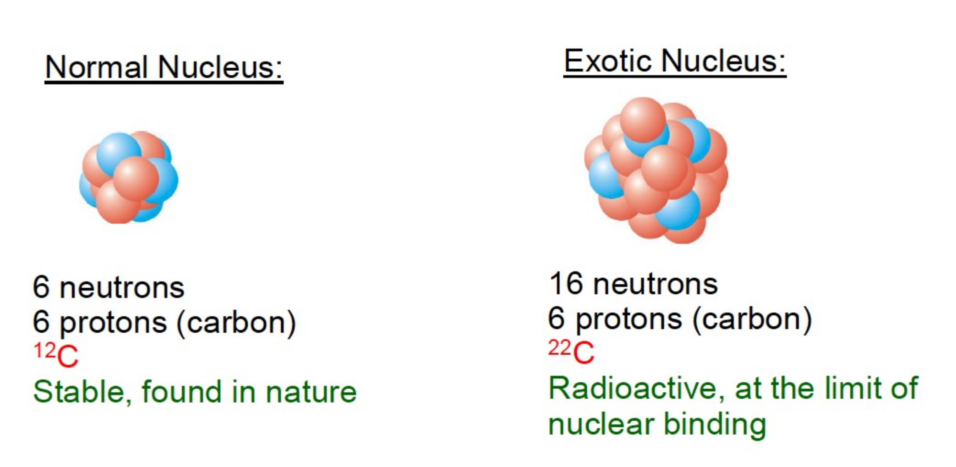
Nuclear landscape limits





- 254 stable isotopes have been observed;
- more than 3000 isotopes have been made in laboratories;
- as many as 6000-8000 are expected to possibly exist

“Exotic” nuclei



“Exotic” nuclei are those which will undergo radioactive decay towards a lower-energy system
They are characterized by:

- excess of protons or neutrons
- short half-lives
- neutron/proton dominated surface
- low binding of nucleons

What binds the nuclear system?

Why / how do nuclei exist?

The Nuclear Force(s)

Properties of the Nuclear Strong Force

Consider a very simple and common nucleus - ^4He

- Two protons, and two neutrons

How is this held together?

- Gravity:
  - Let's consider the gravitational attraction between 1 proton and the other three nucleons
  - Mass of nucleon ~ 1.67×10^{-27}kg
  - Radius of nucleus ~ 10^{-15} m
  - **Force ~ 5.6 x 10^{-34} N (attractive)**

$$F = G\frac{m_1 m_2}{r^2}, G = 6.67 \times 10^{-11} Nm^2kg^{-2}$$

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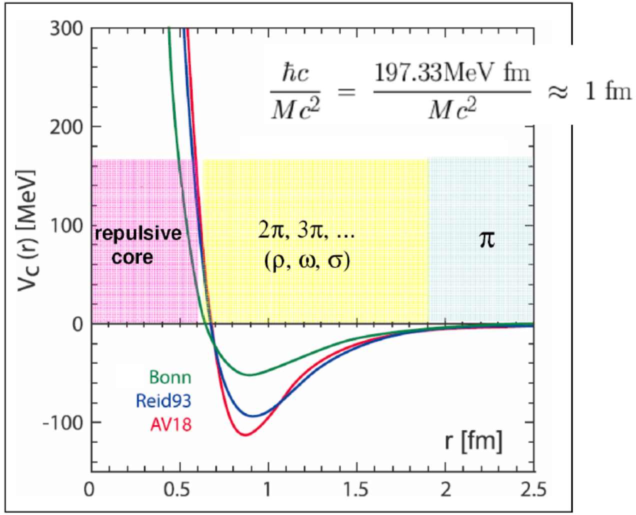
- Coulomb repulsion between the protons:
  - Same radius, each has 1e (1.6×10^{-19}C) charge
  - **Force ~ 231 N (repulsive)**

$$F = k_e \frac{q_1 q_2}{r^2}, k_e = 8.988 \times 10^9 Nm^2 C^{-2}$$

What binds the nuclear system?

Protons and neutrons are bound together by the strong force.

The strong (colour) force between quarks in one proton, and quarks in another proton is sufficient to overcome the electromagnetic repulsion



From T. Hatsuda (Oslo 2008)

Consider the nuclear binding force as:

- a residual strong interaction, or
- the exchange of mesons

The Nuclear Force(s)

Properties of the Nuclear Strong Force

1. Nuclei exist - they are not blown apart by Coulomb repulsion which is **much, much stronger** gravitational attraction.

There is a strong attractive force that binds nucleons into nuclei.

2. The motion of the planets, our interactions with the Earth etc. are explained by gravity.

3. There are 45 stable isotopes with Z=20 or less.
   – 20 of these have an even Z ***and*** and even N
   – 21 have an even Z ***or*** an even N
   – 4 have an odd Z ***and*** odd N

The Nuclear Force(s)

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The strong force must act over a short range only.

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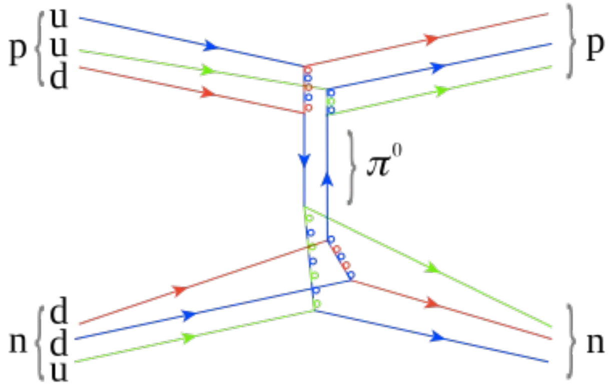
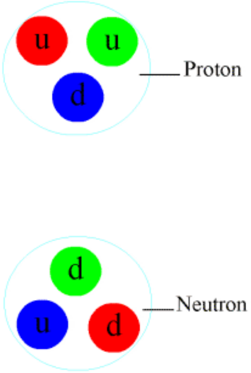
The strong force has a pairing component, which favors pairs of nucleons.

Pion exchange model

Can interpret the nuclear strong force as an exchange force involving neutral pions, π^0

Hideki Yukawa judged the range of the nuclear force to be about 1 fm, and calculated the range of the exchange particle to be of order 100 MeV/c^2 – led to discovery of the pion





→ For a proton to interact with another proton, it must exchange something with it, but quarks are confined, thus exchange quark-antiquark pair (meson); lightest is pion, defining the upper range for the nuclear strong force

Binding energy and mass

Mass $M(N,Z)$ of neutral atom (of order GeV)

Mass excess:

$$\Delta(N,Z) = M(N,Z) - uA \text{ (of order MeV)}$$

Atomic mass unit $u = M(^{12}C)/12 = 931.5\ \mathrm{MeV}/c^2$

$$\rightarrow \Delta(^{12}C) = 0$$

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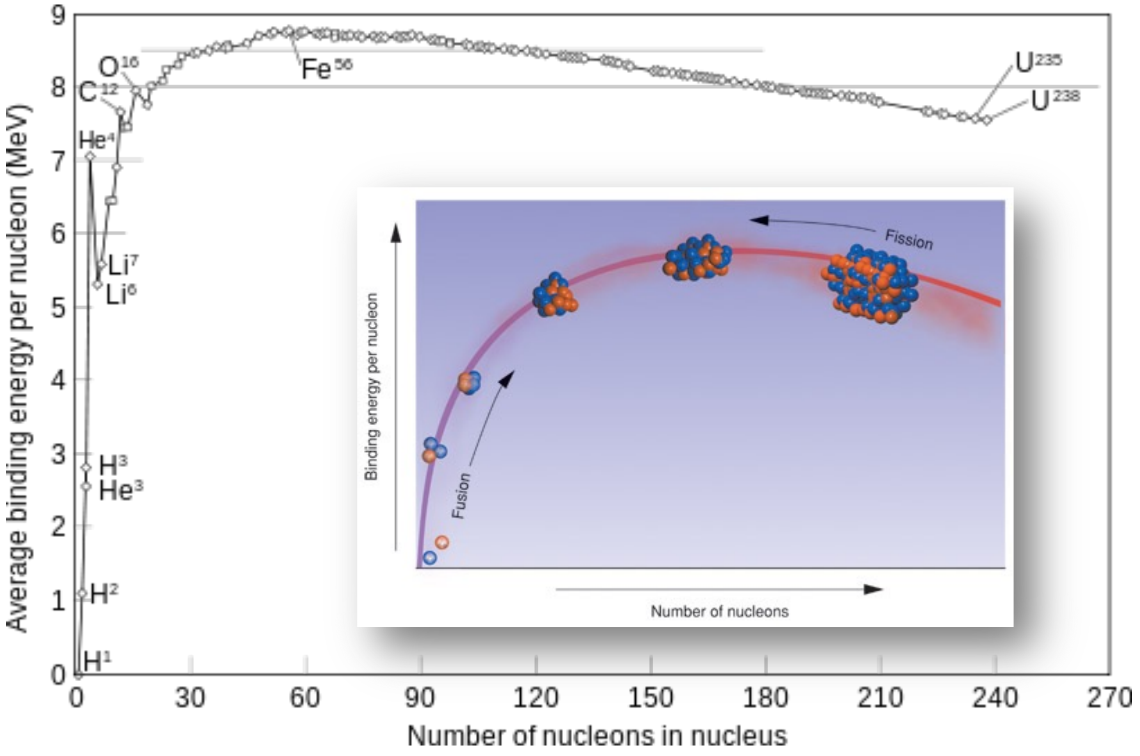
--> $\Delta(^{12}C) = 0$

Binding energy:

$$B(N,Z) = ZM_H c^2 + NM_n c^2 - M(N,Z)c^2$$

$$\boxed{B(N,Z) = Z\Delta_H c^2 + N\Delta_n c^2 - \Delta(N,Z)c^2}$$

Binding of nuclei – Fission and fusion



The liquid-drop model for nuclear binding

$$B(Z,A) = a_V A + a_S A^{2/3} + a_C Z^2/A^{1/3} + a_A (N-Z)^2/A - a_P/A^{1/2}$$

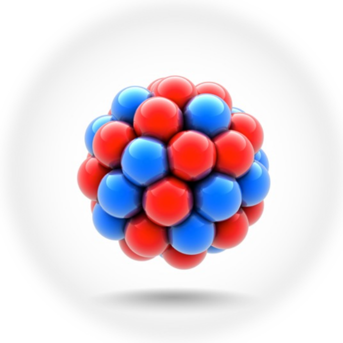
Volume term
a_V = -15.68 MeV

Surface term
a_S = 18.56 MeV

Coulomb term
a_C = 0.717 MeV

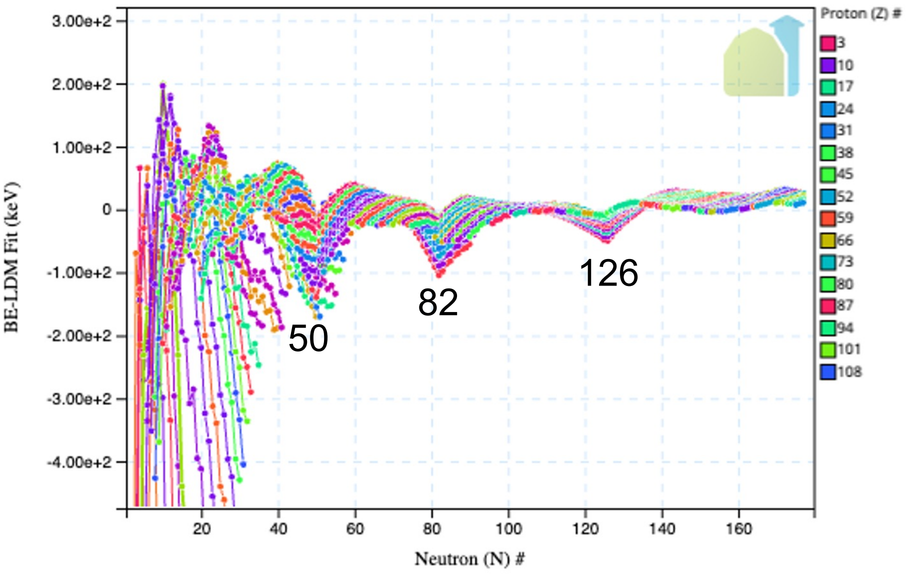
Asymmetry term
a_A = 28.1 MeV

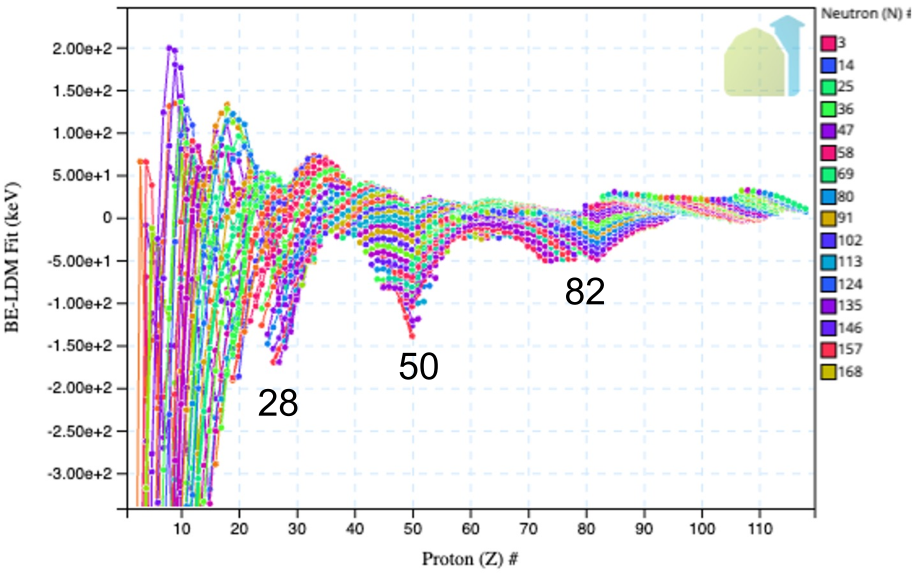
Pairing term
a_P = 12.0 MeV
(+) even-even
(-) odd-odd
(0) even-odd



$$R = r_0 A^{1/3}$$

Semi-empirical (liquid drop) mass vs. reality





$$B(N,Z) = ZM_Hc^2 + NM_nc^2 - M(N,Z)c^2$$

Nucleon separation energies

Ground state masses directly are large; changes in trends are obscured

More useful are differences in masses, the energy required to remove nucleons from a given system

$$\boxed{S_{2n} = B(N,Z) - B(N\text{-}2,Z) = M(N\text{-}2,Z) + 2M_n - M(N,Z)}$$

Question!

Why would nuclear physicists usually consider plots of S_{2n} rather than S_n?

(A) Removal of a single neutron is not allowed in any nuclear system
(B) Mass differences between even-N and odd-N systems are large
(C) Neutrons provide more information than protons for nuclear structure
(D) S_n is more difficult to calculate

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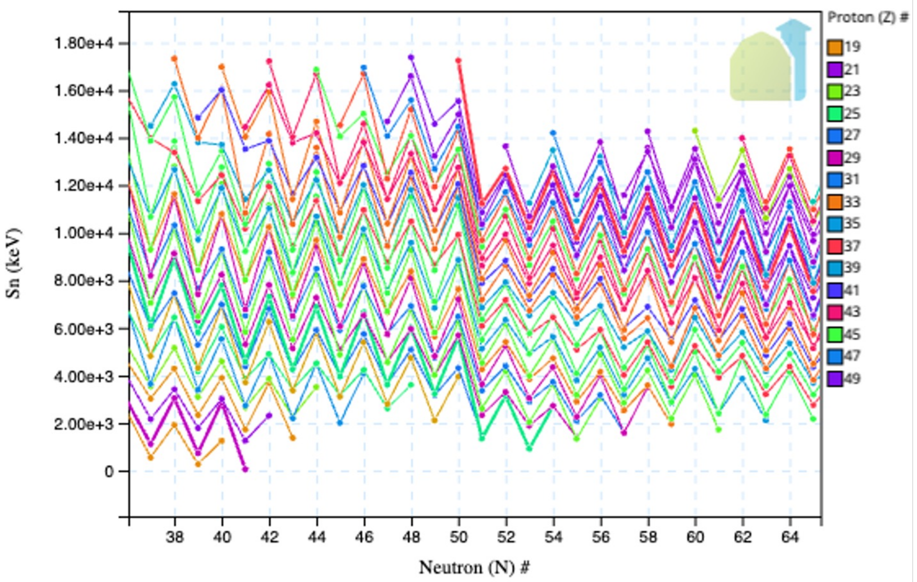
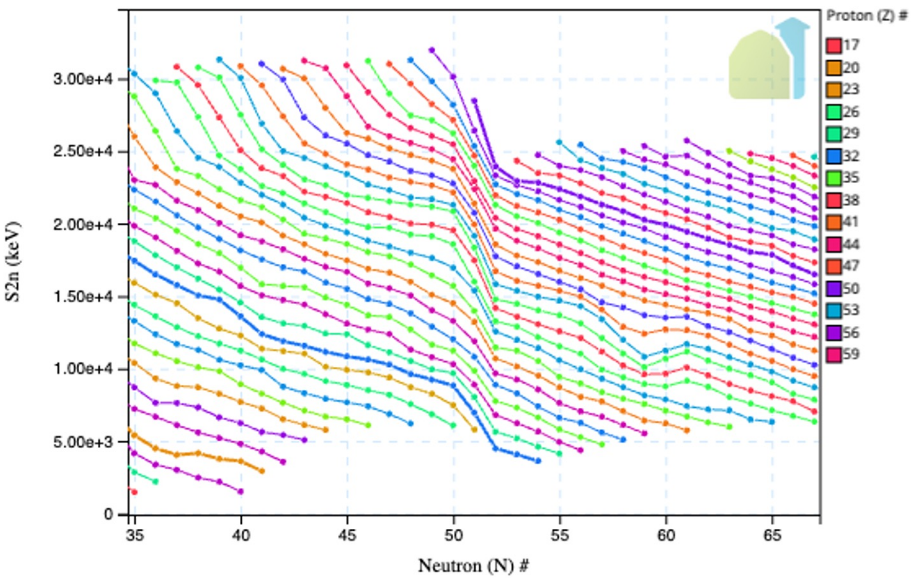
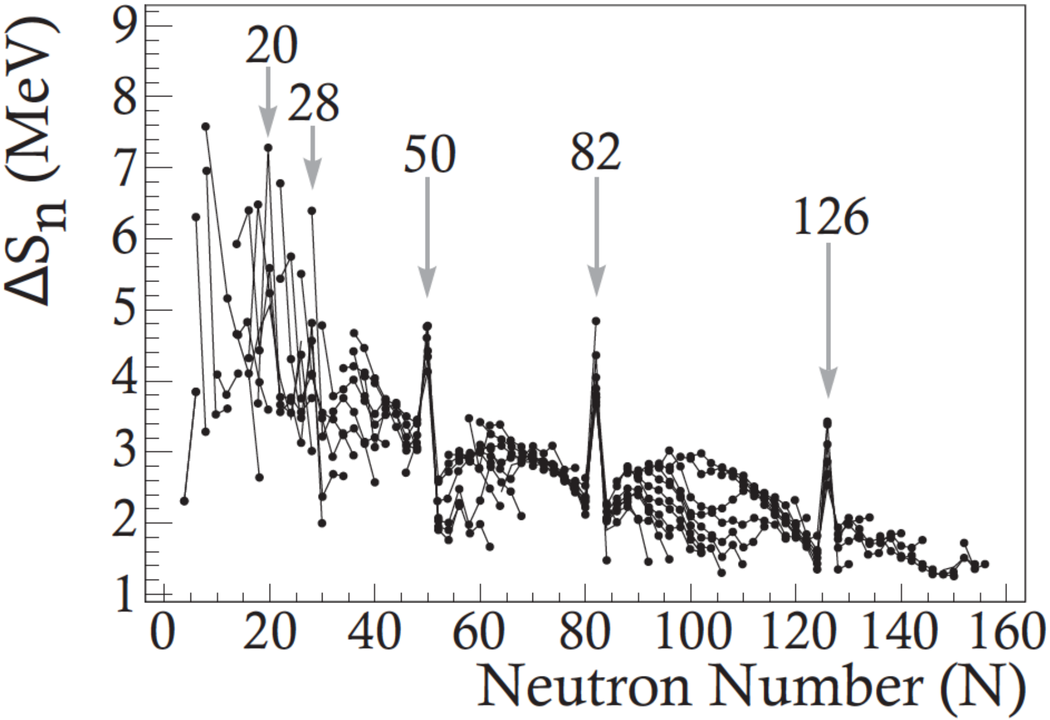
Nucleon separation energies



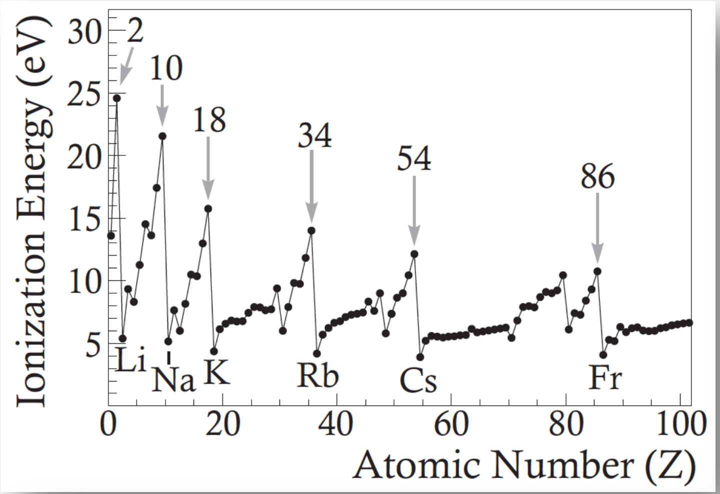


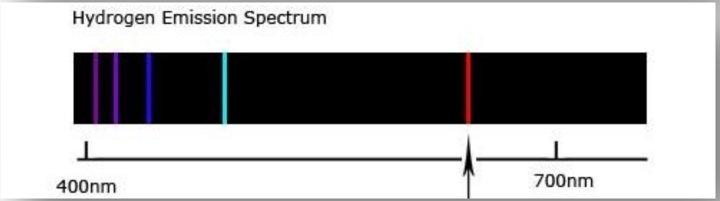
2012 AME Mass evaluation

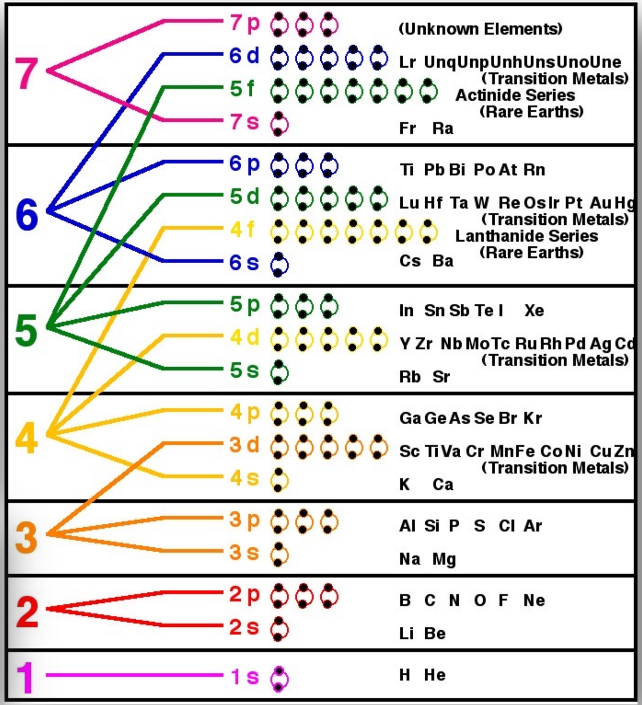
Differences in nucleon separation energies



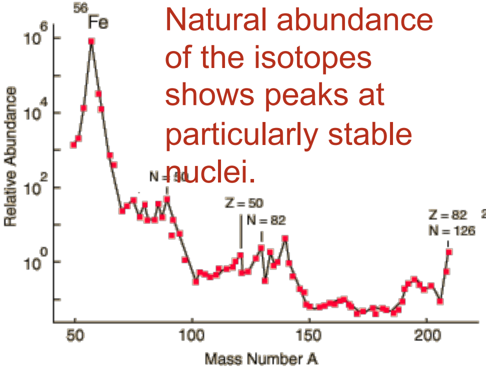
Ionization energies in atoms…





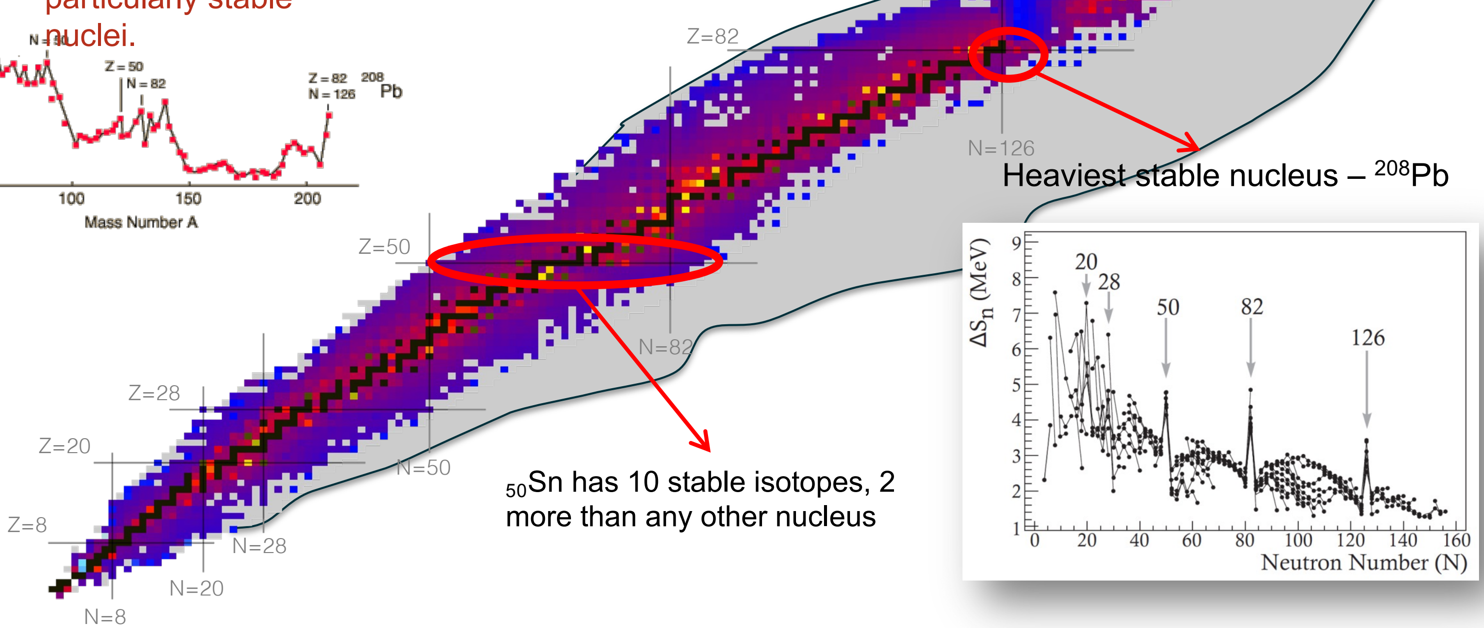


Nuclear magic numbers



Natural abundance of the isotopes shows peaks at particularly stable nuclei.

Similar indicators are found across the nuclear landscape, suggesting a nuclear shell structure.

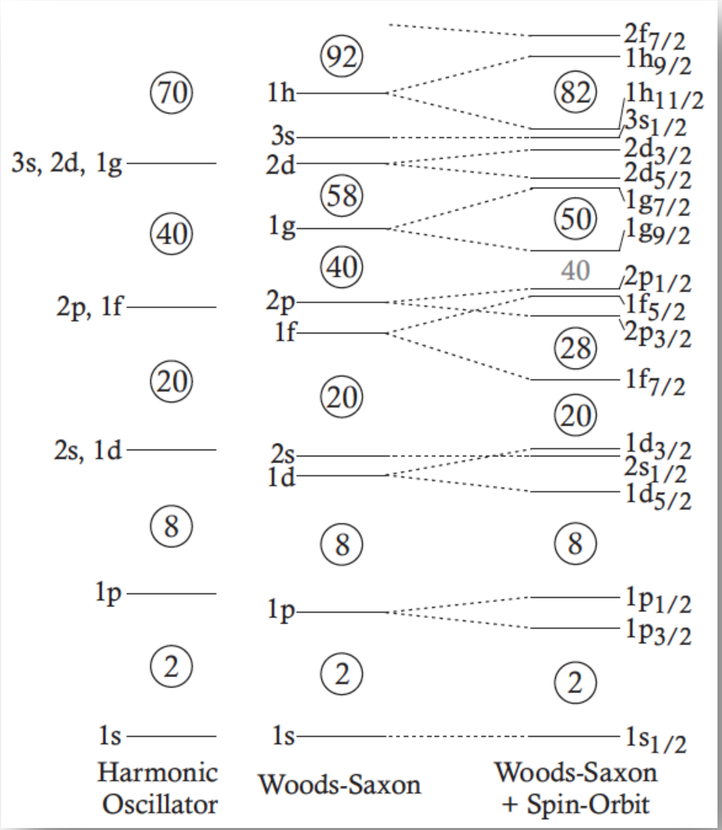


Heaviest stable nucleus – ^{208}Pb

$_{50}Sn$ has 10 stable isotopes, 2 more than any other nucleus

Firestone, R.B. Table of Isotopes. Wiley, New York, 1996.

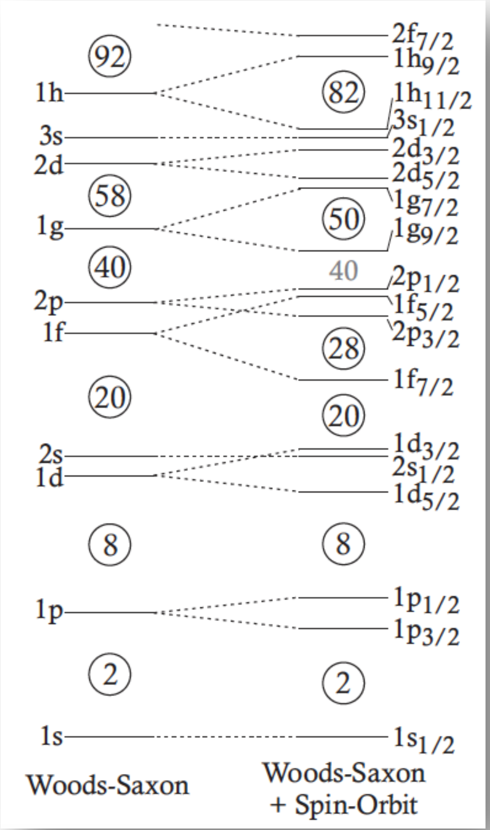
Nuclear shell structure



Maria Goeppert-Mayer & Hans D. Jensen
1963

Maria Goeppert-Mayer, Phys. Rev. 75, 1969 (1949).
O. Haxel, Phys. Rev. 75, 1766 (1949).

Nuclear shell structure



Single-particle levels in nuclei

- Single-particle levels in the fermionic system are grouped into shells, with stabilizing gaps between groups of states at certain occupation numbers with “magic numbers” of protons and neutrons

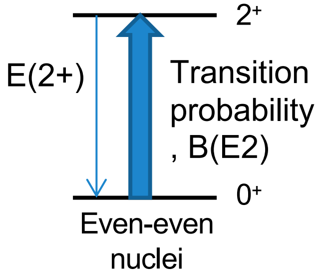
Magic numbers

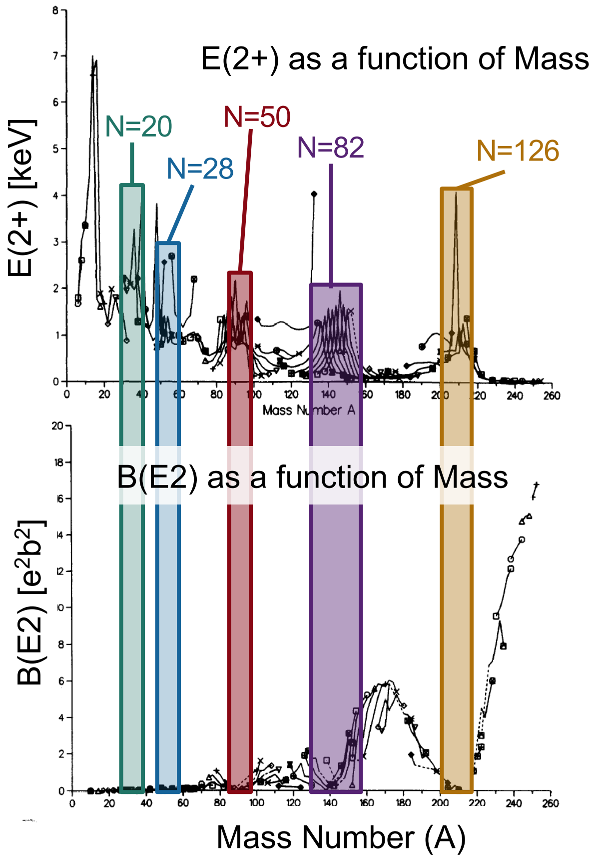
- Magic numbers correspond to particularly stable structures (2, 8, 20, 28, 50, 82,…)

$$H = H_0 + H_{res} = \sum_{i=1}^{A}\left[\frac{\mathbf{p}_i^2}{2m_i} + U_i(\mathbf{r})\right] + H_{res}$$

Nuclear shell structure signatures

To improve our understanding and descriptions of nuclei far from stability, we need to identify the location of shell gaps, and ideally the spacing between single-particle states in the most exotic nuclei.





Structure of Even-Even Nuclei

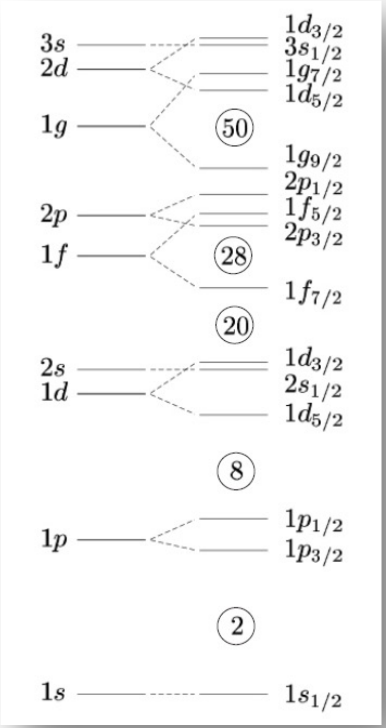
The ground state of even-even nuclei is always 0+, while the first excited state is usually a 2+ state. The energy of this state, and the cross-section to populate it are sensitive to details of the nuclear structure.

$$B(E2; i \rightarrow f) \; = \; \frac{1}{2J_i + 1} \langle \lambda_f J_f || \, E2 \, || \lambda_i J_i \rangle^2$$

"Exotic" shell structure

A driving question in nuclear science:

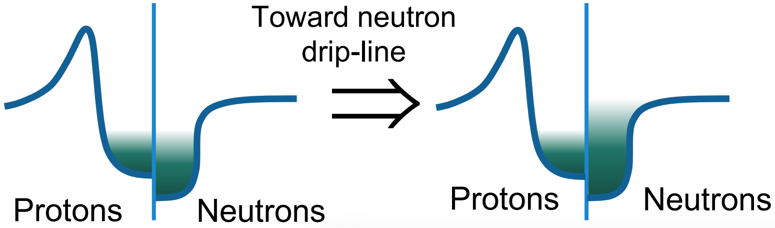
Is the shell-model static across the entire chart of nuclides?

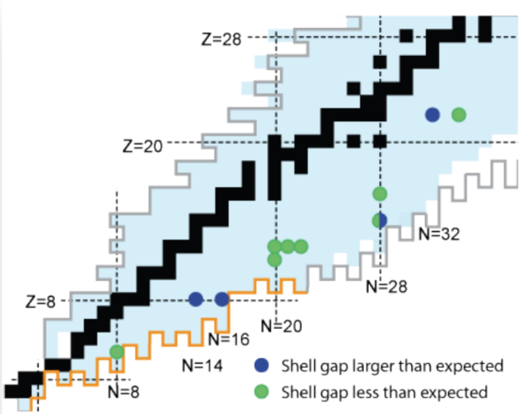


Near the valley of β stability

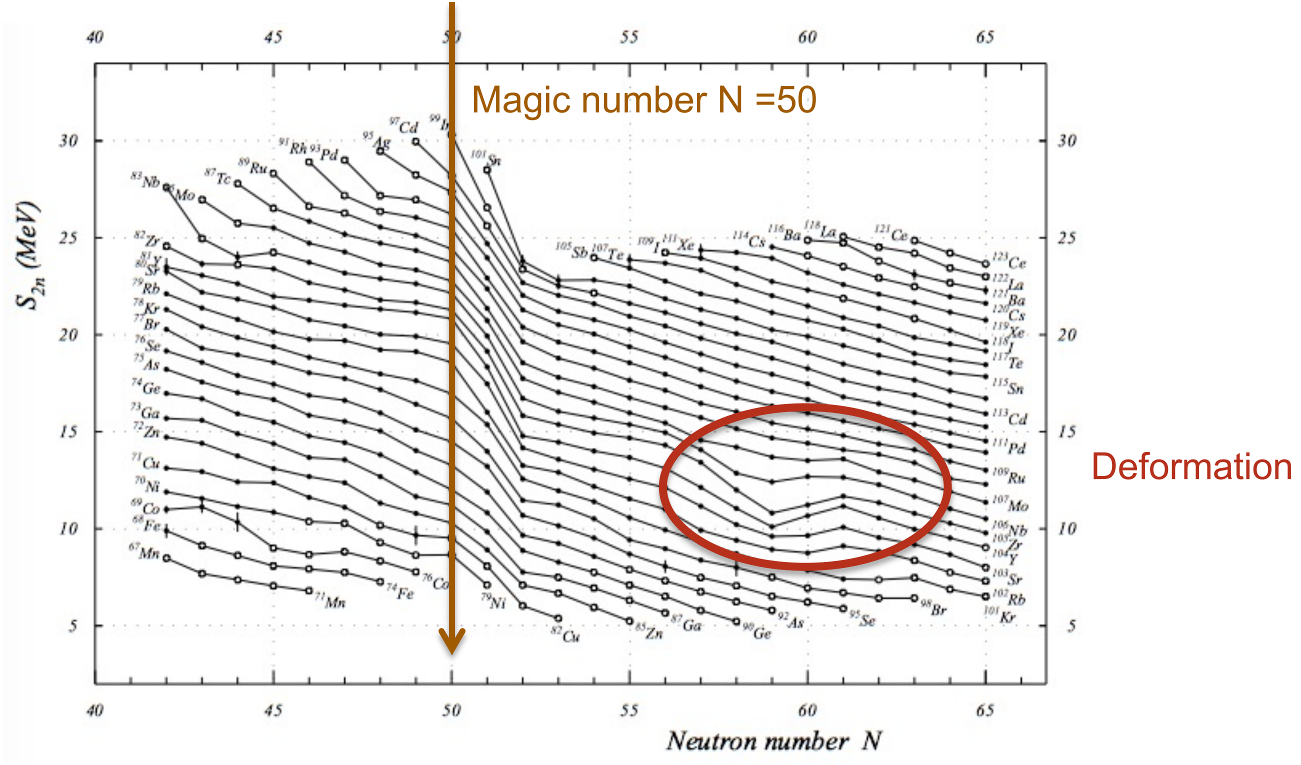
⇒?

Approaching the drip-lines





Masses and shells



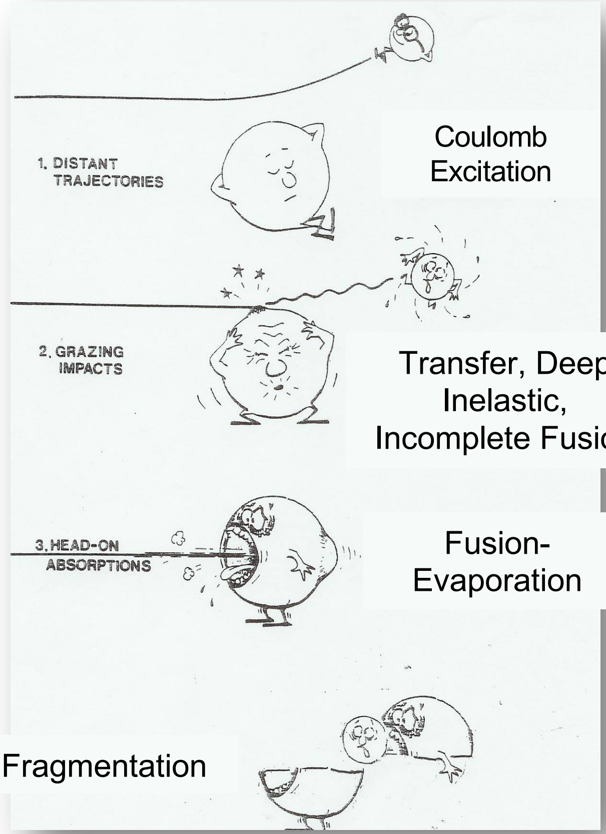
Mass observables

$$S_{2n} = B(N,Z) – B(N\text{-}2,Z) = M(N\text{-}2,Z) + 2M_n – M(N,Z)$$

$$S_{2p} = B(N,Z) – B(N,Z\text{-}2) = M(N,Z\text{-}2) + 2M_H – M(N,Z)$$

Measure nuclear masses for first insight to exotic nuclei. But we need to produce them.

Making exotic nuclei



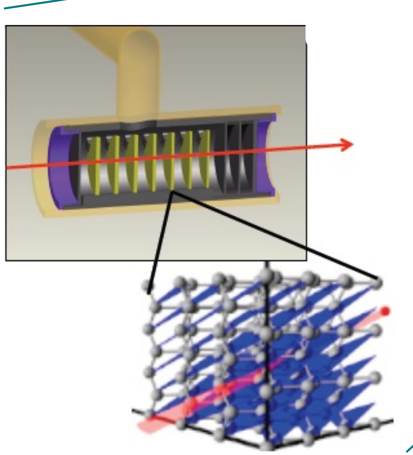
To study the most exotic nuclei we must first produce them. Using stable beam facilities and different combinations of targets + beams, a wide variety of exotic nuclei can be produced and studied.

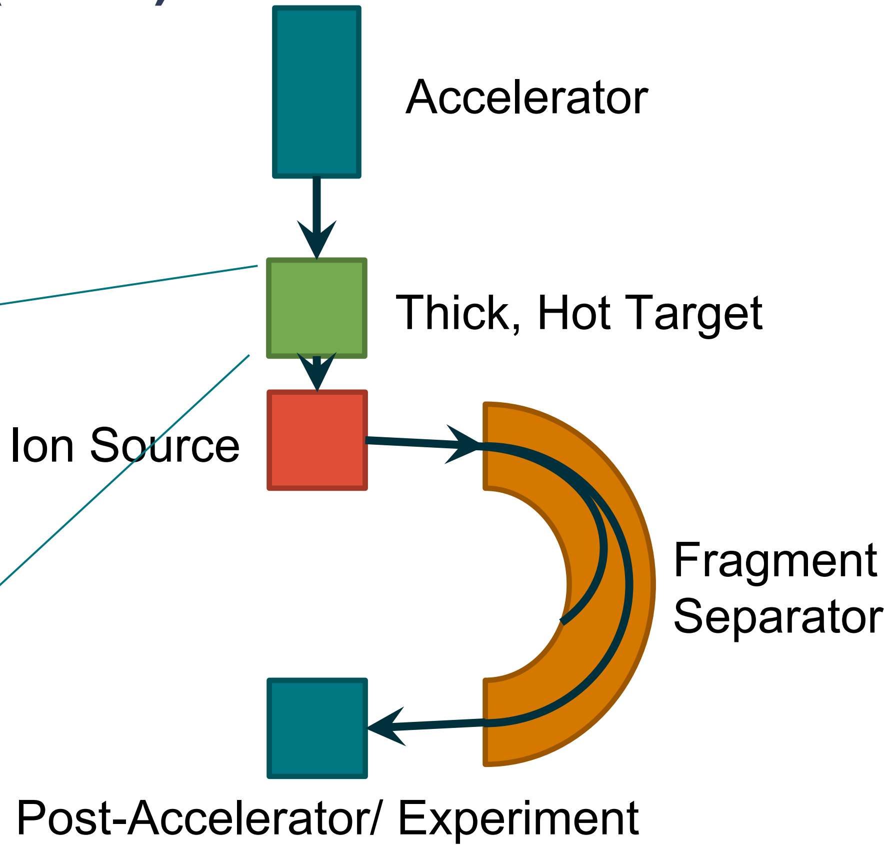
However, there are a finite # of combinations of stable beams + targets → make radioactive beams, and use these

Figure: Borrowed from R.M. Clark, 2007 RIA Summer School

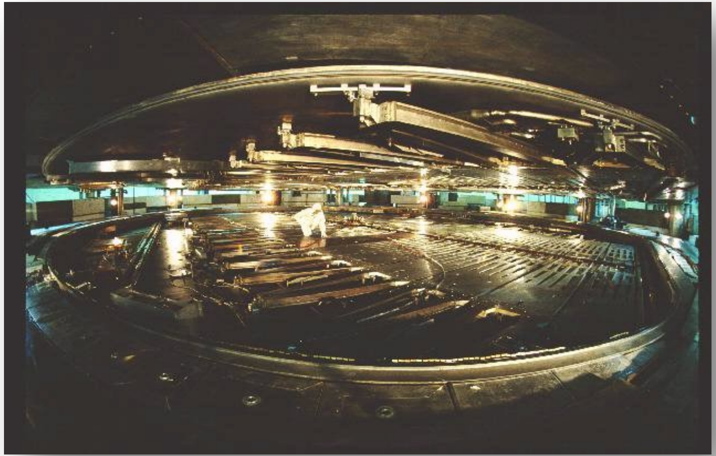
Isotope Separation On-Line (ISOL)

Examples of ISOL facilities:
TRIUMF (Canada)
SPIRAL/SPIRAL2 (France)
HIE-ISOLDE (CERN)
iTHEMBA (South Africa)





ISOL Facilities - TRIUMF

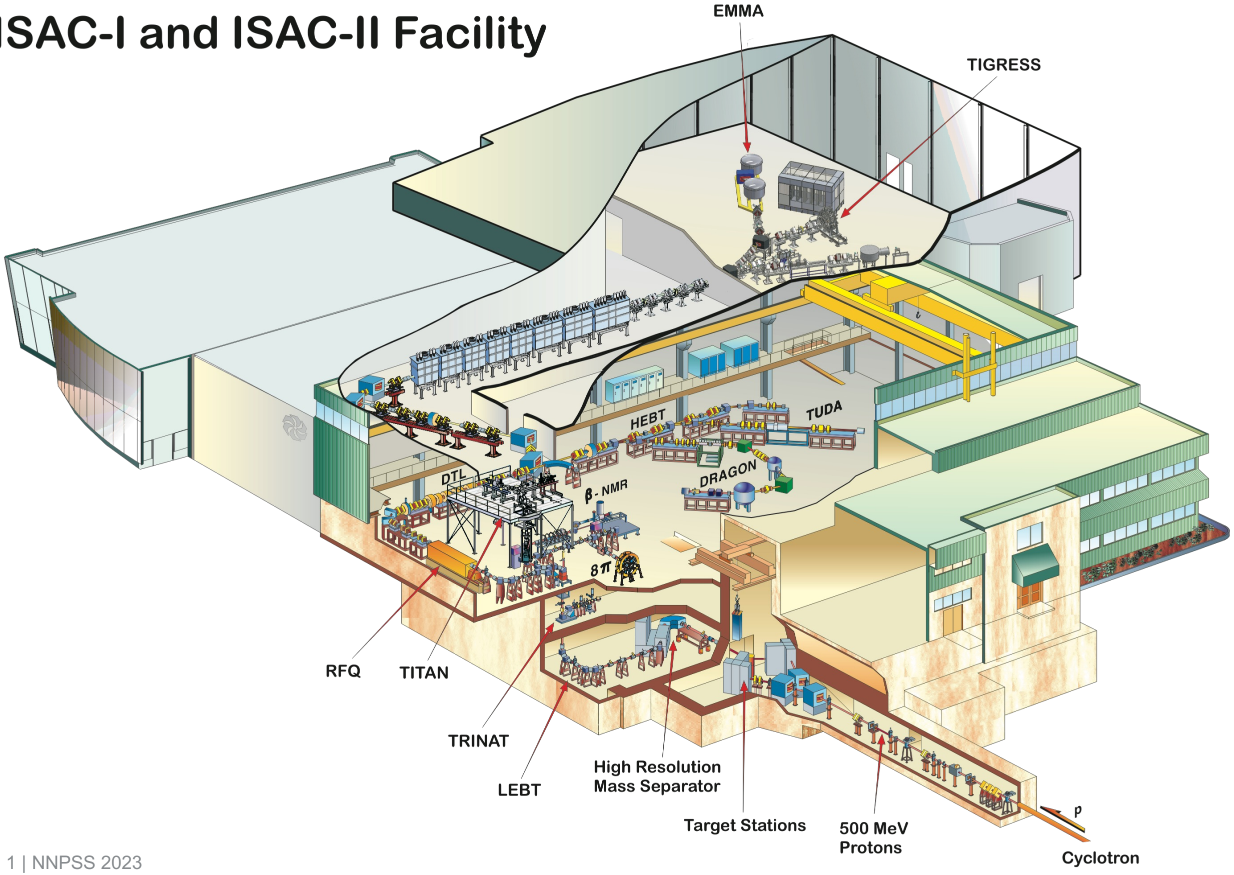


TRIUMF is home to the world's largest cyclotron – accelerates H^- to 520 MeV, extracts as proton beam

Proton beam is sent into target hall, and interacts with thick targets (materials such as UC)

ISOL Facilities - TRIUMF

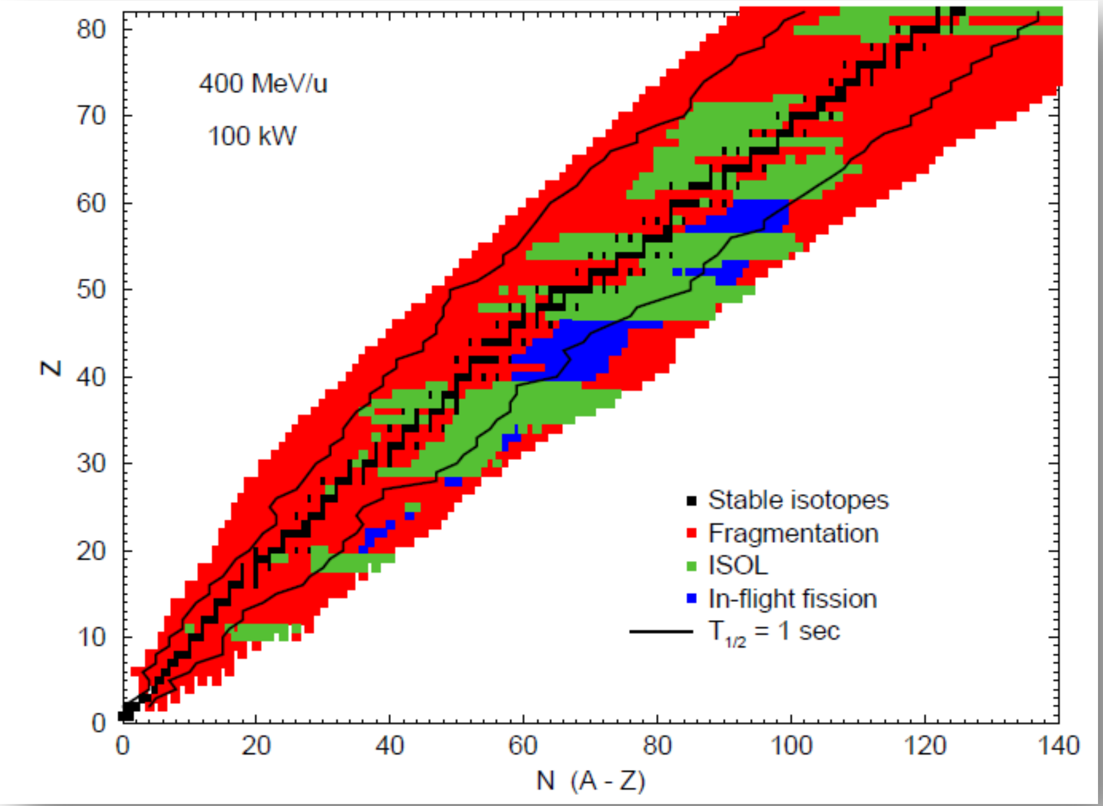
ISAC-I and ISAC-II Facility



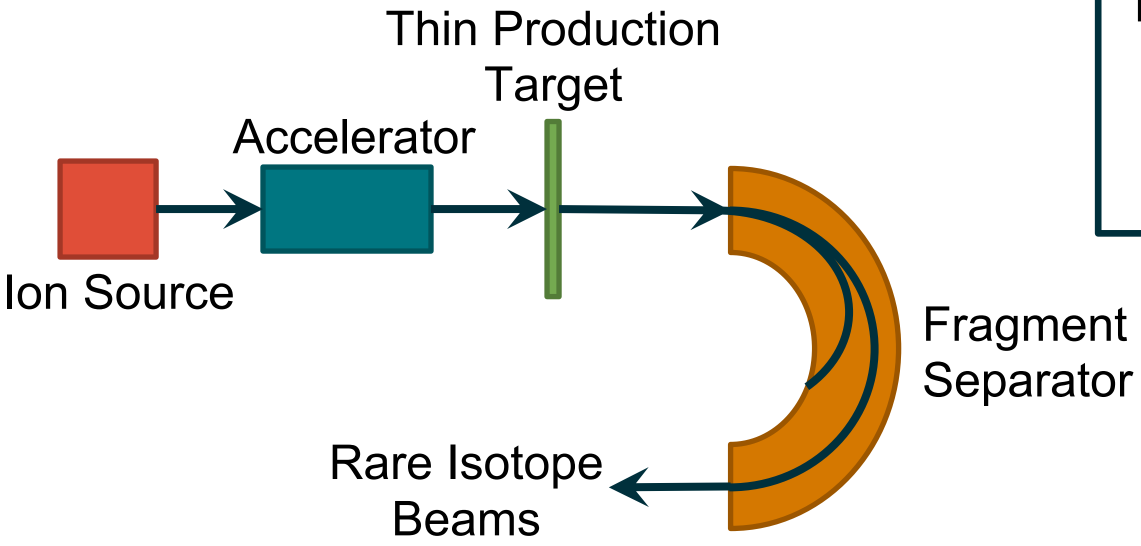
ISOL - Considerations

- Nuclei produced within the crystal lattice of the target must migrate to the surface → very chemically selective process
- Due to extraction time, limited isotopes with long lifetimes → $\tau > 1s$
- Beam ions lose a LOT of energy in the target → targets must withstand very high temperatures, which limits materials available
- Products that diffuse to the surface must still be ionized before they can be used in an experiment → again, chemically selective
- Chemically selectivity results in good Z purity of radionuclides, but you need mass separator ionization to isolate according to A – with this though, can obtain isotopically pure beams
- Reaccelerate rare-isotopes so beams are often very good quality, and for certain elements, very high intensity (i.e. alkali earth)

What can we make? – ISOL

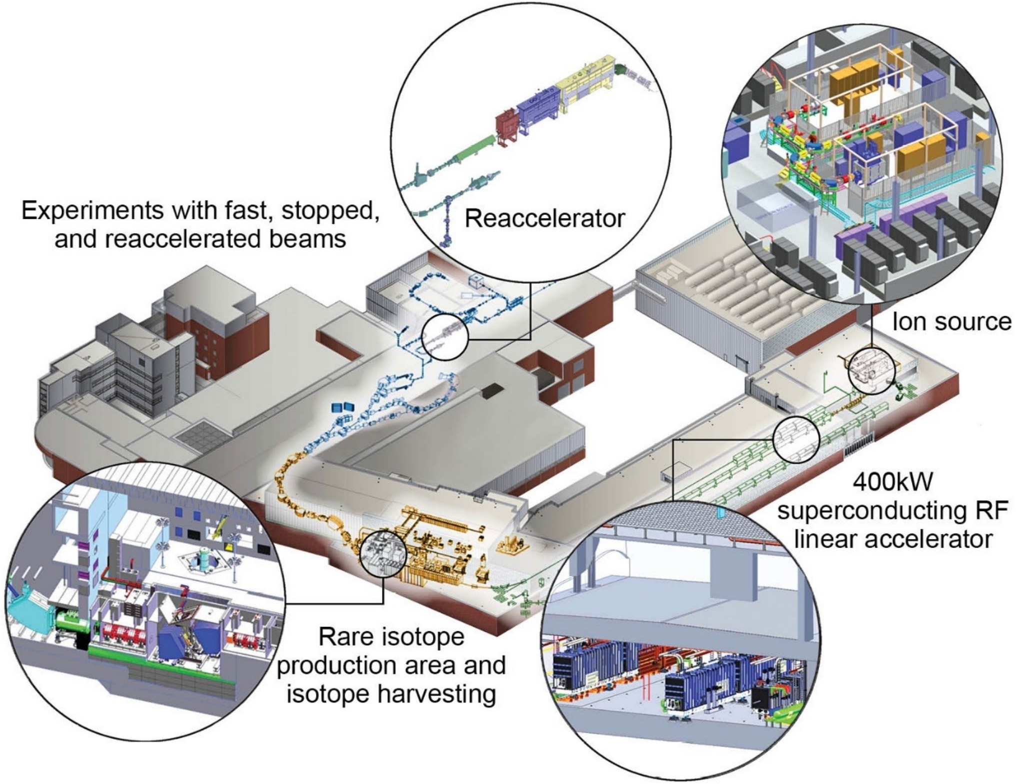


Fragmentation + In-flight separation



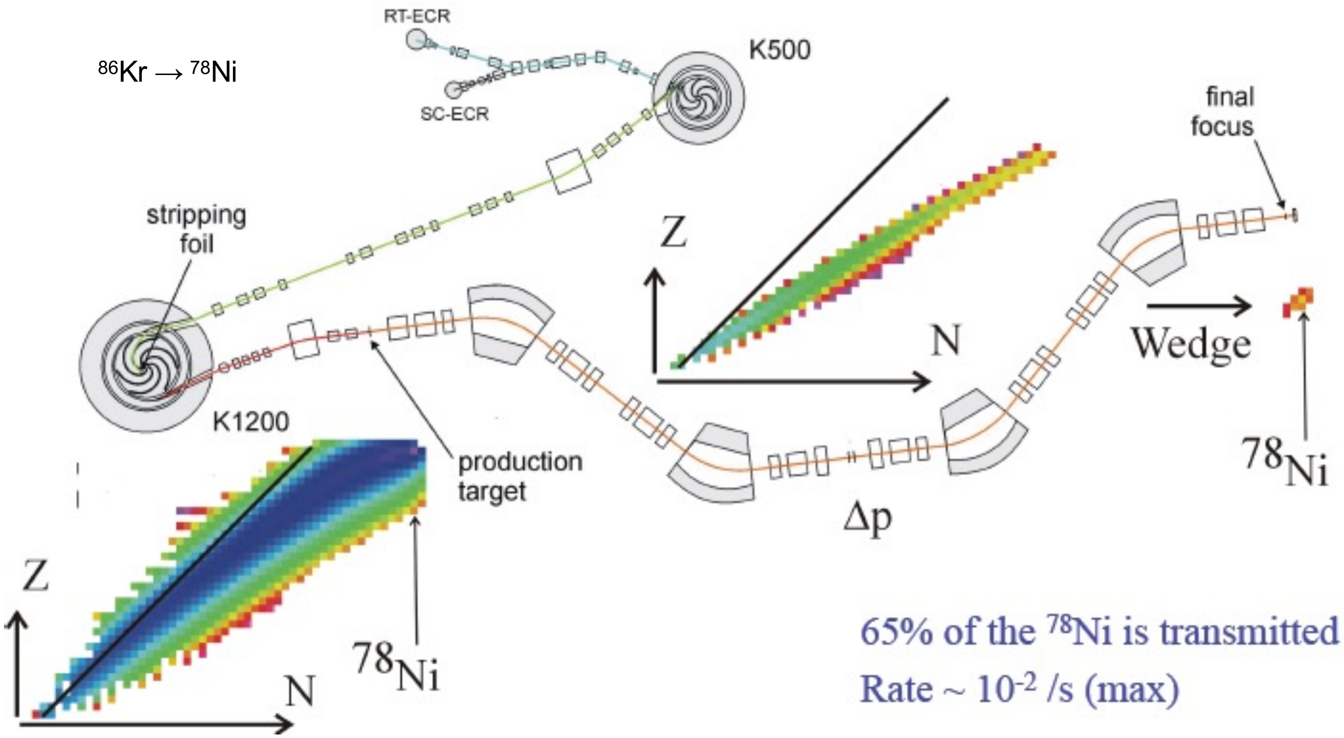
Examples of fragmentation facilities:
FRIB (USA)
RIKEN (Japan)
GANIL (France)
GSI (Germany)

Fragmentation facilities - FRIB



Fragmentation + separation: Example

$^{86}Kr \rightarrow {}^{78}Ni$



65% of the ^{78}Ni is transmitted

Rate ~ 10^{-2} /s (max)

Fragmentation - Considerations

- Production of nuclei is chemically independent – you make everything lighter than your primary beam, and it's moving FAST
- NEED in-flight separation to obtain clean secondary beams, and will still usually not obtain 100% beam purity
- Beams are high energy, momentum spread in reactions means they can have large emittances, etc.
- Certain experimental techniques are either NOT possible, or need to be significantly altered for fast beams*

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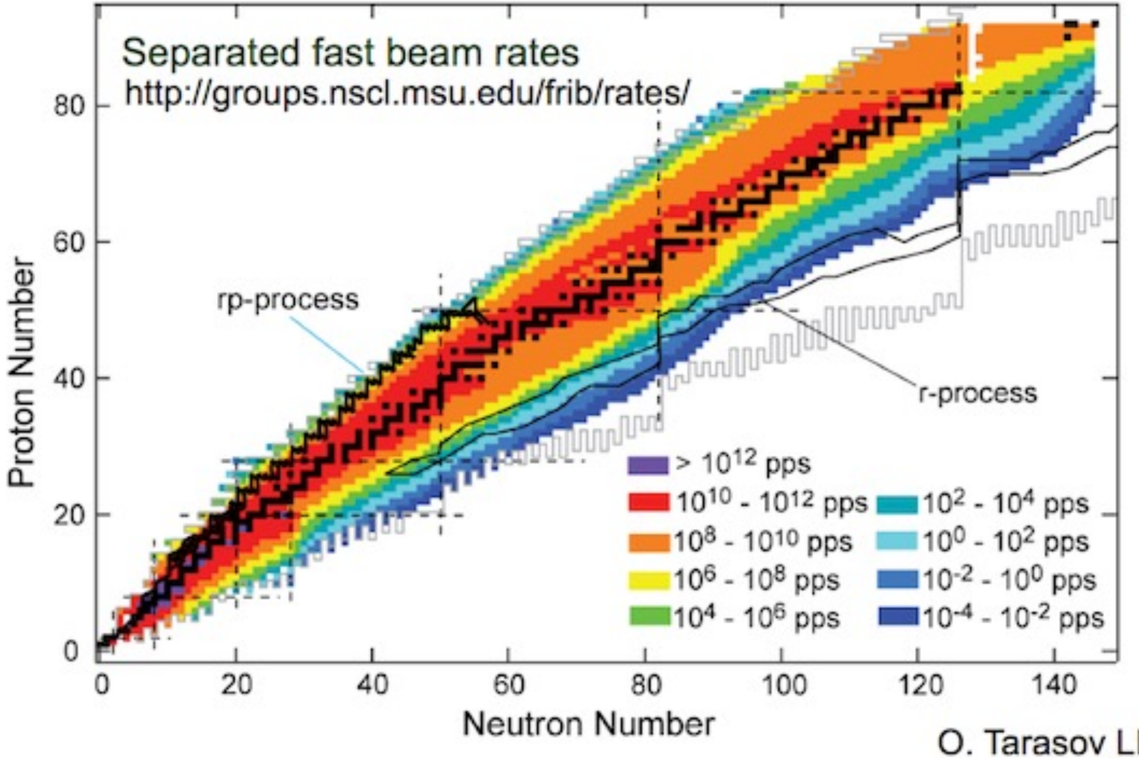
* Reaccelerated beams from at FRIB address this challenge in part, but not completely

What can we make? – Fragmentation

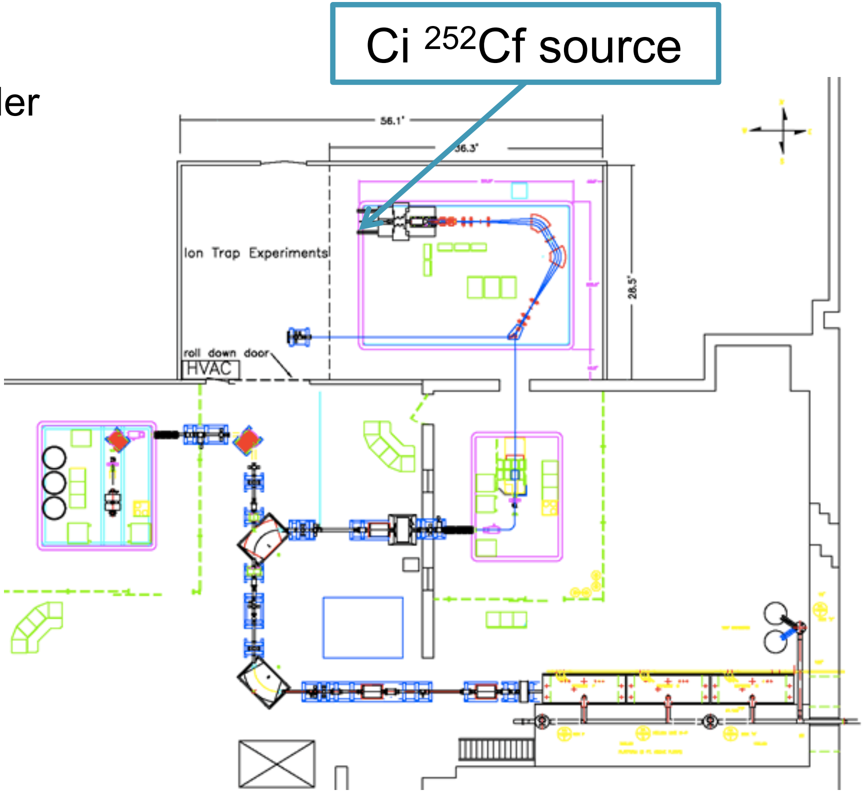


O. Tarasov LISE++

Fission-fragment accelerator

CARIBU (Californium Rare Ion Breeder Upgrade)
@ Argonne National Laboratory

- Turns a source of neutron-rich isotopes, such as a spontaneous fission source, into a low-energy beam using a gas catcher and charge breeder

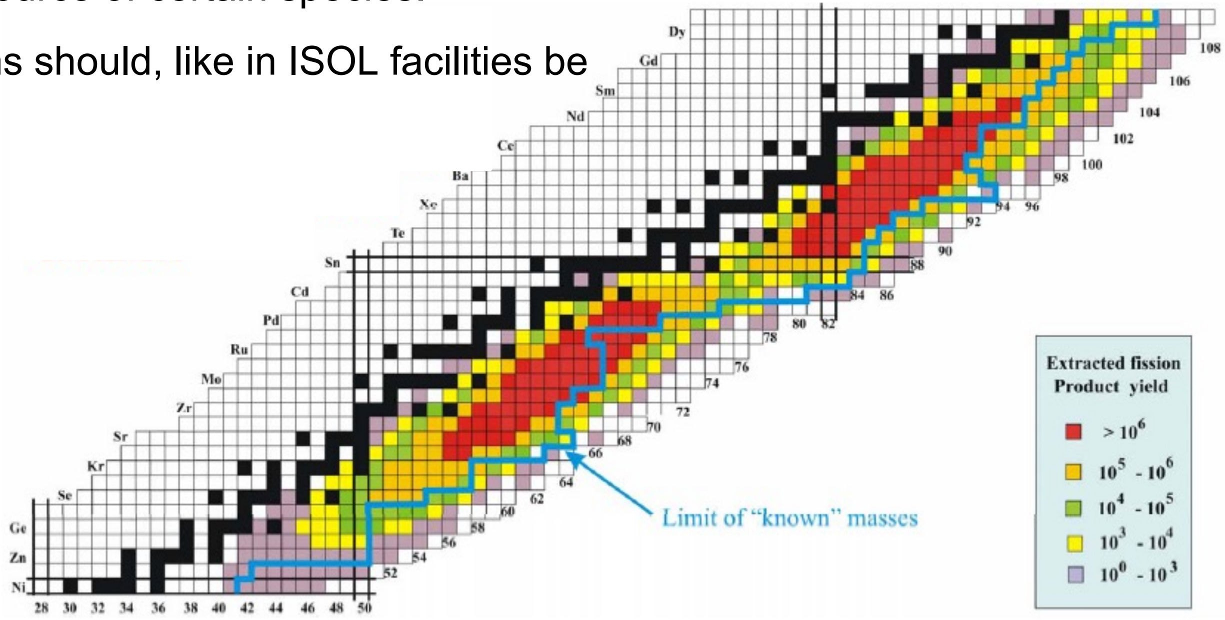


CARIBU: www.phy.anl.gov/atlas/caribu.html

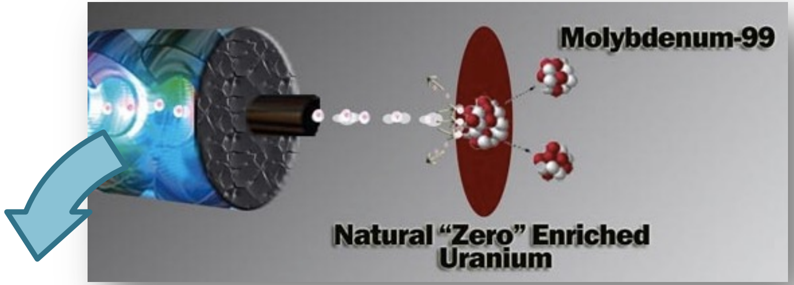
Fission fragment yields

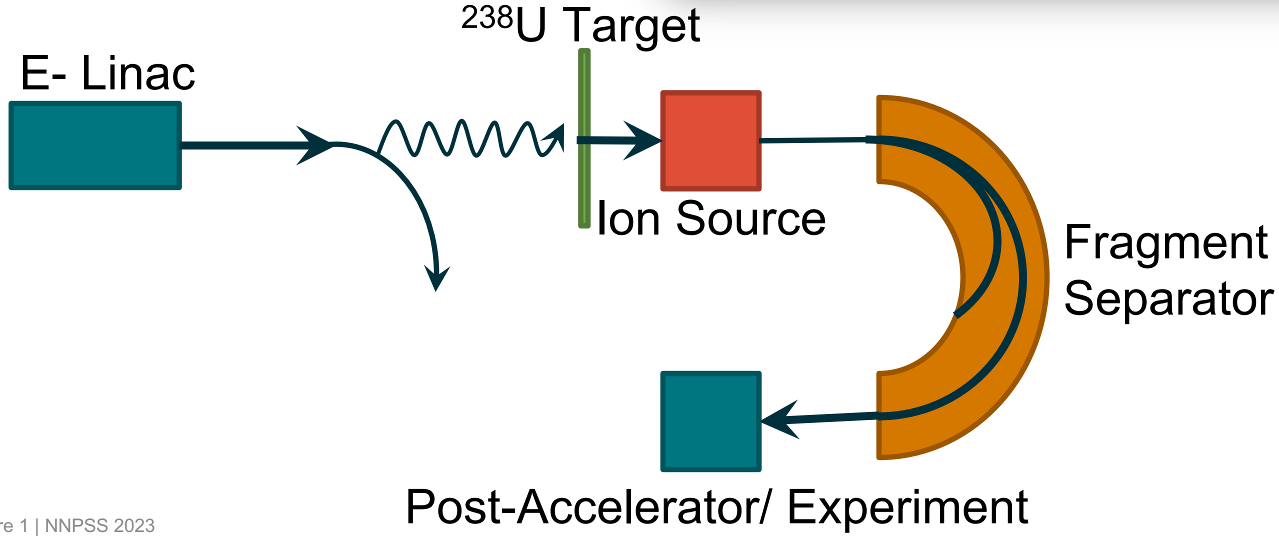
Production has inherent selectivity – cannot produce all nuclei but will provide an intense source of certain species.

Due to reacceleration, beams should, like in ISOL facilities be of very high quality.



On the horizon: Photo-fission production





Question!

If you wanted to run an experiment on ^{42}Ar, where would you go to make the measurement?

(A) TRIUMF (ISOL facility)
(B) NSCL (fragmentation facility)
(C) CARIBU (fission fragments)
(D) RIKEN RIBF (fragmentation facility)

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Mass Measurements

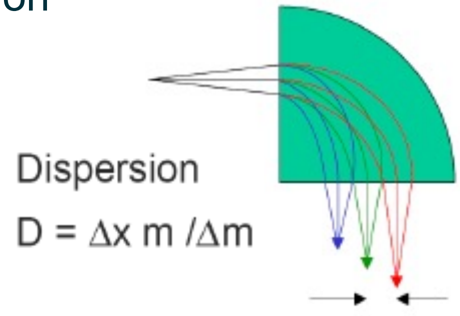
Mass measurements

Direct vs. indirect

- Indirect measurements
  - Q-value measurements – decay and kinematics from two-body reactions
- Direct measurements
  - Conventional mass spectrometry
  - Time-of-flight measurements
    - Spectrometer, multi-turn or multi-reflection
  - Frequency measurements
    - Penning traps, storage rings

A(a, b)B

$$Q = M_A + M_a - M_b - M_B$$



TOF mass measurements

Obtain mass based on equations of motion for charged particles through a magnetic system.

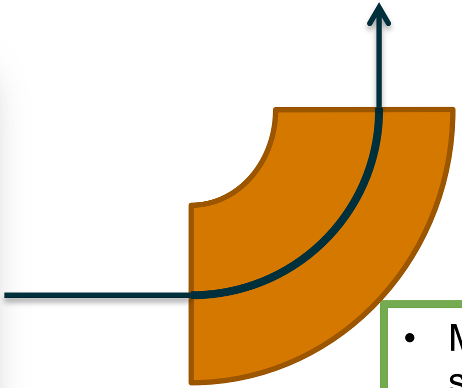
$$F_{Lorentz} = F_{centrifugal}$$

$$\rightarrow qvB\sin(\theta) = \frac{mv^2}{\rho}$$

$$m = \gamma m_0$$

$$v = \frac{L_{path}}{TOF}$$

$$\Rightarrow m_0 = \frac{TOF}{L_{path}} \frac{q(B\rho)}{\gamma}$$



- Measure multiple masses simultaneously
- Accuracy $\Delta m/m \dashrightarrow 10^{-6}$
- Lifetimes down to ms

Measurement requires precision knowledge of TOF and magnetic rigidity ($B\rho$).

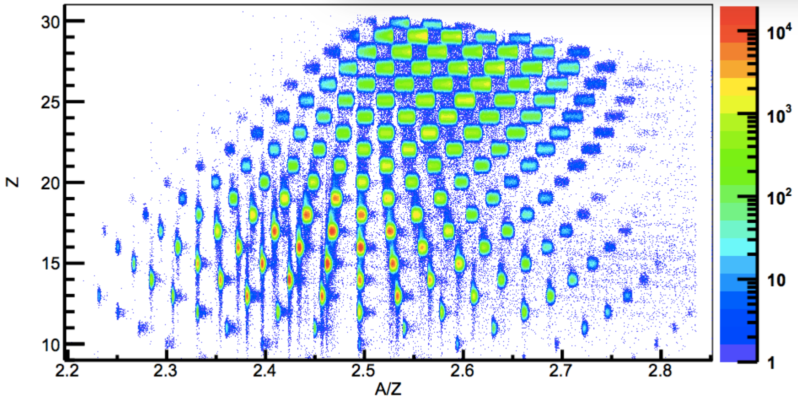
→ In practice, measure known masses to calibrate TOF measurements

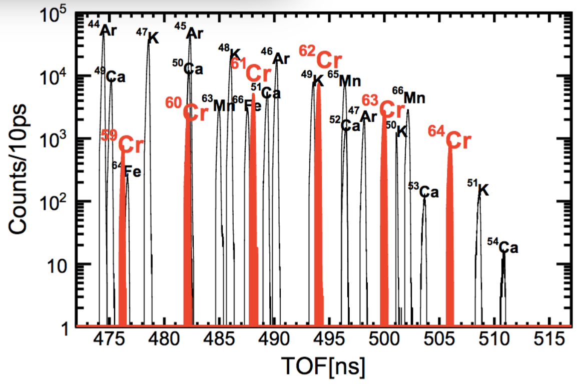
Experimental equipment – long flight-path magnetic separator

→ TOFI @ LANL, SPEG @ GANIL, ARIS+S800 @ FRIB…

TOF mass measurements

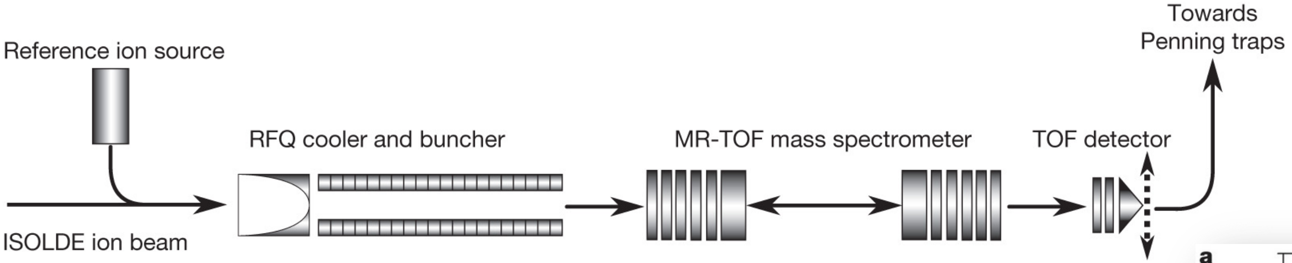
Solid angle	Ω = 20msr
Momentum acceptance	$\delta p/p \approx 1$ % (due to MCP)
Max Rigidity	$B\rho \approx$ 4Tm
Central flight Path	L_0 = 59m
Achieved Mass Resolution	$M/\Delta M \approx 5500$



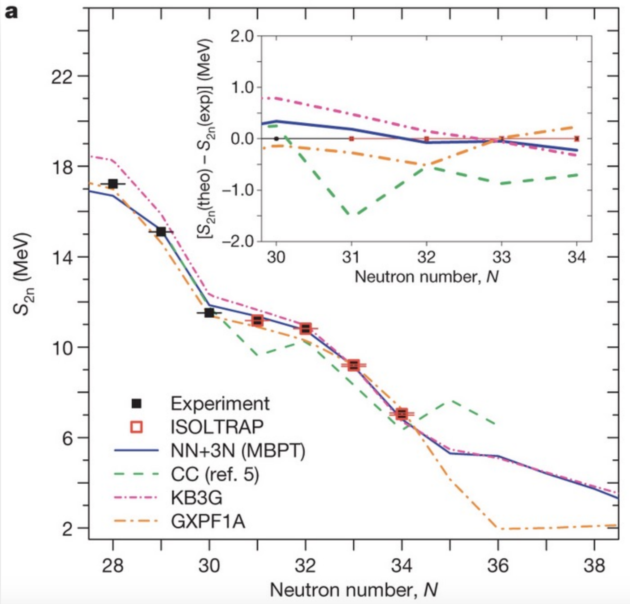


Z. Meisel, PhD Thesis, 2015.

MR-TOF – ^{54}Ca at ISOLDE

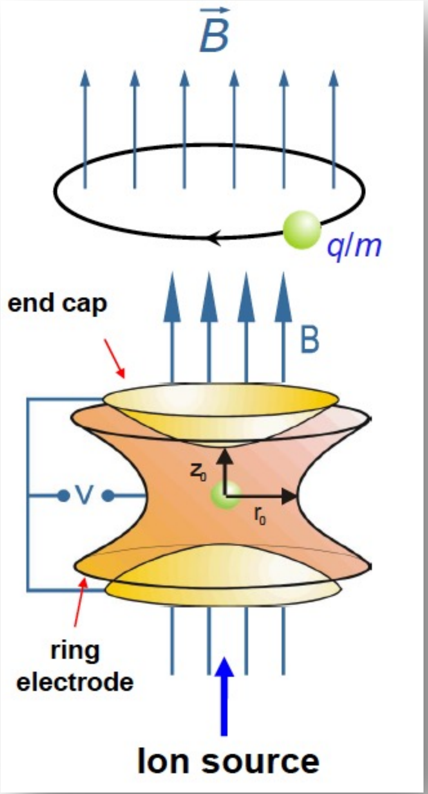


TOF measurements require long flight path lengths; a method around this is to “reuse” the same flight distance – multi-reflection TOF

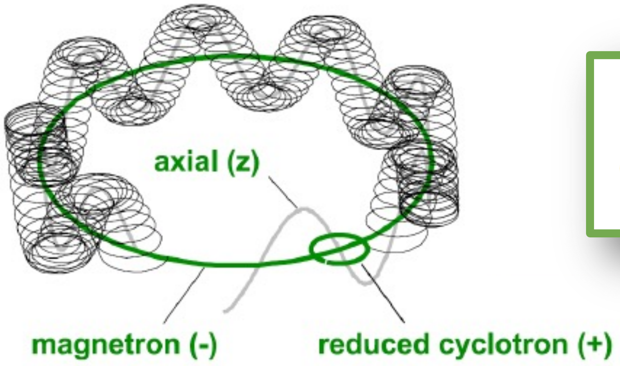


F Wienholtz et al. Nature 498, 346-349 (2013).

Penning trap mass measurements



Mass measurement comes from determination of the cyclotron frequency for the characteristic motion of the stored ions



$$f_c = \frac{1}{2\pi} \cdot \frac{q}{m} \cdot B$$

Motion is superposition of three fundamental motions:

- axial motion (f_Z)
- magnetron motion (f_-)
- modified cyclotron motion (f_+)

$\rightarrow f_C = f_+ + f_-$

Penning trap mass measurements

Strong homogenous B field provides radial confinement; weak electric quadrupole field provides axial confinement

↓

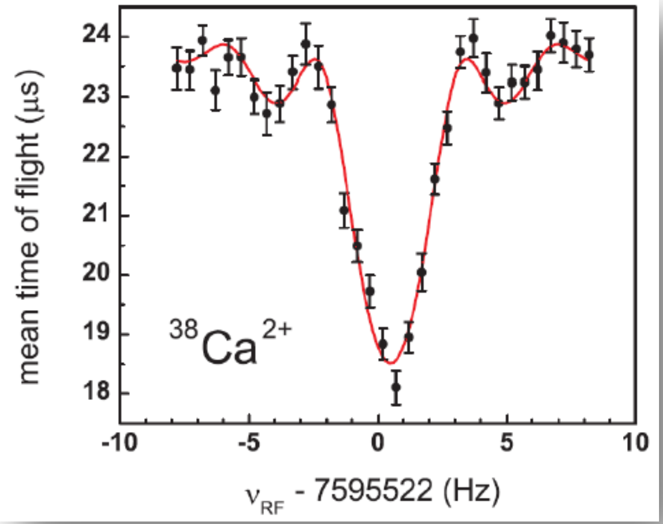
Excite cyclotron motion with multipolar RF (to excite cyclotron motion resonance)

↓

Transform radial to axial energy and eject ions toward particle detector

↓

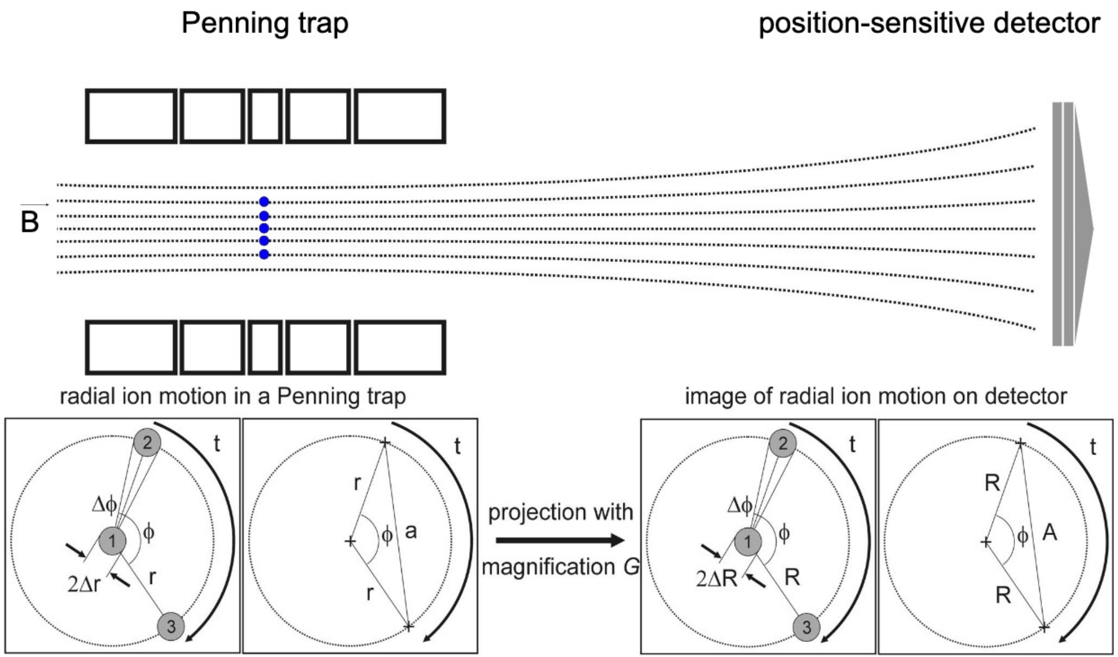
Measure TOF to detector

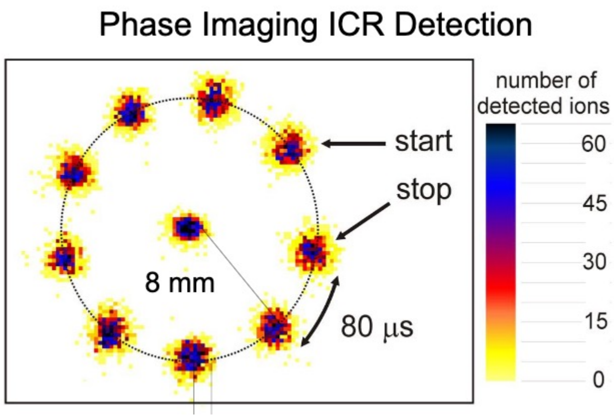


Constantly measure reference ions as well to reduce systematic error in measurements (varying B field, etc.)

G. Bollen et al., Phys. Rev. Lett. 96, 152501 (2006).

Phase Imaging Ion Cyclotron Resonance





Describing masses

Algebraic descriptions:

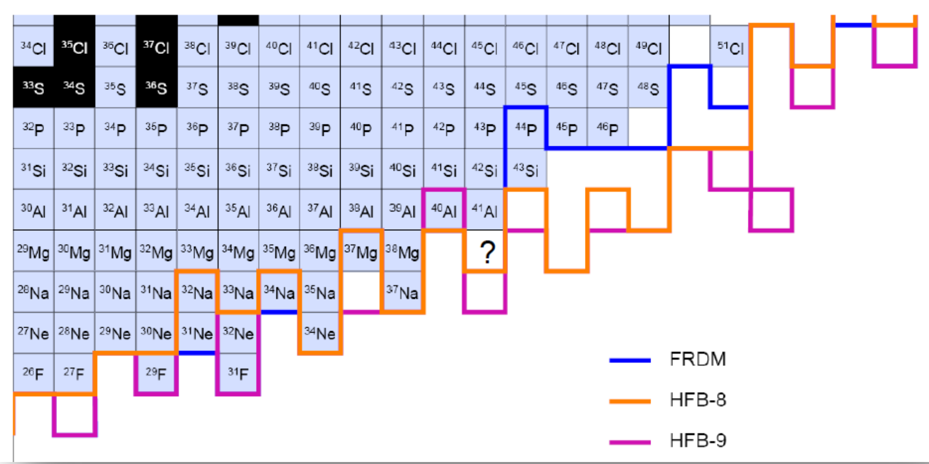
- Garvey-Kelson (GK) relationships between sums and differences between masses

Microscopic-macroscopic:

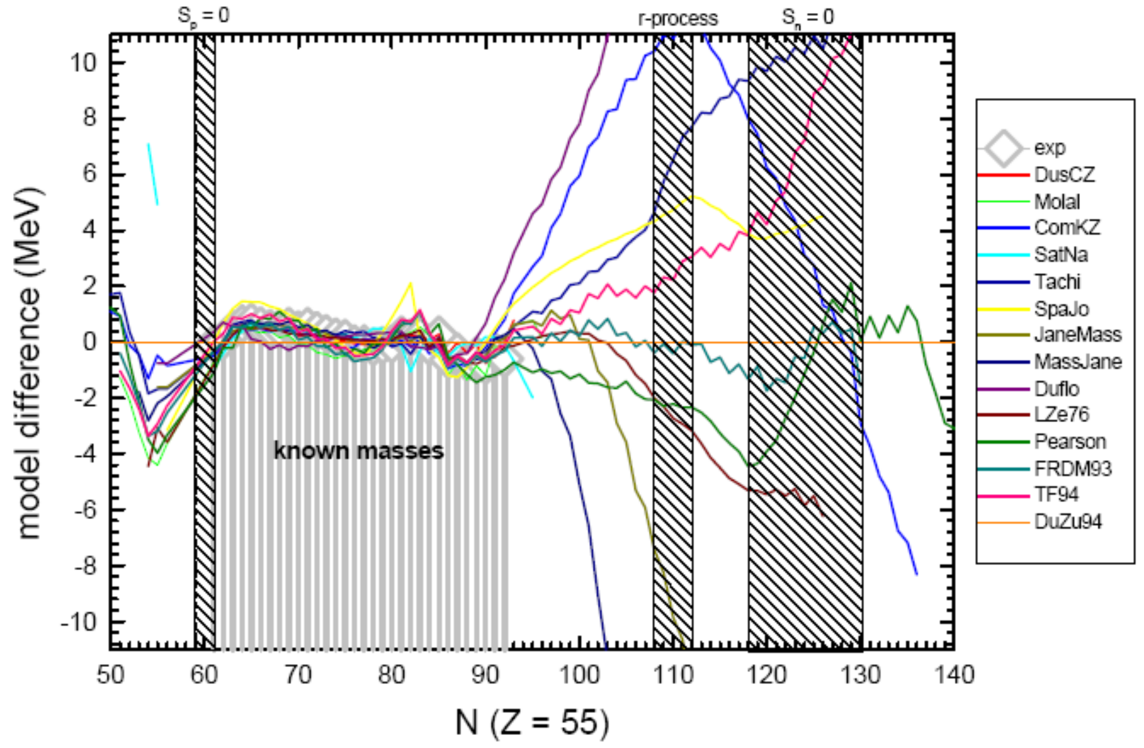
- Finite-range droplet model (FRDM) – 31 parameters fit to data
  - Bulk part from liquid drop + shell and pairing corrections

Microscopic:

- Relativistic mean-field (RMF) and Hartree-Fock Bogoliubov (HFB); use effective nucleon-nucleon interactions



What can we predict?



Away from available data, predictions still vary widely.

Question!

What technique would be best suited for measuring the (most accurate) mass of an exotic nucleus with a lifetime of about 10 ms?

(A) Penning trap mass spectroscopy
(B) TOF measurement at ISOL facility
(C) TOF measurement at fragmentation facility
(D) Decay Q-value measurement

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Thank You