UC RIVERSIDE | 2023 National Nuclear Physics Summer School

July 10-22, 2023

Lattice QCD (selected topics)

Lecture 3

Martha Constantinou

Temple University

July 12, 2023

OUTLINE OF LECTURE 3

- ★ Proton spin crisis
- ★ Proton mass puzzle
- ★ Key points of Lectures 3

OUTLINE OF LECTURE 3

★ **Proton spin crisis** But not really

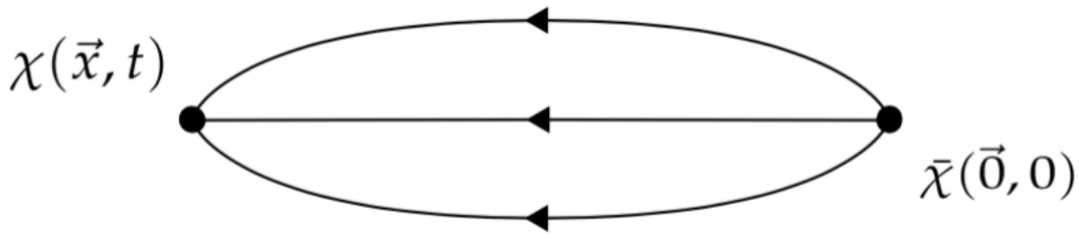
★ **Proton mass puzzle**

★ **Key points of Lectures 3**

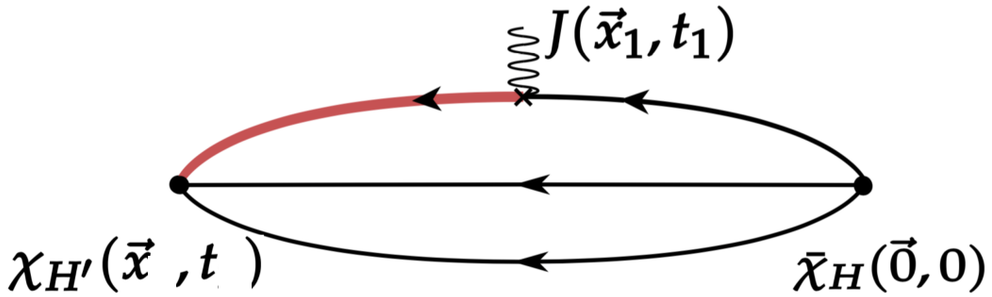
How to study Hadron Structure

Correlation functions in lattice QCD

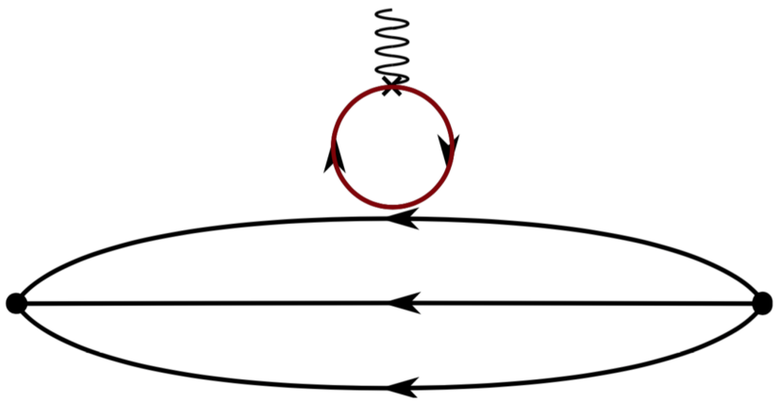
2pt-functions



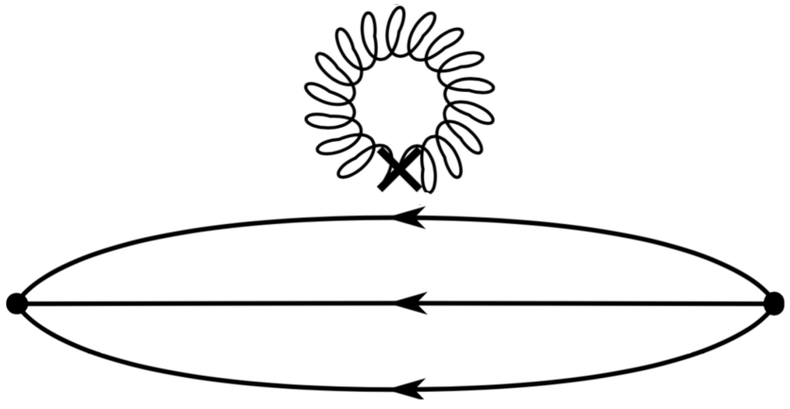
3pt-functions



Connected



Disconnected Quark loop



Disconnected Gluon loop

Physics investigation: the proton spin

Proton spin “puzzle”

- ★ Fundamental degree of freedom (from space-time symmetry) Proton spin:1/2
- ★ Spin plays an important role in determining the structure of composite particles, like the proton
- ★ Simple models predict that the 3 quarks responsible for the proton’s quantum numbers carry 1/3 of its spin
- ★ DIS experiments (1988) show surprising results for proton spin

[J. Ashman et al., Phys. Lett., vol. B206 (1988) 364]

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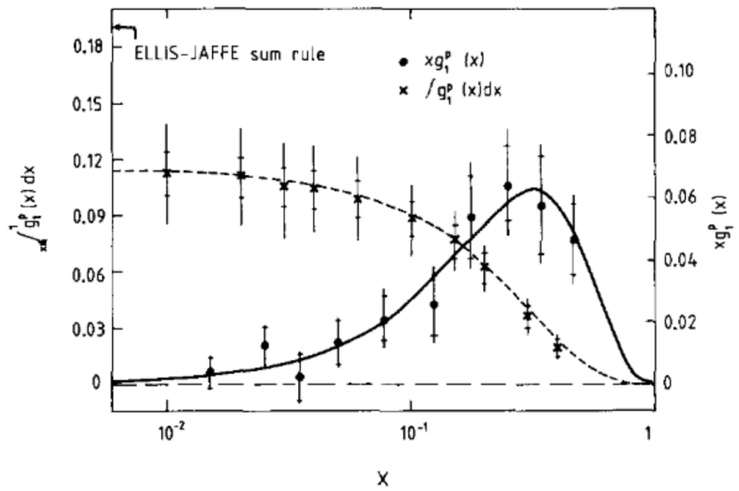

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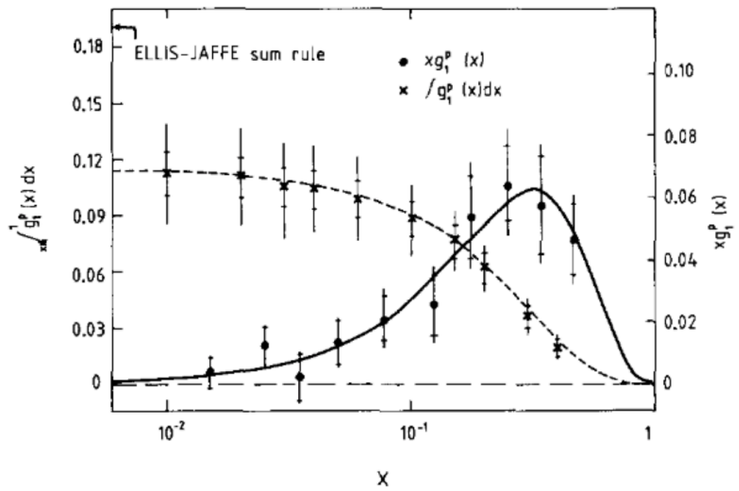

SPIN CRISIS !

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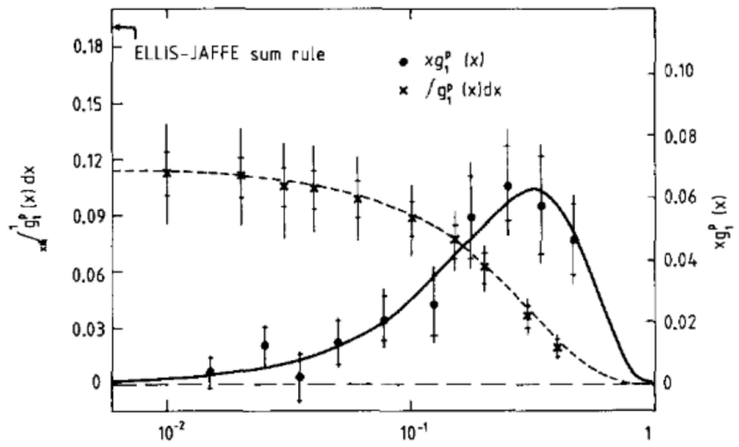
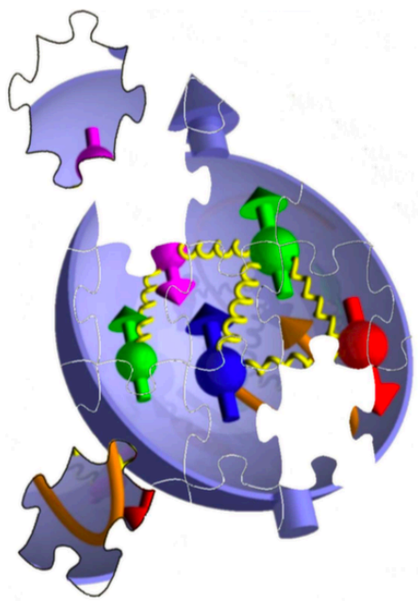


SPIN CRISIS !

Still open questions:

- Sea quark and gluon contributions
- Parton orbital angular momentum



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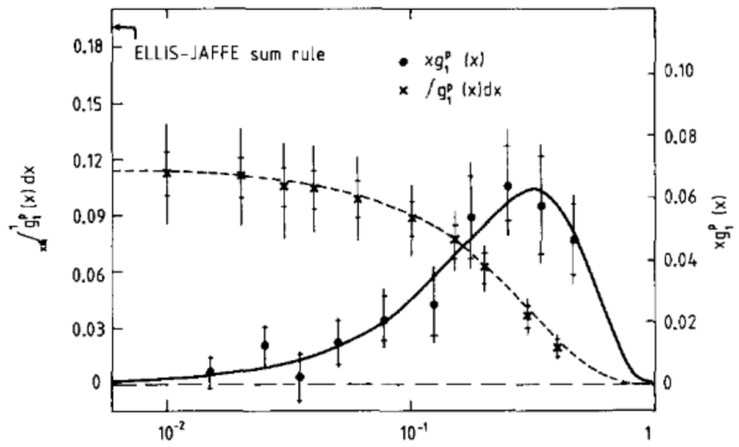

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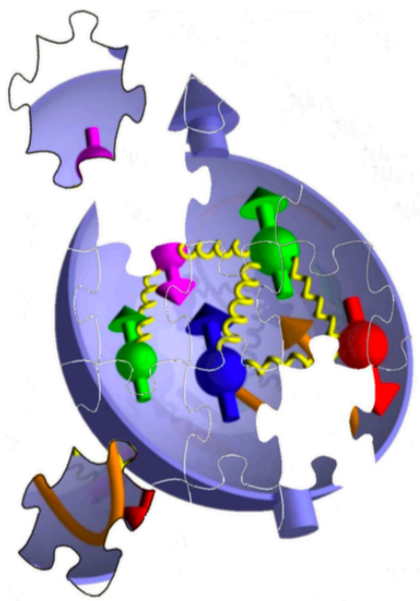

We must quantify the proton spin decomposition

Spin structure from first principles

Lattice QCD can provide important information on the spin

Ji's Spin Decomposition

$$\frac{1}{2} = \sum_q J^q + J^G = \sum_q \left(L^q + \frac{1}{2}\Delta\Sigma^q \right) + J^G$$

L_q: Quark orbital angular momentum

$\Delta\Sigma_q$: Intrinsic spin

J_g: Gluon spin

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Extraction from Lattice QCD:

$$J^q = \frac{1}{2}\left(A^q_{20} + B^q_{20} \right)$$

$$L^q = J^q - \Sigma^q$$

$$\Sigma^q = g^q_A$$

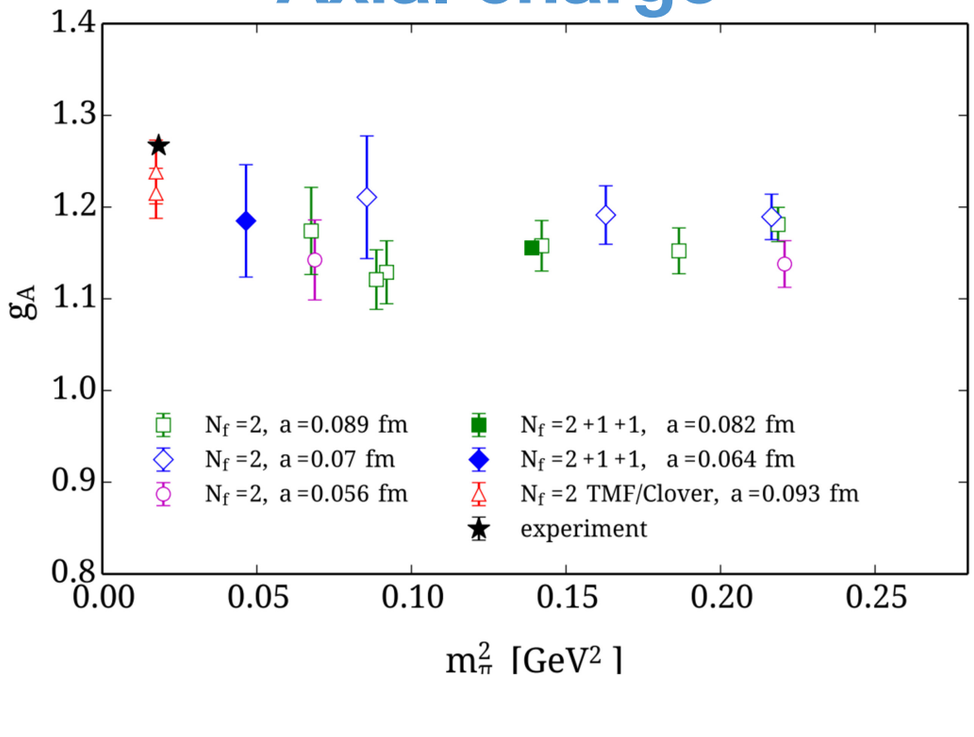
Necessary computations:

- Axial Charge
- Quark momentum fraction
- Gluon momentum fraction

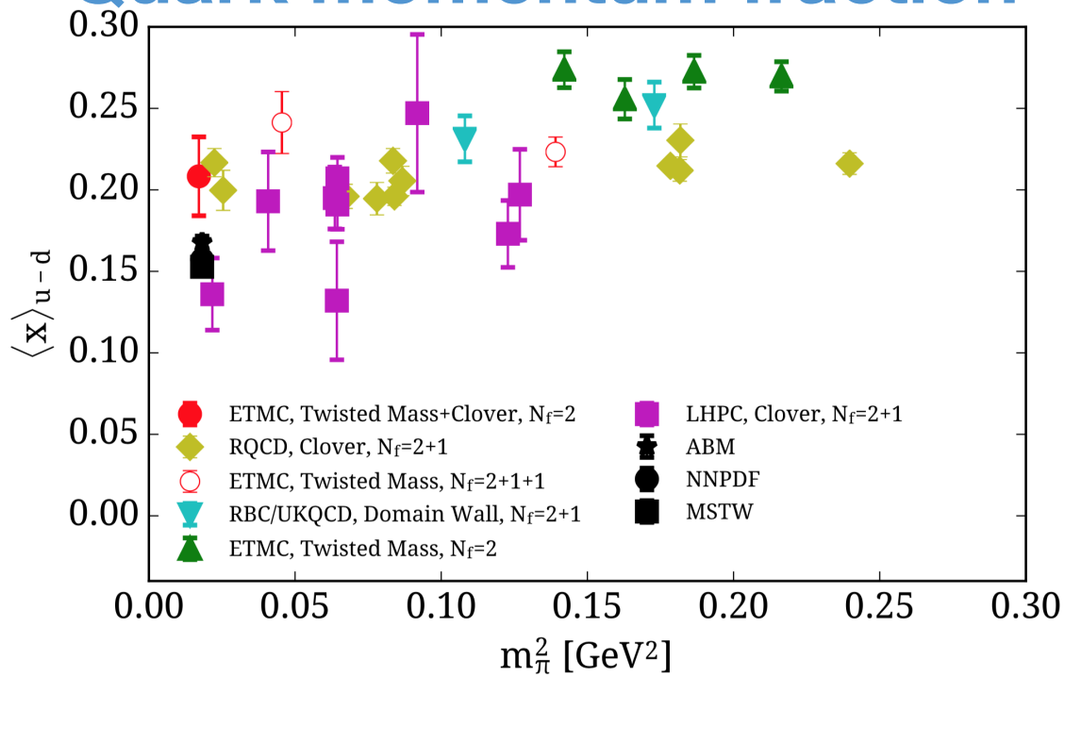
Quark Orbital Angular extracted indirectly

Spin components in Lattice QCD

Axial charge



Quark momentum fraction



Spin components in Lattice QCD

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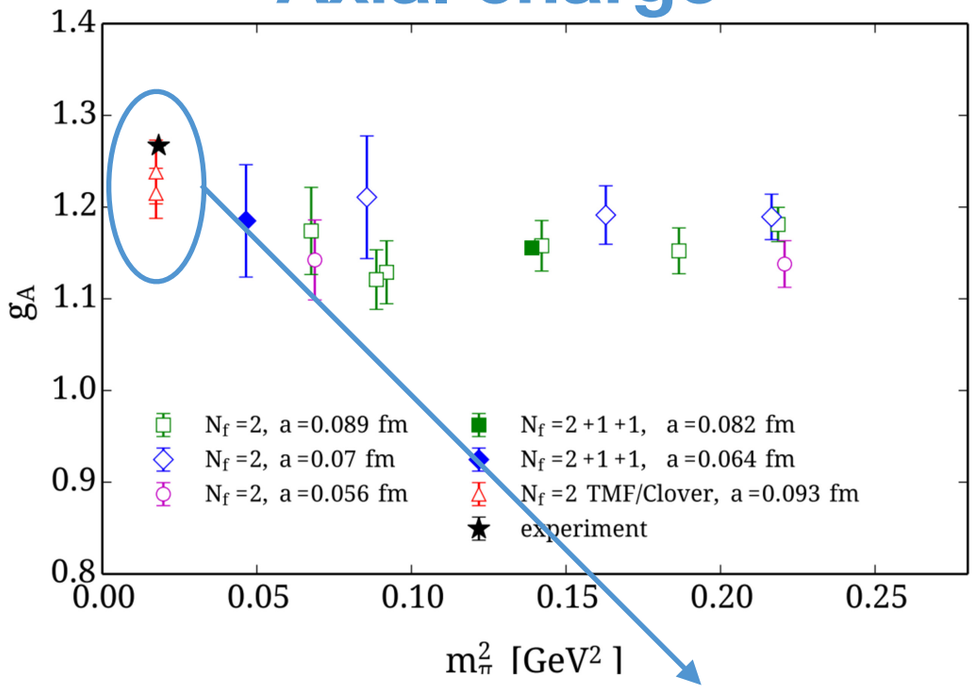
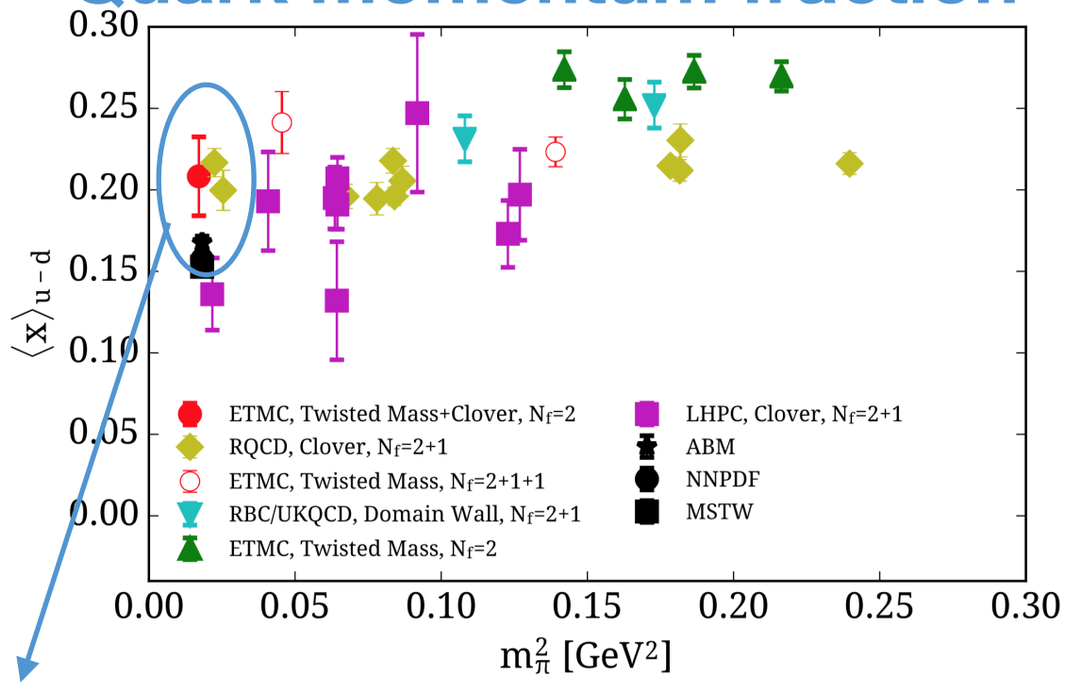


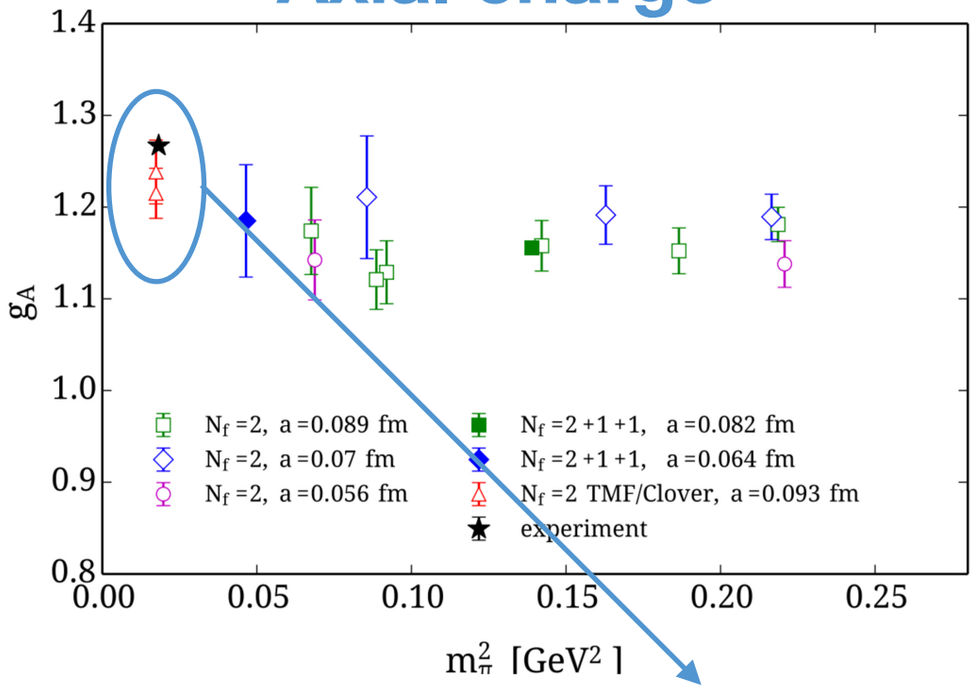
Quark momentum fraction



Simulations of nature enabled from new methods, algorithms, computer architecture

Spin components in Lattice QCD

Axial charge



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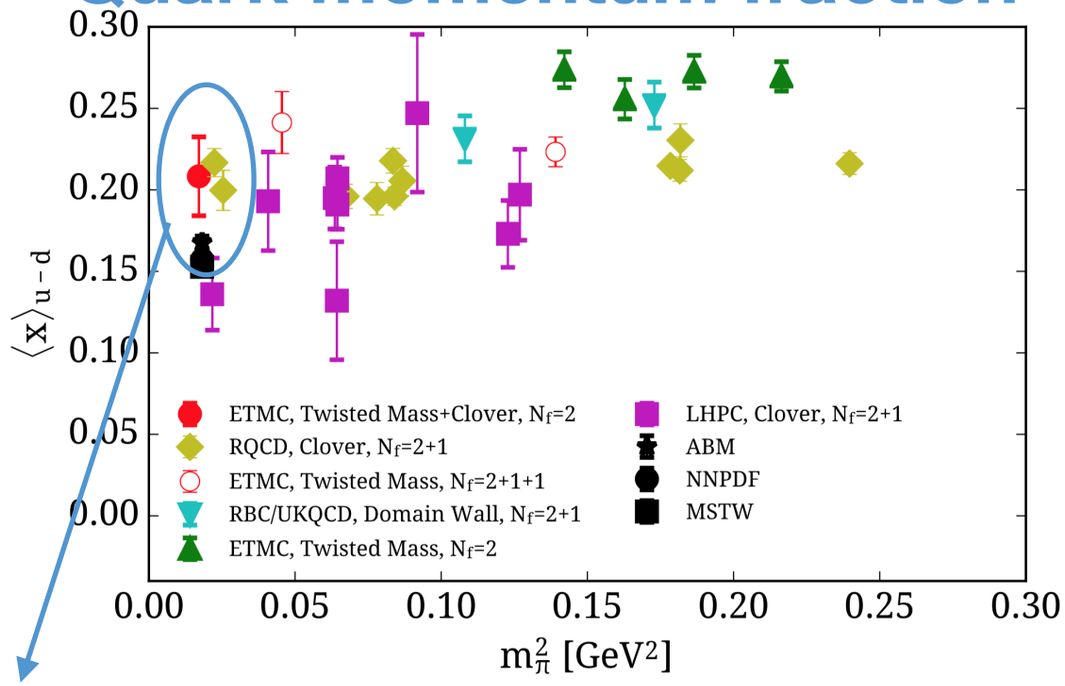
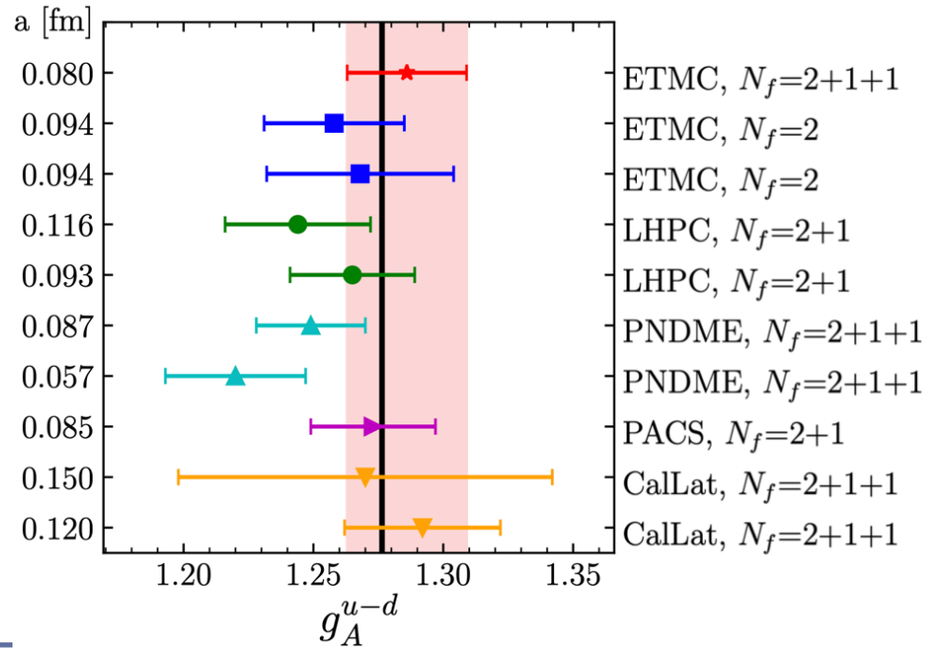


Simulations of nature enabled from new methods, algorithms, computer architecture

Summary of results

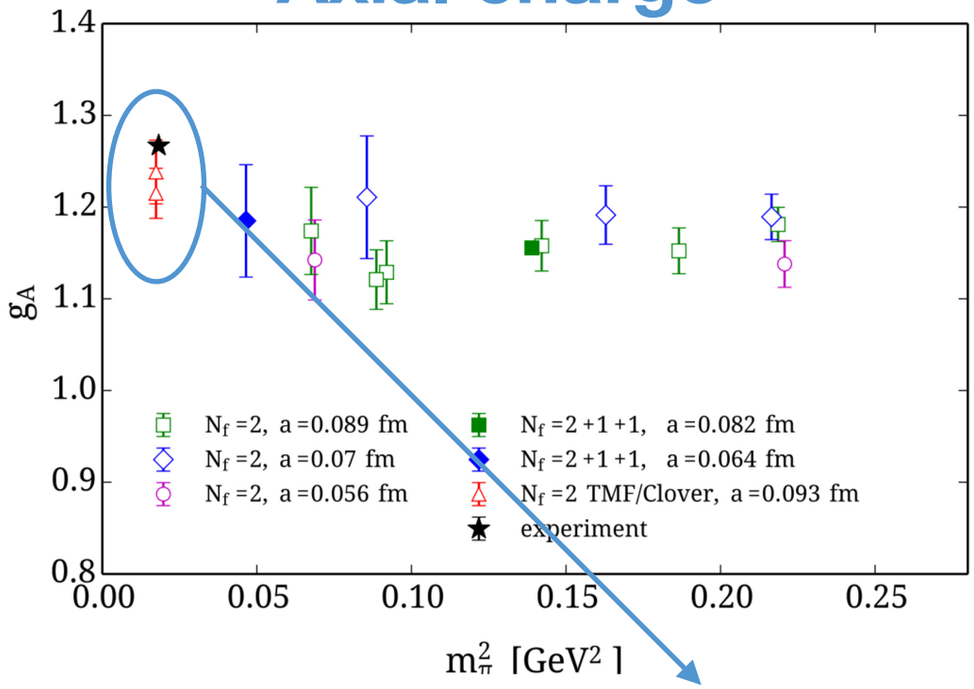


	u	d	s	c
g_A	0.862(17)	-0.424(16)	-0.0458(73)	-0.0098(34)

Taking into account the disconnected contributions is crucial for the spin

Spin components in Lattice QCD

Axial charge



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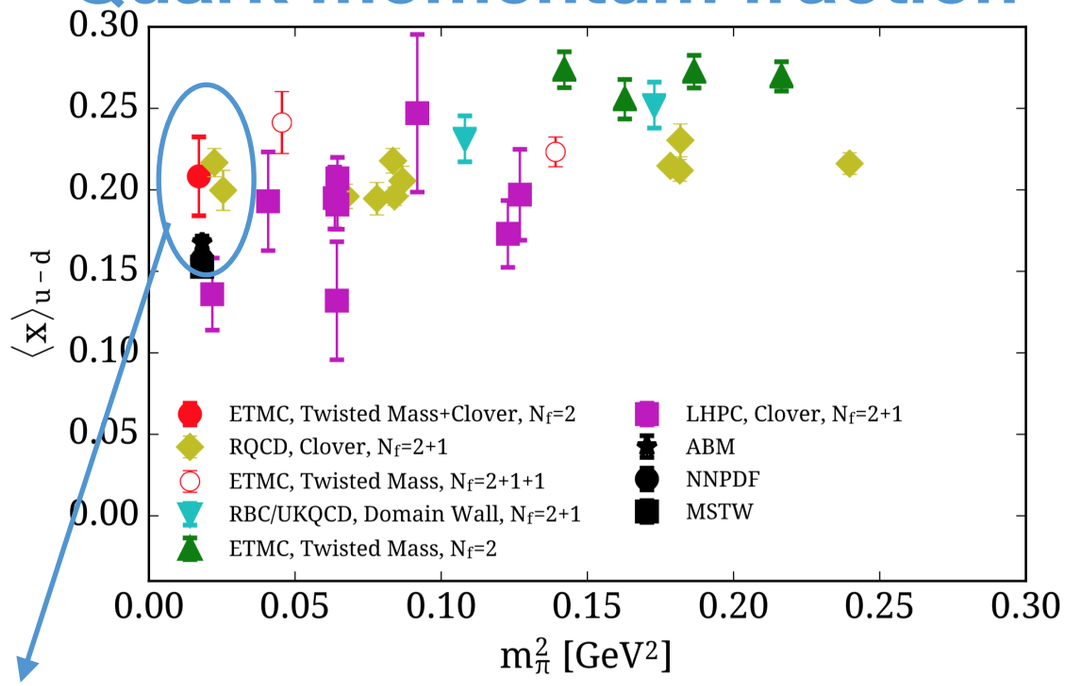

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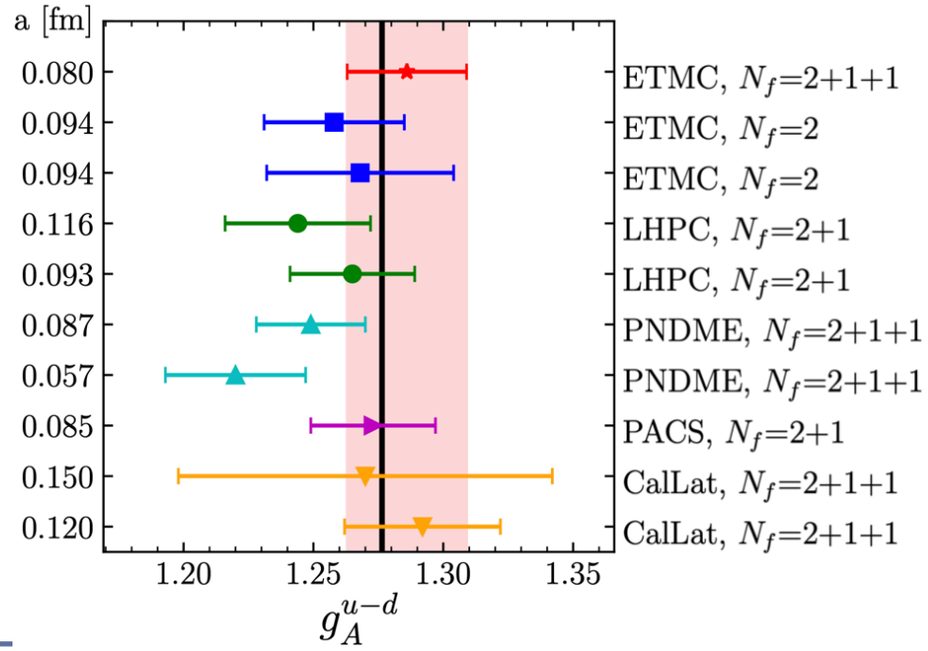

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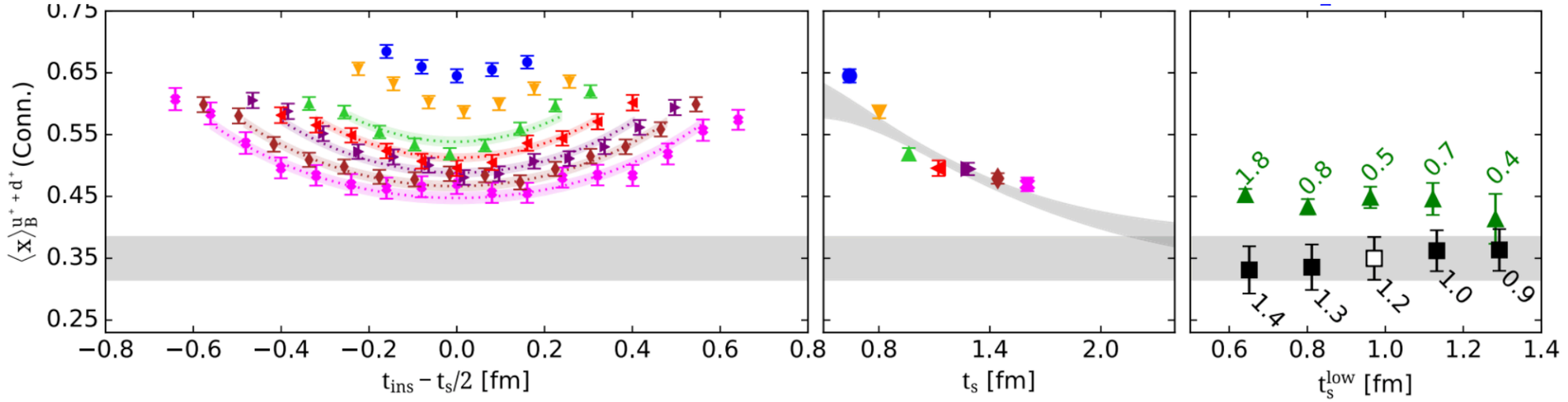
Quark momentum fraction

★ **connected contributions**



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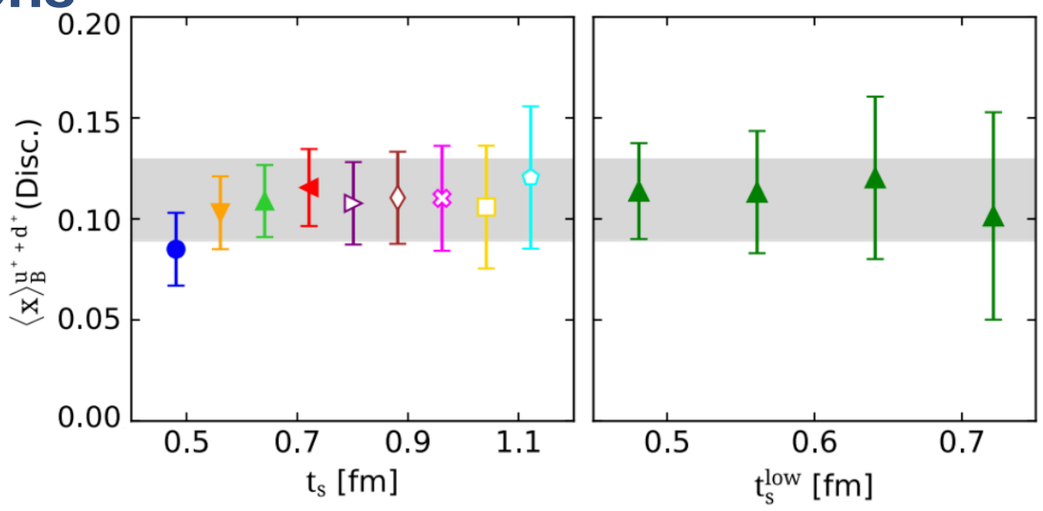

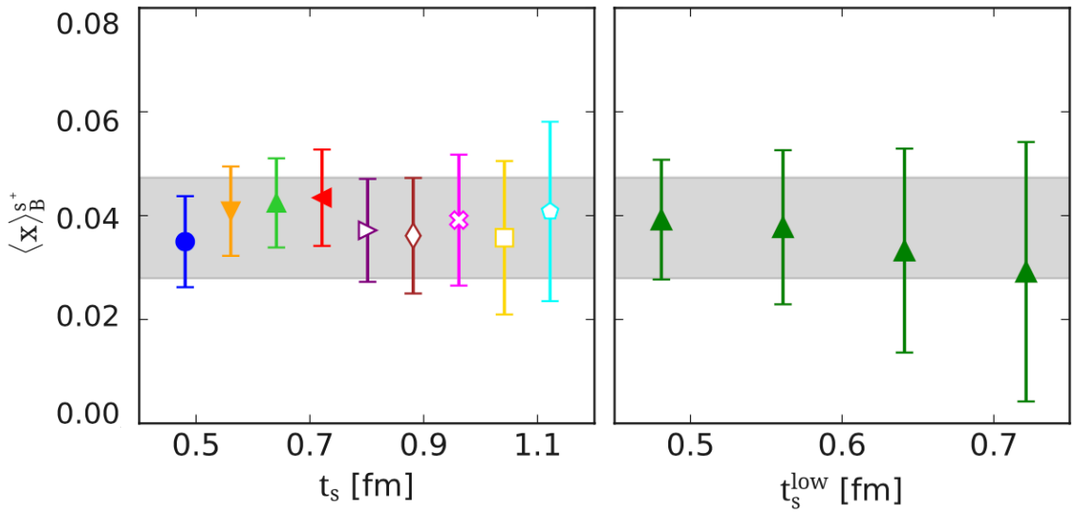

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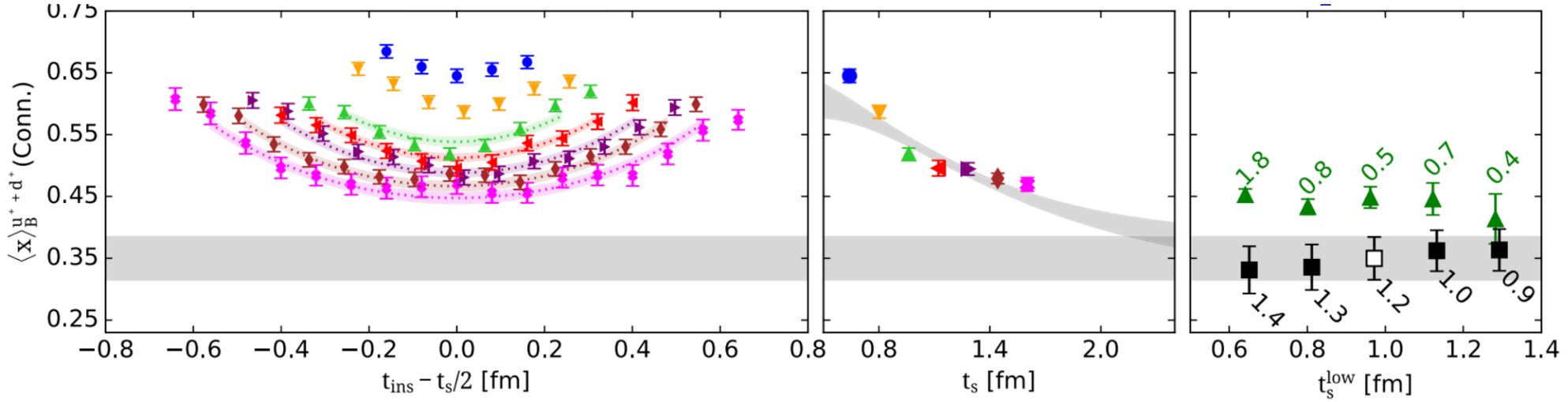

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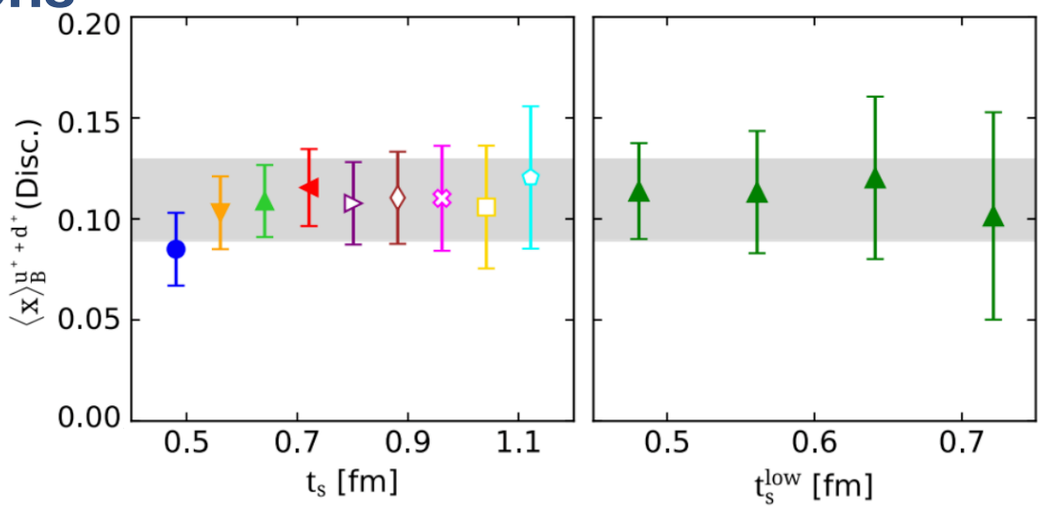
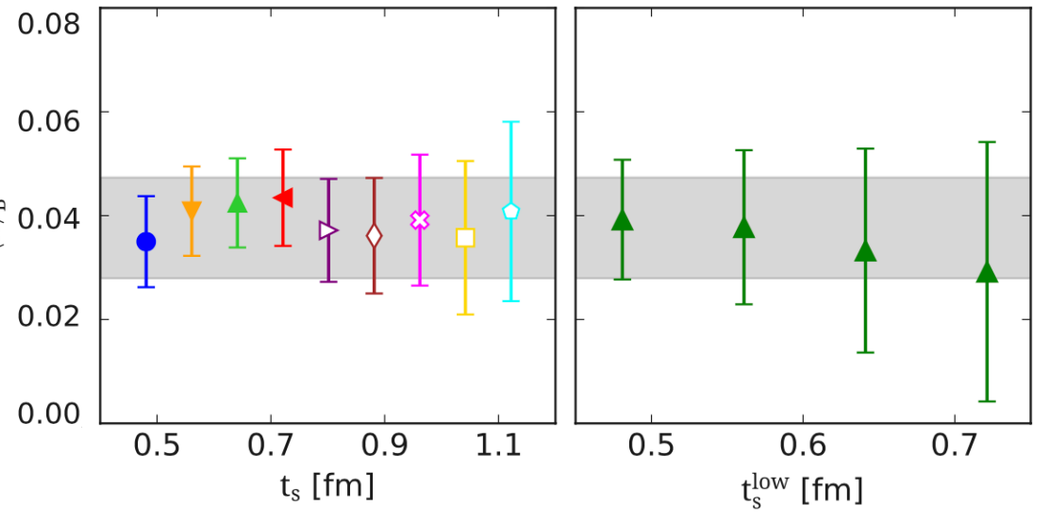
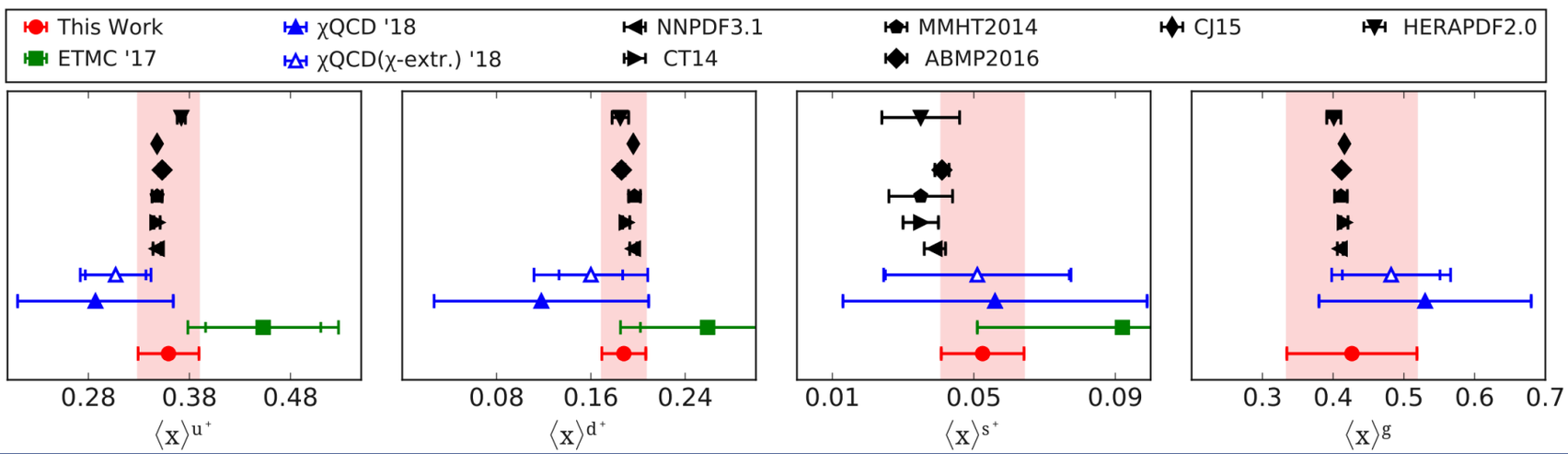


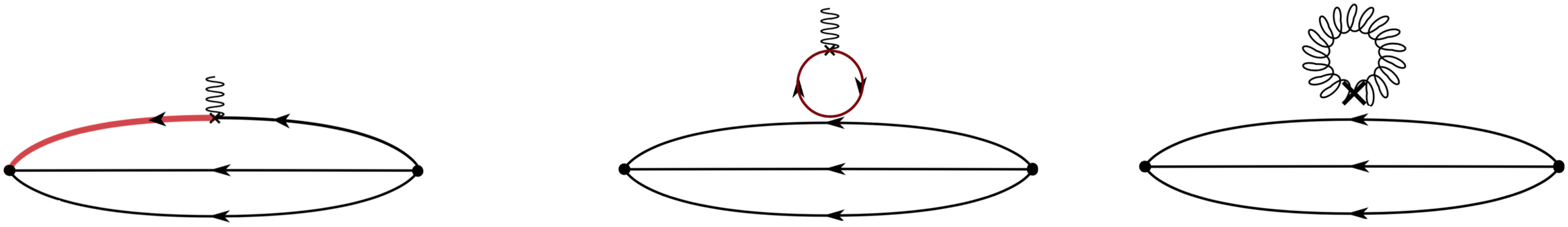


Collection of various results



Results @ physical pion mass

$\overline{MS}(2\text{GeV})$



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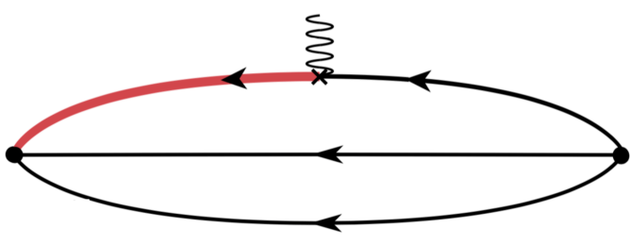
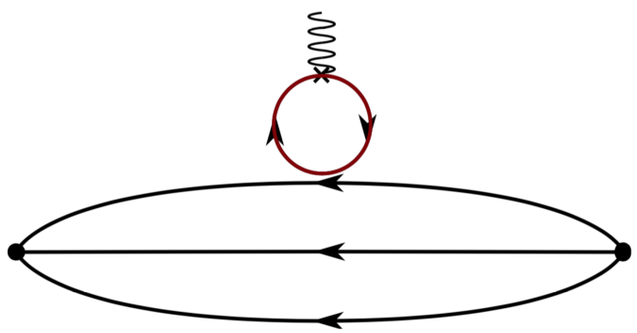


$\langle x \rangle^B_{u+d} = 0.350(35)$



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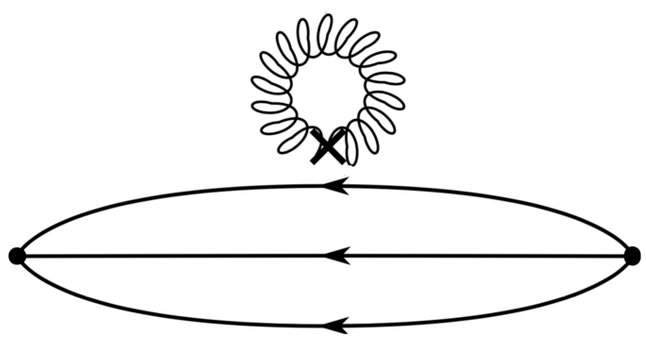

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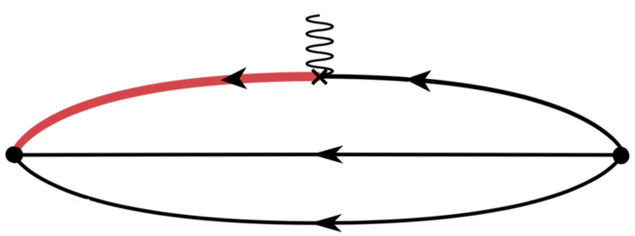

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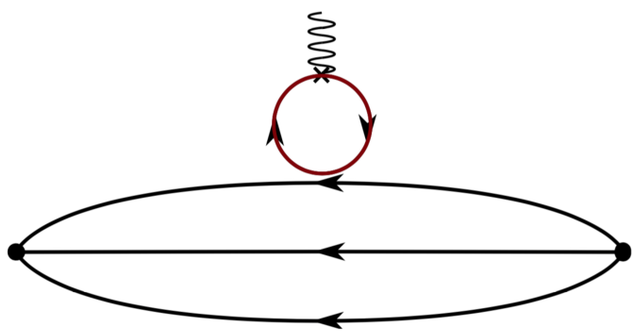

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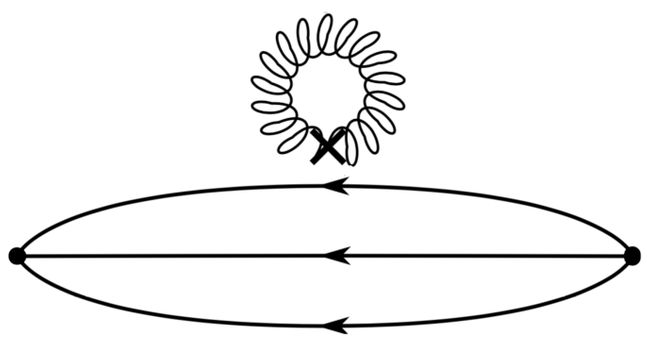

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★ Mixing between quark and gluon contributions to $\langle x \rangle$

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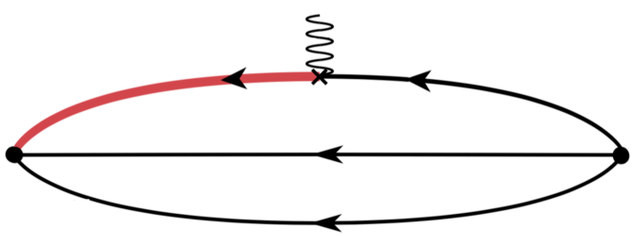

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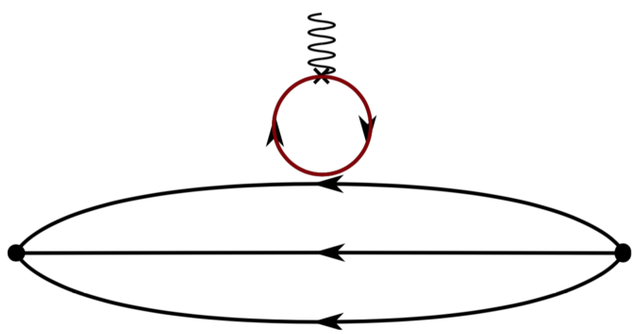

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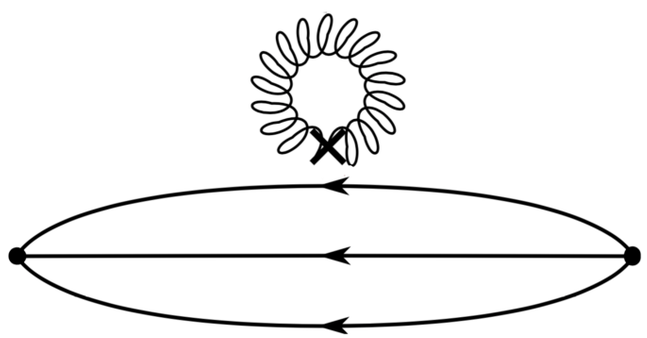

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$\langle x \rangle_u = 0.359(30)$ $\langle x \rangle_d = 0.188(19)$ $\langle x \rangle_s = 0.052(12)$ $\langle x \rangle_c = 0.019(9)$ $\langle x \rangle_g = 0.427(92)$


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M. Constantinou
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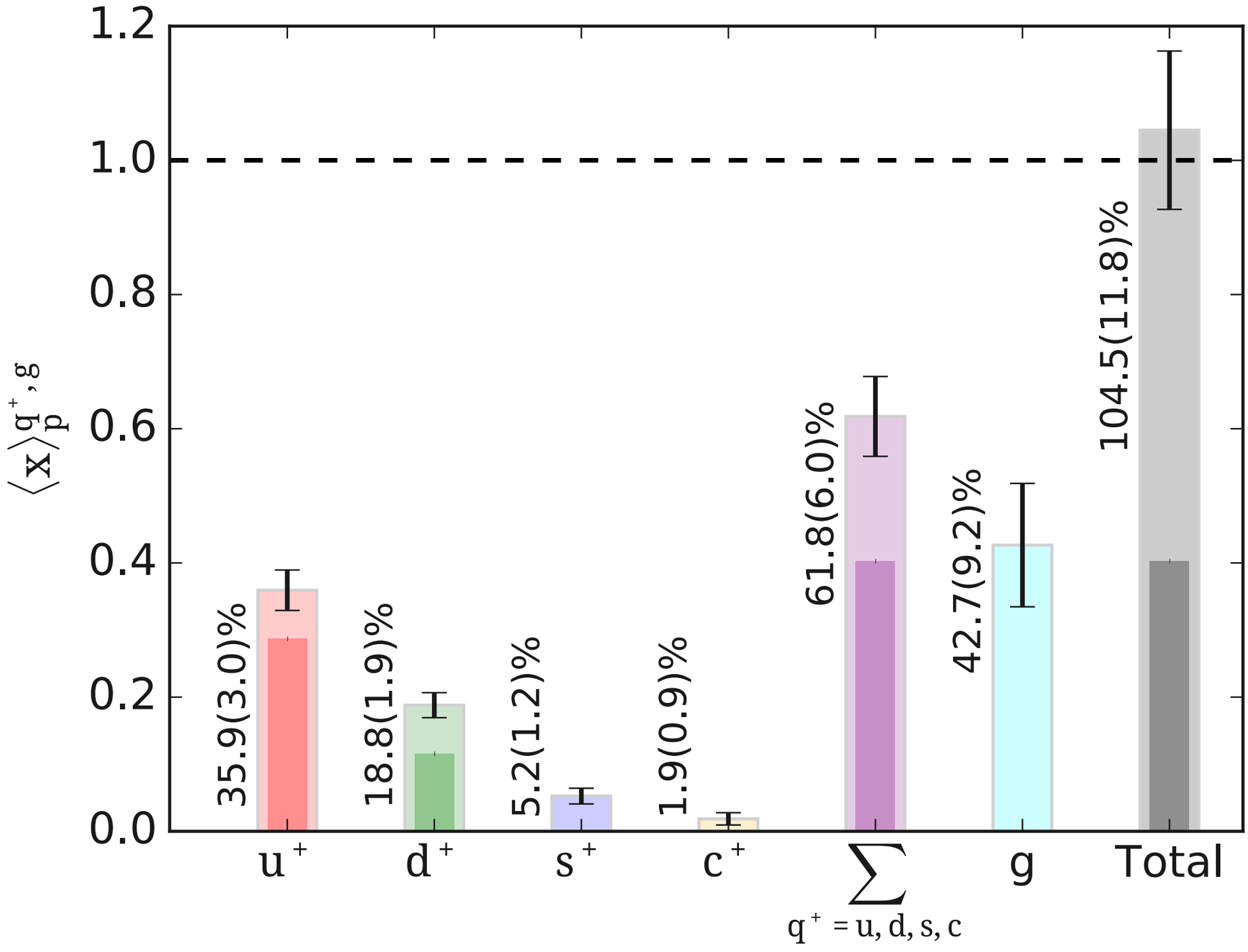
Percentage of momentum



Momentum sum rule satisfied!

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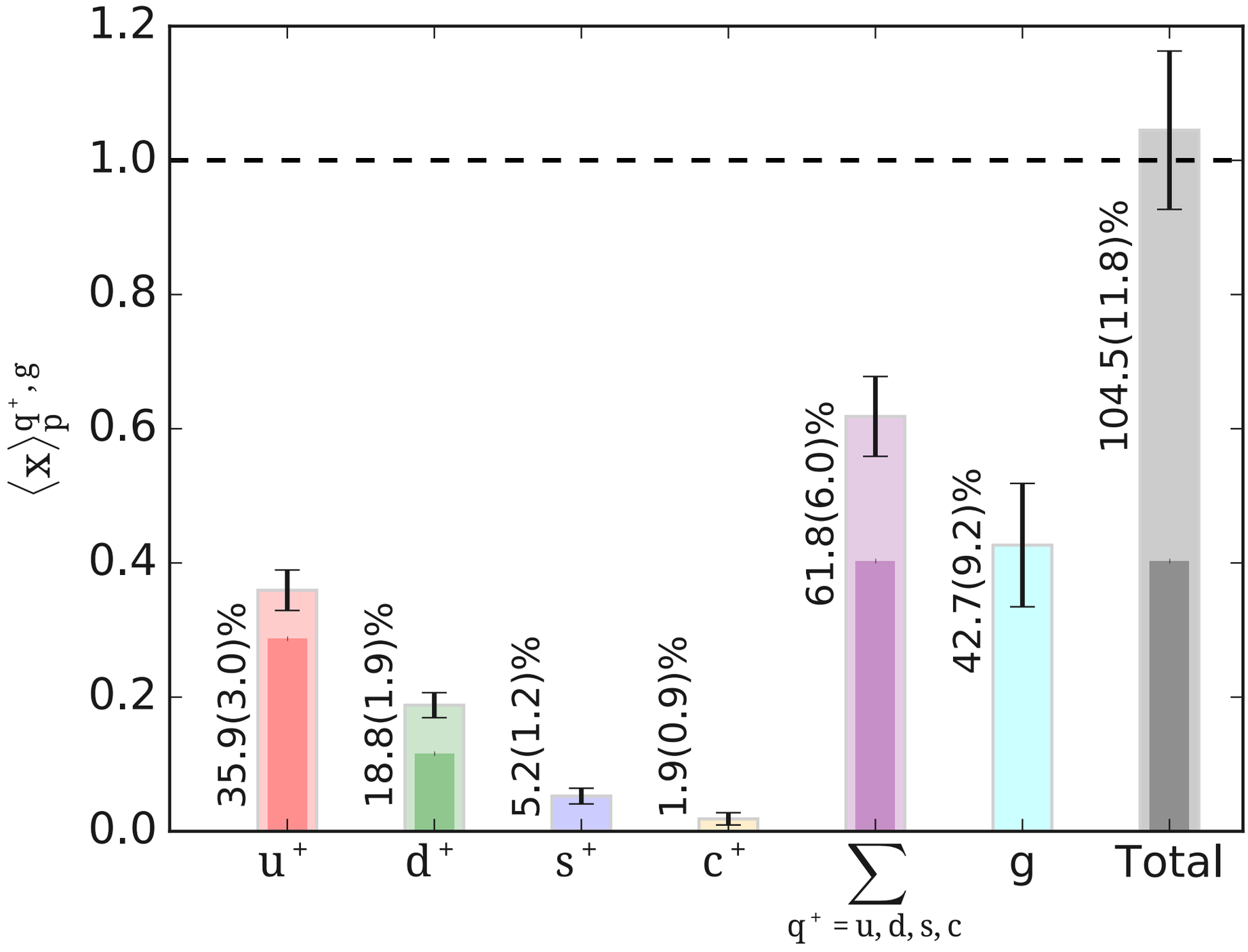

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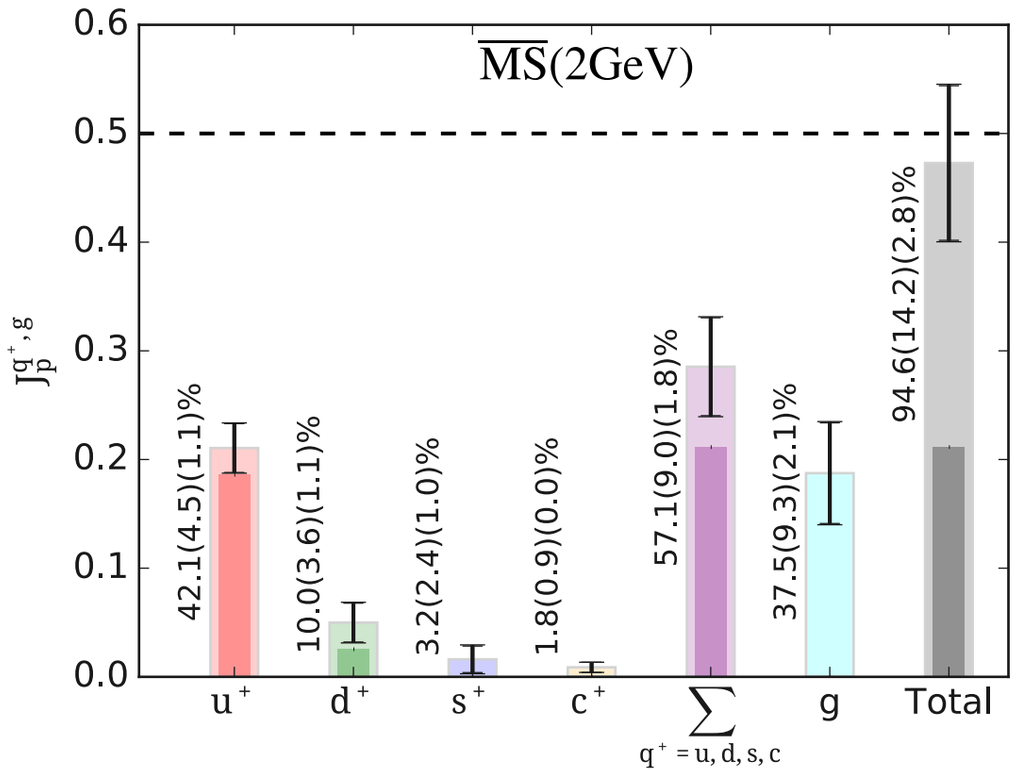
Spin decomposition

[C. Alexandrou et al., Phys. Rev. D 101, 094513 (2020), arXiv:2003.08486]



- ★ Inner bars: connected contributions
- ★ Outer - Inner bars: disconnnected contributions

Spin decomposition

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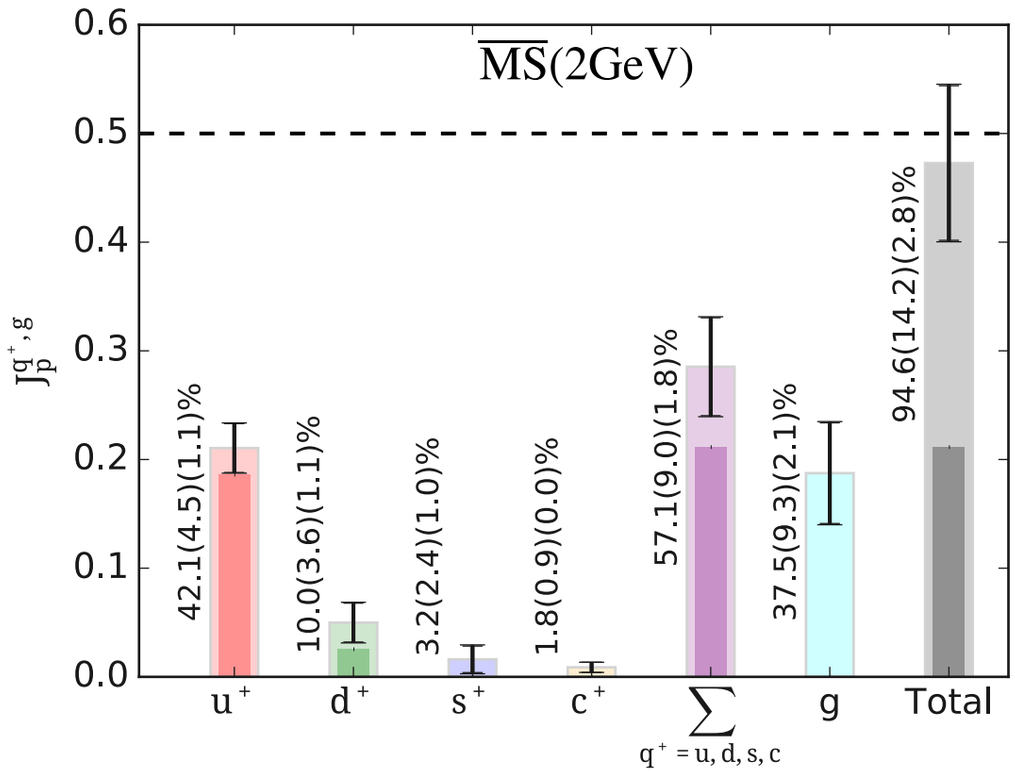

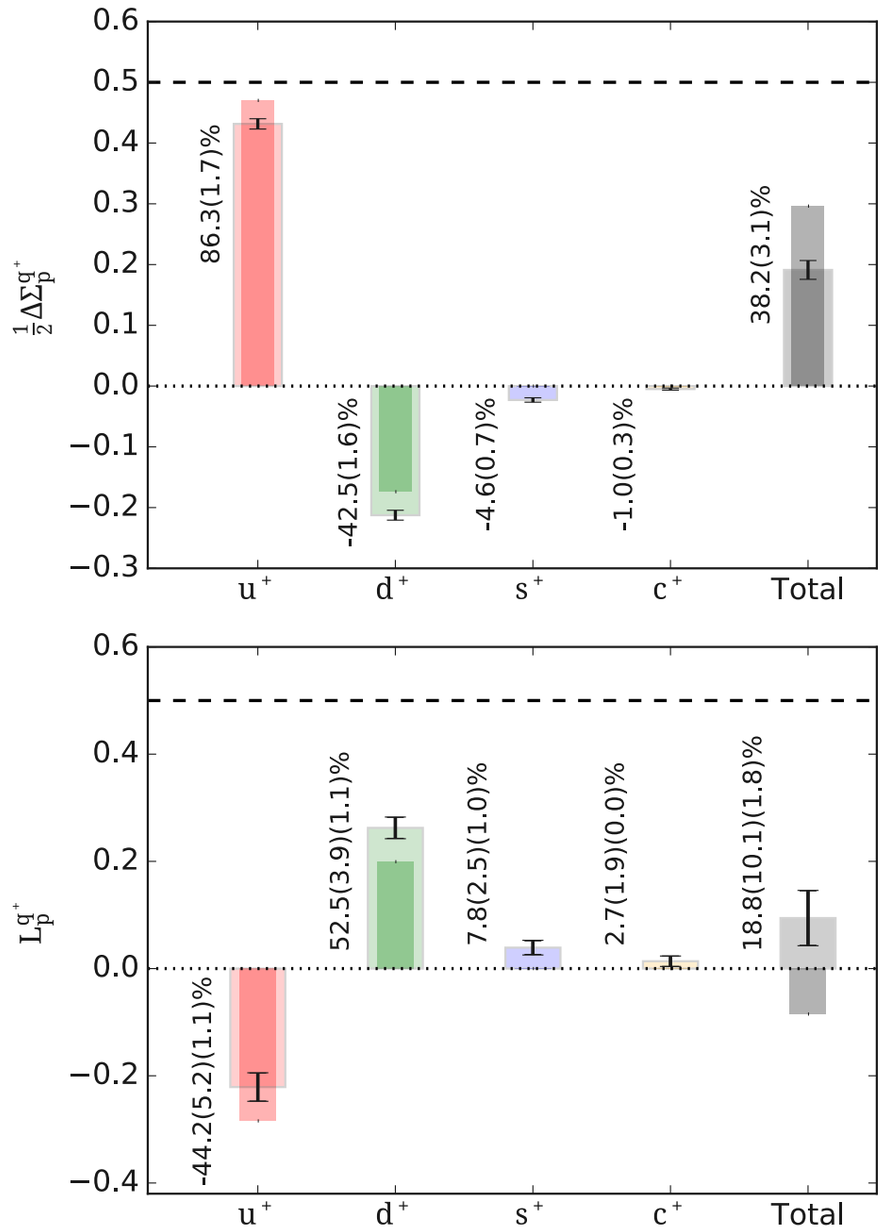

- ★ **Inner bars: connected contributions**
- ★ **Outer - Inner bars: disconnnected contributions**

Quark orbital angular momentum extracted indirectly ($L_q = J_q - \Sigma_q$)

Spin decomposition

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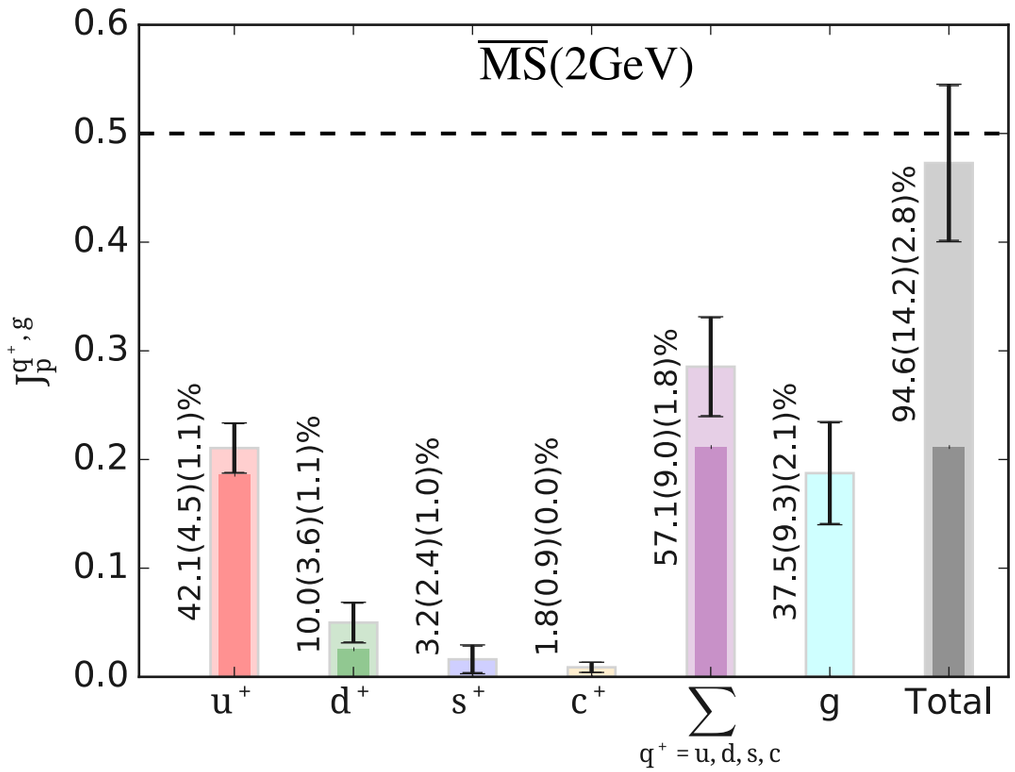
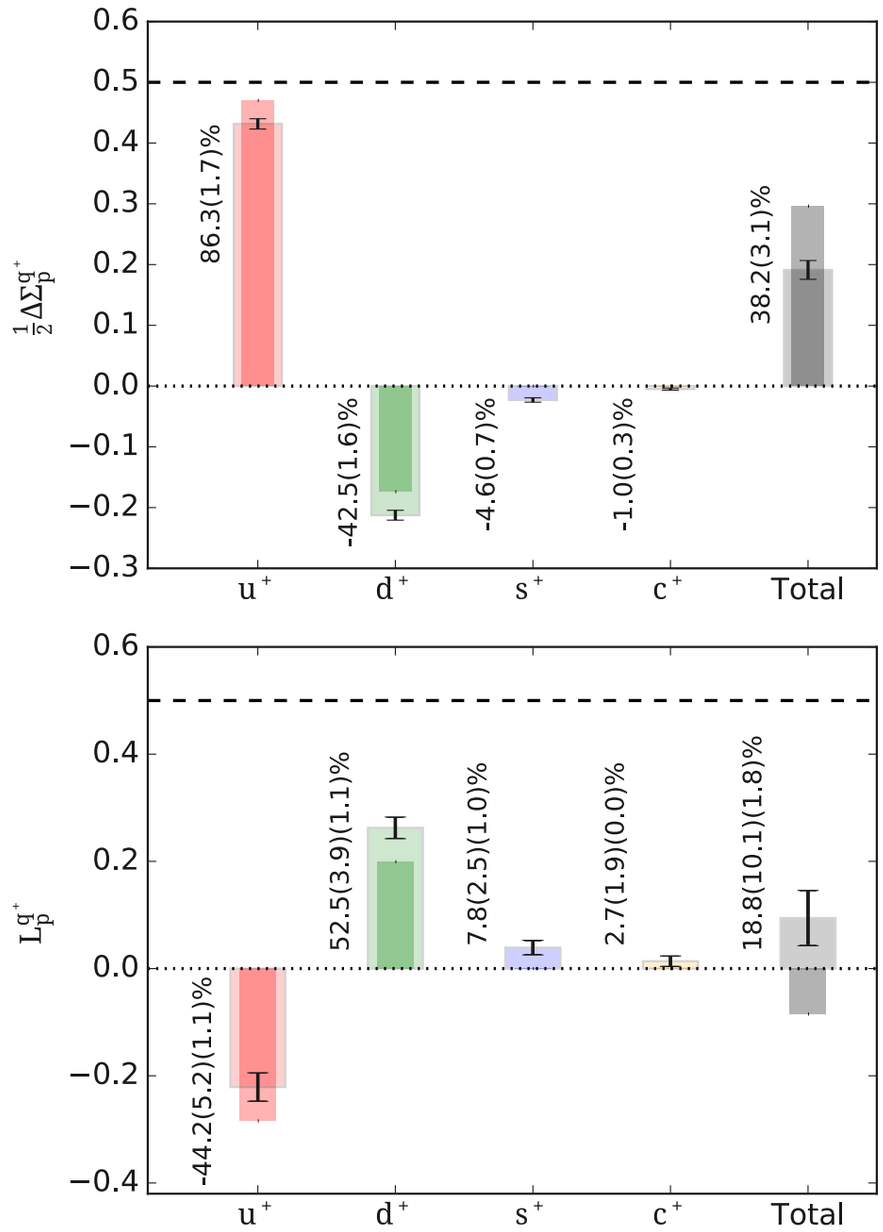




- ★ **Inner bars: connected contributions**
- ★ **Outer - Inner bars: disconnnected contributions**

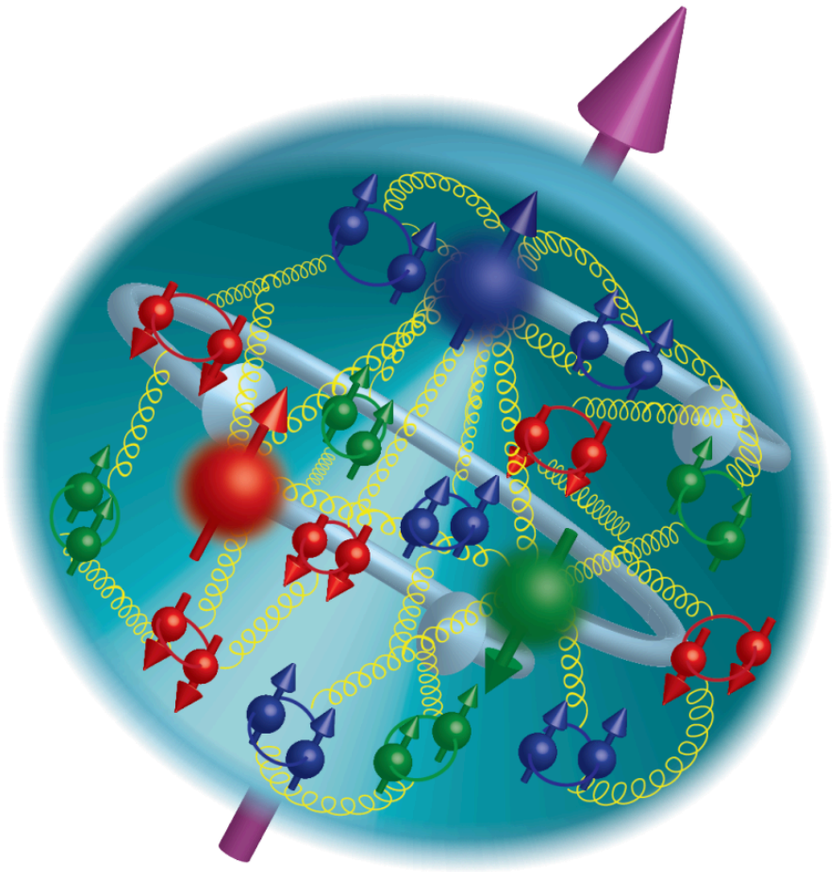
Quark orbital angular momentum extracted indirectly ($L_q = J_q - \Sigma_q$)

Satisfaction of spin and momentum sum rule is not forced

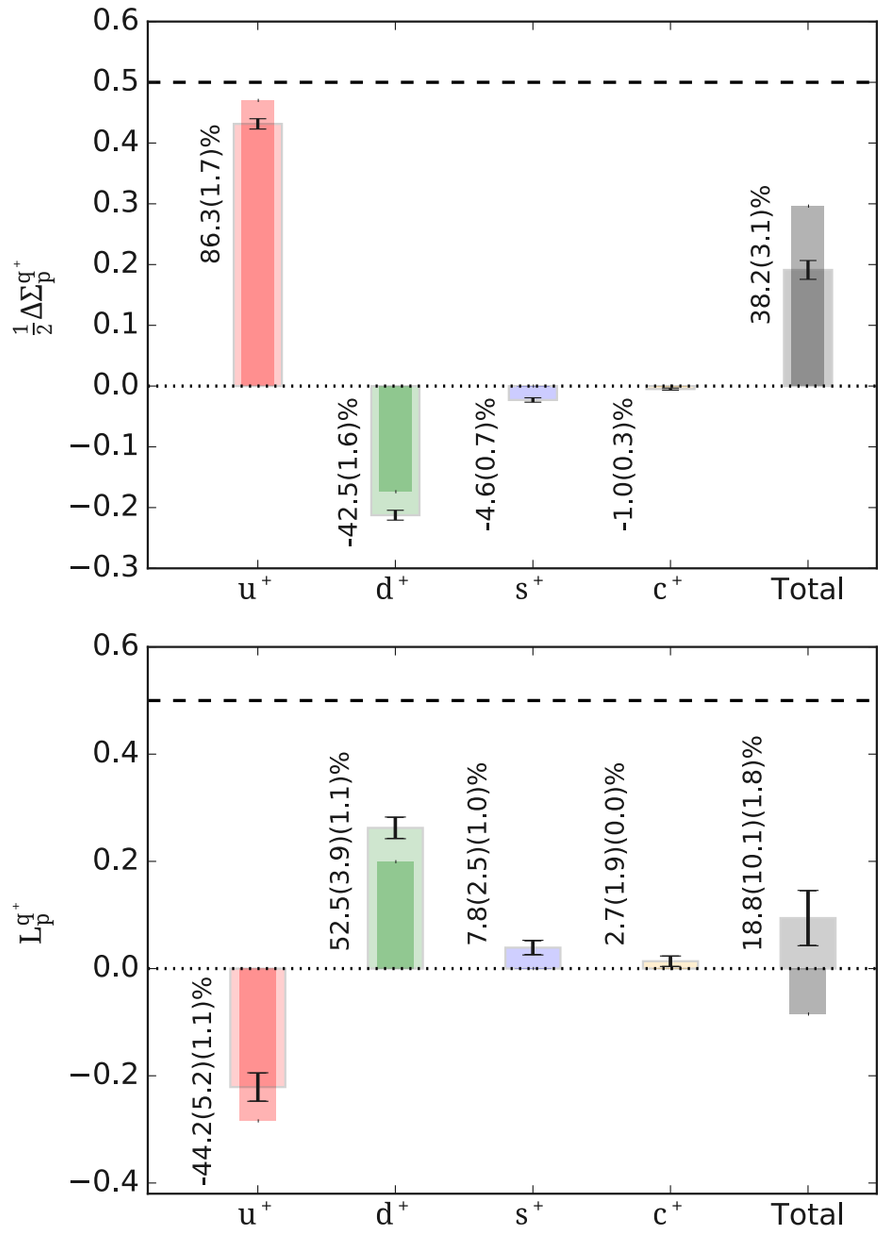
Spin decomposition

[C. Alexandrou et al., Phys. Rev. D 101, 094513 (2020), arXiv:2003.08486]

Better understanding of the spin distribution



Designed by Z.-E. Meziani



Satisfaction of spin and momentum sum rule is not forced

Proton Mass

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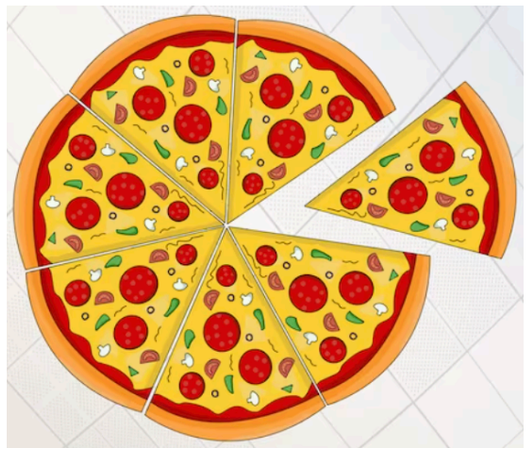
Higgs mechanism responsible for a fraction of the proton mass

Despite major advances the mass decomposition is not well-understood

Proton Mass Decomposition

Based on sum rules
(not unique)

$$\frac{\langle T^{\mu}_{\mu}\rangle}{\langle N|N\rangle} = M\,, \qquad \frac{\langle T^{00}\rangle}{\langle N|N\rangle} = M$$



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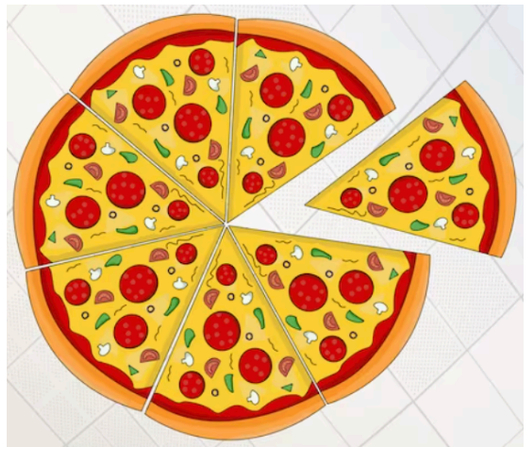

Nature's interest

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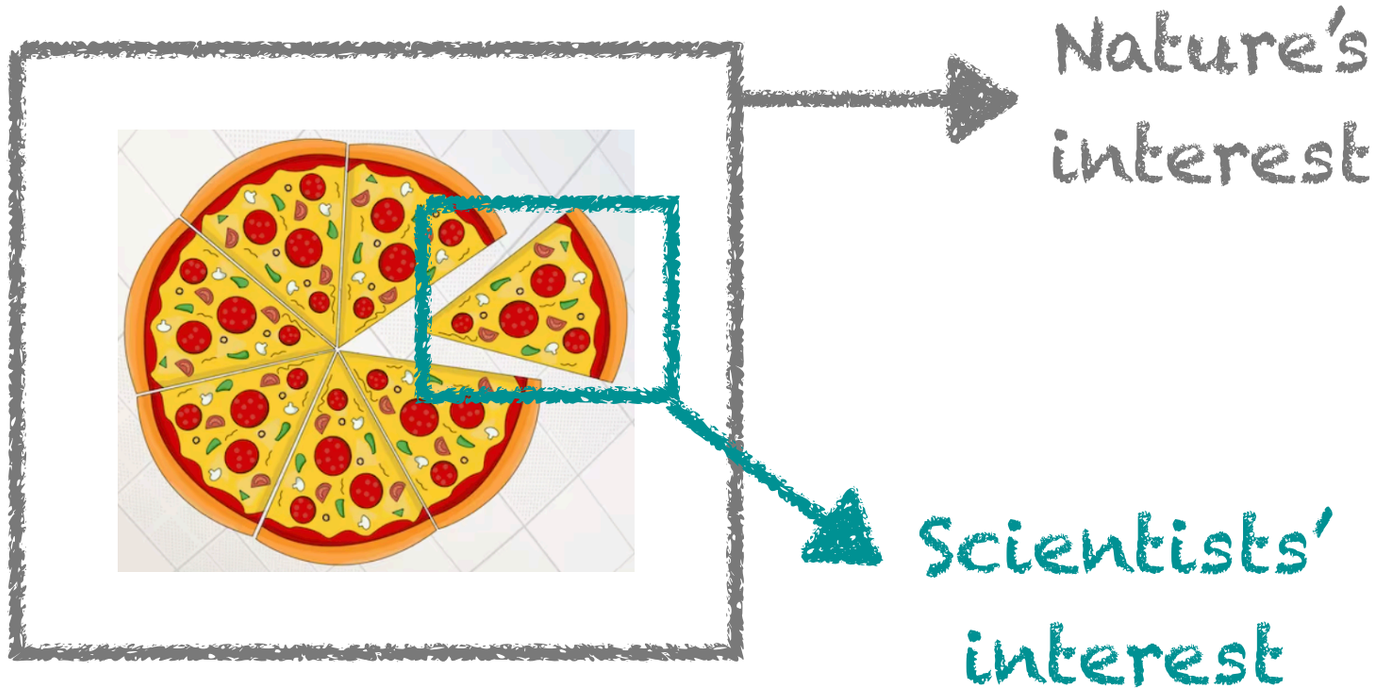

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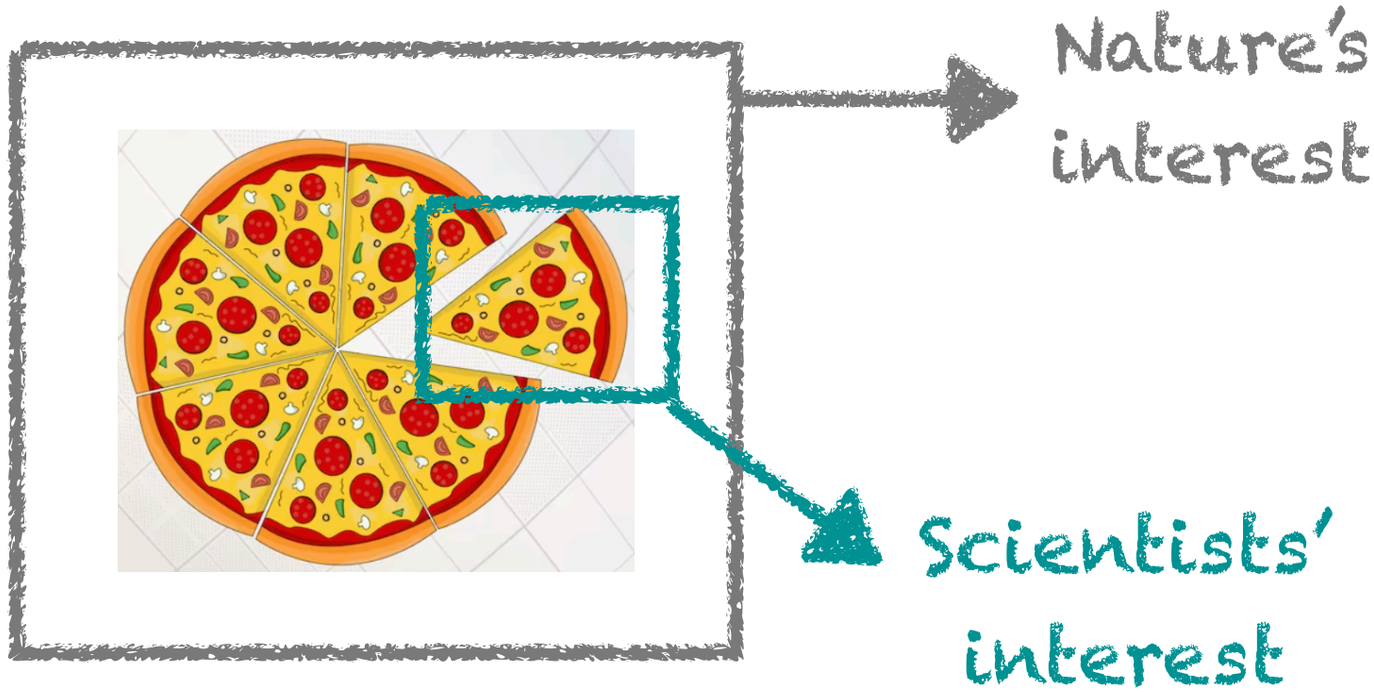

★ **Trace Decomposition**
see, e.g., [M. Shifman et al., Phys. Lett. 78B (1978); D. Kharzeev, Proc. Int. Sch. Phys. Fermi 130 (1996)]

★ **Decomposition of T^{00} in trace and traceless parts in rest frame**
[X.D. Ji, Phys. Rev. Lett. 74, 1071 (1995); X. D. Ji, Phys. Rev. D 52, 271 (1995)]

★ **Decomposition of T^{00} with pressure effects**
[C. Lorce´, Eur. Phys. J. C78 (2018) 2, arXiv:1706.05853]

★ **Quark/Gluon decomposition of trace T^{μ}_{μ}**
[Y. Hatta, A. Rajan, K. Tanaka, JHEP 12, 008 (2018) arXiv:1810.05116; K. Tanaka, JHEP 01, 120 (2019), arXiv:1811.07879]

Ji’s Decomposition

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$$m = M_m + M_q + M_g + M_a$$

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$$m = M_m + M_q + M_g + M_a$$

- M_m: quark mass
- M_q: quark energy
- M_g: gluon energy
- M_a: trace anomaly

Ji's Decomposition

Proton Mass Budget

★ **Available contributions:**

Ji's Decomposition

Proton Mass Budget

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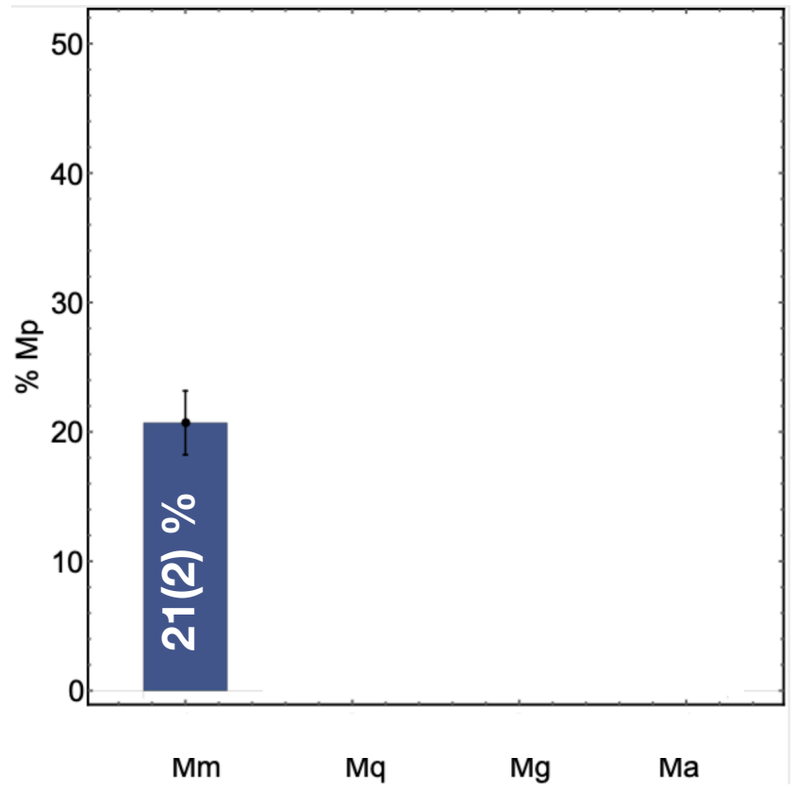
- quark mass (σ-terms)



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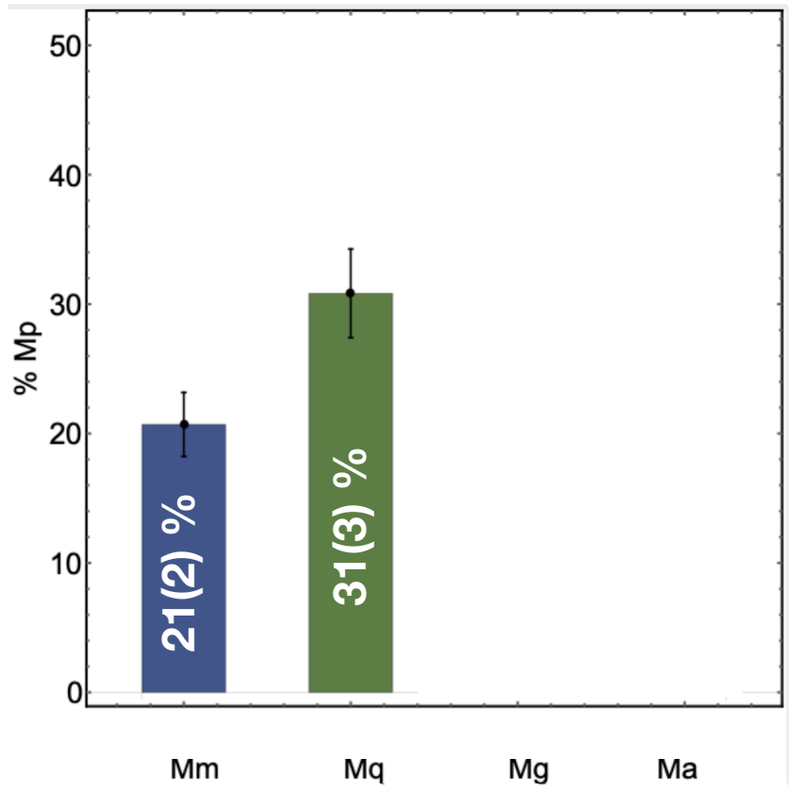

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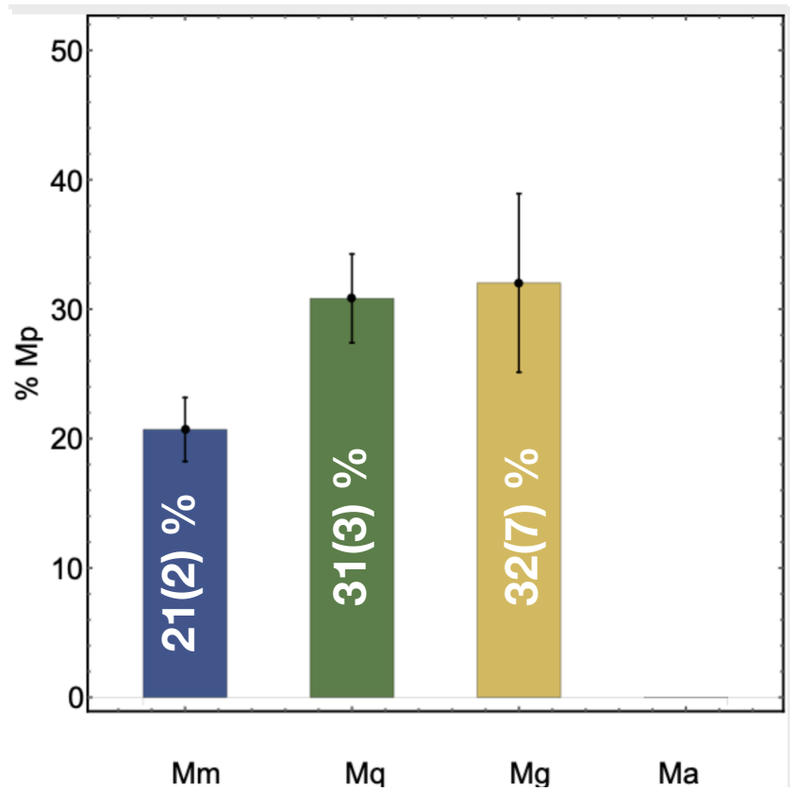

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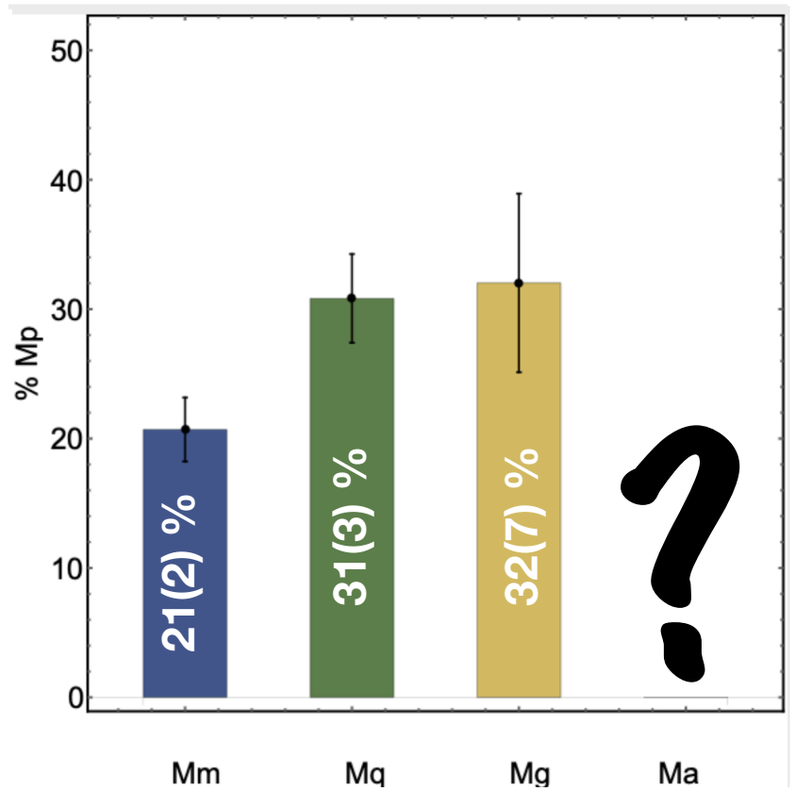

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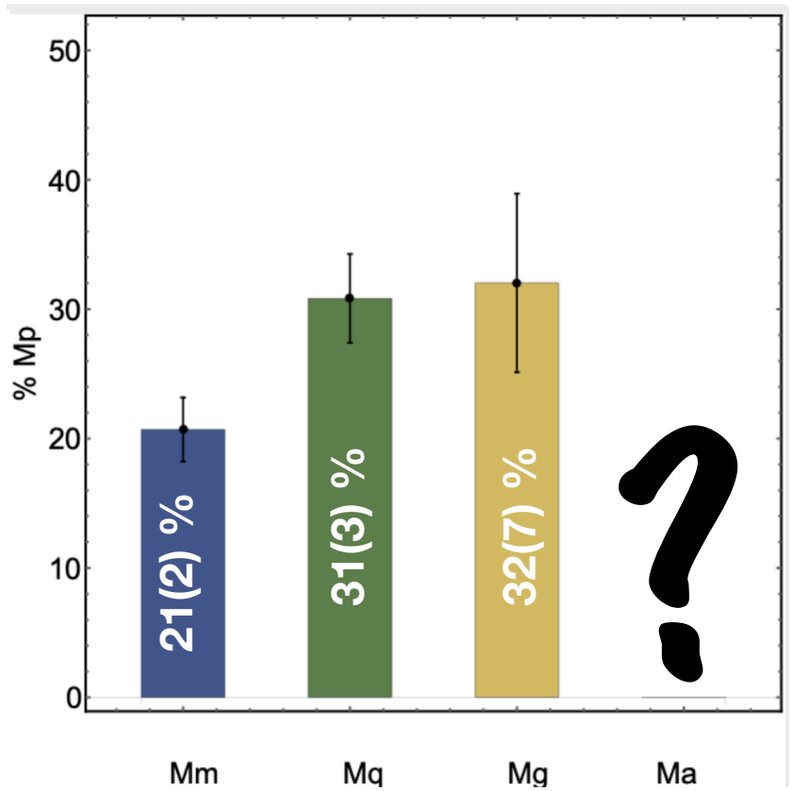

Currently not available

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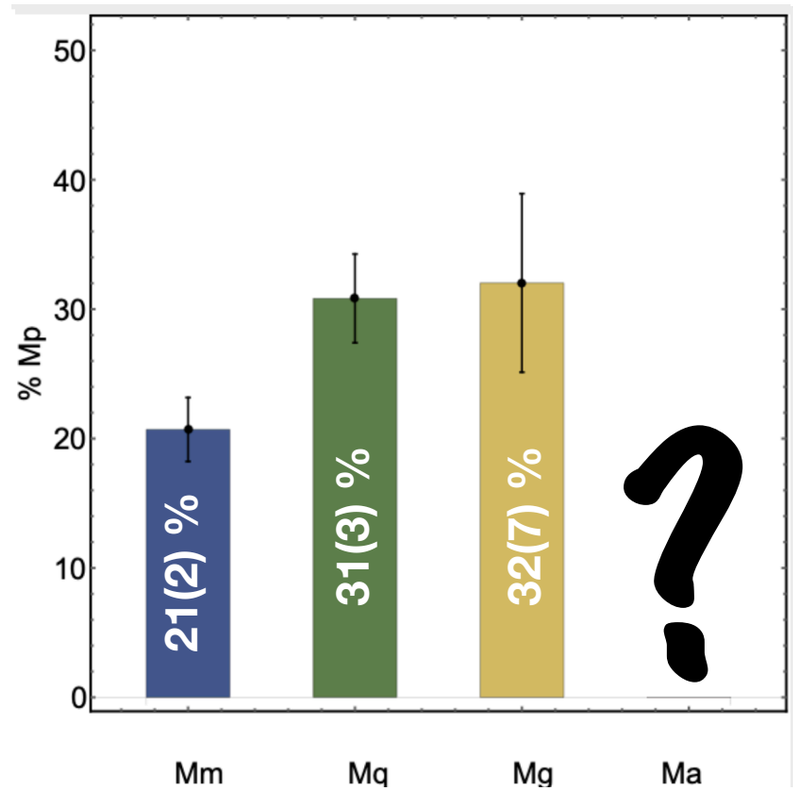


Currently not available

★ **Possibility to access trace anomaly indirectly from sum rules**

$$M_a = \frac{M}{4} - \sum_q \frac{\sigma_q}{4} \qquad M_a = M - \sum_{i=m,q,g} M_i$$

Ji's Decomposition

Proton Mass Budget

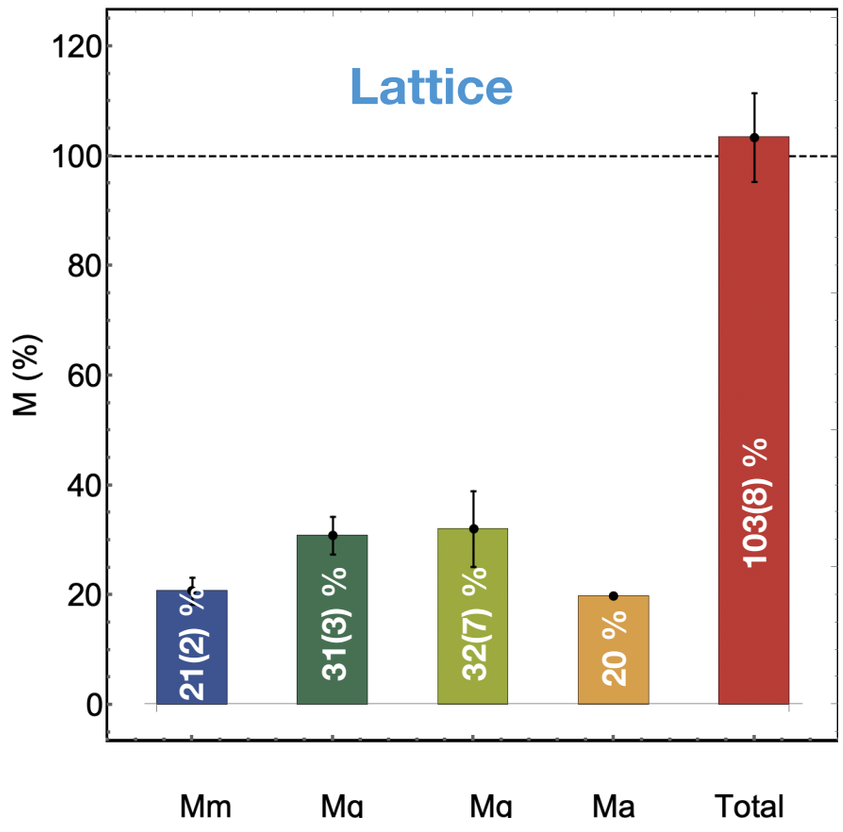
Ji's Decomposition

Proton Mass Budget

Approach A

$$M_a = \frac{M_p}{4} - \sum_q \frac{\sigma_q}{4} \sim 19.83(0.07)\,\%$$

$$M_p = M_m + M_q + M_g + M_a = 103.39(8.09)\,\%$$



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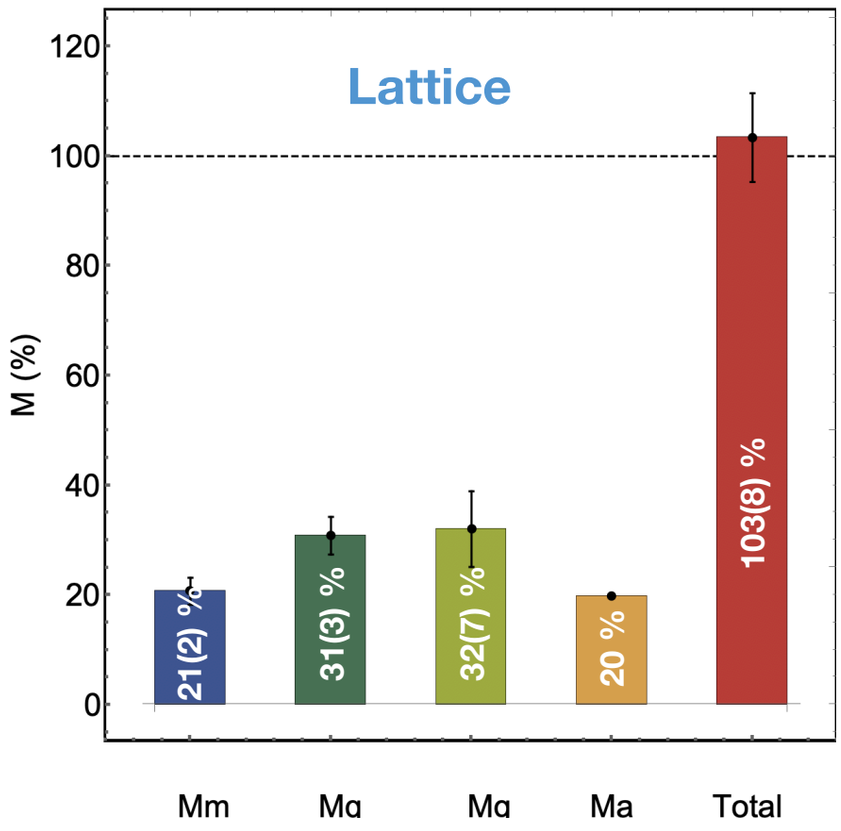
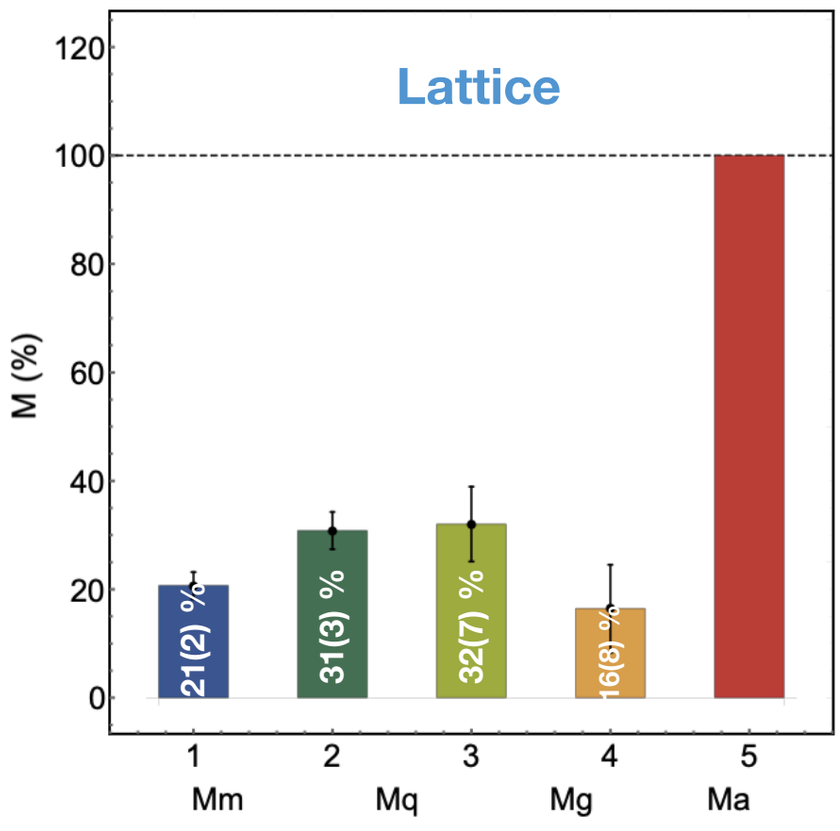


Approach B

$$M_a = M_p - \sum_{i=m,q,g} M_i \sim 16.45(8.09)\,\%$$

M_p : sum rule enforced



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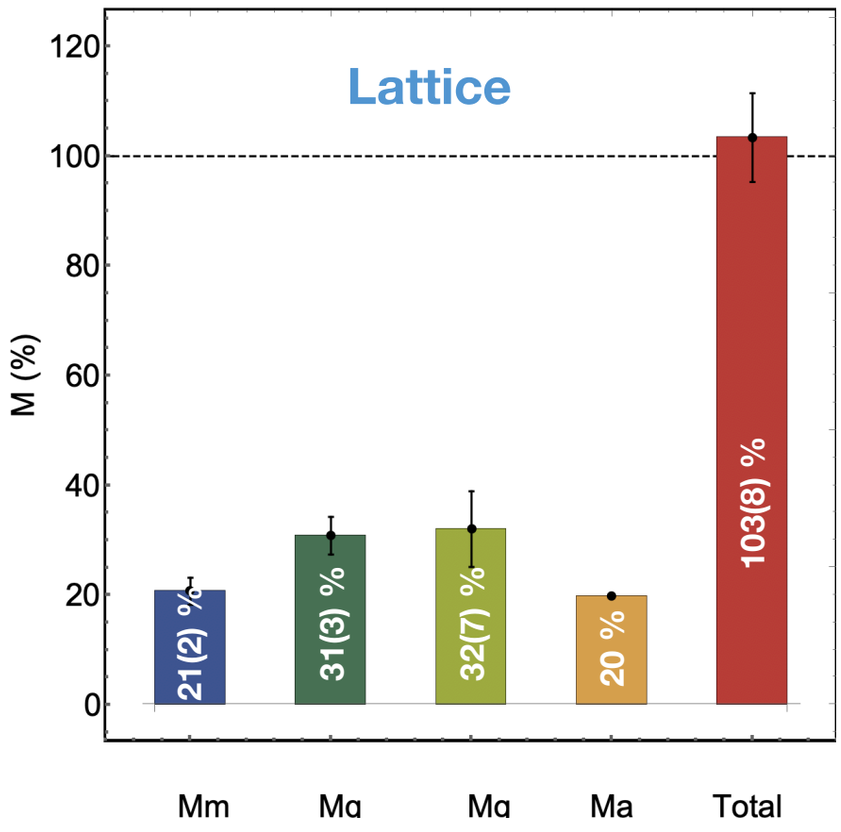

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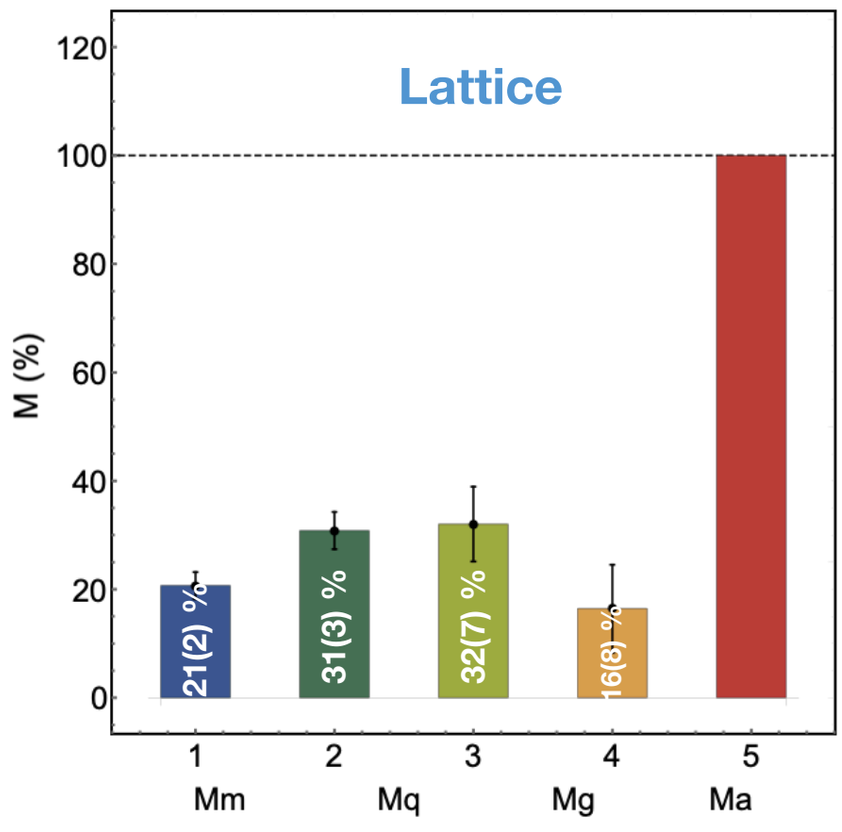
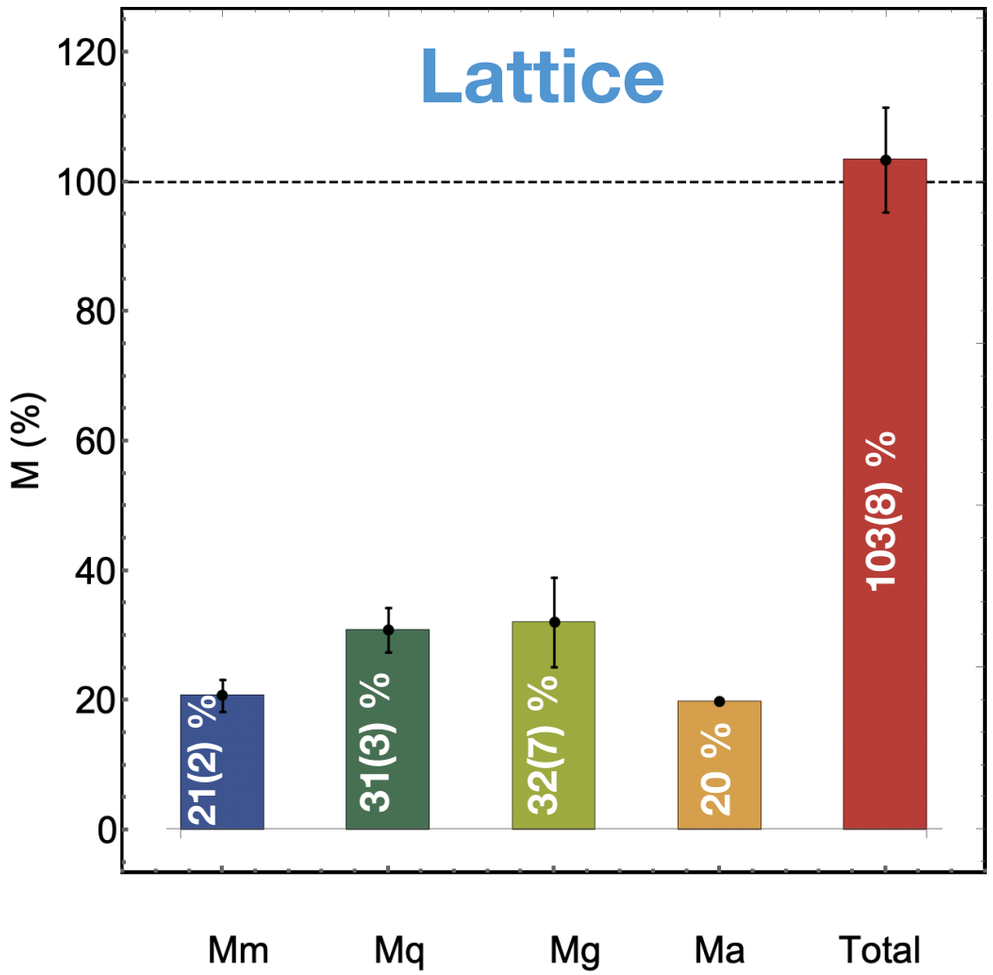
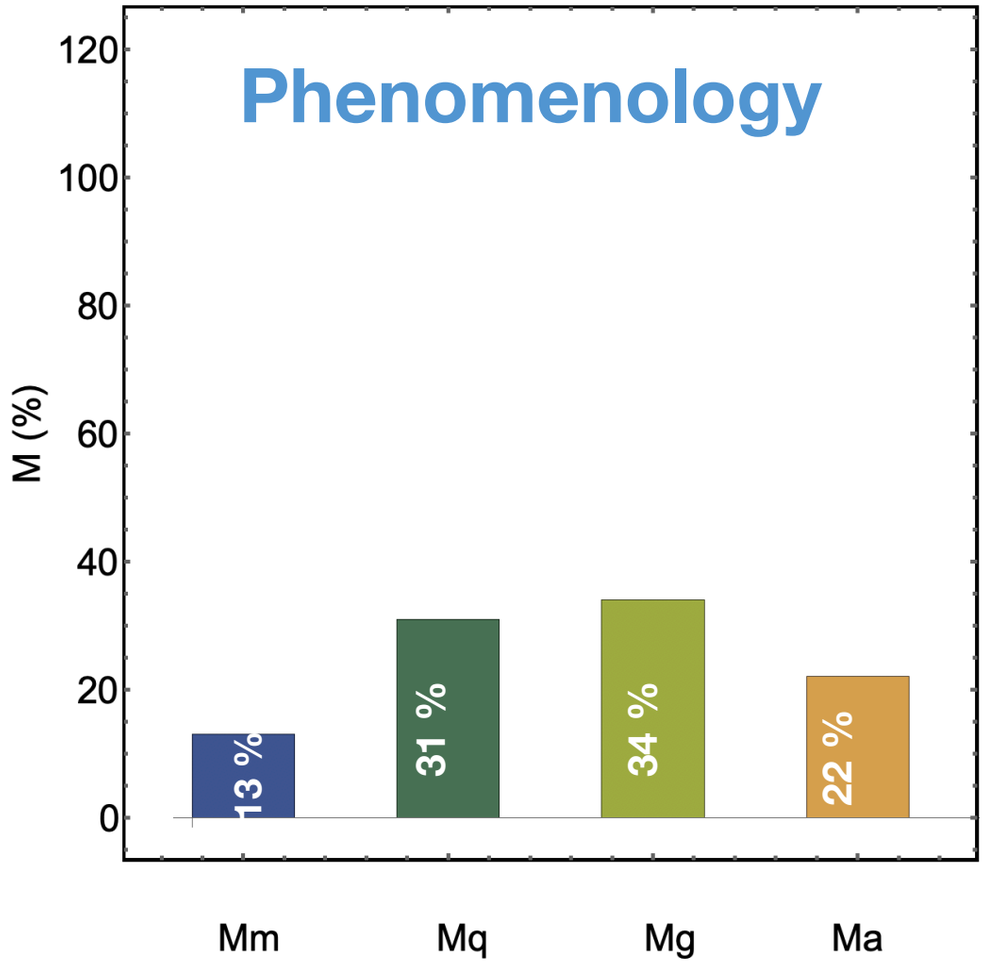


- ★ M_a compatible but different systematic uncertainties
- ★ Uncertainties of trace anomaly term depend on the sum rule

Ji's Decomposition

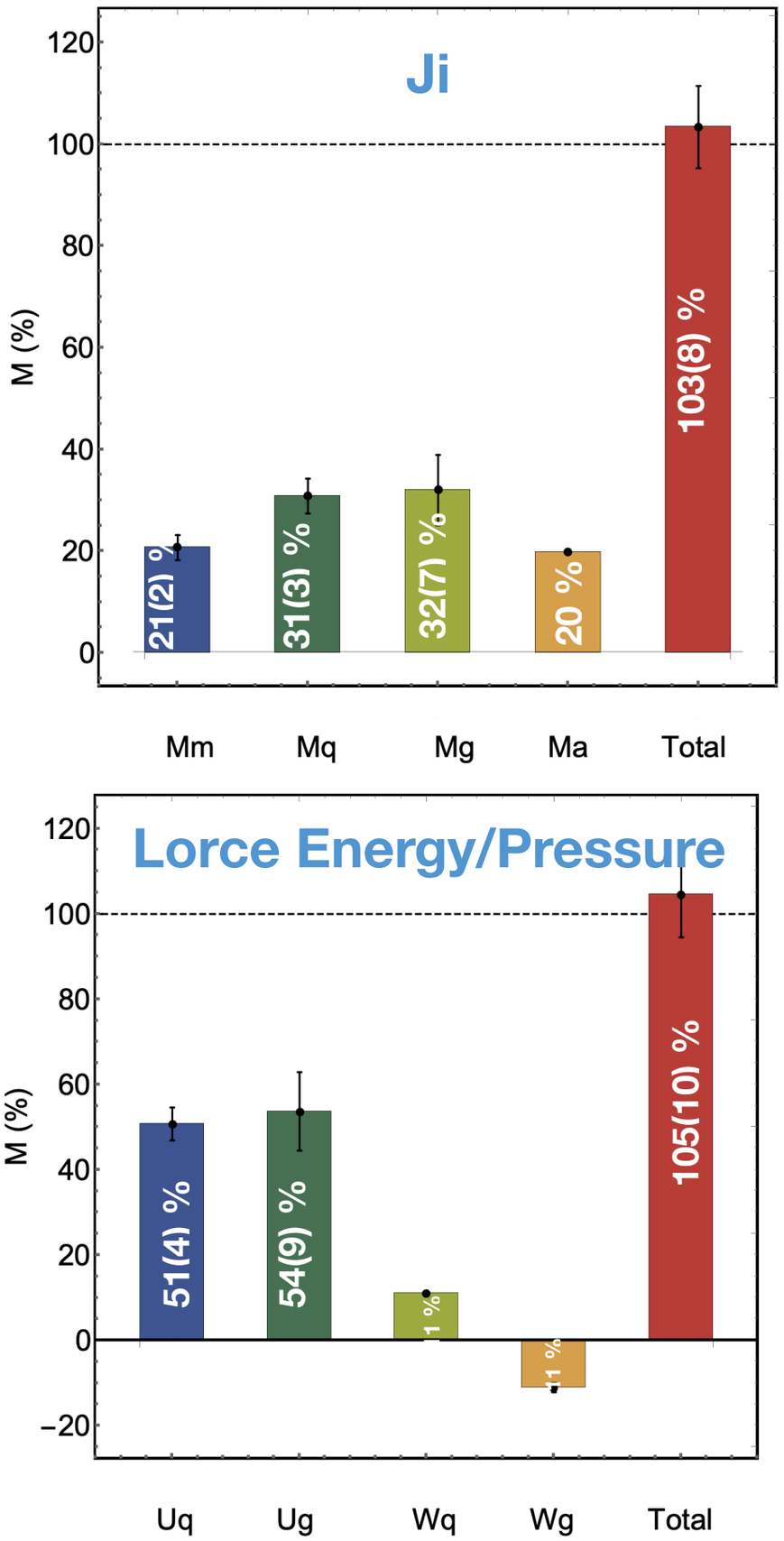


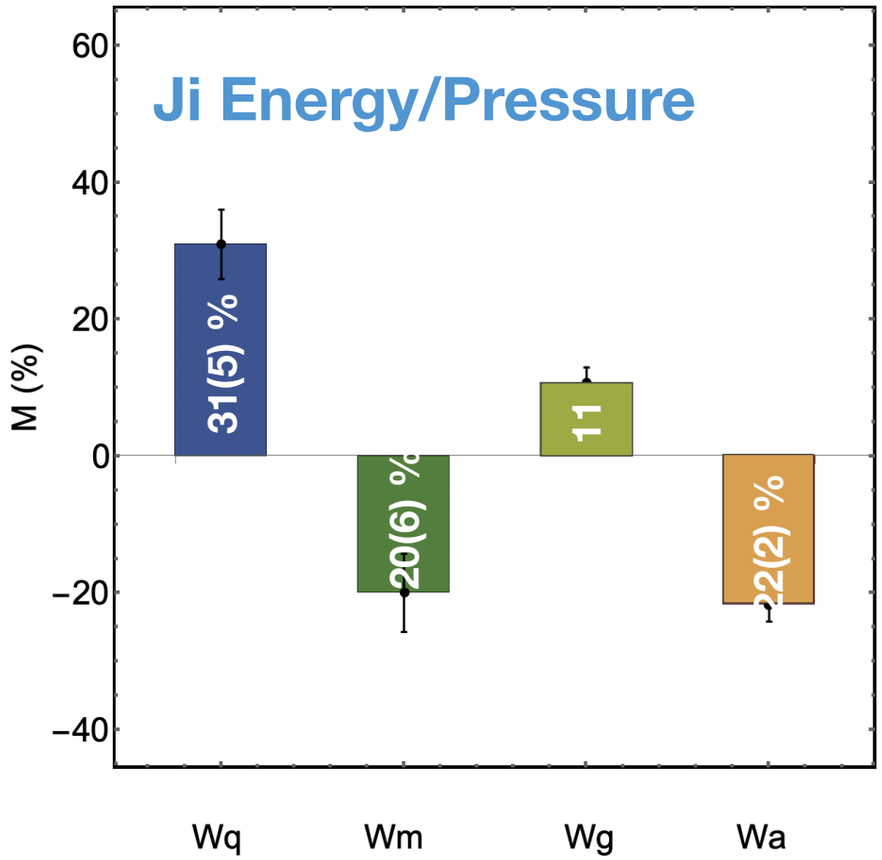


[C. Lorce´, EPJ. C78 (2018) 2]
[L. Harland-Lang et al., EPJ. C 75 (2015)]
[M. Hoferichter et al., PRL 115 (2015)]

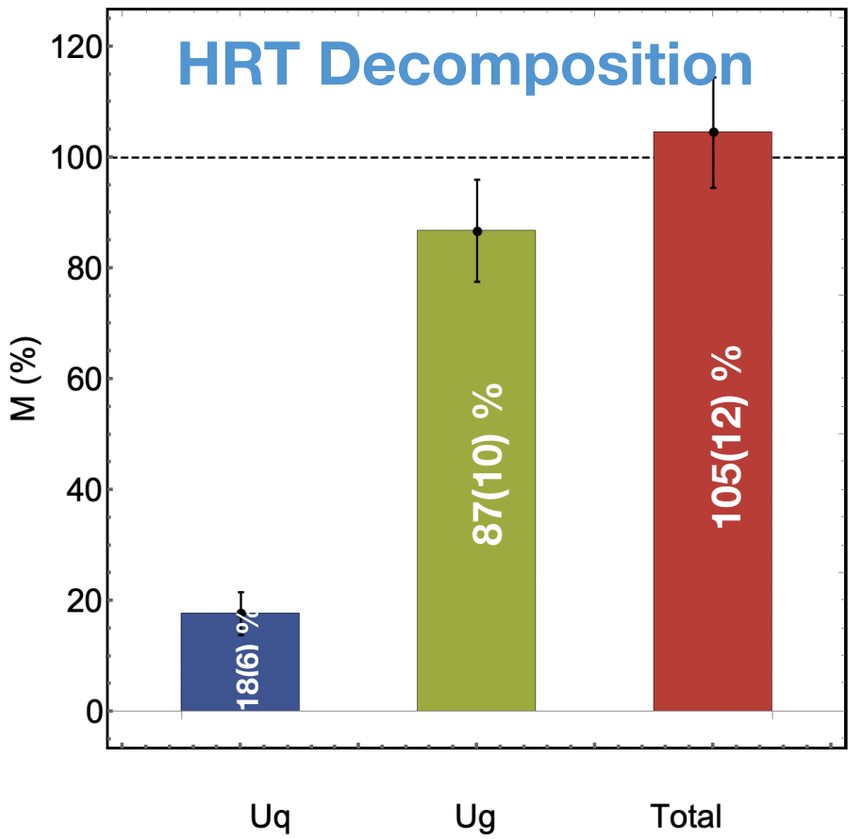
- ★ Lattice and pheno data give similar picture
- ★ The tension in the sigma terms affects M_m
- ★ Contributions are of similar order

Summary

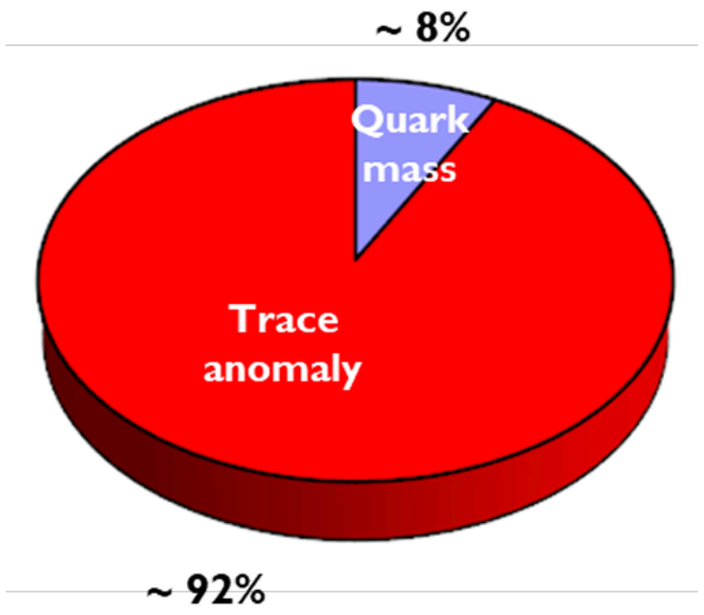




Such a detailed understanding is not available from experiments



Trace decomposition



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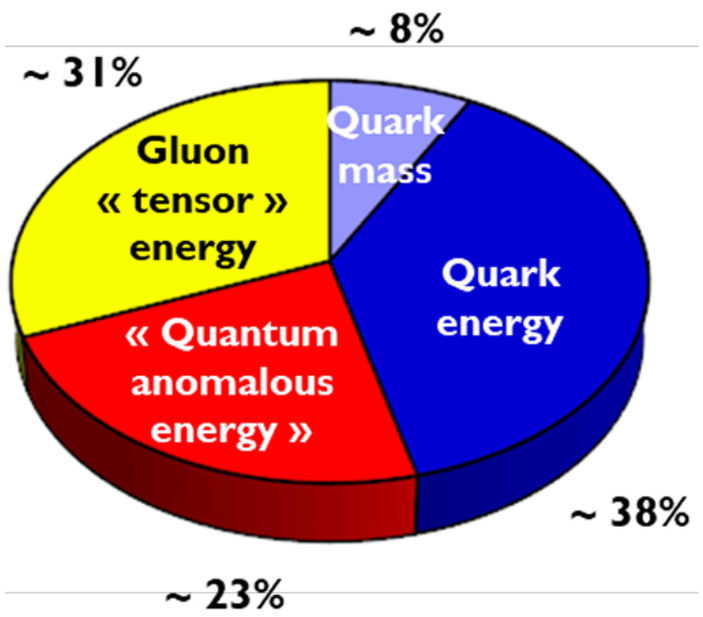
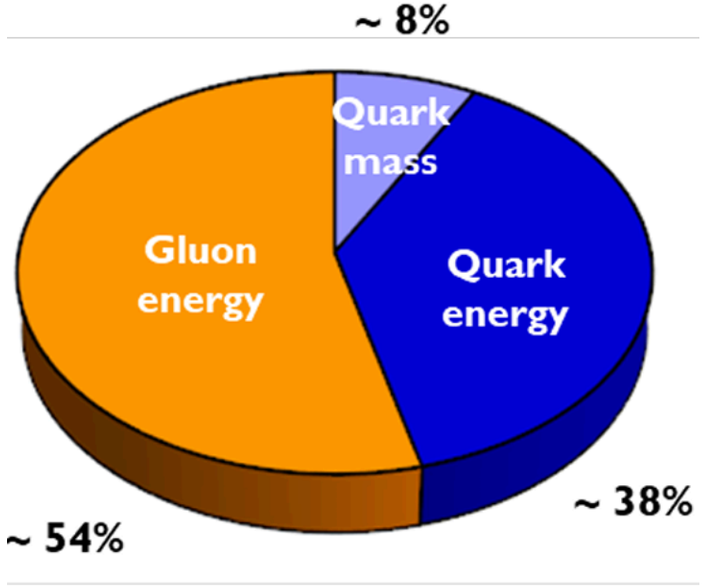


Rest energy decomposition



Summary of Lecture 3

Key points of Lecture 3

★ Hadron structure studies are critical for understanding the immensely rich and complex properties of the visible matter

★ The lattice formulation can provide first principle results for a very broad research program in Hot and Cold QCD, and beyond!

★ Many opportunities for synergies and complementarity.

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Thank you