UC RIVERSIDE | 2023 National Nuclear Physics Summer School

July 10-22, 2023

Lattice QCD (selected topics)

Lecture 2

Martha Constantinou

Temple University

July 11, 2023

OUTLINE OF LECTURE 2

★ Just a bit of Renormalization within Lattice QCD

- perturbatively
- non-perturbatively

★ Hadron spectroscopy

★ Hadron structure

★ Key points of Lecture 2

Origin of ultra-violet divergences in field theories is explained by the theory of renormalization. It also gives a prescription on how to systematically remove them so they don't appear in physical quantities.

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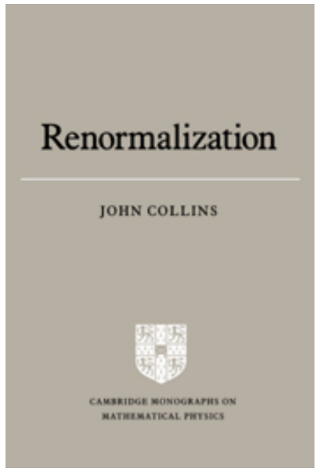
★ An Introduction to Renormalization, the Renormalization Group & OPE

J. Collins

https://doi.org/10.1017/CBO9780511622656



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e.g., calculating scattering amplitudes within QED leads to terms

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A theory is renormalizable if all UV divergence can be canceled with a finite number of counter terms

Renormalization

★ Lattice QCD is renormalizable, thus QCD must recover upon continuum limit (removal of regulator)

★ Lattice regularization has a consequence of that (bare) lattice quantities depend on lattice spacing, α

★ However, physical quantities cannot depend on regulator, thus bare quantities must be tuned with α, so that observables are not affected

★ Renormalization:
– UV divergences must be removed prior continuum limit
– Divergences canceled by adjusting the parameters of the action
– physical results are expressed via measurable parameters
(not via parameters in bare Lagrangian)

Renormalization on the Lattice

★ Lattice perturbation theory was extensively used to renormalize lattice data of matrix elements of operators, and parameters of the QCD Lagrangian in the past

★ In 1995 ideas for non-perturbative renormalization have been implemented
[Martinelli et al., Nucl. Phys. B445, 81, arXiv:hep-lat/9411010]

★ Currently, non-perturbative renormalization prescriptions are mostly used

★ Lattice perturbation theory is still a useful tool for several reasons

Perturbative Renormalization

RI-MOM scheme

★ Regularization Independent momentum subtraction (RI-MOM) schemes naturally defined in perturbation theory

★ Calculation of Green functions of operators at given off-shell external states with momentum p

★ A condition is applied on the Green functions to match them with their tree-level value

★ Examples of RI-type conditions:

For fermion field

$$Z_q^{\mathrm{RI}} = \frac{1}{12}\mathrm{Tr}\left[(S^L)^{-1}(p)\, S^{\mathrm{tree}}(p)\right]\Big|_{p^2=\mu^2}$$

For operator $\bar{\psi}\Gamma\psi$

$$(Z_q^{\mathrm{RI}})^{-1}\, Z_\Gamma^{\mathrm{RI}}\, \mathrm{Tr}\left[G_\Gamma^L(p)\, G_\Gamma^{\mathrm{tree}}(p)\right]\Big|_{p^2=\mu^2} = \mathrm{Tr}\left[G_\Gamma^{\mathrm{tree}}(p)\, G_\Gamma^{\mathrm{tree}}(p)\right]\Big|_{p^2=\mu^2}$$

What should we first study in Lattice QCD?

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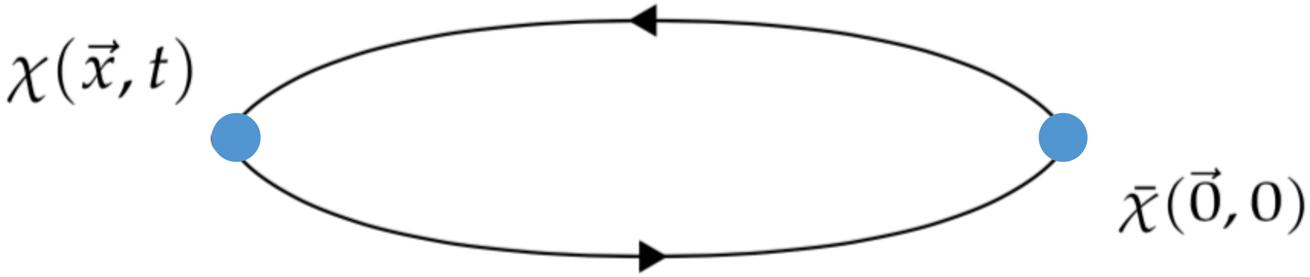
Start from quantities that are (relatively) easy to compute, and can be compared against experimental data

First goals of Lattice QCD

Reproduce the low-lying spectrum

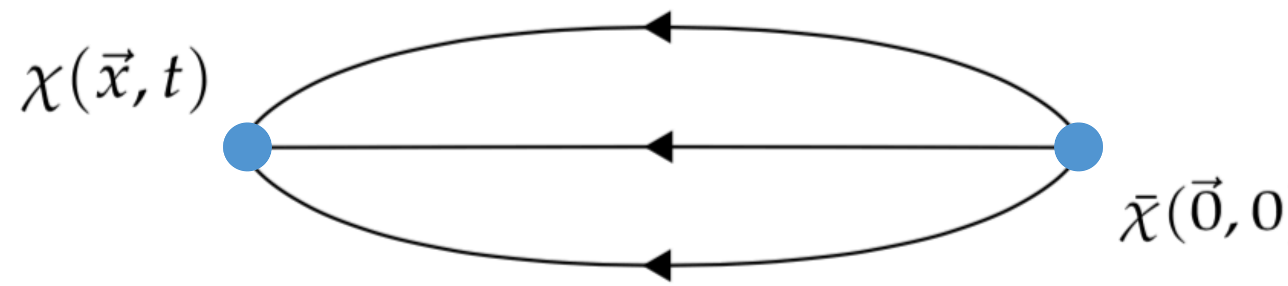
Mesons

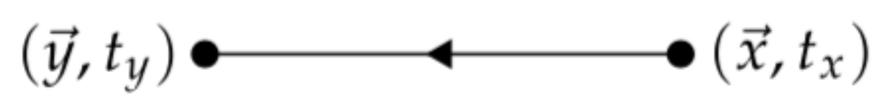
e.g. pion, kaon



Baryons

e.g. proton





$$= G(\vec{y},t_y;\vec{x},t_x)$$

Quark propagator

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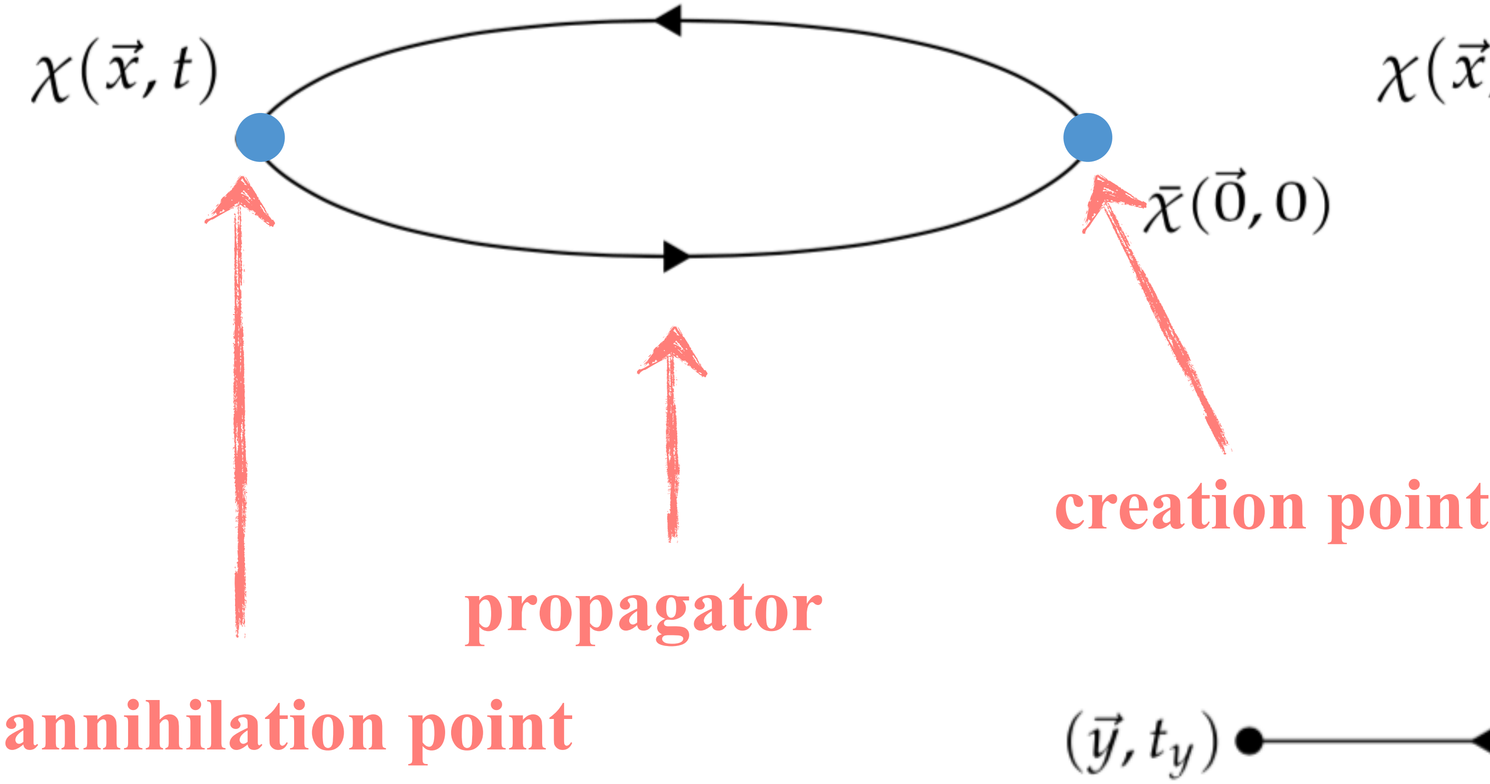

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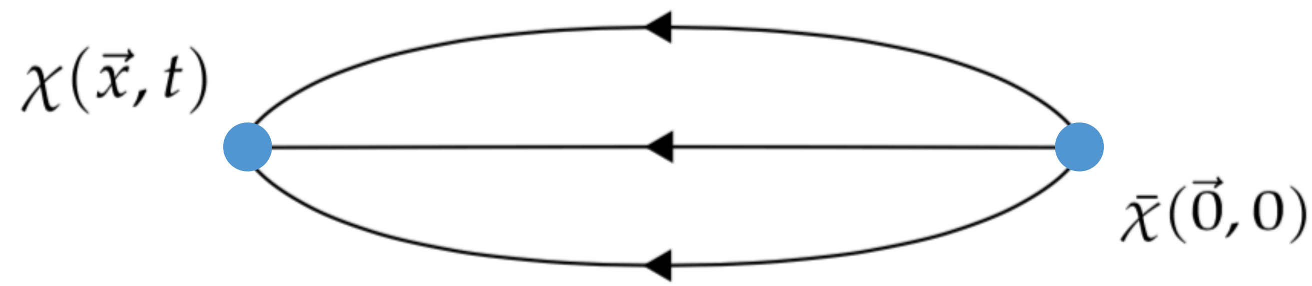

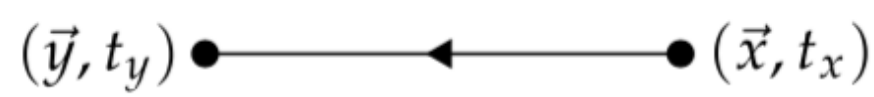

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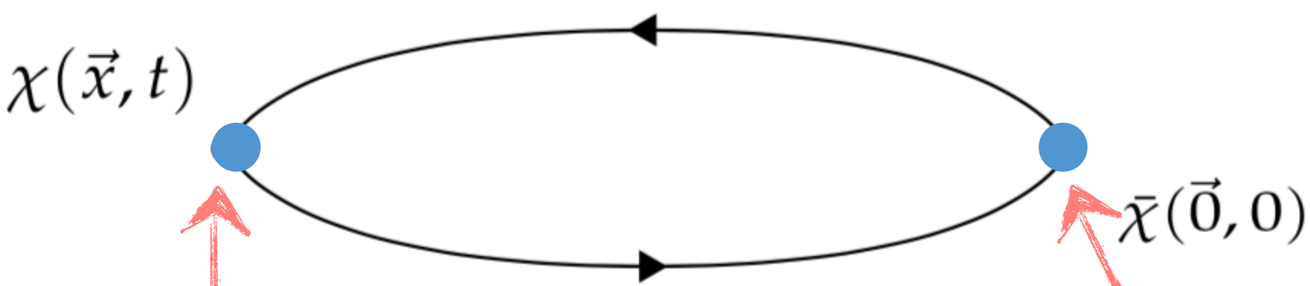

creation point

propagator

annihilation point

Baryons

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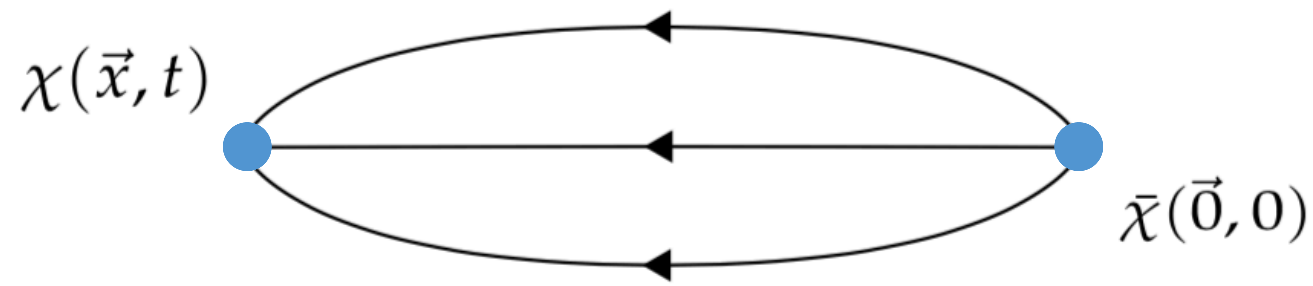

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Quark propagator

Most costly part of calculation

Calculation of Hadron mass

Extraction of a hadron's mass from its propagator:

★ Two-point correlator (hadron level, Heisenberg picture):

$$C(t) = \sum_{\vec{x}} \langle \Omega | \, \chi(\vec{x},t)\bar{\chi}(\vec{0},0) \, |\Omega\rangle = \sum_{\vec{x}} \langle \Omega | \, e^{-i\hat{\vec{p}}\cdot\vec{x}} e^{\hat{H}t} \chi(\vec{0},0) e^{-\hat{H}t} e^{i\hat{\vec{p}}\cdot\vec{x}} \bar{\chi}(\vec{0},0) \, |\Omega\rangle$$

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Insertion of complete set of momentum and energy states:

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★ The mass of the hadron appears, for the n^{th} state

Calculation of Hadron mass

★ Overlap with ground state, excitations, other hadron states. Thus:

$$C(t) = \sum_{n'} \frac{1}{2E_n(\vec{k})} |\langle \Omega|\chi(\vec{0},0)|(n',\vec{0})\rangle|^2 e^{-m_{n'}t}$$

★ For large enough t the exponential for excited states and multi-hadron states, becomes very small, thus ground-state dominance.

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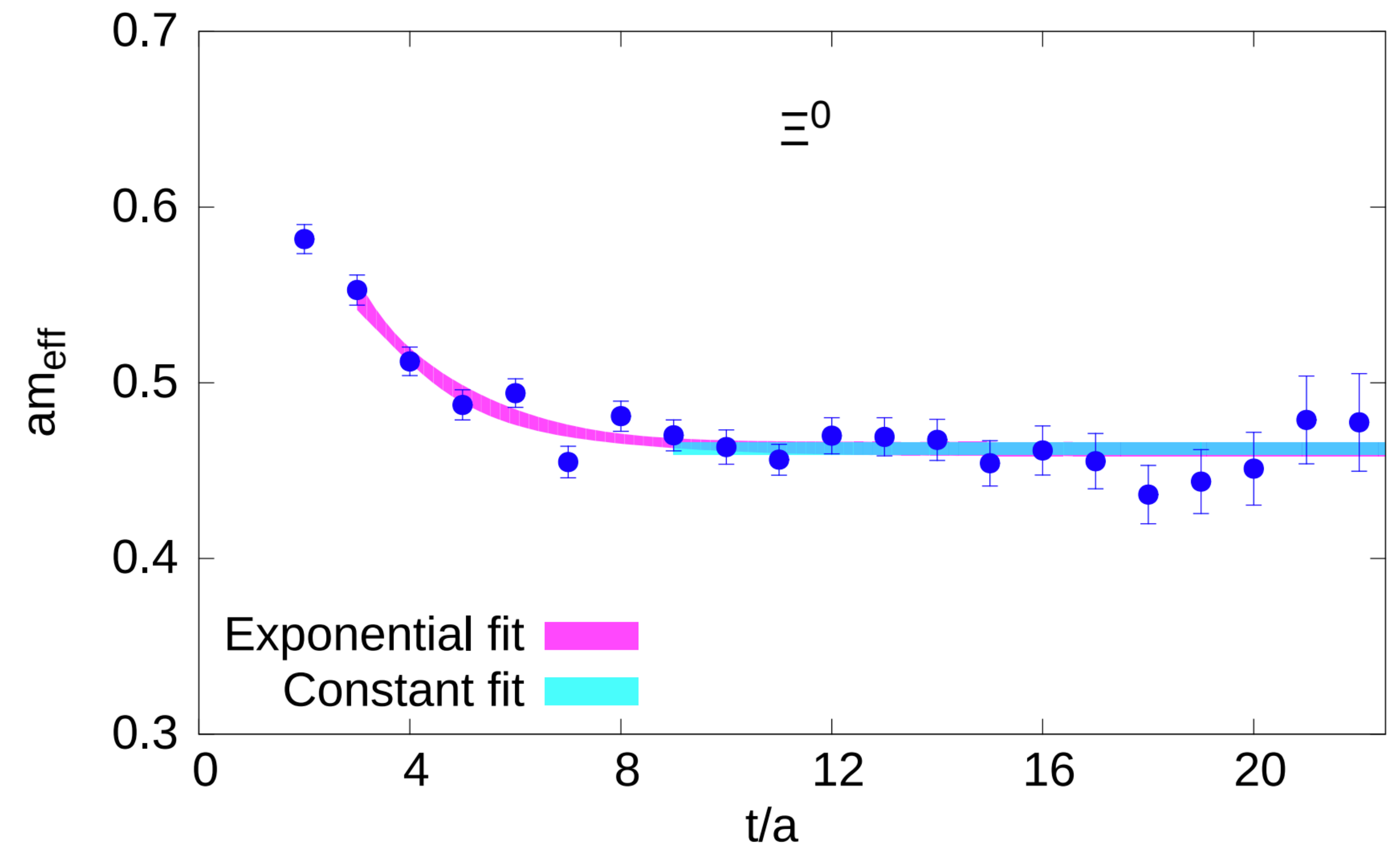

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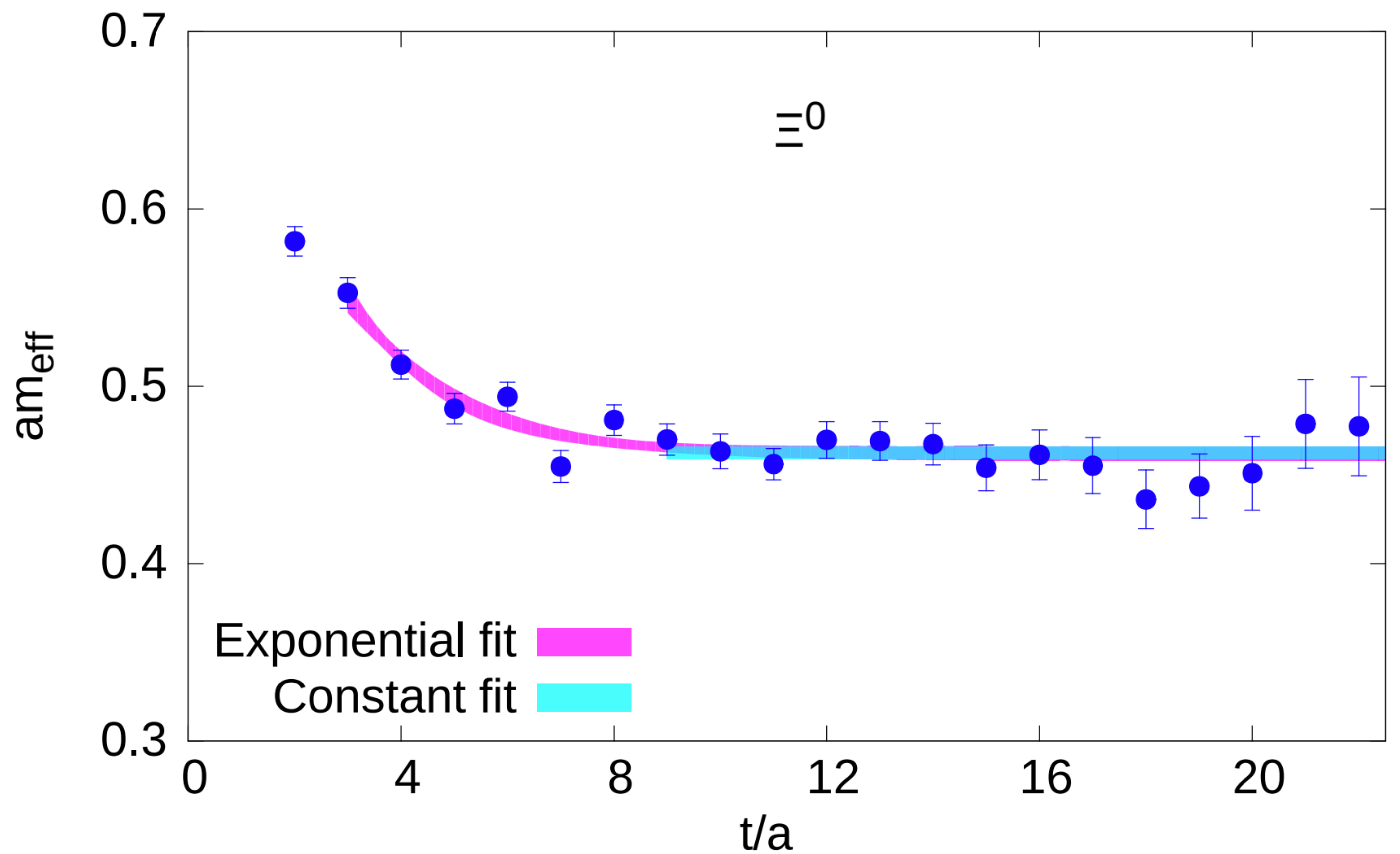
One may proceed with a constant or multi-state fit



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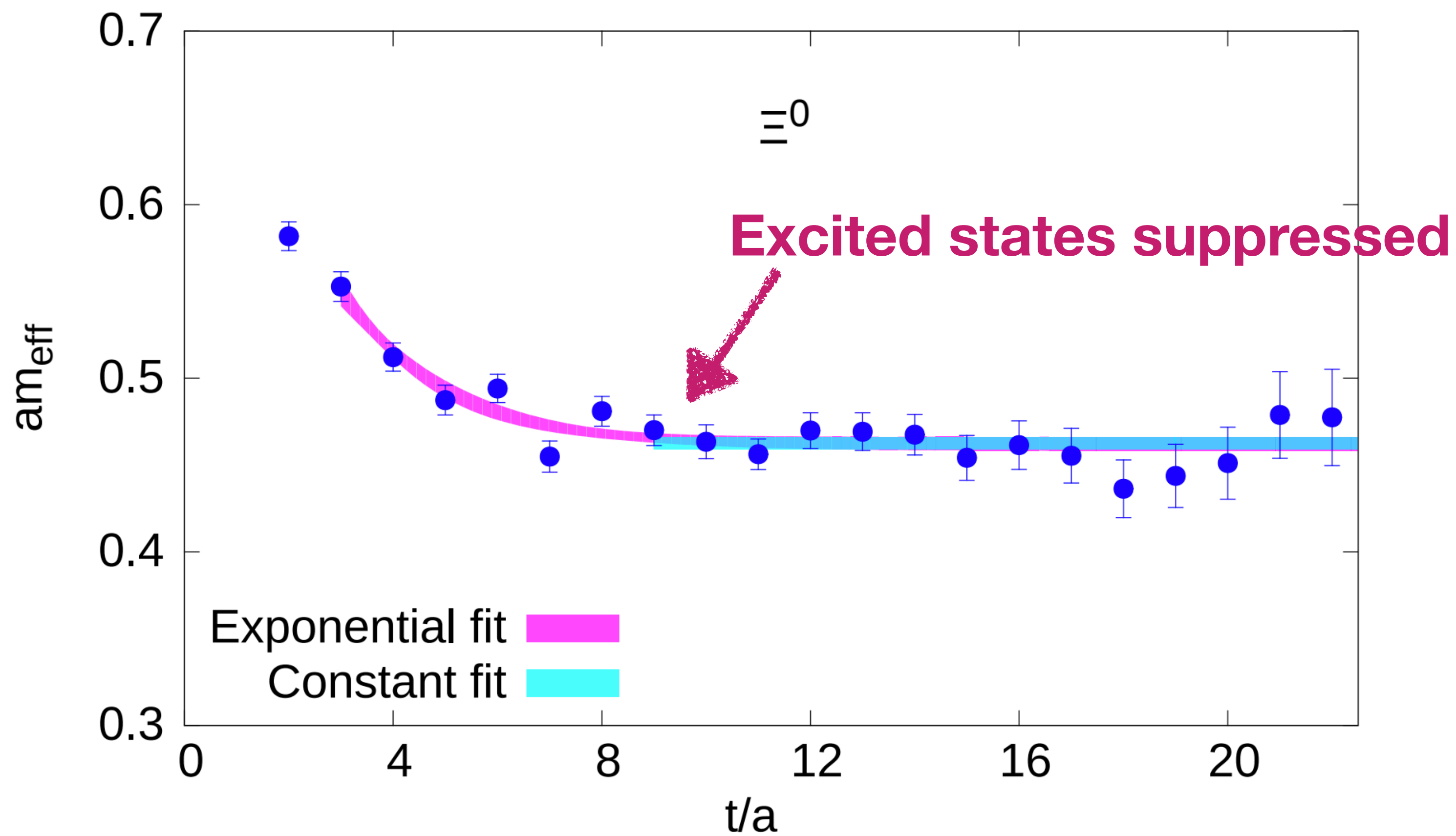
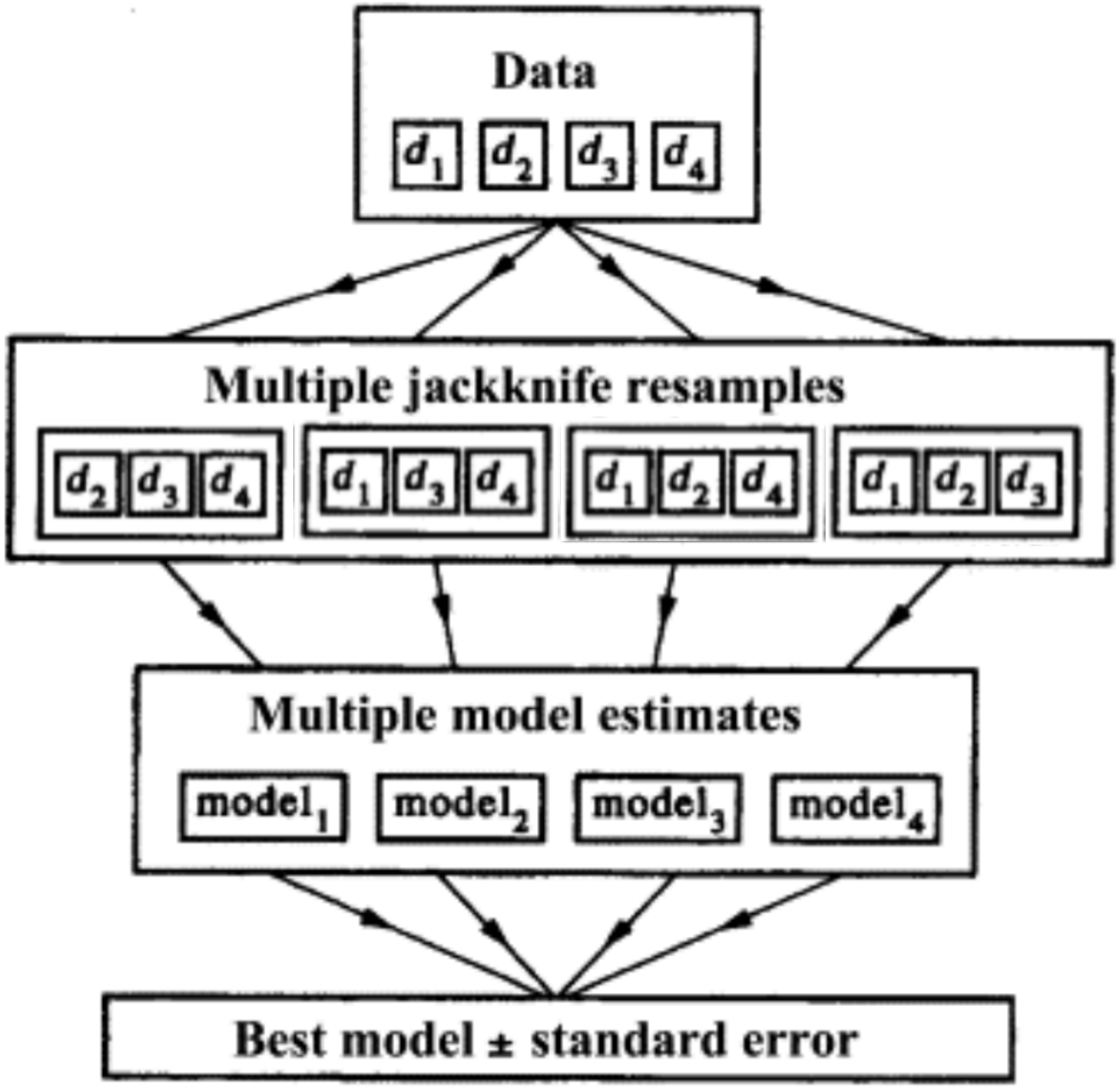


Calculation of Hadron mass

Results MUST be accompanied by uncertainties

Jackknife resampling
for variance and bias estimation



- ★ Choose the number of omitted data in each bin (defines # bins)
- ★ Calculate the average over remaining data in each bin
- ★ Calculate the average of the bins
- ★ Calculate the statistical error of the above average

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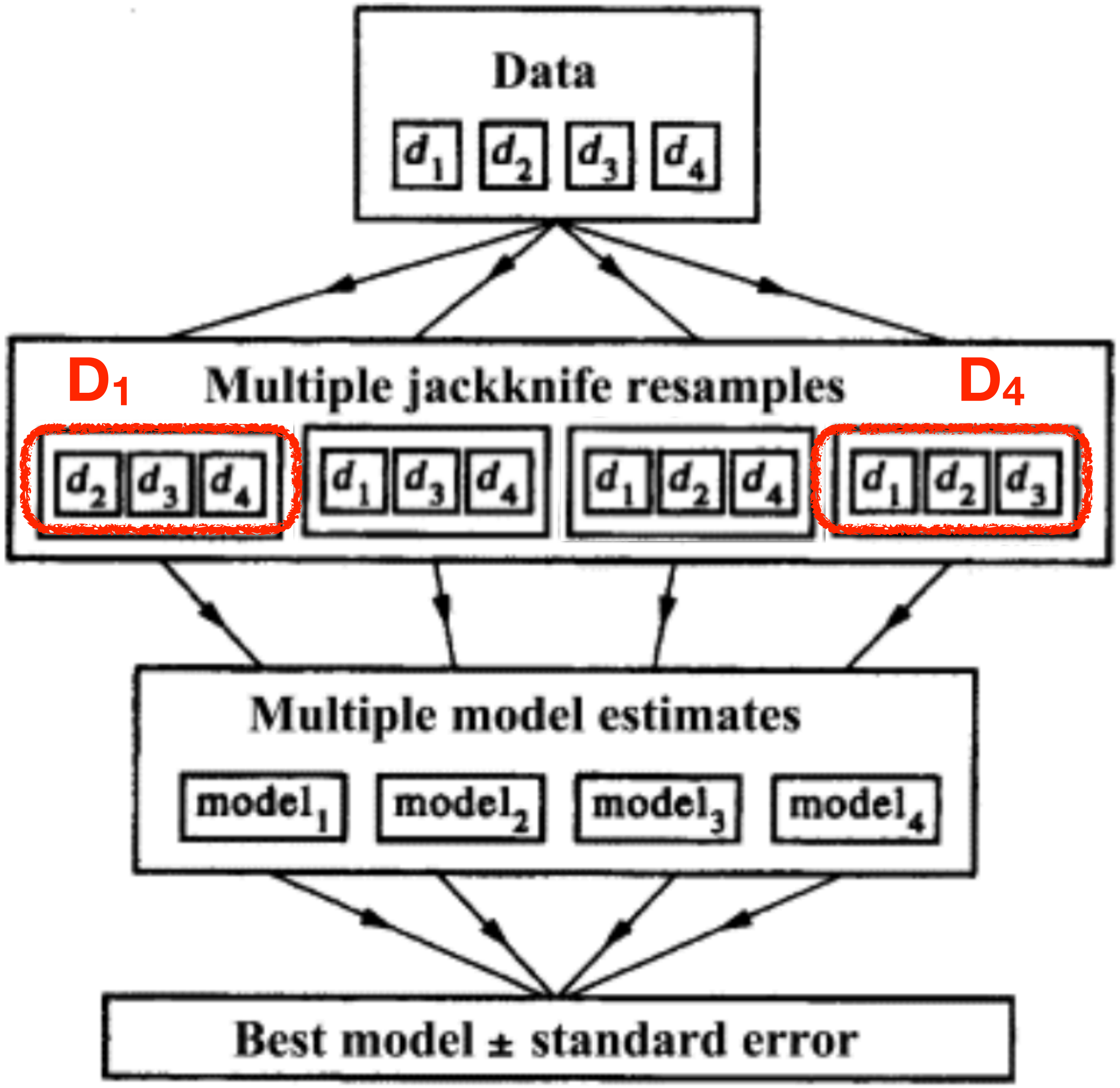

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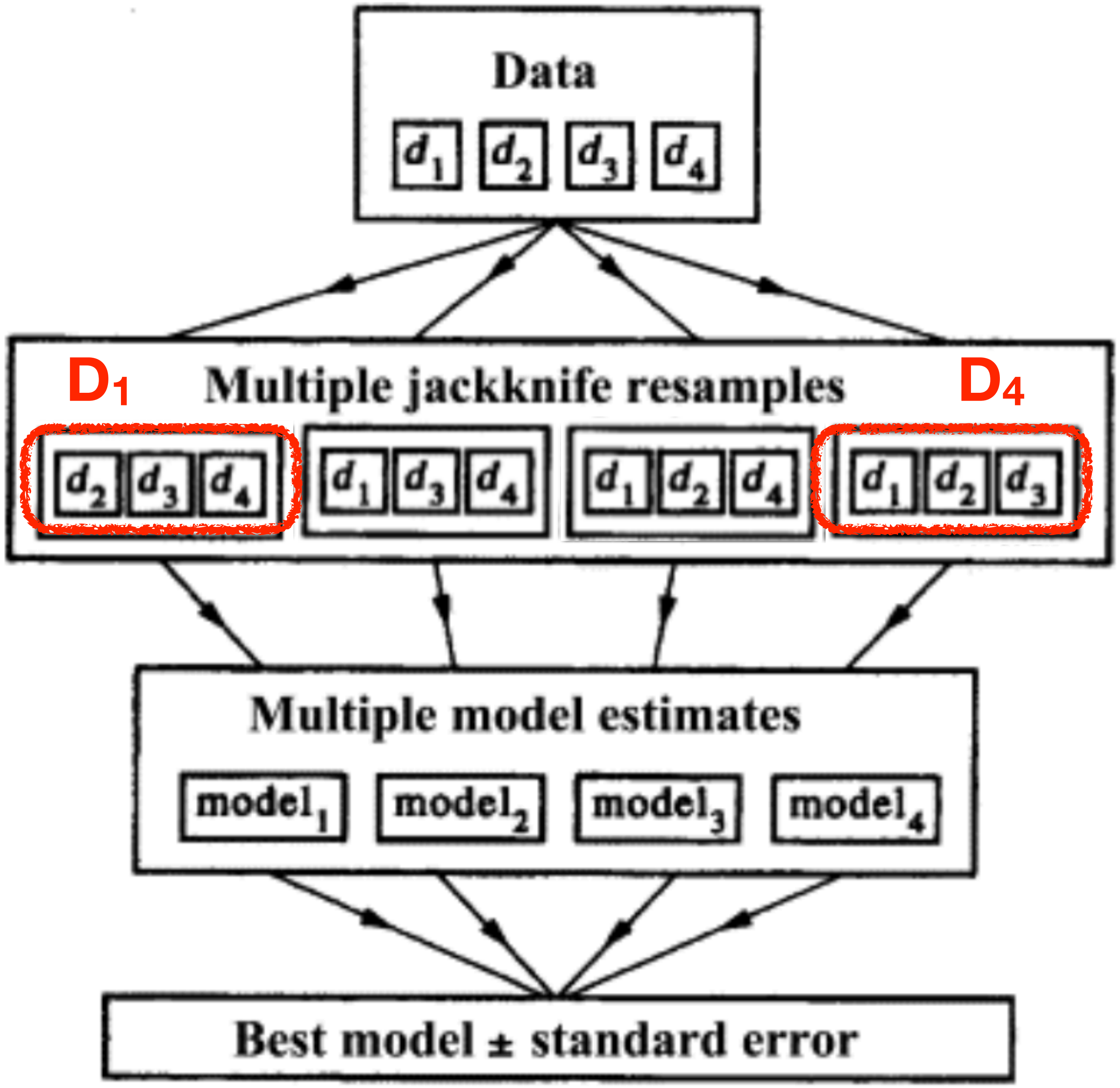

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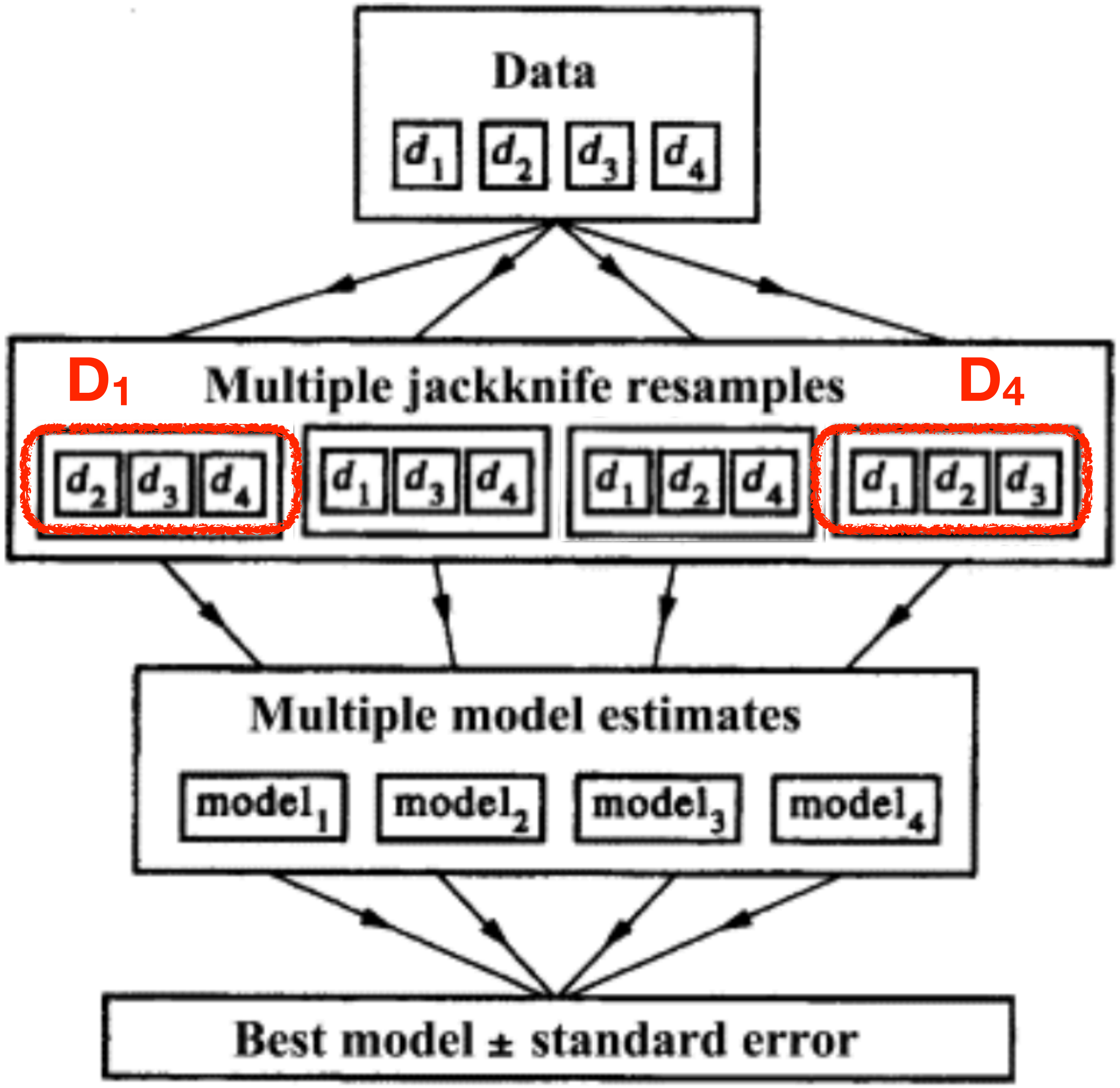

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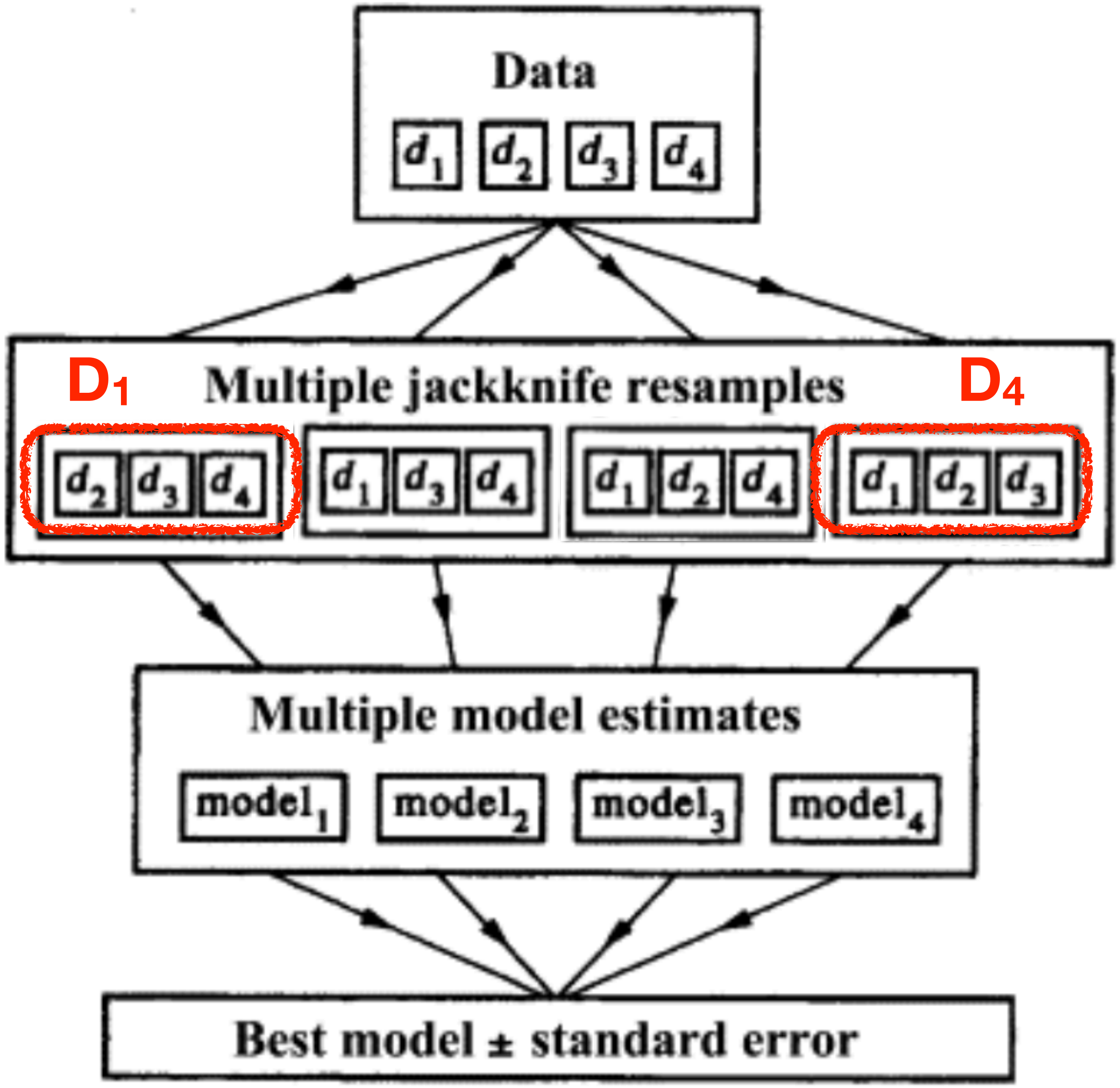

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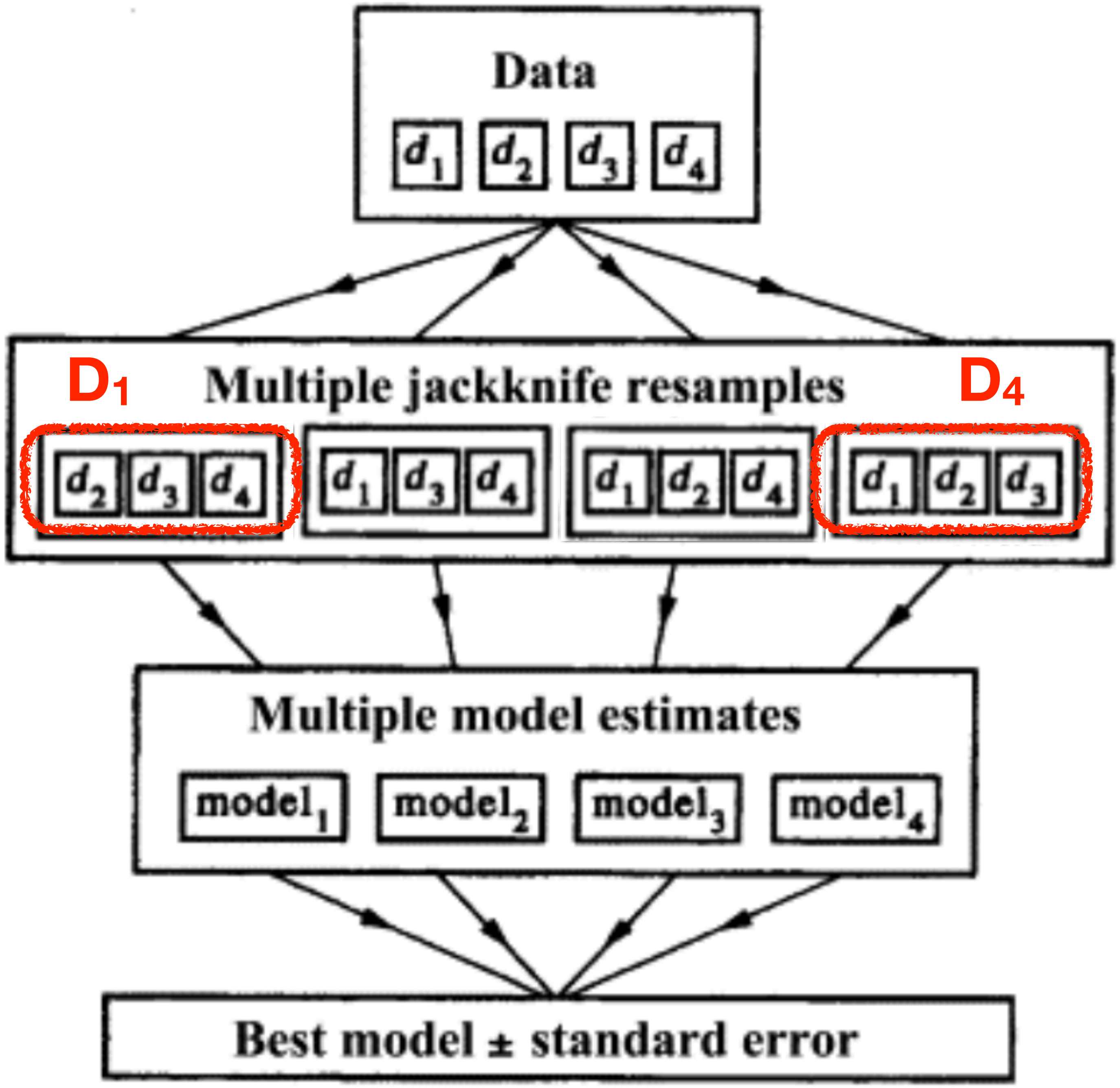

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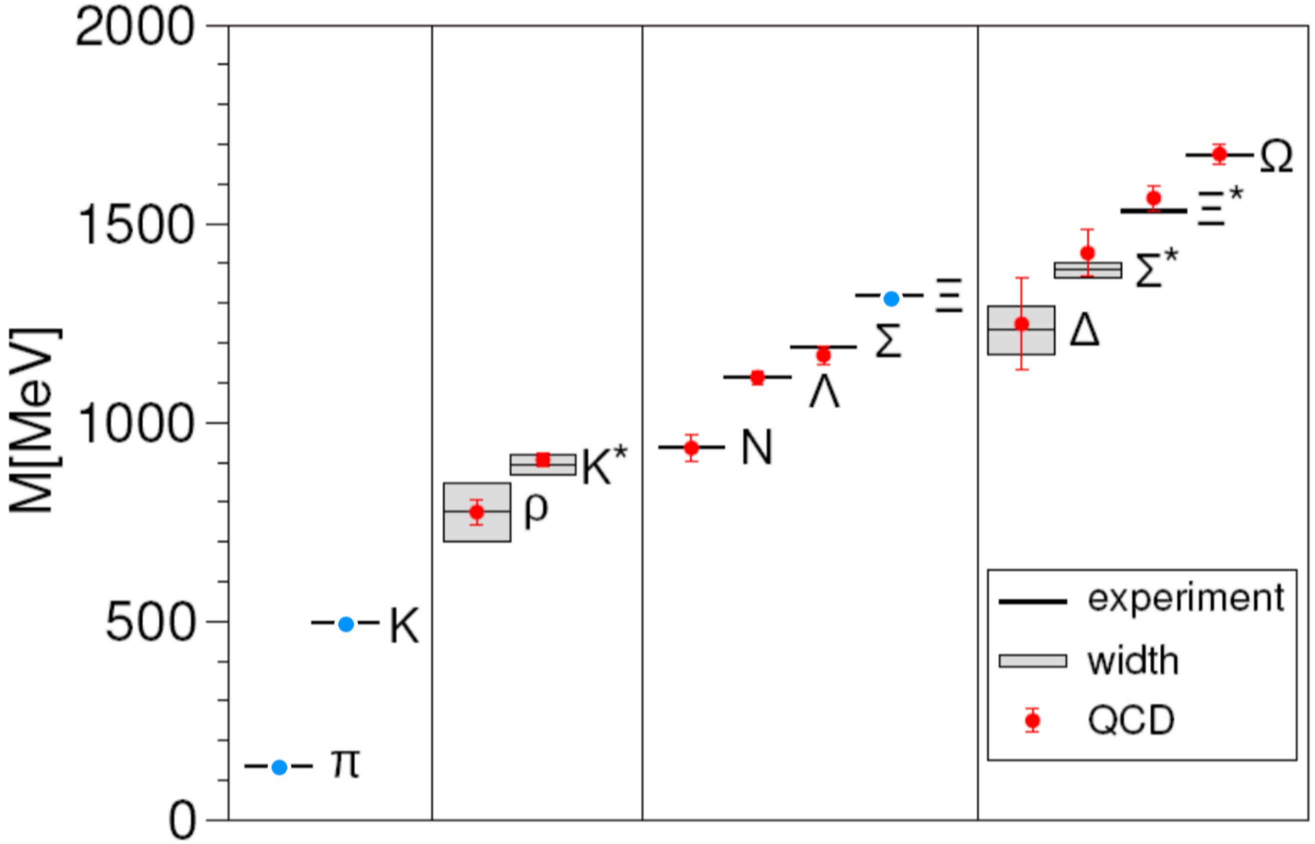
★ Calculate the statistical error of the above average

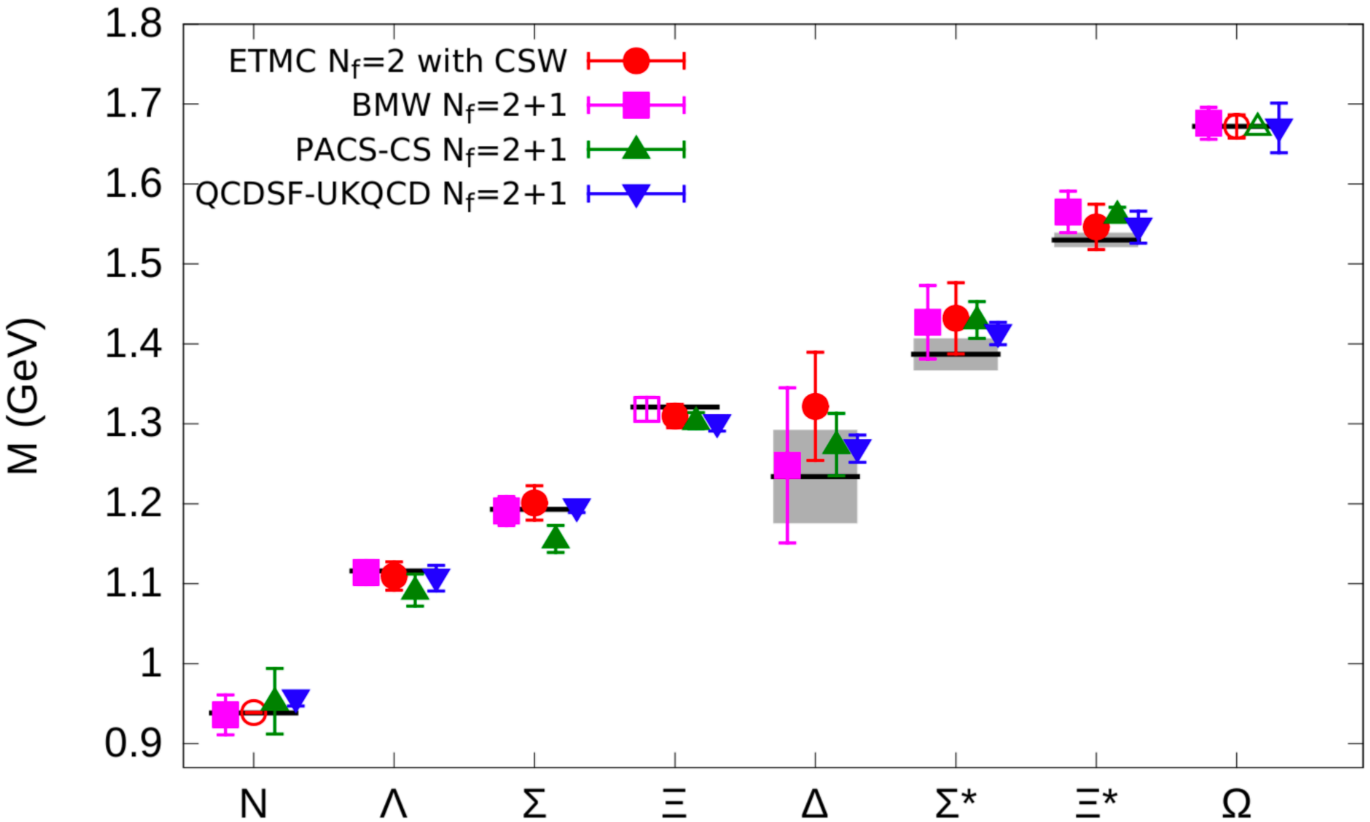
$$d\bar{D} = \sqrt{\sum_i (D_i - \bar{D})^2} \sqrt{\frac{N_{\text{bin}} - 1}{N_{\text{bin}}}}$$

Hadron Spectroscopy

Low-lying meson and baryon states



BMW collaboration, Science 322, 1224 (2008)



[C. Alexandrou et al.,Phys. Rev. D 90, 074501 (2014)]

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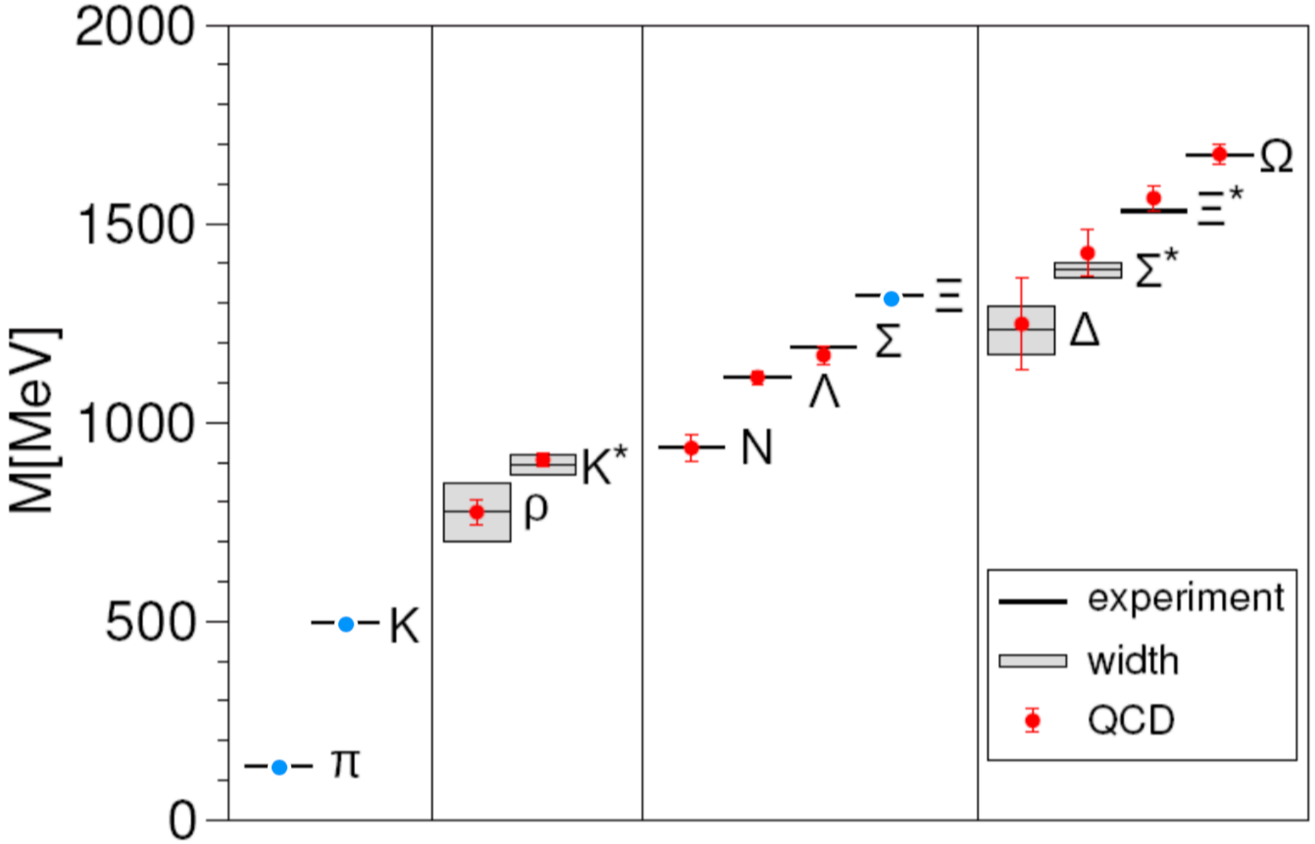

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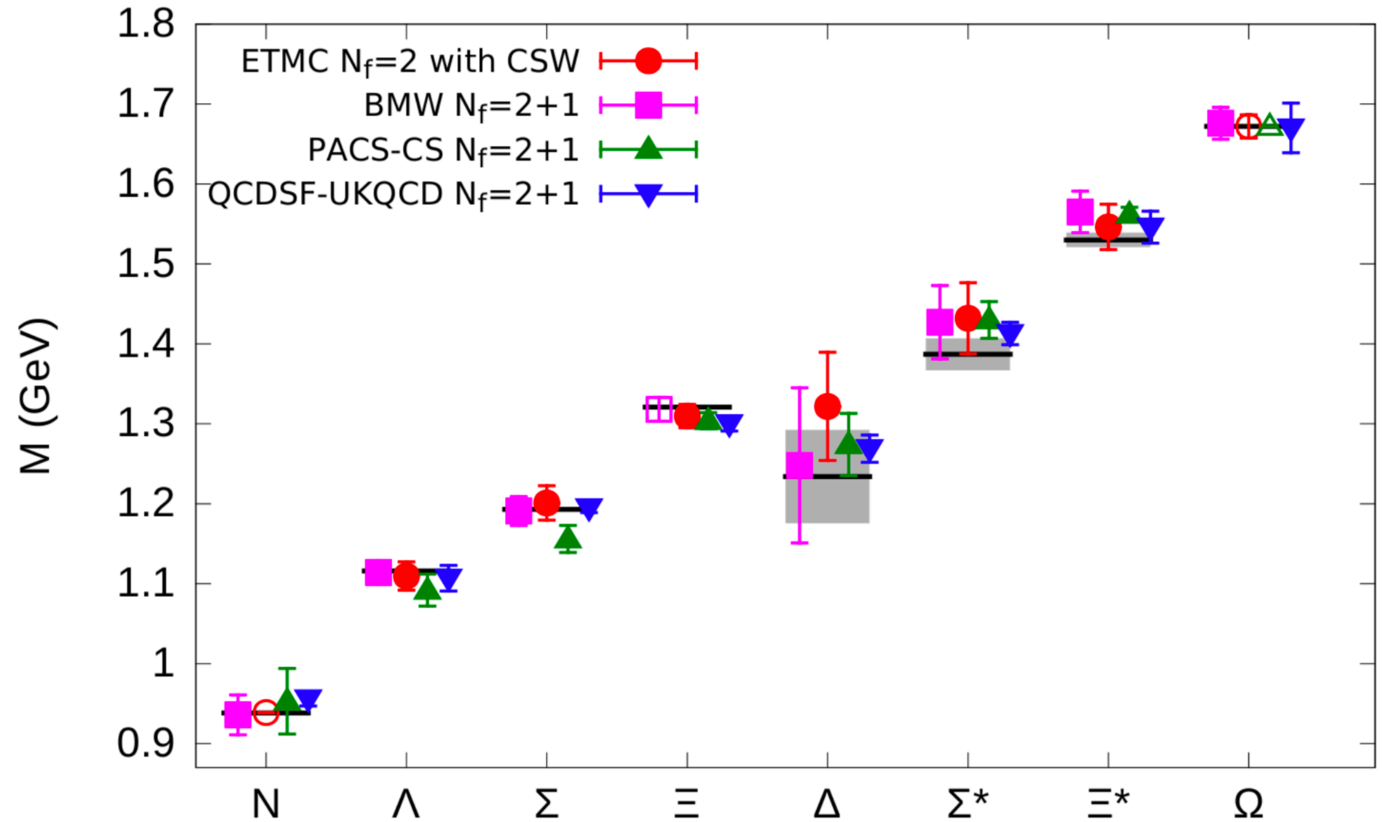

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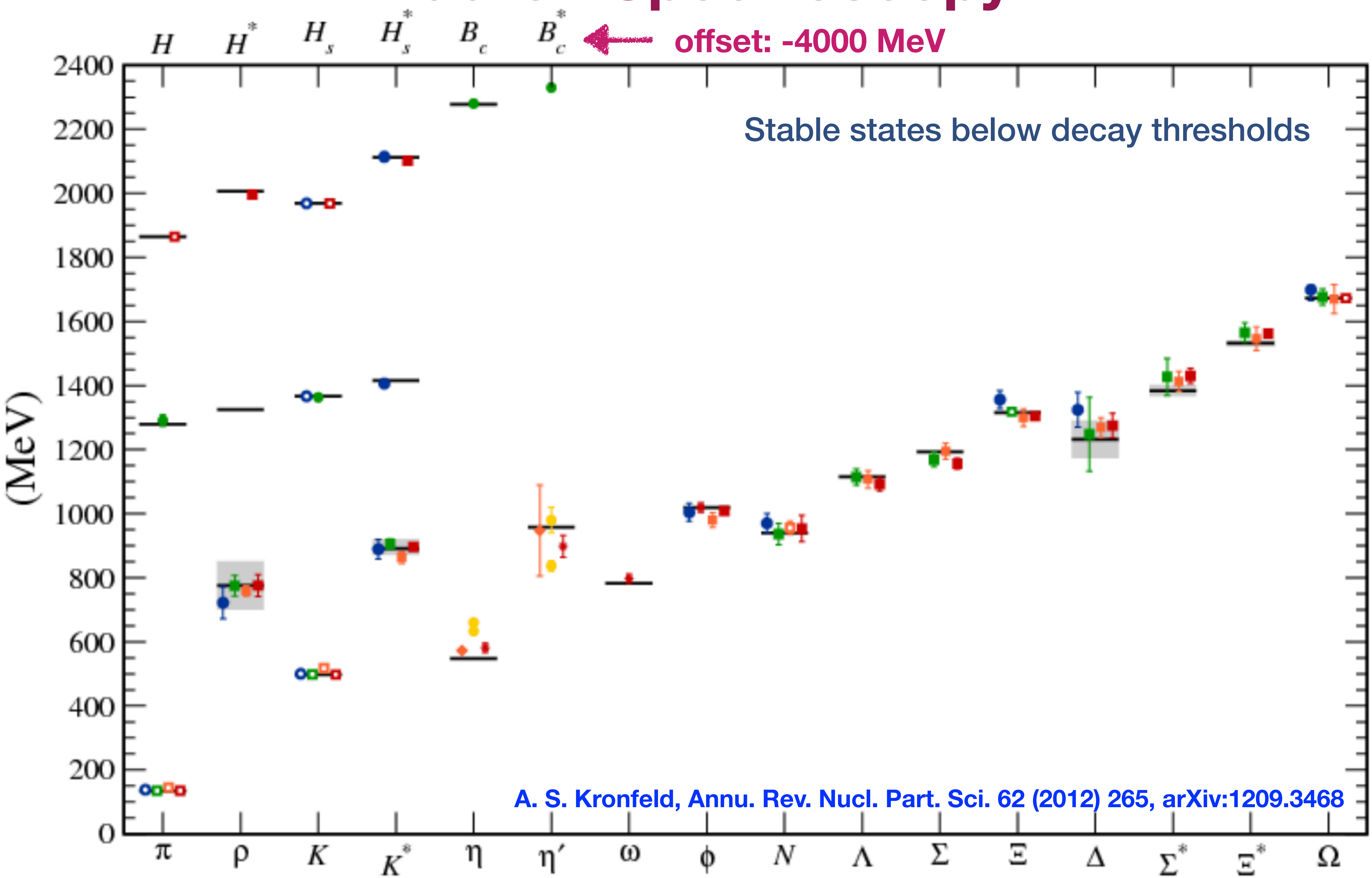
Lattice results reproduce experimental values

Hadron Spectroscopy



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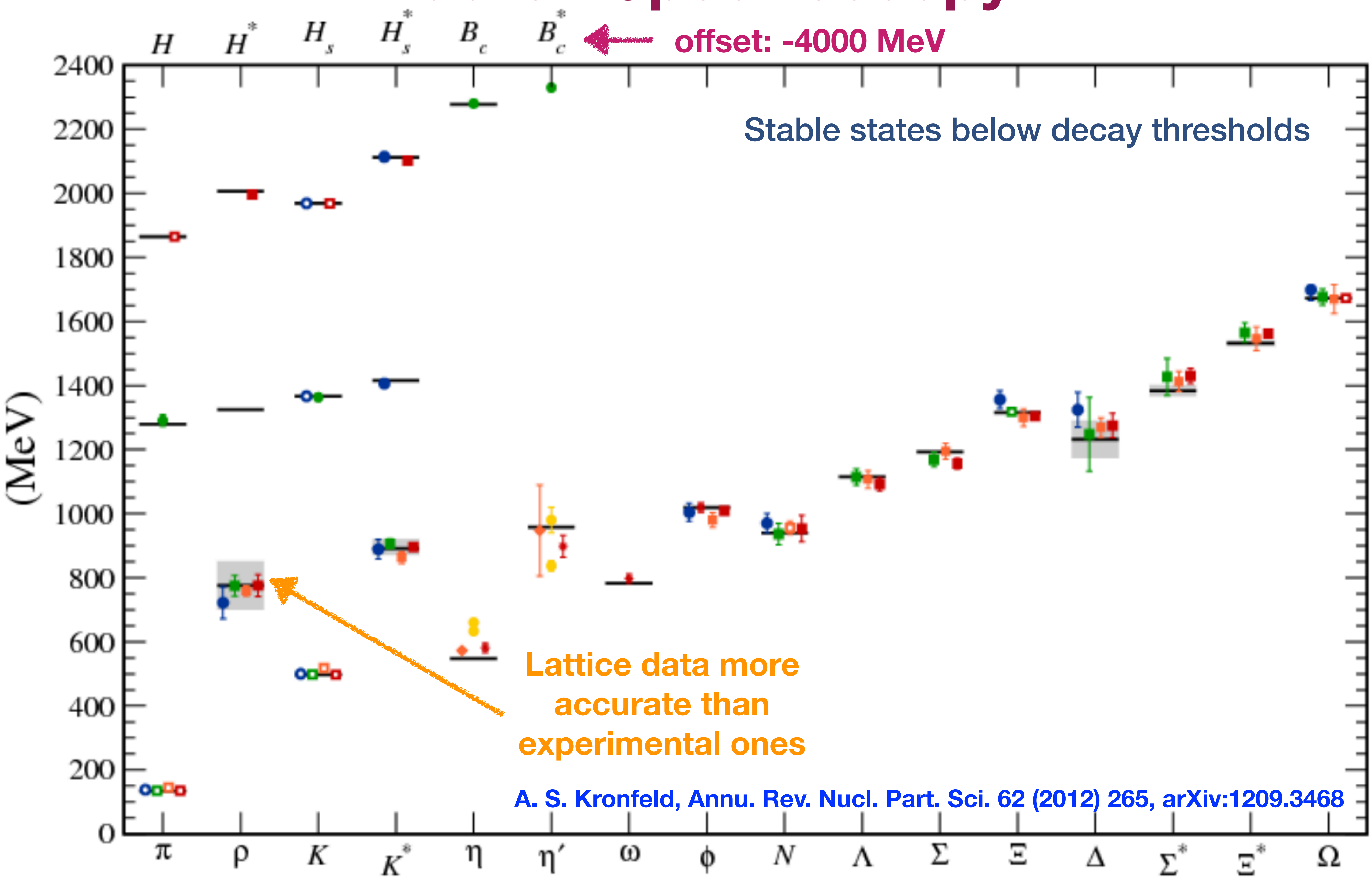

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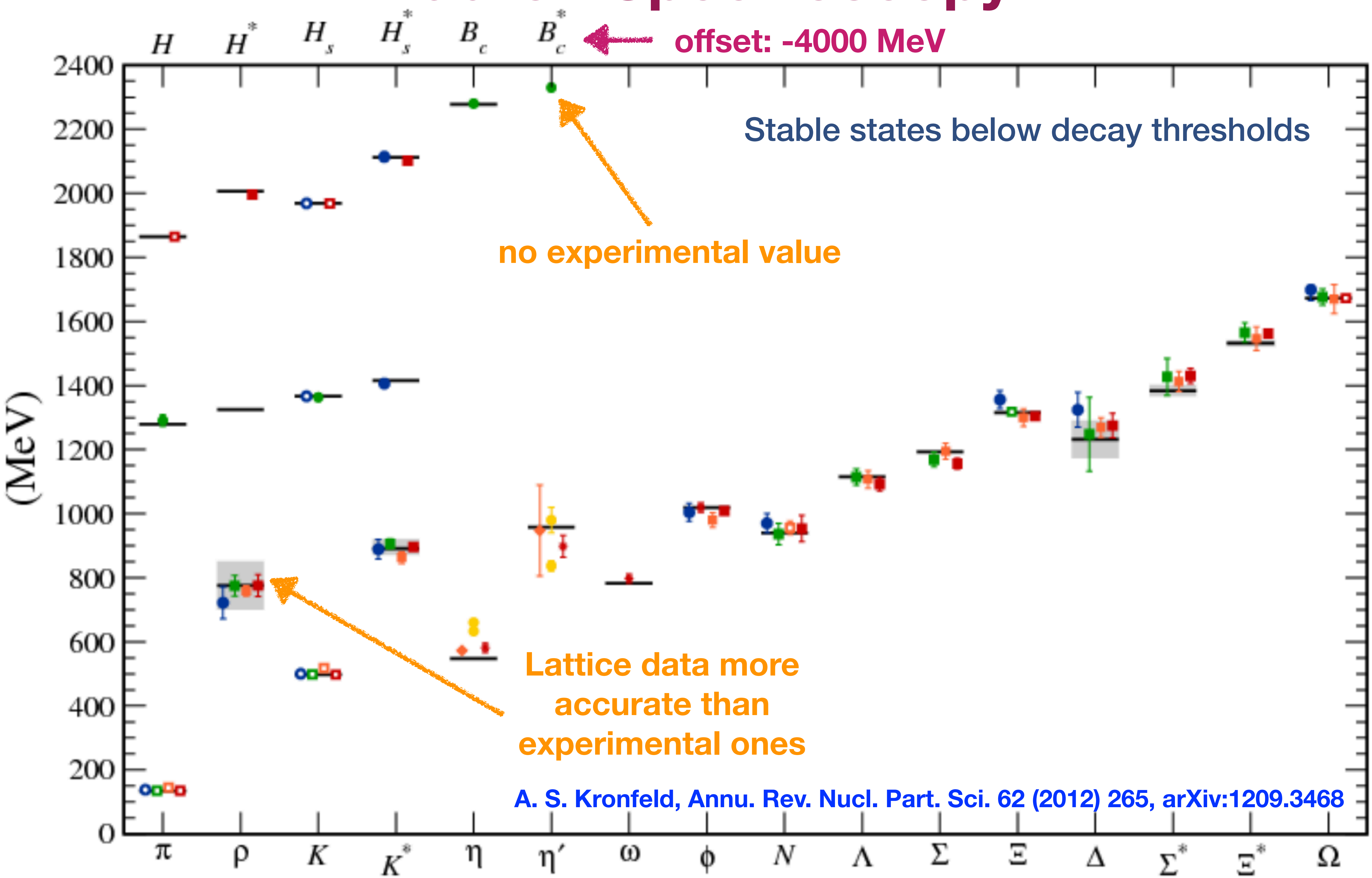

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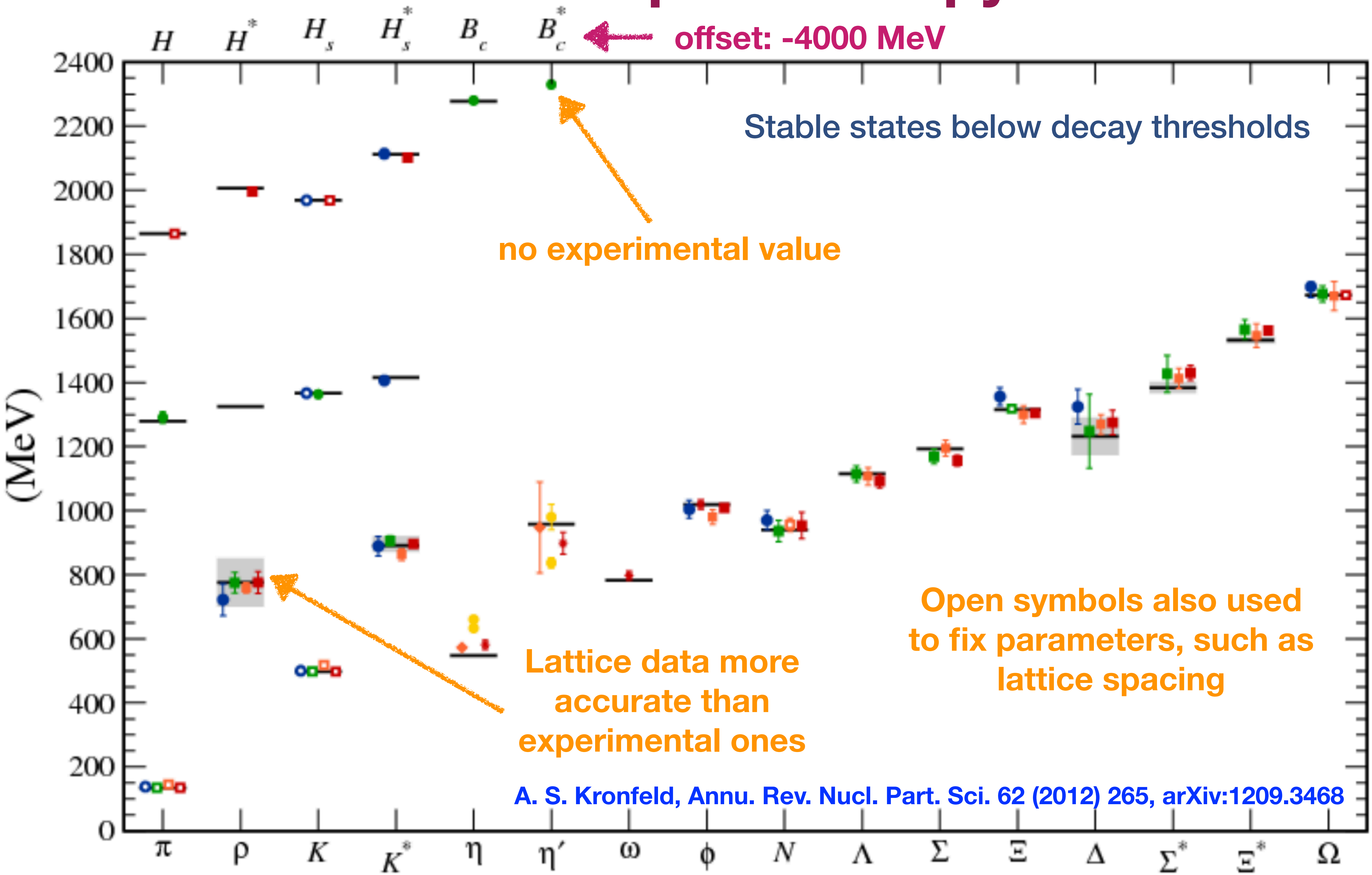
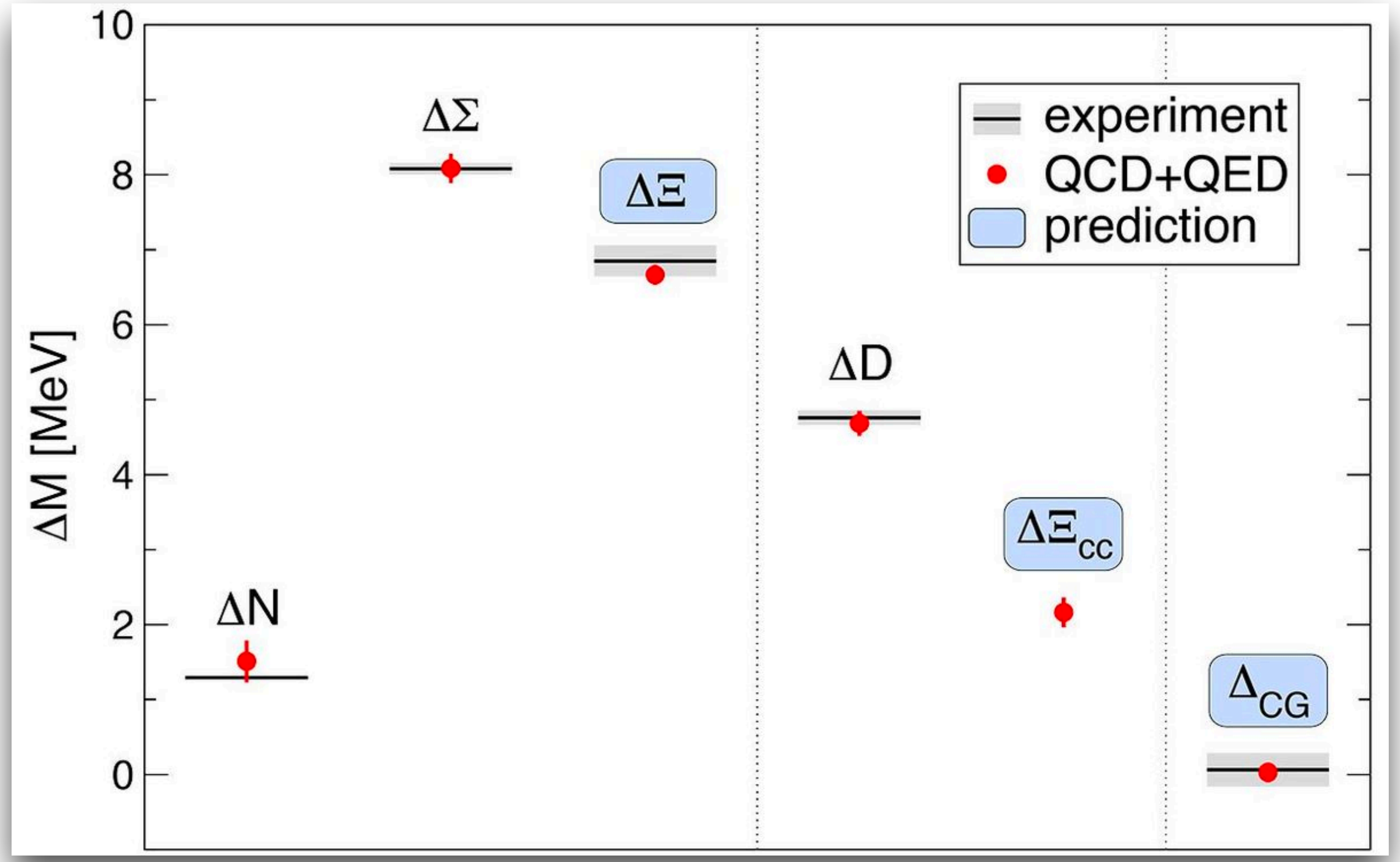


Lattice results reproduce experimental values

Hadron Spectroscopy

★ QCD + QED effects: mass splitting between, e.g., proton and neutron

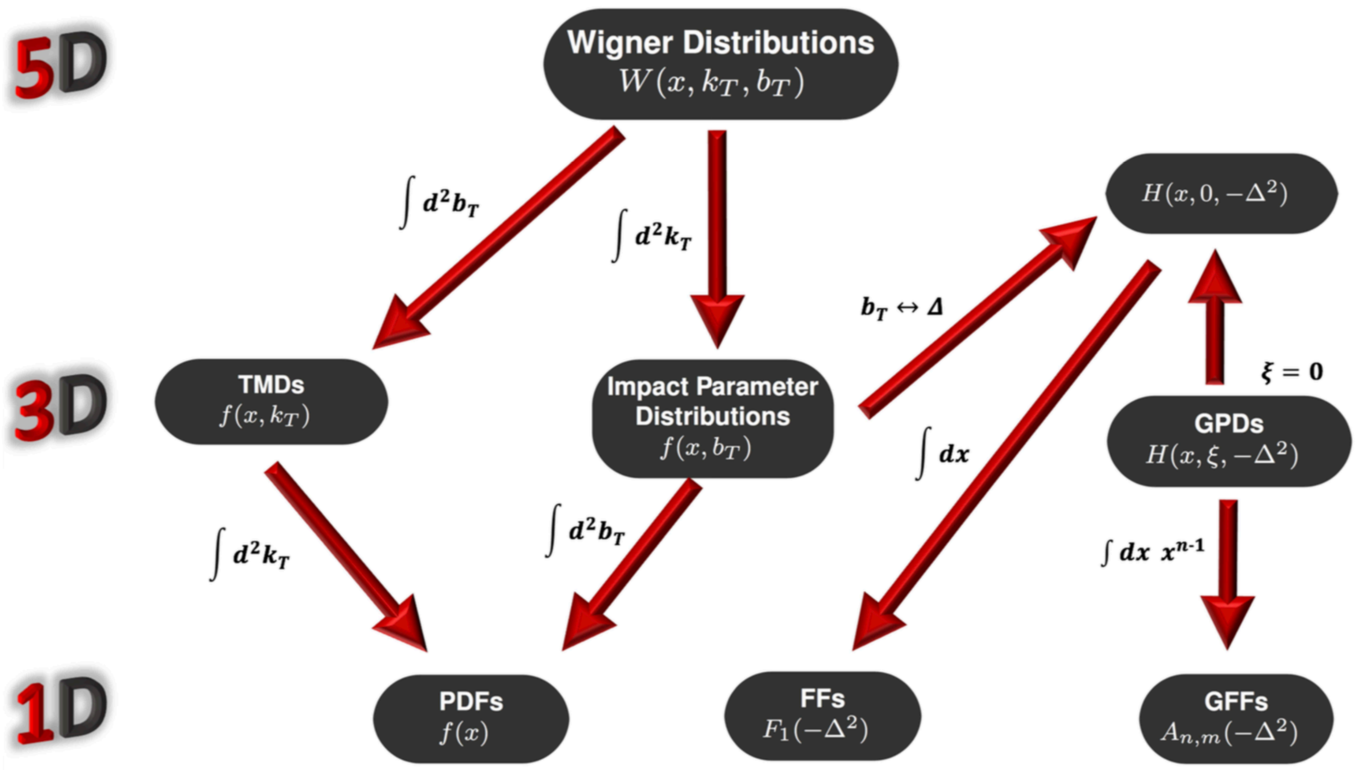


Borsanyi et al, Science 347, 14521455 (2015)

How to study Hadron Structure

Hadron Characterization

[X. Ji, D. Mueller, A. Radyushkin]



$x = \frac{k^+}{p^+}$: longitudinal momentum fraction

Δ : momentum transfer

$\xi = \frac{\Delta^+}{2P^+}$: (fraction of longitudinal momentum)

b_T : impact parameter

k_T : parton transverse momentum

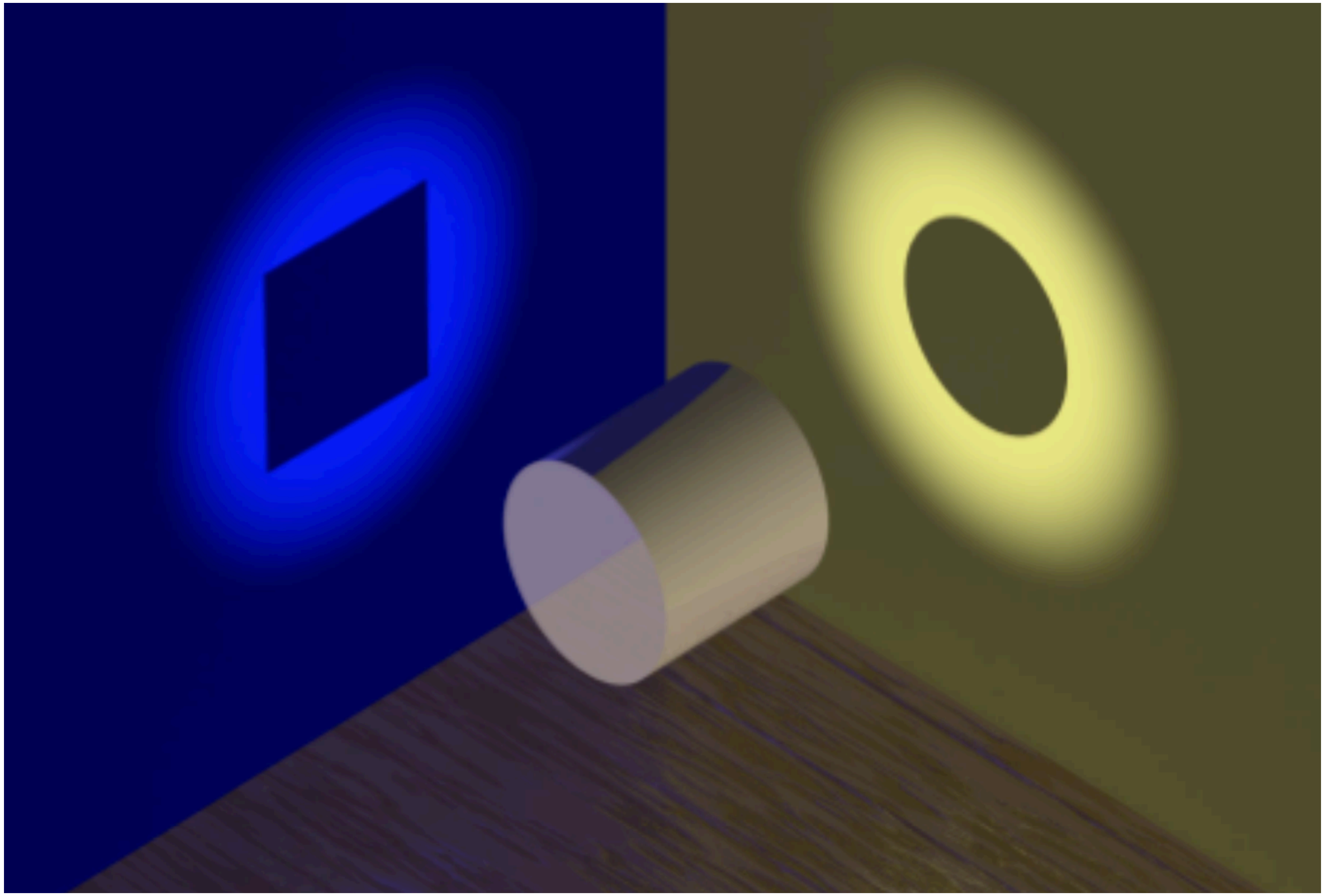
Hadrons structure studied in terms of:

★ **parton distribution functions (PDFs)**
- Deep Inelastic Scattering (DIS)

★ **generalized distributions (GPDs)**
- Semi-Inclusive Deep Inelastic Scattering (SIDIS)

★ **transverse momentum dependent distributions (TMDs)**
- Deeply Virtual Compton Scattering (DVCS)

All Distributions necessary to understand the complexity of internal hadron structure

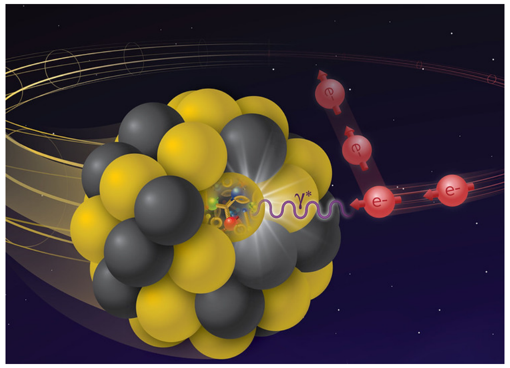


"Studying GPDs at JLab", S. Stepanyan, GHP 2015

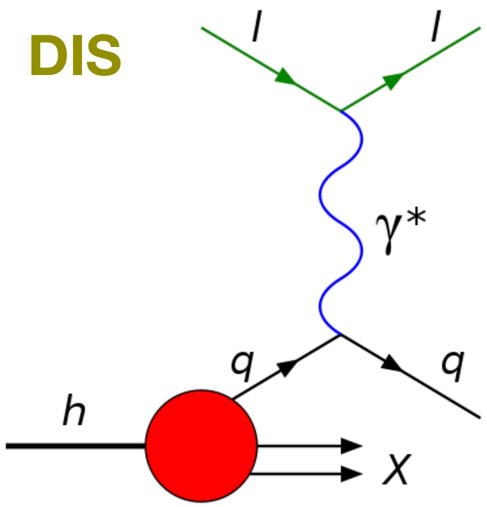
Hadron Structure

- **Structure of hadrons explored in high-energy scattering processes**

Collisions @ EIC



DIS



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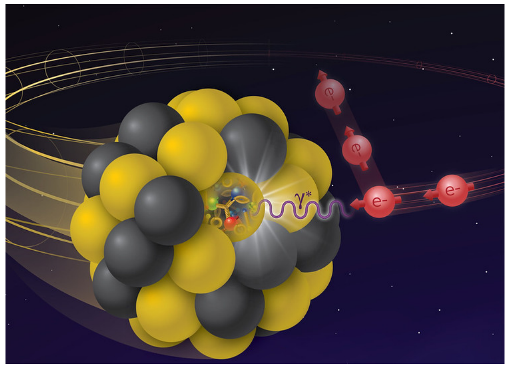
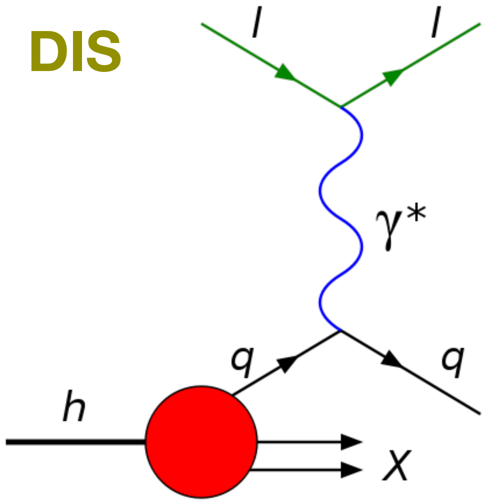
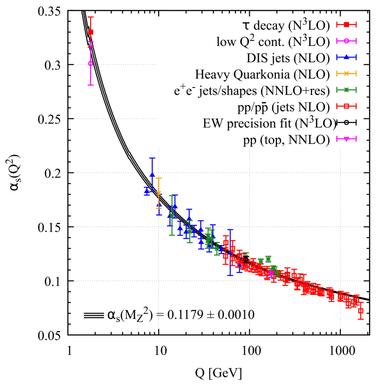




- **Due to asymptotic freedom, e.g.**



$$\sigma_{\mathrm{DIS}}(x, Q^2) = \sum_i \left[H^i_{\mathrm{DIS}} \otimes f_i\right](x, Q^2)$$

$$[a \otimes b](x) \equiv \int_x^1 \frac{d\xi}{\xi}\, a\left(\frac{x}{\xi}\right)\, b(\xi)$$

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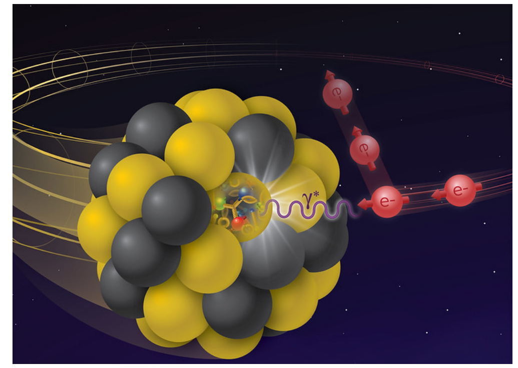

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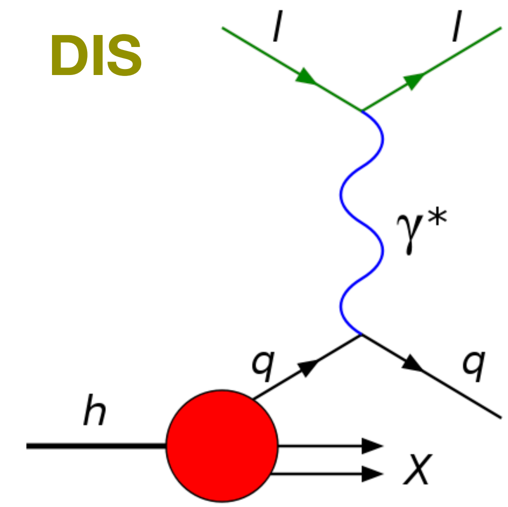

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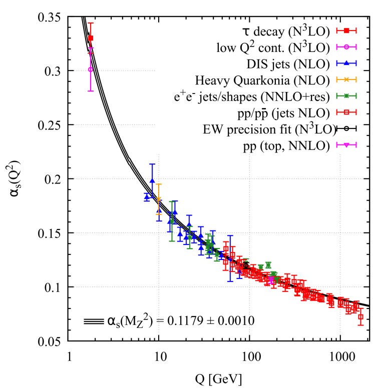

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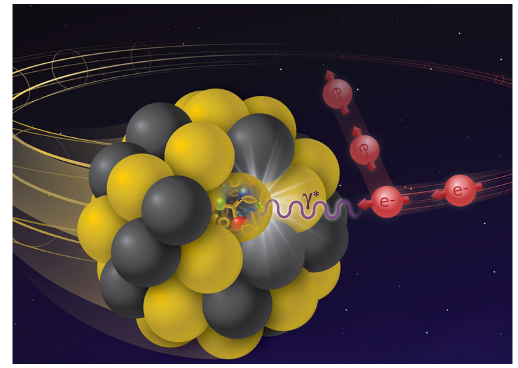

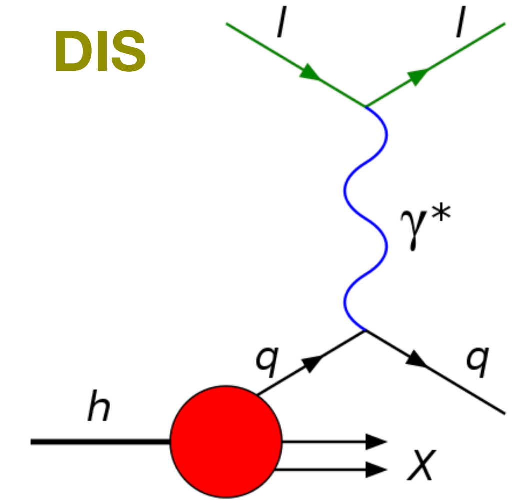

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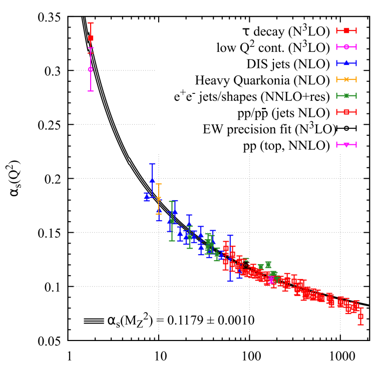

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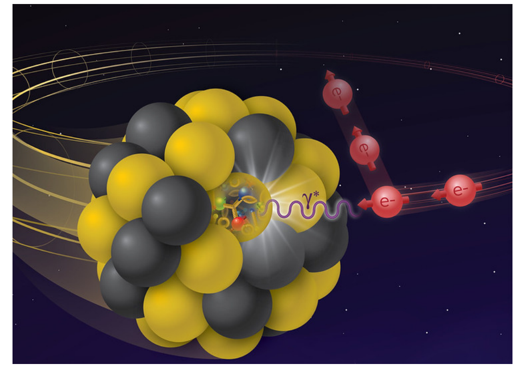

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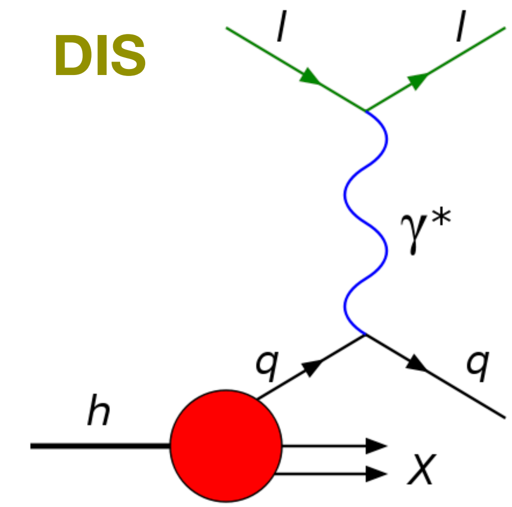

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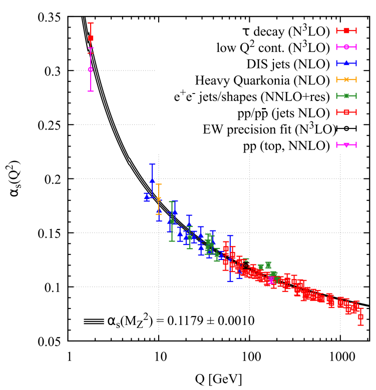

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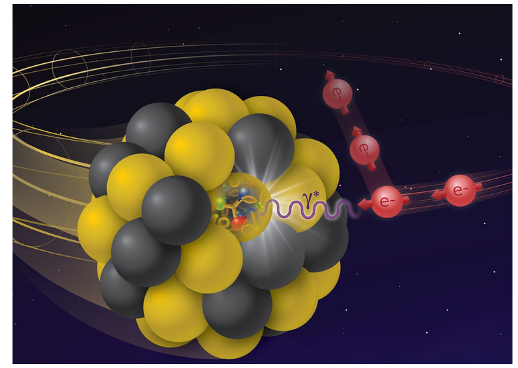

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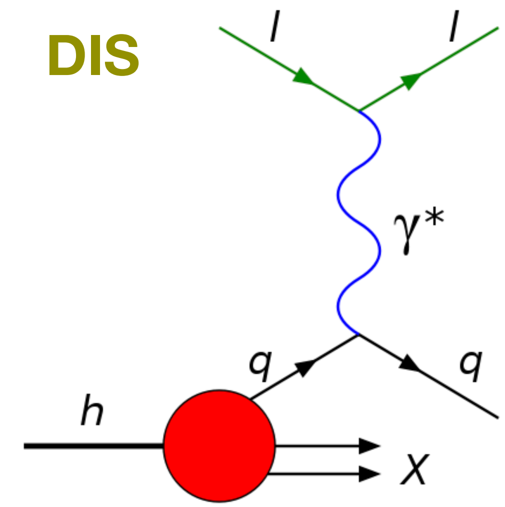

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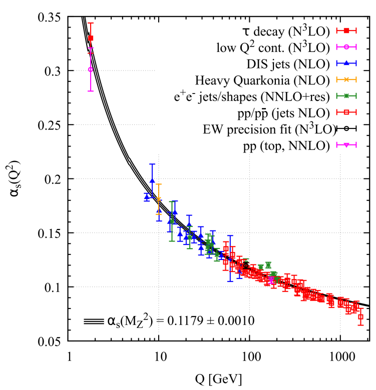

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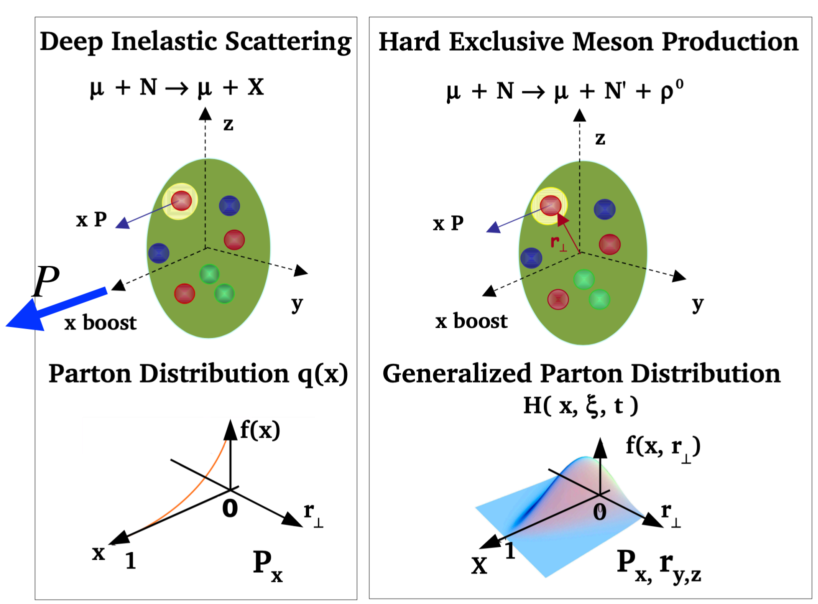
Perturb. part
(process dependent)

Non-Perturb. part
(process “independent”)

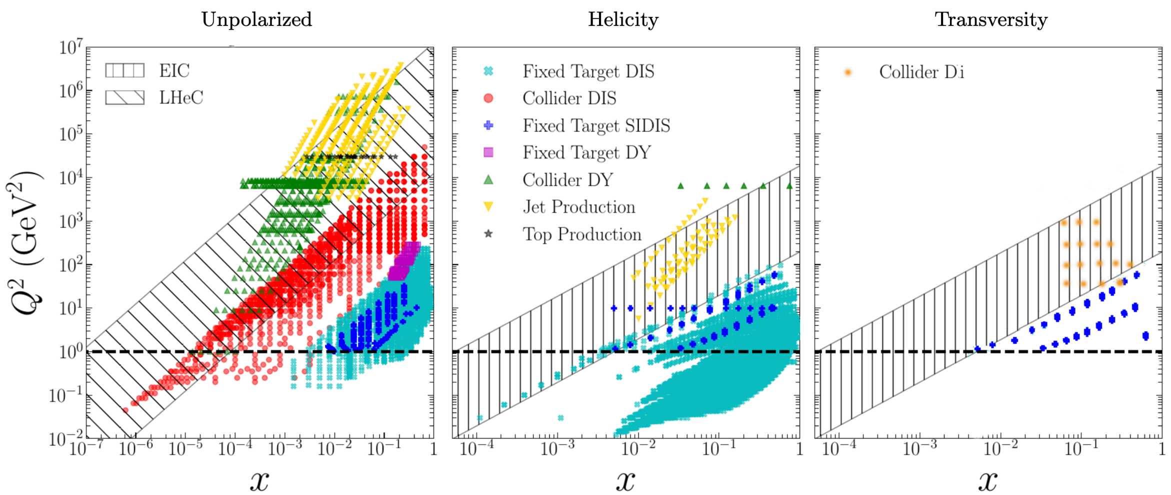
- Non perturb. part provides information on partonic structure of hadrons



Distribution Functions

★ **Key universal non-perturbative tools for study of hadron structure**

★ **Global fit analyses of DIS data: main source of information for PDFs**



[Ethier & Nocera, *Ann. Rev. Nucl. Part. Sci.* 70 (2020) 1, arXiv:2001.07722]

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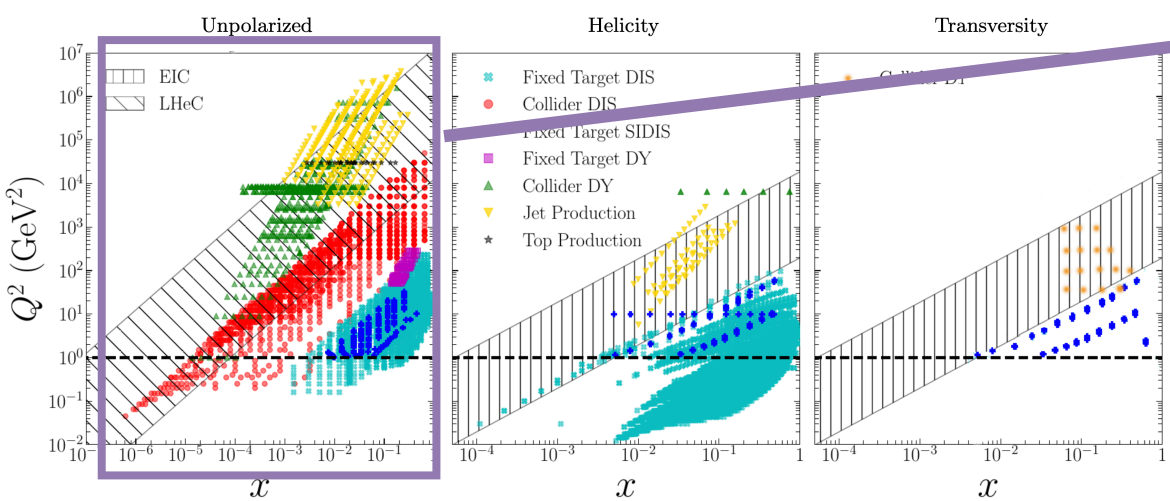

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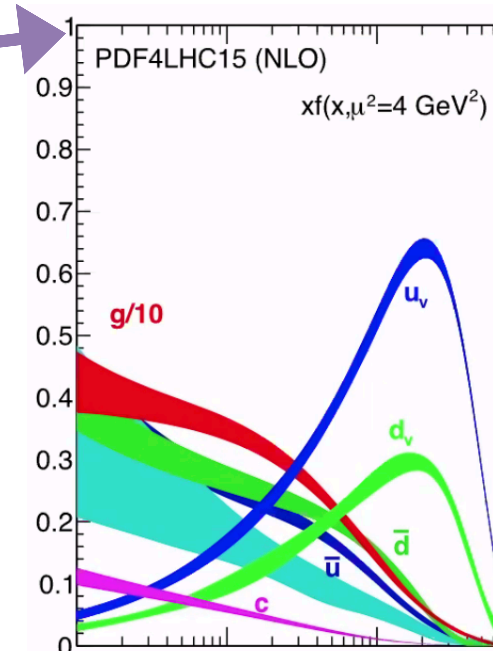

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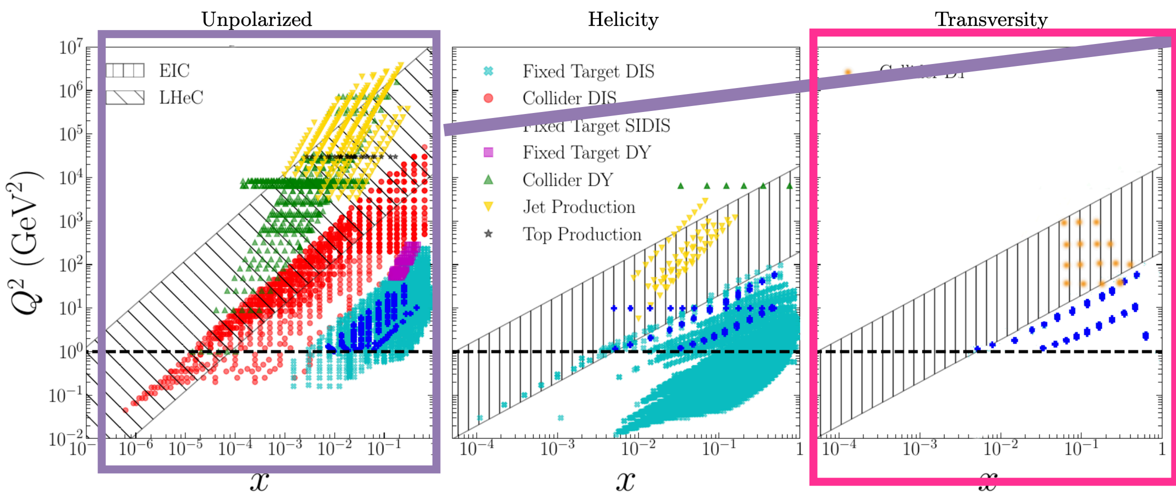

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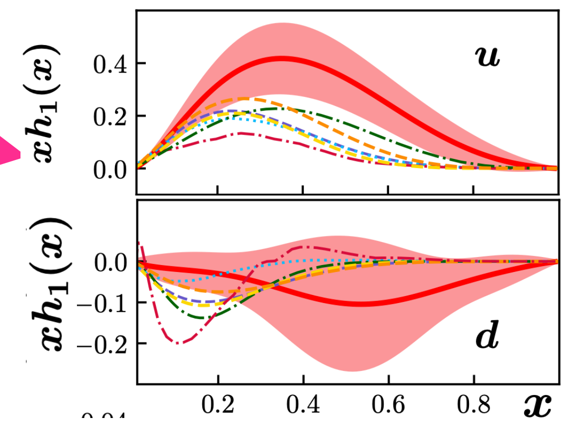

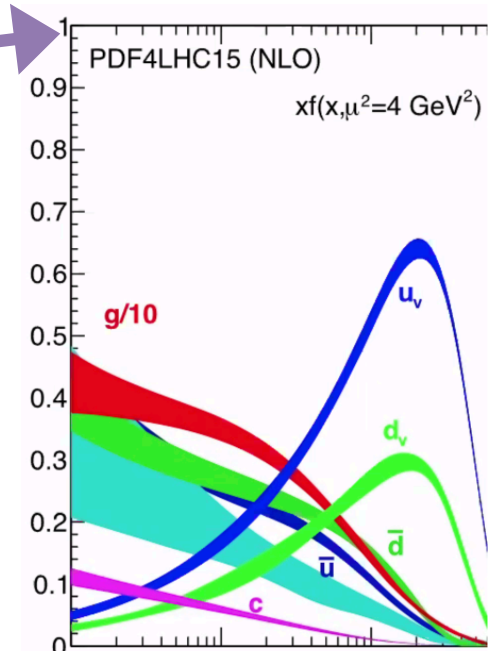

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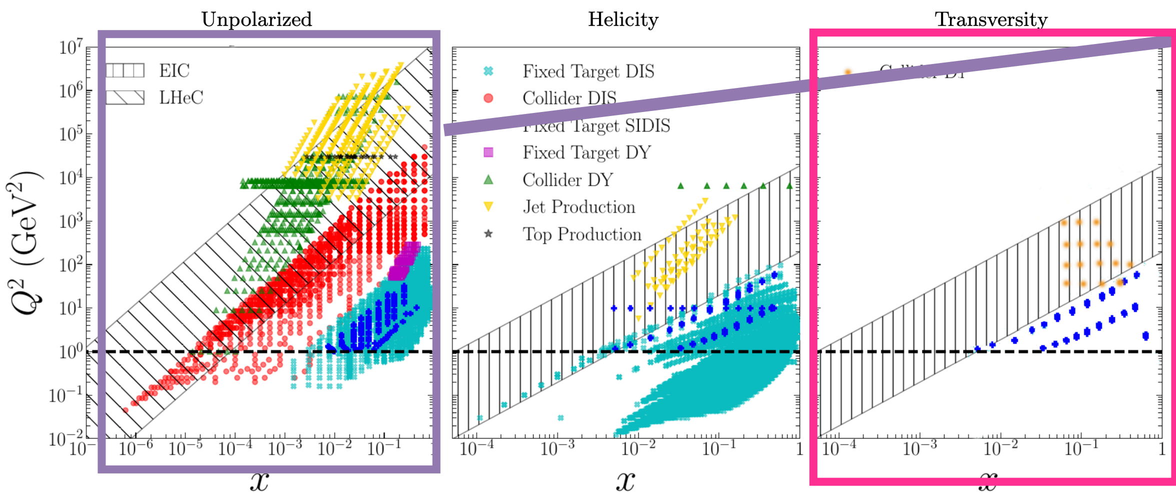

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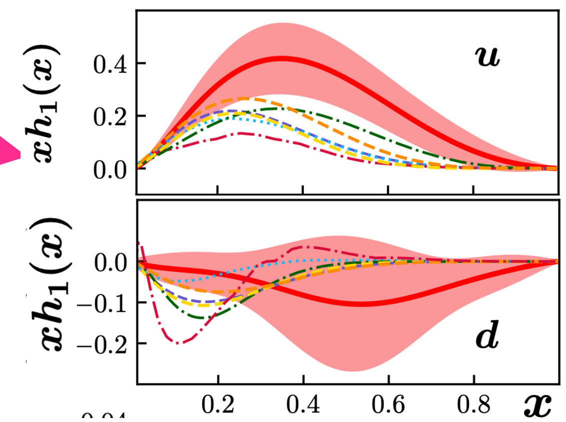

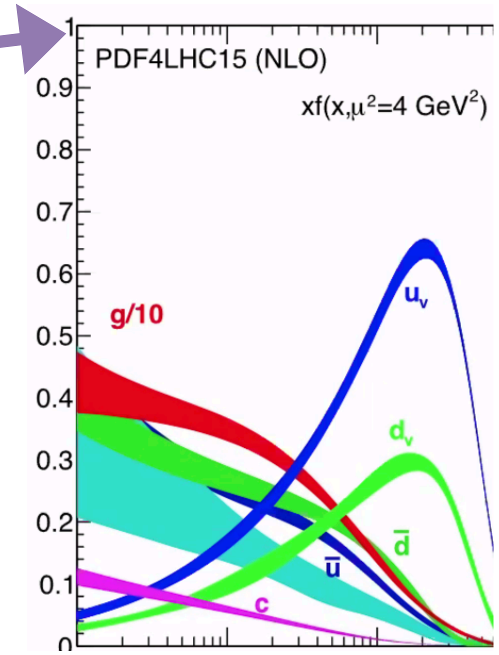

▶ **Global fits improved: theoretical advances & new data**

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Calculation from first principle (lattice QCD) can help in the reliable extraction of physical quantities such as the proton spin and mass (related to PDFs)

Distribution Functions

★ **PDFs parameterized in terms of off-forward matrix elements of light-cone operators**

$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p' \,|\, \bar{\psi}(-\lambda n/2)\, \mathcal{O}\, \underbrace{\mathcal{P} e^{ig \int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(n\alpha)}}_{\text{gauge invariance}} \psi(\lambda n/2) \,|\, p\rangle$$

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★ **On lattice: moments of PDFs (reconstructed via OPE)**

$$\langle x^{n-1} \rangle = \int_{-1}^{+1} x^{n-1} f(x)\, dx$$

★ **Reconstruction difficult task:**

- n>3: operator mixing
- Statistical noise increases with high moments

★ **Moments of PDFs have physical interpretation and may serve as benchmark**

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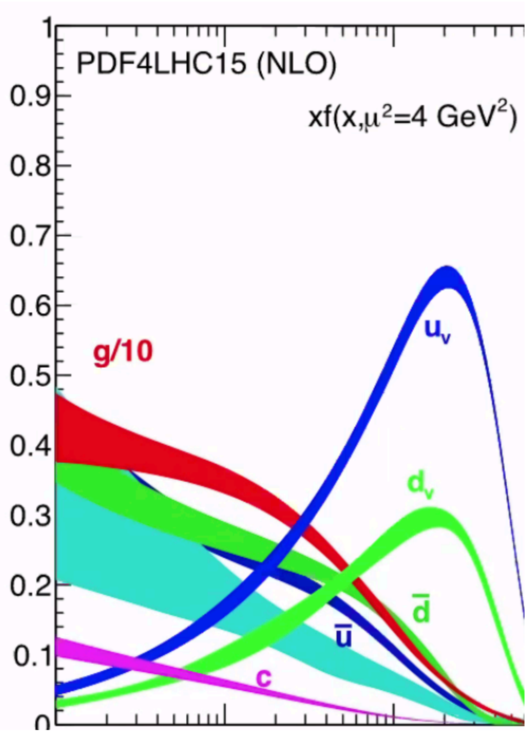

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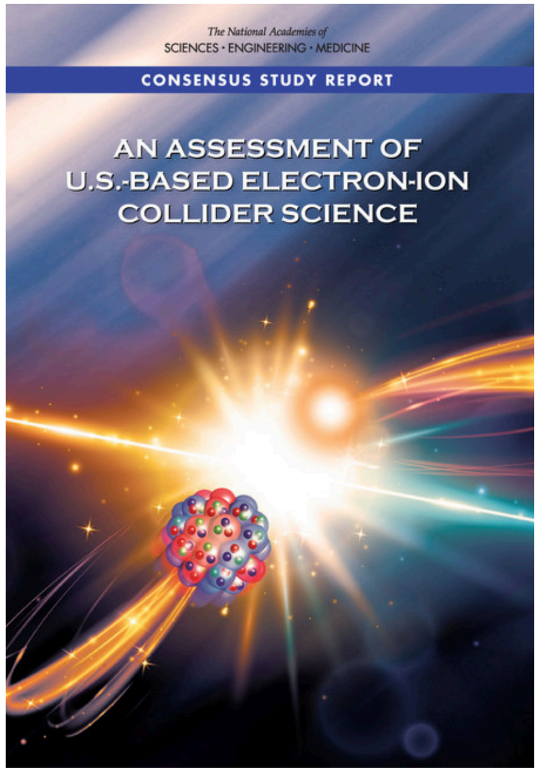
Hadron Structure investigations are timely

Lattice QCD in synergy with Experiments

Advances of Lattice QCD are timely

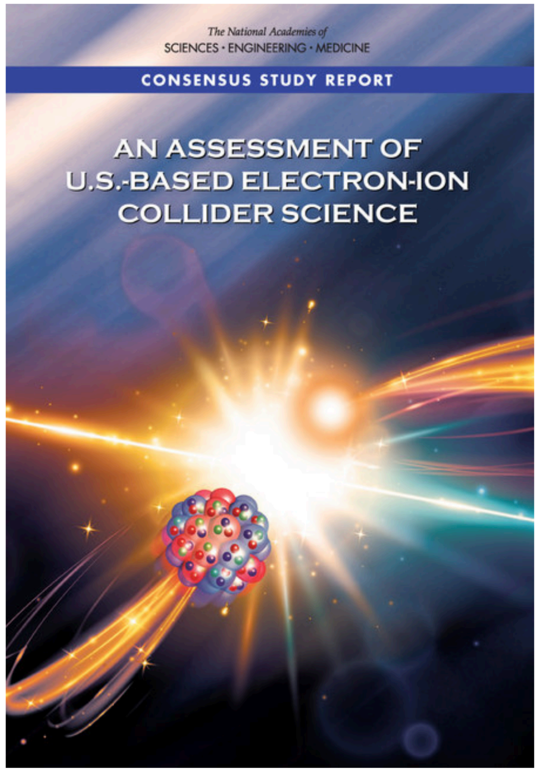


Main Pillar of NAS
Assessment report for EIC

Finding 1: An EIC can uniquely address three profound questions about nucleons—neutrons and protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?

Advances of Lattice QCD are timely



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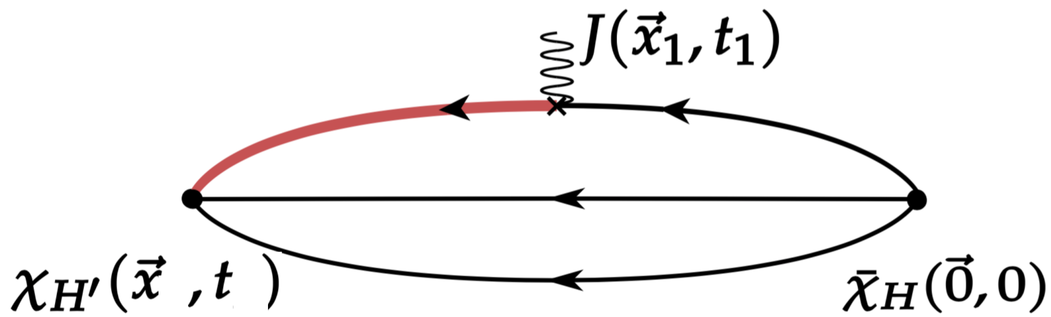
Lattice QCD is featured in the EIC Yellow Report
- 900-page document
- scientist from 151 Institutions

Lattice QCD can provide valuable input in understanding the proton mass and spin decomposition from *first principles*

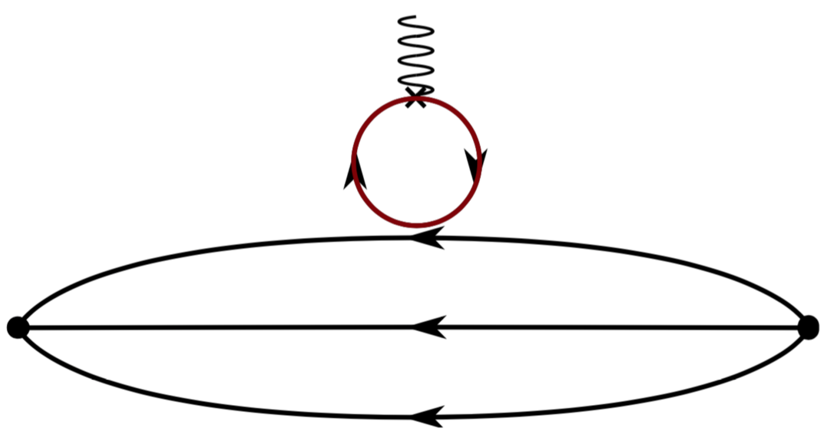
Hadron Structure From Lattice QCD

Hadrons on the Lattice

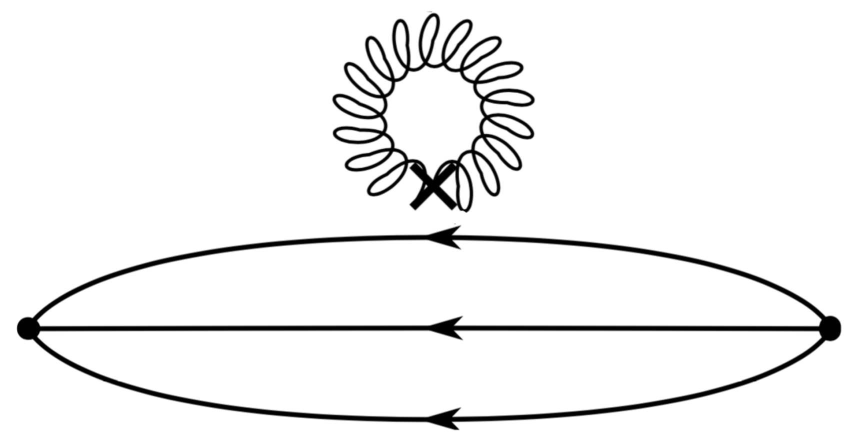
Diagrams for baryons



Connected



Disconnected Quark loop



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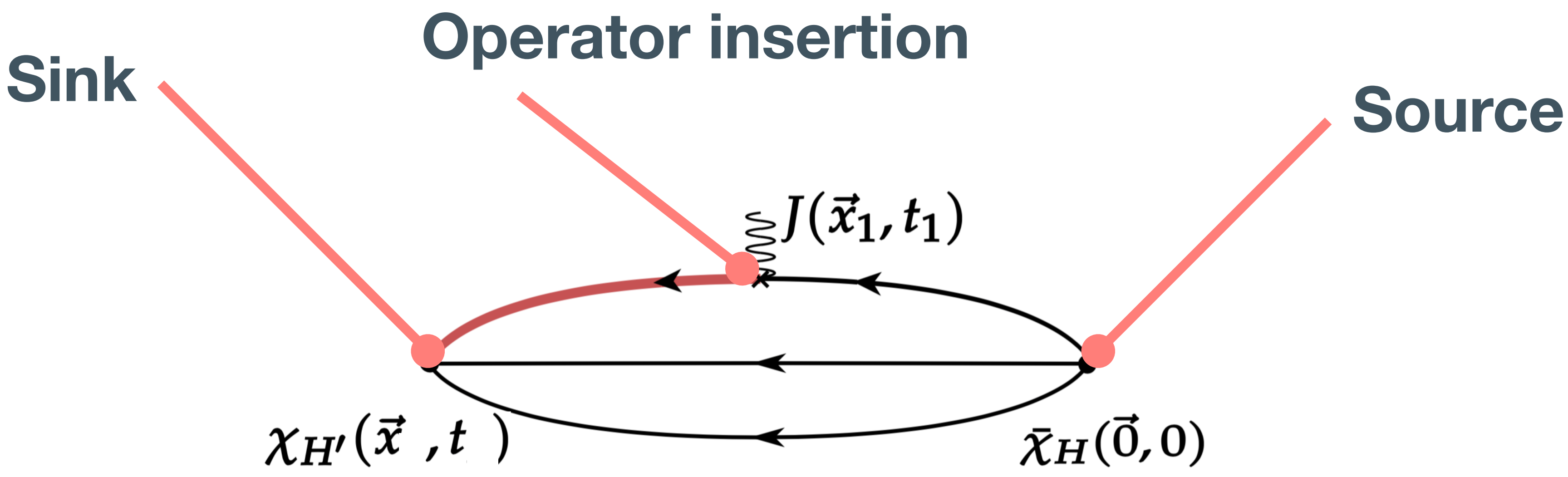

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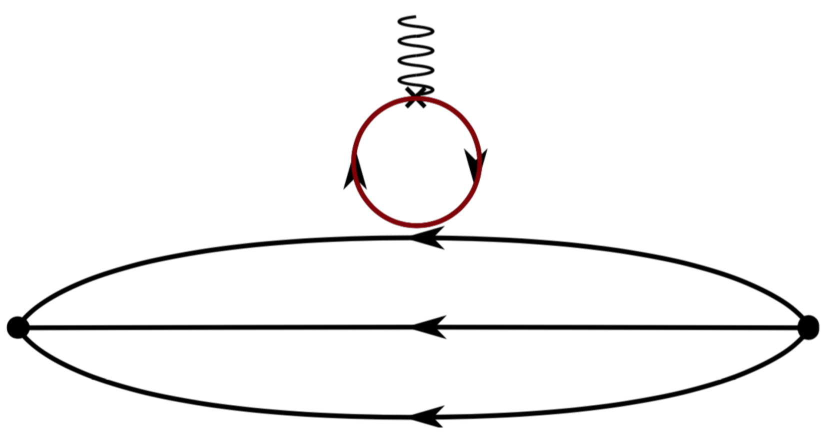

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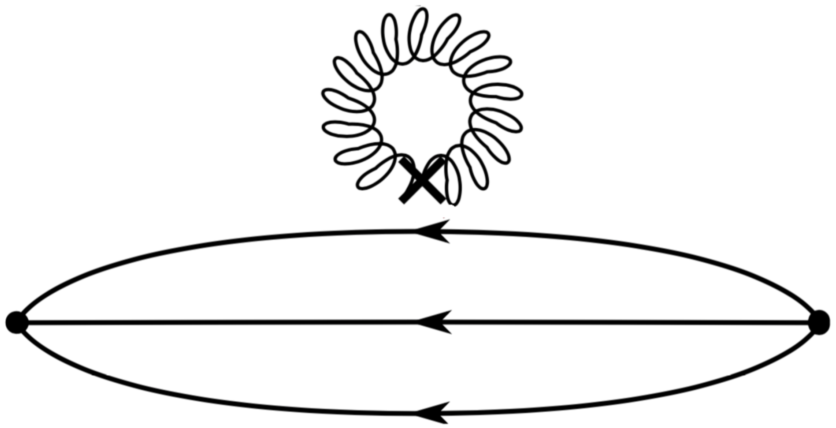

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Hadrons on the Lattice

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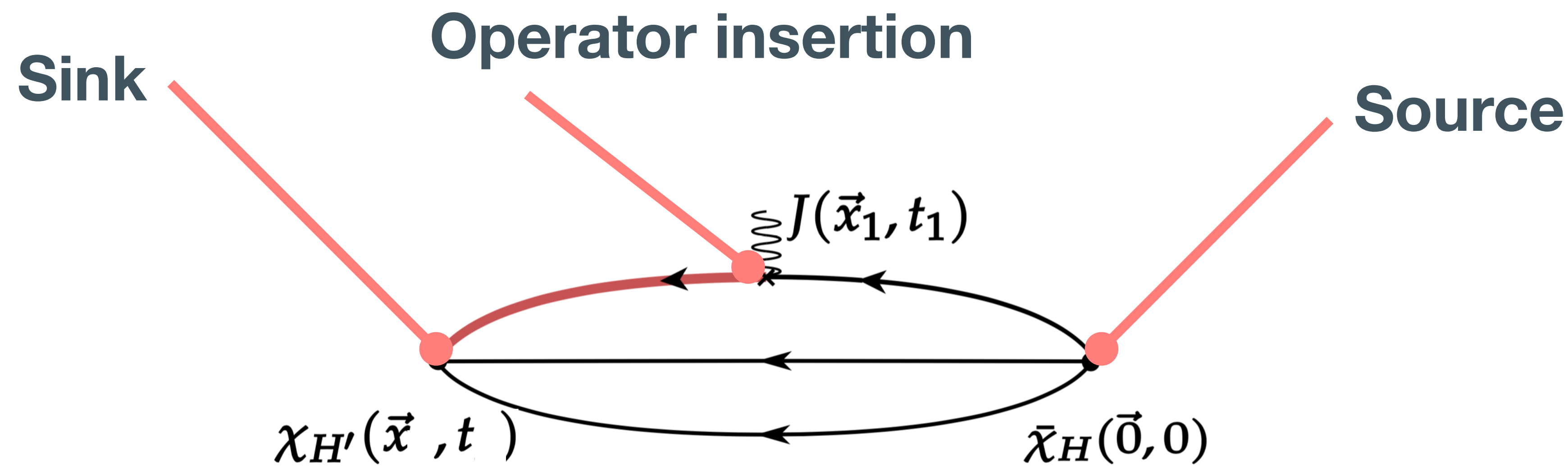

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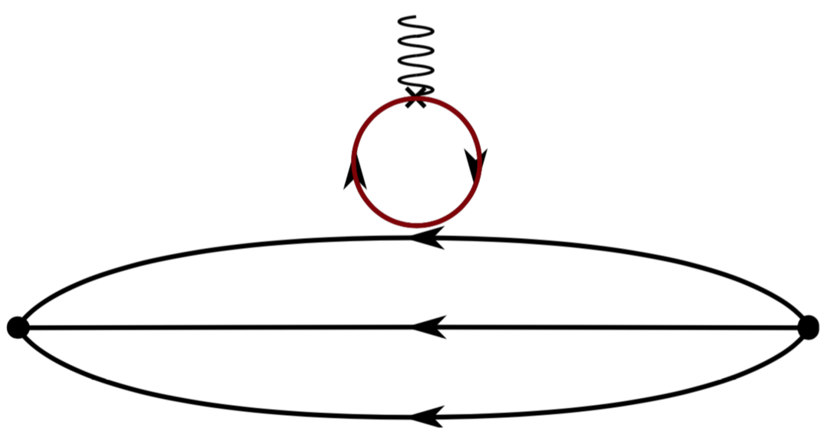

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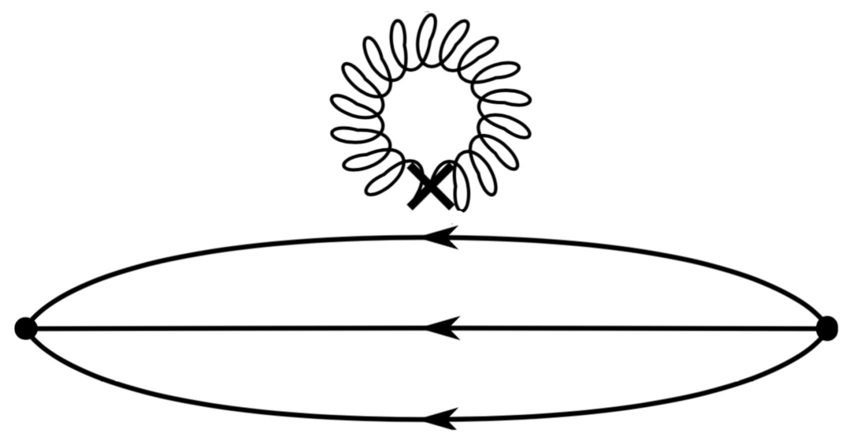

Disconnected Gluon loop

★ Separation between source and sink: excited states investigation

★ Type of current insertion gives different observable

★ Extraction of each contribution has its own challenges (statistical and systematic uncertainties)

Hadrons on the Lattice

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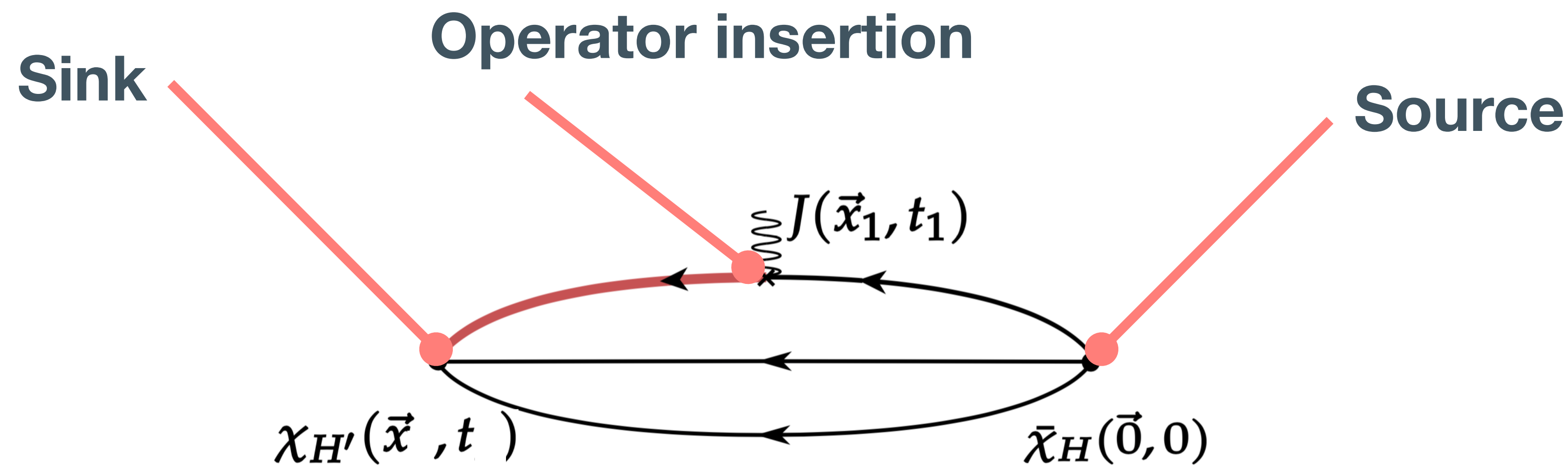

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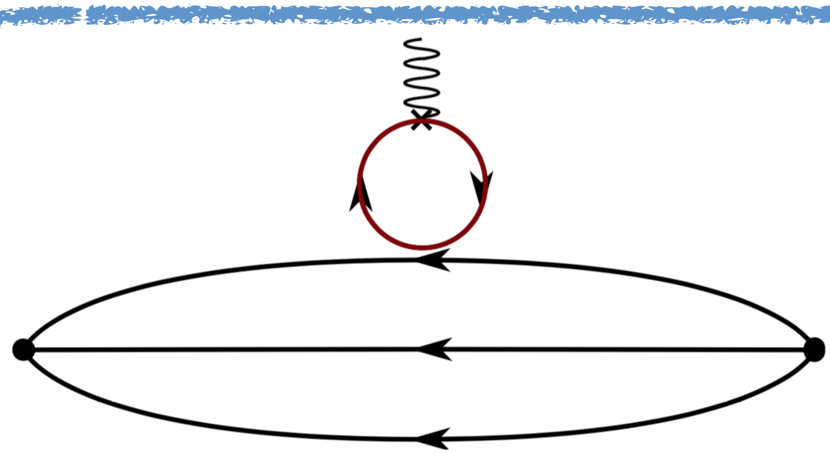

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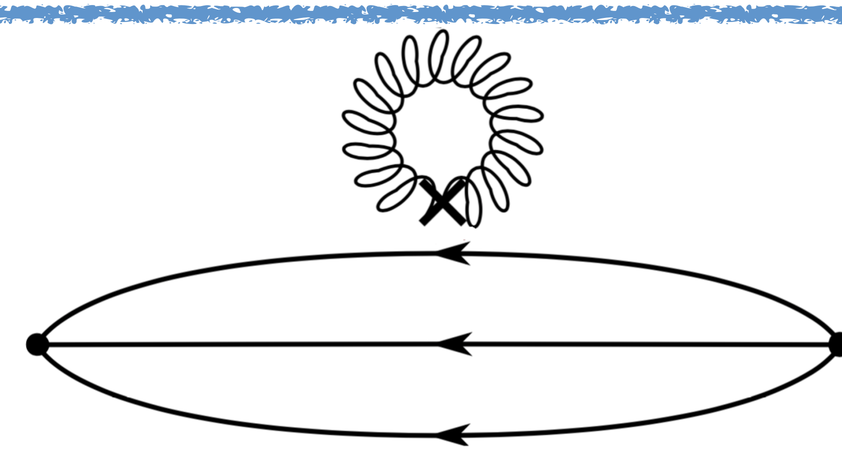

Disconnected Gluon loop

Particularly interesting for EIC physics

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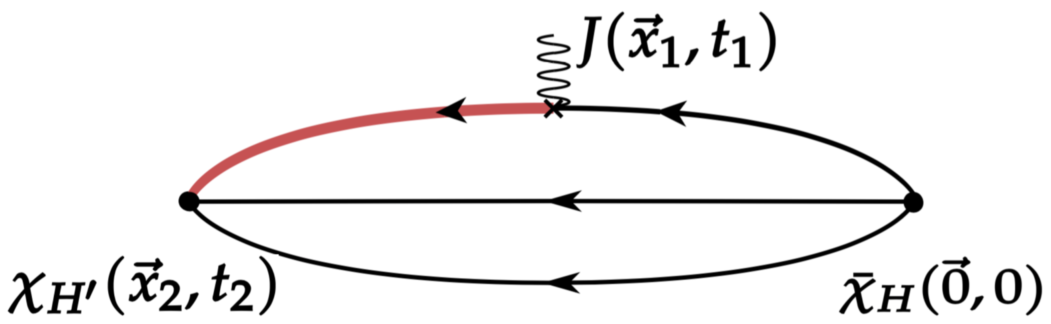
Hadrons on the Lattice

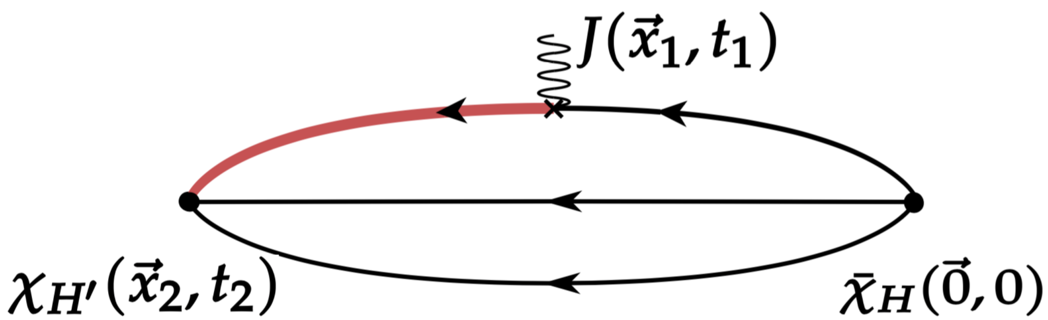


Connected

3pt correlation function

$$G^{\mathcal{H}'\mathcal{J}^\mu\mathcal{H}}(x;x_1) = \langle \Omega \, | \chi_{\mathcal{H}'}(x) \mathcal{J}^\mu(x_1) \bar{\chi}_{\mathcal{H}} | \, \Omega \rangle$$

Hadrons on the Lattice



Connected

3pt correlation function

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Insertion of two set of complete eigenstate

$$\sum_{\vec{x},\vec{x}_1} e^{-i\vec{x}\cdot\vec{p}'} G^{\mathcal{H}'\mathcal{J}^\mu\mathcal{H}}(\vec{x}, t; \vec{x}_1, t_1) e^{-i\vec{x}_1\cdot\vec{p}_1} = \sum_{\vec{x},\vec{x}_1} e^{-i\vec{x}\cdot\vec{p}'} \langle \Omega \, |\chi_{\mathcal{H}'} e^{-\hat{H}t} e^{i\vec{x}\cdot\hat{\vec{p}}'} e^{-i\vec{x}_1\cdot\hat{\vec{p}}'} e^{\hat{H}t_1} \mathcal{J}^\mu e^{-\hat{H}t_1} e^{i\vec{x}_1\cdot\hat{\vec{p}}'} \bar{\chi}_{\mathcal{H}}| \, \Omega \rangle \, e^{-i\vec{x}_1\cdot\vec{p}_1} =$$

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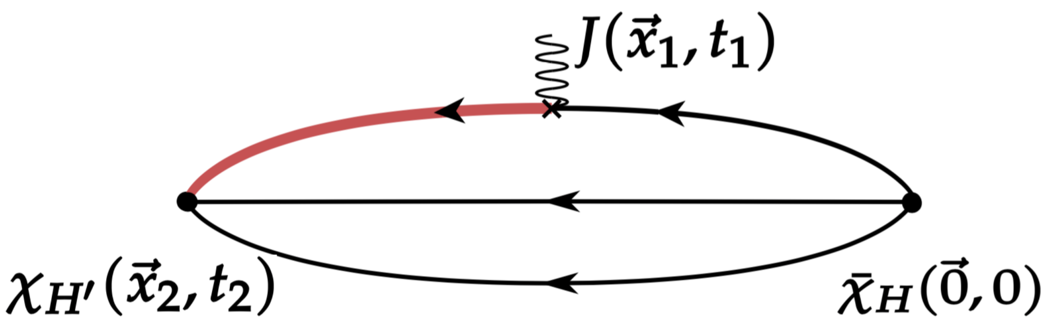

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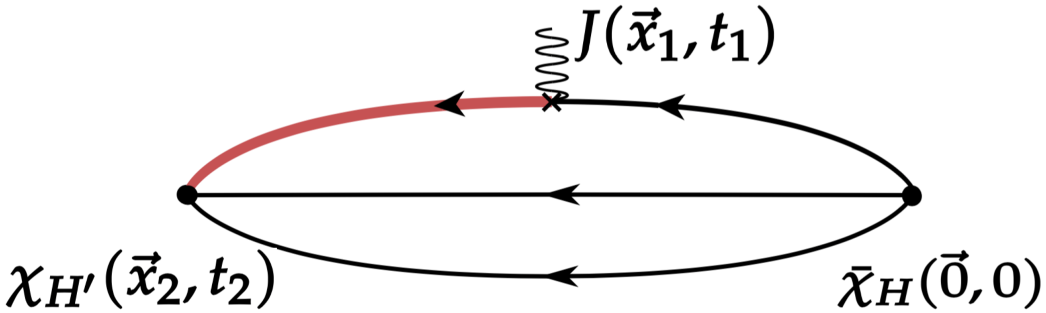

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Limit t-t_1 >>1, t_1>>1: ground state dominance

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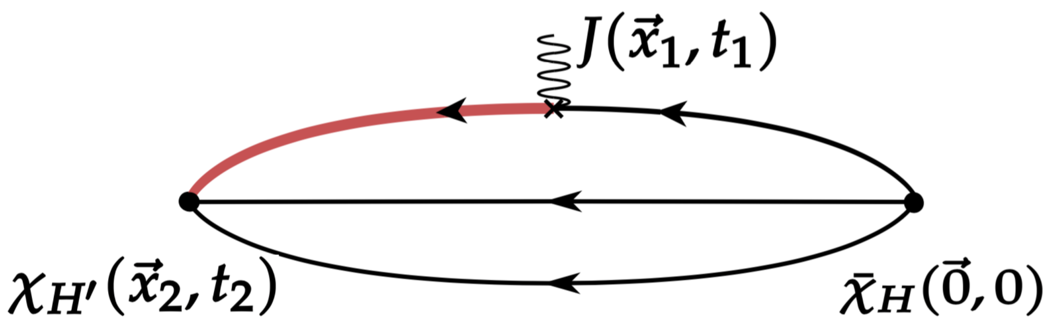
Hadrons on the Lattice



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- $H' \neq H$: transition amplitudes and transition form factors
- $H' = H$: hadron structure
  - $p_f = p_i$: no momentum transfer (charges)
  - $p_f \neq p_i$: momentum transfer (form factors)

Nucleon on the Lattice

A. Calculation of matrix elements with appropriate currents for the quantities under study (e.g., vector-axial current)

$$C^{2pt} = \langle N \,|\, N \rangle \qquad C_\Gamma^{3pt} = \langle N \,|\, \overline{\psi}(0)\Gamma\psi(0) \,|\, N \rangle$$

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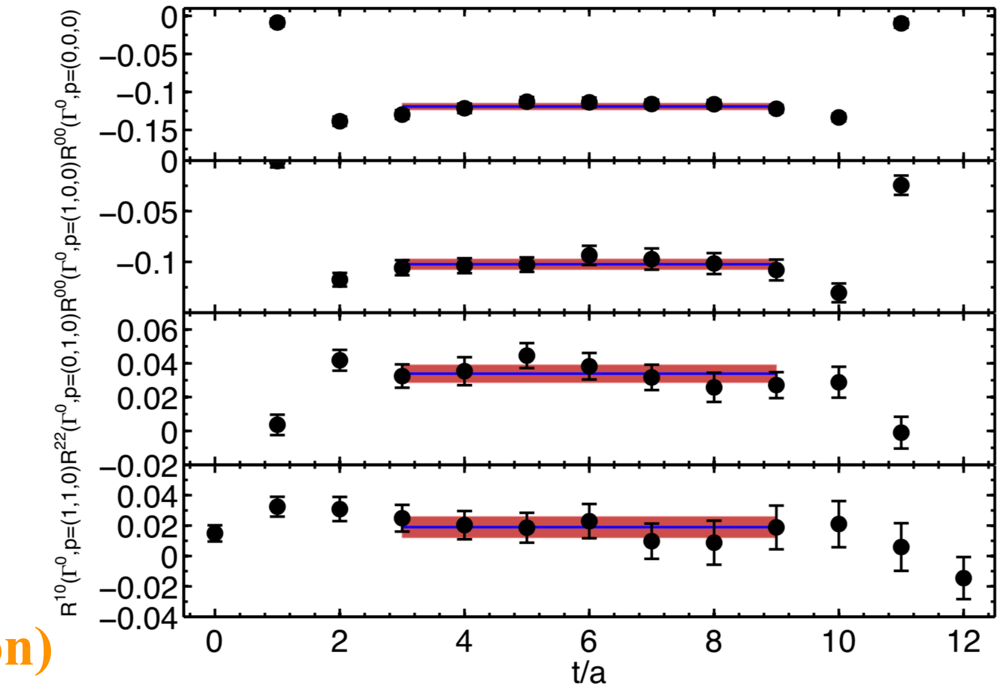
B. Construction of optimized ratios and identify ground state



$$R^{\mu}_{\mathcal{O}}(\Gamma,\vec{q},t) = \frac{G_{\mathcal{O}}(\Gamma,\vec{q},t)}{G(\vec{0},t_f)}\sqrt{\frac{G(-\vec{q},t_f-t)G(\vec{0},t)G(\vec{0},t_f)}{G(\vec{0},t_f-t)G(-\vec{q},t)G(-\vec{q},t_f)}} \overset{0<<\tau<<t}{=} \Pi_\Gamma$$

Multiply analysis techniques available (single- & multi-state, summation)

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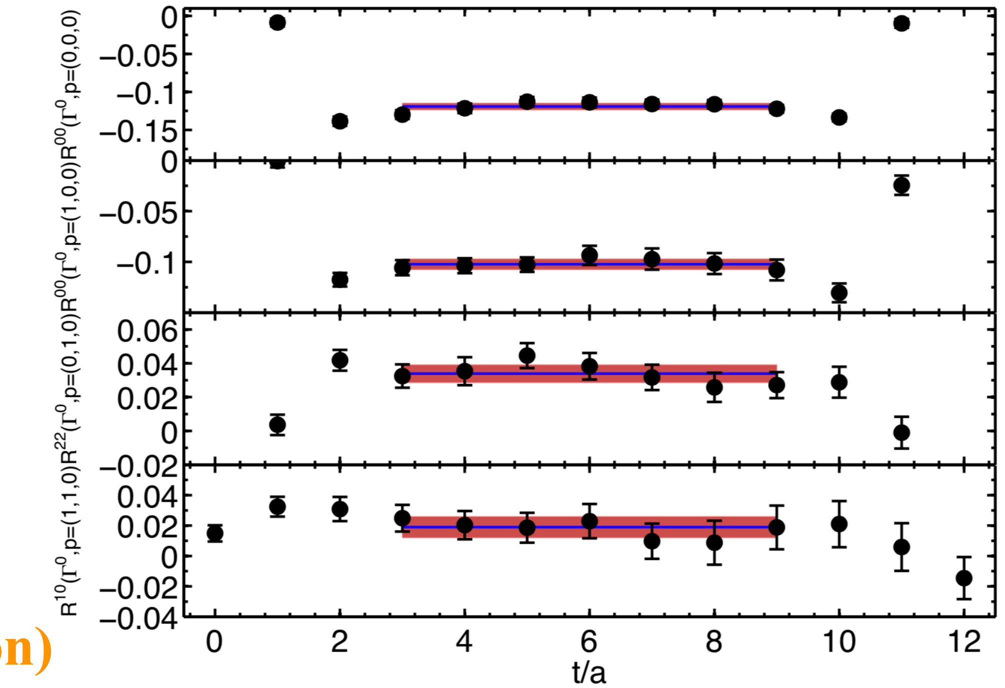

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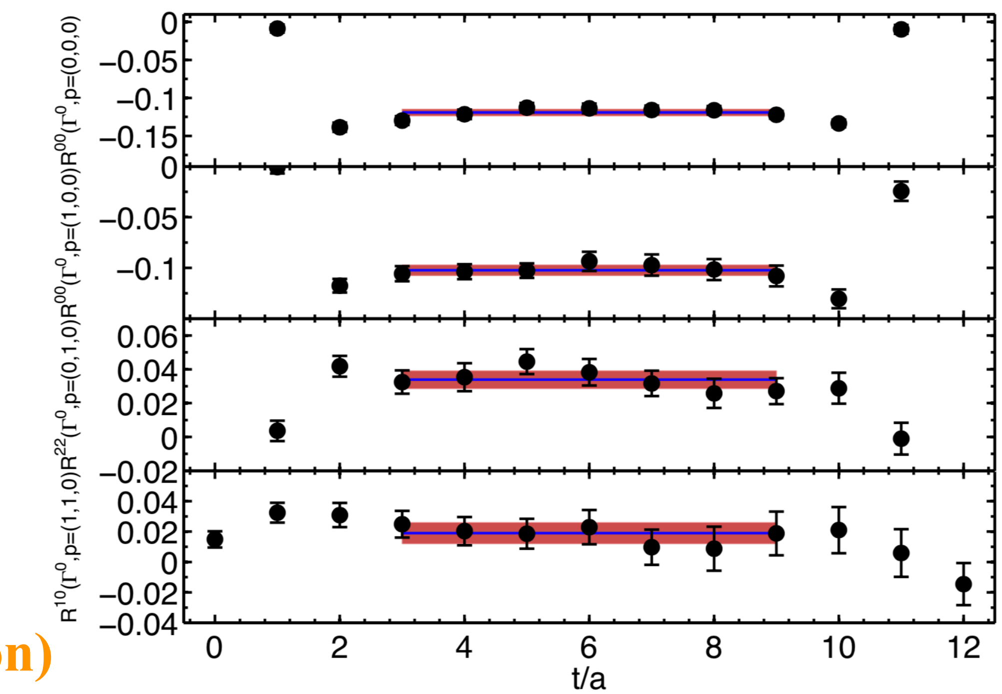

Multiply analysis techniques available (single- & multi-state, summation)

C. Renormalization (usually multiplicative)

$$\Pi^{R}_{\Gamma} = Z\Pi_{\Gamma}$$

D. Kinematic factors based on symmetry properties, e.g.

$$A^3_{\mu} \equiv \bar{\psi}\,\gamma_{\mu}\gamma_5\frac{\tau^3}{2}\psi \Rightarrow \bar{u}_N(p')\left[G_A(q^2)\,\gamma_{\mu}\gamma_5 + G_p(q^2)\frac{q_{\mu}\gamma_5}{2\,m_N}\right]u_N(p)$$

Inherited uncertainties in lattice calculations

Laborious effort to eliminate uncertainties

Statistical errors significantly increase with:

★ decrease of pion mass

★ increase of momentum transfer between initial-final state

★ increase of source-sink separation T_{sink}

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Sources of systematic uncertainties:

★ cut-off effects (finite lattice spacing)

★ finite volume effects

★ contamination from other hadron states

★ chiral extrapolation for unphysical pion mass

★ renormalization and mixing

Investigation of systematic uncertainties

On a single ensemble:

- ★ Excited states contamination
- ★ Pion mass (with simulations at physical point)
- ★ Renormalization and mixing

Using multiple ensembles:

- ★ Cut-off effects due to finite lattice spacing
- ★ Finite volume effects
- ★ Pion mass dependence

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★ Pion mass (with simulations at physical point)

★ Renormalization and mixing

Using multiple ensembles:

★ Cut-off effects due to finite lattice spacing

★ Finite volume effects

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Effects reduced in single ensemble with appropriate parameters

Summary of Lecture 2

Key points of Lecture 2

★ Renormalization is an indispensable part of lattice calculations

★ Well-defined perturbative and non-perturbative renormalization procedures

★ Calculation of nucleon and pion mass has been an important starting point for lattice QCD

★ Hadron Spectroscopy has advanced tremendously and can provide predictions and input for experiments

★ Hadron structure studies are critical for understanding the immensely rich and complex properties of the visible matter

Thank you