Nuclear Matter and Nuclear Structure III

Thomas Papenbrock, The University of Tennessee, tpapenbr@utk.edu

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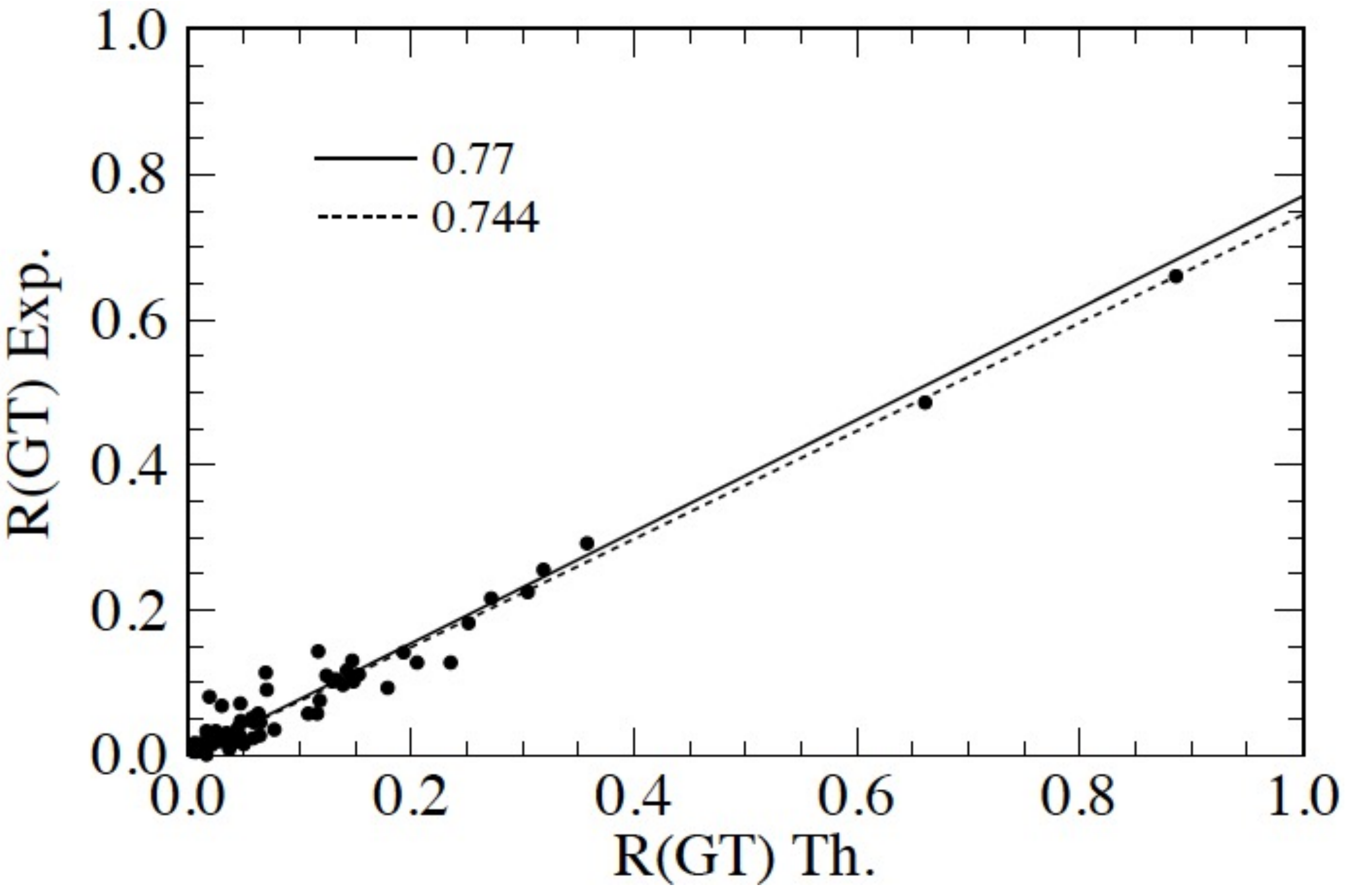
Two-body currents, continued …

Two-body currents solve 50-year-old puzzle of quenched β-decays

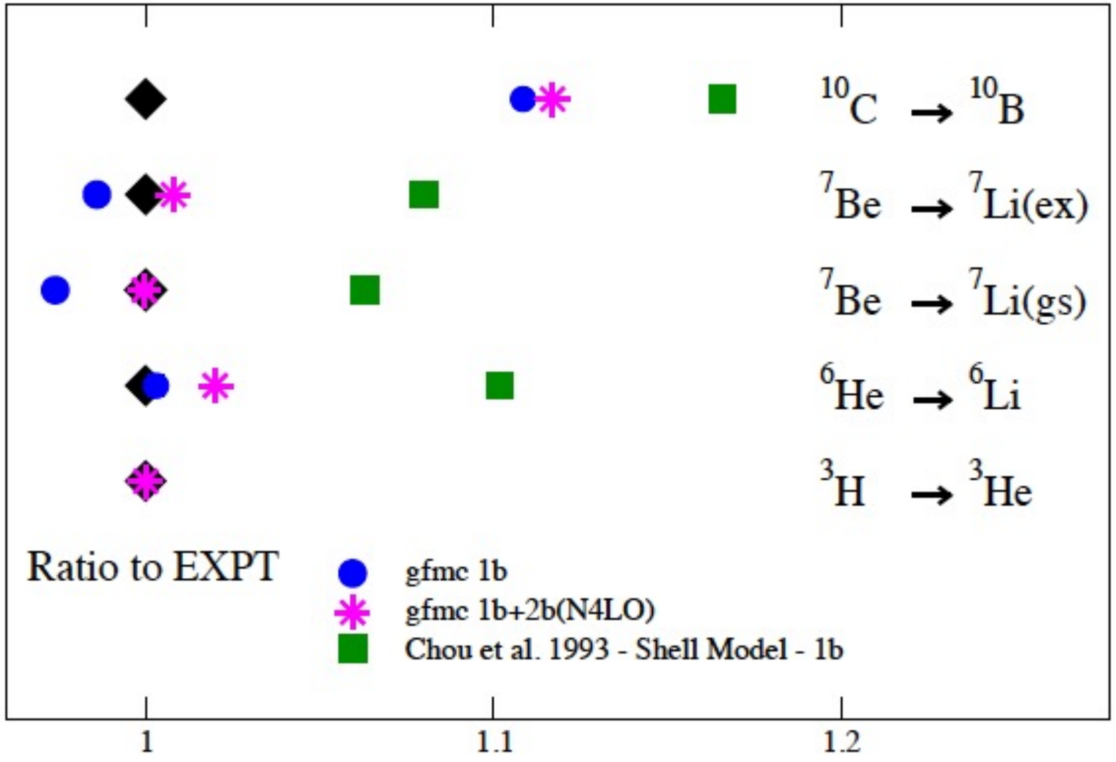
Puzzle: The strengths of Gamow-Teller transitions (operator $\propto g_A \vec{\sigma} \tau^{\pm}$) in nuclei are smaller ("quenched") than what is expected from the β-decay of the free neutron.

- Wilkinson (1973): quenching factor $q^2 \approx 0.90$ for nuclei with $A = 17 \dots 21$
- Brown & Wildenthal (1985): quenching factor $q^2 \approx 0.77$ for nuclei with $A = 17 \dots 40$
- Martinez-Pinedo et al. (1996): quenching factor $q^2 \approx 0.74$ for nuclei with $A = 40 \dots 60$

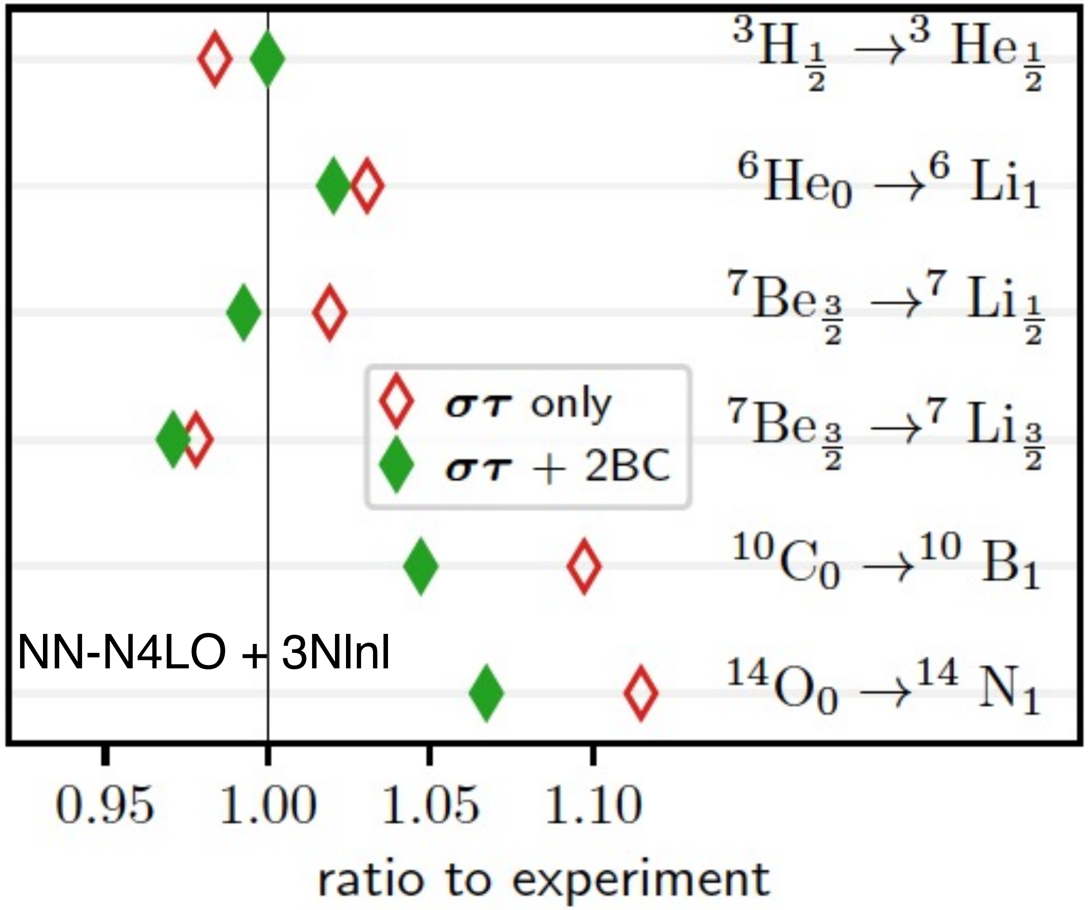


Martinez-Pinedo, Poves, Caurier, and Zuker, Phys. Rev. C **53**, R2602 (1996)

β decays in light nuclei, including two-body currents



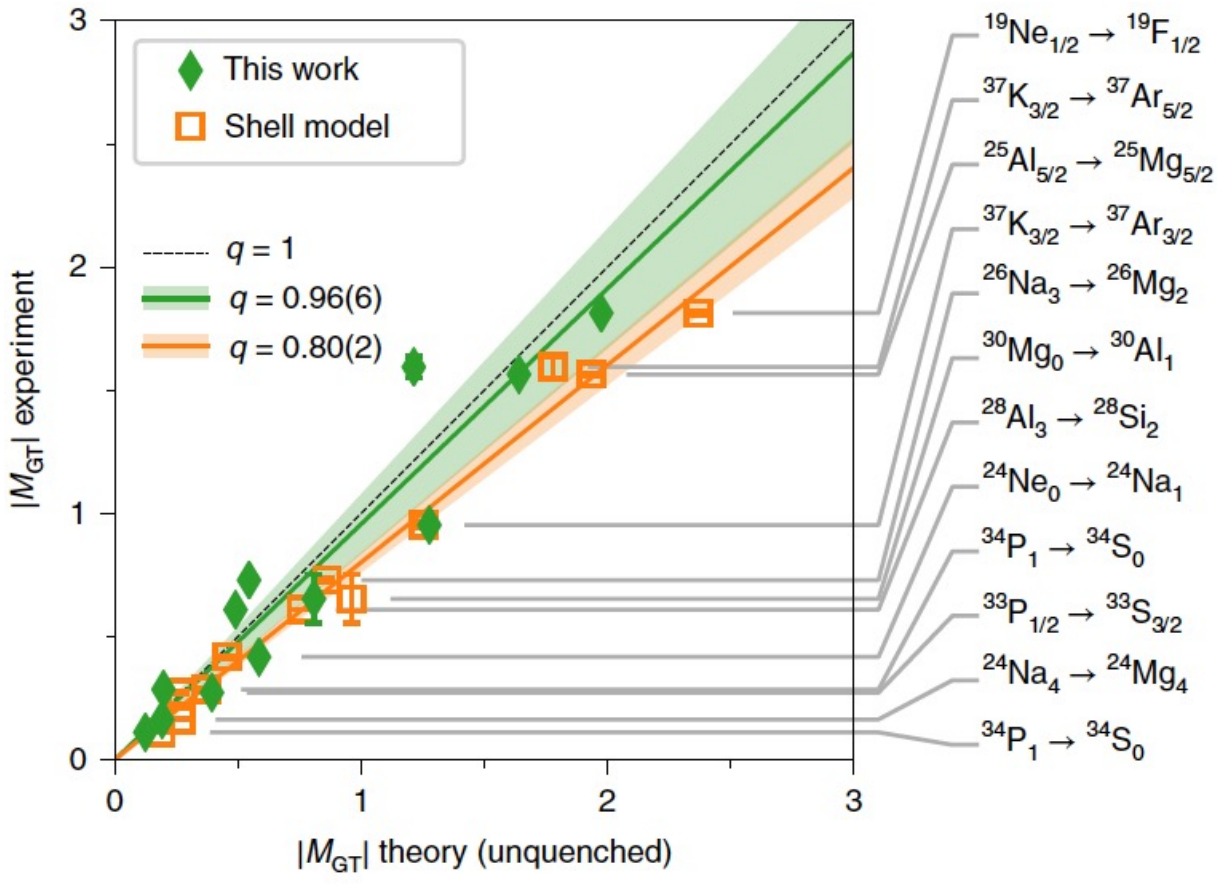
Pastore, Baroni, Carlson, Gandolfi, Pieper, Schiavilla & Wiringa, Phys. Rev. C (2018)

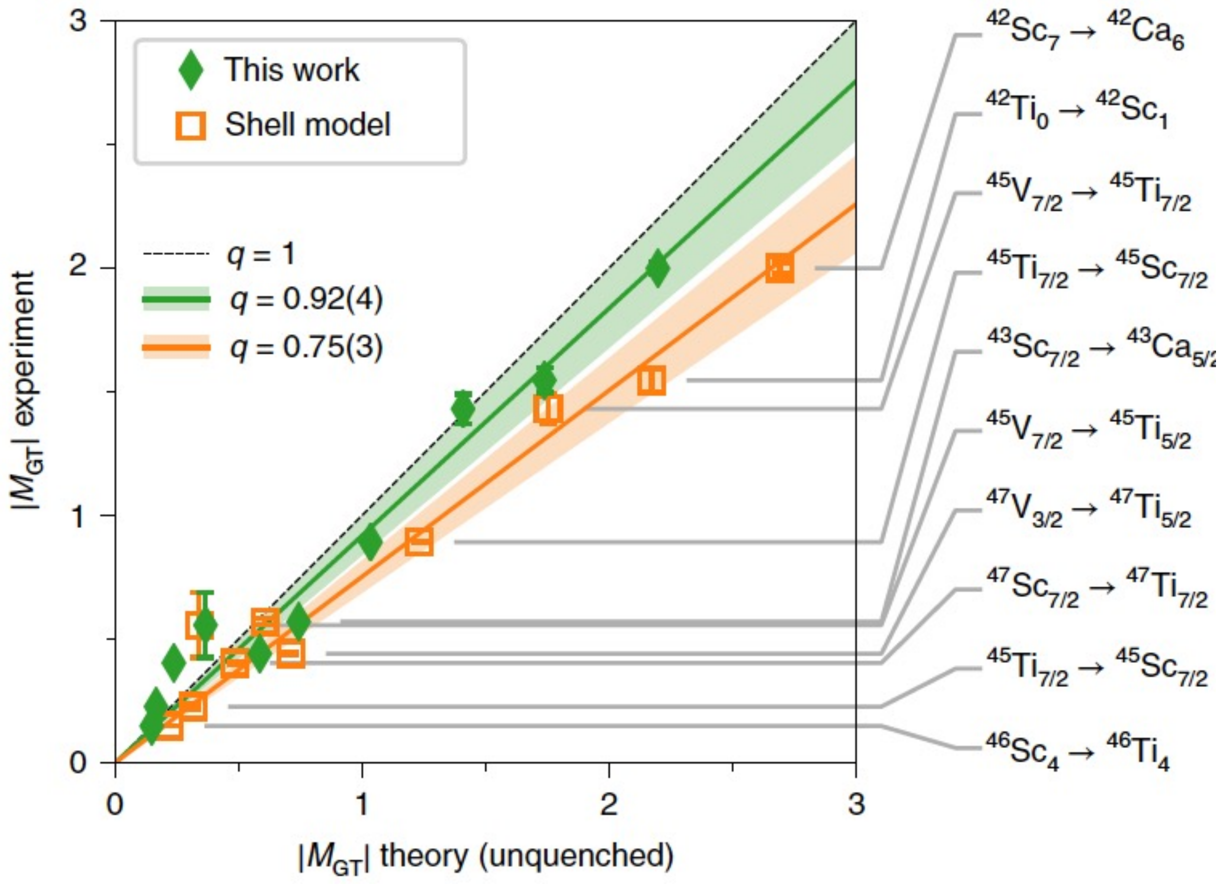


Gysbers, Hagen, Holt, Jansen, Morris, Navratil, TP, Quaglioni, Schwenk, Stroberg & Wendt, Nature Physics (2019)

In light nuclei, two-body currents of β decay play a smaller role;
some tension between quantum Monte Carlo and no-core shell model computations, though.

β decays in medium-mass nuclei, including two-body currents

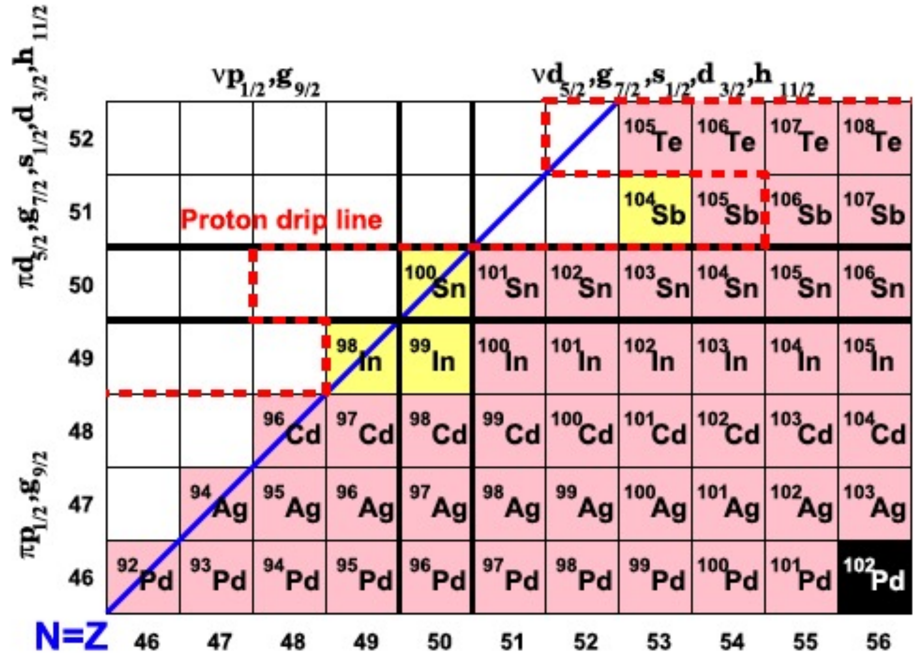




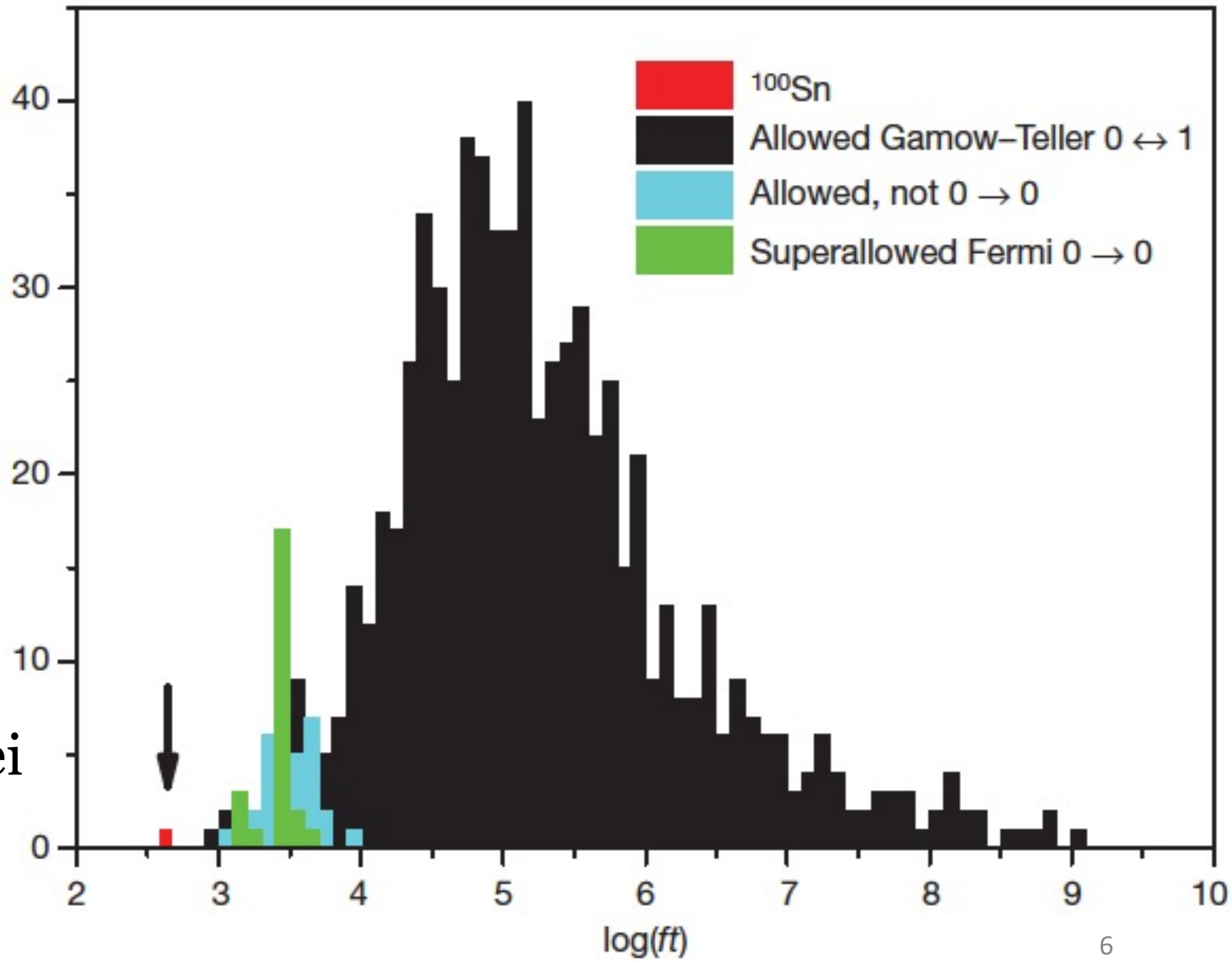
IMSRG computations with NN-N^{4}LO + 3NlnI interaction

Gysbers, Hagen, Holt, Jansen, Morris, Navratil, TP, Quaglioni, Schwenk, Stroberg & Wendt, Nature Physics (2019); arXiv:1212.3375

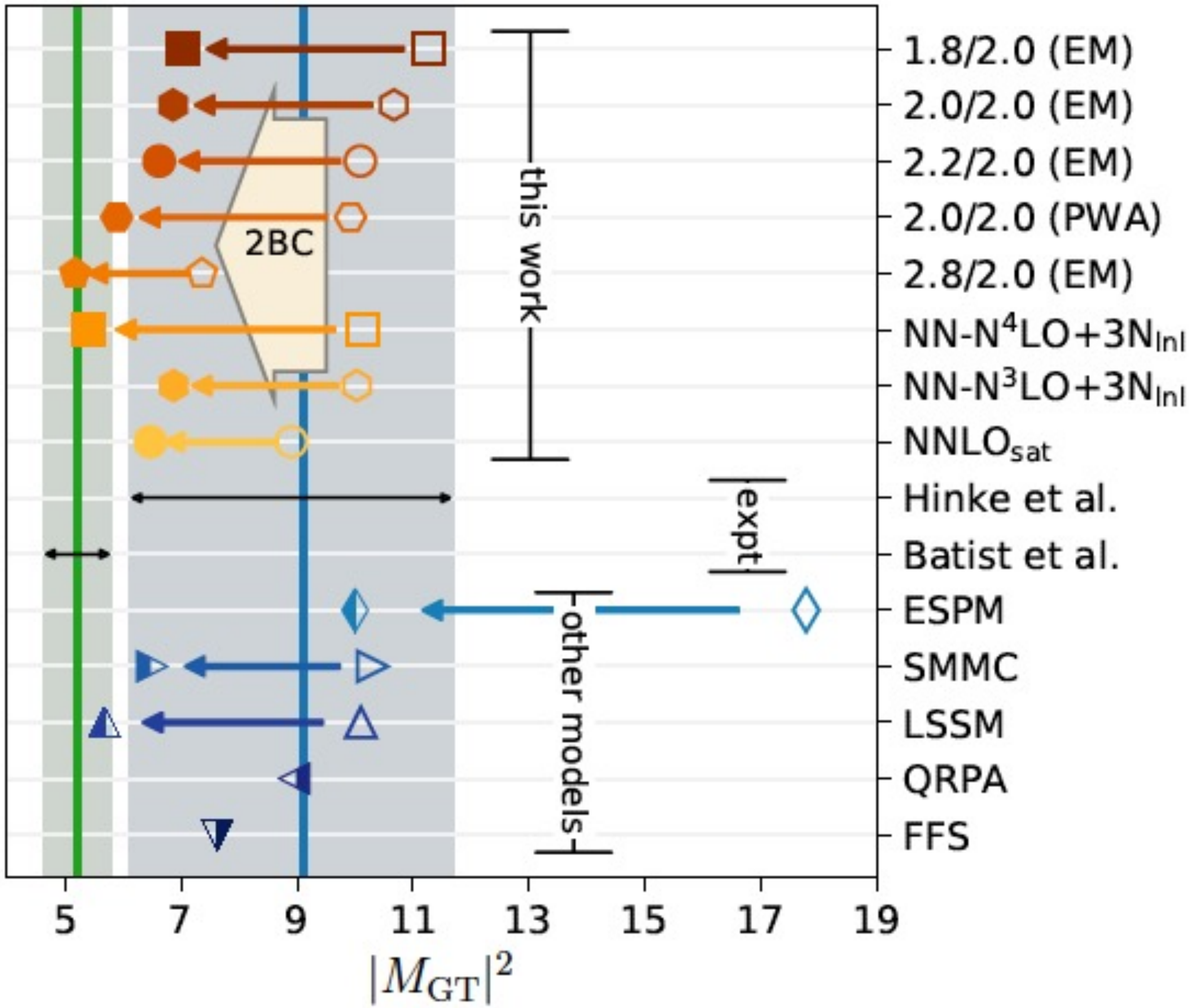
β decay of ^{100}Sn



^{100}Sn has strongest Gamow-Teller matrix-element strength of all nuclei [Hinke et al., Nature (2012)]



β decay of ^{100}Sn, including two-body currents



Coupled-cluster computations based on various potentials from chiral EFT

Open symbols: no two-body currents

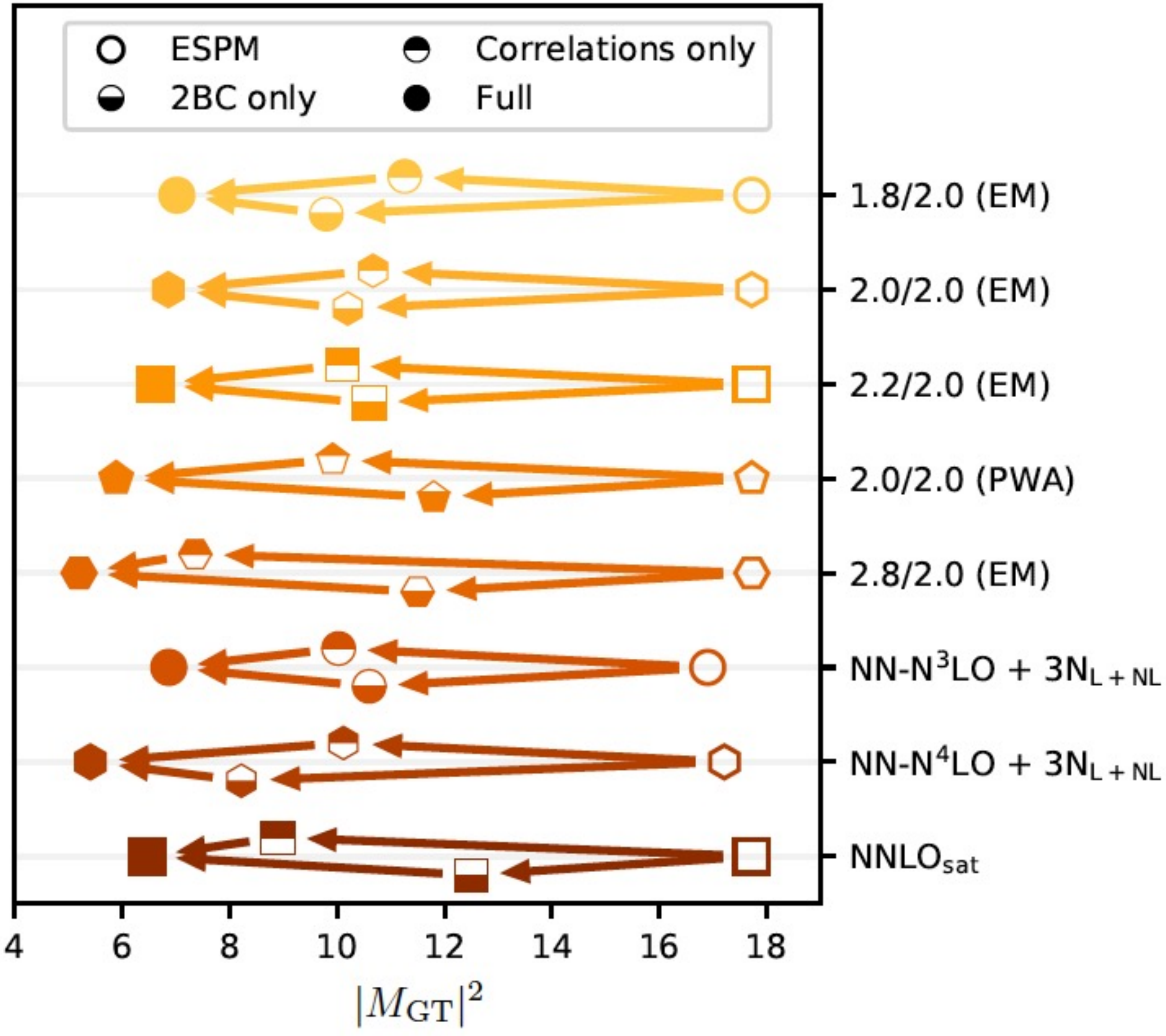
Full symbols: with two-body currents

Two-body currents reduce the systematic uncertainty from the set of chiral interactions.

Traditional models need quenching factors to describe data.
(open symbols: no quenching).

Gysbers, Hagen, Holt, Jansen, Morris, Navratil, TP, Quaglioni, Schwenk, Stroberg & Wendt, Nature Physics (2019); arXiv:1212.3375

Resolution-scale dependence of correlations and two-body currents in ^{100}Sn



- Starting from the extreme single-particle model (ESPM), the contributions from correlations and two-body currents depend on the order in which they are included
- The contributions of two-body currents also depend on the resolution (harder interactions yield stronger correlations and smaller two-body currents)

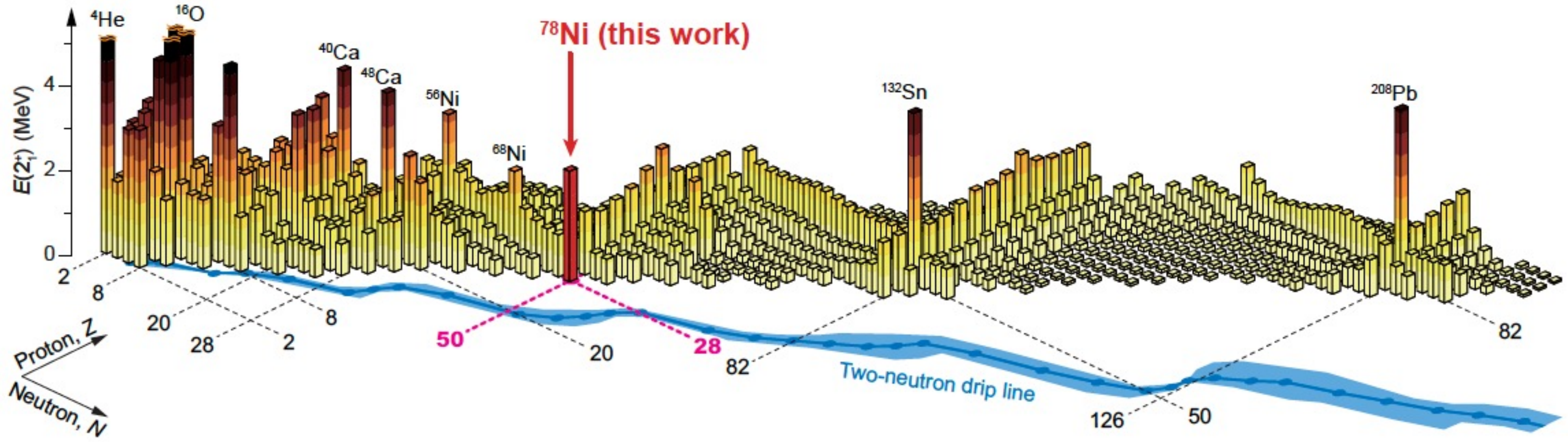
Gysbers, Hagen, Holt, Jansen, Morris, Navratil, TP, Quaglioni, Schwenk, Stroberg & Wendt, Nature Physics (2019); arXiv:1212.3375

Summary two-body currents

- Two-body currents (2BCs) naturally arise in theories with three-body forces
  - In chiral EFTs, these are subleading corrections
- 2BCs deliver visible contributions to nuclear magnetic moments
- 2BCs provide us with a solution to the long-standing puzzle of quenched β decays

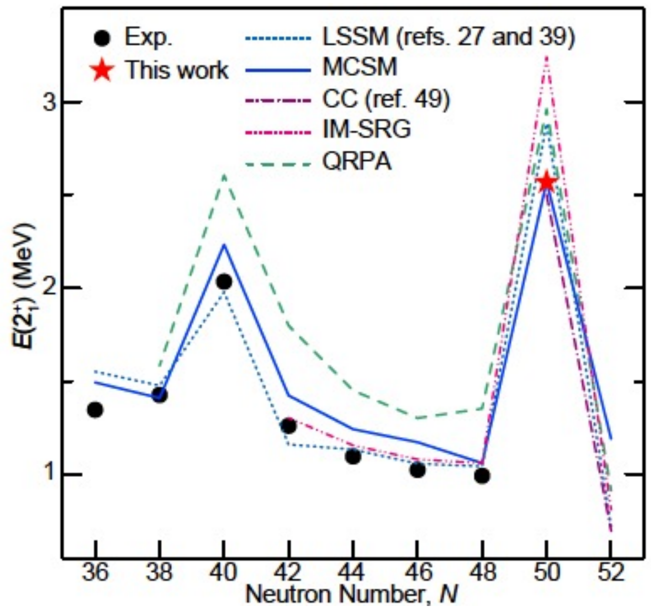
A few more success stories of computations of nuclei using EFT Hamiltonians

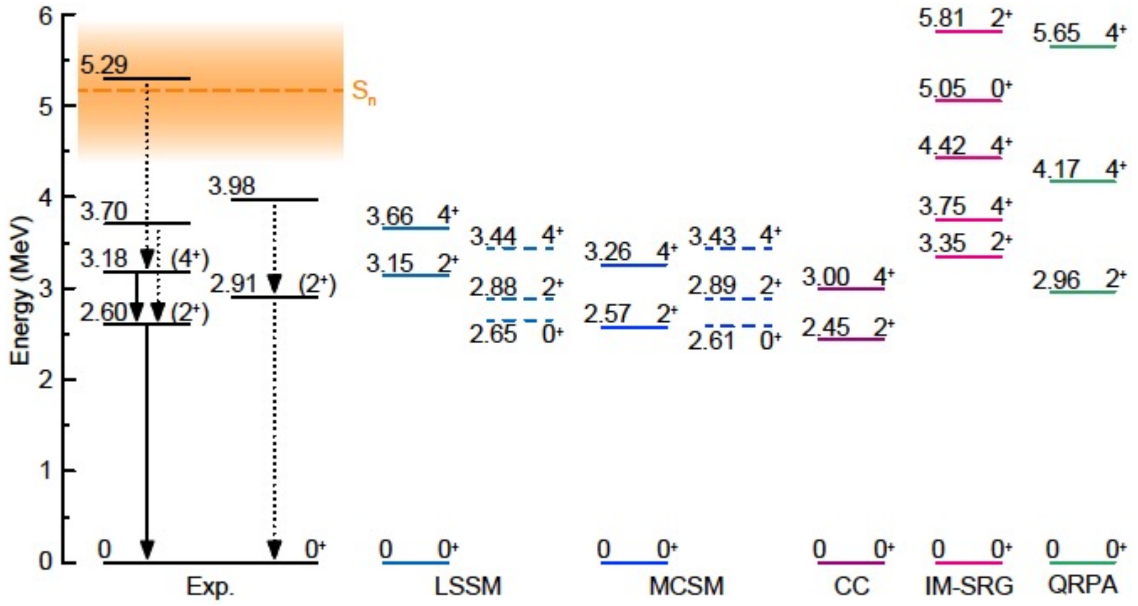
^{78}Ni (Z=28, N=50) is a neutron-rich doubly magic nucleus



Doubly magic nuclei are more strongly bound, and more difficult to excite, than their neighbors

They are the cornerstones for understanding entire regions of the nuclear chart





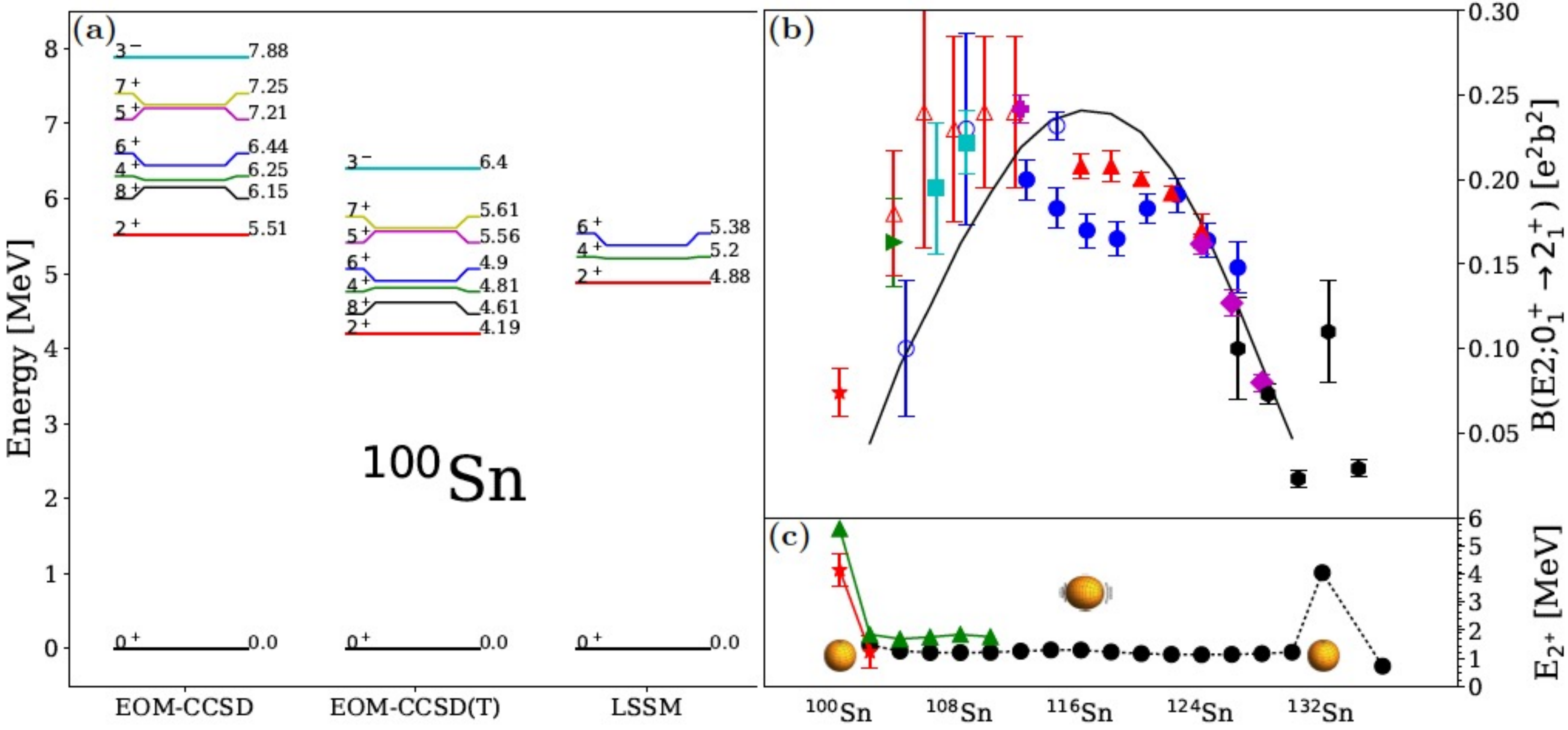
Predictions from 2016

LSSM: shell model

CC: EFT Hamiltonian, adjusted to 2,3,4 nucleons only

R. Taniuchi, C. Santamaria, P. Doornenbal, A. Obertelli, K. Yoneda et al., Nature 569, 53-58 (2019); arXiv:1912.05978

Theory predicts that ^{100}Sn (N=Z=50) is a doubly magic nucleus



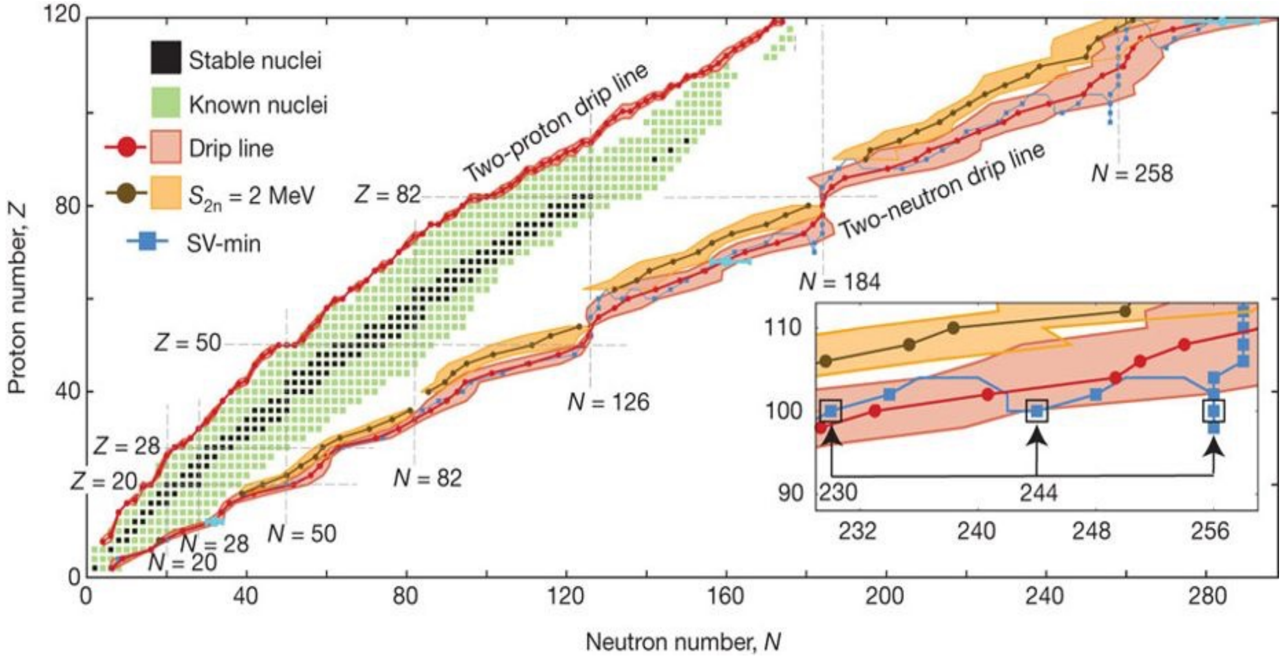
Coupled cluster based on interaction 1.8/2.0(EM); LSSM: Large Scale Shell Model [Faestermann, Gorska & Grawe (2013)]

Doubly magic nuclei are hard to excite (gap in the spectrum) and exhibit small electric quadrupole strength B(E2)

Morris, Simonis, Stroberg, Stumpf, Hagen, Holt, Jansen, TP, Roth & Schwenk, Phys. Rev. Lett. (2018)

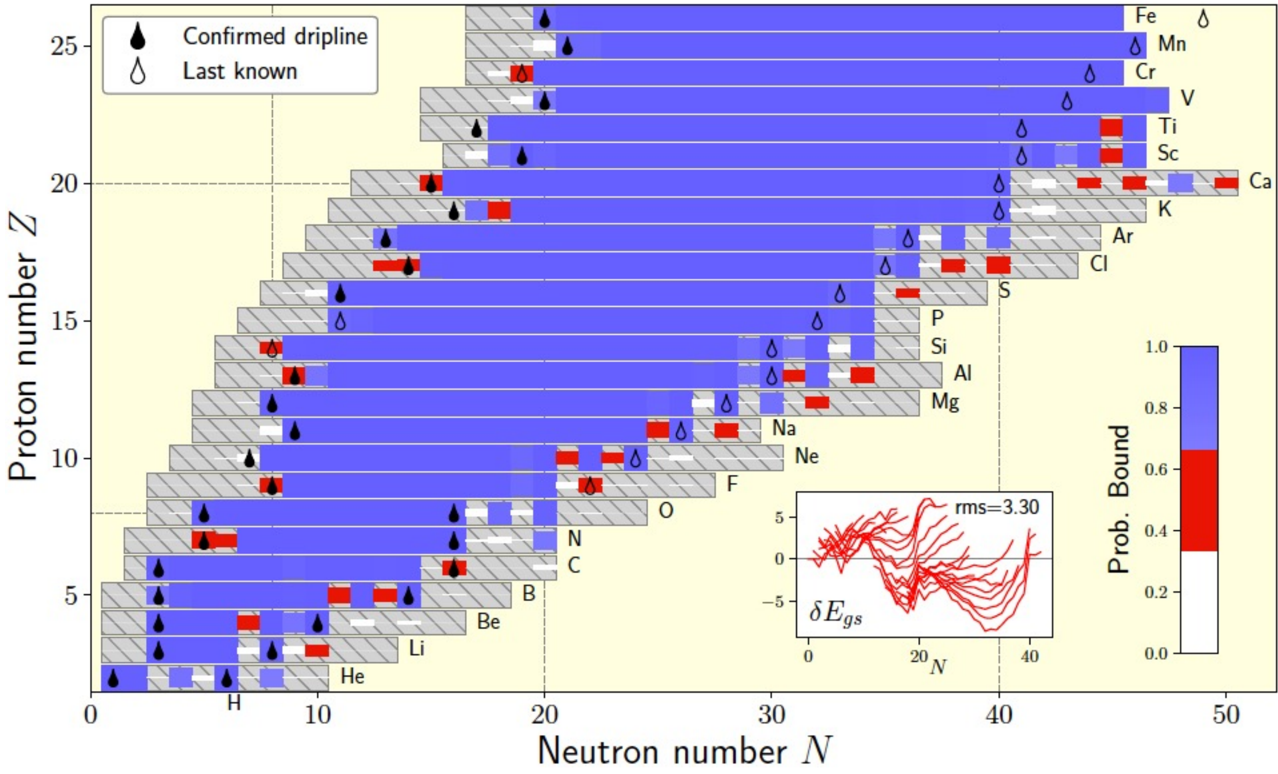
Limits of the nuclear landscape …
… coming within the limits of Hamiltonian-based methods

Nuclear DFT: Erler et al, Nature (2012)



$6{,}900 \pm 500_{syst}$ nuclei with $Z \leq 120$

EFT Hamiltonian: Holt, Stroberg, Schwenk & Simonis (2019)



Renaissance and development of methods that scale polynomially with mass number

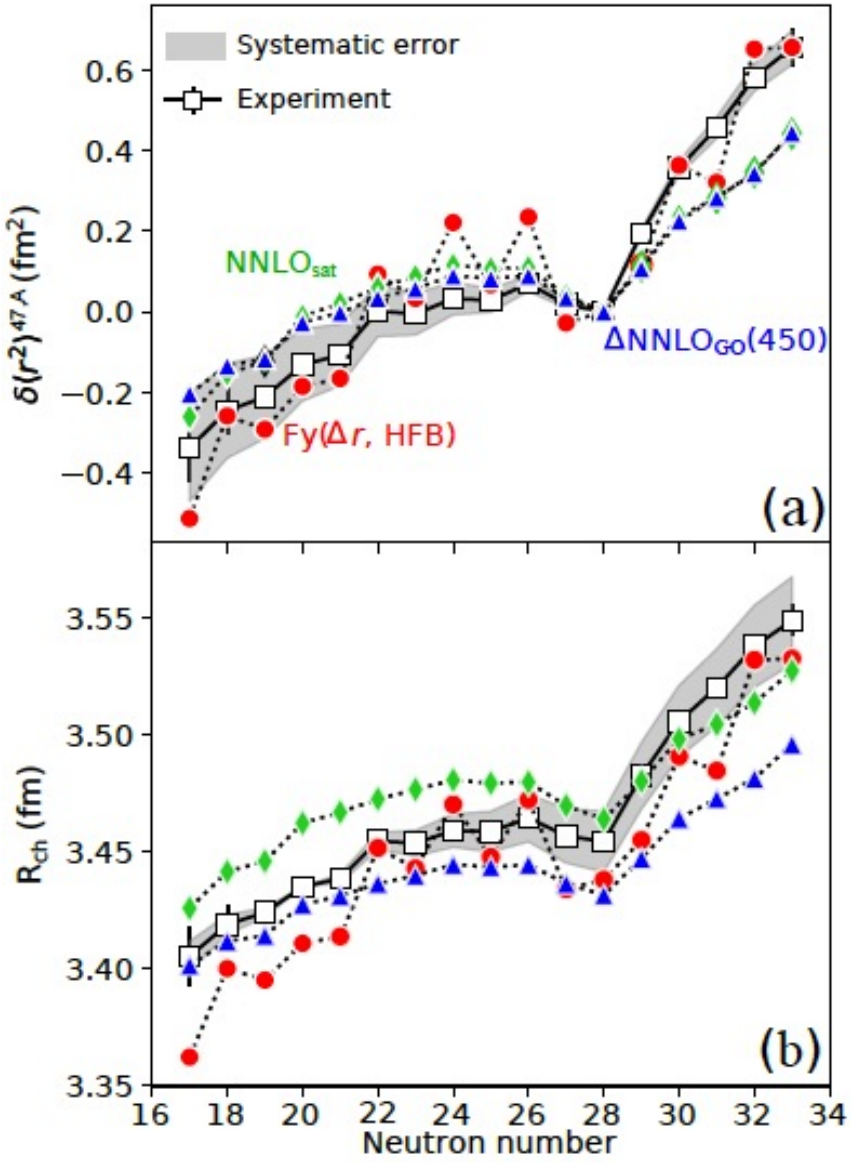
[Dickhoff & Barbieri; Dean & Hjorth-Jensen; Hagen, Jansen & TP; Tsukiyama, Bogner, Hergert & Schwenk; Elhatisari, Epelbaum, Lee, Lähde, Lu, Meissner; Soma & Duguet; Holt & Stroberg…]

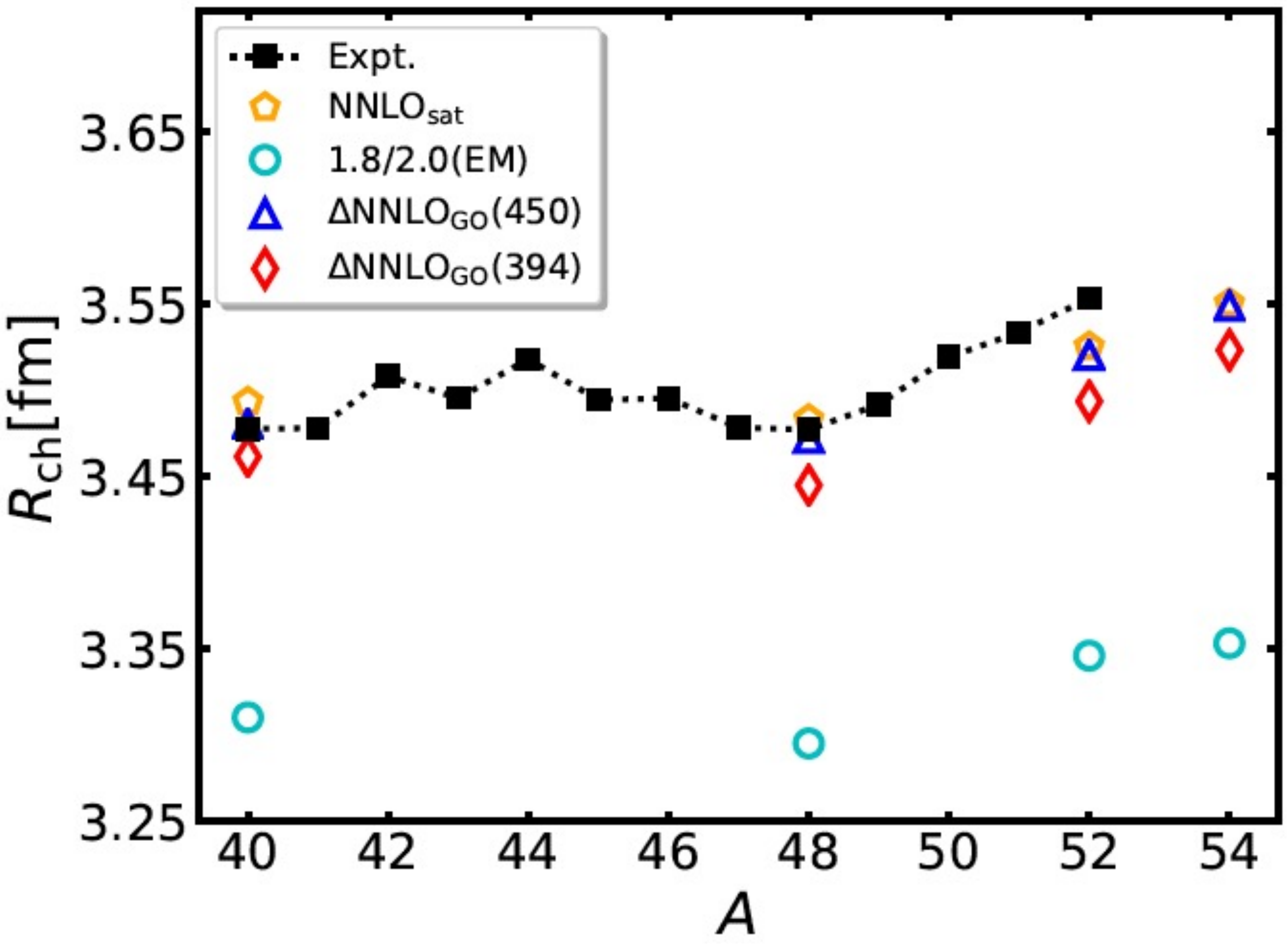
→ Review: H. Hergert, Front. Phys. 8, 379 (2020); arXiv:2008.05061

Challenges and open problems

(You might contribute to solving these ☺)

Challenges: Charge radii challenge nuclear theory





W.G. Jiang et al, arXiv:2006.16774

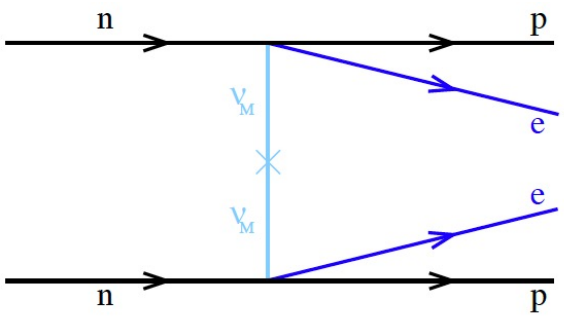
Sharp increase beyond N=28 not reproduced by EFT Hamiltonians

A. Koszorus, X. F. Yang et al, Nature Physics 17, 439 (2021); arXiv:2012.01864

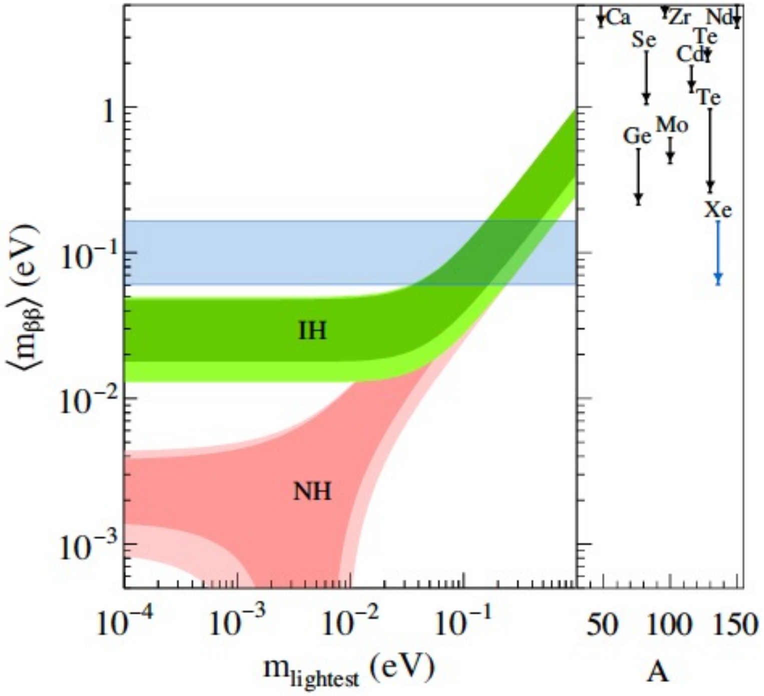
Challenges: Nuclear matrix element for neutrinoless $\beta\beta$ decay

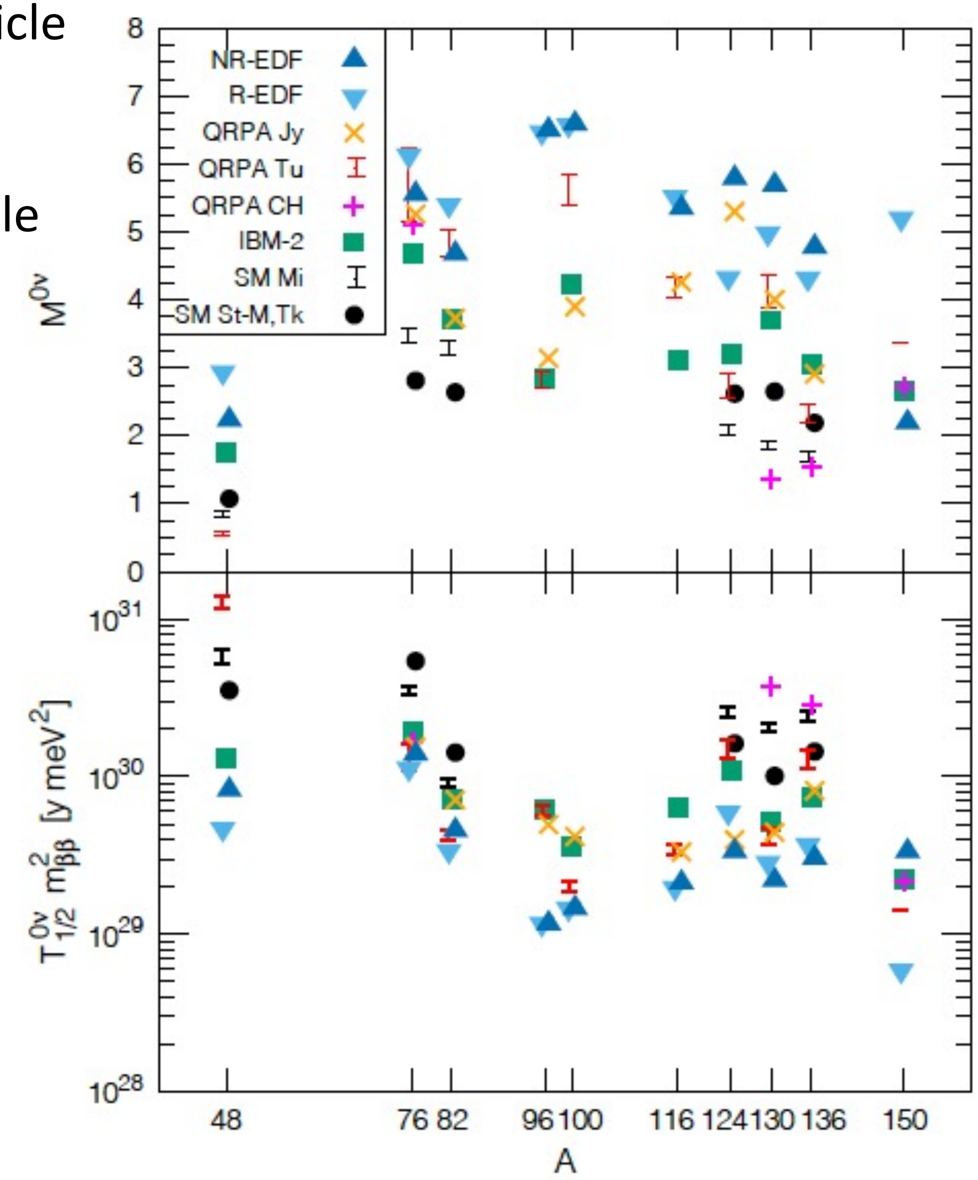
Hypothesis: The neutrino is a Majorana fermion, i.e. its own antiparticle
→ Search for neutrinoless $\beta\beta$ decay
Interest: Next-generation experiments will probe inverted hierarchy
Need: Nuclear matrix element to relate lifetime to neutrino mass scale

Light Majorana-neutrino exchange in $\beta\beta$ decay



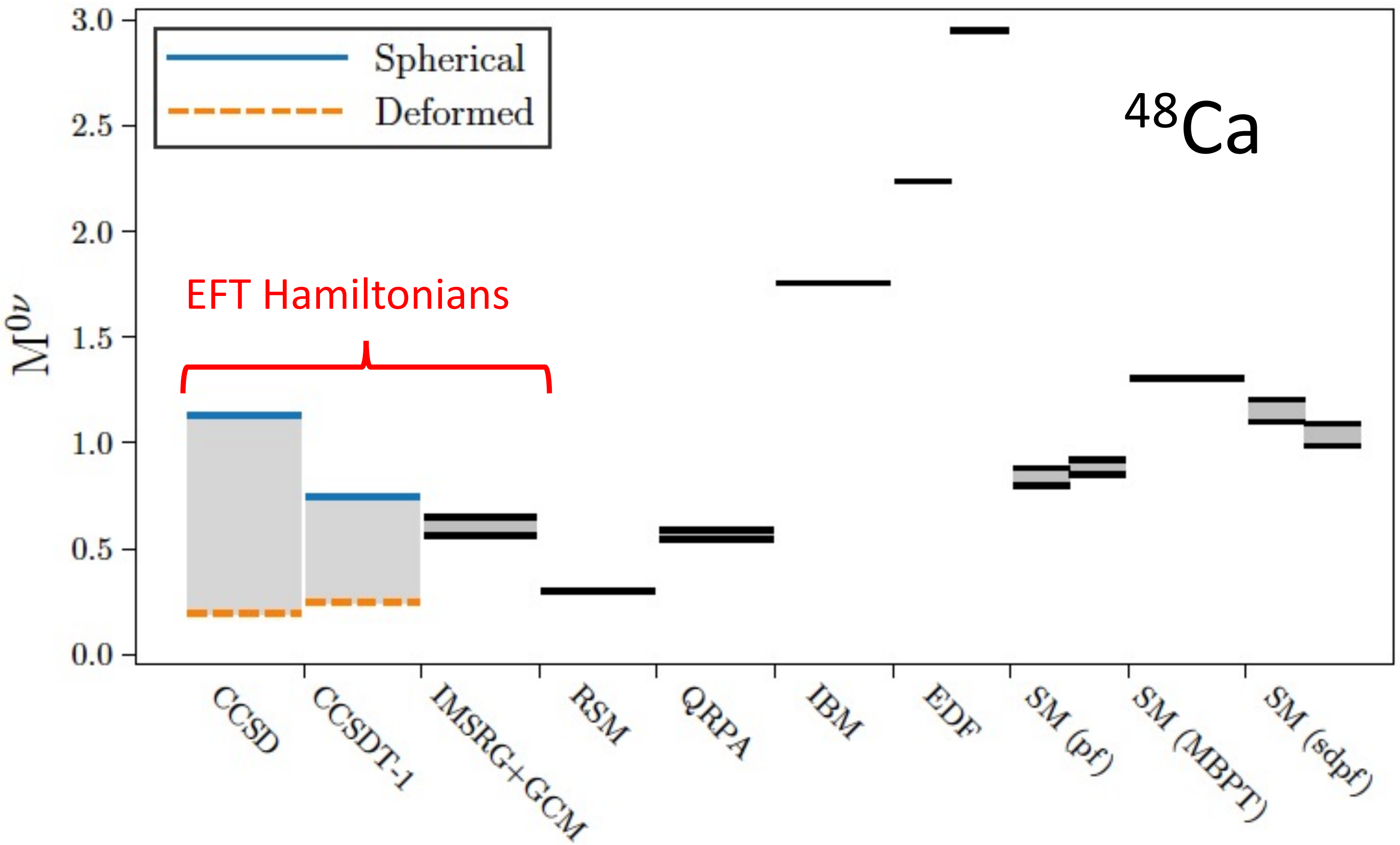
IH inverted hierarchy
NH normal hierarchy





Engel & Menéndez, Rep. Prog. Phys. 80, 046301 (2017); arXiv:1610.06548

Challenges: Nuclear matrix element for neutrinoless $\beta\beta$ decay



J. M. Yao et al., Phys. Rev. Lett. 124, 232501 (2020); arXiv:1908.05424.
S. J. Novario et al., Phys. Rev. Lett. 126, 182502 (2021); arXiv:2008.09696

Challenges:

- Higher precision
- ^{76}Ge, mass 130 nuclei are used in detectors (and not ^{48}Ca)
- Contact of unknown strength also enters (to keep RG invariance), [Cirigliano, Dekens, de Vries, Graesser, Mereghetti, Pastore, van Kolck, Phys. Rev. Lett. 120, 202001 (2018); arXiv:1802.10097]

Summary successes and challenges

- 🙂 Computations based EFT Hamiltonians now reach mass numbers $A \sim 100$
- 🙂 Link nuclear structure to forces between 2 and 3 nucleons

- 🤔 What causes the dramatic increase of charge radii beyond neutron number $N = 28$?
- 🤔 What is the nuclear matrix element for neutrinoless $\beta\beta$ decay?
- 🤔 How does nuclear binding depend on the pion mass?
- 🤔 What is the nuclear equation of state at multiples of the saturation energy?
- 🤔
- 🤔
- 🤔

Effective field theories for heavy nuclei

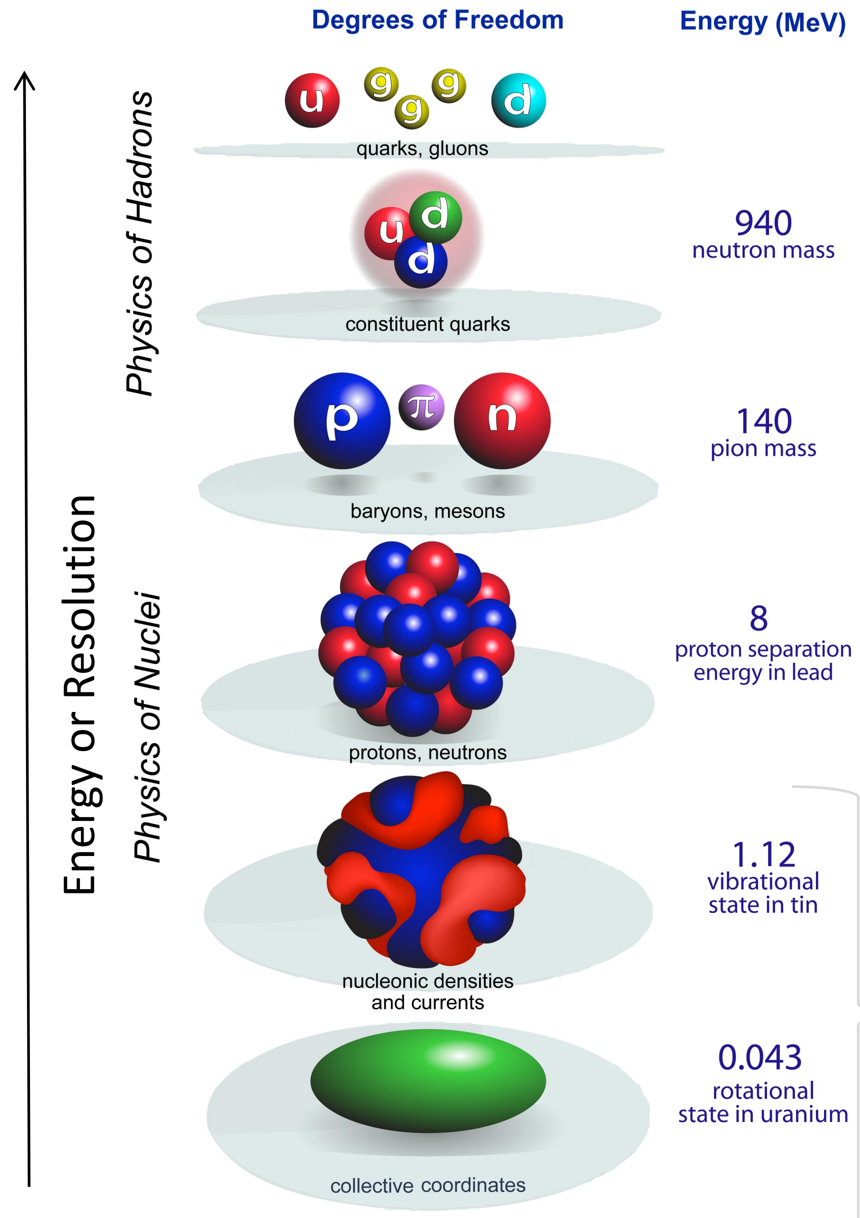
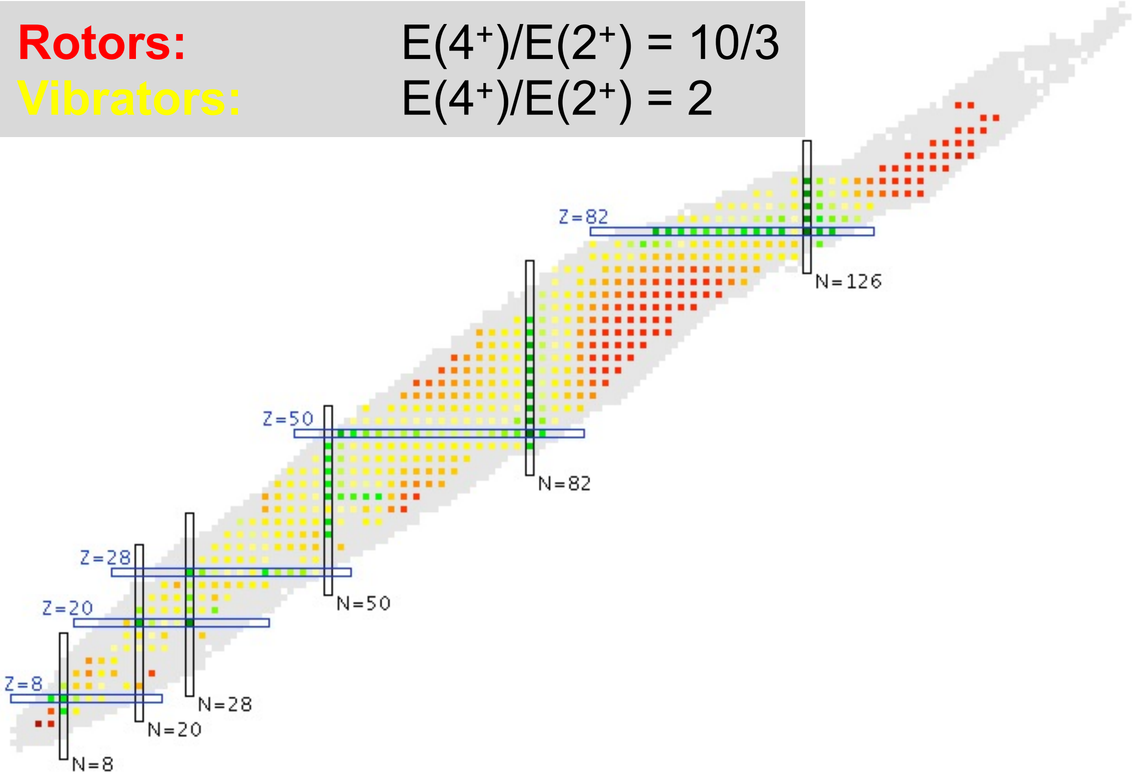


Fig.: Bertsch, Dean, Nazarewicz (2007)

Rotors: $E(4^+)/E(2^+) = 10/3$

Vibrators: $E(4^+)/E(2^+) = 2$



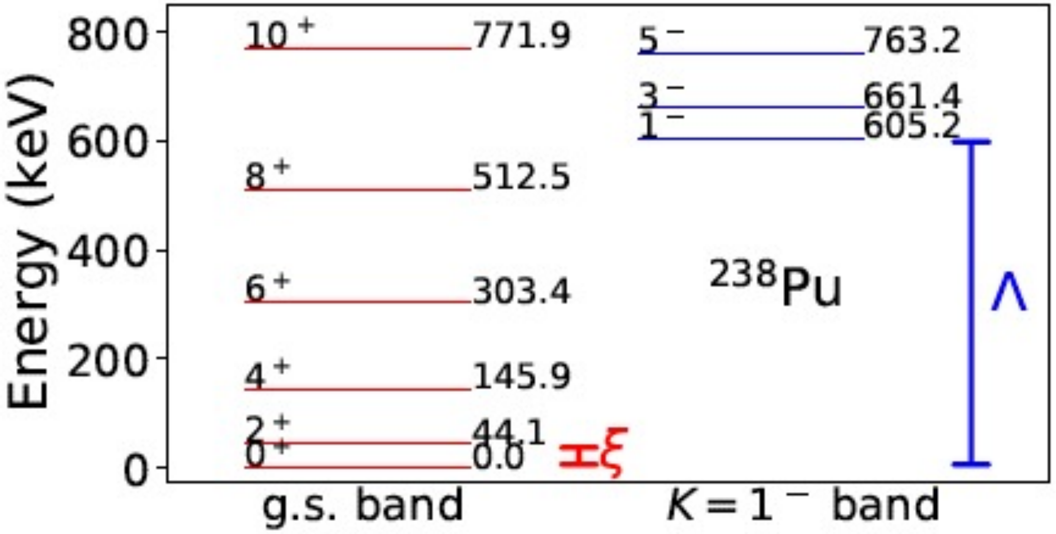
Vibrators: EFT based on linear (Wigner/Weyl) realization [Coello Pérez & TP 2015; 2016; Coello Pérez, Menéndez & Schwenk 2018]

Rotors: EFTs based on non-linear realization of SO(3)

Axially symmetric nuclei: [TP 2011; TP & Weidenmüller 2014; Coello Pérez & TP 2015; TP & Weidenmüller arXiv:2005.11865]

Triaxial deformation: [Chen, Kaiser, Meißner, Meng 2017; 2018]

Scales in heavy deformed nuclei

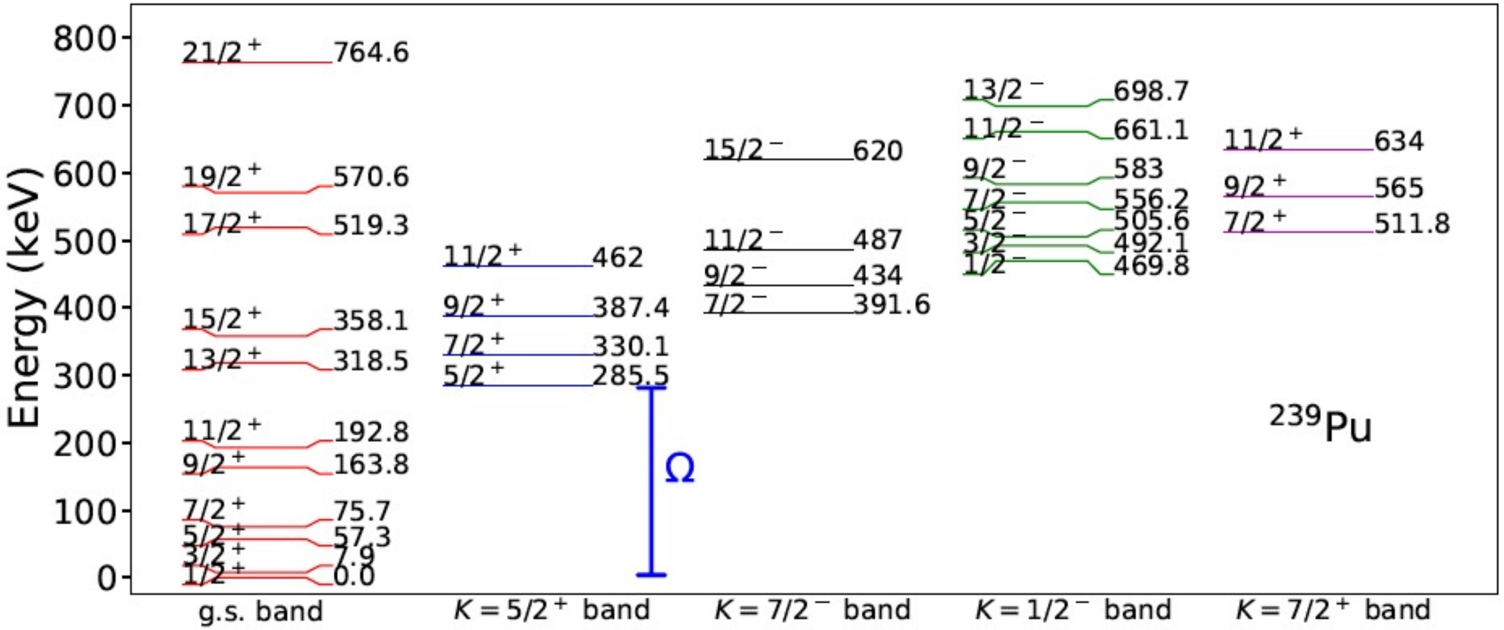


ξ = the small energy scale of interest

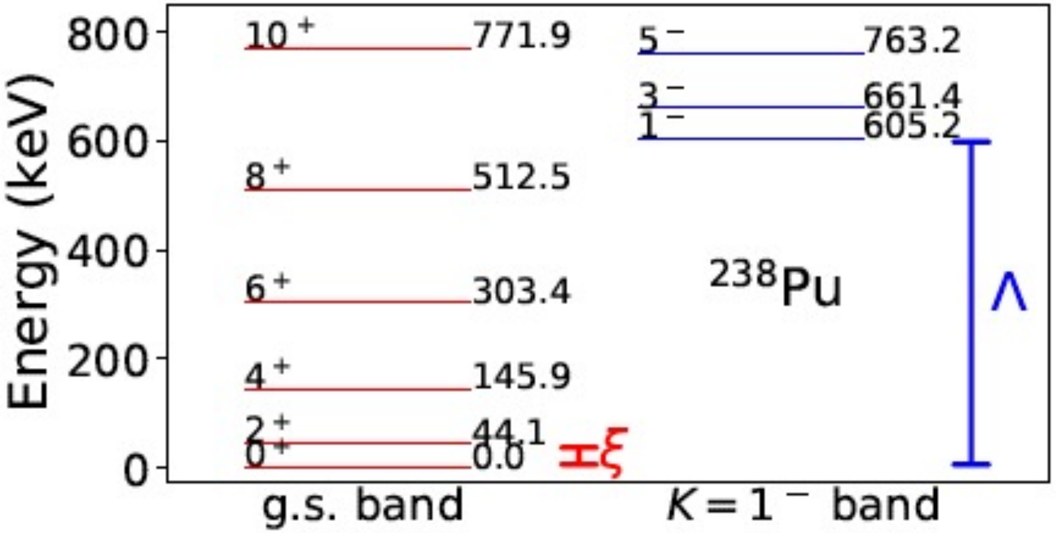
Λ = breakdown scale

Ω = single-particle scale of fermion

EFT exploits the small ratio $\frac{\xi}{\Lambda} \ll 1$



Deformed nuclei: emergent symmetry breaking



ξ = the small energy scale of interest
Λ = breakdown scale
Ω = single-particle scale of fermion

EFT exploits the small ratio $\frac{\xi}{\Lambda} \ll 1$

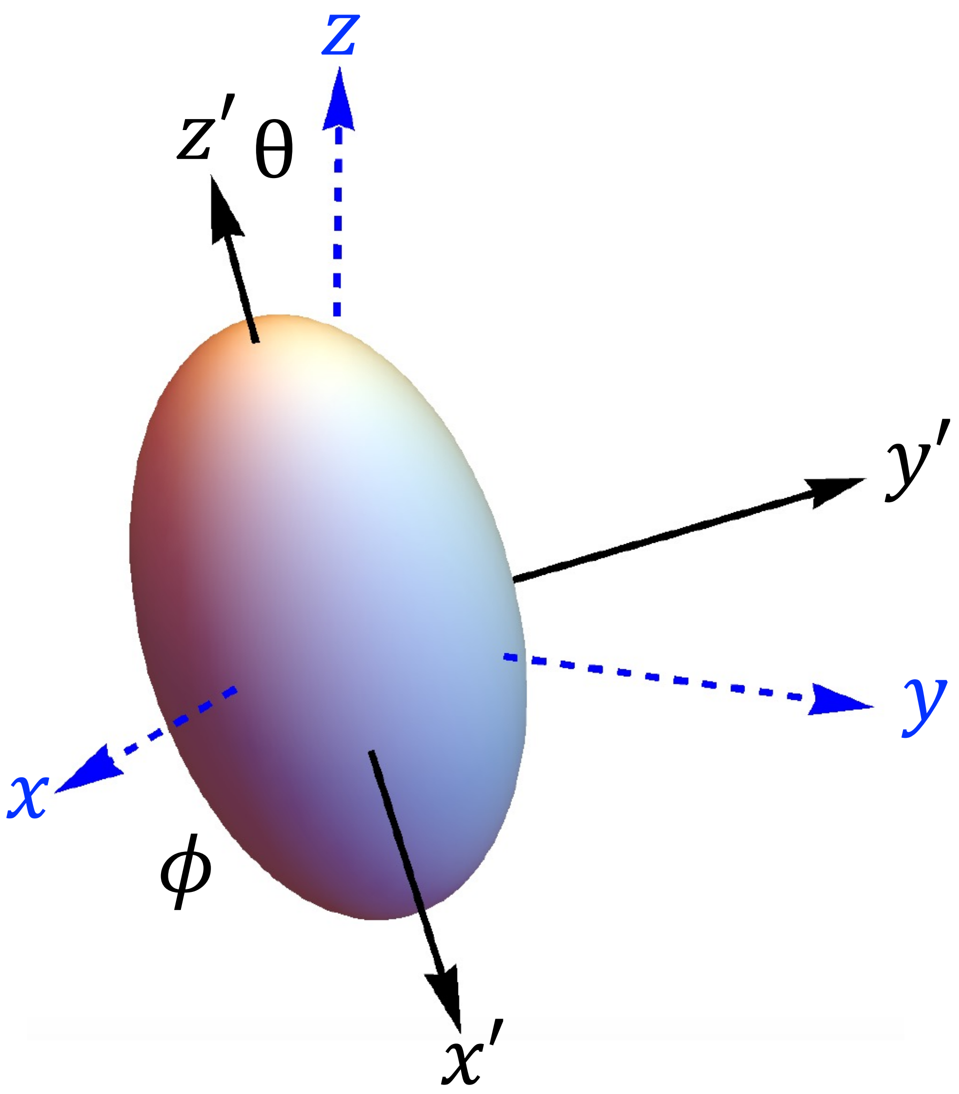
- The small size of ξ is reminiscent of an “almost” zero mode, similar to spontaneous symmetry breaking (SSB)
- But: finite systems (such as atomic nuclei) cannot exhibit SSB
- We deal with ”emergent symmetry breaking” instead [Yannouleas & Landman, Rept. Prog. Phys. 70, 2067 (2007); arXiv:0711.0637]
- Standard tools from SSB can be extended to deal with this case [Gasser & Leutwyler, Nucl. Phys. B 307, 763 (1988) ; TP & H. A. Weidenmüller, Phys. Rev. C 89, 014334 (2014); arXiv:1307.1181]

Nice review of EFT construction in presence of SSB: T. Brauner, Symmetry 2, 609 (2010); arXiv:1001.5212

EFT of emergent symmetry breaking

- Emergent symmetry breaking from rotational group $G = SO(3)$ down to axial symmetry $H = SO(2)$
- As in SSB, the degrees of freedom parameterize the coset ${}^{G}/_{H} = S^2$, i.e the surface of the unit sphere: $e_r(\theta, \phi) = (\cos\phi \sin\theta\,, \sin\phi \sin\theta\,, \cos\theta)^T$
  - Note: Emergent SB completely defines degrees of freedom
- Here (and in contrast to SSB), $\theta = \theta(t), \phi = \phi(t)$ are only time dependent
  - In SSB, NG fields $\psi(x, t)$ must also depend on position
- As in SSB, only derivative couplings can enter the Lagrangian (same as for Nambu-Goldstone bosons)
  - simplest Lagrangian is $L = \frac{C}{2}(\partial_t e_r)^2 = \frac{C}{2}\left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta\right)$
  - Note: Emergent SB severely constrains possible form of interaction
  - The Lagrangian is that of a rotor; quantized energies are $E = \frac{I(I+1)}{2C}$
- Higher-order terms are powers of $(\partial_t e_r)^2$; suppression is in powers of $\left(\frac{\xi}{\Lambda}\right)^2$
- Most general Lagrangian also contains gauge potentials (more below)

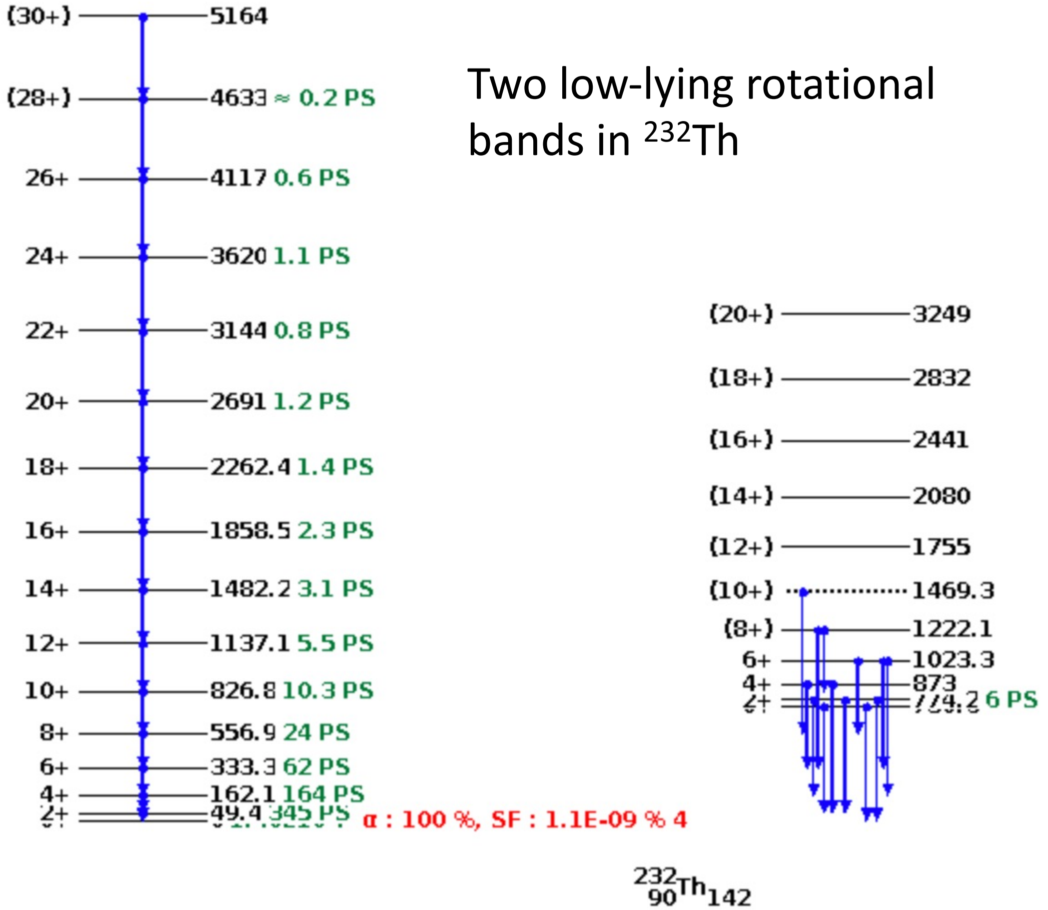
Axially symmetric even-even nucleus



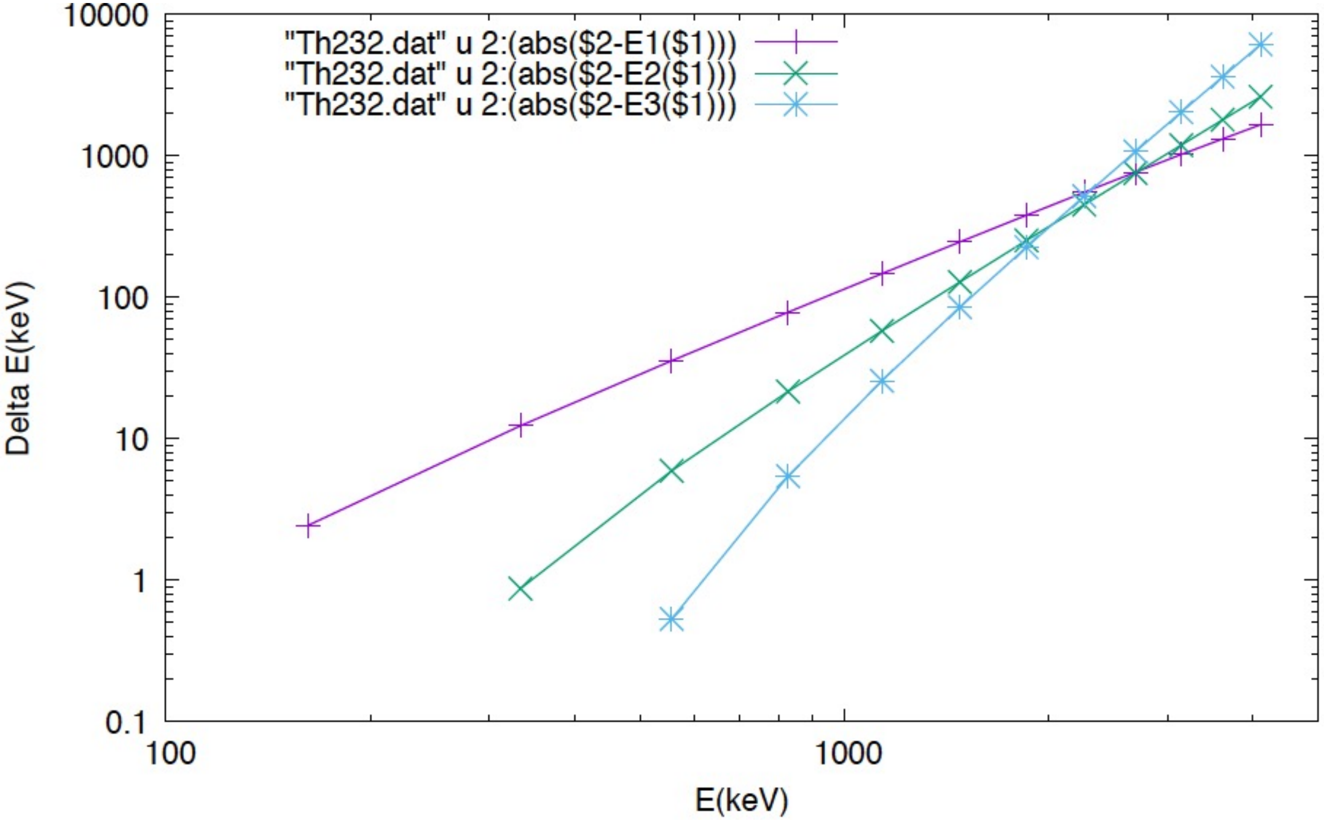
- Effective field theory: Nonlinear realization of SO(3) in case of spontaneous symmetry breaking down to axial SO(2) [Weinberg 1968, Callan, Coleman, Wess & Zumino 1969]: Degrees of freedom parameterize the unit sphere, i.e. the coset SO(3)/SO(2) ~ S^2
- Traditional NP: We have an axially symmetric rotor, and its orientation is in direction of the angles (θ, ϕ).
- Berry: The body-fixed system is only defined up to rotations around the body-fixed symmetry (z’) axis → gauge freedom

^{232}Th as an example

Two low-lying rotational bands in ^{232}Th



"Lepage plot," → Peter Lepage, arXiv:nucl-th/9706029



LO: $E_{LO} = aI(I+1)$

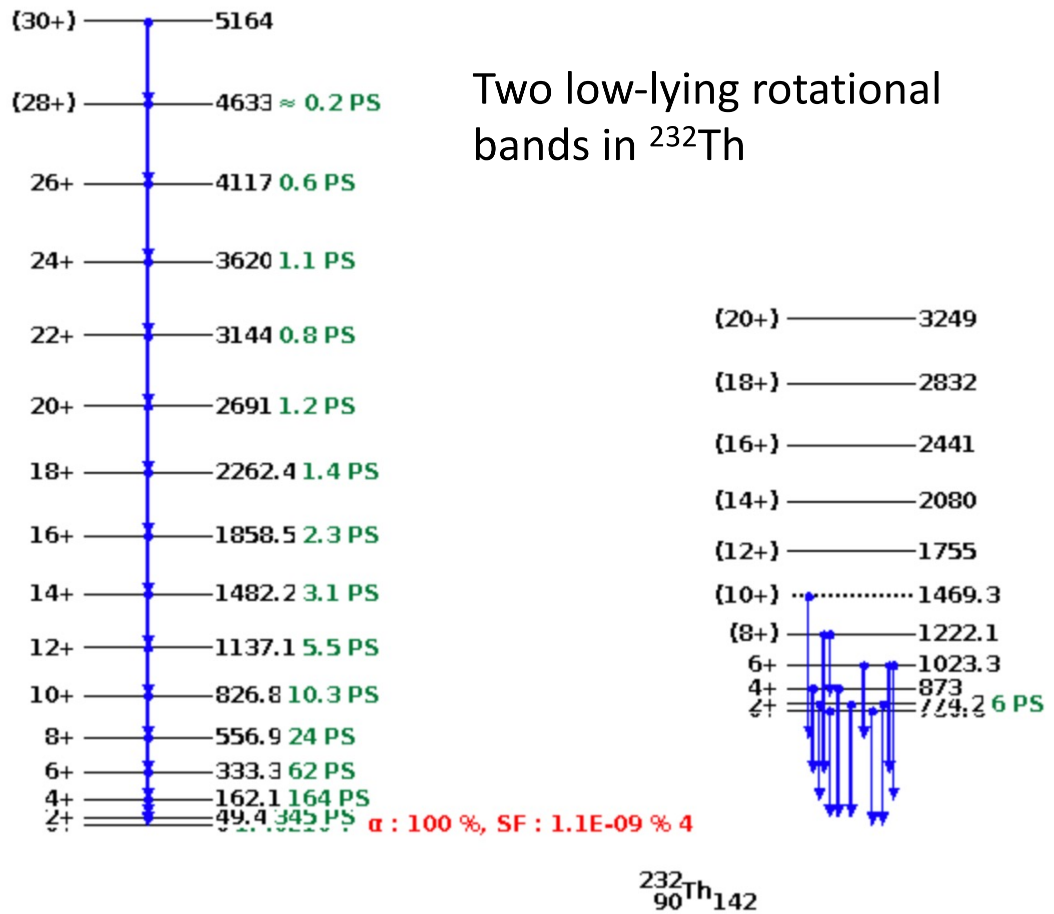
NLO: $E_{NLO} = aI(I+1) + b[I(I+1)]^2$

NNLO: $E_{NNLO} = aI(I+1) + b[I(I+1)]^2 + c[I(I+1)]^3$

Q1: What is the low-energy scale ξ?

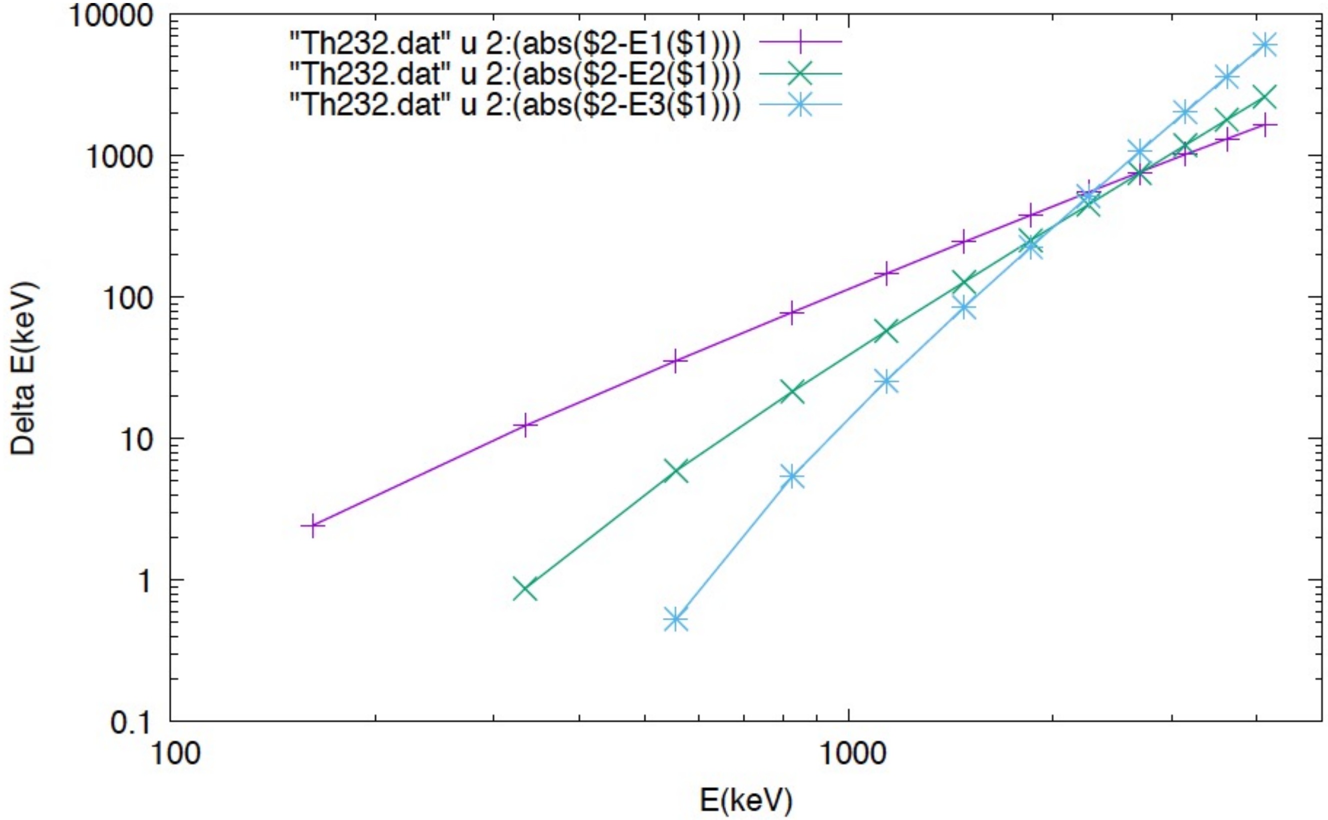
Q2: What is the breakdown scale Λ?

^{232}Th as an example



Two low-lying rotational bands in ^{232}Th

"Lepage plot," → Peter Lepage, arXiv:nucl-th/9706029



LO: $E_{LO} = aI(I+1)$

NLO: $E_{NLO} = aI(I+1) + b[I(I+1)]^2$

NNLO: $E_{NNLO} = aI(I+1) + b[I(I+1)]^2 + c[I(I+1)]^3$

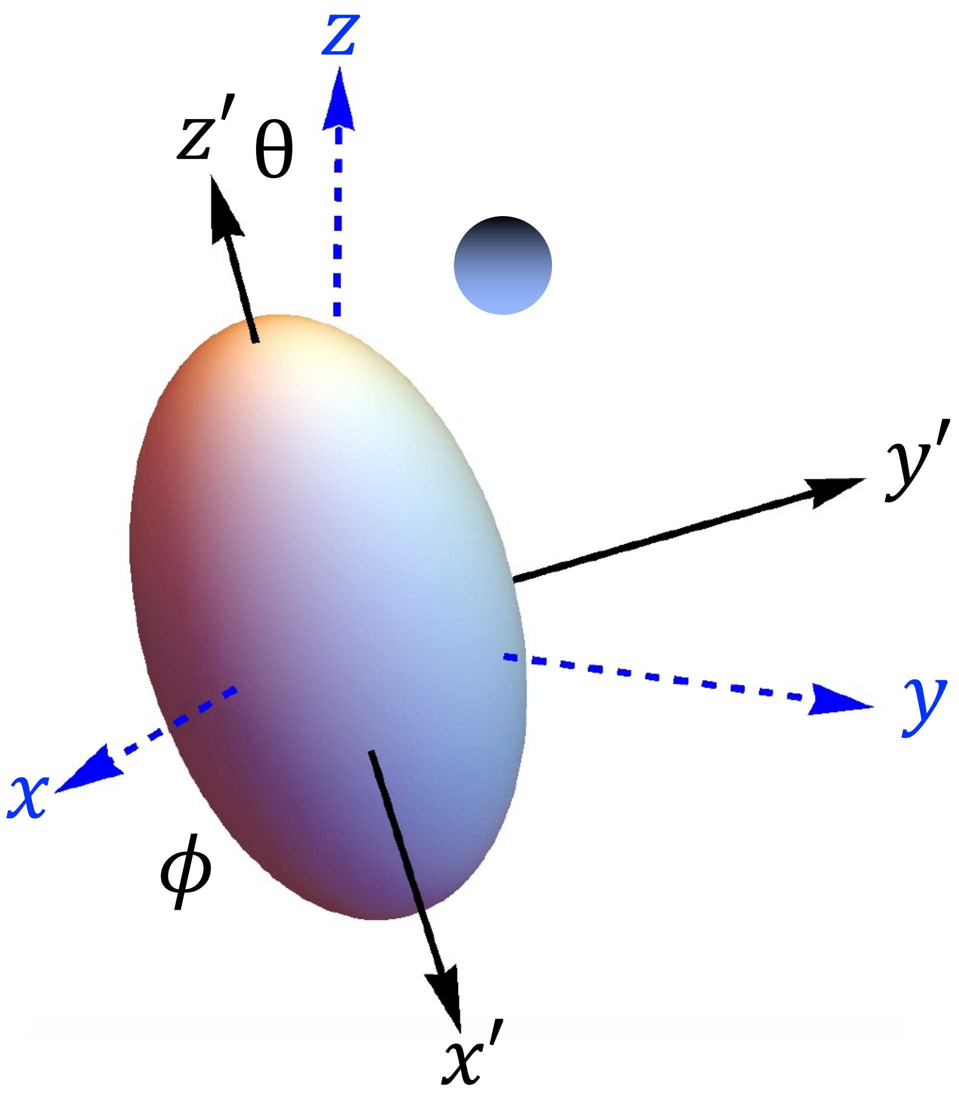
Q1: What is the low-energy scale ξ?

A1: $\xi \approx 50$ keV

Q2: What is the breakdown scale Λ?

A2: (from Lepage plot) $\Lambda \approx 2400$ keV

Odd nucleon coupled to even-even rotor



- Effective field theory: Non-linear realization of broken SO(3): The dynamics of the odd nucleon is defined in the body-fixed system; it introduces a covariant derivative.

- Traditional NP: This is the "strong" coupling limit; Coriolis forces appear in the co-rotating body-fixed system

- Berry: The nucleon is much faster than the rotor. The adiabatic approximation introduces gauge potentials

Gauge potentials

The fermion's total spin is $\widehat{K} = (\widehat{K}_{x\prime}, \widehat{K}_{y\prime}, \widehat{K}_{z\prime})$ with components in the body-fixed frame.
It generates a gauge potential that couples to the angular velocity of the even-even rotor.

Abelian gauge potential (from covariant derivative on the unit sphere)

$$\mathbf{A}_{\mathrm{a}}(\theta, \phi) \equiv \mathbf{e}_\phi \cot\theta \hat{K}_{z'}$$

Corresponding Berry curvature ("magnetic field") is of monopole type, quantized

$$\mathbf{B}_{\mathrm{a}}(\theta, \phi) \equiv \nabla_\Omega \times \mathbf{A}_{\mathrm{a}} = -\mathbf{e}_r \hat{K}_{z'}$$

Non-Abelian gauge potential (also allowed by symmetries):

$$\mathbf{A}_{\mathrm{n}}(\theta, \phi) = g\mathbf{e}_r \times \mathbf{K} = g\left(\mathbf{e}_\phi \hat{K}_{x'} - \mathbf{e}_\theta \hat{K}_{y'}\right)$$

Corresponding Berry curvature ("magnetic field") is again of monopole type

$$\begin{aligned}\mathbf{B}_{\mathrm{tot}} &\equiv \nabla_\Omega \times \mathbf{A}_{\mathrm{tot}} - i\mathbf{A}_{\mathrm{tot}} \times \mathbf{A}_{\mathrm{tot}} \\ &= (g^2 - 1)\mathbf{e}_r \hat{K}_{z'}\end{aligned}$$

Manifestations of gauge potentials in Hamiltonians

Lagrangian

$$L = \frac{C_0}{2}\mathbf{v}^2 + \mathbf{v}\cdot(\mathbf{A}_\mathrm{a} + \mathbf{A}_\mathrm{n}) + L_\Psi$$

$$\mathbf{v} \equiv \frac{d}{dt}\mathbf{e}_r = v_\theta \mathbf{e}_\theta + v_\phi \mathbf{e}_\phi$$

Hamiltonian

$$H = H_\Psi + \frac{g^2}{2C_0}\left(\hat{K}_{x'}^2 + \hat{K}_{y'}^2\right) + \frac{\mathbf{I}^2 - \hat{K}_{z'}^2}{2C_0} + \frac{g}{C_0}\left(I_{x'}\hat{K}_{x'} + I_{y'}\hat{K}_{y'}\right)$$

Manifestations of gauge potentials in Hamiltonians

Lagrangian

$$L = \frac{C_0}{2}\mathbf{v}^2 + \mathbf{v}\cdot(\mathbf{A}_{\mathrm{a}} + \mathbf{A}_{\mathrm{n}}) + L_\Psi$$

$$\mathbf{v} \equiv \frac{d}{dt}\mathbf{e}_r = v_\theta \mathbf{e}_\theta + v_\phi \mathbf{e}_\phi$$

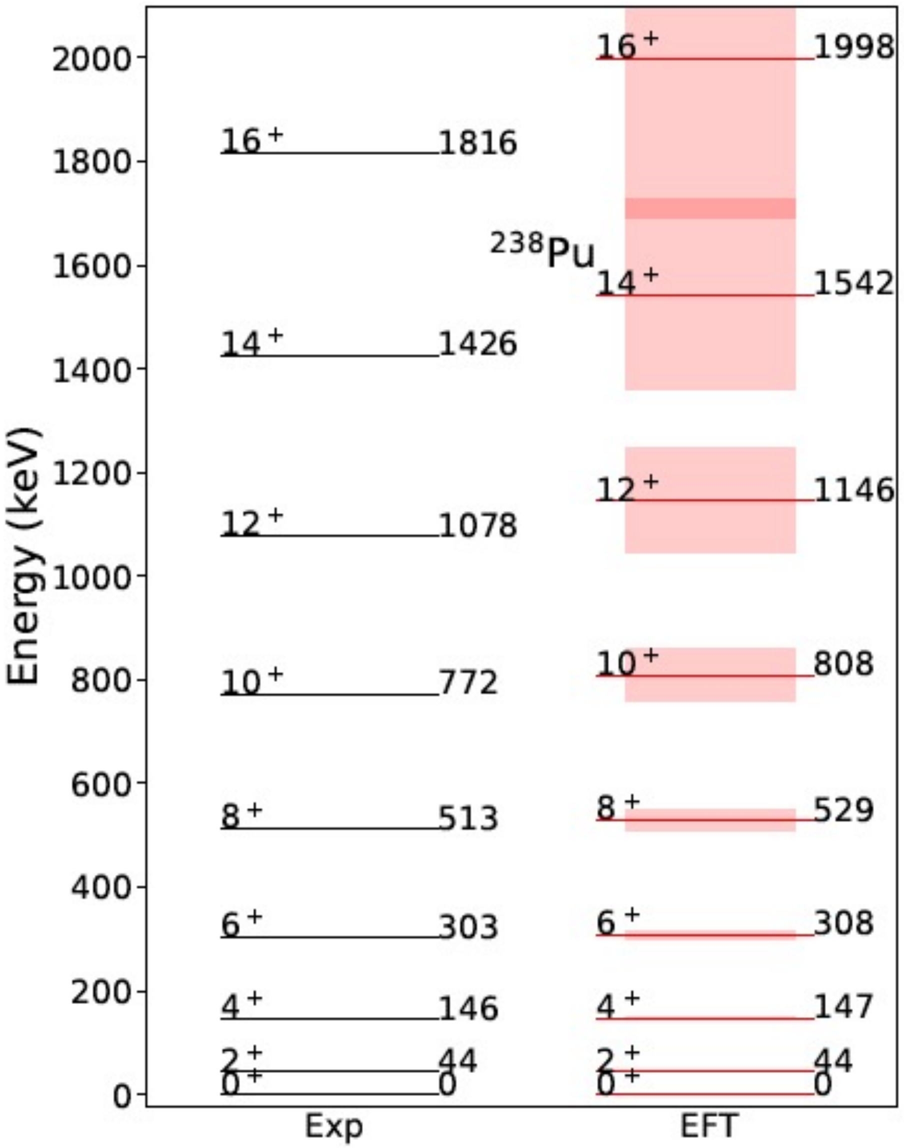
From non-Abelian gauge potential

Hamiltonian

$$H = H_\Psi + \frac{g^2}{2C_0}\left(\hat{K}_{x'}^2 + \hat{K}_{y'}^2\right) + \frac{\mathbf{I}^2 - \hat{K}_{z'}^2}{2C_0} + \frac{g}{C_0}\left(I_{x'}\hat{K}_{x'} + I_{y'}\hat{K}_{y'}\right)$$

From Abelian gauge potential

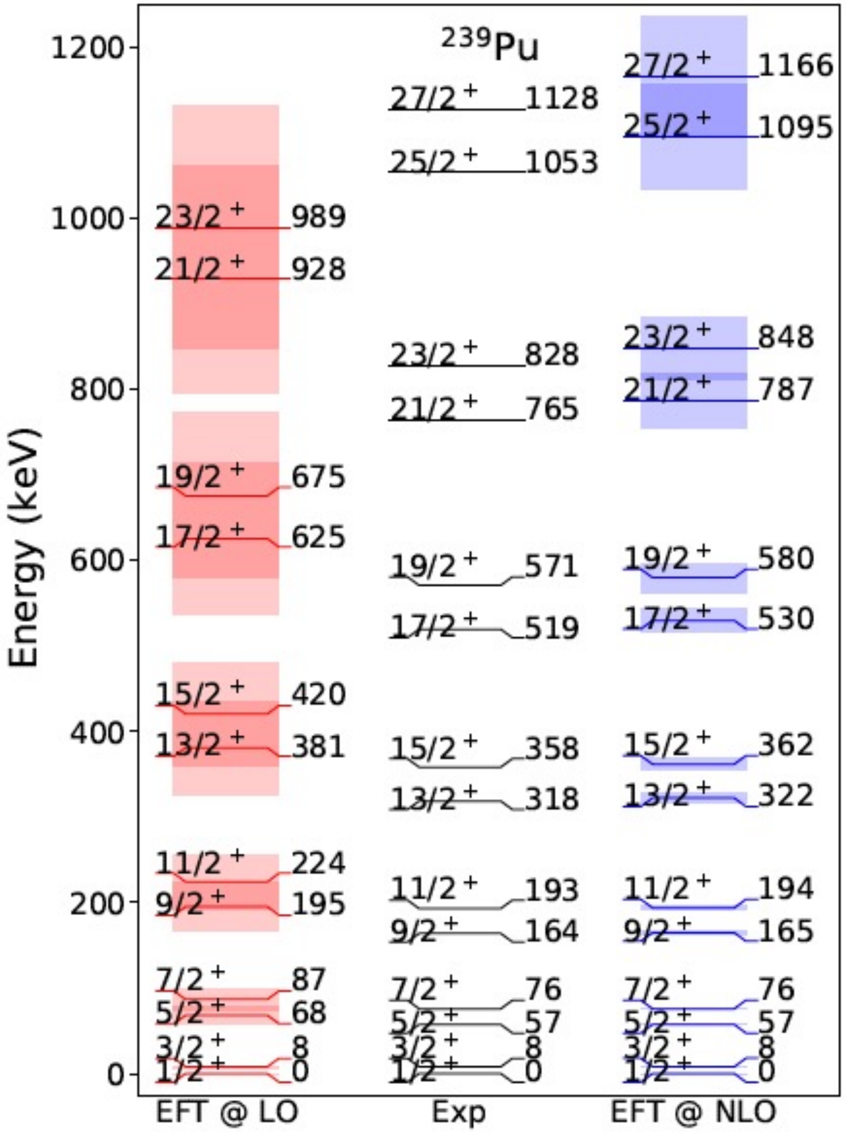
^{239}Pu as a neutron coupled to ^{238}Pu



Uncertainty estimates based on power counting

Leading order: Take moment of inertia (MOI) from ^{238}Pu and adjust decoupling coefficient

Next-to-leading order: re-adjust MOI for ^{239}Pu



Gauge potentials, Berry phases, and Coriolis forces

Different interpretations of the velocity–dependent rotor–nucleon couplings

1. **Coriolis forces** enter in rotating frames: Velocity-dependent forces are present in rotating nuclei [Bohr, Kerman, Mottelson, Nilsson 1950s].

2. **Molecular Aharonov-Bohm effect**: In rotating molecules, the nuclei are slow (and the electrons are fast), and the adiabatic decoupling (à la Born Oppenheimer) introduces Berry phases and gauge potentials [Mead & Truhlar 1979; Wilczek & Zee 1984; Kuratsuji & Iida 1985; Nazarewicz 1996].

3. **Covariant derivative**: In presence of spontaneous symmetry breaking, the rotational symmetry is realized non-linearly for the rotor's degrees of freedom. This introduces a covariant derivative $iD \equiv i\partial_t + \mathbf{v} \cdot A_a$ [Weinberg 1968; Callan, Coleman, Wess & Zumino 1969].

4. **Gauge invariance**: The ambiguities in defining a body-fixed frame, i.e. separating rotational and intrinsic degrees of freedom, imply a gauge invariance [Littlejohn & Reinsch 1997]. Our case: ambiguities regarding rotations around the *z'* axis.

[Leutwyler 1994; Roman & Soto 1999; Hofmann 1999; Chandrasekharan et al. 2008; Brauner 2010; TP 2011, TP & Weidenmüller 2014; …]

Falling Cat Problem

Q: How does a cat change its orientation, i.e. its angular momentum, without an external torque?

A: Changes in its shape (intrinsic degrees of freedom) induce a change in the external orientation.

Q: What does this has to do with odd-mass deformed nuclei?

A: In both cases, non-Abelian gauge potentials arise that describe the internal dynamics and couple it to the overall orientation. (In the nucleus, the odd nucleon causes the internal dynamics.)

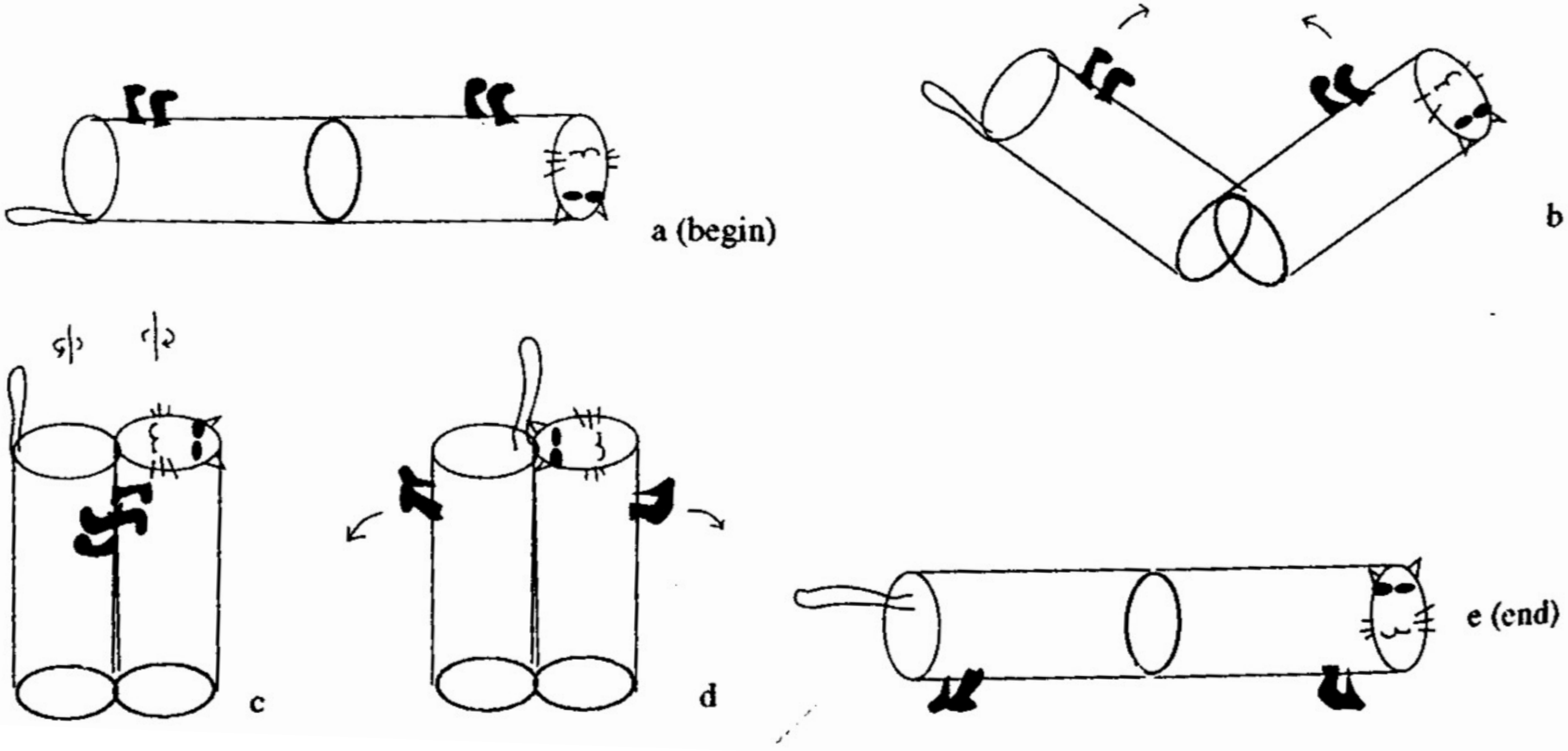
→ Gauge theory of deformable bodies

Shapere & Wilczek, *Geometric Phases in Physics* (1989); Littlejohn & Reinsch, Rev. Mod. Phys. (1997)

“Gauge theory of the falling cat,” Montgomery (1993)

6. Some Specific Reorientations and Steering Strategies

6.1. A Cartoon. Probably the simplest path resulting in the cat flip is the one depicted below.



“Bend, twist, unbend” makes a closed loop in internal configuration space while leading to a rotation.

Summary EFT for deformed nuclei

- Develop EFT for emergent symmetry breaking guided by standard approach in spontaneous symmetry breaking
- Systematically improvable approach
  - Re-discovers venerable models
  - Gives uncertainty estimates
- Odd nuclei naturally introduce gauge potentials and Berry phases
  - These relate odd-mass deformed nuclei to the falling cat

Thank you for your attention, participation, and questions!

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