Using GW Data to Constrain Neutron Star EOS

Mohammad Al-Mamun and Andrew W. Steiner

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Department of Physics and Astronomy
University of Tennessee, Knoxville

Introduction

Intro, Goal and Complications

Neutron Stars(NS) are the remnant of **supernova** explosions.

- **NS**'s are the densest object(except black holes).
- Typical **NS** has mass 1.4 $M_\odot$ with radius 10-15 km.
- Density of a **NS** core is likely to be 3 to 4 times the nuclear saturation density.

The ultimate goal here is to find the **EOS** which can describe dense matter composition inside a **NS**. But,

- At present, Nuclear experiments can constrain the **EOS** up to the saturation density.
- Nuclear theory also cannot yet provide a proper description for the dense matter of **NS** core.

Fortunately, **Tolman-Oppenheimer-Volkoff (TOV) equations** provide an one-to-one correspondence between **mass-radius** curve and the **EOS**. So, we'll be using this relationship in our model to constrain the desired quantities.

TOV equations

Tolman-Oppenheimer-Volkoff (TOV) equations, a set of coupled ODE, constrains the structure of a spherically symmetric body of isotropic material which is in static gravitational equilibrium.
For a spherically symmetric metric,

$$ds^2 = -e^{2\phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2 \tag{1}$$

the **TOV** equations has the form of

$$\frac{dP}{dr} = -\frac{G\epsilon m}{r^2}\left(1 + \frac{P}{\epsilon}\right)\left(1 + \frac{4\pi P r^3}{m}\right)\left(1 - \frac{2Gm}{r}\right)^{-1} \tag{2}$$

$$\frac{dm}{dr} = 4\pi r^2 \epsilon \tag{3}$$

Equation of State(EOS) relates the energy density(ϵ) with pressure P. A polytropic **EOS** has the form of

$$\epsilon = \kappa P^{1+1/n} \tag{4}$$

where

- κ = fitting const.
- n = polytropic index.

The **EOS** also depends on the number density of **protons**, **electrons** and **neutrons** as well as some exotic high energy particles. Adding these interactions to the **EOS** can produce a realistic model of the star.

Methods

Priors

Quiescent low mass X-ray binaries(**qLMXB**) are the primary sources of spectral data for **NS** M and R data. List of **qLMXB**s used for parameter properties and atmosphere informations are given below[1]:

- X5
- ω cen
- NGC6397
- NGC6304
- M13
- M30
- M28

We also have couple of inferences of mass-radius from PRE X-ray bursts[2],

- SAX J1810.8−429
- 4U 1702−429
- 4U 1724−307

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[1] **Steiner et. al.(2018)**; **https://doi.org/10.1093/mnras/sty215** .

[2] **J. Nattila et. al.**; **https://doi.org/10.1051/0004-6361/201527416**.

Priors

Observables used in the prior distribution of **GW170817**[3] data source properties paper are as follows:

$$\textit{chirp mass, } M_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} \tag{5}$$

$$\textit{mass ratio, } q = \frac{m_2}{m_1} \tag{6}$$

and linear combination of tidal deformabilities($\tilde{\Lambda}$),

$$\tilde{\Lambda} = f\,(m_1, m_2)\,\Lambda_1 + g\,(m_1, m_2)\,\Lambda_2 \tag{7}$$

here, m_1, m_2, Λ_1 *and* Λ_2 are the masses and tidal deformabilities of the **NS** binaries respectively. And for our models,

- we've selected low spin priors from the **GW170817** inspiral.
- we've done bayesian analysis with these joint priors to produce the posterior distribution of the observables.

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[3] **GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral**; **https://link.aps.org/doi/10.1103/PhysRevLett.119.161101**.

Results

Constrains on Parameters

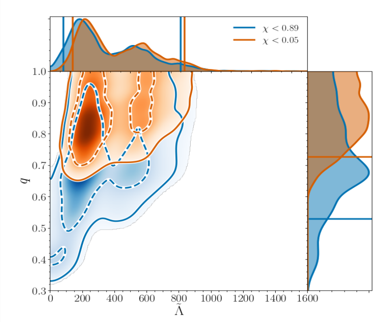

Figure 1: q vs $\tilde{\Lambda}$ from LIGO

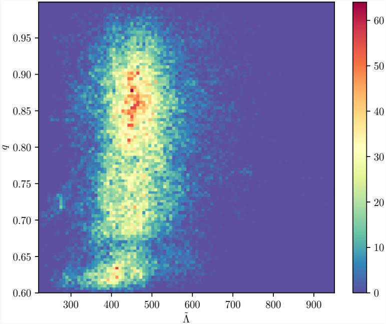

Figure 2: q vs $\tilde{\Lambda}$

Figure 2 represent the 2d-hitogram for q and $\tilde{\Lambda}$. The peak densities occur at an appoximate value of 0.85 for q, which is in good agreement with **LIGO** results shown in figure 1.

Constrains on Parameters

2D-histogram plot for Λ2 vs Λ1 is given in figure 3 which is compared with the LIGO's version in figure 4. We can see that Λ_2 is shifted to a higher value with our combined prior.

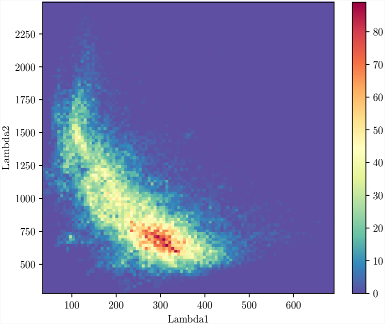

Figure 3: Λ_2 vs Λ_1

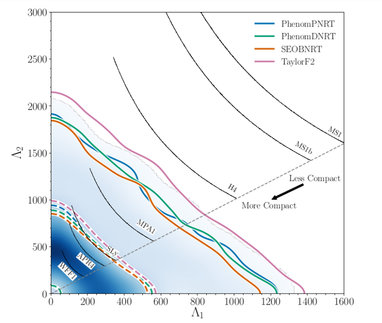

Figure 4: Λ_2 vs Λ_1 form LIGO

Mass-Radius Curve for NS

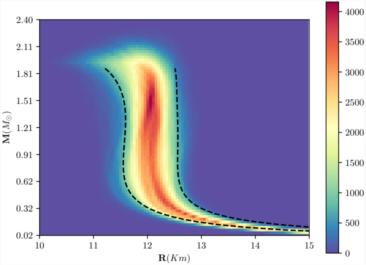

Figure 5: Mass-Radius Curve

In figure:5, we've generated **Mass-Radius** distribution with the dashed black lines showing the 90% confidence intervals. We can see that the radial range for a 1.4 solar mass **NS** is in between 11.5-12.8 km.

And in figure:6, we've shown the posterior distribution of pressure with varying energy densities. Here we can see that the maximum pressure for NS occurs at $\epsilon = 550 Mev/fm^3$.

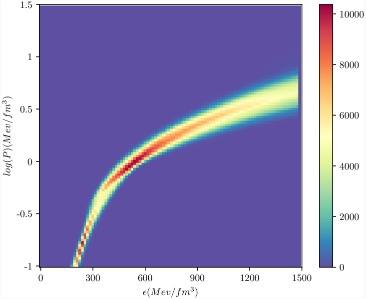

Figure 6: log pressure vs energy density

Concluding Remarks

- This work is one of the first joint X-ray and LIGO analysis to constrain **NS EOS**.
- Constrains on parameter space from our posterior distribution is in good agreement with LIGO results.
- Radial range for 1.4$M_\odot$ **NS** is approximately 11.5-12.8 km.
- We would also like to use **GW** data as standalone prior and compare the results with our recent work.

Thank you