Fundamental Symmetries
and
Weak Interaction through Parity Violation

(Particularly with Polarized Electron Scattering)

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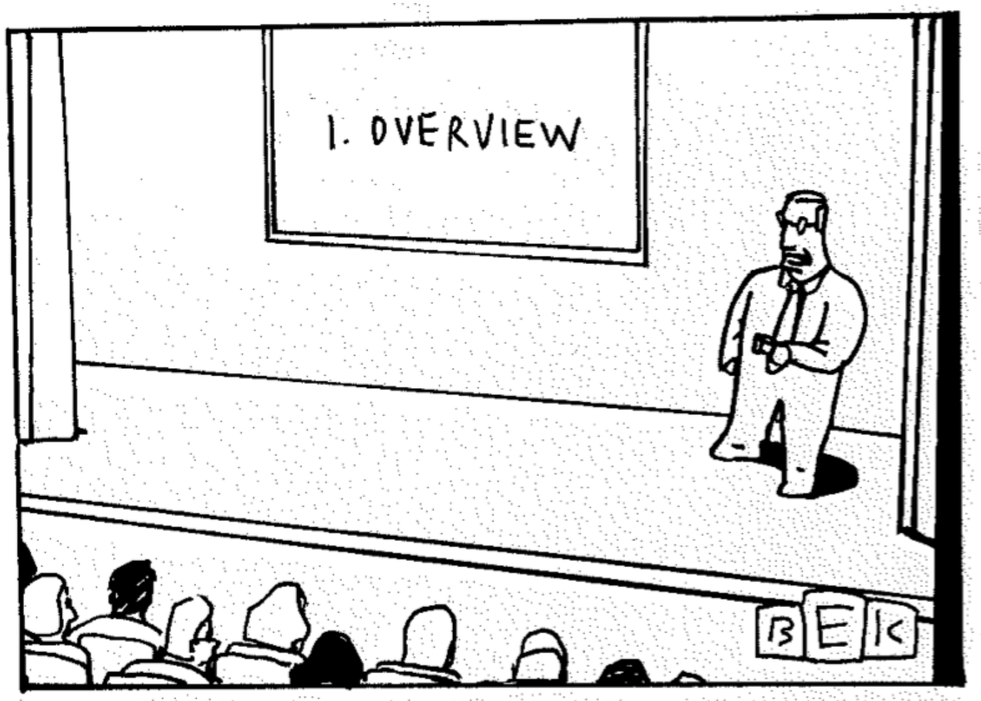
Outline

- Overview
  - Intro
  - History of PV experiments
  - Symmetries
- Theory
  - Classical Mechanics → QM → QED
  - EM as gauge theory
  - Conservation Laws and symmetries
  - SM Lagrangian
  - How to measure?
- Experiment
  - Qweak
  - MOLLER
  - Other measurements



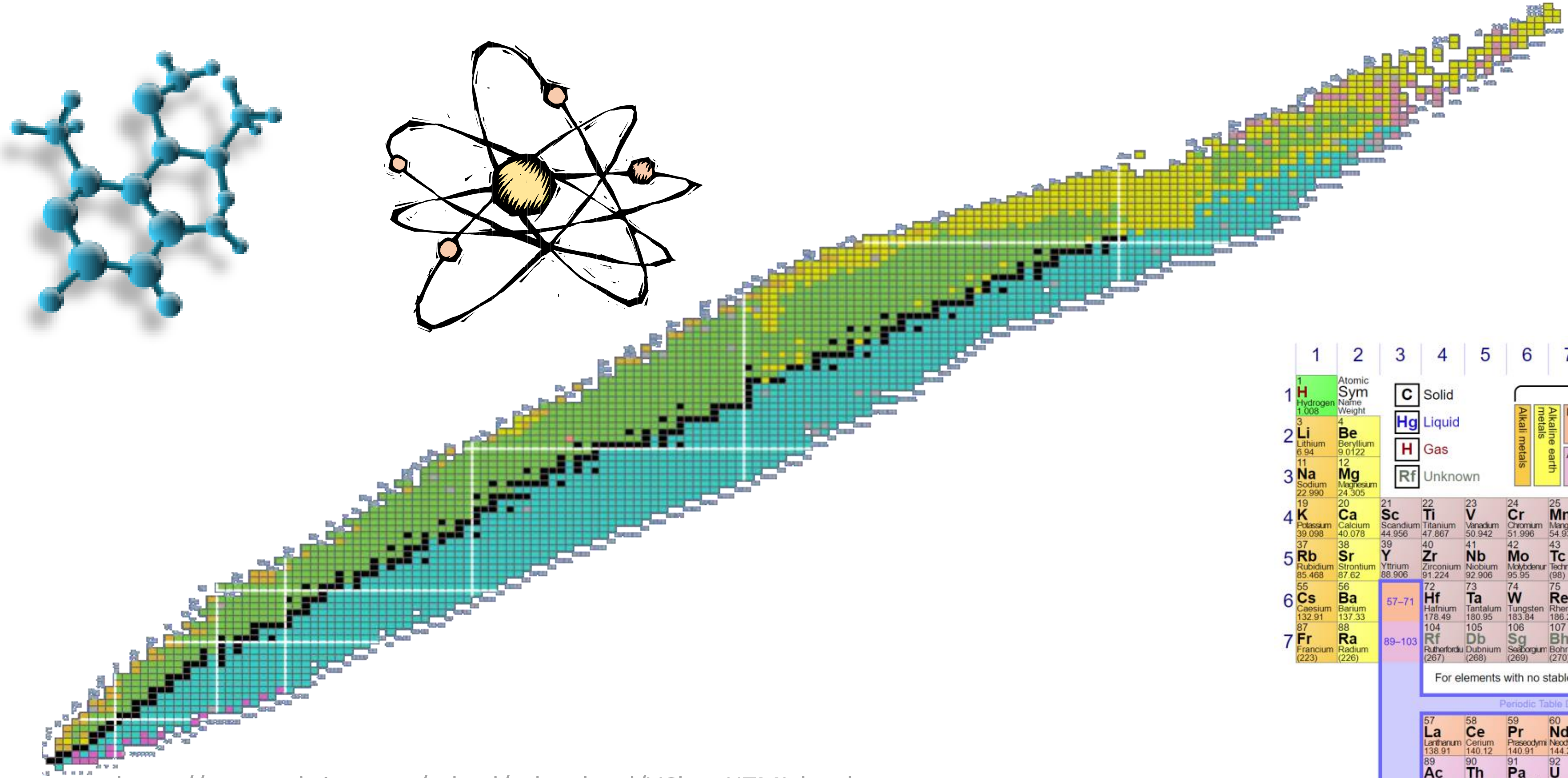
"First, I want to give you an overview of what I will tell you over and over again during the entire presentation."

What are we made of?

What is everything made of?

How does everything interact?

hadrons
- mesons
- baryons



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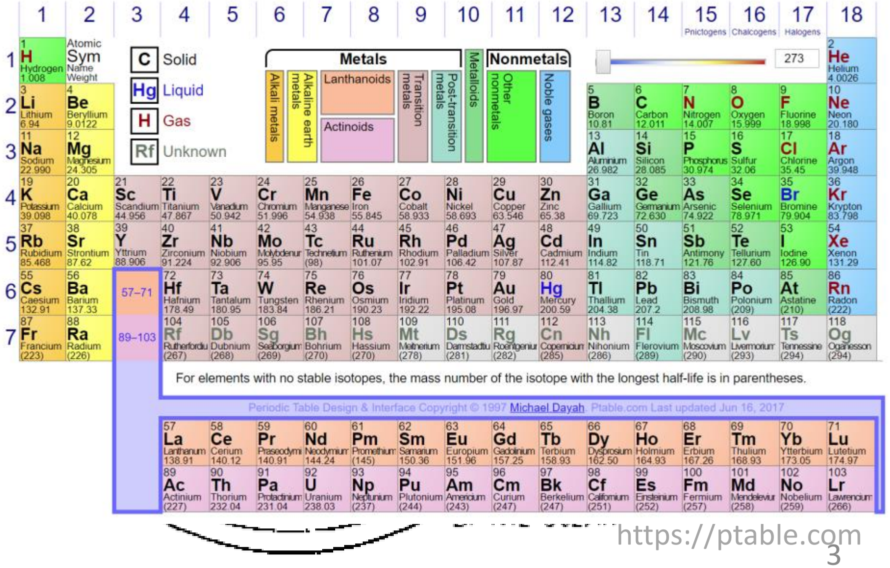
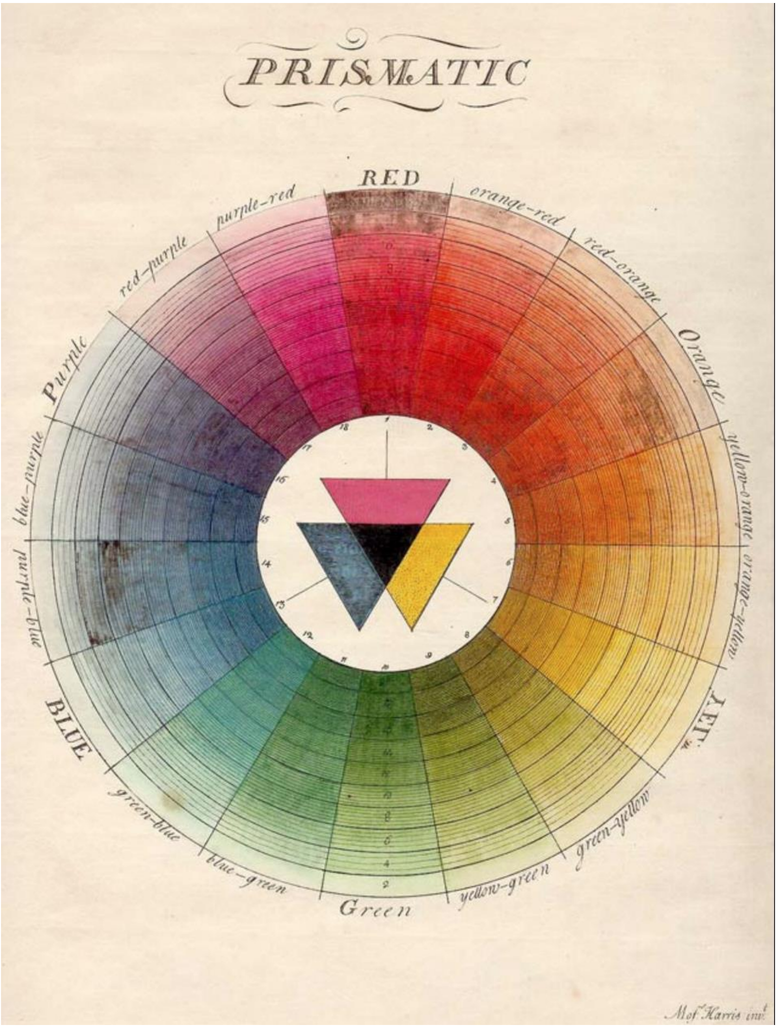


https://ptable.com

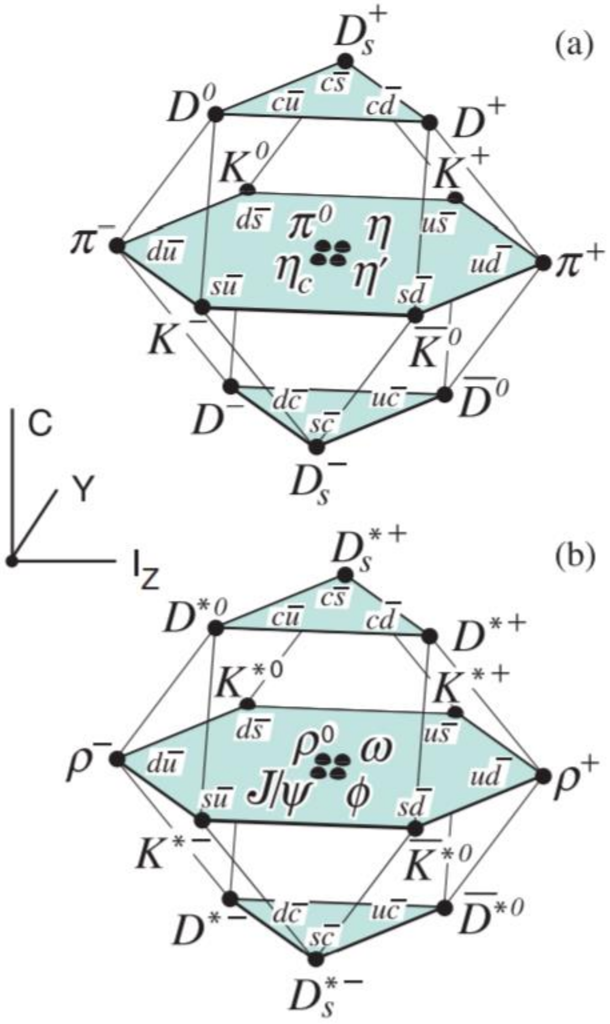
What is the Standard Model?



Moses Harris, in his book *The Natural System of Colours* (1776)

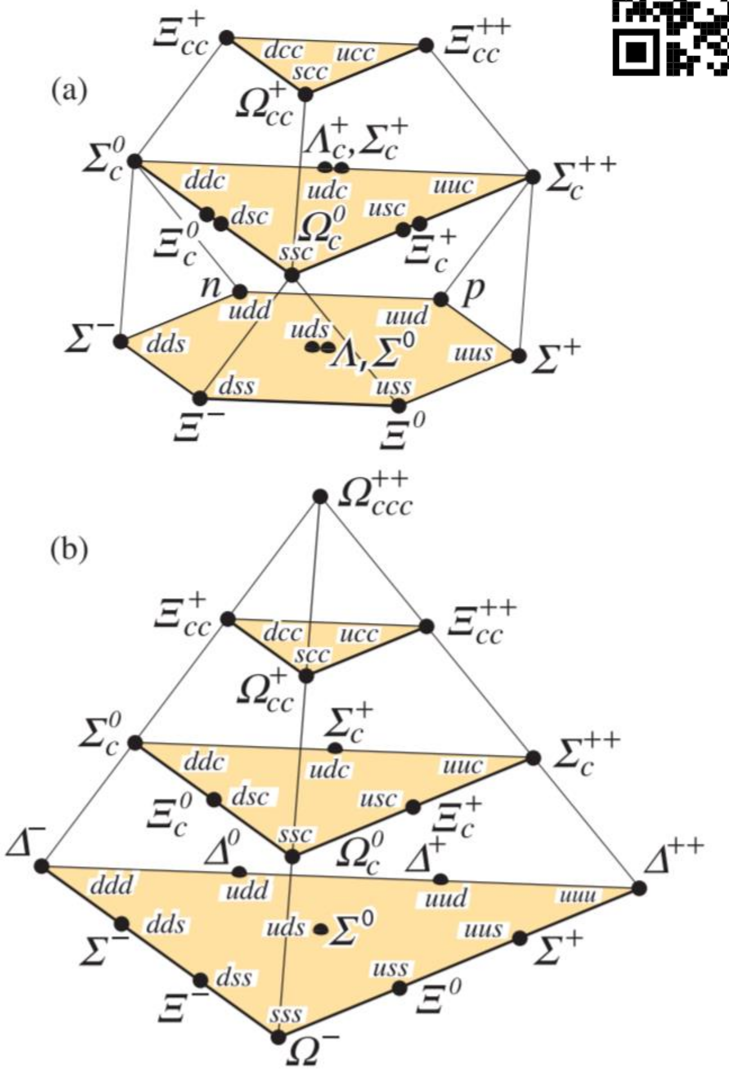
Particle Zoo to the Quark Model

http://pdg.lbl.gov/2019/reviews/rpp2018-rev-quark-model.pdf

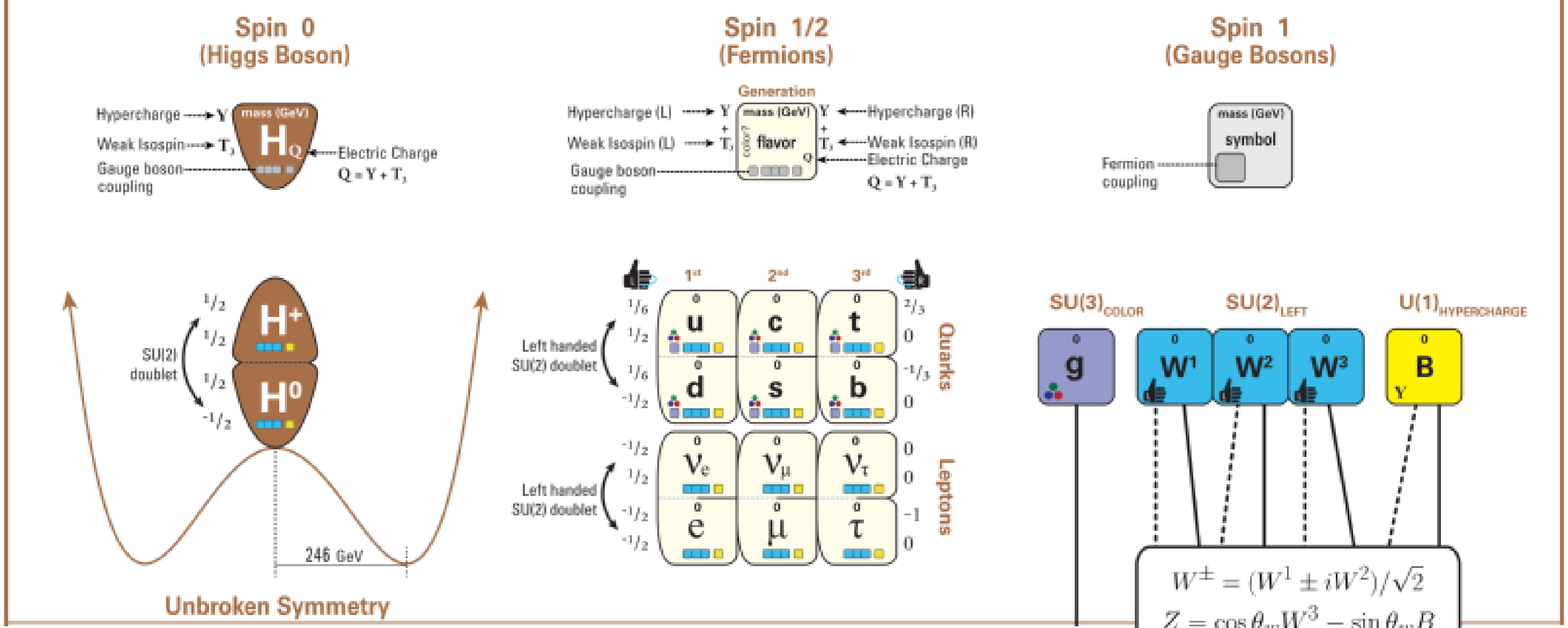


Mesons
(u, d, s, c)

Baryons
(u, d, s, c)



The Standard Model of Particle Physics

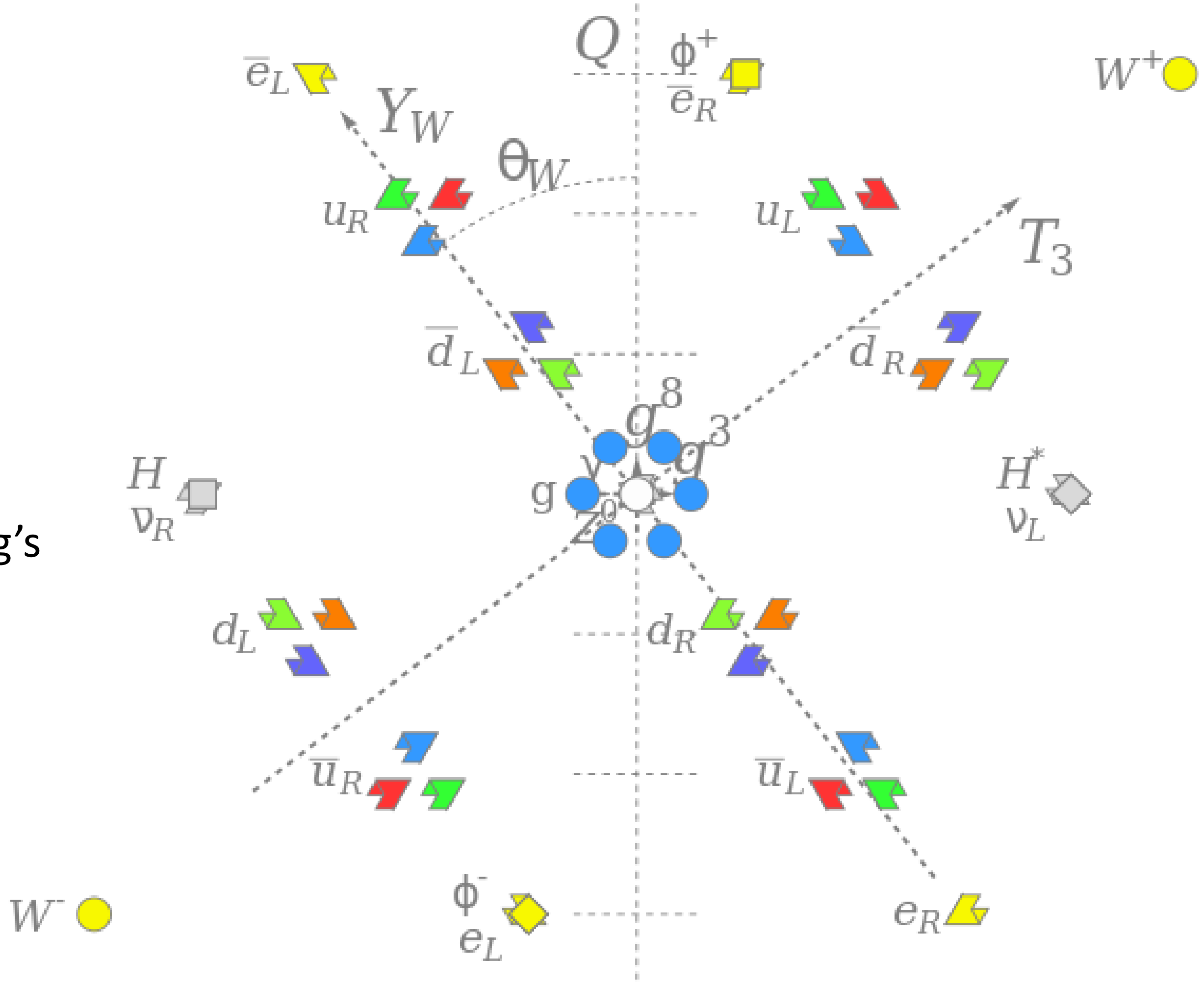


$$W^{\pm} = (W^1 \pm iW^2)/\sqrt{2}$$
$$Z = \cos\theta_w W^3 - \sin\theta_w B$$
$$\gamma = \sin\theta_w W^3 + \cos\theta_w B$$

Still a zoo?

Each lepton has an anti-particle with opposite charge and handedness

Need to include the Higg's bosons



Each quark flavor can have red, green, blue and anti-red, anti-green, anti-blue color charge

They also can be right or left-handed

There are 8 gluons (not just one)

Q – electric charge

Y_w – hypercharge (offset by weak mixing angle)

What we don’t know

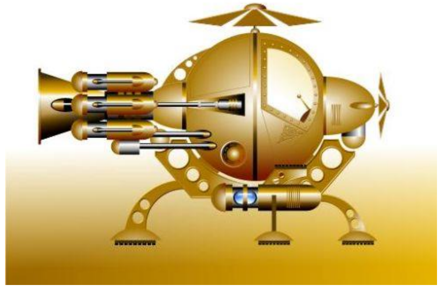
how to make a cloaking device

how to tell a joke

why people don’t laugh at our jokes

how to write pop hits

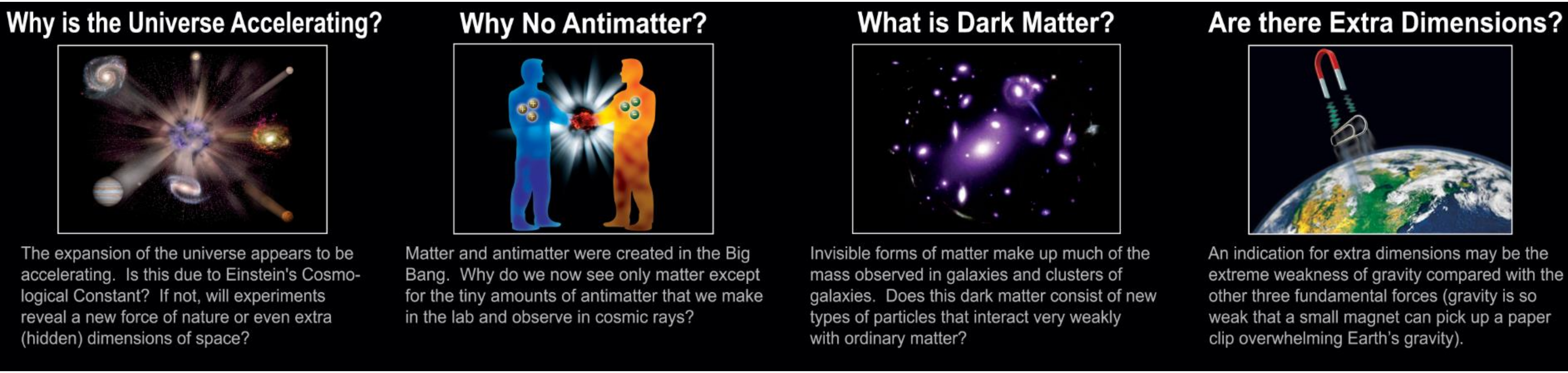
how to make a light saber



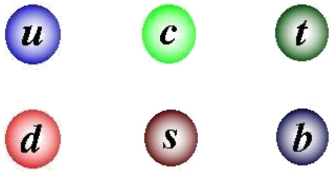
how to make a time machine

why people can’t say nuclear

Why are we testing the SM?

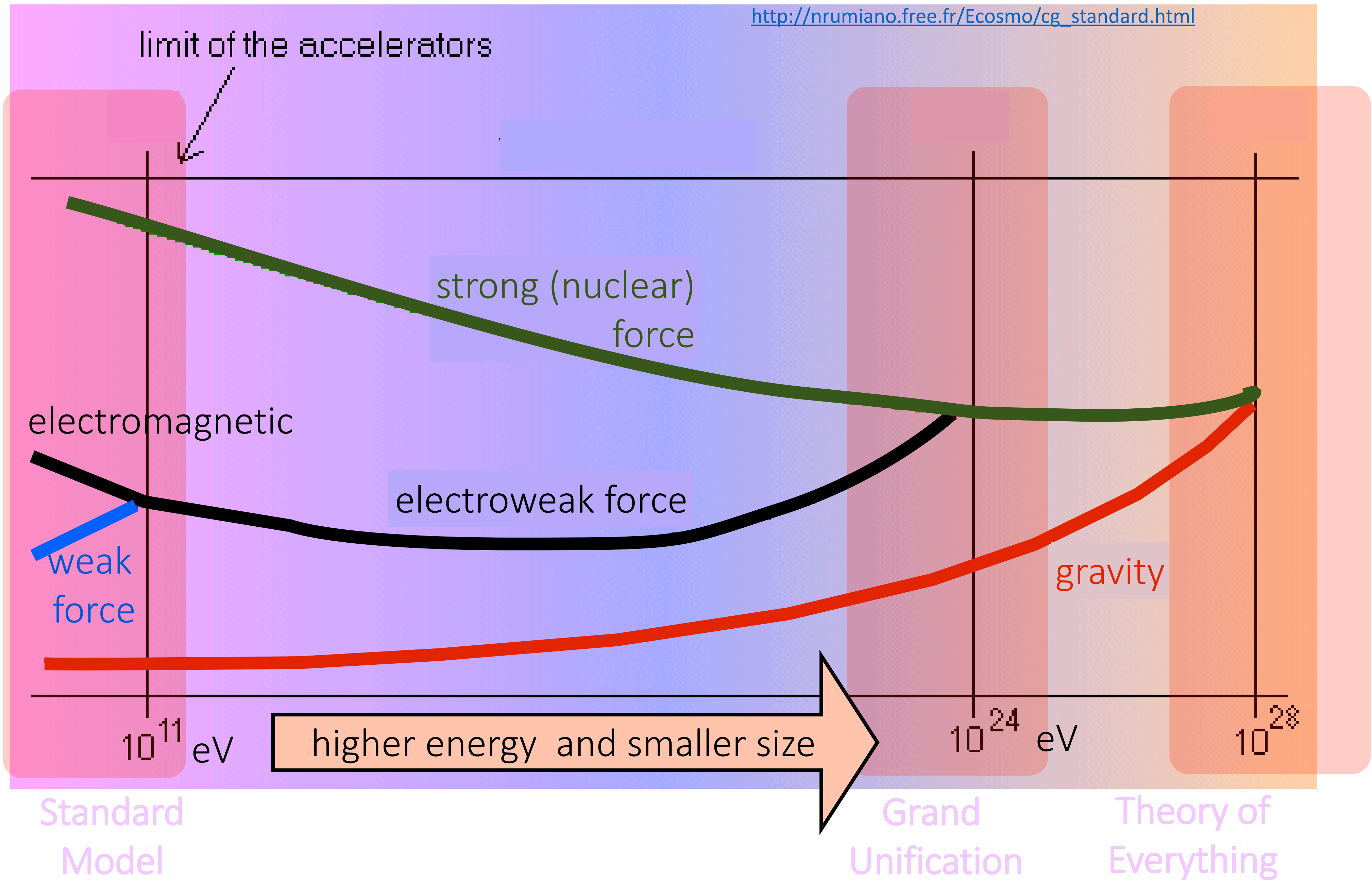


http://www.cpepweb.org/images/Unsolved.jpg



are quarks made of
something smaller

The 4 forces have very different strength — and it changes !



The Standard Model Lagrangian

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{8}tr(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}) - \frac{1}{2}tr(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}) && \text{(U(1), SU(2) and SU(3) gauge terms)} \\
& +(\bar{\nu}_L, \bar{e}_L)\,\tilde{\sigma}^\mu i D_\mu \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} + \bar{e}_R \sigma^\mu i D_\mu e_R + \bar{\nu}_R \sigma^\mu i D_\mu \nu_R + \text{(h.c.)} && \text{(lepton dynamical term)} \\
& -\frac{\sqrt{2}}{v}\left[(\bar{\nu}_L, \bar{e}_L)\,\phi M^e e_R + \bar{e}_R \bar{M}^e \bar{\phi} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}\right] && \text{(electron, muon, tauon mass term)} \\
& -\frac{\sqrt{2}}{v}\left[(-\bar{e}_L, \bar{\nu}_L)\,\phi^* M^\nu \nu_R + \bar{\nu}_R \bar{M}^\nu \phi^T \begin{pmatrix} -e_L \\ \nu_L \end{pmatrix}\right] && \text{(neutrino mass term)} \\
& +(\bar{u}_L, \bar{d}_L)\,\tilde{\sigma}^\mu i D_\mu \begin{pmatrix} u_L \\ d_L \end{pmatrix} + \bar{u}_R \sigma^\mu i D_\mu u_R + \bar{d}_R \sigma^\mu i D_\mu d_R + \text{(h.c.)} && \text{(quark dynamical term)} \\
& -\frac{\sqrt{2}}{v}\left[(\bar{u}_L, \bar{d}_L)\,\phi M^d d_R + \bar{d}_R \bar{M}^d \bar{\phi} \begin{pmatrix} u_L \\ d_L \end{pmatrix}\right] && \text{(down, strange, bottom mass term)} \\
& -\frac{\sqrt{2}}{v}\left[(-\bar{d}_L, \bar{u}_L)\,\phi^* M^u u_R + \bar{u}_R \bar{M}^u \phi^T \begin{pmatrix} -d_L \\ u_L \end{pmatrix}\right] && \text{(up, charmed, top mass term)} \\
& +\overline{(D_\mu \phi)} D^\mu \phi - m_h^2[\bar{\phi}\phi - v^2/2]^2/2v^2. && \text{(Higgs dynamical and mass term)} \quad (1)
\end{aligned}
$$

https://www.quora.com/What-is-the-Lagrangian-of-the-Standard-Model

The SM Lagrangian

1 Specific to gluons
2 Interactions between bosons
3 Interactions via the weak force
4 Ghosts
5 Faddeev-Popov ghosts (weak force)

1

$$-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc}f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \frac{1}{2} i g_s^2 (\bar q^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu + \bar G^a \partial^2 G^a + g_s f^{abc}\partial_\mu \bar G^a G^b g^c_\mu$$

2

$$- \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \frac12 \partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \frac12 \partial_\mu A_\nu \partial_\mu A_\nu - \frac12 \partial_\mu H \partial_\mu H - \frac12 m_h^2 H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - M^2\phi^+\phi^- - \frac12\partial_\mu\phi^0\partial_\mu\phi^0 - \frac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h\left[\frac{2M^2}{g^2} + \frac{2M}{g}H + \frac12(H^2+\phi^0\phi^0+2\phi^+\phi^-)\right] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \frac12 g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac12 g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \frac18 g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2] - gMW^+_\mu W^-_\mu H - \frac12 g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \frac12 ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \frac12 g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)] + \frac12 g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_wMA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac14 g^2 W^+_\mu W^-_\mu[H^2 + (\phi^0)^2 + 2\phi^+\phi^-] - \frac14 g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2 + (\phi^0)^2 + 2(2s_w^2-1)^2\phi^+\phi^-] - \frac12 g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \frac12 ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac12 g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) + \frac12 ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- - g^1 s_w^2 A_\mu A_\mu\phi^+\phi^-$$

$$- \bar e^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar\nu^\lambda\gamma\partial\nu^\lambda - \bar u^\lambda_j(\gamma\partial+m_u^\lambda)u^\lambda_j -$$

https://www.symmetrymagazine.org/article/the-deconstructed-standard-model-equation

$$g^1 s_w^2 A_\mu A_\mu\phi^+\phi^- - \bar e^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar\nu^\lambda\gamma\partial\nu^\lambda - \bar u^\lambda_j(\gamma\partial+m_u^\lambda)u^\lambda_j -$$

3

$$\bar d^\lambda_j(\gamma\partial + m_d^\lambda)d^\lambda_j + igs_wA_\mu\left[-(\bar e^\lambda\gamma^\mu e^\lambda) + \frac23(\bar u^\lambda_j\gamma^\mu u^\lambda_j) - \frac13(\bar d^\lambda_j\gamma^\mu d^\lambda_j)\right] + \frac{ig}{4c_w}Z^0_\mu\left[(\bar\nu^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar e^\lambda\gamma^\mu(4s_w^2-1-\gamma^5)e^\lambda) + (\bar u^\lambda_j\gamma^\mu(\tfrac43 s_w^2 - 1 - \gamma^5)u^\lambda_j) + (\bar d^\lambda_j\gamma^\mu(1-\tfrac83 s_w^2-\gamma^5)d^\lambda_j)\right] + \frac{ig}{2\sqrt2}W^+_\mu[(\bar\nu^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) + (\bar u^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \frac{ig}{2\sqrt2}W^-_\mu[(\bar e^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar d^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1+\gamma^5)u^\lambda_j)]$$

4

$$+\frac{ig}{2\sqrt2}\frac{m_e^\lambda}{M}[-\phi^+(\bar\nu^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar e^\lambda(1+\gamma^5)\nu^\lambda)] - \frac{g}{2}\frac{m_e^\lambda}{M}[H(\bar e^\lambda e^\lambda) + i\phi^0(\bar e^\lambda\gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt2}\phi^+[-m_d^\kappa(\bar u^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m_u^\lambda(\bar u^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j)] + \frac{ig}{2M\sqrt2}\phi^-[m_d^\lambda(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m_u^\kappa(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j)] - \frac g2\frac{m_u^\lambda}{M}H(\bar u^\lambda_j u^\lambda_j) - \frac g2\frac{m_d^\lambda}{M}H(\bar d^\lambda_j d^\lambda_j) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar u^\lambda_j\gamma^5u^\lambda_j) - \frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar d^\lambda_j\gamma^5 d^\lambda_j)$$

5

$$+\bar X^+(\partial^2 - M^2)X^+ + \bar X^-(\partial^2-M^2)X^- + \bar X^0\left(\partial^2 - \frac{M^2}{c_w^2}\right)X^0 + \bar Y\partial^2 Y + igc_wW^+_\mu(\partial_\mu\bar X^0X^- - \partial_\mu\bar X^+X^0) + igs_wW^+_\mu(\partial_\mu\bar YX^- - \partial_\mu\bar X^+Y) + igc_wW^-_\mu(\partial_\mu\bar X^-X^0 - \partial_\mu\bar X^0X^+) + igs_wW^-_\mu(\partial_\mu\bar X^-Y - \partial_\mu\bar YX^+) + igc_wZ^0_\mu(\partial_\mu\bar X^+X^+ - \partial_\mu\bar X^-X^-) + igs_wA_\mu(\partial_\mu\bar X^+X^+ - \partial_\mu\bar X^-X^-) - \frac12 gM\left[\bar X^+X^+H + \bar X^-X^-H + \frac{1}{c_w^2}\bar X^0X^0H\right] + \frac{1-2c_w^2}{2c_w}igM[\bar X^+X^0\phi^+ - \bar X^-X^0\phi^-] + \frac{1}{2c_w}igM[\bar X^0X^-\phi^+ - \bar X^0X^+\phi^-] + igMs_w[\bar X^0X^-\phi^+ - \bar X^0X^+\phi^-] + \frac12 igM[\bar X^+X^+\phi^0 - \bar X^-X^-\phi^0]$$

Why parity-violating electron scattering (PVES)?

- Search for physics ***Beyond the Standard Model*** (BSM) with low energy ($Q^2 << M^2$) precision tests complementary to high energy measurements

  - ***Neutrino mass and their role in the early universe*** — 0νββ *decay*, θ_{13}, β *decay,…*
  - ***Matter-antimatter asymmetry in the present universe*** — *EDM, DM, LFV,* 0νββ, θ_{13}
  - ***Unseen Forces of the Early Universe*** — *Weak decays,* ***PVES****,* g_μ*-2,…*

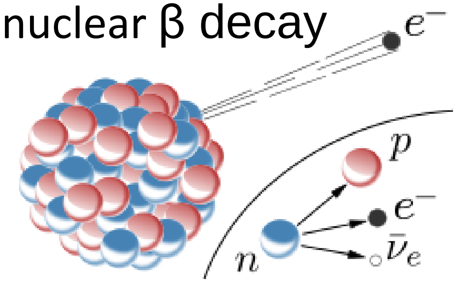
- ***LHC new physics signals likely will need additional indirect evidence***

  - ***Neutrons:*** *Lifetime, P- & T-Violating Asymmetries (LANSCE, NIST, SNS...)*
  - ***Muons:*** *Lifetime, Michel parameters, g-2, Mu2e (PSI, TRIUMF, FNAL, J-PARC...)*
  - ***PVES:*** *Low energy weak neutral current couplings, precision weak mixing angle (SLAC, Jefferson Lab, Mainz)*

- Study nuclear and nucleon properties
  - Strange quark content of nucleon
  - Neutron radii of heavy nuclei

A brief history of parity violation

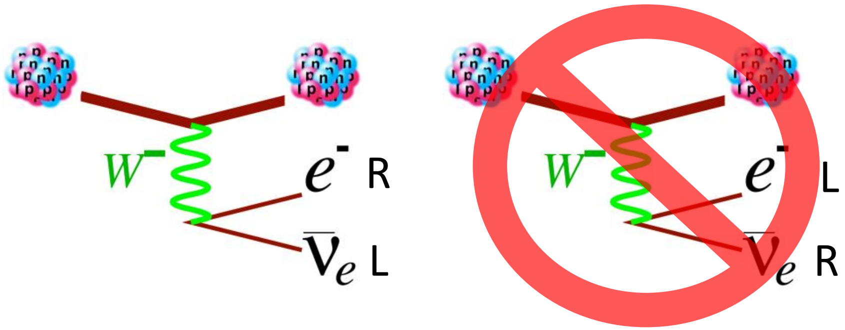
1930s – weak interaction needed to explain nuclear β decay

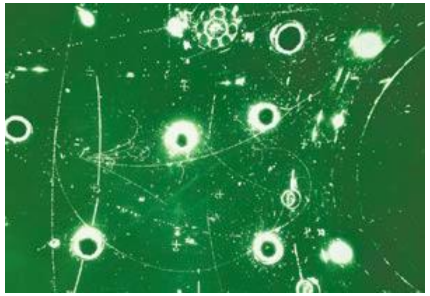


1950s – parity violation in weak interaction;

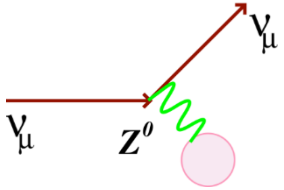
V-A theory

to describe ^{60}Co decay





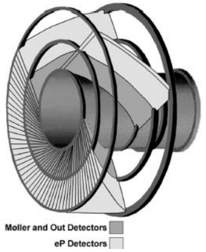
1970s – neutral weak current events seen at Gargamelle



late 1970s – parity violation observed in electron scattering - SLAC E122

1996 – atomic parity violation observed in atomic transitions of Cs

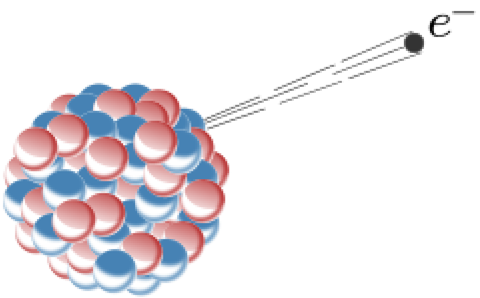
2005 – first measurement of the weak charge of the electron, Q_W^e

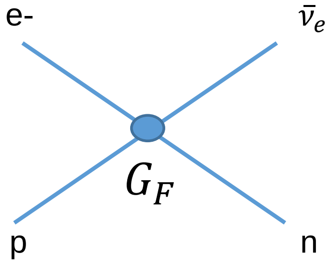


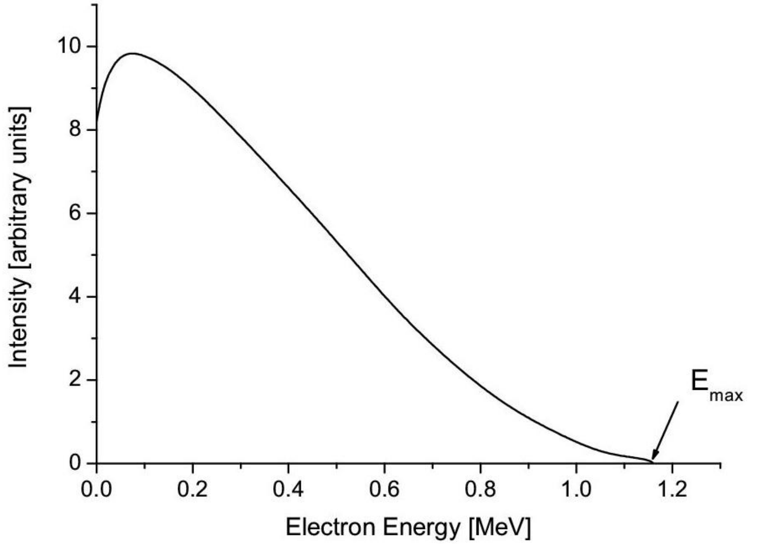
2013 – first measurement of the weak charge of the proton, Q_W^p

Precision Frontier

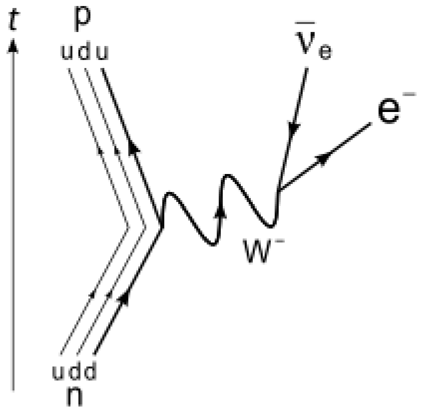
Nuclear beta decay



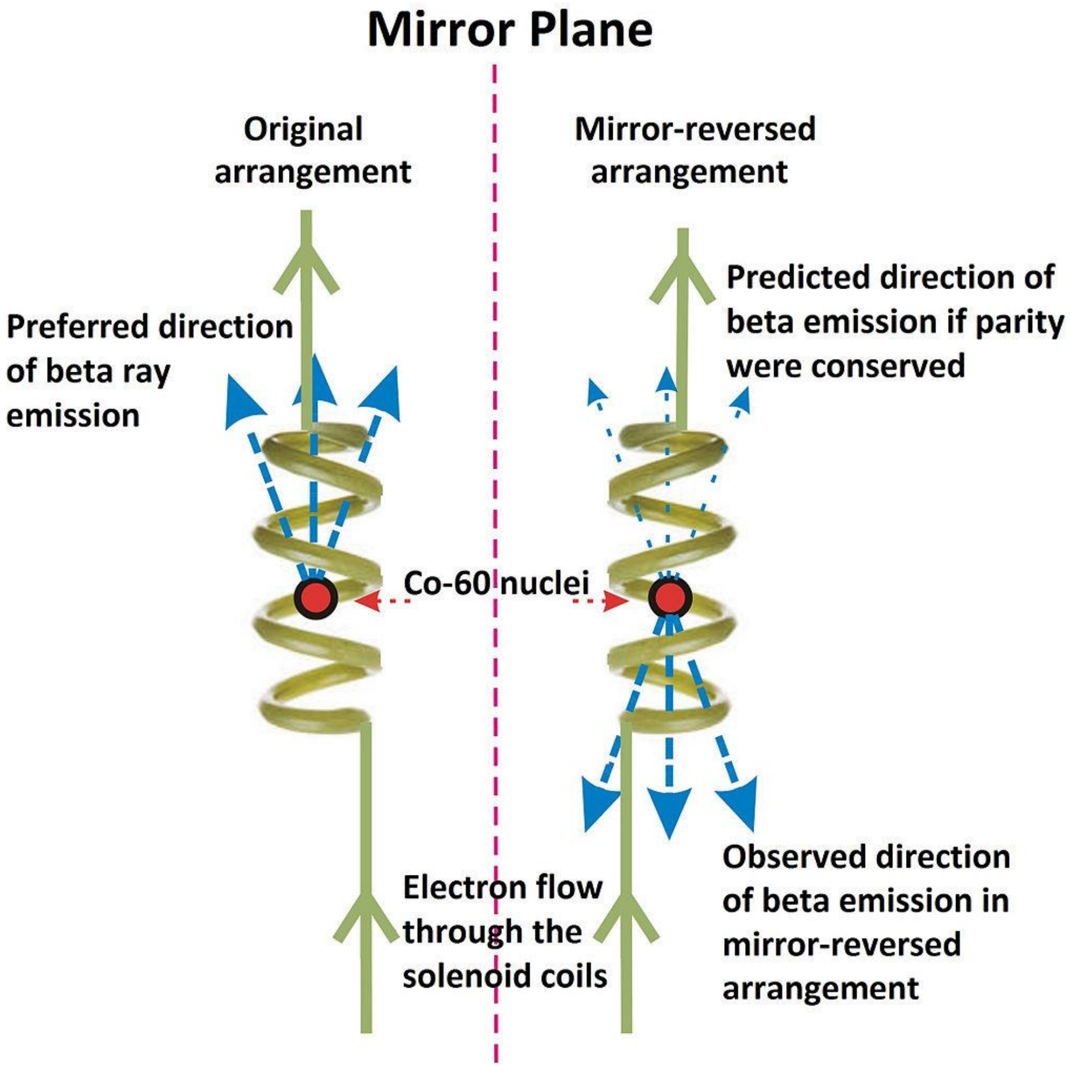




G_F



Parity-violation in charge current - maximal



Bleckneuhaus, with English language captions by Stigmatella aurantiaca

$$^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e + 2\gamma$$

electrons favored the direction opposite to that of the nuclear spin

Madam Wu

What about a neutral weak current?

LETTERS TO THE EDITOR

PARITY NONCONSERVATION IN THE FIRST ORDER IN THE WEAK-INTERACTION CONSTANT IN ELECTRON SCATTERING AND OTHER EFFECTS

Ya. B. ZEL' DOVICH

Submitted to JETP editor December 25, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 964-966 (March, 1959)

WE assume that besides the weak interaction that causes beta decay,

$$g\,(\bar{P}ON)\,(\bar{e}^-O\nu) + \text{Herm. conj.}, \quad (1)$$

there exists an interaction

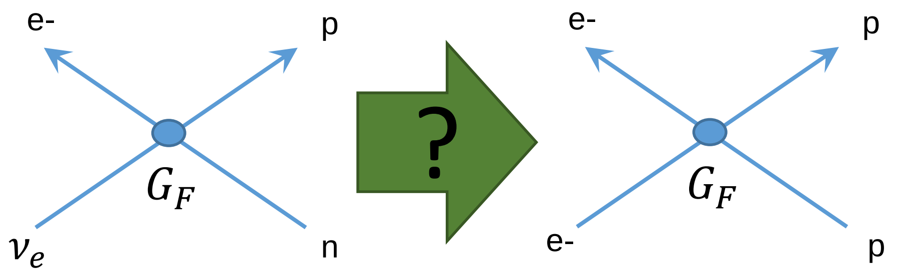
$$g\,(\bar{P}OP)\,(\bar{e}^-Oe^-) \quad (2)$$

with $g \approx 10^{-49}$ and the operator $O = \gamma_\mu(1 + i\gamma_5)$ characteristic[1] of processes in which parity is not conserved.*

Zel'dovich – 1959

Is there a neutral analog to β decay?

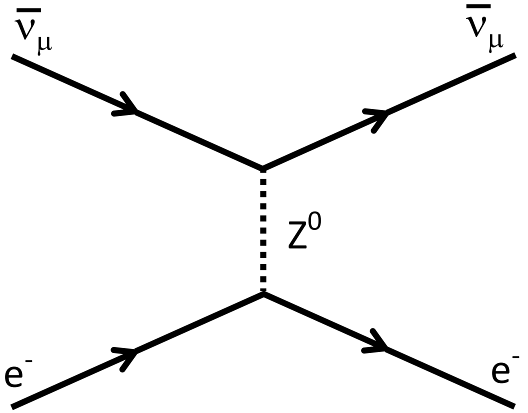
Would determine the sign of G_F



Neutral weak currents observed

The prediction of the Z^0 implied the existence of previously unobserved neutral current processes like:

$$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$$



These processes were first discovered in 1973:

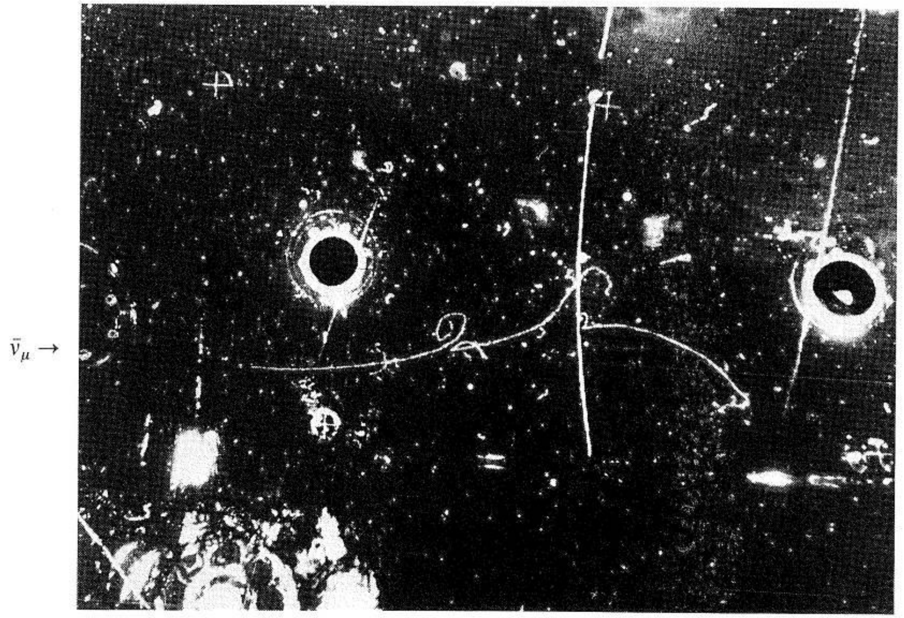


Fig. 6.19 One of the historic pictures of high-energy physics. The event was found in the bubble chamber Gargamelle, filled with freon (CF_3Br) and exposed to an antineutrino ($\bar{\nu}_\mu$) beam at the CERN PS (Hasert *et al.* 1973). It consists of a single electron of energy 400 MeV projected at an angle of $1.5 \pm 1.5°$ to the beam (see Problem 6.13). It is identified by the characteristic bremsstrahlung energy loss and pair production along the track. The event constituted the first evidence for a weak neutral current in the process $\bar{\nu}_\mu + e^- \rightarrow e^- + \bar{\nu}_\mu$. A total of three such events were observed in 1.4 million pictures (with $\sim 10^9$ antineutrinos per pulse) over a two-year period. Even today (1980) only about 100 events exist from six experiments.

What about parity violation?

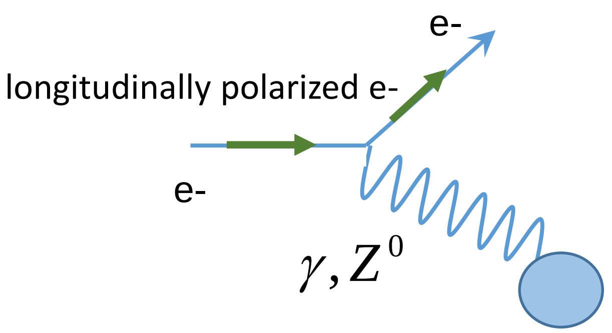
Parity-violating asymmetry

Then in the scattering of electrons by protons the interaction (2) will interfere with the Coulomb scattering, and the nonconservation of parity will appear in terms of the first order in the small quantity g. Owing to this it becomes possible to test the hypothesis used here experimentally and to determine the sign of g.

The matrix element of the Coulomb scattering is of the order of magnitude e^2/k^2, where k is the momentum transferred ($\hbar = c = 1$). Consequently, the ratio of the interference term to the Coulomb term is of the order of gk^2/e^2.

In the scattering of fast ($\sim 10^9$ ev) longitudinally polarized electrons through large angles by unpolarized target nuclei it can be expected that the cross-sections for right-hand and left-hand electrons (i.e., for electrons with $\sigma \cdot p > 0$ and $\sigma \cdot p < 0$) can differ by 0.1 to 0.01 percent. Such an effect is a specific test for an interaction not conserving parity.

parity non-conservation via weak – EM interference



parity-violating asymmetry

$$A_{PV} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} \approx \frac{M_{weak}}{M_\gamma}$$

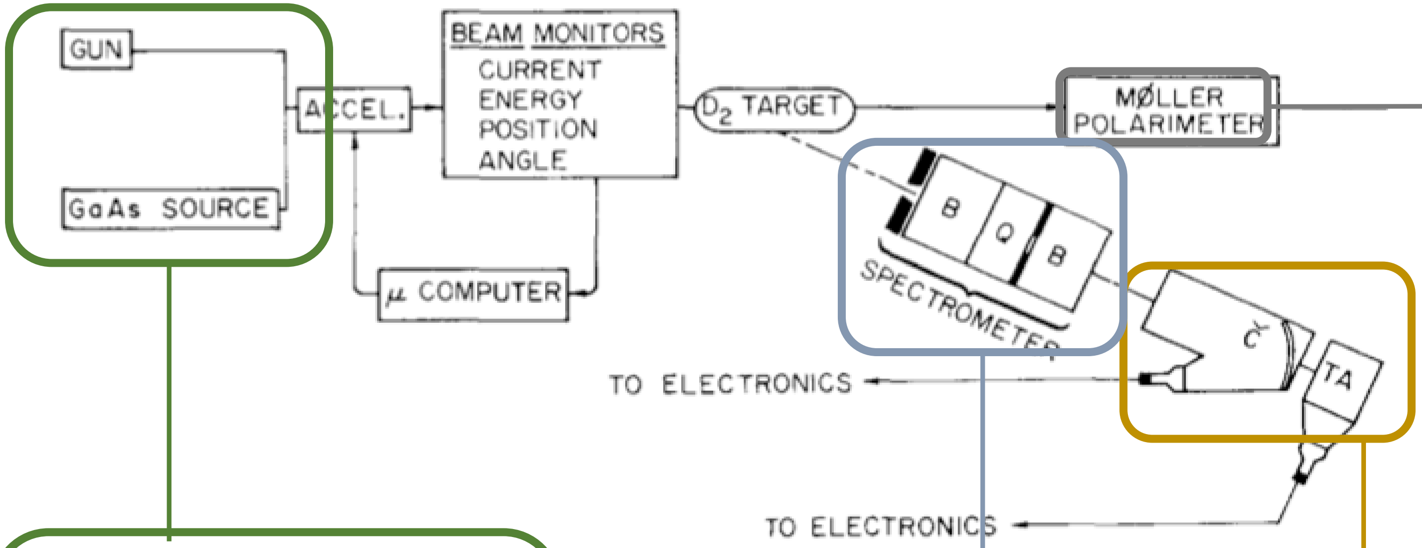
$$\approx \frac{G_F Q^2}{4\pi\alpha}$$

Four drops of ink in a 55-gallon barrel of water would produce an "ink concentration" of 1 ppm!!!

$$Q^2 \sim 0.1 - 1\ GeV^2$$

$$A_{PV} \leq 10^{-6} - 10^{-4}$$

SLAC Experiment E122

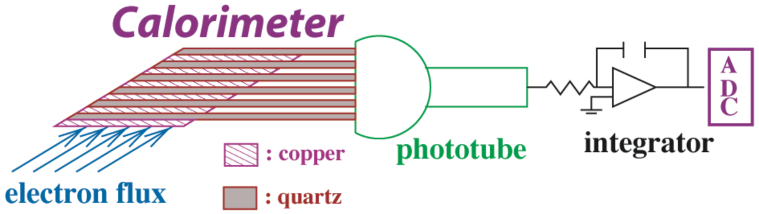


Polarimetry

- High luminosity from photoemission from NEA GaAs cathode
- Rapid helicity-flip (sign of e- polarization)

Magnetic spectrometer Background and kinematic separation

Integrating detectors



Huge achievement!
Highest P^2I ever, by far. Developed for this experiment at SLAC and used ever since

SLAC Experiment E122

*Parity Non-Conservation in Inelastic Electron Scattering, **C.Y. Prescott et. al, 1978***

	Left	Right
γ Charge	0, ±1, ±1/3, ±2/3	0, ±1, ±1/3, ±2/3
W Charge	T=±1/2	0
Z Charge	T-qsin$^2\theta_W$	-qsin$^2\theta_W$

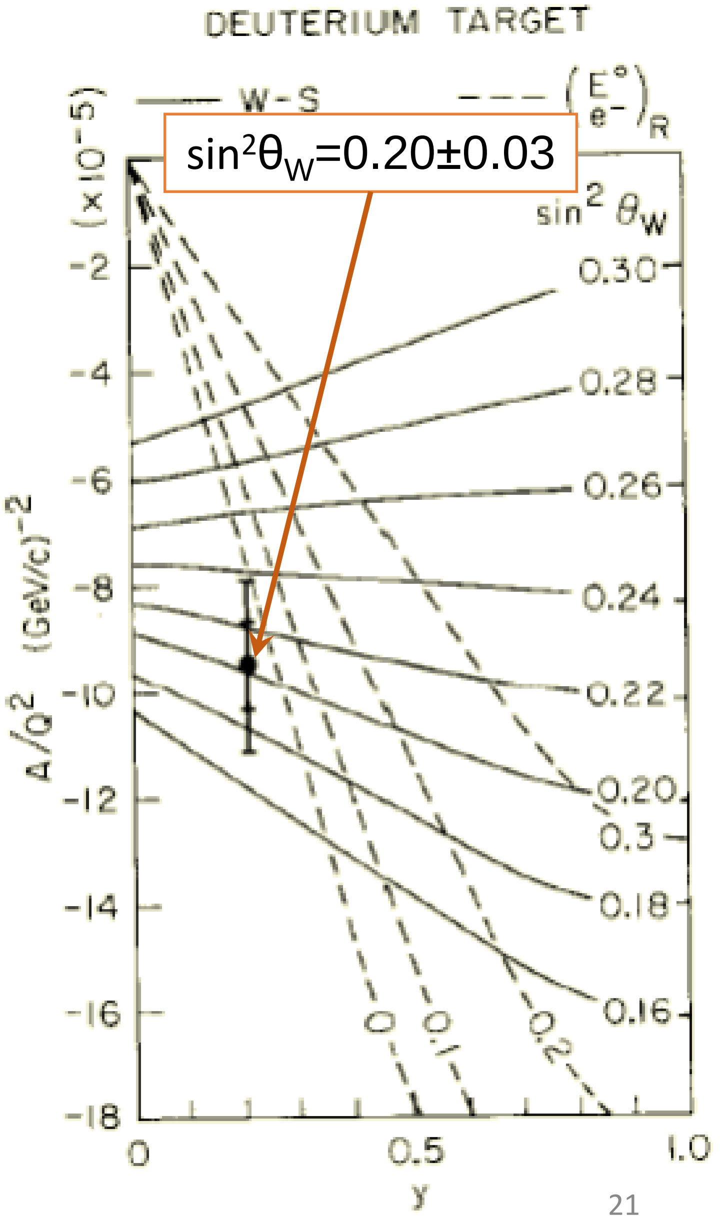
A_{PV} ~ 100 ± 10 ppm

GWS --- Nobel Prize 1979

Deep inelastic scattering:
Y dependence reflects quark axial/electron vector coupling strength

$$A_{PV} \propto \frac{3G_F Q^2}{10\sqrt{2}\pi\alpha}\left[a(x)+b(x)Y\right]$$

At high x
$$a(x) = 2g_V^u g_A^e - g_V^d g_A^e$$
$$b(x) = 2g_A^u g_V^e - g_A^d g_V^e$$



Standard Model of Electroweak Interactions

Glashow-Weinberg-Salam Model (1967): unified EM and weak forces as an electroweak force

$\rightarrow$ $SU(2)_L$ x $U(1)$ gauge theory with spontaneous symmetry breaking

The Nobel Prize in Physics 1979

"for their contributions to the theory of the unified weak and electromagnetic interaction between elementary particles, including, inter alia, the prediction of the weak neutral current"

Sheldon Lee Glashow

1/3 of the prize

USA

Harvard University, Lyman Laboratory Cambridge, MA, USA

b. 1932

Abdus Salam

1/3 of the prize

Pakistan

International Centre for Theoretical Physics Trieste, Italy; Imperial College London, United Kingdom

b. 1926
d. 1996

Steven Weinberg

1/3 of the prize

USA

Harvard University Cambridge, MA, USA

b. 1933

$\sin^2\theta_W$ – "weak mixing angle", parameterizes the mixing between the two neutral currents

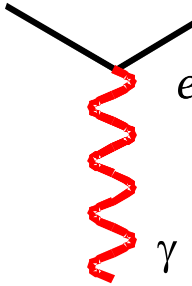
$$\begin{pmatrix} \gamma \\ Z^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B^0 \\ W^0 \end{pmatrix}$$

fermions:

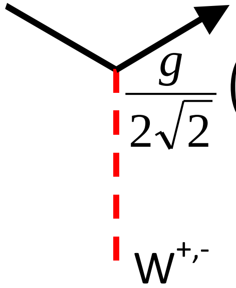
$$\begin{pmatrix} \upsilon_e \\ e^- \end{pmatrix}_L , \begin{pmatrix} u \\ d' \end{pmatrix}_L , \ldots$$

$$e_R^- , u_R , d'_R , \ldots$$

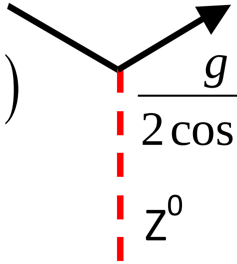
Interaction of fermions with gauge bosons:



$e\gamma^\mu$



$$\frac{g}{2\sqrt{2}} \left(\gamma^\mu - \gamma^\mu \gamma^5\right)$$



$$\frac{g}{2\cos\theta_W} \left(c_V^f \gamma^\mu - c_A^f \gamma^\mu \gamma^5\right)$$

$$c_V^f = t_3^f - 2\sin^2\theta_W \, Q_f$$

$$c_A^f = t_3^f$$

The Standard Model

is a

$$SU(3) \times SU(2) \times \mathrm{U}(1)$$

Strong Weak EM

Gauge Theory

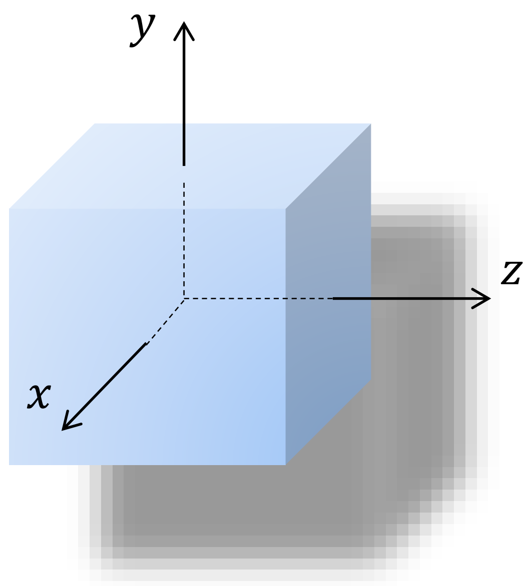
What the heck does that mean?

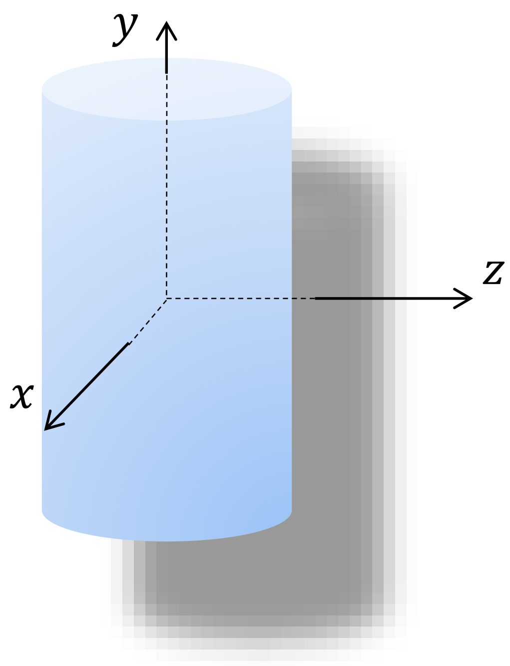
How does that relate to the other things we've talked about?

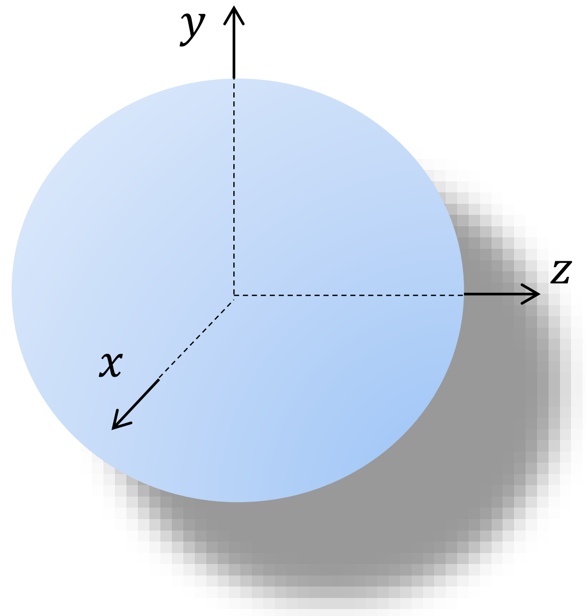
What is a “gauge”?

- A thing that measures
- The coordinate system we choose
  - Transforms a certain way
- What is the meaning of
  - contravariant?
  - covariant?

Symmetries and hidden variables







What kinds of symmetry does this object have?

What about this one?

Or this one?

How would I describe the translation of the objects?

What about a translation or rotation of the coordinates instead of the objects?

Symmetries and Conservation Laws

Conservation laws imply symmetries

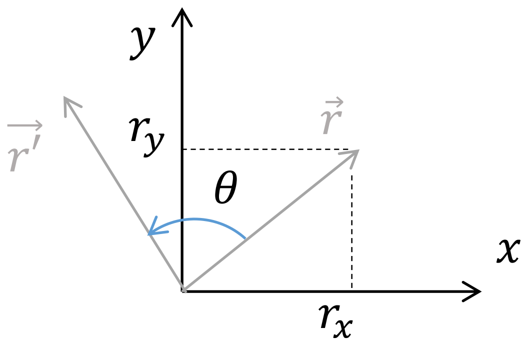
Conservation of:	Implies:
Energy	Time invariance
Linear momentum	Translational invariance
Angular momentum	Rotational invariance

Symmetries imply "ignorable" coordinates. We look for conserved quantities in order to identify the coordinates that we can eliminate to reduce the dimension of the system

Emmy Noether (1882-1935)

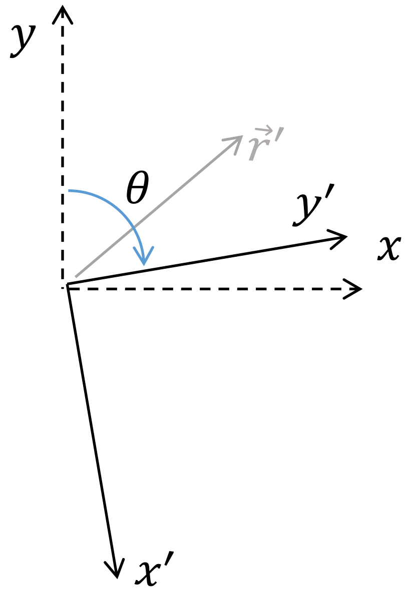
Rotation matrix (2D)

$$\vec{r} = r_x\hat{\imath} + r_y\hat{\jmath}$$



Active — vector orientation changes (use to describe motion of a point)

$$\vec{r'} = R\vec{r} \qquad R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$



Passive — rotate the coordinate axes instead – transform the basis vectors with the inverse of the rotation matrix R

Note: Could use x, y *or* r, θ coords depending on the *symmetry* of the system (Translate between with matrices that change the basis)

Tables, chairs and beer mugs

Hilbert is credited with saying "One must be able to say at all times – instead of points, straight lines and planes – tables, chairs and beer mugs."

"Vectors" can refer to more than just positions in Euclidean or other geometries; abstract vector spaces are sets that are closed under finite vector addition and scalar multiplication

He knew Minkowski and Einstein; he is also quoted as saying that "physics is too hard for physicists" – he worked to bring mathematical rigor to physics

David Hilbert (1862-1943)

Gauge Theory

- U(n) is the group of $n \times n$ unitary matrices with a defined matrix multiplication
  - In the simple case $n = 1$, the group U(1) corresponds to 1x1 (complex) unitary matrices (complex numbers with absolute value 1)
  - Could be parameterized by the angle theta in the complex plane
- SU(n) Lie group of $n \times n$ (complex) unitary matrices with determinant 1
  - Pauli matrices* form the basis of SU(2)
  - Gell-Man matrices* form the basis of SU(3)
  - $A^{-1} = A^{\dagger}$ and determinant = 1
- SO(n) is the group of all real rotations about the origin of *n*D Euclidean space
  - $A^{-1} = A^{T}$ and determinant = 1

* traceless, Hermitian (so they can generate unitary matrix group elements through exponentiation)

The Basis Vectors

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Pauli matrices

Gell-Mann matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

$$\begin{array}{ll} (r\bar{b} + b\bar{r})/\sqrt{2} & -i(r\bar{b} - b\bar{r})/\sqrt{2} \\ (r\bar{g} + g\bar{r})/\sqrt{2} & -i(r\bar{g} - g\bar{r})/\sqrt{2} \\ (b\bar{g} + g\bar{b})/\sqrt{2} & -i(b\bar{g} - g\bar{b})/\sqrt{2} \\ (r\bar{r} - b\bar{b})/\sqrt{2} & (r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6}. \end{array}$$

What is unitarity and why is it required?

$$\langle \phi | \psi \rangle = \langle \phi' | \psi' \rangle$$

Observables are actually the product of states
Need to be able to recover the observable after a change of basis

The adjoint times the operator must be 1:

$$U^{\dagger} U = 1$$

CKM matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Spontaneous symmetry breaking

Why do the weak bosons have mass?

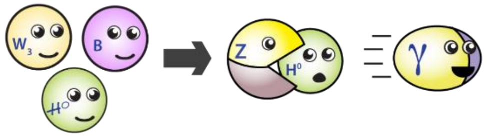
$$\begin{pmatrix} W^1 \\ W^2 \end{pmatrix}, \begin{pmatrix} W^3 \\ B^0 \end{pmatrix} \qquad \begin{pmatrix} W^+ \\ W^- \\ Z^0 \end{pmatrix}, \gamma$$

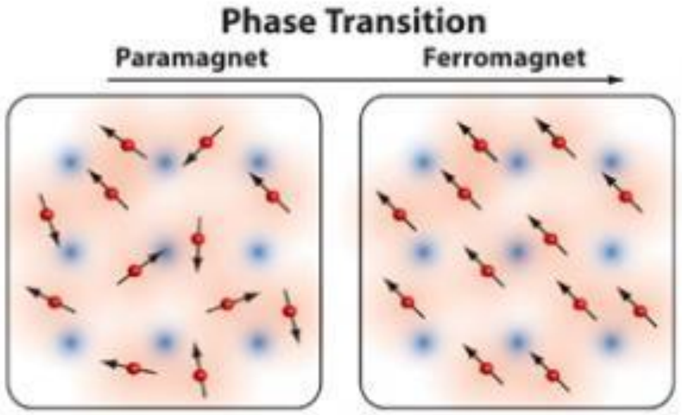
Higgs mechanism

$$\begin{pmatrix} \gamma \\ Z^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B^0 \\ W^0 \end{pmatrix}$$

Higgs field – scalar (not a vector) field that permeates all of space

As universe cooled, symmetry was broken and 3 of the electroweak bosons absorbed 3 of the Higgs bosons, gaining mass





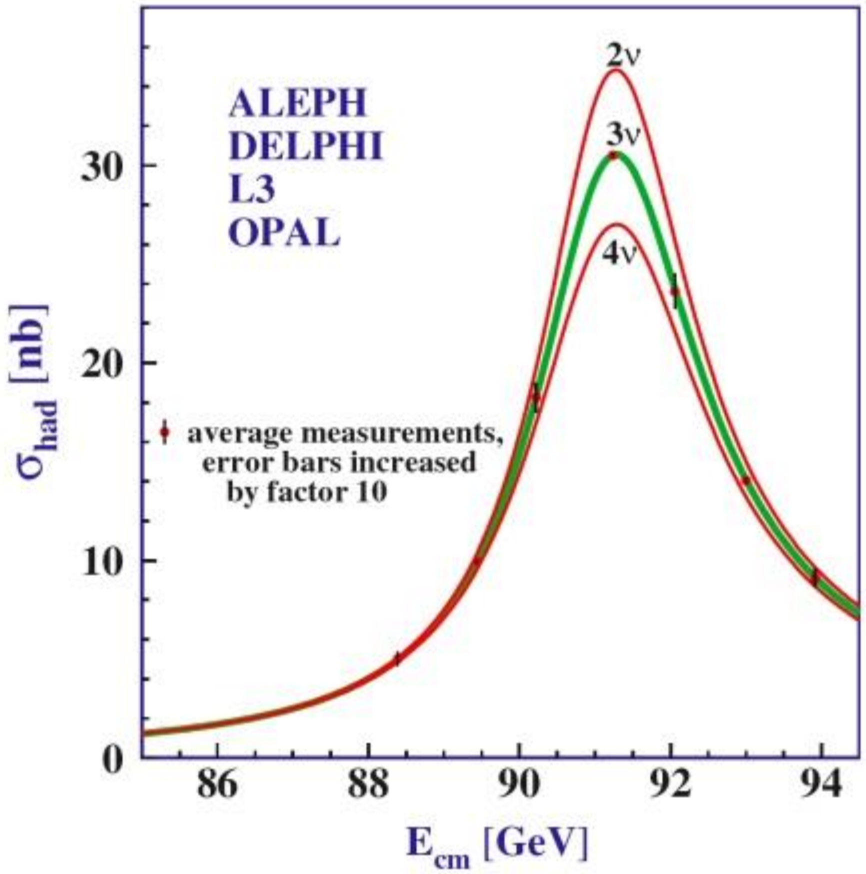
but leaving the photon massless

and one Higgs boson to be discovered at CERN

High energy - width of the Z^0

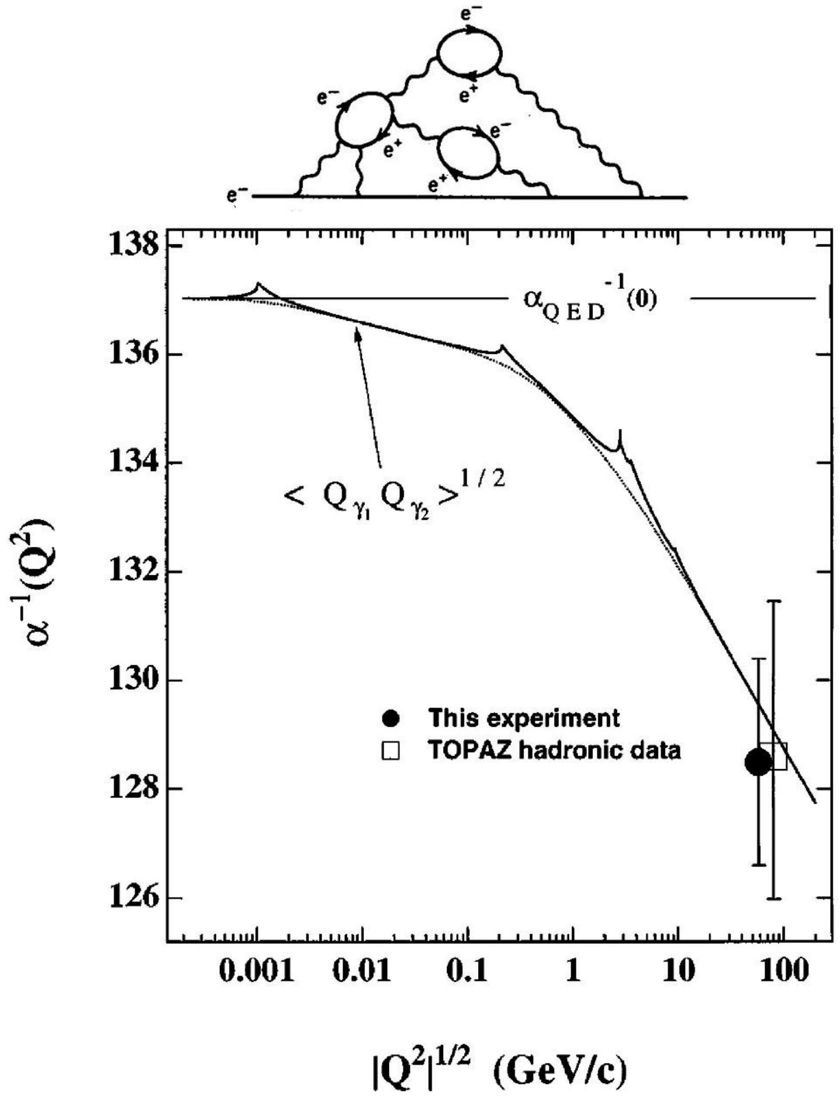
Production of real Z^0 bosons in e^+e^- annihilation

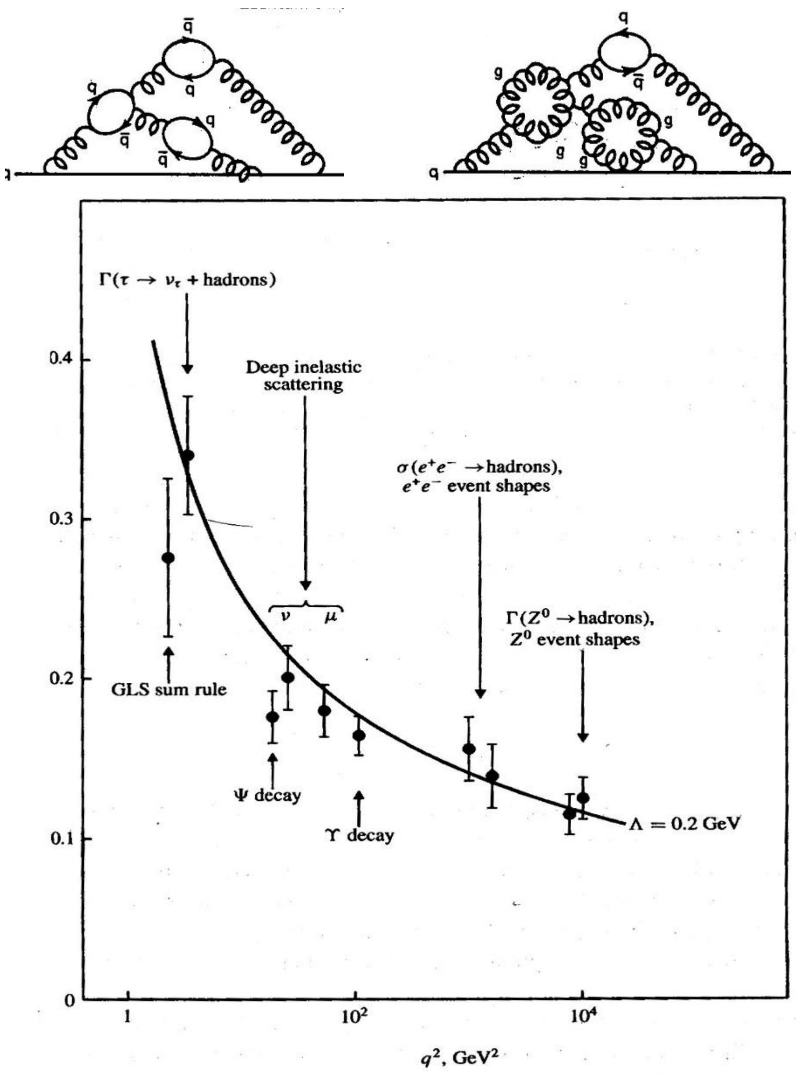
$$\Gamma_{tot}(Z^0) = \sum_{all\ fermions\ f} \Gamma\left(Z^0 \to f\bar{f}\right)$$



- Measure a variety of electroweak processes with couplings to all possible fermions
- Extract values of $(\sin^2\theta_W)_{eff}$ in a consistent renormalization scheme from all processes

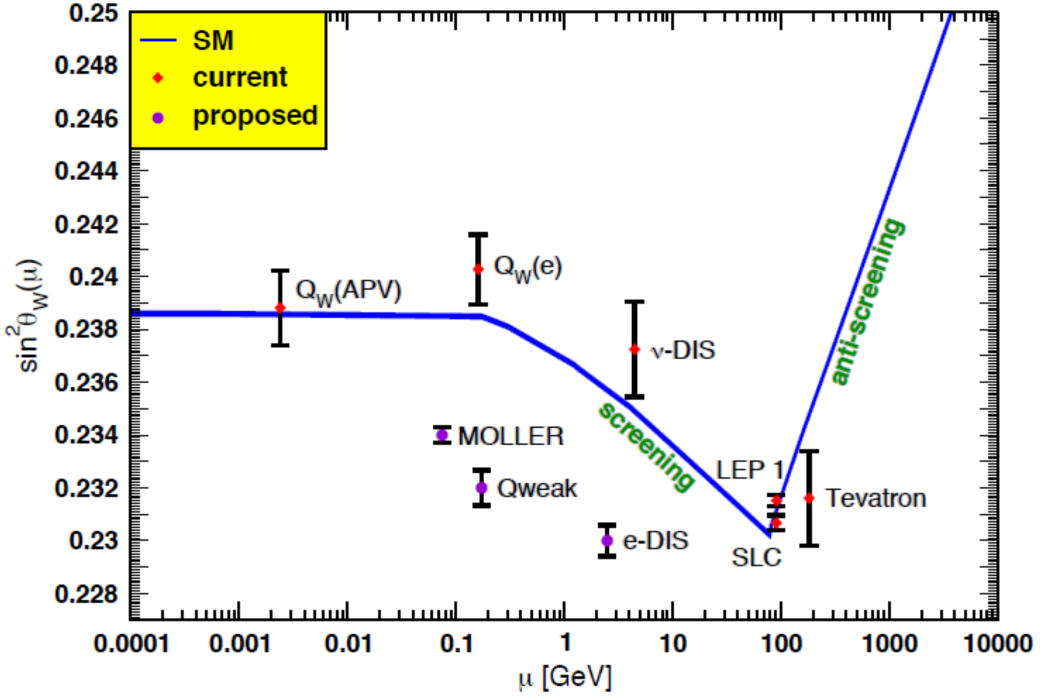
Running of coupling constants

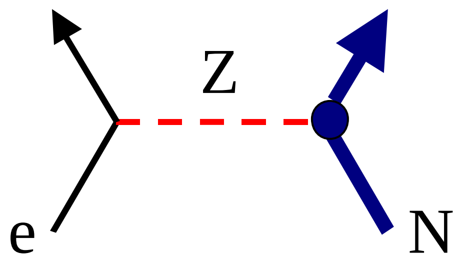


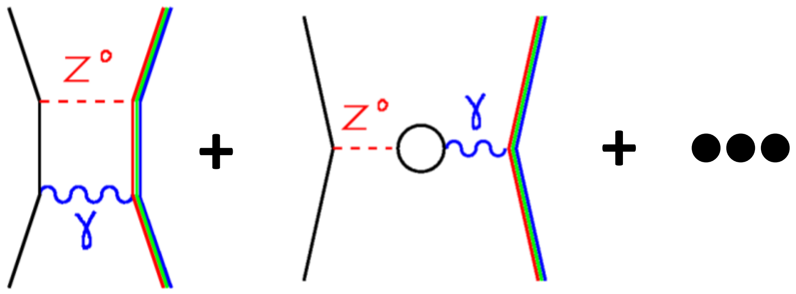


What about $\sin^2\theta_W$?

Running of $\sin^2\theta_W$



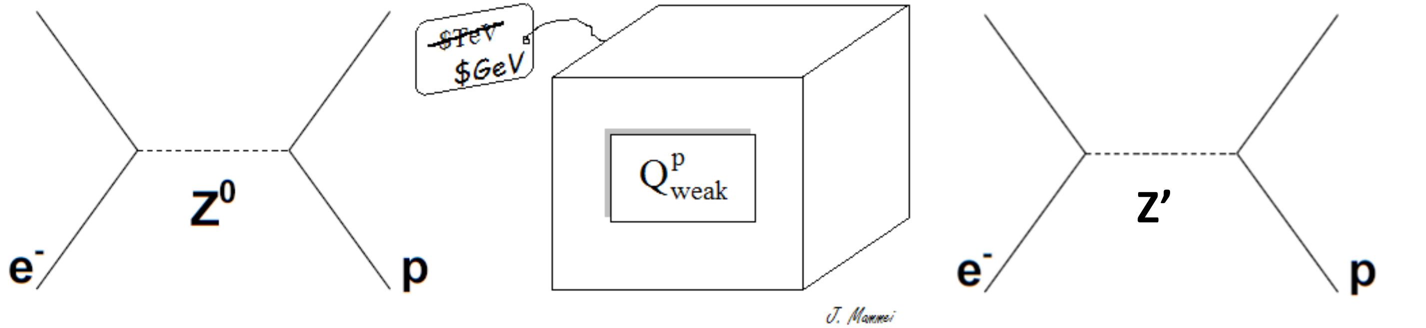




We know how to calculate this within the SM for the particles we know about…

Sensitivity to New Physics

Precision measurements well below the Z-pole have more sensitivity (for a given experimental precision) to new types of tree level physics, such as additional heavier Z' bosons.



$$A_Z \propto \frac{g^2}{-q^2 + M_Z^2 + iM_Z\Gamma_Z}$$

$$A_{Z'} \propto \frac{g^2}{-q^2 + M_{Z'}^2 + iM_{Z'}\Gamma_{Z'}} \xrightarrow{q^2 << M_{Z'}^2} \frac{g^2}{M_{Z'}^2}$$

$$\text{At Z-pole, } q^2 \sim M_Z^2,\ A \sim \frac{1}{M_Z\Gamma_Z} + \frac{1}{M_{Z'}^2}, \quad \sim 0.1\%\ \text{precision} \rightarrow M_{Z'} < 500\,\text{GeV}$$

$$\text{At low energy, } q^2 << M_Z^2,\ A \sim \frac{1}{M_Z^2} + \frac{1}{M_{Z'}^2}, \quad \sim 0.1\%\ \text{precision} \rightarrow M_{Z'} < 2.5\,\text{TeV}$$

Low Energy Weak Neutral Current SM Tests

Low energy weak charge "triad" (M. Ramsey-Musolf) probed in weak neutral current experiments

parity-violating
Moller scattering
$\vec{e} + e \rightarrow e + e$

$$Q_W^e \approx -(1 - 4\sin^2\theta_W)$$

SLAC E158 ~6σ

MOLLER ~25σ

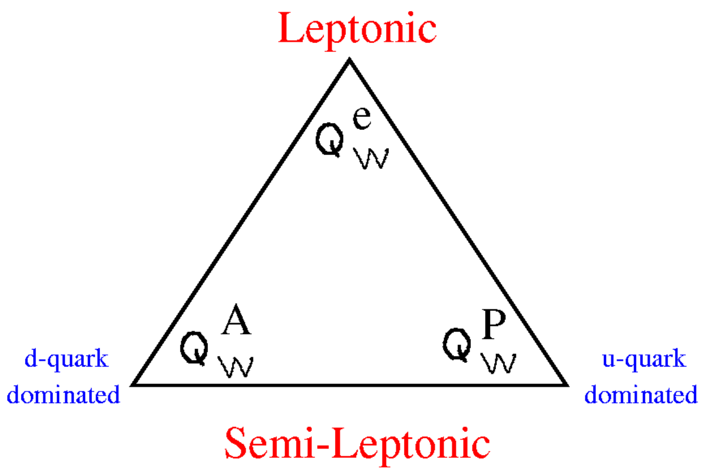
a complementary set for exploring new physics possibilities well below the Z pole

A_{PV} primarily sensitive to neutron weak charge

$$Q_W^A \approx -N + Z(1 - 4\sin^2\theta_W) \approx -N$$

Cesium A_{PV} ~4σ

Francium A_{PV}



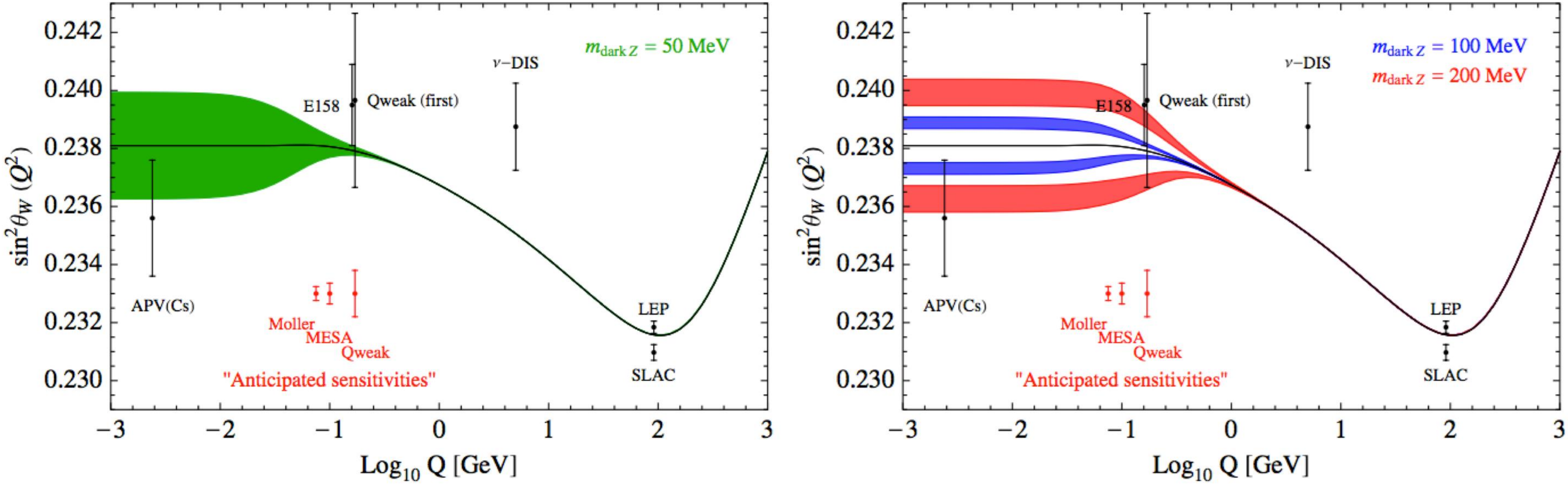
parity-violating e-p elastic scattering
$e + p \rightarrow e + p$

$$Q_W^p \approx 1 - 4\sin^2\theta_W$$

Q_{weak}

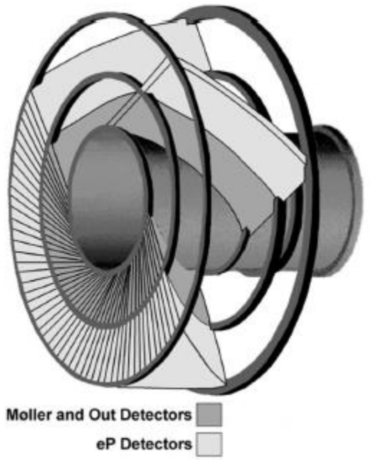
P2

Tests of the SM via low E PV measurements



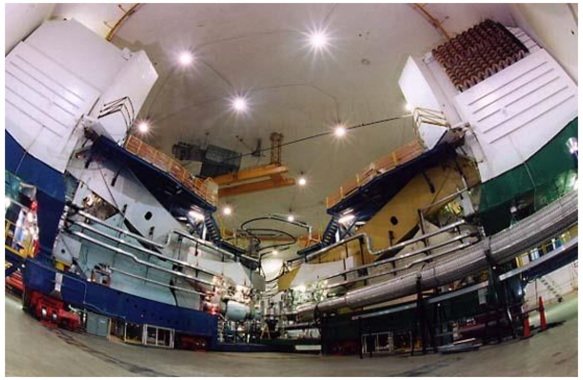
Credit: W. Marciano

Some PVES Experiments

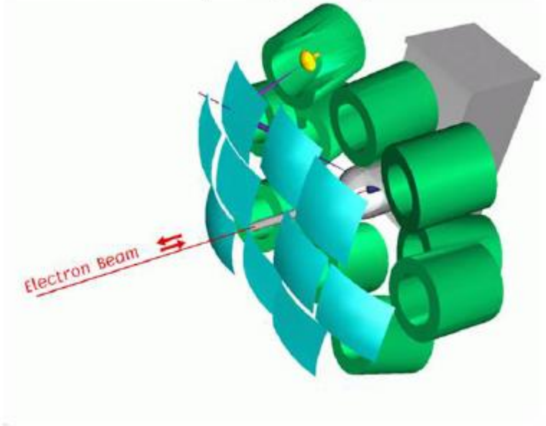


a (E158, SLAC)

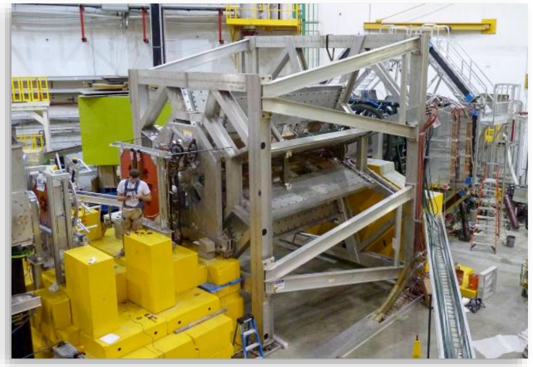
b (PVA4, Mainz)

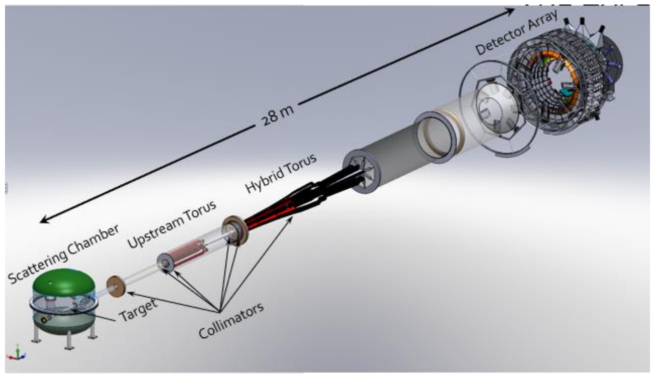


c (HAPPEx, JLab)



d (SAMPLE, MIT-Bates)





SOLID, P2…

Homework – Refresh your memory!

- Linear algebra
  - Hermitian conjugate
  - Tensors - https://www.youtube.com/watch?v=CliW7kSxxWU
- Classical Mechanics
  - Hamiltonian
- Electricity and Magnetism
  - Maxwell's Equations
  - Scalar and vector potentials
- Calculate uncertainty on asymmetry

An atom walks into a bar

says to the bartender

"I think I lost one of my electrons"

and the bartender says

"Are you sure?"

and the atom says

"I'm POSITIVE"

Stuff we don't know

You know Ampere, and Kirchoff, and Bunsen, and Maxwell,
Fraunhofer, Faraday, they knew their facts well!
But do you recall... the most famous discovery of all?

Rudolph the bright red photon
Had momentum h-bar k,
But if you ever saw him,
He'd collapse his state they say.
Michelson interference
Used to make him scratch his head --
"Should I take this or that path?
Maybe I'll take both instead!"

To avoid catastrophe,
Max Planck had to say,
"Ho Ho Ho --
Quantize energy of light --
h times nu will work just right!"

Later, in 1913,
Niels Bohr shouted out with glee,
"Photons can cause transitions --
Give that Nobel Prize to me!"

http://www.haverford.edu/physics-astro/songs/
"Rudolph the Bright Red Photon", words by Walter F. Smith, 11-5-05
(sung to "Rudolph the Red-nosed Reindeer", tune by Johnny Marks, words by Robert May)

collide

Stuff we don't know