

# *Low Energy Nuclear Experiments*

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*National Nuclear Physics Summer School, 8-19 July 2019*

# Overview, part 1 (*general properties of nuclei, mostly macroscopic*)

*What can experimentalists determine about a nuclear system in the lab?*

- *History ... the isotopes, the facilities we use*
- *What can we measure/is observable?*
- *Questions to ask about the nucleus*
  - *How much do they weigh?*
  - *What size are they?*
  - *What shape are they?*

*Attempt to give many accessible examples from recent literature, leaning towards the study of exotic nuclei where possible*

# Overview, part 2 *(mostly direct reactions, not so exotic)*

*The connection between direct reactions and nuclear structure*

- *History*
- *Reactions, reaction types, direct reactions*
- *Observables*
- *Energies, momentum*
- *Spectroscopic factors, occupancies (in context of 'modern' [but stable-beam] examples)*

*Attempt to steer clear of reactions for reaction's sake, rather using them as a meaningful tool to gain insights into topical nuclear structure properties*

# Overview, part 3 *(mostly direct reactions, quite exotic, microscopic)*

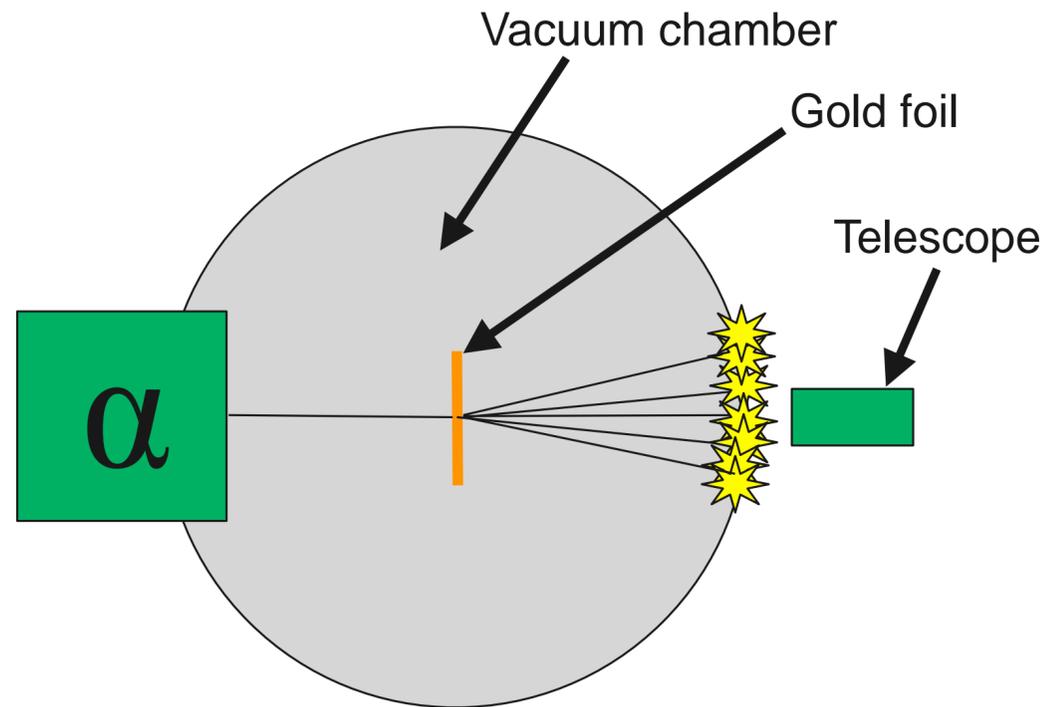
*The connection between direction reactions and nuclear structure*

- *History*
- *Exotic beams*
- *Kinematics*
- *Spectrometers (with a focus on solenoidal spectrometers)*
- *A few examples from the last few years (2014, 2017, 2017, current) (what drove them, reaction choices, results, commentary)*

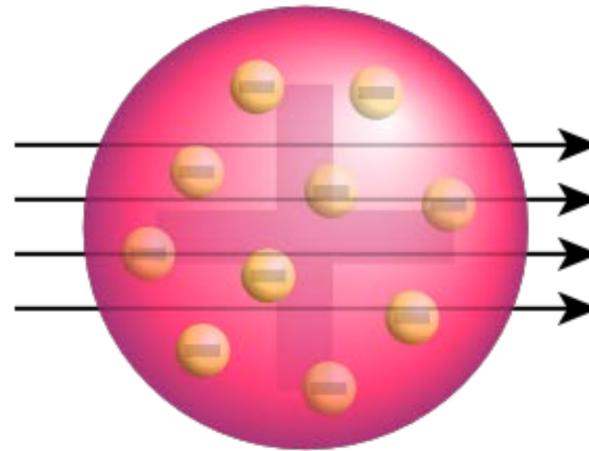
***Part 2: Mostly direct reactions,...***  
***not so exotic***

# To begin at the beginning ...

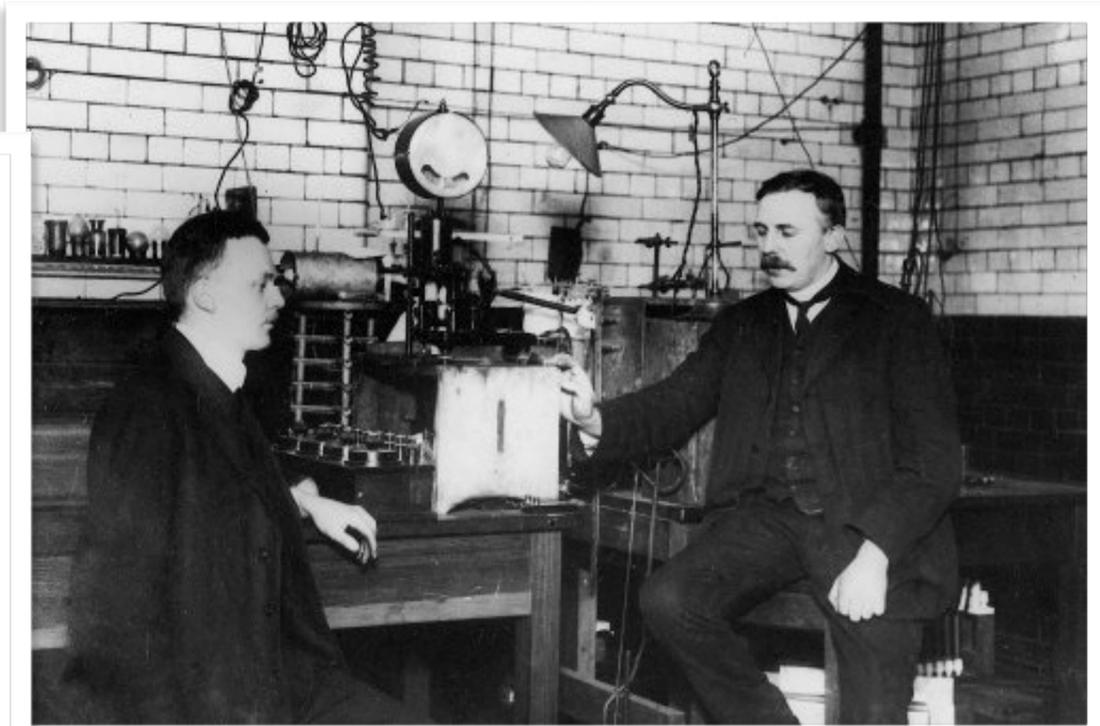
## The Geiger-Marsden experiment



*The plum-pudding idea seemed reasonable: this result would fit expectations*



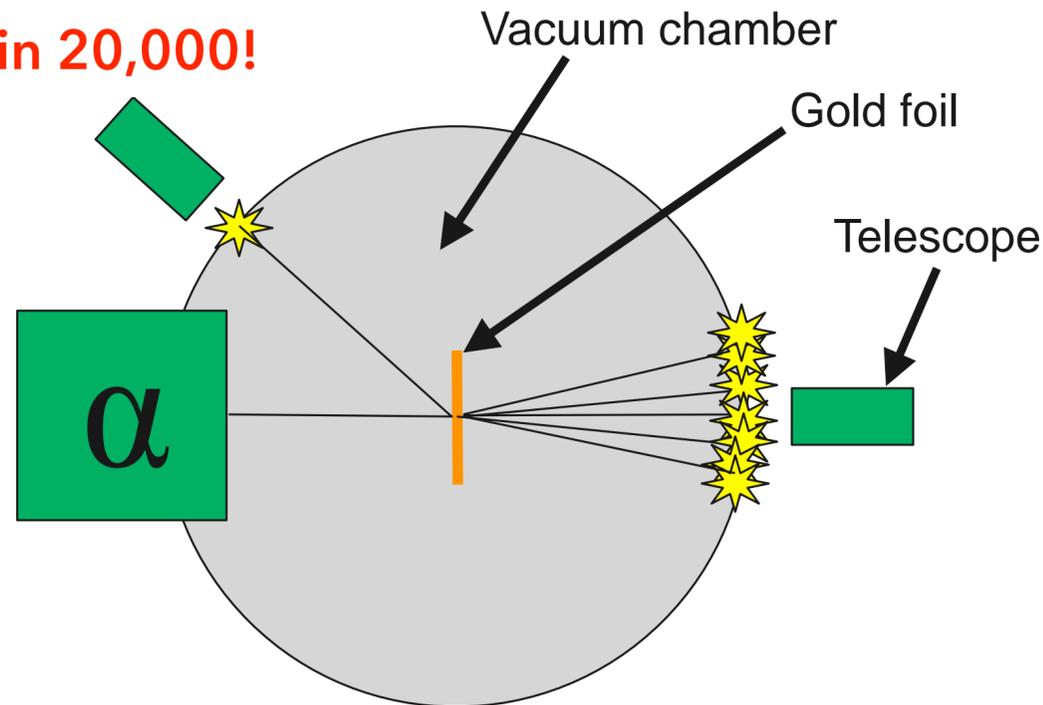
- A 0.1 Ci radium source
- $\sim 10^{10}$   $\alpha$  particles per second ( $\sim 1$  nA of  $^4\text{He}$ )
- $\alpha$  particles of 7.7 MeV ( $\sim 1.9$  MeV/u)
- A gold foil of 0.00004 cm thick ( $\sim 0.8$  mg/cm<sup>2</sup>)
- A telescope was used to look at flashes of light on a zinc sulphide screen



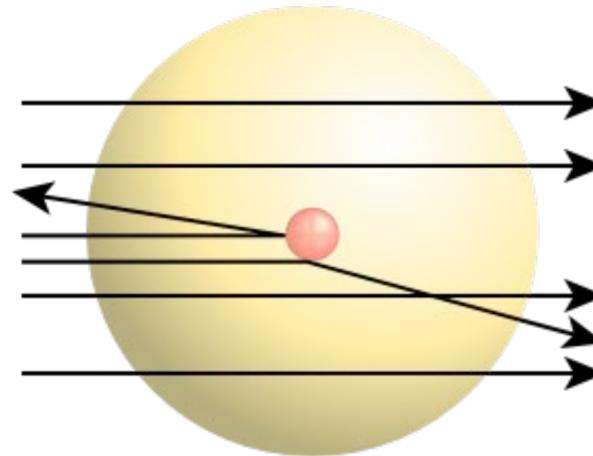
# Elastic scattering $\Rightarrow$ nucleus

## The Geiger-Marsden experiment

<1 in 20,000!



The atom has a dense, positive mass at the centre ... it is mostly empty!



- A 0.1 Ci radium source
- $\sim 10^{10}$   $\alpha$  particles per second ( $\sim 1$  nA of  $^4\text{He}$ )
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- A telescope was used to look at flashes of light on a zinc sulphide screen

"It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you." – E. Rutherford.

This has *all the same* ingredients a modern nuclear reaction experiment:

- A beam
- A target
- A chamber
- Reaction products
- A detector
- ...

### § 7. *General Considerations.*

In comparing the theory outlined in this paper with the experimental results, it has been supposed that the atom consists of a central charge supposed concentrated at a point, and that the large single deflexions of the  $\alpha$  and  $\beta$  particles are mainly due to their passage through the strong central field. The effect of the equal and opposite compensating charge supposed distributed uniformly throughout a sphere has been neglected. Some of the evidence in support of

This has *all the same* ingredients a modern nuclear reaction experiment:

- A beam
- A target
- A chamber
- Reaction products
- A detector
- ... *thus inferring something about the target nucleus*

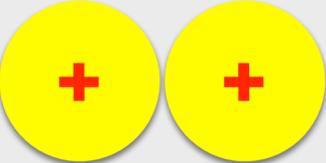
# History

Nuclear reactions and structure share an intertwined history between technological / facilities advances, theoretical advances, and insights ... **and it still is (hence this school)!**

- Rutherford observed the  $^{14}\text{N} + \alpha \rightarrow ^{17}\text{O} + p$  reaction (again, using an  $\alpha$  source)
- Cockcroft and Walton used "swift" protons to "split" the atom, carrying out the first artificial nuclear reaction with **600-keV** protons via the  $^7\text{Li}(p,\alpha)^4\text{He}$  (**reaction notation ... soon**)

**MORE ENERGY** was the eagerly sought (to overcome the Coulomb barrier)

- Bigger Cockcroft-Walton generators (2 million volts, and above)
- Ernest **Lawrence** developed cyclotrons (**more energy**)
- Van Der Graaff accelerators led to tandem Van de Graaff accelerators (**more energy**)
- Then all sorts: linac, superconducting linacs, coupled cyclotrons, etc. (**more and more energy**)



$Z_1, A_1$        $Z_2, A_2$

$$V_{\text{barrier}} \approx \frac{1.44 \times Z_1 Z_2}{1.25(A_1^{1/3} + A_2^{1/3}) + 2} \text{ MeV}$$

# Accelerators for everyone ...

Tables from a 1974 retrospective by D. A. Bromley charting the growth of electrostatic accelerators (*this omits a comparatively long list of cyclotrons [sorry LBNL] also appearing at a similar time*)

Location of the HVEC-CN accelerators.

Serial number	Location	Delivery date
C-1	Oak Ridge National Laboratory	5/5/51
C-2	Rice University	4/53
C-3	Columbia University	6/30/55
C-4	Imperial College of Science & Technology	10/14/55
C-5	Atomic Weapons Research Est., England	6/27/56
C-6	University of Strasbourg, France	11/1/56
C-7	Pennsylvania State University	9/30/57
C-8	Atomic Energy Establishment, India	6/15/58
C-9	University of Freiburg, Germany	7/15/58
C-10	Atomic Energy Commission, Sweden	1/15/65
C-11	University of Zurich, Switzerland	8/1/59
C-12	University of Frankfurt, Germany	2/7/61
C-13	University of Padua, Italy	4/61
C-14	Japan Atomic Energy Research Institute	11/25/61
C-15	University of Laval, Quebec	4/62
C-16	University of Texas, Austin	3/1/63
C-17	Southern Universities Nuclear Inst., S. Afr.	2/1/63
C-18	State University of Iowa	8/20/63
C-19	Ohio State University	6/62
C-20	University of Alberta, Canada	4/1/64
C-21	Hahn-Meitner Institute, Germany	10/13/65
C-22	University of Virginia	12/26/64
C-23	University of Kentucky	7/1/63
C-24	Lowell Institute of Technology	7/1/64
C-25	Institute of Nuclear Energy Research, Taiwan	6/3/68
C-26	University of Arizona	6/15/66

Typically 6 - 6.5 MV

Location of the HVEC-EN accelerators.

Serial number	Location	Delivery date
E-1 <sup>a</sup>	University of Montreal	9/58
E-2	University of Wisconsin	6/1/59
E-3 <sup>a</sup>	Florida State University	8/1/59
E-4	California Institute of Technology	1/15/60
E-5	Australian National University	2/15/60
E-6	Eidg. Technische Hochschule, Zurich	9/60
E-7	Max-Planck-Institut für Kernphysik, Germany	5/31/61
E-8 <sup>a</sup>	Niels Bohr Institute, Copenhagen	5/1/61
E-9	University of Liverpool	5/27/61
E-10	Rice Institute	6/30/61
E-11 <sup>a</sup>	Argonne National Laboratory	6/30/61
E-12	Oak Ridge National Laboratory	6/30/61
E-14	University of Pennsylvania	2/1/62
E-15	University of Texas	11/1/62
E-16 <sup>a</sup>	Centre D'Études Nucléaires, Saclay, France	11/2/62
E-17	University of Erlangen	6/10/66
E-18	University of Oxford	7/63
E-19	Département Atomique Militaire, France	7/30/63
E-20	University of Pittsburgh	11/63
E-21	Weizmann Institute of Science, Israel	12/31/62
E-22	University of Pittsburgh	11/63
E-23	Comision Nacional de Energia Nuclear, Mexico	3/15/68
E-24	University of Utrecht, The Netherlands	1967
E-25	University of Western Michigan	3/17/69
E-26	University of Uppsala, Sweden	8/1/68
E-27	Kansas State University	3/1/69
E-28	University of California, Livermore	3/12/71
E-29	University of Aarhus, Denmark	1972
E-30	University of the Witwatersrand, S. Africa	1973

5 MV tandems and above

# ... literally

Location of the HVEC-FN accelerators.

Serial number	Location	Delivery date
FN-1	Rutgers, The State University, New Jersey	12/63
FN-2	Los Alamos Scientific Laboratory	10/63
FN-3	University of Washington	12/63
FN-4	Stanford University	8/64
FN-5	University of Washington	11/64
FN-6	Edgewood Arsenal	11/30/65
FN-7	University of Cologne	12/1/66
FN-8	State University, Stony Brook, New York	8/67
FN-9	McMaster University	9/67
FN-10	Duke University	9/28/68
FN-11	Argonne National Laboratory	6/20/67
FN-12	Notre Dame University	2/29/68
FN-13	Purdue University	9/68
FN-14	Centre d'Études Nucléaires, Saclay, France	3/30/69
FN-15	Institut de Physique Atomique, Romania	1/71
FN-16	Niels Bohr Institute, Copenhagen	10/3/69
FN-17	Florida State University	

~9 MV tandems

Location of the HVEC-MP accelerators.

Serial number	Location	Delivery date
M-1	Yale University	3/1/65
M-2	University of Minnesota	7/1/65
M-3	Atomic Energy of Canada, Ltd.	8/30/65
M-4	University of Rochester	8/13/66
M-5	Max-Planck-Institut für Kernphysik, Heidelberg	7/67
M-6	Brookhaven National Laboratory	10/31/69
M-7	Brookhaven National Laboratory	10/31/69
M-8	University of Munich	5/15/70
M-9	Institute of Physics, Orsay, France	1973
M-10	University of Strasbourg, France	1973

~14 MV tandems

Some still in use, hear about these two later

Later there came a small number of remarkable "one offs" such as the Yale, Daresbury (UK), and Oak Ridge tandems, which were capable of terminal voltages greater than 20 MV (now all extinct).

A concurrent development of magnetic spectrometers with high resolving power.



# Aside: reaction basics



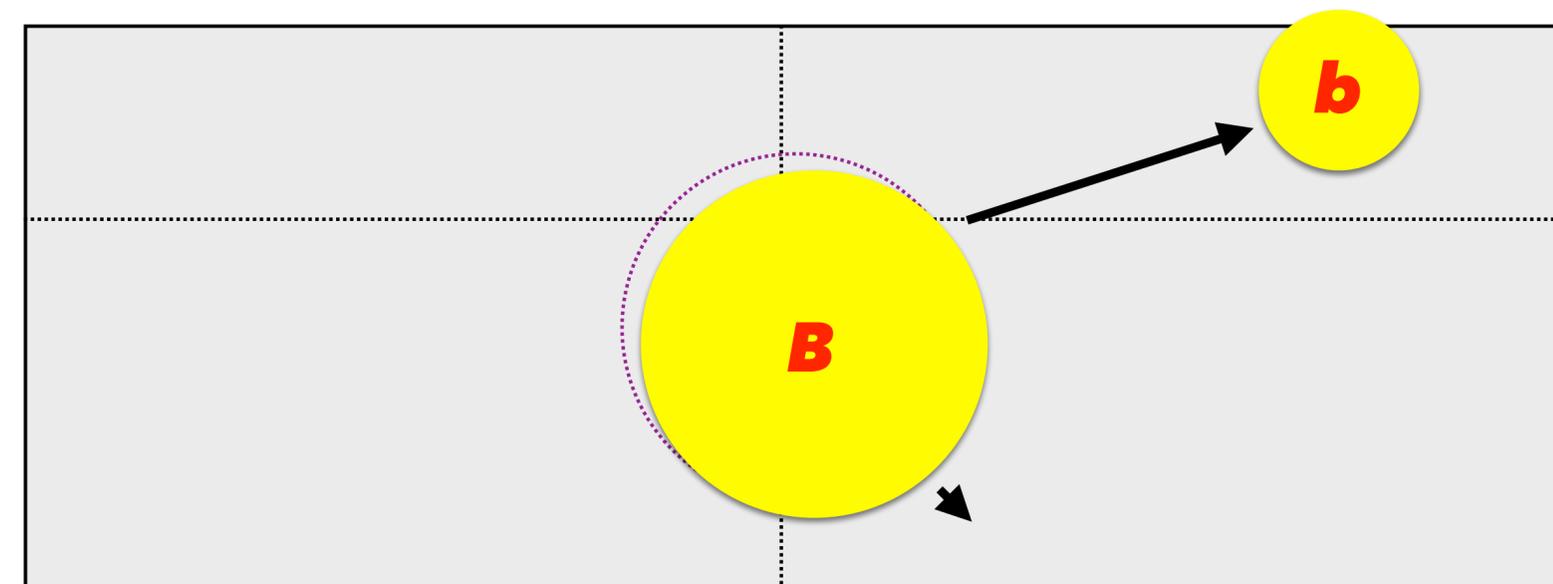
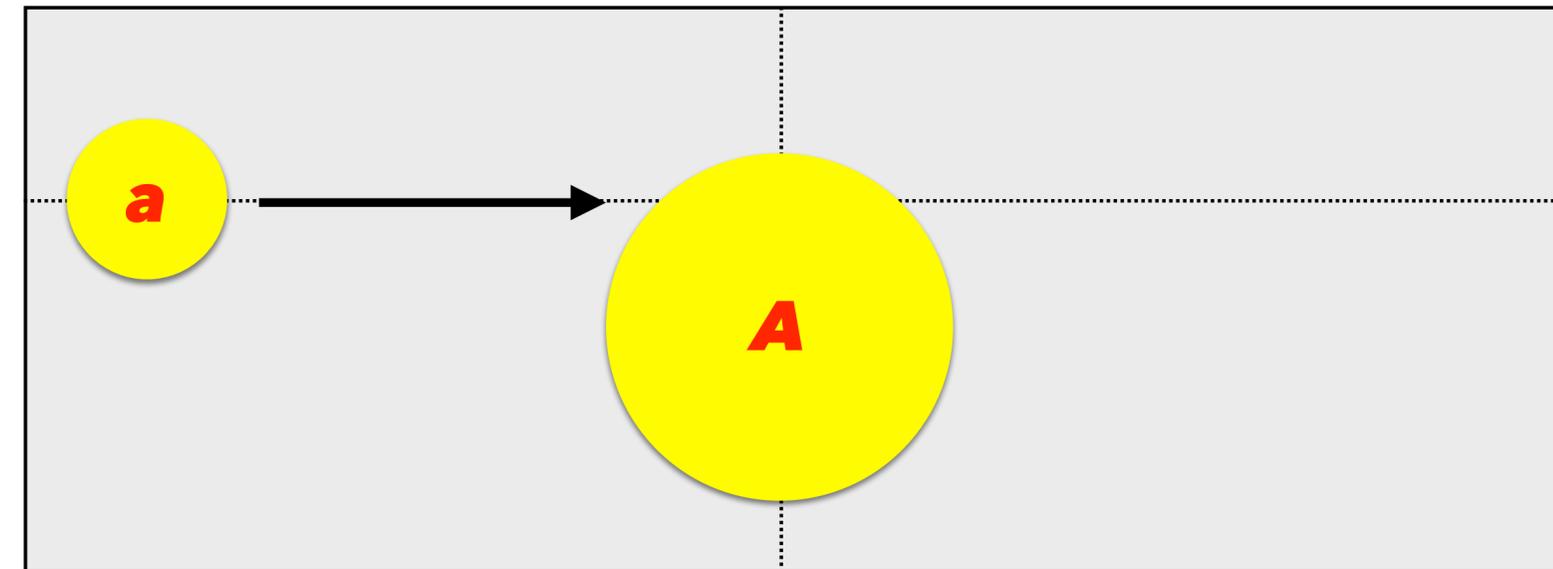
## The ingredients

- Target (**A**)
- Projectile (**a**)
- Beam-like outgoing ion (**b**)
- Target-like outgoing ion (recoil) (**B**)

## What can be measured

- **Count numbers** of **b** and/or **B**
- Energy of **b** and/or **B**
- Type of **b** and/or **B**
- Angle of **b** and/or **B**
- And also in coincidence with ...  
anything

**A(a,b)B**



N.B. the **beam** has  $E$ ,  $I$ , size, spread, purity, the **target** has thickness, purity, etc.

# Reaction types

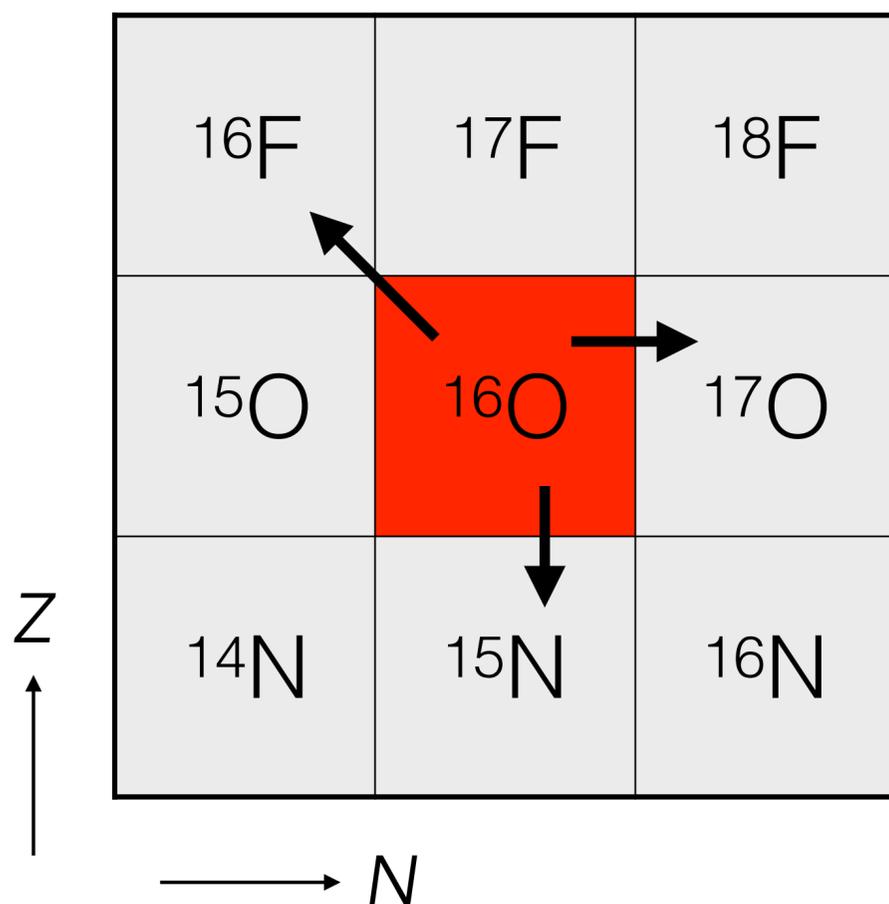
Many, many types ... elastic, compound, and *direct*

For most reactions it is the  $(a,b)$  of  $A(a,b)B$  that is used to label the reaction

The probe ( $a$ ) can be hadrons, electrons, nuclei, pions, photons, ..., etc.

We'll stick mostly to hadrons, and mostly to *direct* reactions

# Reaction types $A(a,b)B$



$^{16}\text{O}(e,e)^{16}\text{O}$  – **elastic** scattering,  $Q = 0$

$^{16}\text{O}(d,d)^{16}\text{O}$  – **elastic** scattering,  $Q = 0$

$^{16}\text{O}(d,d')^{16}\text{O}^*$  – **inelastic** scattering,  $Q \sim E_B$

$^{16}\text{O}(d,p)^{17}\text{O}$  – neutron adding (**transfer**),  $Q +ve$

$^{16}\text{O}(d,^3\text{He})^{15}\text{N}$  – proton removing (**transfer**),  $Q -ve$

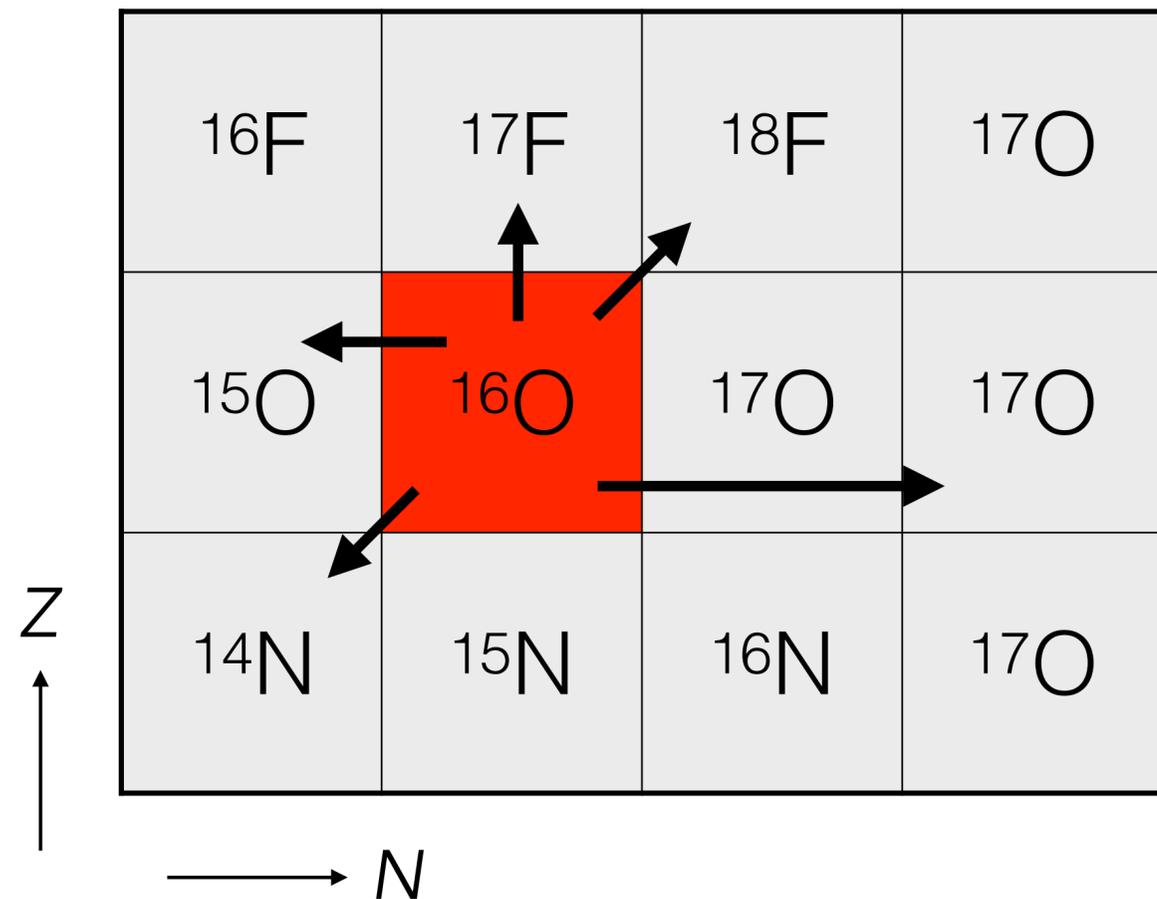
$^{16}\text{O}(e,e'p)^{15}\text{N}$  – proton **knockout**,  $Q -ve$

$^9\text{Be}(^{16}\text{O},^{15}\text{N})X$  – proton **knockout**,  $Q -ve$

$^{16}\text{O}(^3\text{He},t)^{16}\text{F}$  – **charge-exchange** ( $\beta^-$ ),  $Q -ve$

$$Q = m_A c^2 + m_a c^2 + m_b c^2 + m_B c^2 - E_x(\mathbf{b}) - E_x(\mathbf{B})$$

# Reaction types $A(a,b)B$



$^{16}\text{O}(p,d)^{15}\text{O}$  – neutron removing (**transfer**),  $Q$  -ve

$^{16}\text{O}(^3\text{He},\alpha)^{15}\text{O}$  – neutron removing (**transfer**),  $Q$  +ve

$^{16}\text{O}(^3\text{He},d)^{17}\text{F}$  – proton adding (**transfer**),  $Q$  +ve

$^{16}\text{O}(d,\alpha)^{14}\text{N}$  – np-pair removal (**pair transfer**),  $Q$  +ve

$^{16}\text{O}(\alpha,d)^{18}\text{F}$  – np-pair adding (**pair transfer**),  $Q$  -ve

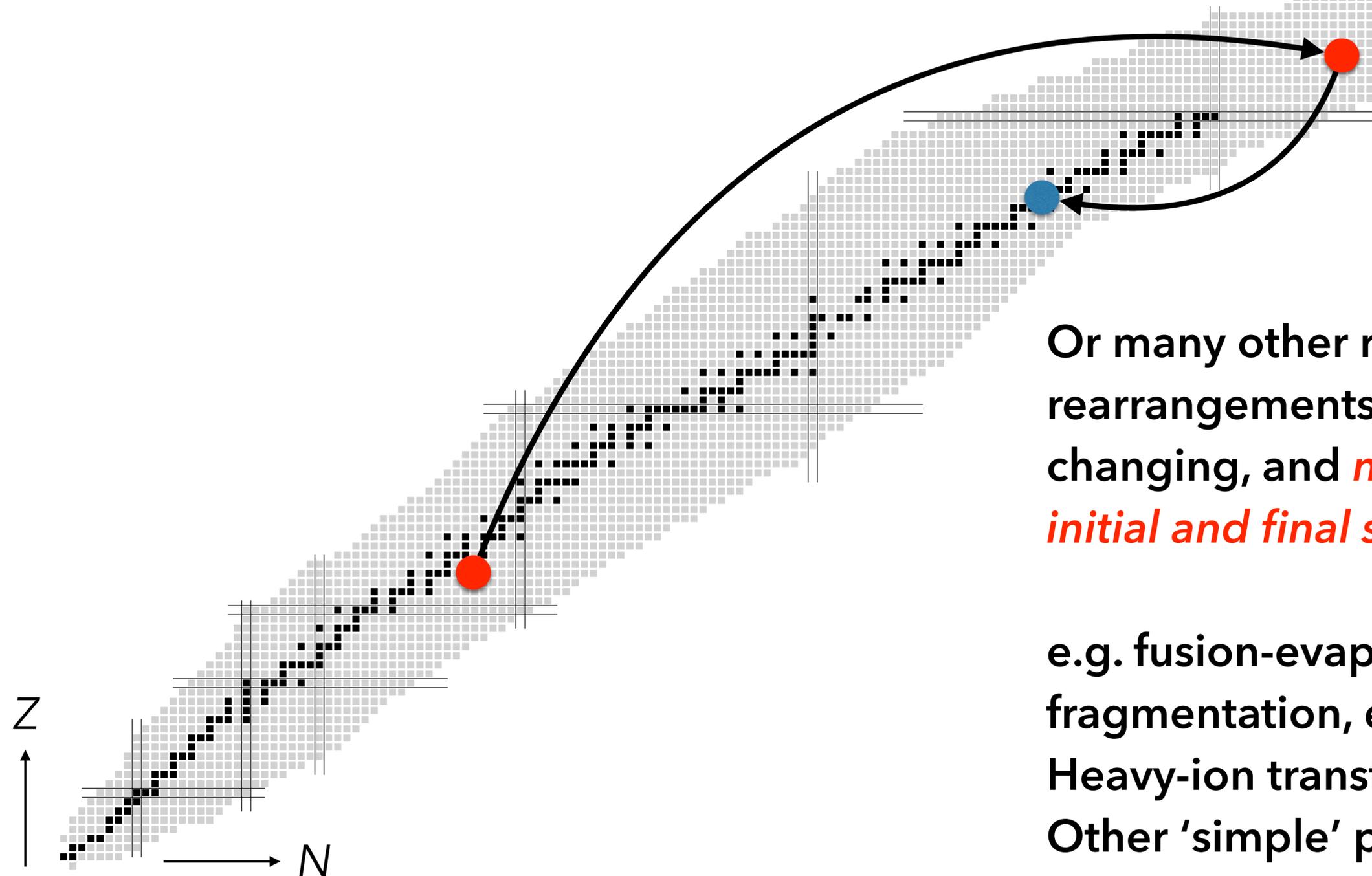
$^{16}\text{O}(t,p)^{16}\text{F}$  – two-neutron (**pair transfer**),  $Q$  +ve

...

$$Q = m_A c^2 + m_a c^2 + m_b c^2 + m_B c^2 - E_x(\mathbf{b}) - E_x(\mathbf{B})$$

# Reaction types $A(a,b)B$

e.g.,  $^{238}\text{U} + ^{76}\text{Ge} \rightarrow ^{180}\text{W} + 58$  other nucleons of stuff



Or many other reactions that lead to huge rearrangements, with many, many nucleons changing, and *no connection between the initial and final states* ...

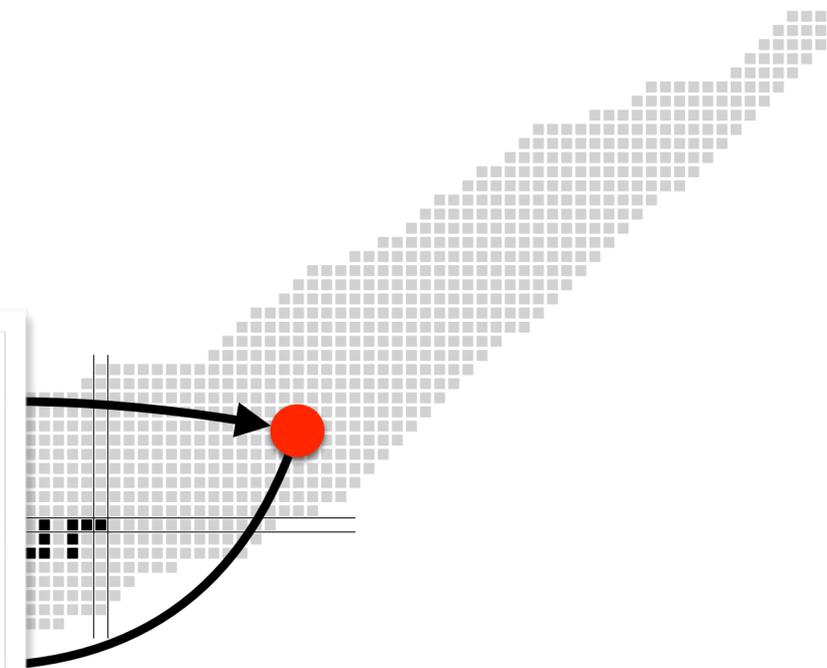
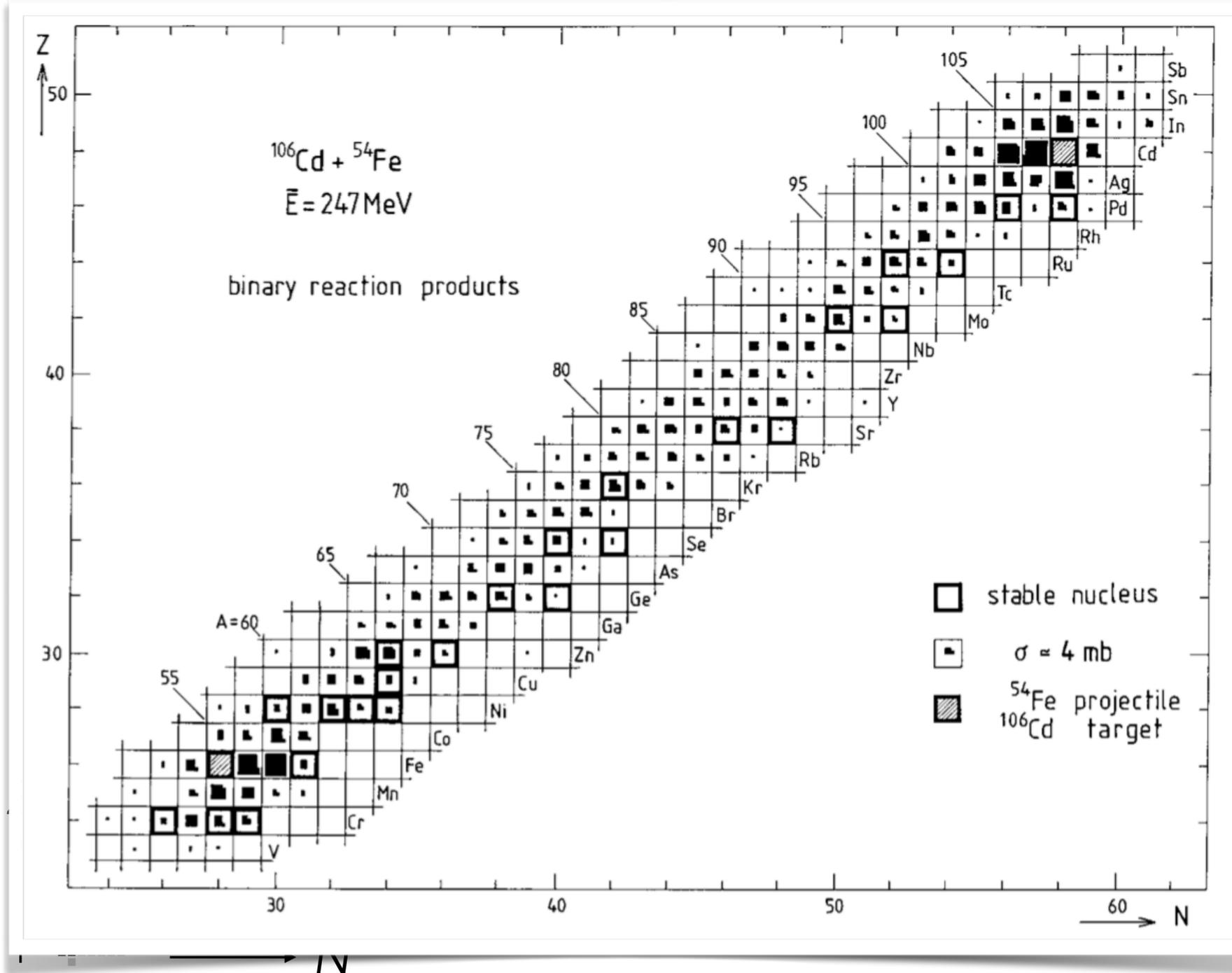
e.g. fusion-evaporation, deep-inelastic, fragmentation, etc

Heavy-ion transfer, fusion, etc.

Other 'simple' probes ignored too, Coulex, etc.

# Reaction types $A(a,b)B$

e.g.,  $^{238}\text{U} + ^{76}\text{Ge} \rightarrow ^{180}\text{W} + 58$  other nucleons of stuff



any other reactions that lead to huge  
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fusion-evaporation, deep-inelastic,  
 fragmentation, etc

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# 2nd poorly-ordered historical preamble

In the early days ('50s), it was recognized that the **angular distributions of protons following a deuteron-induced reaction** showed characteristic shapes that reflected the angular momentum of the **transferred neutron**.

*(This led to / was coupled with a remarkable amount of activity, both experimentally and theoretically. Tandems, cyclotrons, and magnetic spectrographs, all developed at extraordinary pace.)*

Building on earlier works studying resonances (Briet and Wigner, 1936; Wigner 1946) the conceptual framework was there to develop **a model that projected the interior wave function of the nucleus onto the surface of the nucleus and connect the surface to the outside (lab)**.

Thus theoretical developments quickly led to the definition of spectroscopic overlaps, spectroscopic factors (reduced cross sections). Provided an inference of the single-particle content of nuclear excitations. Dramatically aided by the advent of 'fast' (60s fast) computers.

The data were highly instructive, and arguably formed the **skeleton of our understanding of single-particle nuclear structure as we know it today**.

# A simple yet profound observation

$5/2^+, \ell = 2$

$1/2^+, \ell = 0$

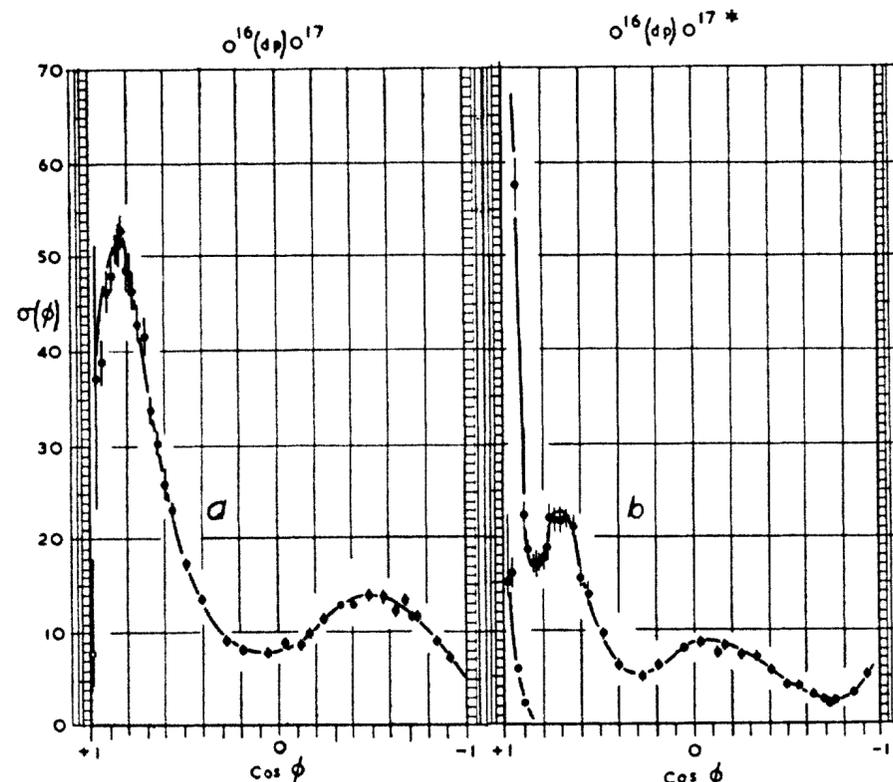


FIG. 1.  $O^{16}(d, p)O^{17}$  angular distributions in the center-of-mass (c.m.) system:  $\phi = \text{c.m. angle}$ ,  $\sigma(\phi) = \text{c.m. differential cross section in arbitrary units}$ . Curve *a* is for formation of  $O^{17}$  in the ground state, and curve *b* is for the 0.88-Mev excited state.

## 8-MeV deuterons from the UoL cyclotron

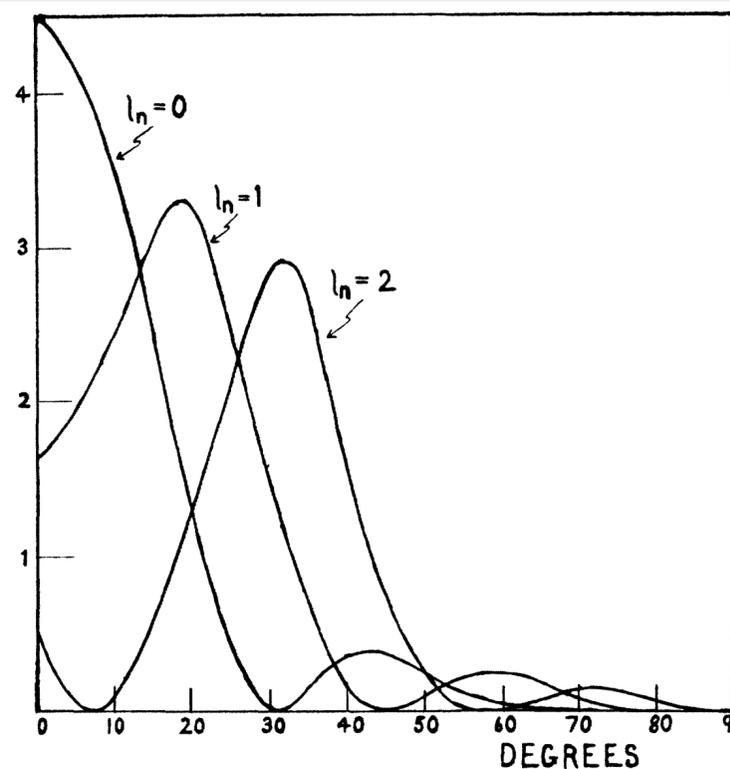


FIG. 1. Theoretical angular distributions for  $(d, p)$  and  $(d, n)$  reactions for different angular momentum transfers to the initial nucleus.

The distinctive patterns in the angular distribution of outgoing ions informs us about the spins and parities of energy levels in the residual nucleus through the use of the Born approximation.

H. B. Burrows et al., *Phys. Rev.* **80**, 1095 (1950), S. T. Butler *ibid.*

### Letters to the Editor

*PUBLICATION of brief reports of important discoveries in physics may be secured by addressing them to this department. The closing date for this department is five weeks prior to the date of issue. No proof will be sent to the authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents. Communications should not exceed 600 words in length.*

#### Angular Distributions of Protons from the Reaction $O^{16}(d, p)O^{17}$

HANNAH B. BURROWS  
*University of Liverpool, Liverpool, England*  
W. M. GIBSON  
*University of Bristol, Bristol, England*  
AND  
J. ROTBLAT  
*Medical College of St. Bartholomew's Hospital, London, England*  
October 30, 1950

THE reaction  $O^{16}(d, p)O^{17}$  gives a number of groups of protons, of which the two corresponding to the ground state and first excited state of  $O^{17}$  have  $Q$ -values of 1.925 Mev and 1.049 Mev (Buechner *et al.*<sup>1</sup>). The intensities of these two groups have been measured at seven angles by Heydenburg and Inglis,<sup>2</sup> using deuteron energies between 0.65 Mev and 3.05 Mev.

We have used the 8-Mev deuteron beam from the University of Liverpool cyclotron, and a scattering camera in which photographic plates record particles emitted from a gas target at all angles from  $10^\circ$  to  $165^\circ$ , to obtain detailed angular distributions for the charged particles emitted in a number of deuteron-induced reactions. A full account of the method and results will be published elsewhere, but because of their theoretical interest (Butler<sup>3</sup>), the angular distributions of the two groups of protons from the reaction  $O^{16}(d, p)O^{17}$  are presented here.

Tracks of protons from the two groups were identified by their ranges in the photographic emulsion, and the number of protons in each group, found in a given area, was determined for a series of angles from  $10^\circ$  to  $160^\circ$ . Ordinarily, measurements were made at  $5^\circ$  intervals, but at the more critical angles the interval was reduced to  $2.5^\circ$  or even to  $1.25^\circ$ . Using these numbers and the geometry of the apparatus, we calculated the angular distributions

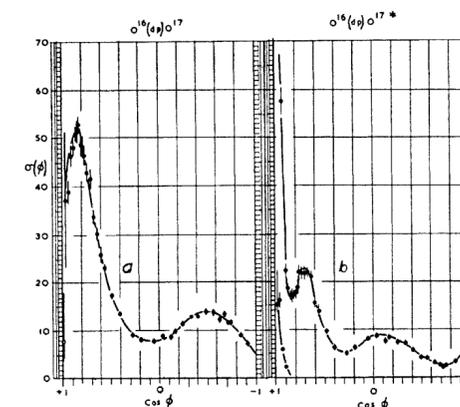


FIG. 1.  $O^{16}(d, p)O^{17}$  angular distributions in the center-of-mass (c.m.) system:  $\phi = \text{c.m. angle}$ ,  $\sigma(\phi) = \text{c.m. differential cross section in arbitrary units}$ . Curve *a* is for formation of  $O^{17}$  in the ground state, and curve *b* is for the 0.88-Mev excited state.

of the two proton groups in the center-of-mass system. These are shown in Fig. 1, in which the ordinates are proportional to the cross sections per unit solid angle in the center-of-mass system, at a center-of-mass angle  $\phi$ , and the abscissae are  $\cos\phi$ .

Figure 1a shows that when the  $O^{17}$  nucleus is formed in its ground state, there is a definite maximum in the intensity at  $\cos\phi = 0.83$  ( $\phi = 34^\circ$ ). At higher angles, the intensity falls to a minimum at about  $85^\circ$ , rises to a smaller maximum at  $120^\circ$ , and falls again towards  $180^\circ$ . Below  $34^\circ$  the intensity falls, apparently tending to zero in the forward direction, although it is not excluded that it may rise again at very small angles; it is hoped that further experiments will show the behavior at angles too small to be studied with this apparatus.

In contrast to this, the intensity of protons from the formation of  $O^{17}$  in its excited state at 0.88 Mev (Fig. 1b) has a peak at  $\cos\phi = 0.7$  ( $\phi = 45^\circ$ ) and a minimum at  $\cos\phi = 0.84$  ( $\phi = 33^\circ$ ), rising steeply as the angle decreases from  $33^\circ$ .

The most interesting feature of these results is the difference in behavior of the two groups at angles below  $50^\circ$ . Butler<sup>3</sup> has shown that a stripping process, in which no compound nucleus is formed, can give one of several characteristic angular distributions, according to the spins and parities of the reacting nuclei. The observed results for small angles fit very well with the theoretical predictions, and it appears that  $(d, n)$  and  $(d, p)$  angular distributions may be of use in determining the spins and parities of ground and excited states in many nuclei.

<sup>1</sup> Buechner, Strait, Sperduto, and Malm, *Phys. Rev.* **76**, 1543 (1949).  
<sup>2</sup> N. P. Heydenburg and D. R. Inglis, *Phys. Rev.* **73**, 230 (1948).  
<sup>3</sup> S. T. Butler, *Phys. Rev.* **80**, 1095 (1950). Following letter.

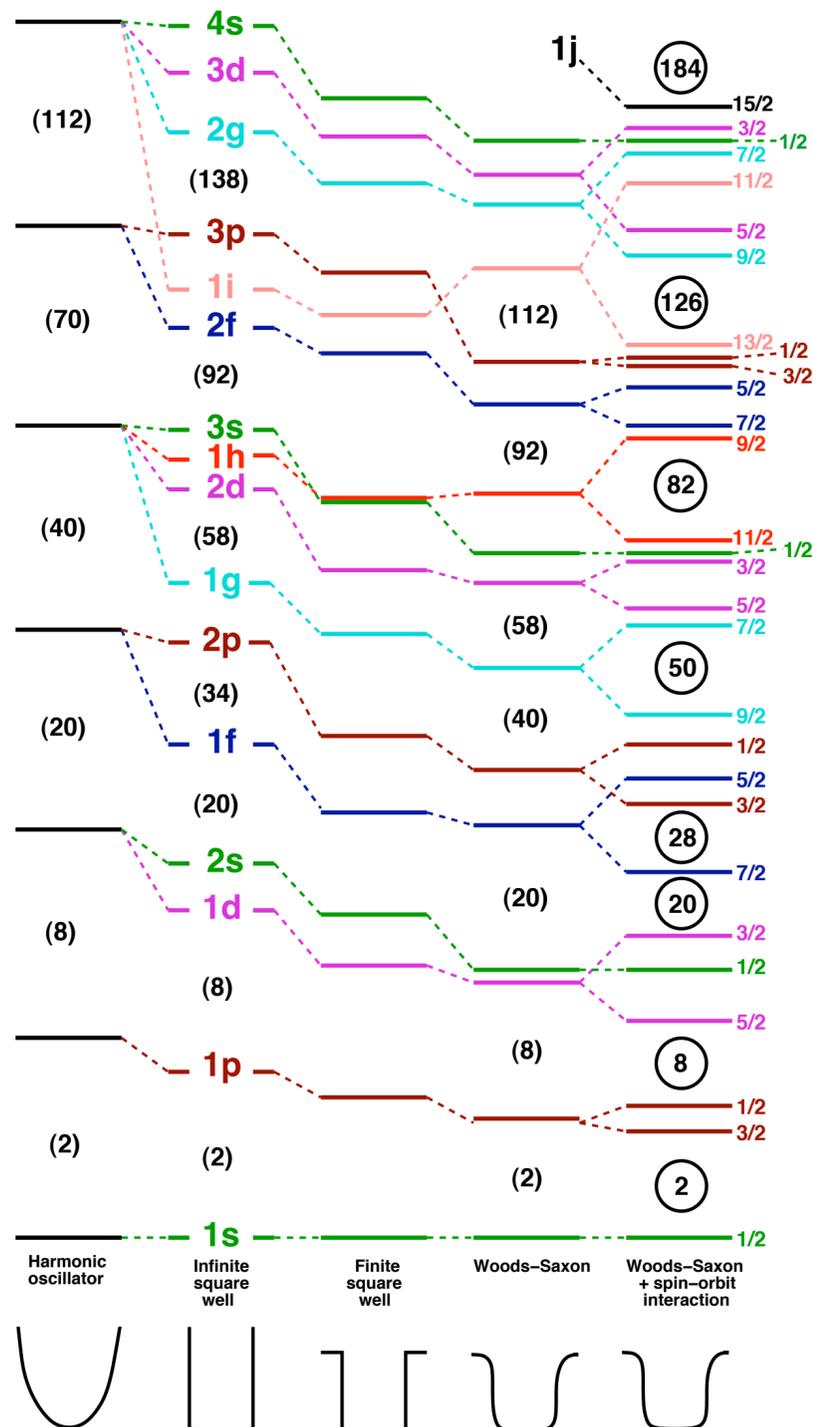
#### On Angular Distributions from $(d, p)$ and $(d, n)$ Nuclear Reactions

S. T. BUTLER\*  
*Department of Mathematical Physics, University of Birmingham, Birmingham, England*  
October 30, 1950

THE purpose of this note is to report the results of calculations which show how information regarding the spins and parities of nuclear energy levels can be obtained from angular distributions from nuclear reactions of the type  $X(d, p/n)Y$  without the necessity of assuming properties of resonance levels of a compound nucleus. This work was commenced, at the suggestion of Professor Peierls, when experimental angular distributions for certain  $(d, p)$  reactions<sup>1</sup> were made available to him some time ago by Professor Rotblat. All exhibited a pronounced structure at small angles, and the work of Holt and Young<sup>2</sup> gives similar results. Such a structure must arise from contributions from high incident angular momenta of classical impact parameters larger than the nuclear radius. The obvious conclusion is that the reactions proceed, at least in part, by a stripping process in which one of the particles of the deuteron is absorbed into the nucleus, while the other merely carries off the balance of energy and momentum. Such a process is possible in the case of  $(d, p)$  and  $(d, n)$  reactions because of the low binding energy and large diameter of the deuteron.

I have calculated angular distributions resulting from such a stripping process by equating, at the nuclear surface, the exact wave function for a particle outside the nucleus to the interior wave function. After some simplification the resulting boundary equations can be solved in such a way that unknown properties of the nuclear wave functions affect the important parts of the distributions merely as a constant multiplying factor. The resulting curves show a pronounced maximum near the forward direction, the position of which is determined in each case by the spins and parities of the nuclear states involved. This is due to the fact that the requirements of conservation of angular momentum and of parity allow the nucleus to accept a particle (say a neutron) with only very limited values of angular momenta  $l_n$ , and the angular distribution depends very sensitively on these

# ... at around the same time



- **1949** (Haxel, Jensen, and Suess, and independently Mayer) – the nuclear shell model – the surprising dominance of **relatively unimpeded independent particle motion**.
- These are 'simple' phenomenological models that work surprisingly well (some of our **most advanced models** today start out here ... )
- They describe an average central potential in which protons and neutrons execute independent single-particle motions

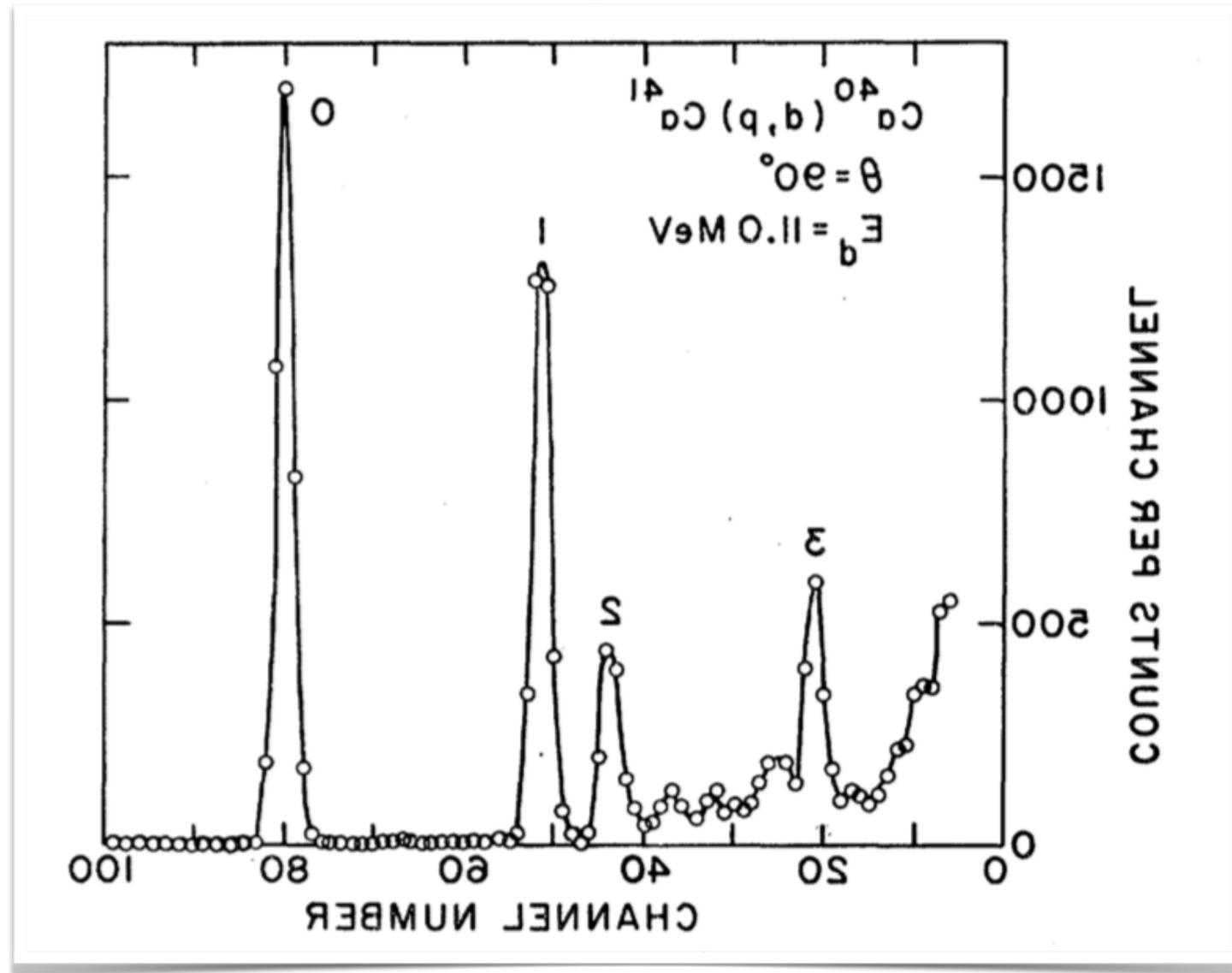
## Enter REALITY

- There is a residual interaction - crudely this is the difference between the central potential and reality
- It is due to the fact nucleons do interact with each other ...

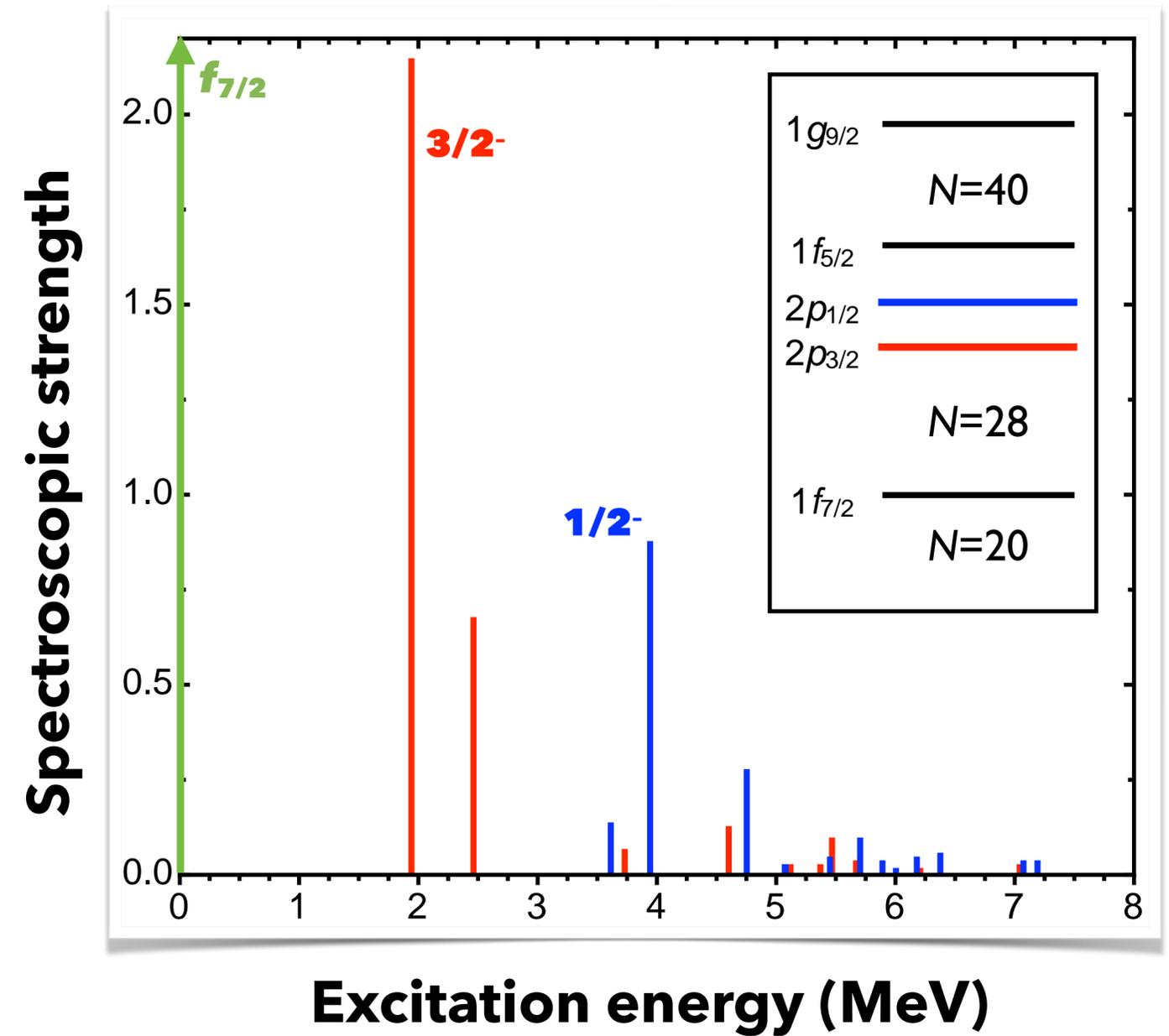
## **RESIDUAL INTERACTIONS**

e.g.  $^{40}\text{Ca}(d,p)^{41}\text{Ca}$   $A(a,b)B$

Single-particle reaction, single-particle structure ... (significant in 'testing' reaction theory – more later)



(flipped)



# Transfer reactions

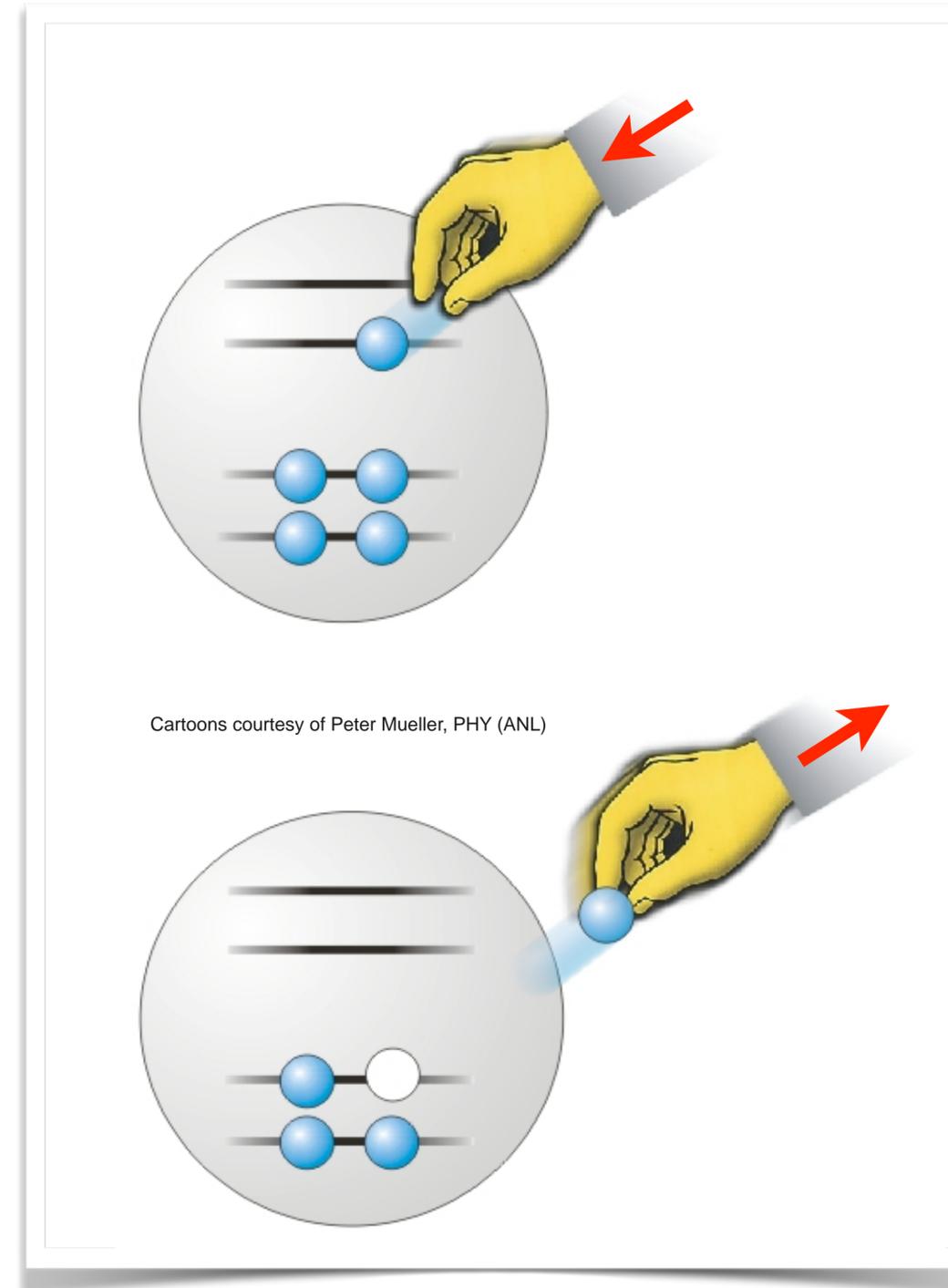
$A(a,b)B$

A well understood probe of nuclear structure, much of the formalism developed in the late 50s / early 60s. Exploited to great effect.

Single-nucleon **ADDING** probes the **EMPTINESS** of the orbital, or the **VACANCY** (cross section proportional to how much 'space' available in the orbital)

Single-nucleon **REMOVAL** probes the **FULLNESS** of the orbital, or the **OCCUPANCY** (cross section proportional to how many particles that are in the orbital)

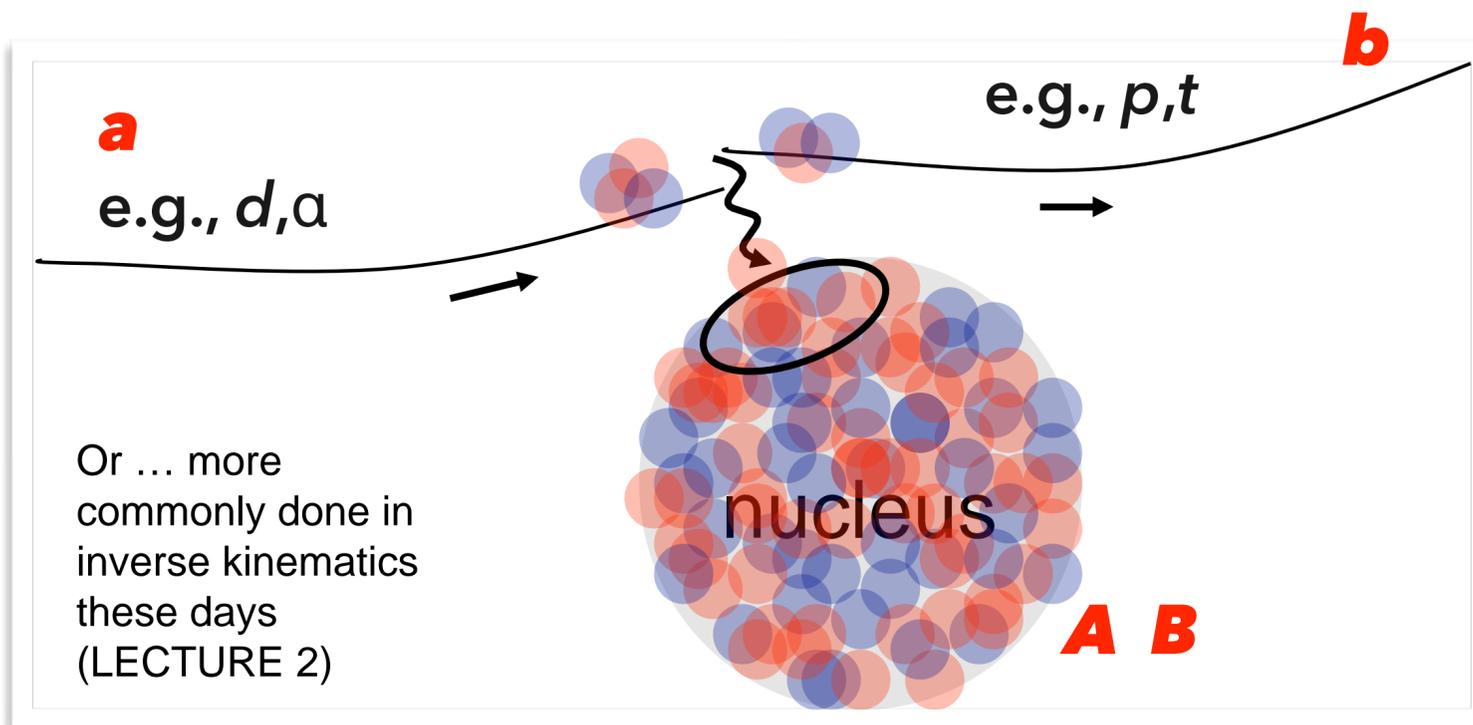
*Requires a few careful considerations...*



# Transfer reactions

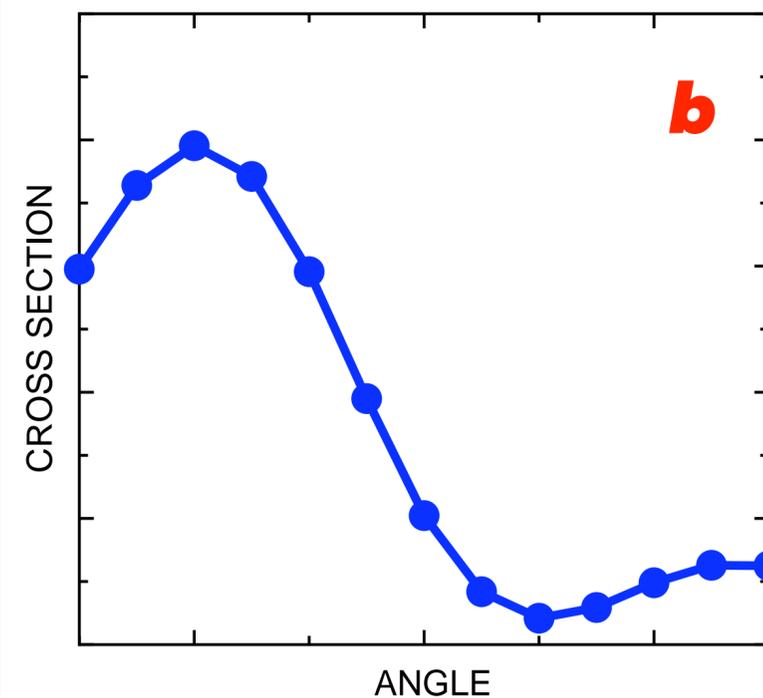


What is measured?



Yield **b**  
(Cross section)  
Momentum  
(Energy)

+



Measure at several angles,  
shapes characteristic of  $\ell$

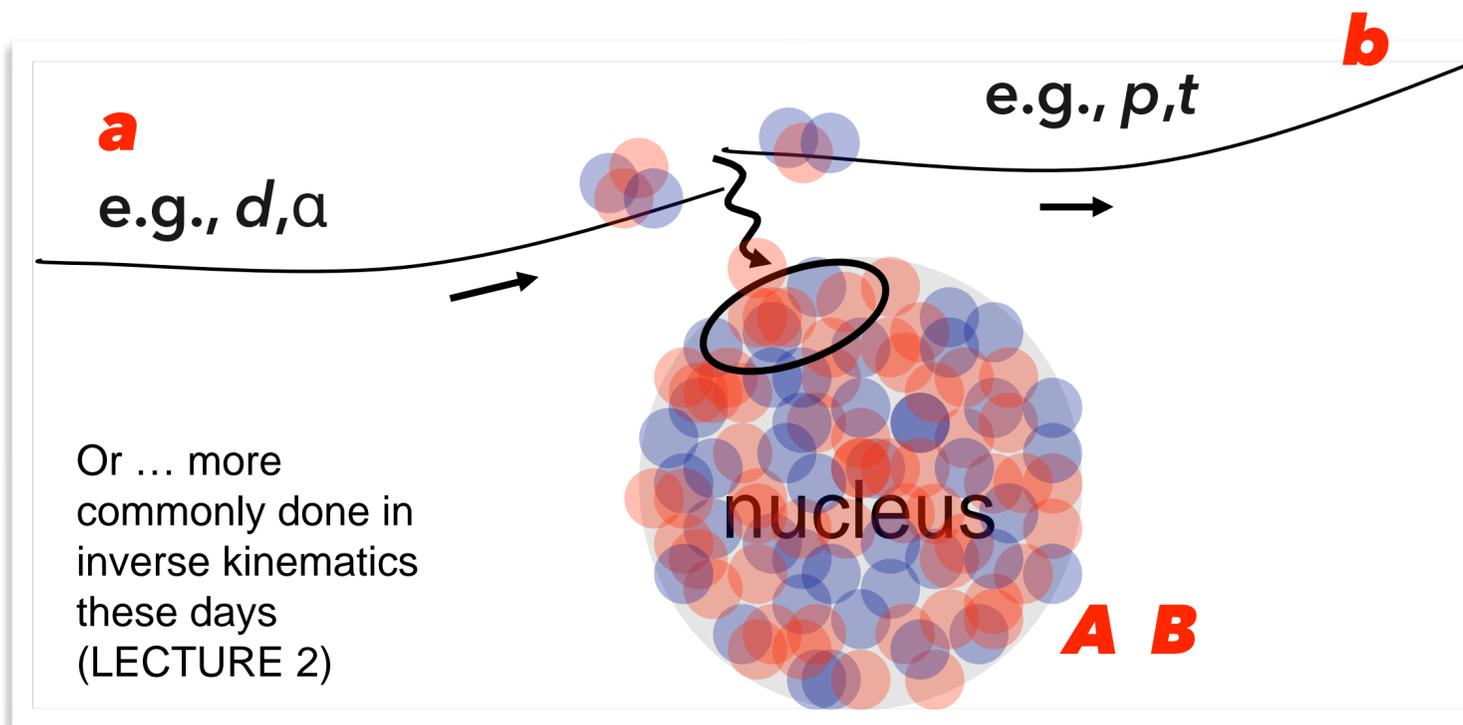
$$\text{Yield} = \#b \#t \Omega \frac{d\sigma}{d\Omega}$$

$$\frac{d\sigma}{d\Omega} = \frac{\text{Yield}}{\#b \#t \Omega}$$

# Transfer reactions

$A(a,b)B$

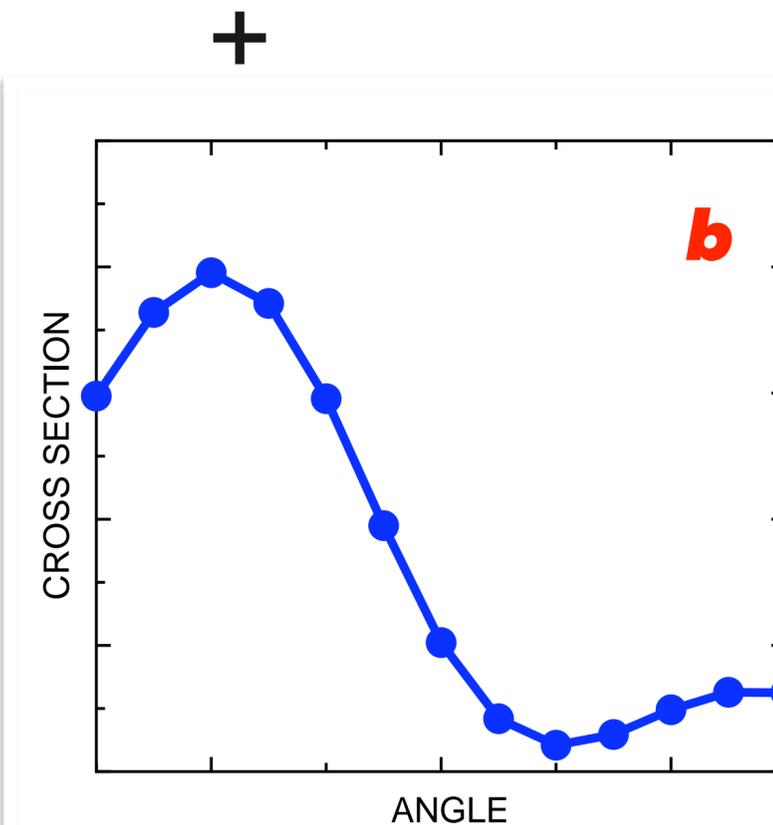
What is measured?



Yield  $b$   
(Cross section)  
Momentum  
(Energy)

$$\text{Yield} = \#b \#t \Omega \frac{d\sigma}{d\Omega}$$

$$\frac{d\sigma}{d\Omega} = \frac{\text{Yield}}{\#b \#t \Omega}$$



Measure at several angles,  
shapes characteristic of  $\ell$

If we have carried out our experiment appropriately we know the transfer can be considered a one-step process happening **dominantly at the nuclear surface**, populating single-particle states in the target nucleus...interpretation follows...which is easier if the experiment is done well!

# Transfer reaction → nuclear structure

Spin and isospin factors. For common reactions on neutron-rich isotopes such as  $(d,p)$  the isospin term is  $1^*$  and the spin term is either 1 or  $(2j+1)$

\*See book chapter by J. P. Schiffer in "Isospin" edited by D. H. Wilkinson, 1969. It can be quite nontrivial! **A thorough example is given in Szwec et al. Phys. Rev. C 94, 054314 (2016).**

**The model**, theory, often requires several parameters and a little bit of respect!

$$\left. \frac{d\sigma}{d\Omega} \right|_{\text{measured}} = g S_j \left. \frac{d\sigma}{d\Omega} \right|_{\text{model}}$$

Calculations typically assume  $S = 1$ , pure single-particle states ... of course this is not reality

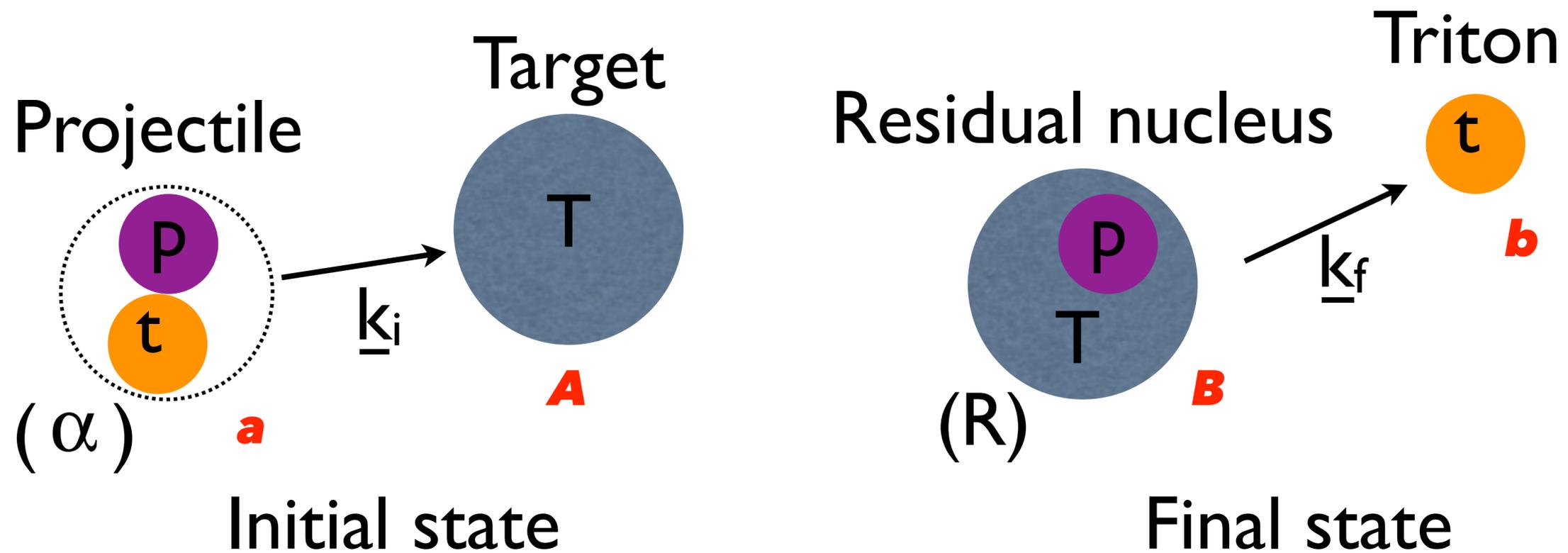
**Spectroscopic factor**: a measure of the *overlap* between the final state and the initial state plus/minus one nucleon

# ***A model, DWBA***

- **DWBA**? distorted-wave Born approximation
- **DW**? Incoming and outgoing waves are distorted by the Coulomb field (optical-model potential required), not plane waves
- **BA**? Transfer considered a perturbation to elastic scattering, often accurate enough to calculate transition rate using the BA

# A model, DWBA

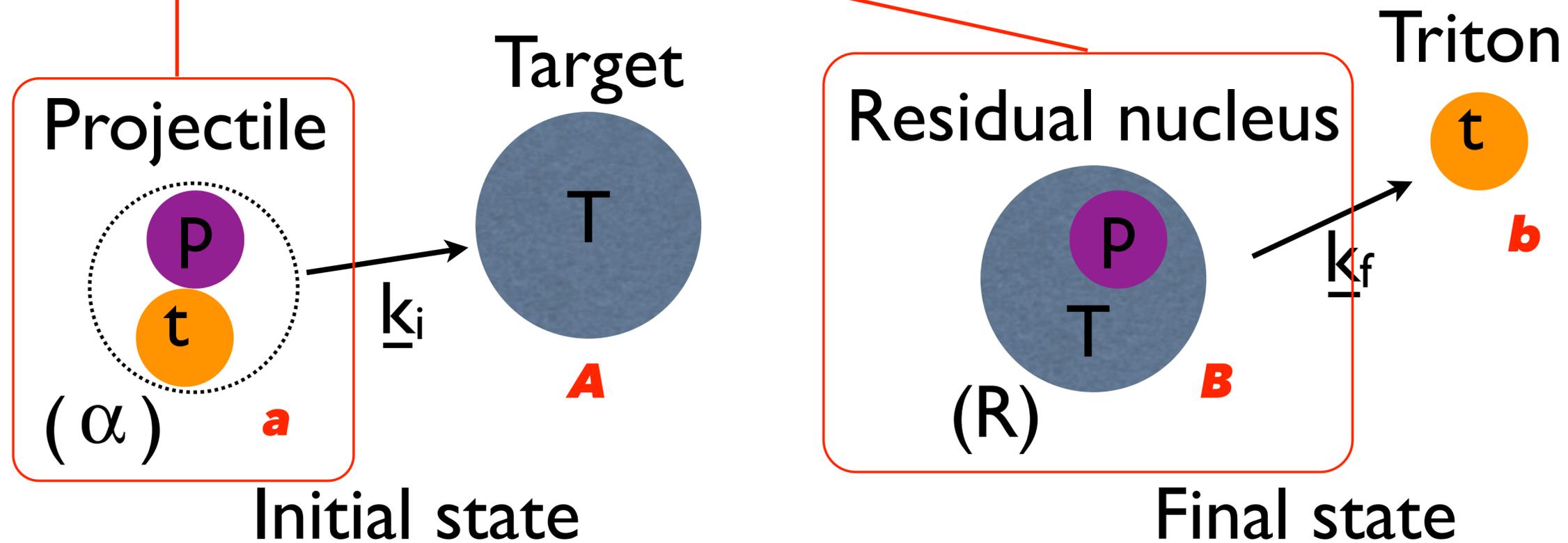
$A(a,b)B$  e.g.  $A(\alpha,t)B$



# A model, DWBA

$A(a,b)B$  e.g.  $A(\alpha,t)B$

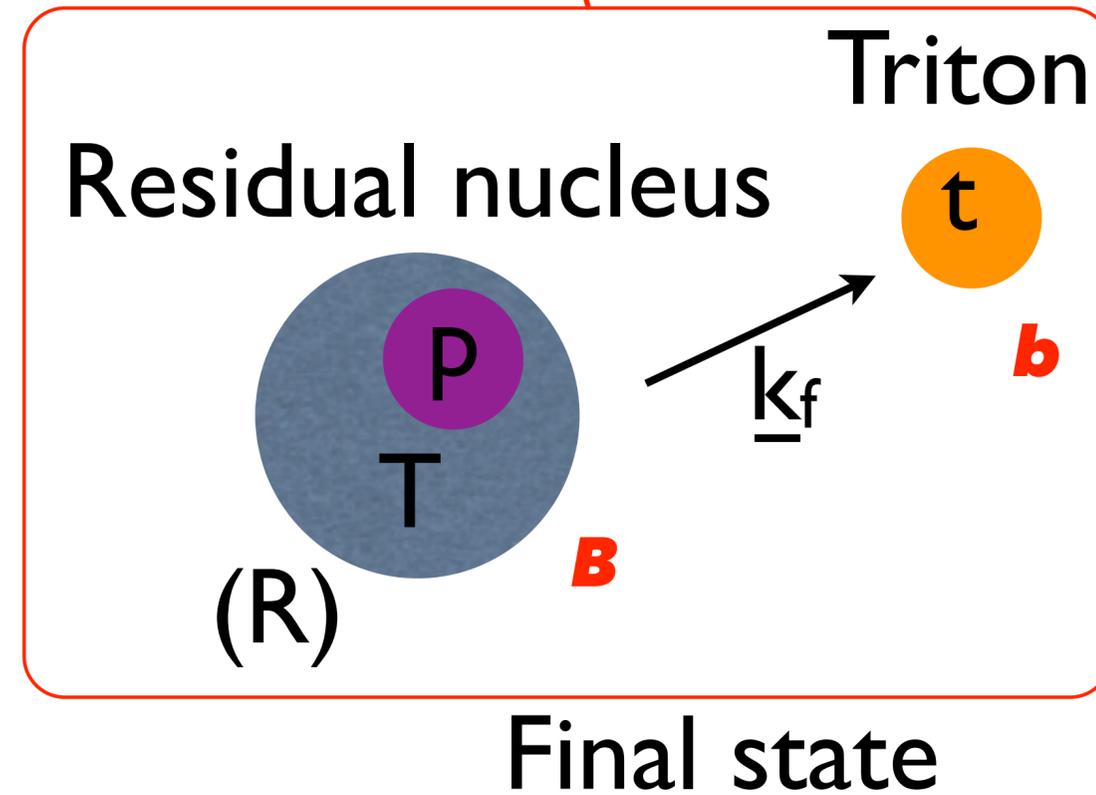
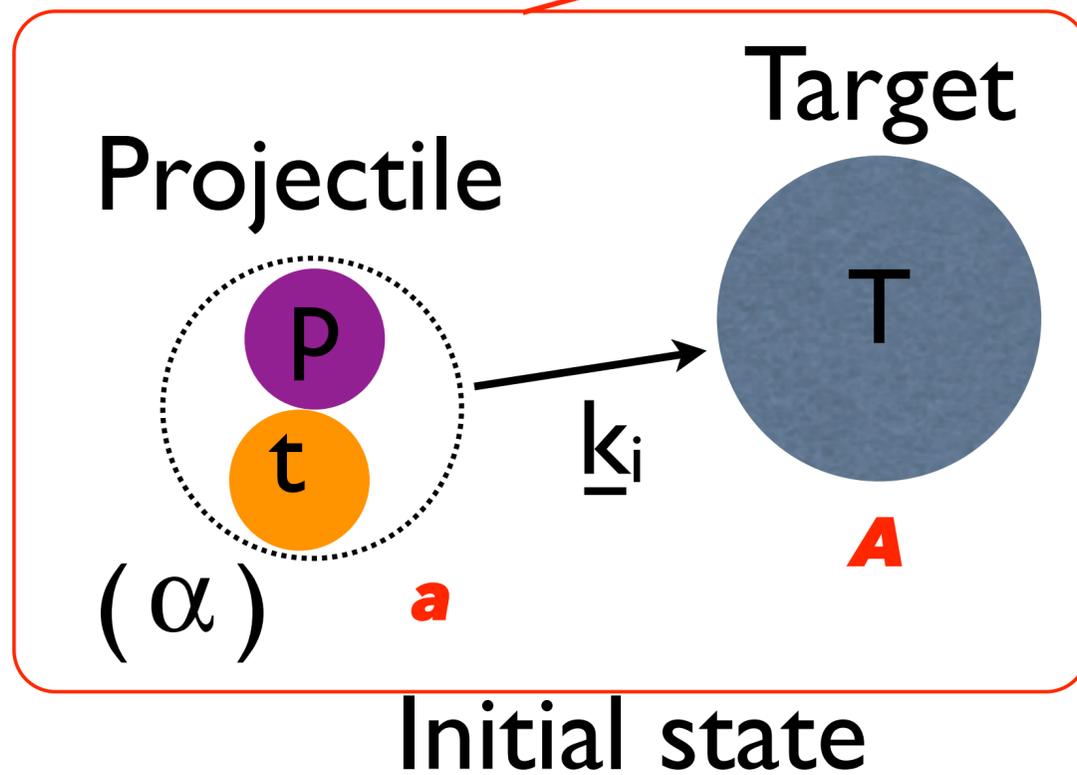
BOUND-STATE  
POTENTIAL (PARAMETERS)  
Projectile: proton bound in alpha particle  
Residual nucleus: proton bound to target nucleus



# A model, DWBA

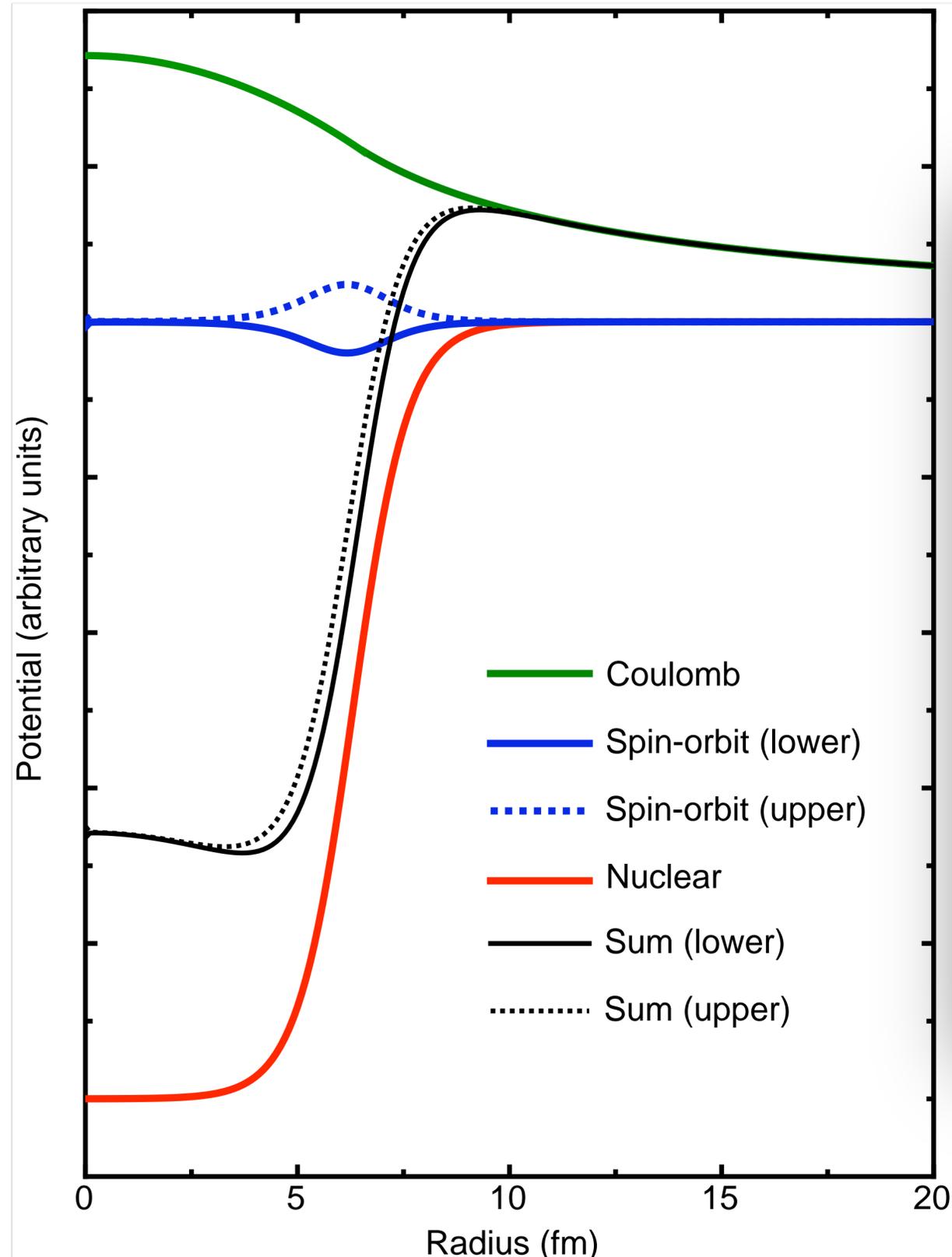
$A(a,b)B$  e.g.  $A(\alpha,t)B$

OPTICAL-MODEL  
POTENTIAL (PARAMETERS)  
Alpha particle moving in average field of the target  
Triton moving in average field of residual nucleus



# A model, DWBA

$A(a,b)B$  e.g.  $A(\alpha,t)B$



```
OPTICAL-MODEL
POTENTIAL (PARAMETERS)
bpk1 — ssh -X
Triton moving in average field of residual nucleus
reset
jbiga=0
REACTION: 132Sn(d,p)133Sn(7/2- 0.00) ELAB=9.545
PARAMETERSET dpsb labangles r0target lstep=1 lmin=0 lmax=30 maxlextrap=0
PROJECTILE
wavefunction av18 r0=1 a=0.5 l=0
;
TARGET
nodes=1 l=3 jp=7/2 r0=1.28 a=.65 vso=6 rso0=1.1 aso=.65 rc0=1.3
;
writens phi2
INCOMING
V = 95.788 R0 = 1.150 A = 0.783
VI = 1.698 RI0 = 1.325 AI = 0.346
VSI = 10.538 RSI0 = 1.364 ASI = 0.847
VSO = 3.557 RS00 = 0.972 ASO = 1.011 RC0 = 1.303
;
OUTGOING
V = 56.286 R0 = 1.224 A = 0.658
VI = 0.579 RI0 = 1.224 AI = 0.658
VSI = 10.286 RSI0 = 1.261 ASI = 0.588
VSO = 5.921 RS00 = 1.059 ASO = 0.590
VS0I = -0.032 RS0I0 = 1.059 ASOI = 0.590 RC0 = 1.227
;
anglemin=0 anglemax=180 anglestep=1
;
reset
```

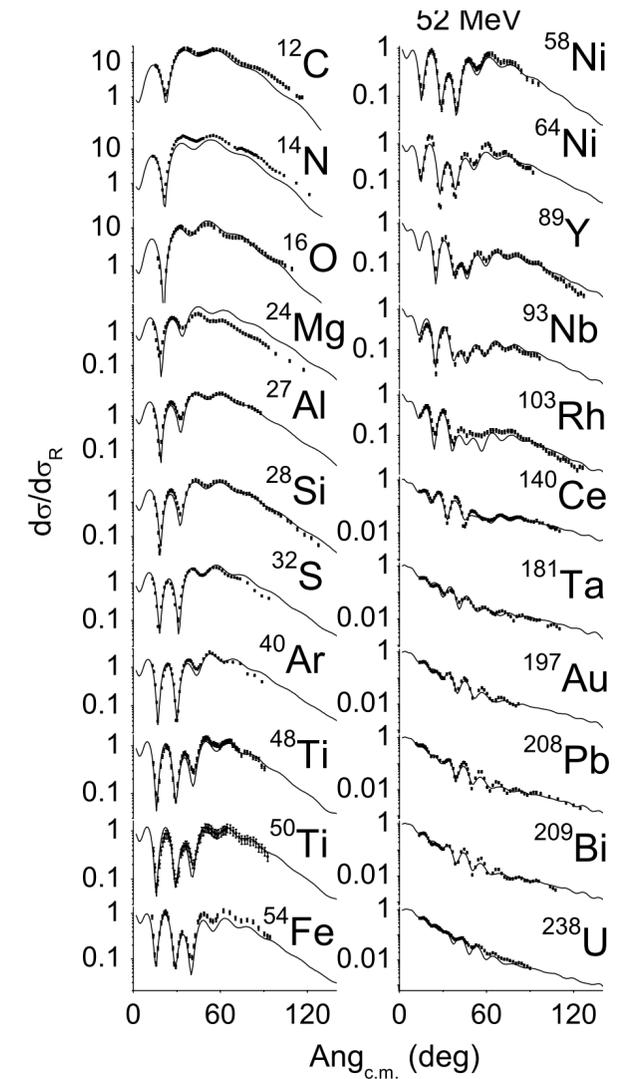
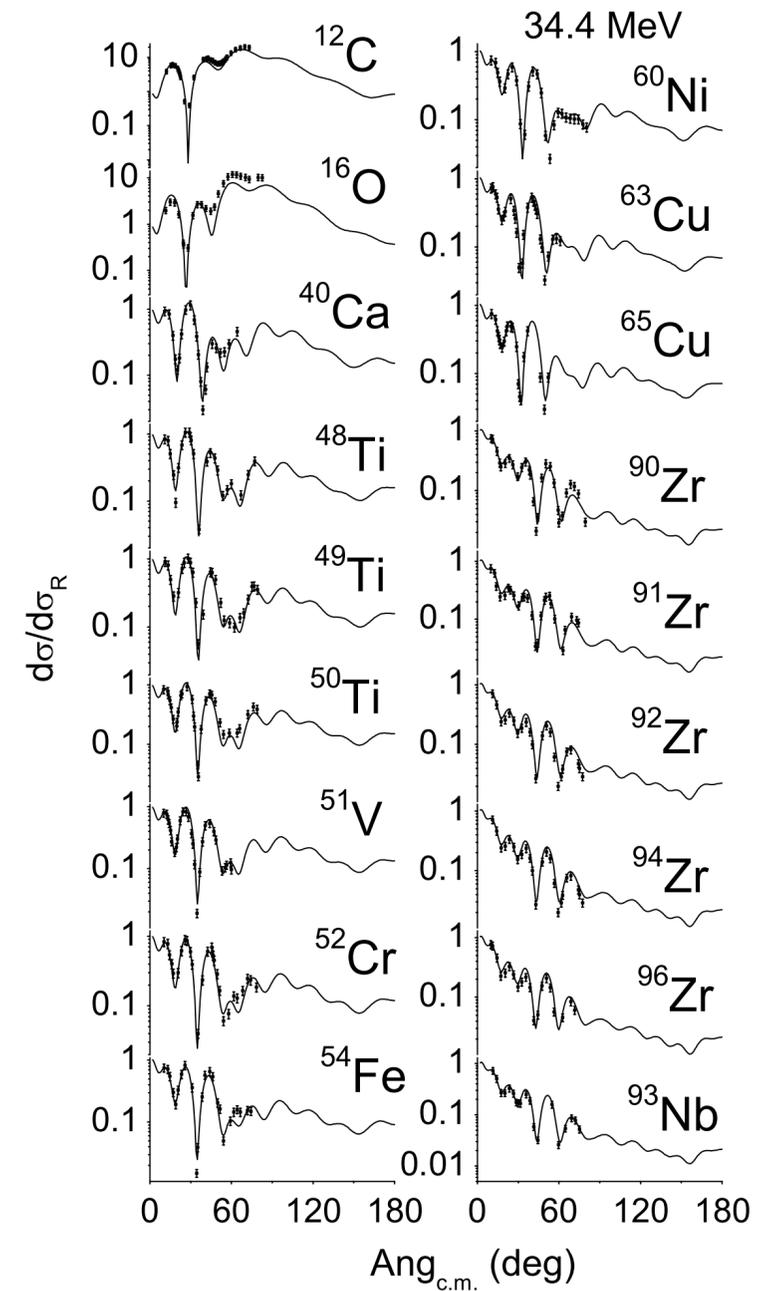
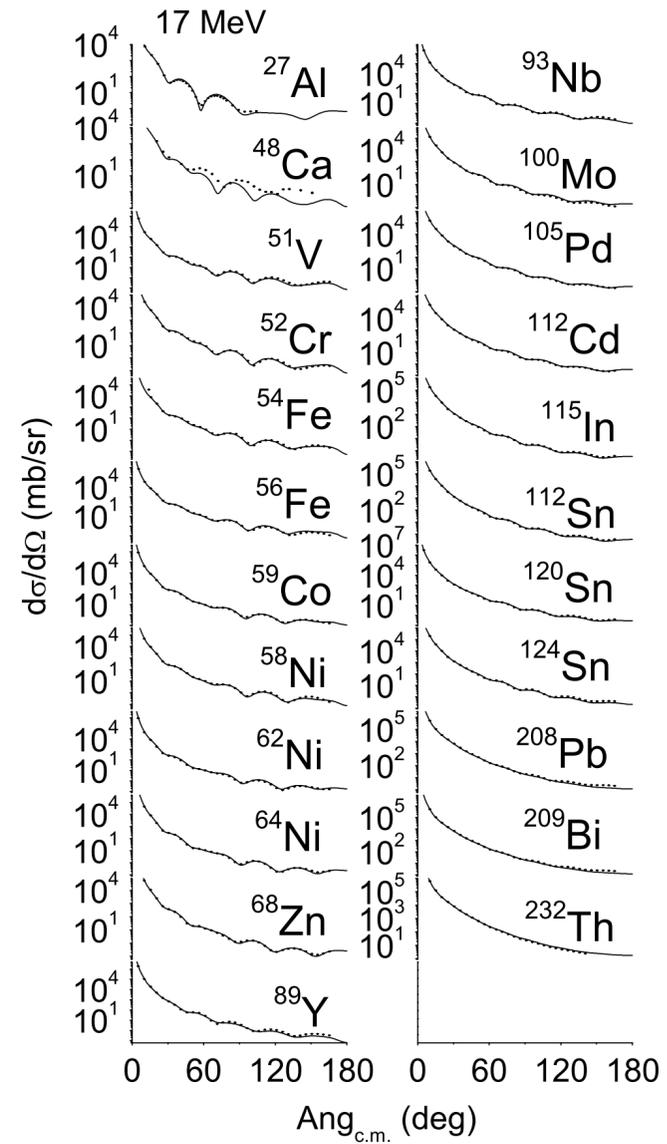
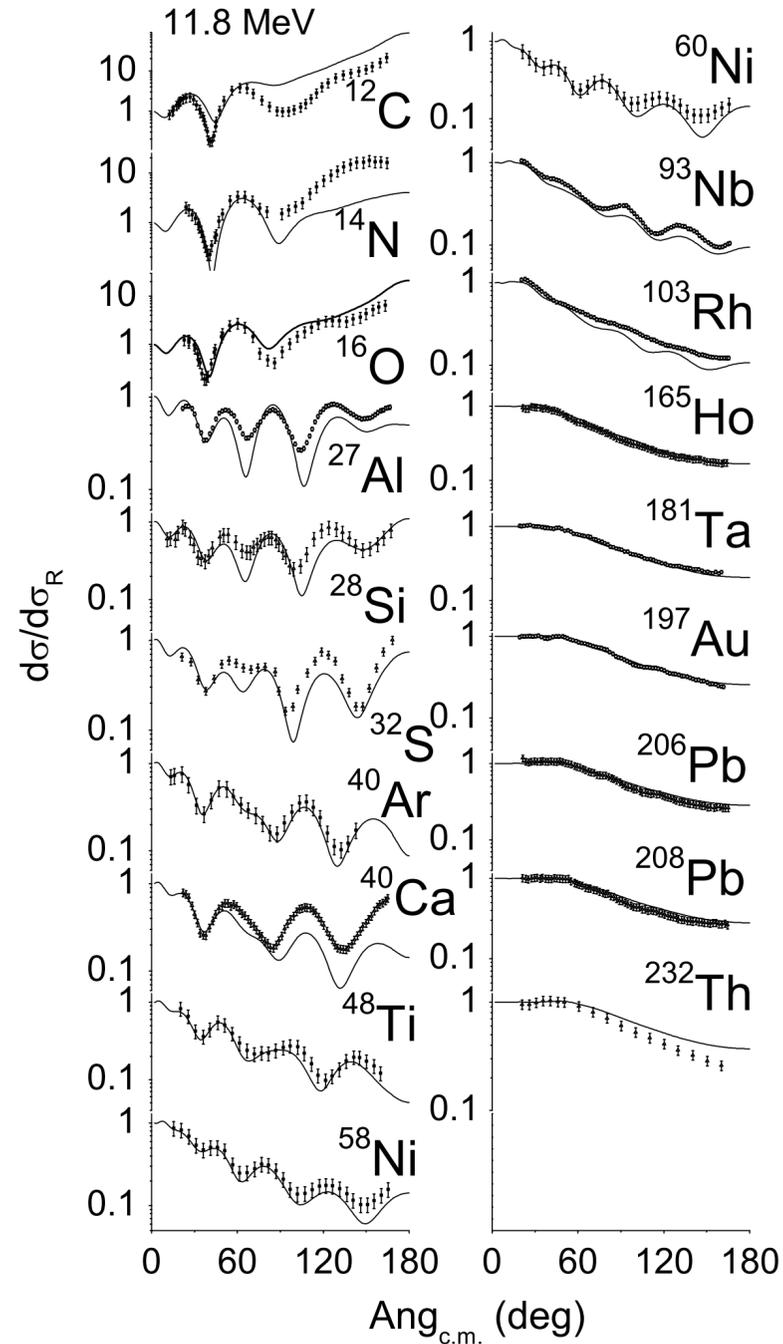
The diagram shows a Triton nucleus (t) moving towards a residual nucleus (B) with radius (R). The final state is labeled with  $k_f$  and  $b$ . The diagram is overlaid on the terminal window output.

Final state

# Optical-model potentials

From elastic scattering data, either specific, or global

$$\left. \frac{d\sigma}{d\Omega} \right|_{\text{Rutherford}} = 1.296 \frac{((Z_1 Z_2)/E_{\text{c.m.}})^2}{\sin^4(\theta/2)}$$



e.g. An and Cai, Phys. Rev. C **73**, 054605 (2006)

# DWBA inputs *(some thoughts, others available) – FOR REFERENCE*

Numerous “modern” finite-range codes available. My experience is limited to **Ptolemy** by M. H. Macfarlane and S. C. Pieper [[ANL-76-11 Rev. 1, ANL Report \(1978\)](#)] and TWOFNR hosted by the University of Surrey. Others include DWUCK5 and FRESCO and so on ([ALL AVAILABLE ONLINE, ask me if interested](#)).

The ingredients are:

- Projectile wave functions:
  - **Argonne  $v_{18}$  potential** for (d,p) and (p,d) [older, but valid, is the Reid wave function]
  - For all other reactions there are new **GFMC parameterizations** of Brida, Pieper, and Wiringa, including spectroscopic overlaps [[Phys. Rev. C 84, 024319 \(2011\)](#)]
- Target wave functions:
  - Potential depth commonly varied to reproduce the relevant binding energy
  - $r_0 = 1.25\text{-}8\text{ fm}$ ,  $a = 0.65\text{ fm}$ ,  $V_{so} = 6\text{ MeV}$ ,  $r_{so0} = 1.1\text{ fm}$ ,  $a_{so} = 0.65\text{ fm}$
  - Radius parameter **consistent** with the average from  $^{16}\text{O}$ - $^{208}\text{Pb}$  from the (e,e'p) work of Kramer, Blok, and Lapikás [[Nucl. Phys. A 679, 267 \(2001\)](#)]
- Optical model potentials:
  - **Protons**, global potential of Koning and Delaroche [[Nucl. Phys. A 713, 231 \(2003\)](#)] with smooth dependence on energy, A, etc.
  - **Deuterons**, global potential of An and Cai [[Phys. Rev. C 73, 054605 \(2006\)](#)]
  - $A = 3$ , recent work of Pang et al. (**GDP08**) [[Phys. Rev. C 79, 024615 \(2009\)](#)]
  - For  **$\alpha$  particles** we used a ‘static’ potential derived from the  $A = 90$  region [[Nucl. Phys. A 131, 653 \(1969\)](#)] (... more later on this)

# ***Doing a direct-reaction experiment***

What reactions?

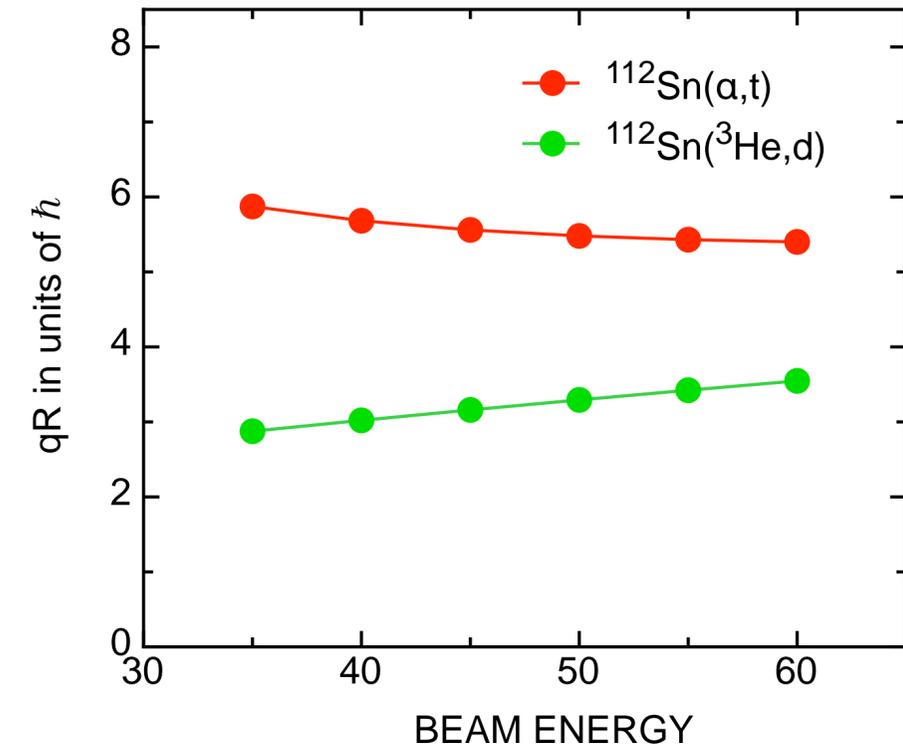
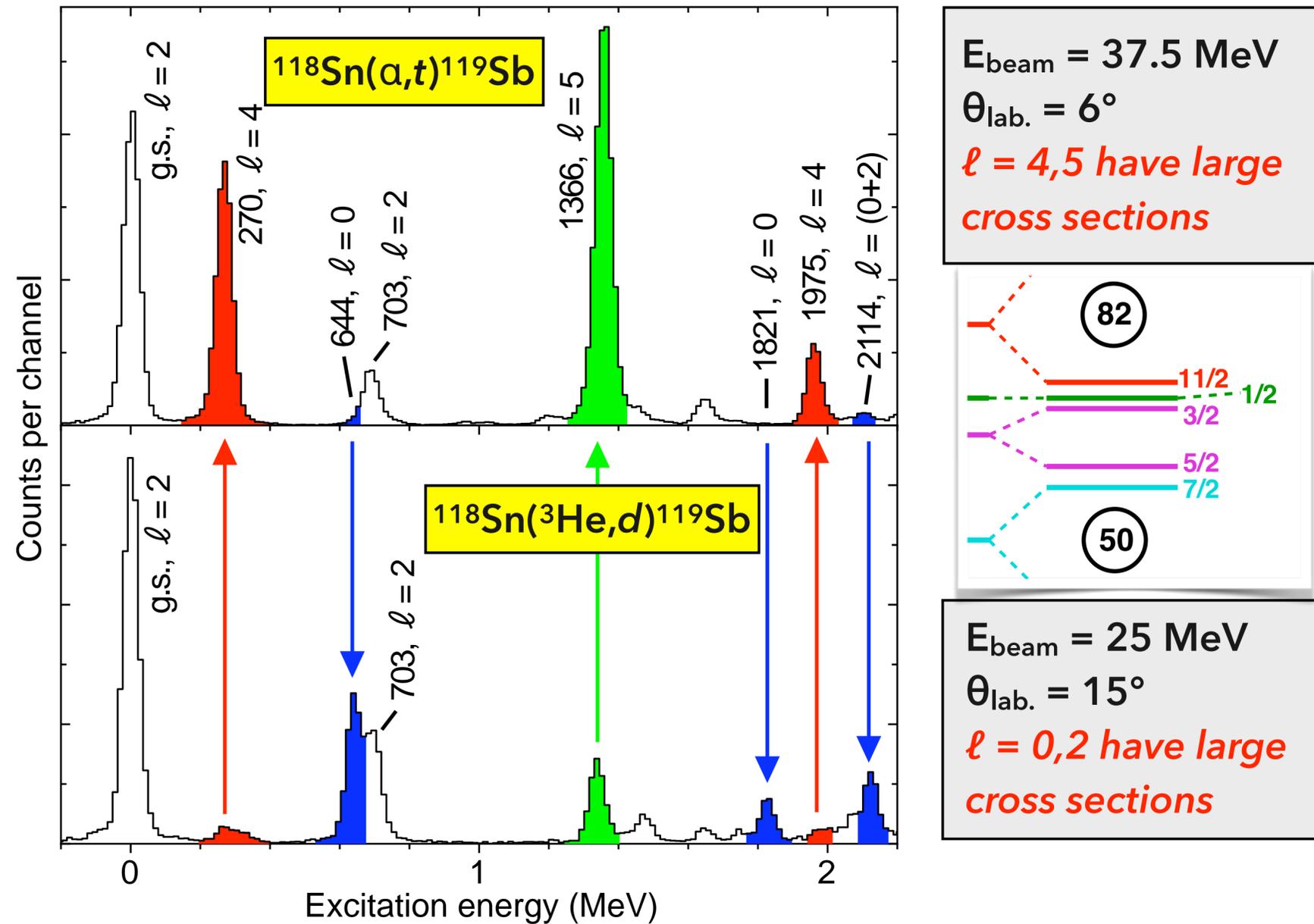
What energy?

What angles?

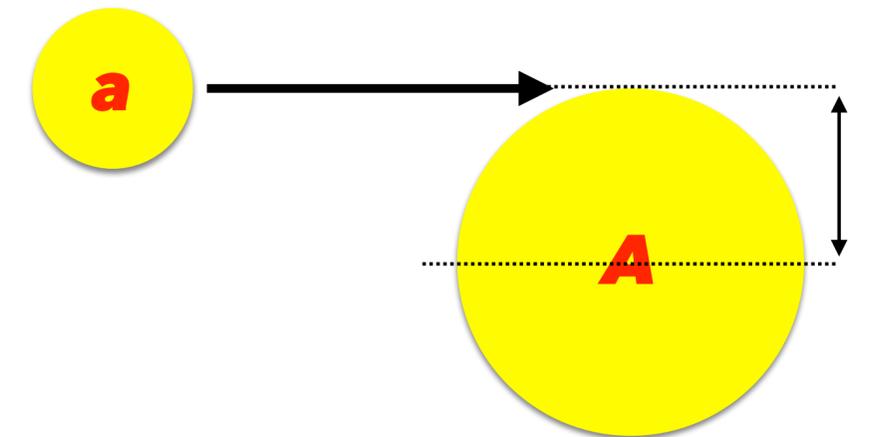
# Angular momentum matching

$$A(a,b)B$$

Proton adding -  $^{118}\text{Sn}(\alpha,t)^{119}\text{Sb}$  versus  $^{118}\text{Sn}(^3\text{He},d)^{119}\text{Sb}$



Simple, semi-classical approx.

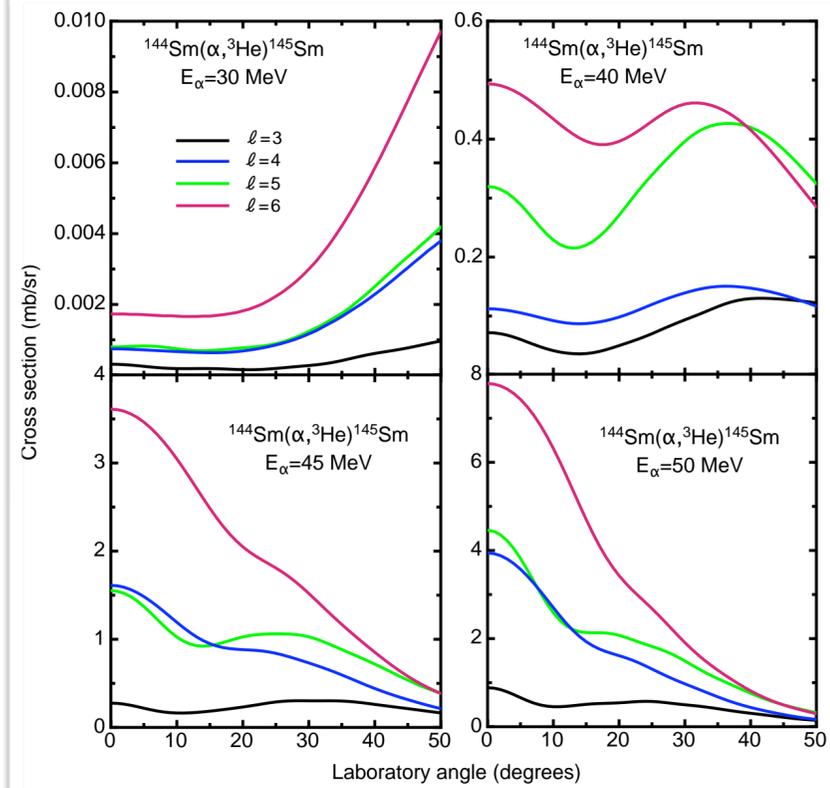


$$\ell = r \times p$$

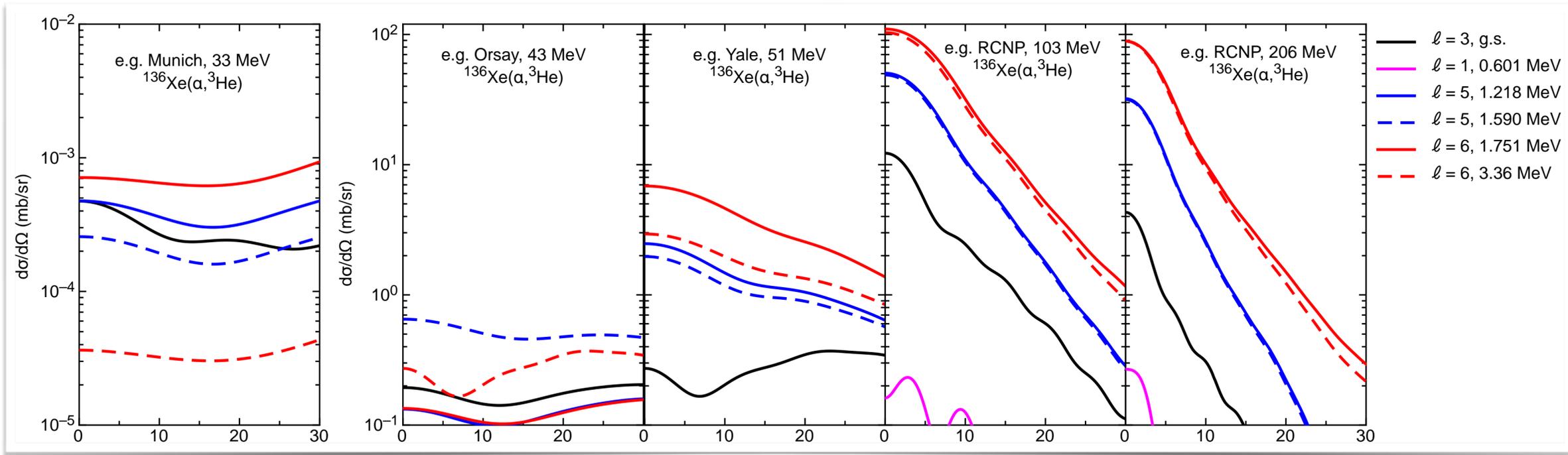
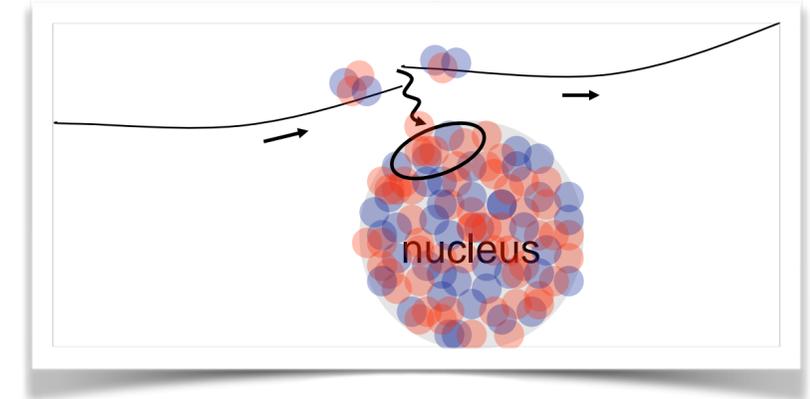
$$\ell \leq qR$$

Data from measurement performed at Yale in March 2010. Part of the thesis work of A. J. Mitchell, University of Manchester  
There are numerous other examples in the literature.

# Incident beam energy ...

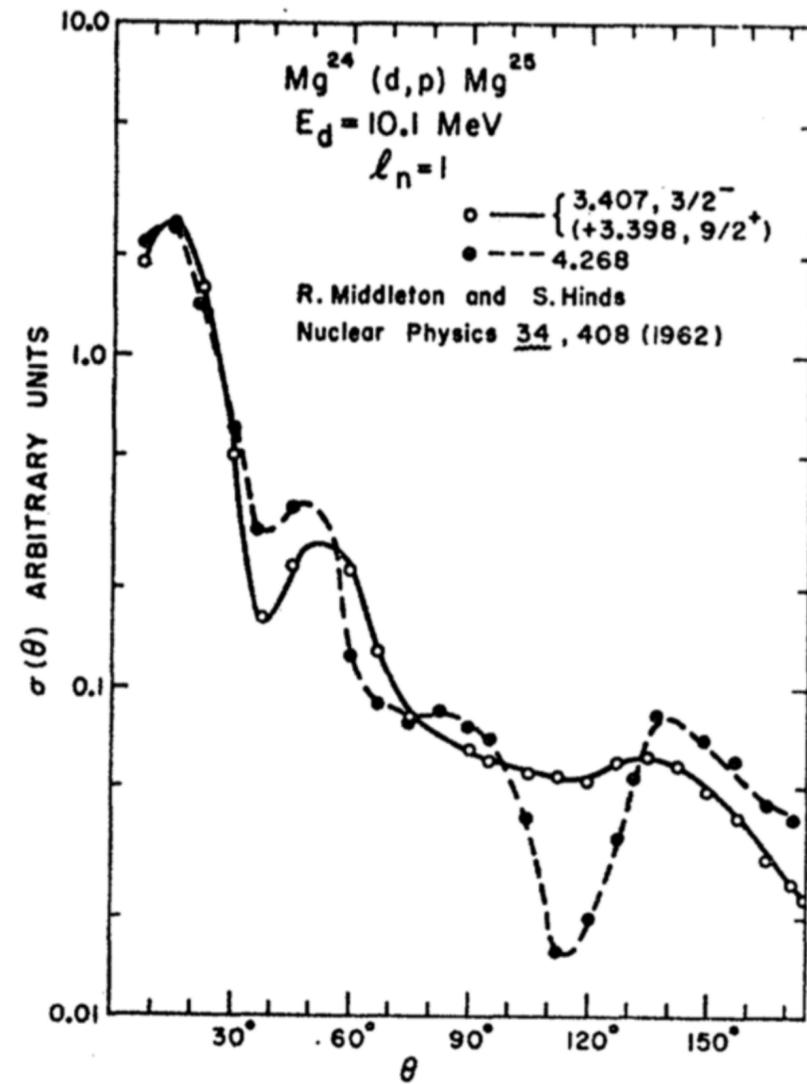


When reactions are carried out at energies a **few MeV/u above** the Coulomb barrier, the resultant **angular distributions are forward peaked**. Note, it is important that **both the incoming and outgoing ions** are a few MeV/u above the barrier.



# Angular distributions

Peak cross sections =  
reliable cross sections



$\ell = 0$

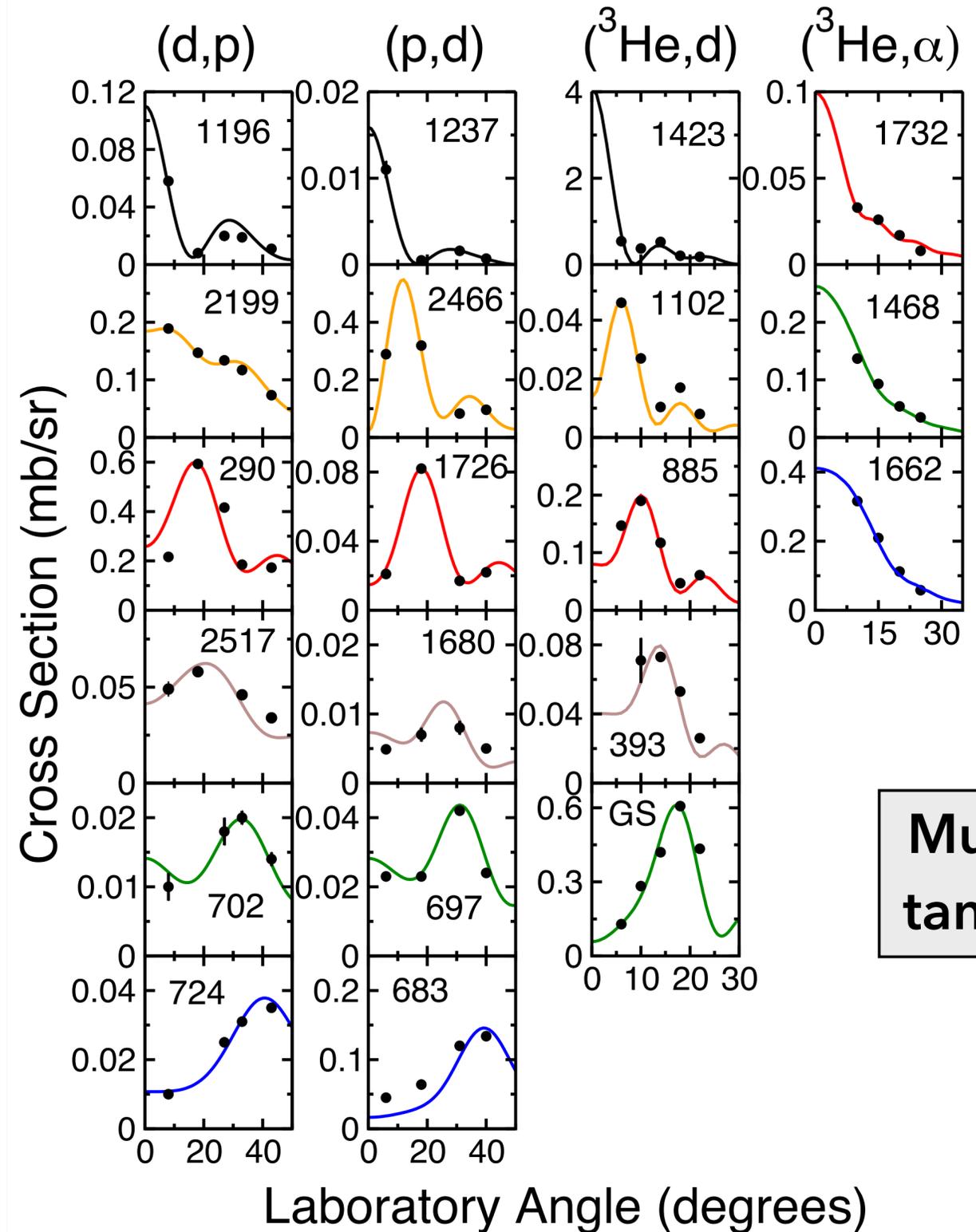
$\ell = 1$

$\ell = 2$

$\ell = 3$

$\ell = 4$

$\ell = 5$



Munich  
tandem

# Direct-reaction check list

Putting it all together, an experiment can be designed (of course with the caveat that compromises are inevitable, especially with exotic-beam studies ...)

- **Energy**

- a few MeV/u above the Coulomb

- **Angles**

- at the first maxima in peaks of the angular

- **Reaction choice**

- momentum matching

- **Spectrometer**

- **Absolute cross sections**

- depends what you want out of your measurement, though always useful. Measure scattering in the Coulomb regime.

- Which model (fixed at DWBA in this talk, but ADWA and so on).

- What consistency checks can be built in to the measurement?

- What systematic uncertainties can be minimized?

- Technique / accelerator / targets / (sometimes no choice) etc

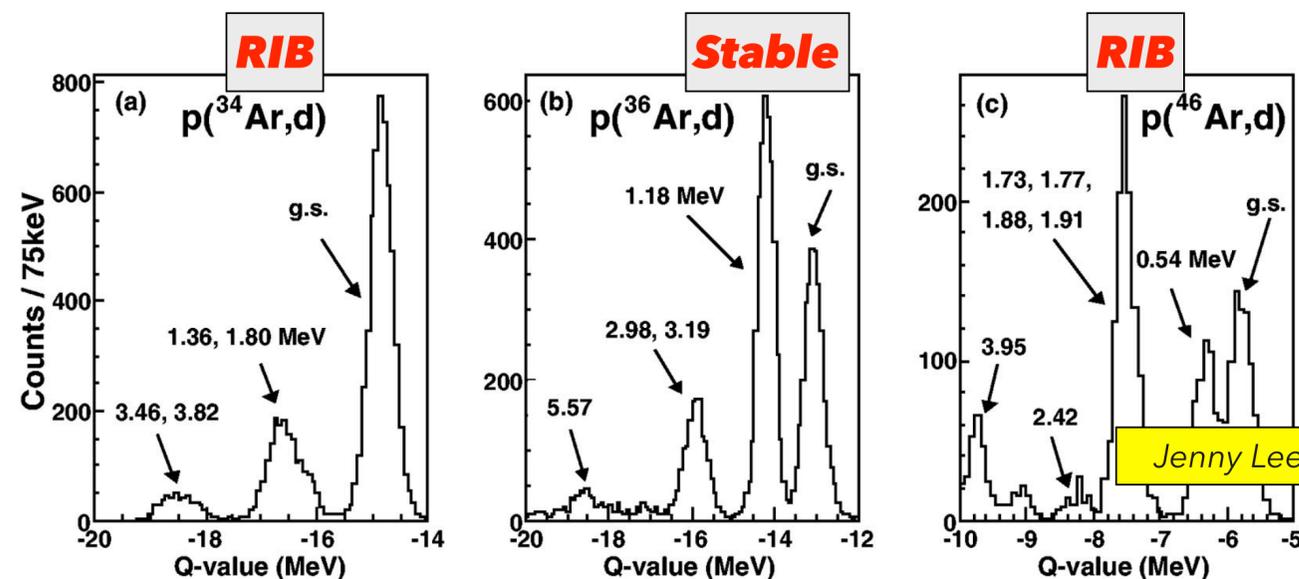
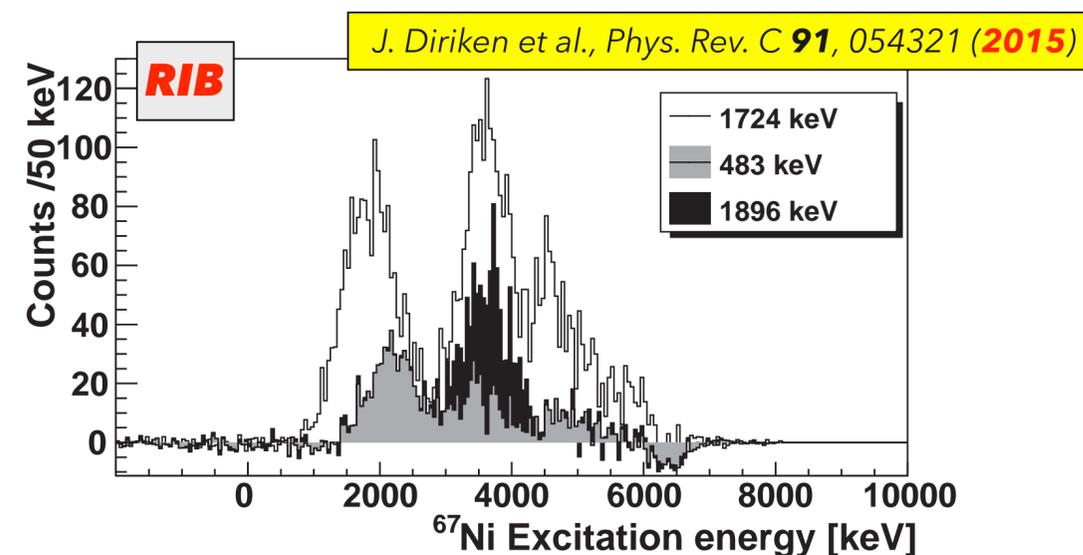
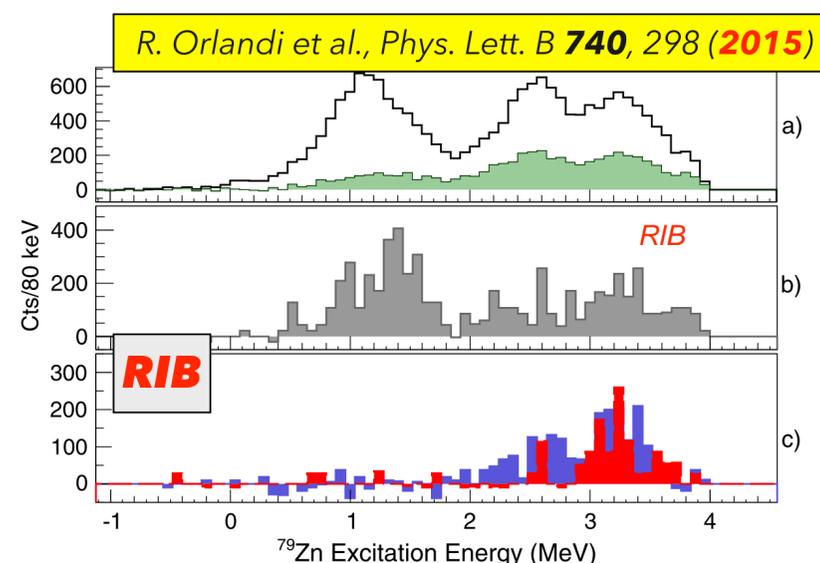
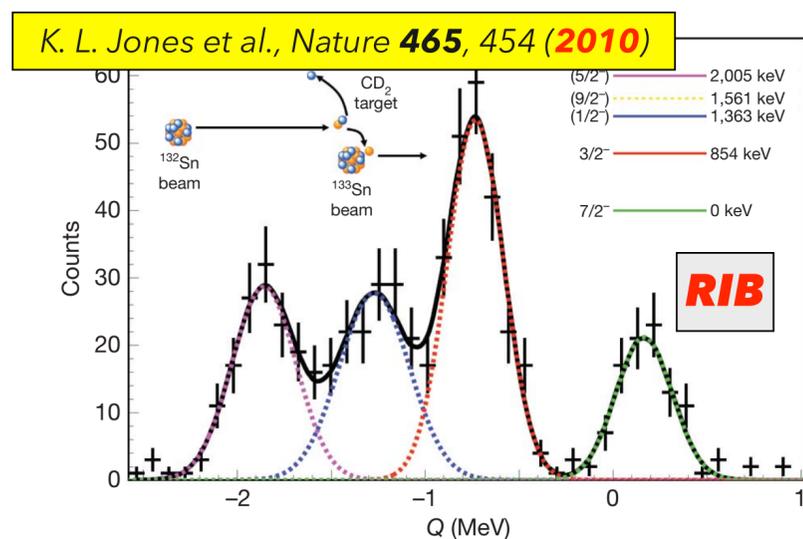
*I will come back to this list several times in the examples section.*

*If the experiment is done appropriately, then the analysis in terms of DWBA will likely be valid.*

# Compromises

One can not always choose the optimal set up, and **compromises are essential for progress**. This may be the case with radioactive ion beam experiments where limited beam energy and intensity are available, or perhaps for classes of reactions or targets (gases, etc).

Great examples of a compromise are the pioneering works below:



Jenny Lee et al., *PRL* **104**, 112701 (2010)

# ***Some examples***

**Introducing single-particle energies**

**Introducing occupancies (vacancies)**

# Single-particle energies – a 'classic' example

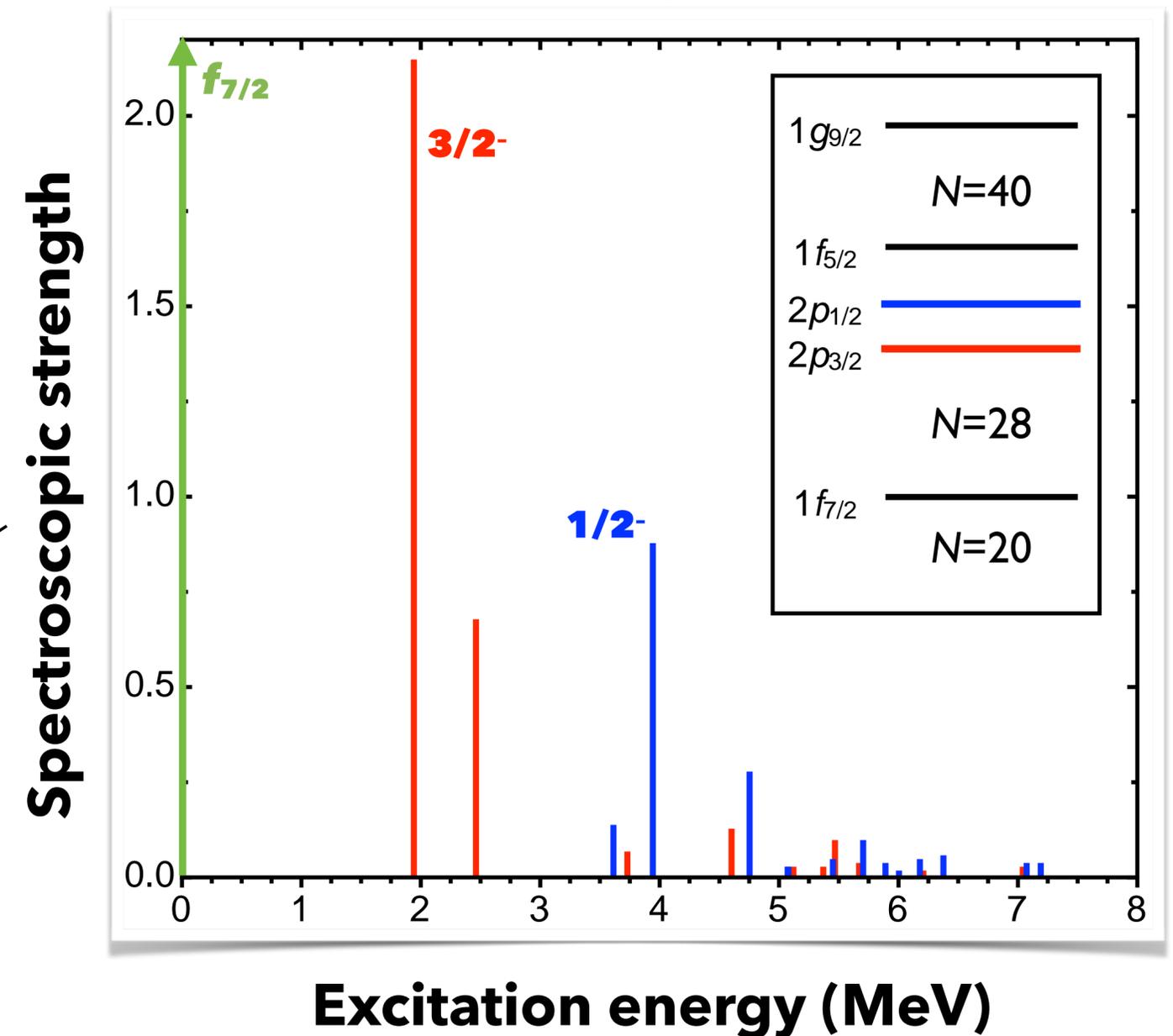
In many cases, single-particle strength is fragmented over several states.  $^{41}\text{Ca}$  is an excellent example of this: just one neutron outside the **doubly-magic**  $^{40}\text{Ca}$  (20 protons, 20 neutrons) ...

For the  $(d,p)$  reaction ...

$$\left. \frac{d\sigma}{d\Omega} \right|_{\text{measured}} = gS_j \left. \frac{d\sigma}{d\Omega} \right|_{\text{model}} \iff \sigma_{\text{exp}} = \frac{(2j+1)C^2 S_j \sigma_{\text{DWBA}}}{\dots}$$

**The centroid of single-particle strength-- weighted by its spectroscopic strength--is a good approximation to the energy of the underlying single-particle orbital.**

**(ESPEs, SPEs in lit., theory)**



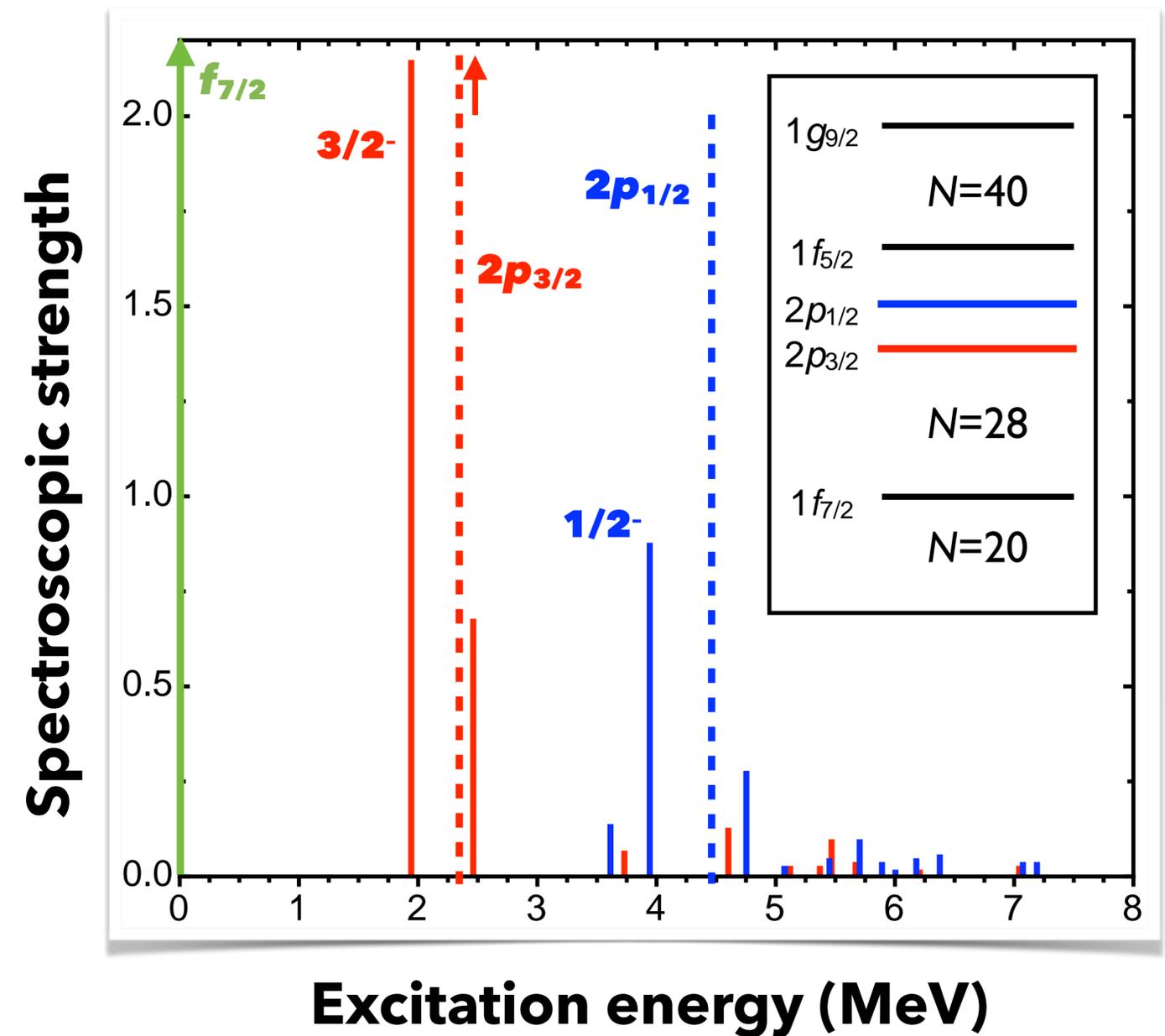
# Single-particle energies – a 'classic' example

In many cases, single-particle strength is fragmented over several states.  $^{41}\text{Ca}$  is an excellent example of this: just one neutron outside the **doubly-magic**  $^{40}\text{Ca}$  (20 protons, 20 neutrons) ...

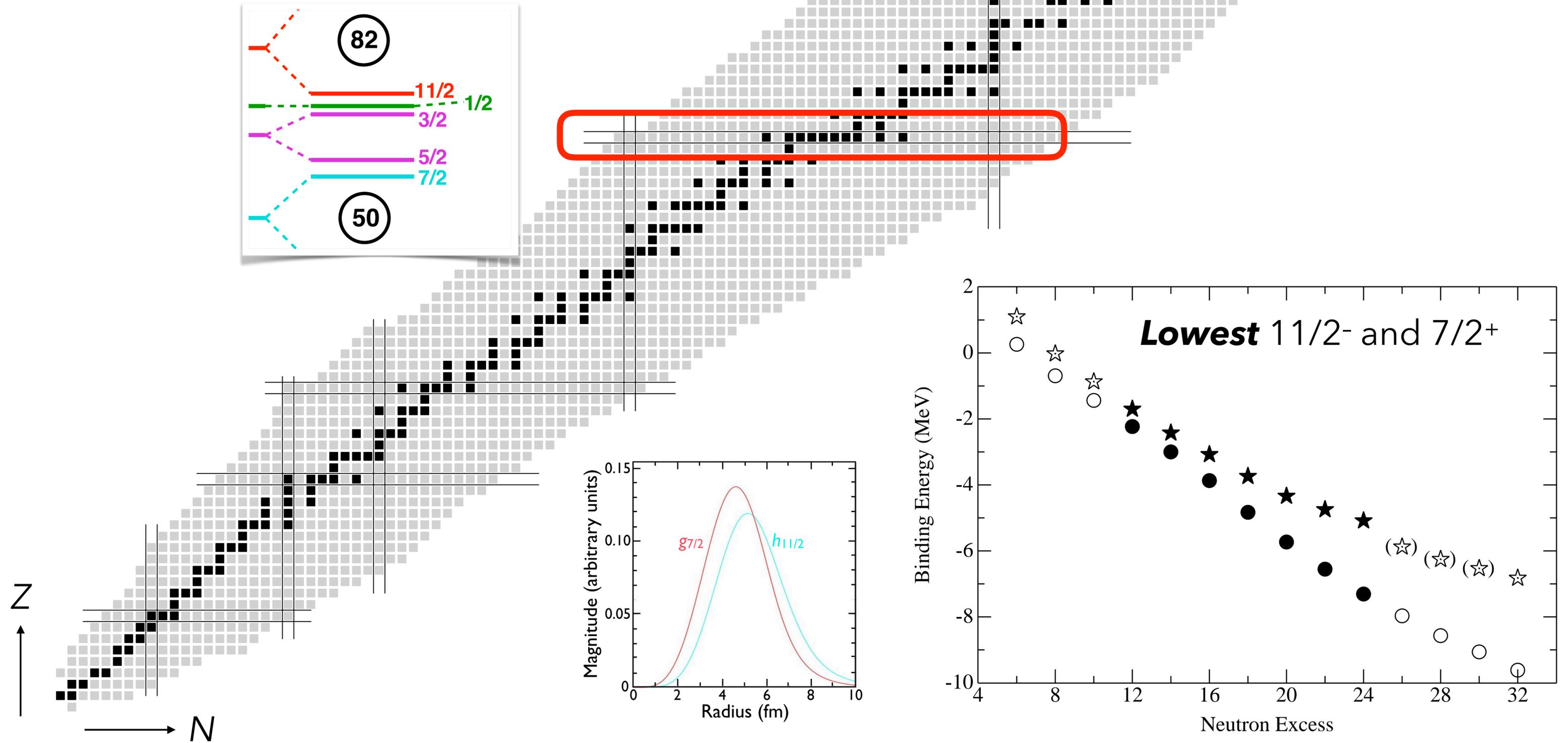
$$E'_j = \frac{\sum_i E_j^*(i) S_j(i)}{\sum_i S_j(i)}$$

The lowest **1/2-** and **3/2-** states lie at **3613.5** and **1942.7** keV, respectively.

The centroid of single-particle strength, the energy of the **2p<sub>1/2</sub>** and **2p<sub>3/2</sub>** orbitals, lie at **4491** and **2327** keV. This is significantly different, a fact often overlooked.



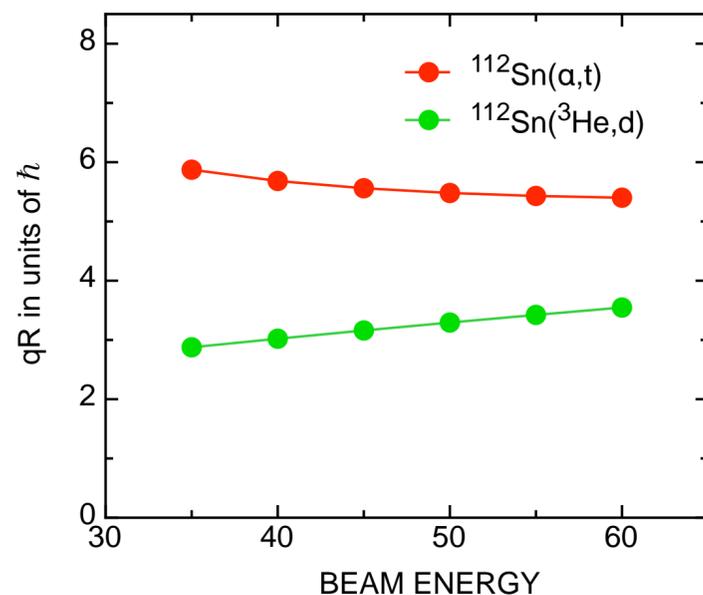
# Another (more modern/relevant) example



J. P. Schiffer et al., Phys. Rev. Lett. **92**, 162501 (2004)

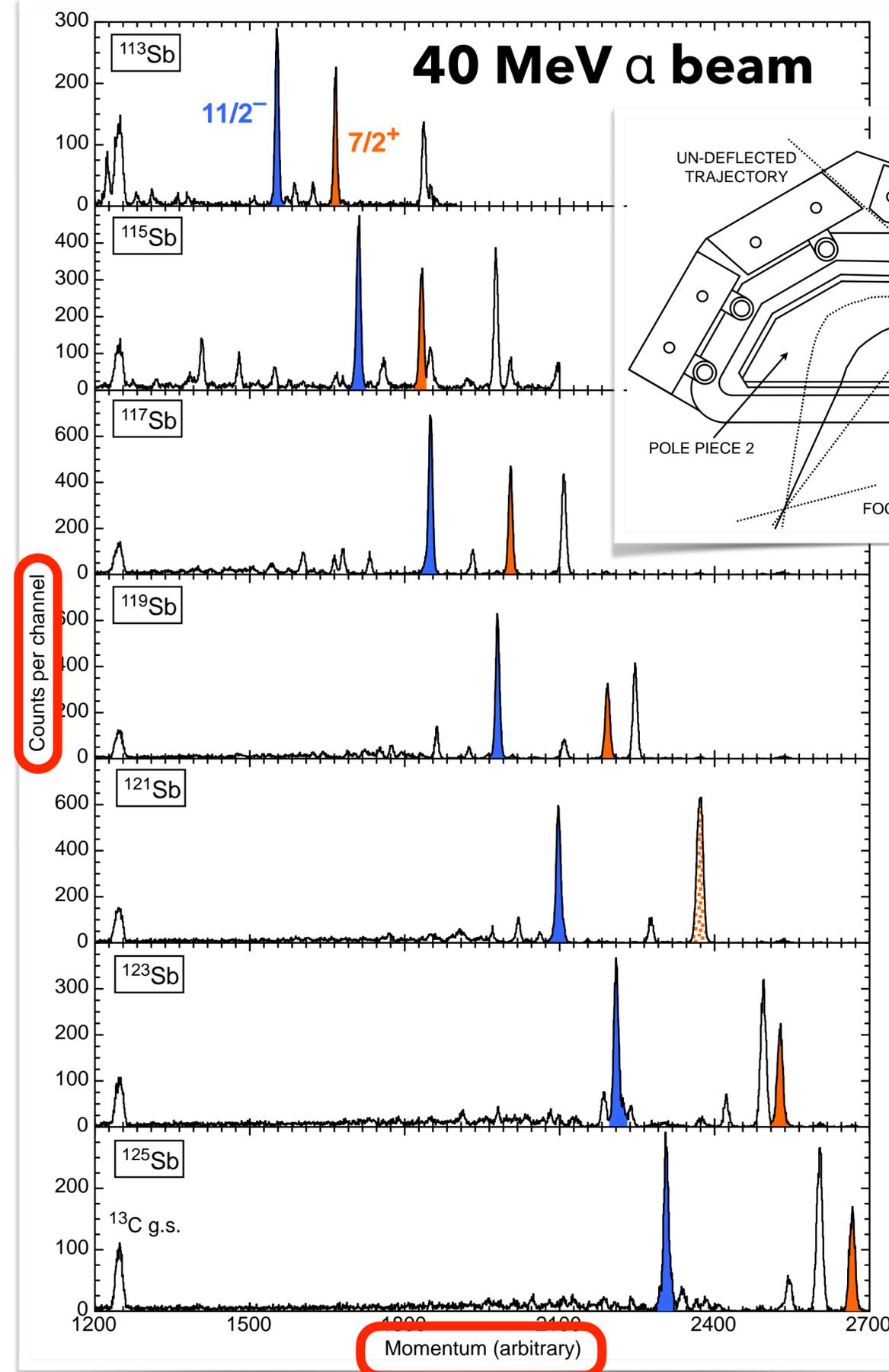
# Sb isotopes

## Transfer to high- $j$ states



- Why  $(\alpha, t)$ ?
- Why 40 MeV?
- Why Yale?
- Why  $200 \mu\text{g}/\text{cm}^2$  targets?
- Why 7 targets?
- What else?

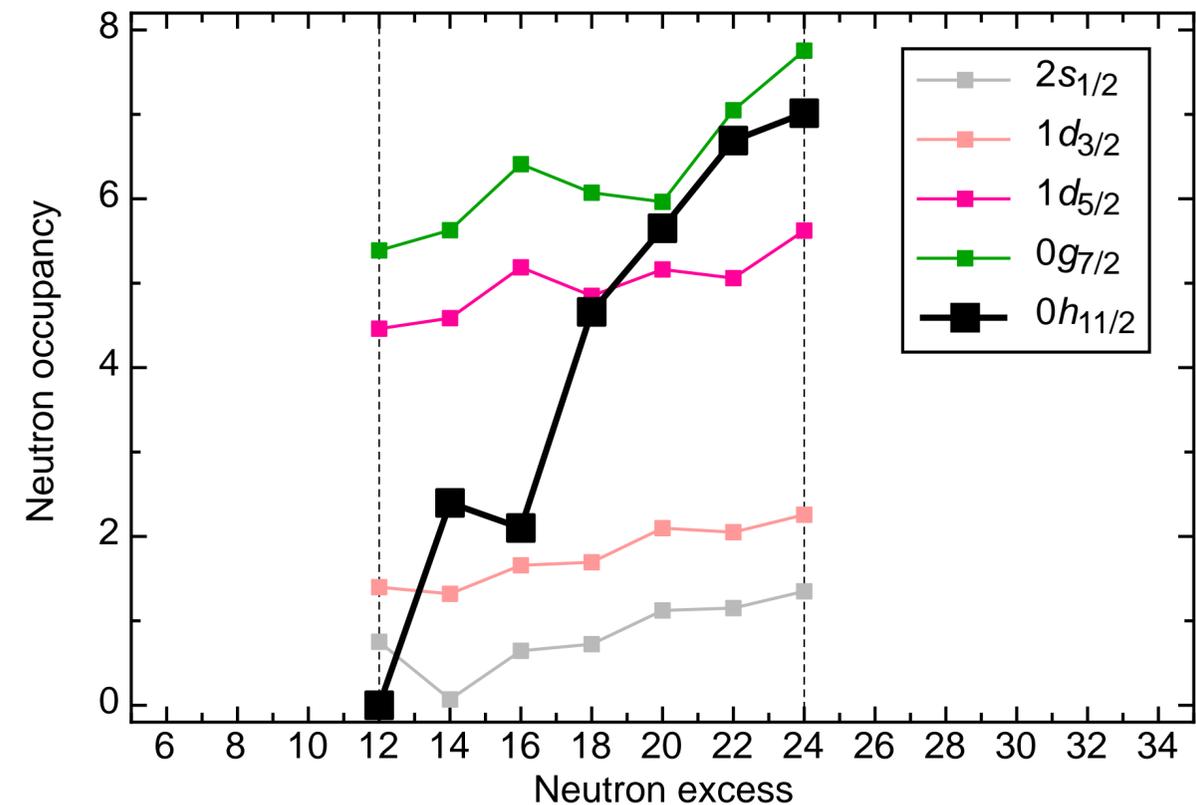
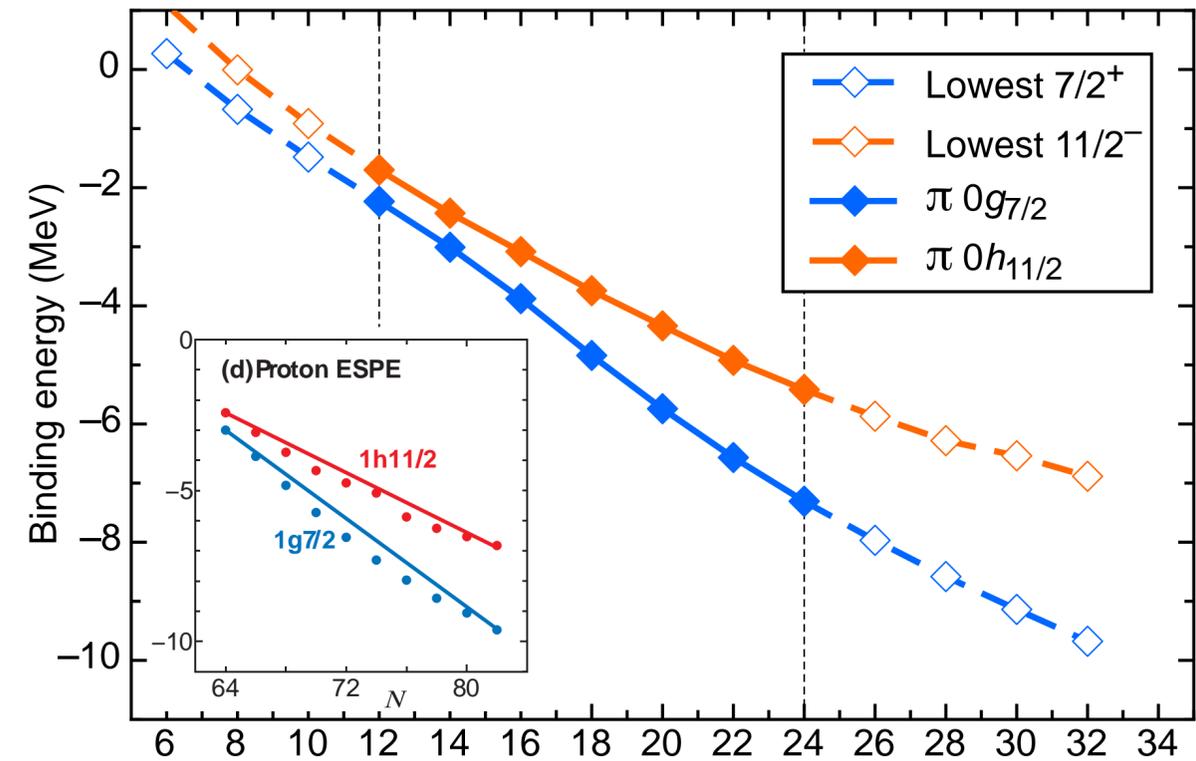
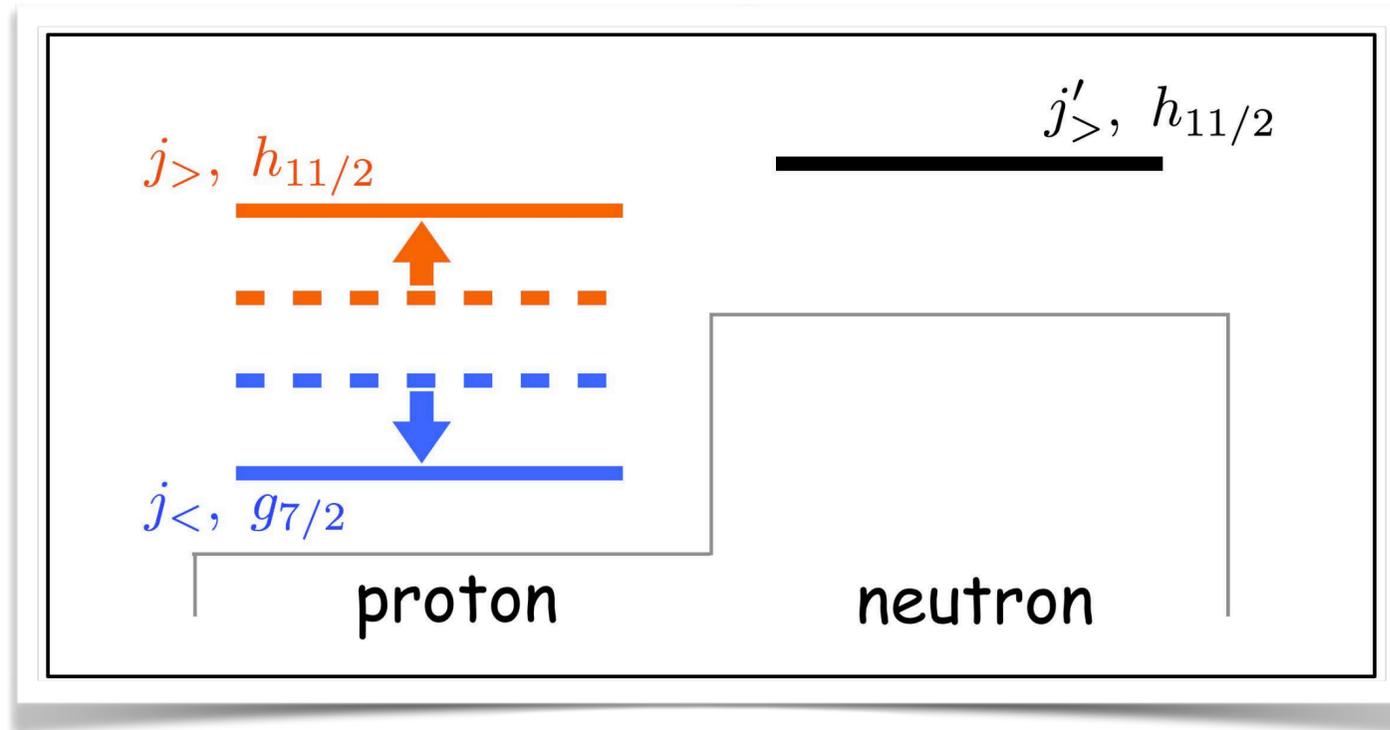
Target	$7/2^+$	$11/2^-$	Ratio	$C^2S_{7/2}$	$C^2S_{11/2}$
$^{112}\text{Sn}$	14.6	21.4	1.47	0.99	0.84
$^{114}\text{Sn}$	19.6	27.3	1.39	1.10	0.93
$^{116}\text{Sn}$	19.7	30.9	1.57	0.95	0.97
$^{118}\text{Sn}$	20.4	33.5	1.64	0.88	0.99
$^{120}\text{Sn}$	27.9	39.4	1.41	1.13	1.12
$^{122}\text{Sn}$	24.6	35.5	1.45	0.98	1.00
$^{124}\text{Sn}$	24.7	39.2	1.59	1.00	1.12



J. P. Schiffer et al., *Phys. Rev. Lett.* **92**, 162501 (2004)

# Explanation? Tensor force

Important data for Otsuka's demonstration of the ubiquitous role of the tensor force in NS.



## Reactions – drive the field forward

Note here that the neutron occupancies are also key ingredient in this story! Often forgotten and here the data is not particularly great – with potential future Exotic Beam studies e.g.  $^{100}\text{Sn}(\alpha, t)$  and  $^{138}\text{Sn}(\alpha, t)$ , it is equally important to understand the behavior of the neutrons too.

# ***Some examples***

Introducing single-particle energies

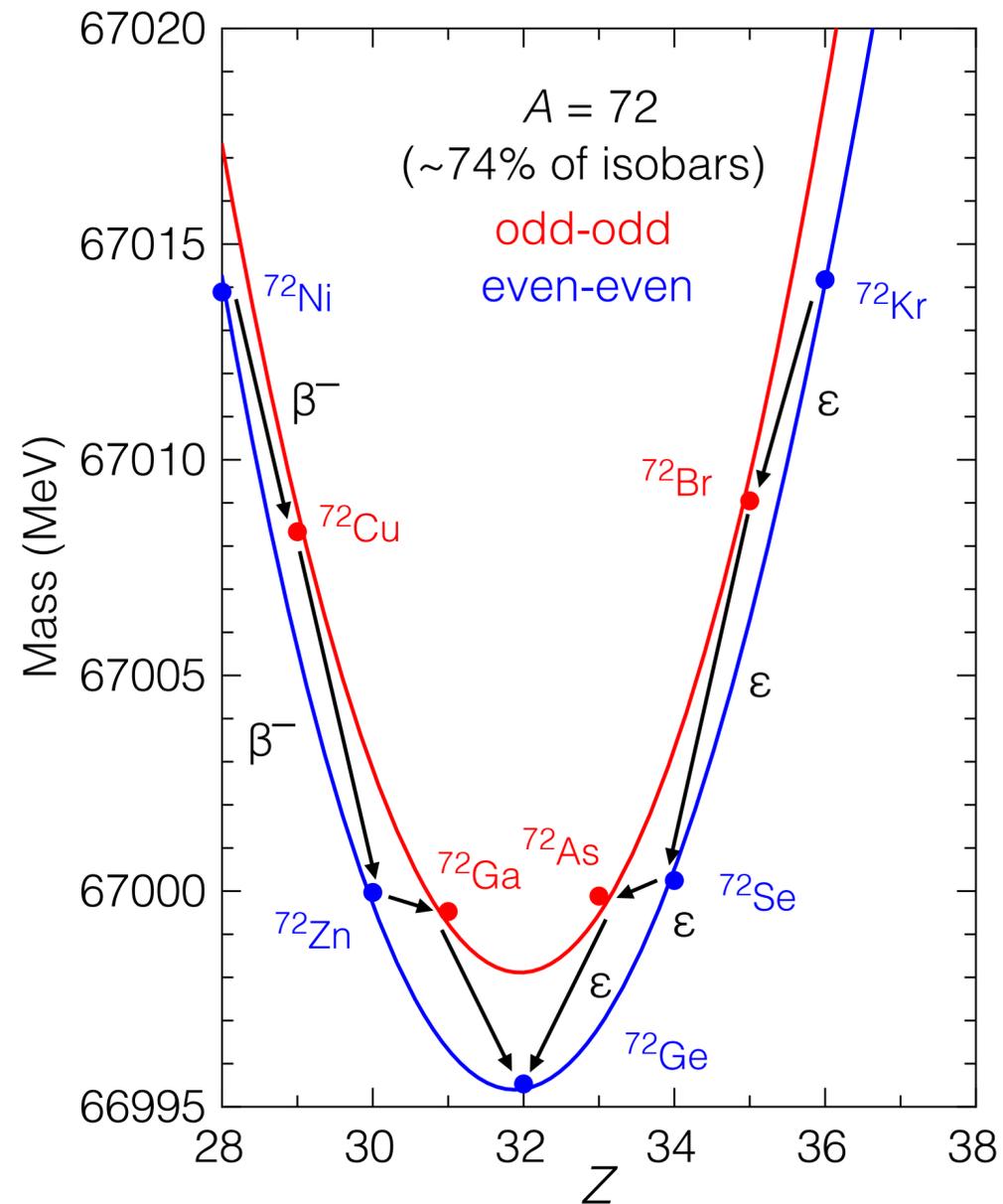
Introducing occupancies (vacancies)

Introducing single-particle energies – *can clearly see the important of this in guiding our understanding of nuclear structure ... many future nuclear-reaction experiments with exotic beams*

Introducing occupancies (vacancies) – *use neutrinoless double- $\beta$  decay as an example*

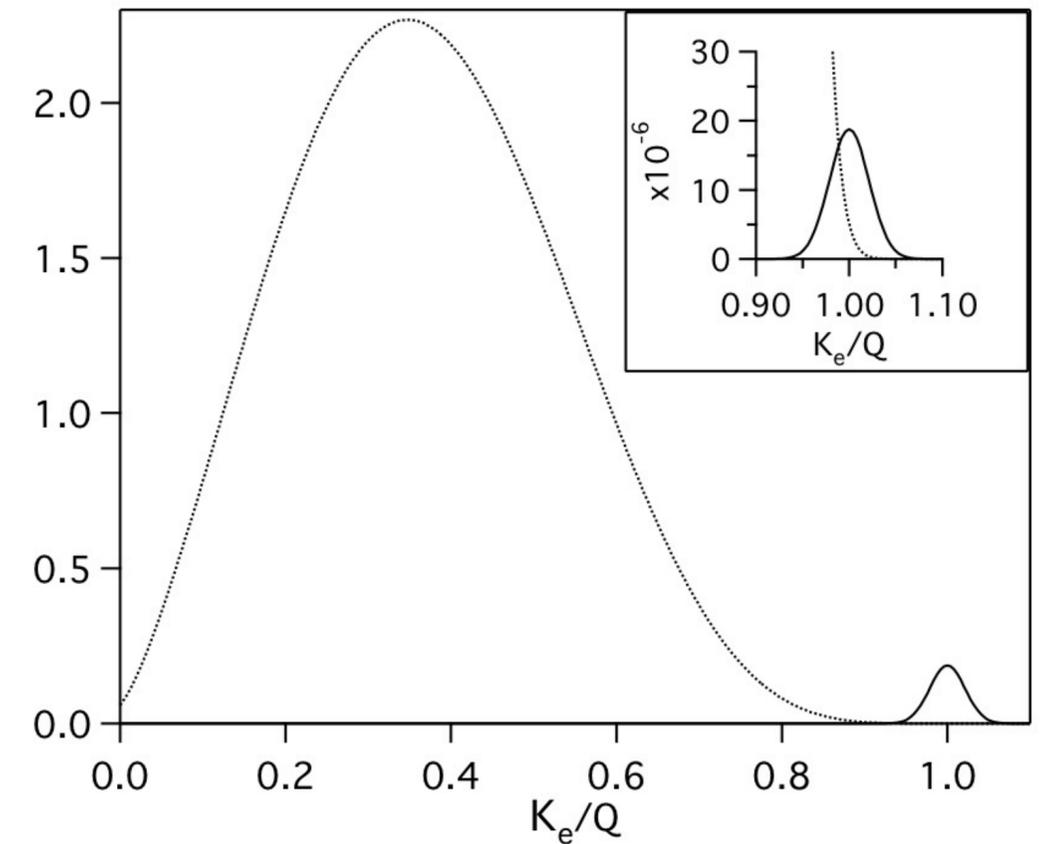
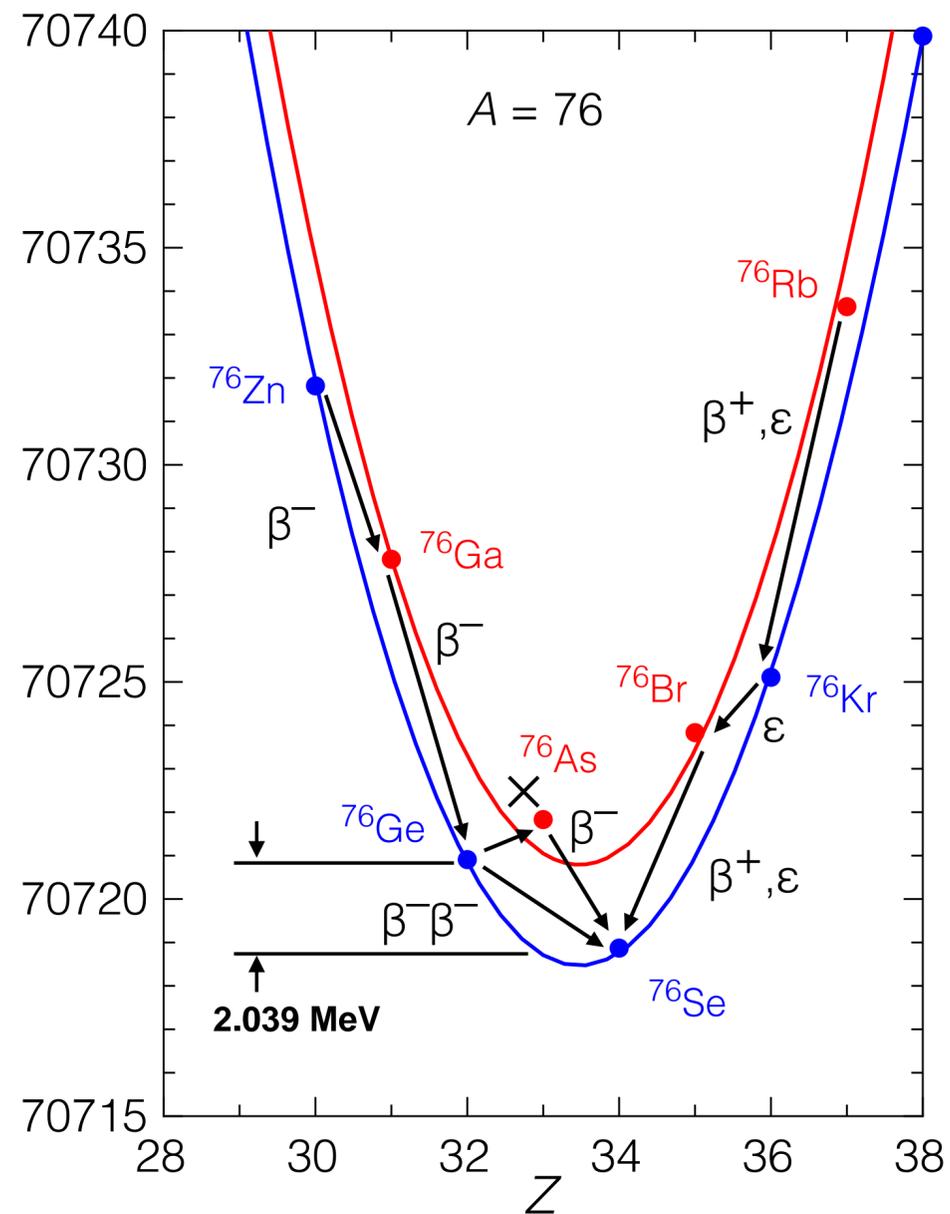
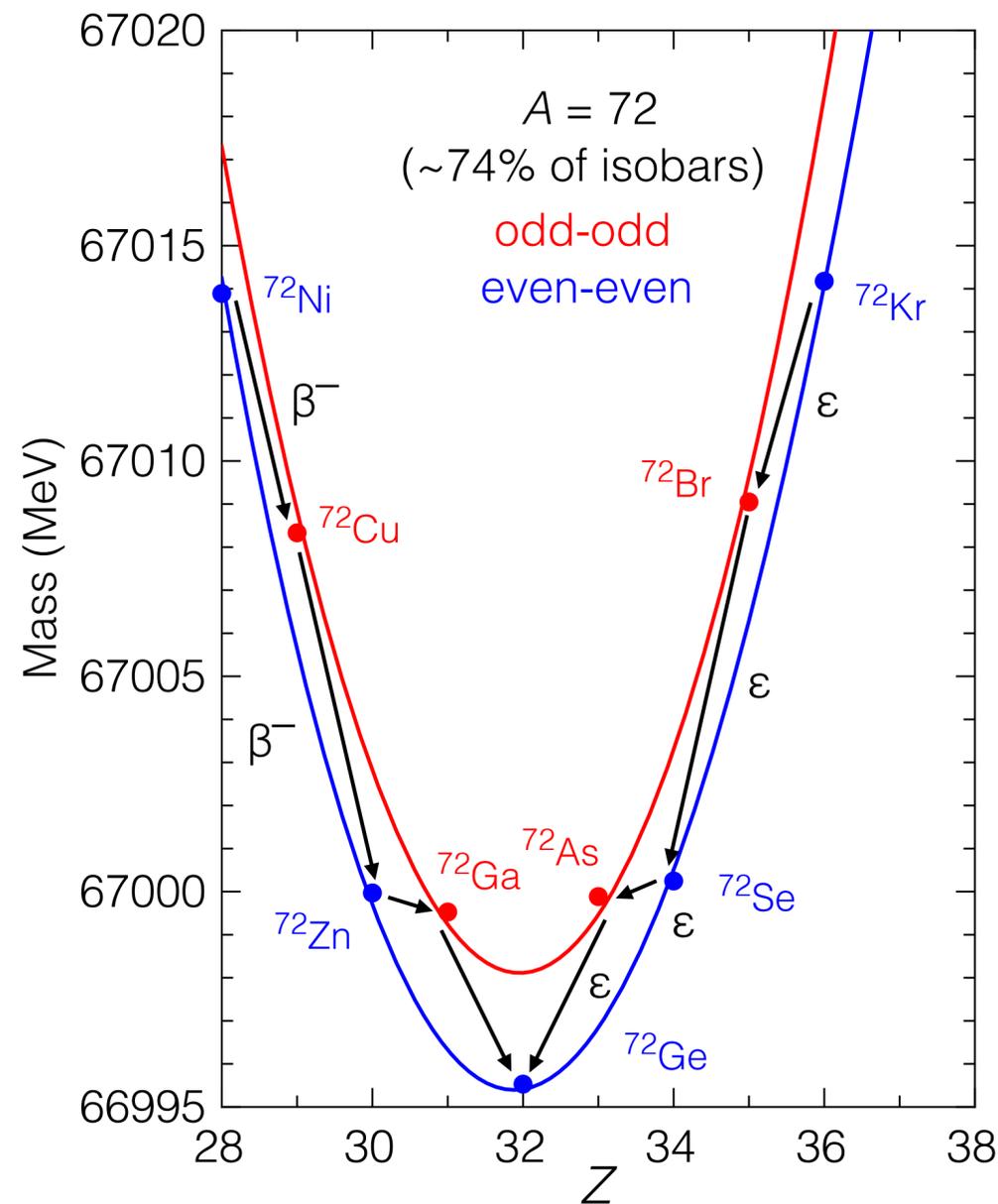
# Neutrinoless double beta decay

A hypothetical decay process ... made 'possible' by pairing in nuclei



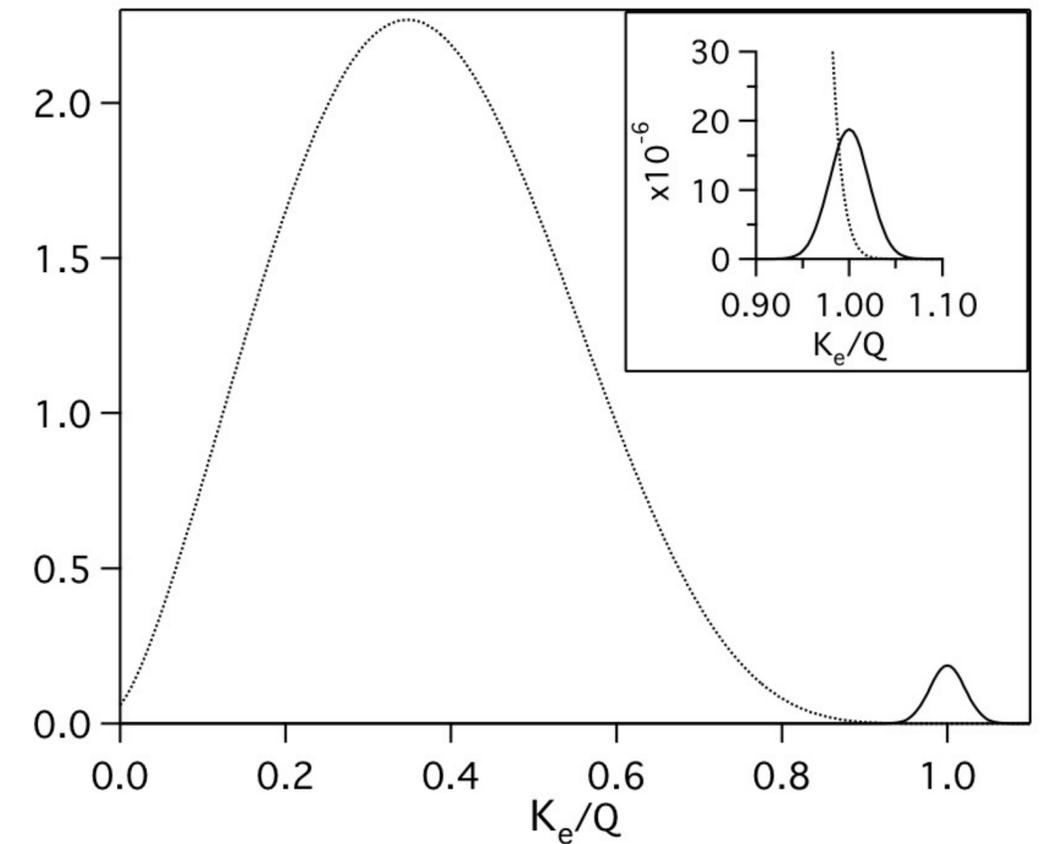
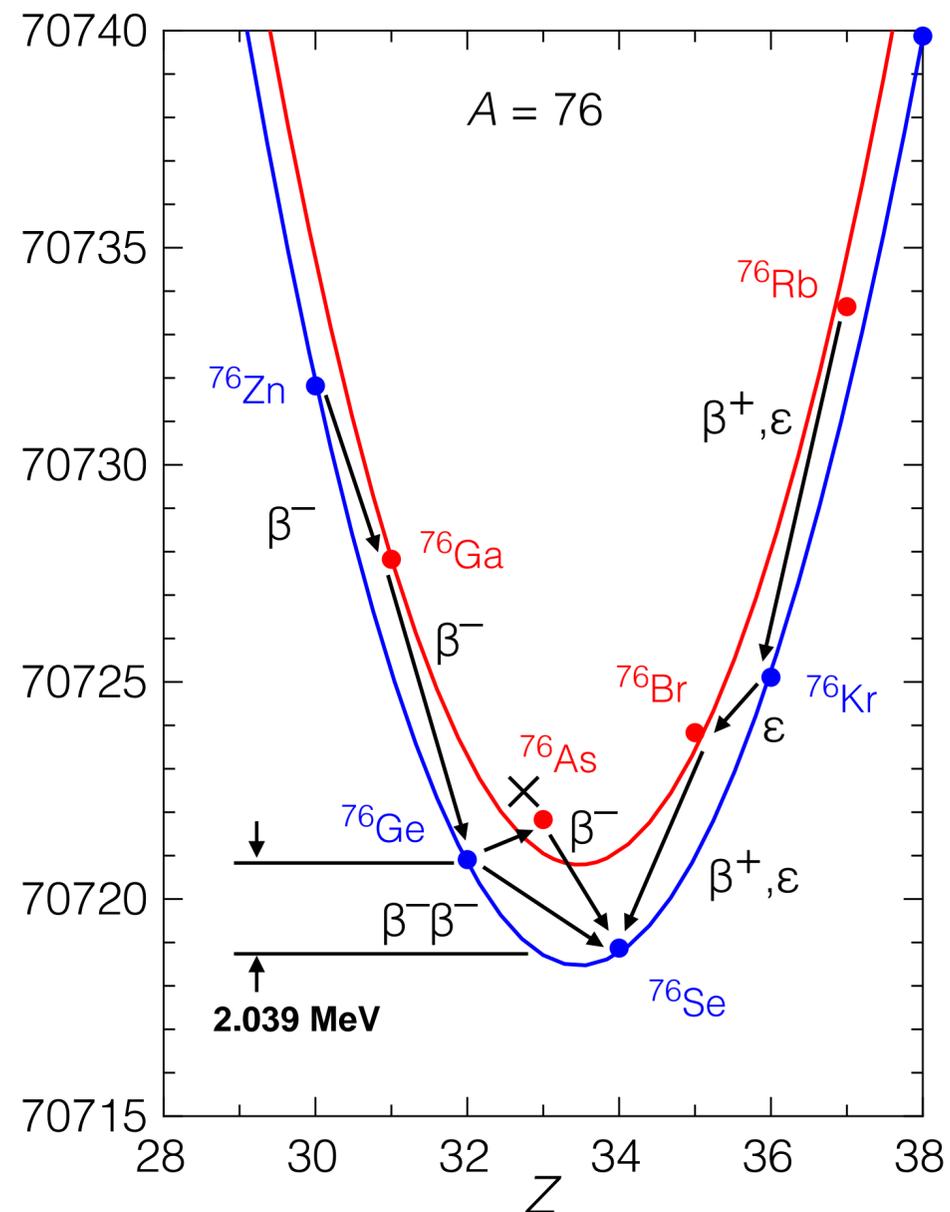
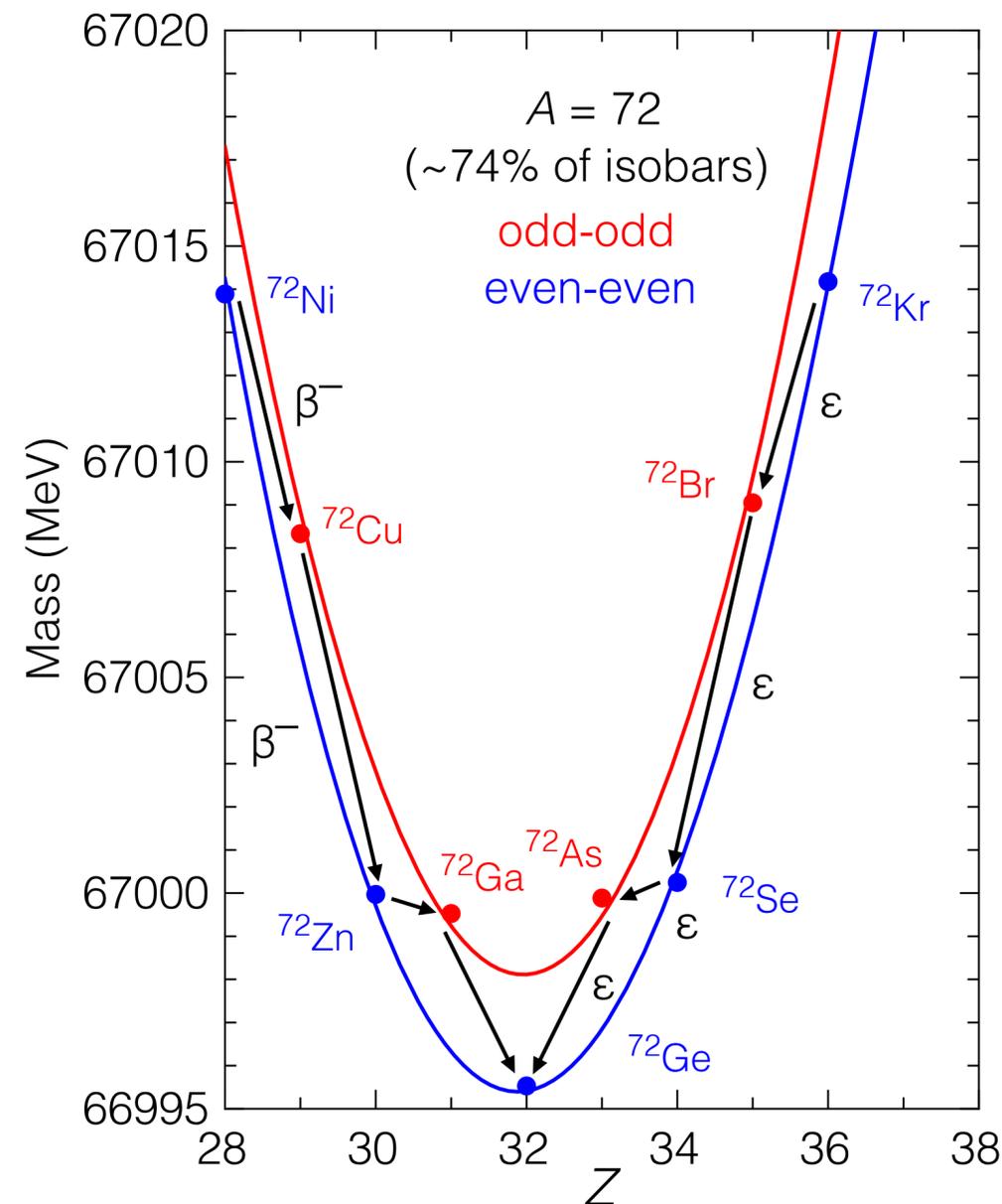
# Neutrinoless double beta decay

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# Neutrinoless double beta decay

A hypothetical decay process ... made 'possible' by pairing in nuclei

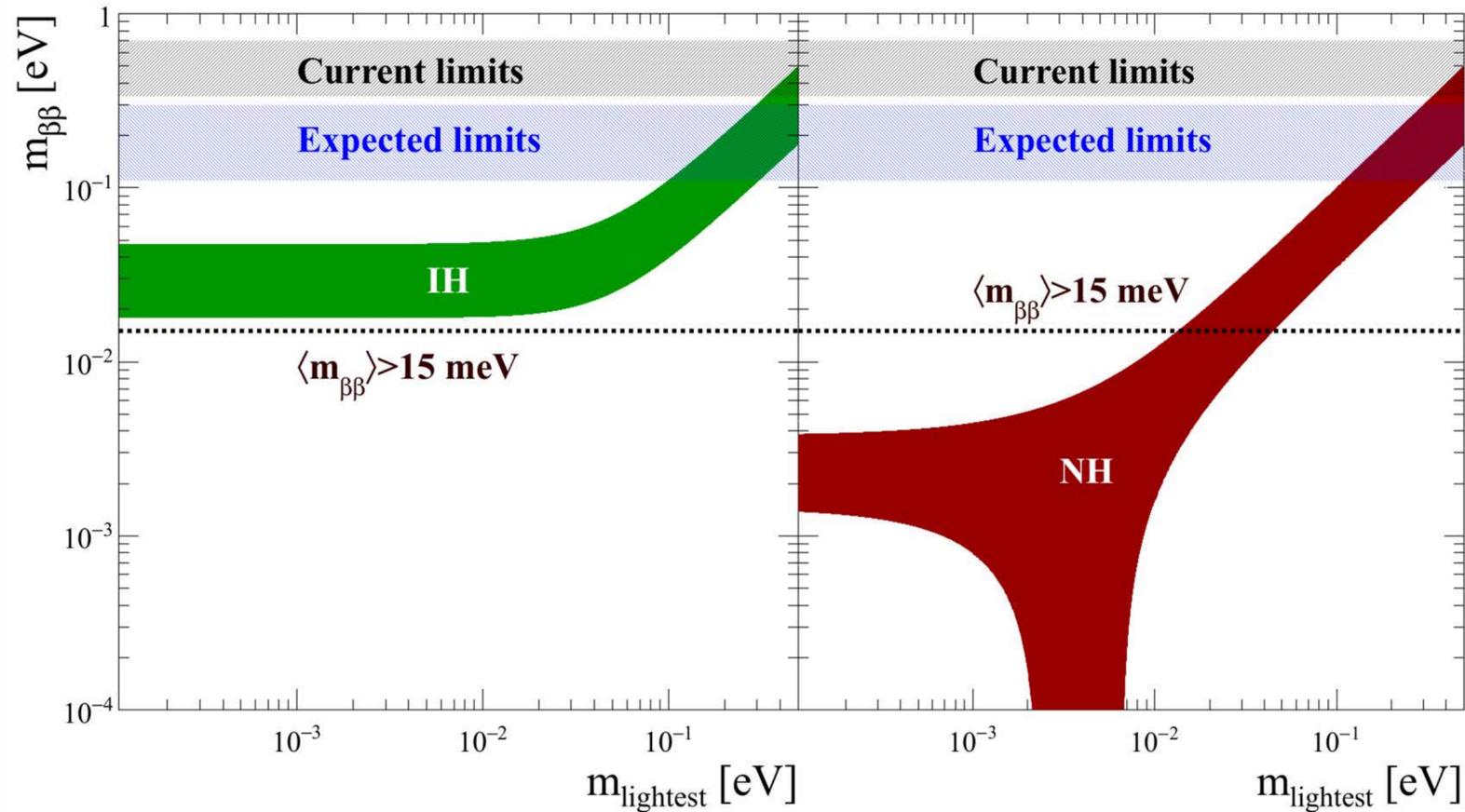


$$[T_{1/2}^{0\nu}]^{-1} = (\text{Phase Space Factor}) \times |\text{Nuclear Matrix Element}|^2 \times |\langle m_{\beta\beta} \rangle|^2$$

# REACHING FOR THE HORIZON

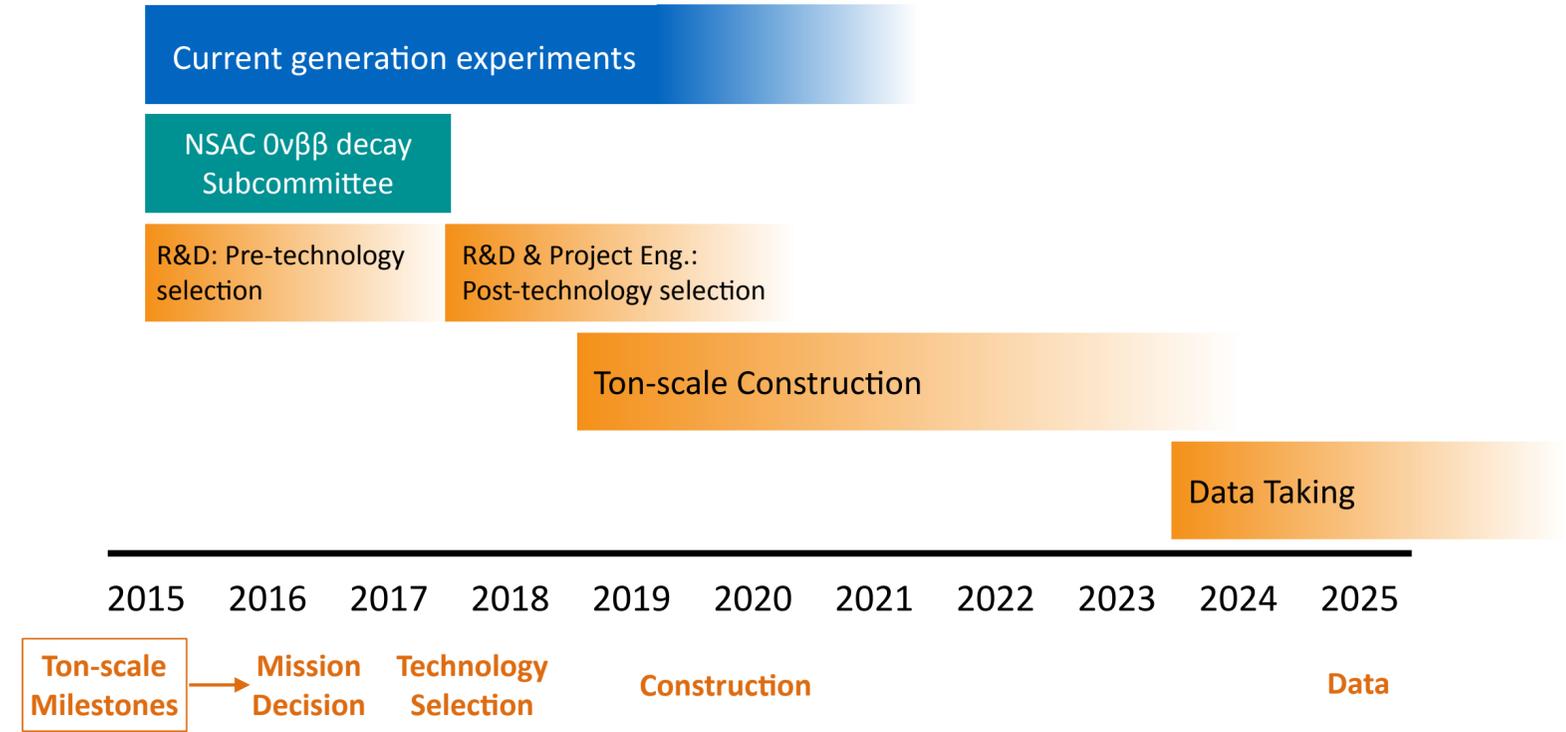
The 2015  
LONG RANGE PLAN  
for NUCLEAR SCIENCE





### Ton-scale Neutrinoless Double Beta Decay ( $0\nu\beta\beta$ ) - A Notional Timeline

*Search for Lepton Number Violation*

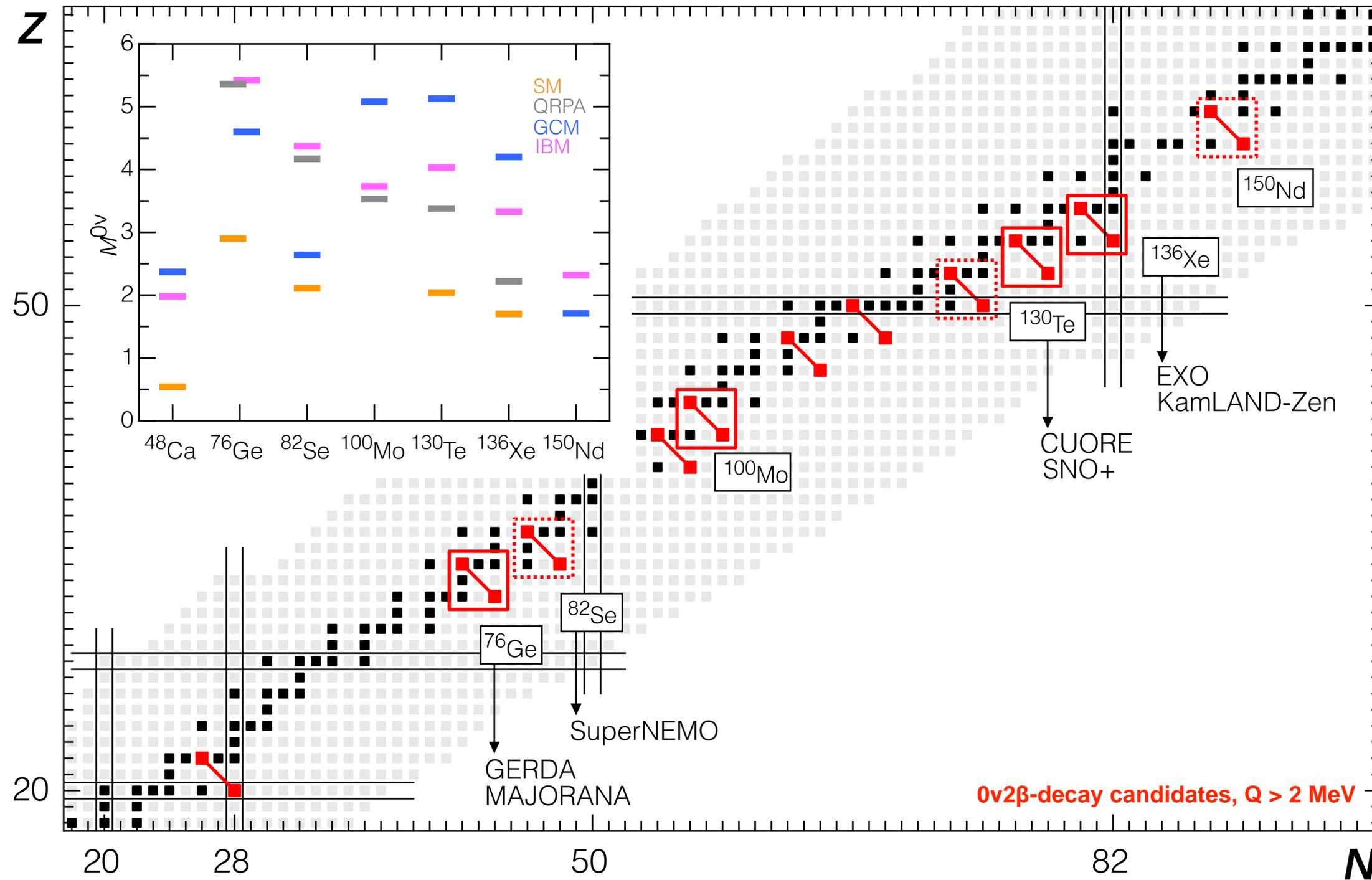


“The **second recommendation** specifically targets the development and deployment of a ton-scale neutrino-less double beta decay experiment. **Demonstration experiments at the scale of 100 kg are currently underway** to identify the requirements and candidate technologies for a larger, next-generation experiment, which is needed to be sensitive to postulated new physics. **An ongoing NSAC subcommittee is helping to guide the process of the down-select, from several current options to one U.S.-led ton-scale experiment.**”

“Construction of this flagship experiment is expected to require five years, with capital investment peaking at about \$50M/year during this period.”

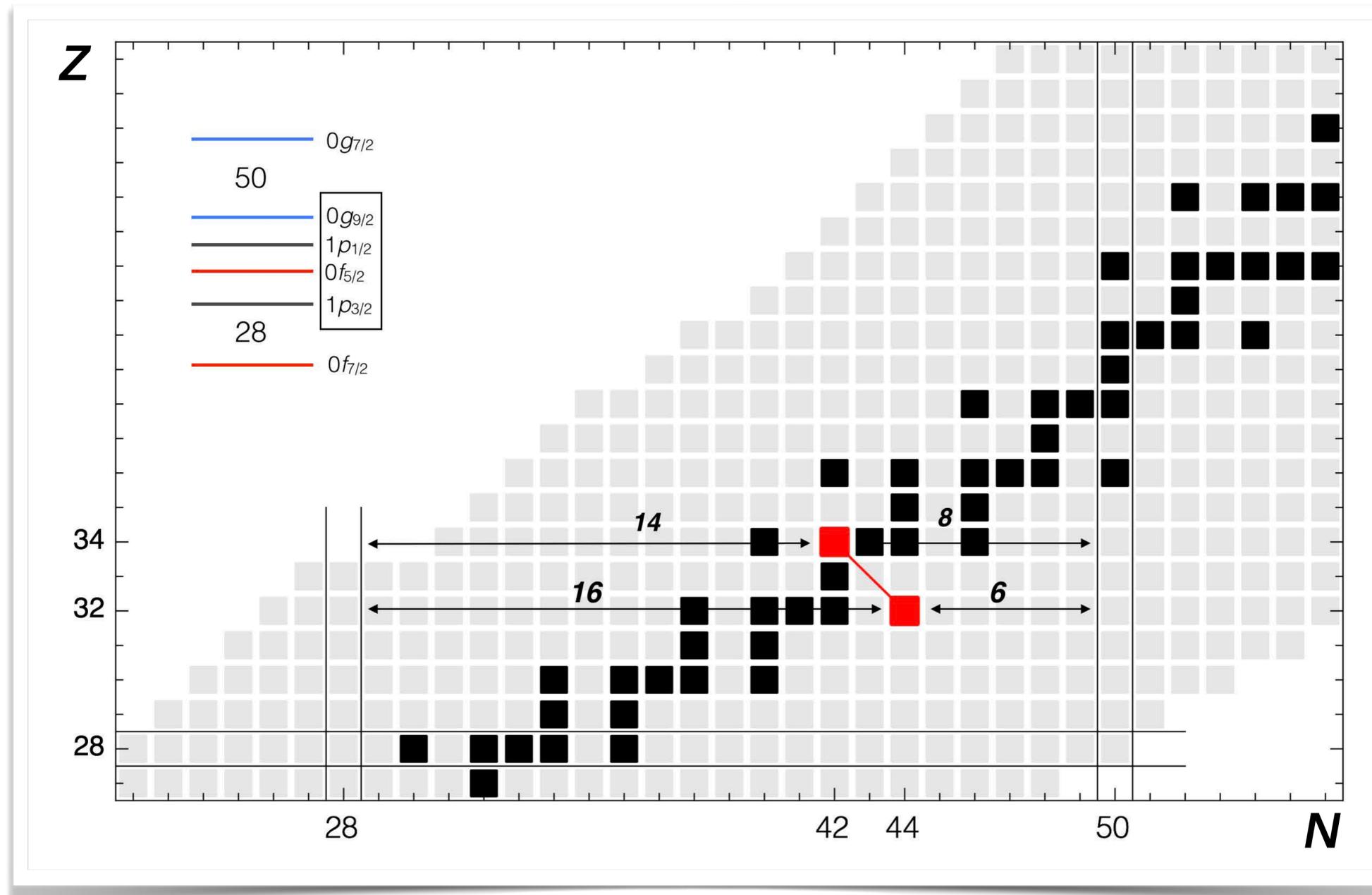
measurements use the atomic nucleus as a laboratory, **nuclear theory is critical in connecting experimental results to the underlying lepton-number violating interactions and parameters through nuclear matrix elements**, which account for the strong interactions of neutrons and protons. Currently, **there exists about a factor of two uncertainty in the relevant matrix elements**, but by the time a ton-scale experiment is ready to take data, **we expect reduced uncertainties as a result of the application to this problem of improved methods to solve the nuclear many-body physics.**”

# Neutrinoless double beta decay



$$[T_{1/2}^{0\nu}]^{-1} = (\text{Phase Space Factor}) \times |\text{Nuclear Matrix Element}|^2 \times |\langle m_{\beta\beta} \rangle|^2$$

# The $^{76}\text{Ge}$ and $^{76}\text{Se}$ isotopes



What is the **occupancy and vacancy** of the active orbitals? How is the proton/neutron strength distributed (nature of the Fermi surface)? How does it **change** from parent to daughter?

-- **NUCLEON TRANSFER REACTIONS** can answer this (let's see how ....!)

# Spectroscopic factors, sum rules

REVIEWS OF MODERN PHYSICS VOLUME 32, NUMBER 3 JULY, 1960

## Stripping Reactions and the Structure of Light and Intermediate Nuclei\*

M. H. MACFARLANE

Argonne National Laboratory, Lemont, Illinois, and University of Rochester, Rochester, New York†

AND

J. B. FRENCH

University of Rochester, Rochester, New York

vacancies + occupancies → valency of the orbit

$$\text{Vacancy}_j = \sum_i (2j + 1) C^2 S_j^{\text{adding}}$$

$$\text{Occupancy}_j = \sum_i C^2 S_j^{\text{removing}}$$

$$S' \equiv \sigma_{\text{exp}} / \sigma_{\text{DWBA}}, \quad N_j \equiv S' / S$$

$$N_j \equiv [\sum S'_{\text{removing}} + \sum (2j + 1) S'_{\text{adding}}] / (2j + 1)$$

**Is the normalization arbitrary?** Well, yes and no. If you have measured **absolute cross sections**, then no (with caveats). Otherwise, yes.

**Does the normalization have a physical meaning?** Or does it just mask things we don't understand? Let's see what value it appears to take before answering this.

# SFs → sum rules → occupancies

## Cross sections to nuclear structure

### $^{76}\text{Ge}(p,d)^{75}\text{Ge}$

E	$\ell$	$S'$
0	1	0.45
191	4	
248	1	0.12
317	3	
457	3	
575	1	1.29
651	3	
885	1	0.10
1137	1	0.11
1250	3	
1410	0	
1451	1	0.37
1580	3	

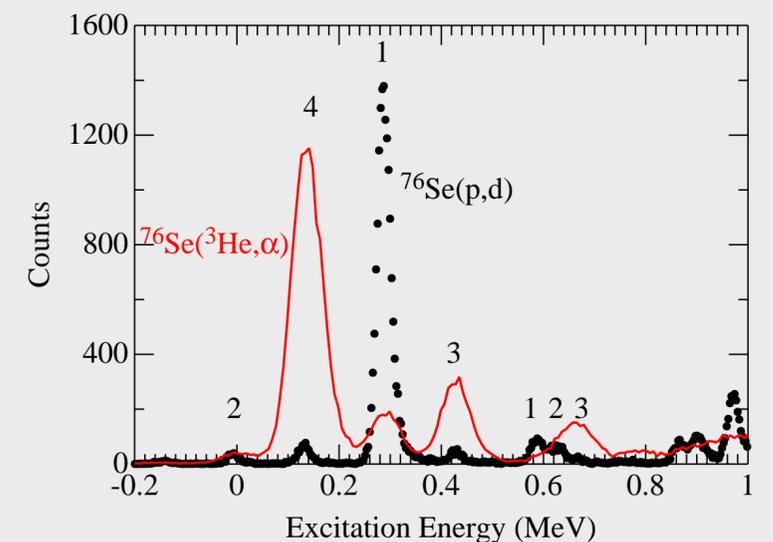
### $^{76}\text{Ge}(d,p)^{77}\text{Ge}$

E	$\ell$	$(2j+1)S$
160	1	0.44
225	4	
421	2	
505	2	
629	1	0.15
884	2	
1021	1	0.12
1048	1	0.04
1250	0	
1385	2	

$$N_j \equiv S' / S$$

## Checks

- (d,p) and (p,d) done at 7.5 MeV/u and 11.5 MeV/u
- ( $\alpha, ^3\text{He}$ ) and ( $^3\text{He}, \alpha$ ) for the  $\ell = 3, 4$ , at 10 MeV/u and 8.7 MeV/u
- 4 targets used (**consistency**)
- Absolute cross sections (Rutherford scattering measured)
- Yale Enge split-pole spectrograph (**now at FSU**)
- Stats (10s nA beams, <1%)
- Targets around 200  $\mu\text{gcm}^2$



# SFs → sum rules → occupancies

## Cross sections to nuclear structure

### $^{76}\text{Ge}(p,d)^{75}\text{Ge}$

E	$\ell$	$S'$	S
0	1	0.45	0.85
191	4		
248	1	0.12	0.23
317	3		
457	3		
575	1	1.29	2.43
651	3		
885	1	0.10	0.19
1137	1	0.11	0.21
1250	3		
1410	0		
1451	1	0.37	0.70
1580	3		

### $^{76}\text{Ge}(d,p)^{77}\text{Ge}$

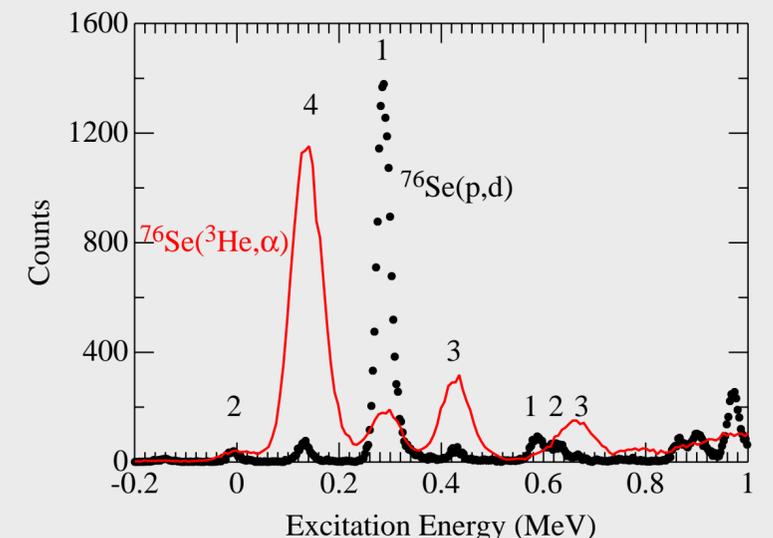
E	$\ell$	$(2j+1)S$	$(2j+1)S$
160	1	0.44	0.82
225	4		
421	2		
505	2		
629	1	0.15	0.28
884	2		
1021	1	0.12	0.22
1048	1	0.04	0.07
1250	0		
1385	2		

$$N_j \equiv S' / S$$

$$N_j \equiv [(0.45 + 0.12 + 1.29 + 0.10 + 0.11 + 0.37) + (0.44 + 0.15 + 0.12 + 0.04)] / (2 + 4) = \underline{\underline{0.53}}$$

## Checks

- (d,p) and (p,d) done at 7.5 MeV/u and 11.5 MeV/u
- (a, $^3\text{He}$ ) and ( $^3\text{He}$ ,a) for the  $\ell = 3,4$ , at 10 MeV/u and 8.7 MeV/u
- 4 targets used
- Absolute cross sections (Rutherford scattering measured)
- Yale Enge split-pole spectrograph (**now at FSU**)
- Stats (10s nA beams, <1%)
- Targets around 200  $\mu\text{gcm}^2$



# Quantitative description of the change

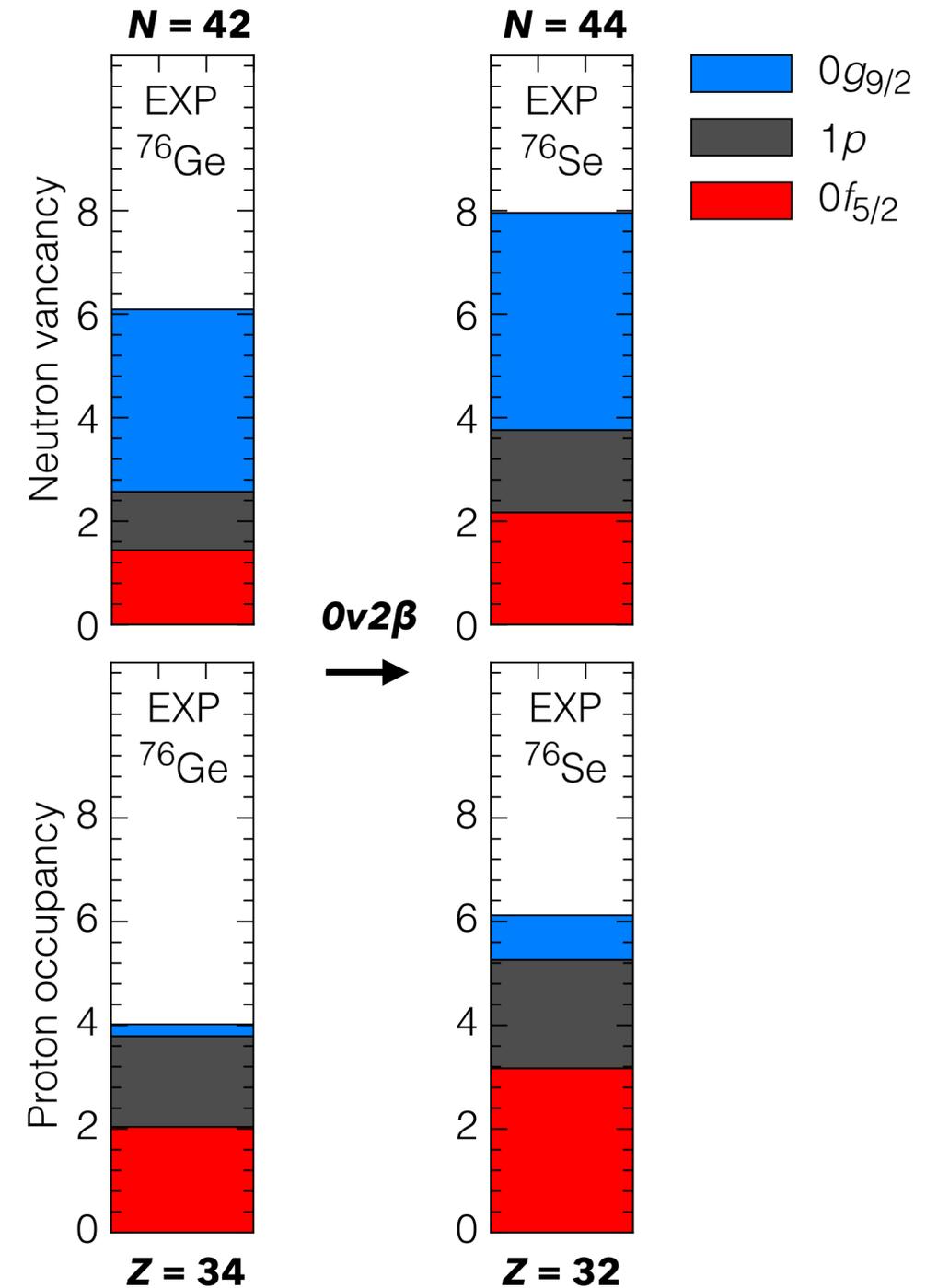
Isotope	$0f_{5/2}$	$1p_{1/2,3/2}$	$0g_{9/2}$	Sum	Expect
$^{74}\text{Ge}$	1.8	1.1	4.3	7.2	8
$^{76}\text{Ge}$	<b>1.4</b>	<b>1.1</b>	<b>3.5</b>	<b>6.0</b>	<b>6</b>
$^{76}\text{Se}$	<b>2.2</b>	<b>1.6</b>	<b>4.2</b>	<b>8.0</b>	<b>8</b>
$^{78}\text{Se}$	2.3	0.9	2.8	6.1	6

Isotope	$0f_{5/2}$	$1p_{1/2,3/2}$	$0g_{9/2}$	Sum	Expect
$^{74}\text{Ge}$	1.89	1.52	0.37	3.78	4
$^{76}\text{Ge}$	<b>1.75</b>	<b>2.04</b>	<b>0.23</b>	<b>4.02</b>	<b>4</b>
$^{76}\text{Se}$	<b>2.09</b>	<b>3.17</b>	<b>0.86</b>	<b>6.12</b>	<b>6</b>
$^{78}\text{Se}$	2.35	1.82	2.05	6.22	6

Normalization factors, average across 4 targets, were **0.53(1)**, **0.56(7)**, and **0.57(4)**, for the  $1p$ ,  $0f$ , and  $0g$  orbitals, respectively. The  $(d,p) + (p,d)$  reactions used for the  $1p$  and the  $(\alpha, ^3\text{He}) + (^3\text{He}, \alpha)$  used for the  $0f$  and  $0g$  states.

**0.1 to 0.3 nucleon uncertainties in the vacancies.**

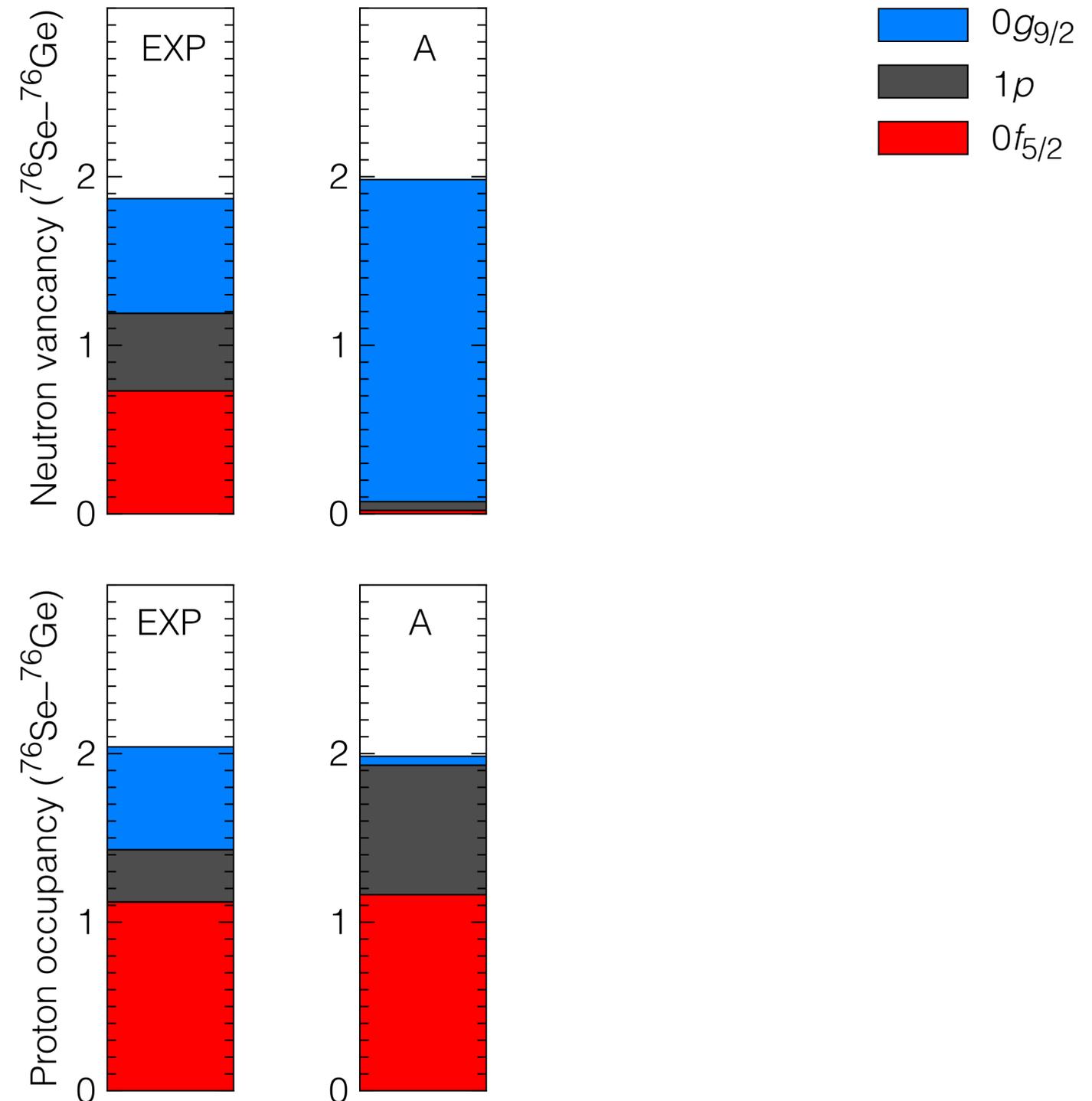


# Quantitative description of the change

This rearrangement must occur in the decay process – **NUCLEAR REACTIONS TELL US SO**

For neutrons, significant changes in the vacancy of all 'active' orbitals—seemingly described quite well. For protons it is similar.

Here is a quick comparison of theory and experiment in the differences ... **before (A)** and **after (B and C)**



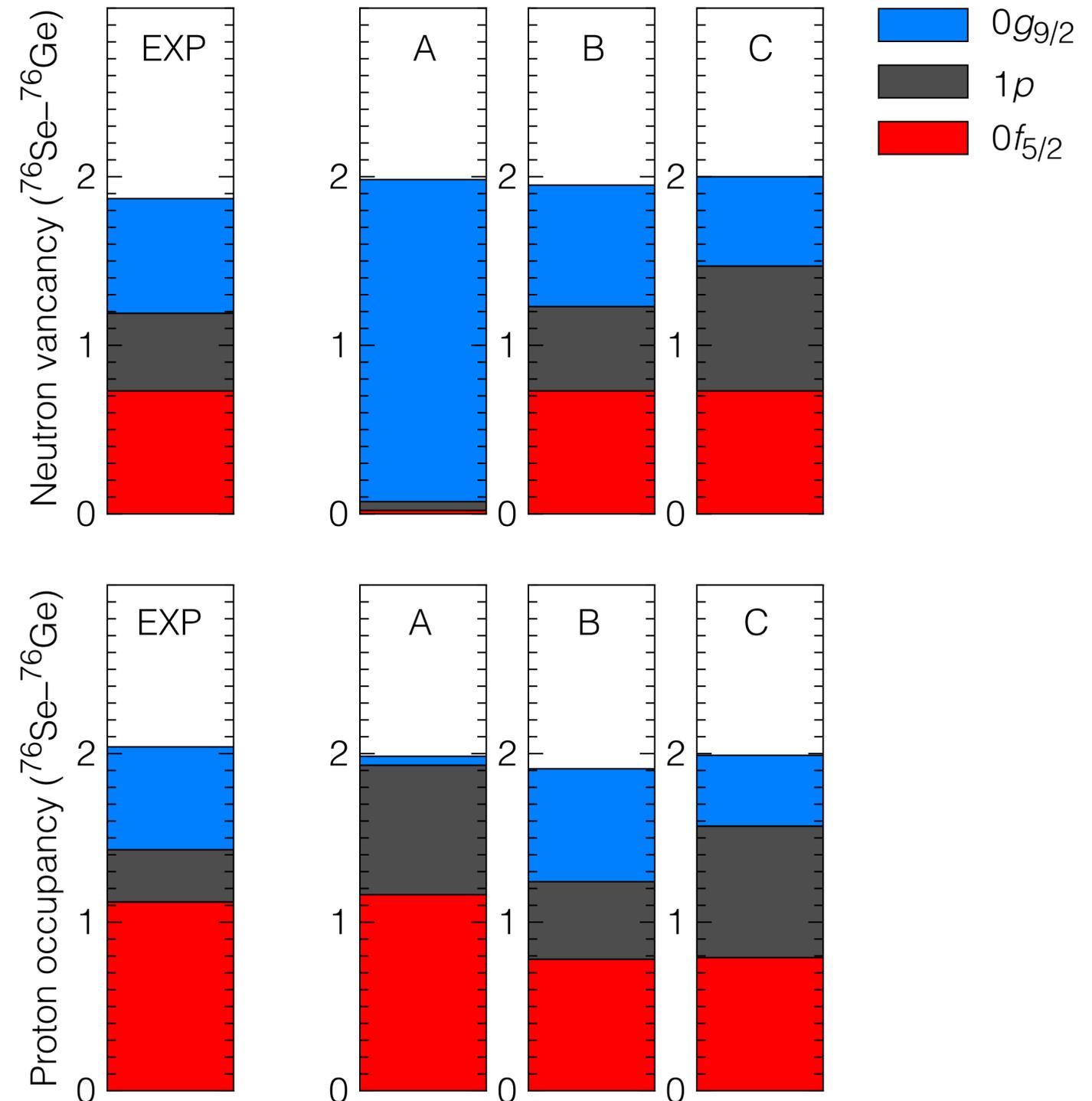
EXP neutrons– J. P. Schiffer et al., Phys. Rev. Lett. **100**, 112501 (2008)  
EXP protons – BPK et al., Phys. Rev. C **79**, 021301(R) (2009)  
A – QRPA by Rodin et al., priv. com., Nucl. Phys. A **766**, 107 (2006)  
B – QRPA by Suhonen et al., priv. com., Phys. Lett. B **668**, 277 (2008)  
C – ISM by Caurier et al., priv. com., Phys. Rev. Lett. **100**, 052503 (2008)

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EXP neutrons– J. P. Schiffer et al., Phys. Rev. Lett. **100**, 112501 (2008)  
EXP protons – BPK et al., Phys. Rev. C **79**, 021301(R) (2009)  
A – QRPA by Rodin et al., priv. com., Nucl. Phys. A **766**, 107 (2006)  
B – QRPA by Suhonen et al., priv. com., Phys. Lett. B **668**, 277 (2008)  
C – ISM by Caurier et al., priv. com., Phys. Rev. Lett. **100**, 052503 (2008)

# Aside: Useful papers

(for a pedagogical discussion of the reduction of the cross-section data)

PRL 108, 022501 (2012)

PHYSICAL REVIEW LETTERS

week ending  
13 JANUARY 2012

## Test of Sum Rules in Nucleon Transfer Reactions

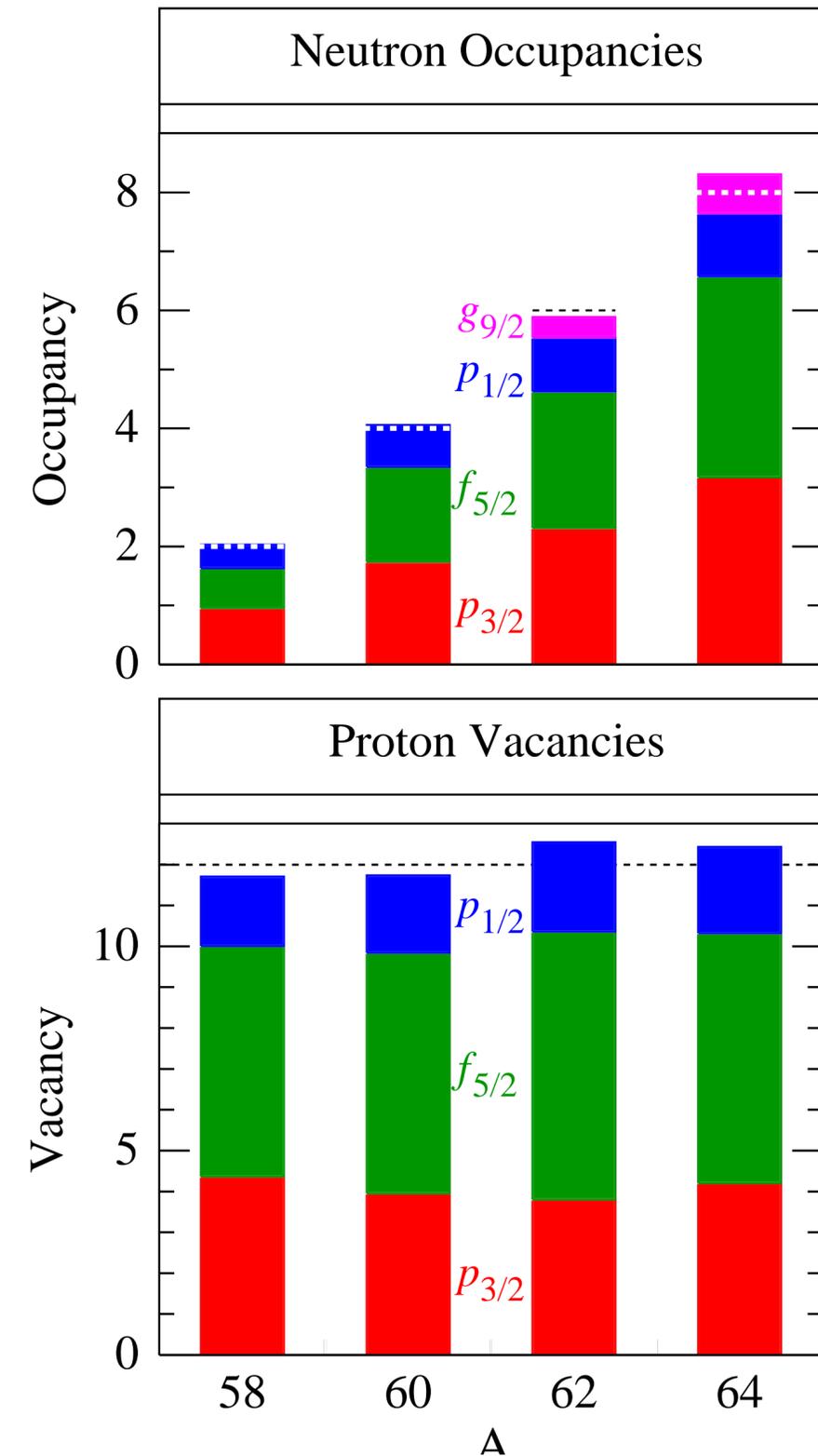
J. P. Schiffer,<sup>1,\*</sup> C. R. Hoffman,<sup>1</sup> B. P. Kay,<sup>1,†</sup> J. A. Clark,<sup>1</sup> C. M. Deibel,<sup>1,2,‡</sup> S. J. Freeman,<sup>3</sup> A. M. Howard,<sup>3,§</sup>  
A. J. Mitchell,<sup>3</sup> P. D. Parker,<sup>4</sup> D. K. Sharp,<sup>3</sup> and J. S. Thomas<sup>3</sup>

PHYSICAL REVIEW C 87, 034306 (2013)

## Valence nucleon populations in the Ni isotopes

J. P. Schiffer,<sup>1,\*</sup> C. R. Hoffman,<sup>1</sup> B. P. Kay,<sup>1,†</sup> J. A. Clark,<sup>1</sup> C. M. Deibel,<sup>1,2,‡</sup> S. J. Freeman,<sup>3</sup> M. Honma,<sup>4</sup> A. M. Howard,<sup>3,§</sup>  
A. J. Mitchell,<sup>3,||</sup> T. Otsuka,<sup>5</sup> P. D. Parker,<sup>6</sup> D. K. Sharp,<sup>3</sup> and J. S. Thomas<sup>3</sup>

See **Calem Hoffman's** EBSS2014 talk at [http://fribusers.org/4\\_GATHERINGS/2\\_SCHOOLS/2014/PRESENTATIONS/hoffman\\_2.pdf](http://fribusers.org/4_GATHERINGS/2_SCHOOLS/2014/PRESENTATIONS/hoffman_2.pdf) for an in-depth discussion on this work.



# So what is that normalization all about?

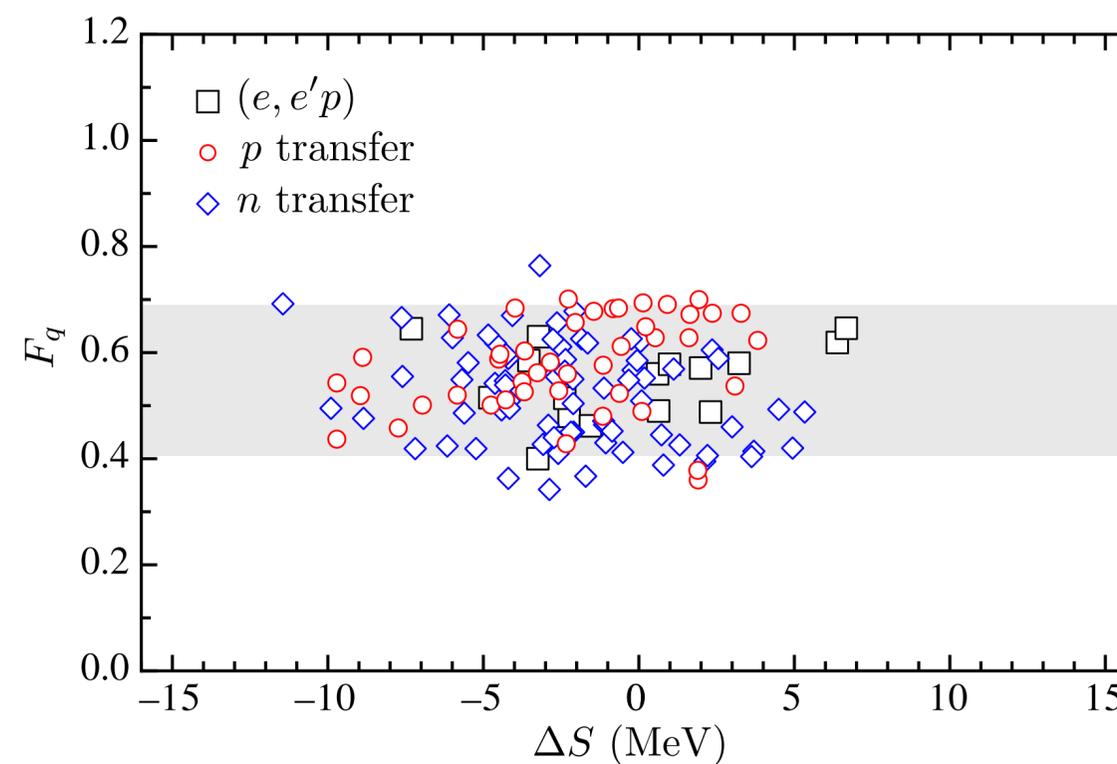
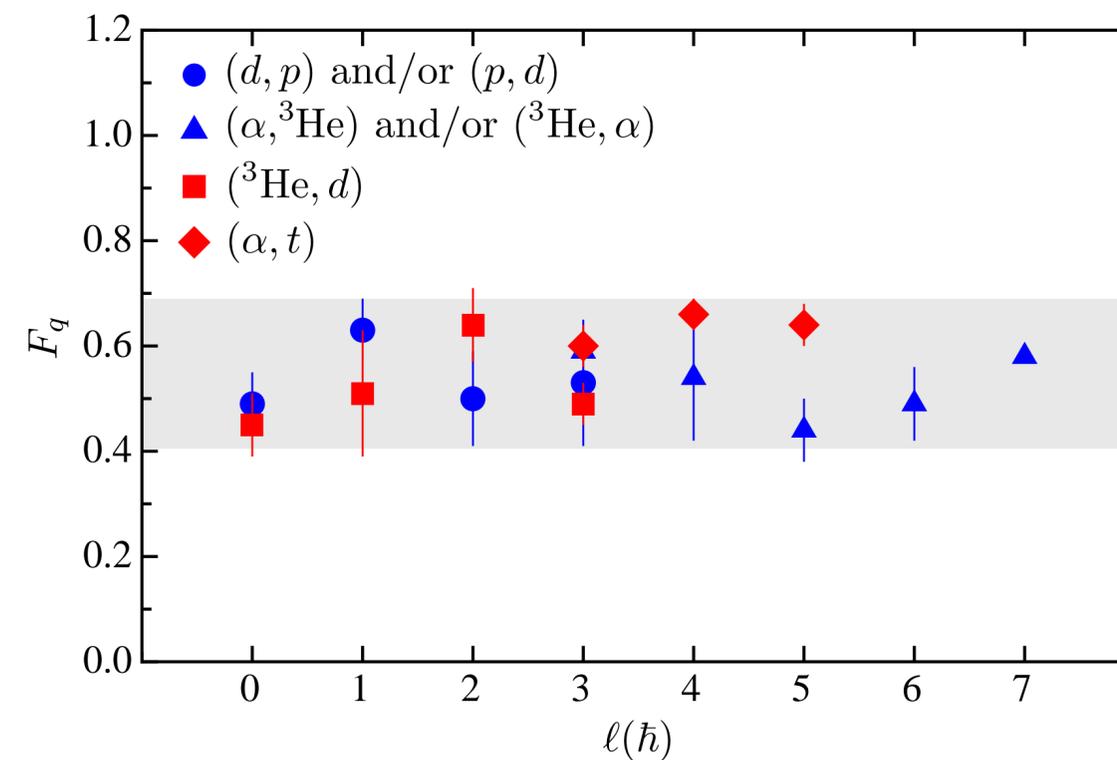
$$N_j \equiv [(0.45 + 0.12 + 1.29 + 0.10 + 0.11 + 0.37) + (0.44 + 0.15 + 0.12 + 0.04)] / (2 + 4) = \underline{\underline{0.53}}$$

**Remember this #**

The normalization appears meaningful, a ubiquitous feature of low-lying single-particle strength, independent of  $A$ ,  $\ell$ , nucleon type, reaction, etc.

Reaction, $\ell$ transfer	Number of determinations	$F_q$	rms spread
$(e, e'p)$ , all $\ell$	16	0.55	0.07
$(d, p)$ , $(p, d)$ , $\ell = 0-2$	40	0.53	0.09
$(d, p)$ , $(p, d)$ , $\ell = 0-3$	46	0.53	0.10
$(\alpha, {}^3\text{He})$ , $({}^3\text{He}, \alpha)$ , $\ell = 4-7$	26	0.50	0.09
$(\alpha, {}^3\text{He})$ , $({}^3\text{He}, \alpha)$ , $\ell = 3-7$	34	0.52	0.09
$({}^3\text{He}, d)$ , $\ell = 0-2$	18	0.54	0.10
$({}^3\text{He}, d)$ , $\ell = 0-4$	26	0.54	0.09
$(\alpha, t)$ , $\ell = 4-5$	14	0.64	0.04
$(\alpha, t)$ , $\ell = 3-5$	18	0.64	0.04
All transfer data <sup>a</sup>	124	0.55	0.10

<sup>a</sup>Rows 3, 5, 7, and 9.

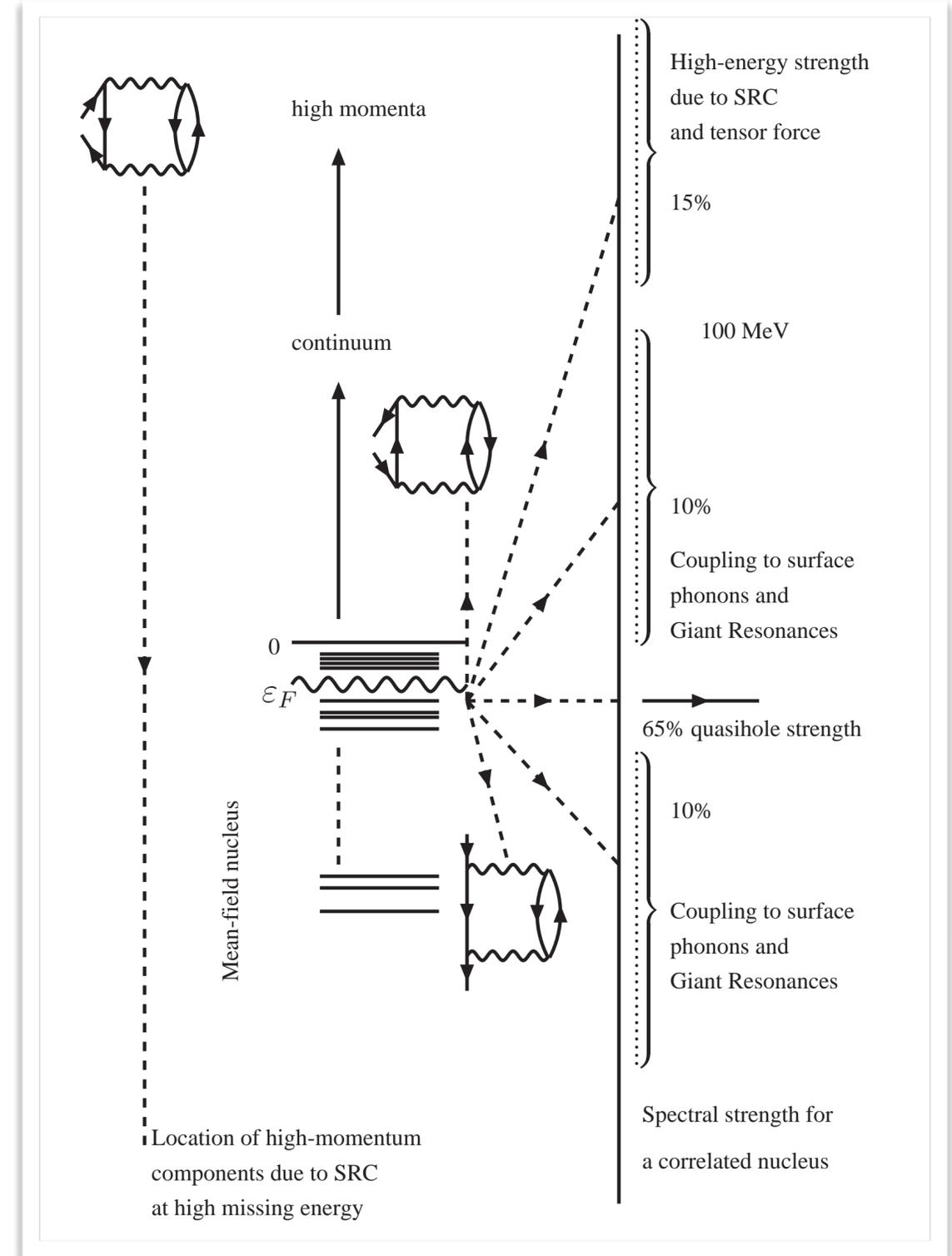


# So what is that normalization all about?

“Thus at any time **only 2/3 of the nucleons in the nucleus act as independent particles** moving in the nuclear mean field. The remaining third of the nucleons are correlated.”\*

## Key points:

- **Can be academic as many studies involve only relative quantities**
- Arguably **essential** in terms of understanding and a ‘hot topic’ these days in the Exotic Beam era ...

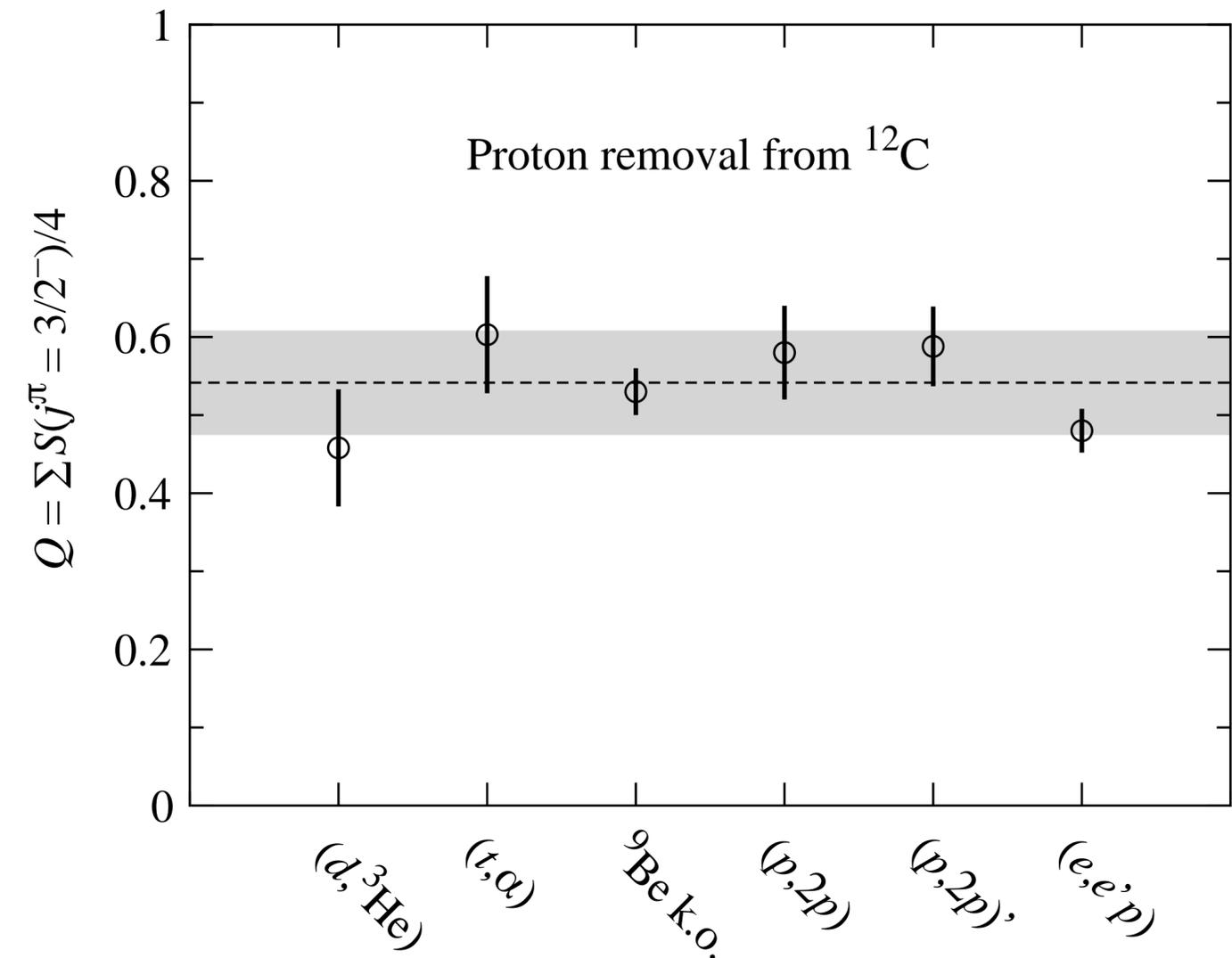
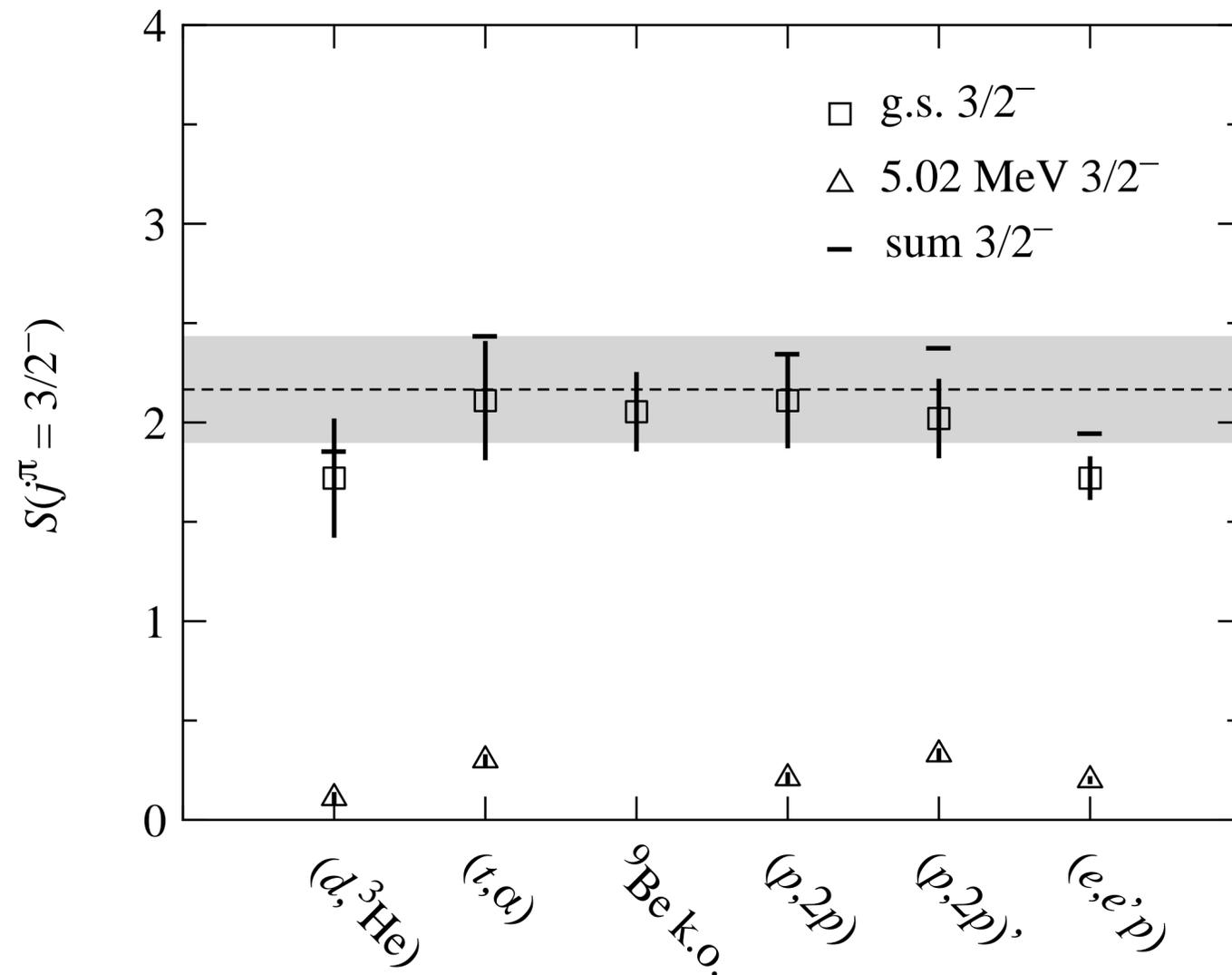


\*V. R. Pandharipande, I. Sick, P. K. A. deWitt Huberts, Rev. Mod. Phys. **69**, 981 (1997)  
W. H. Dickhoff J. Phys. G: Nucl. Part. Phys. **37**, 064007 (2010)

# ... how well understood?

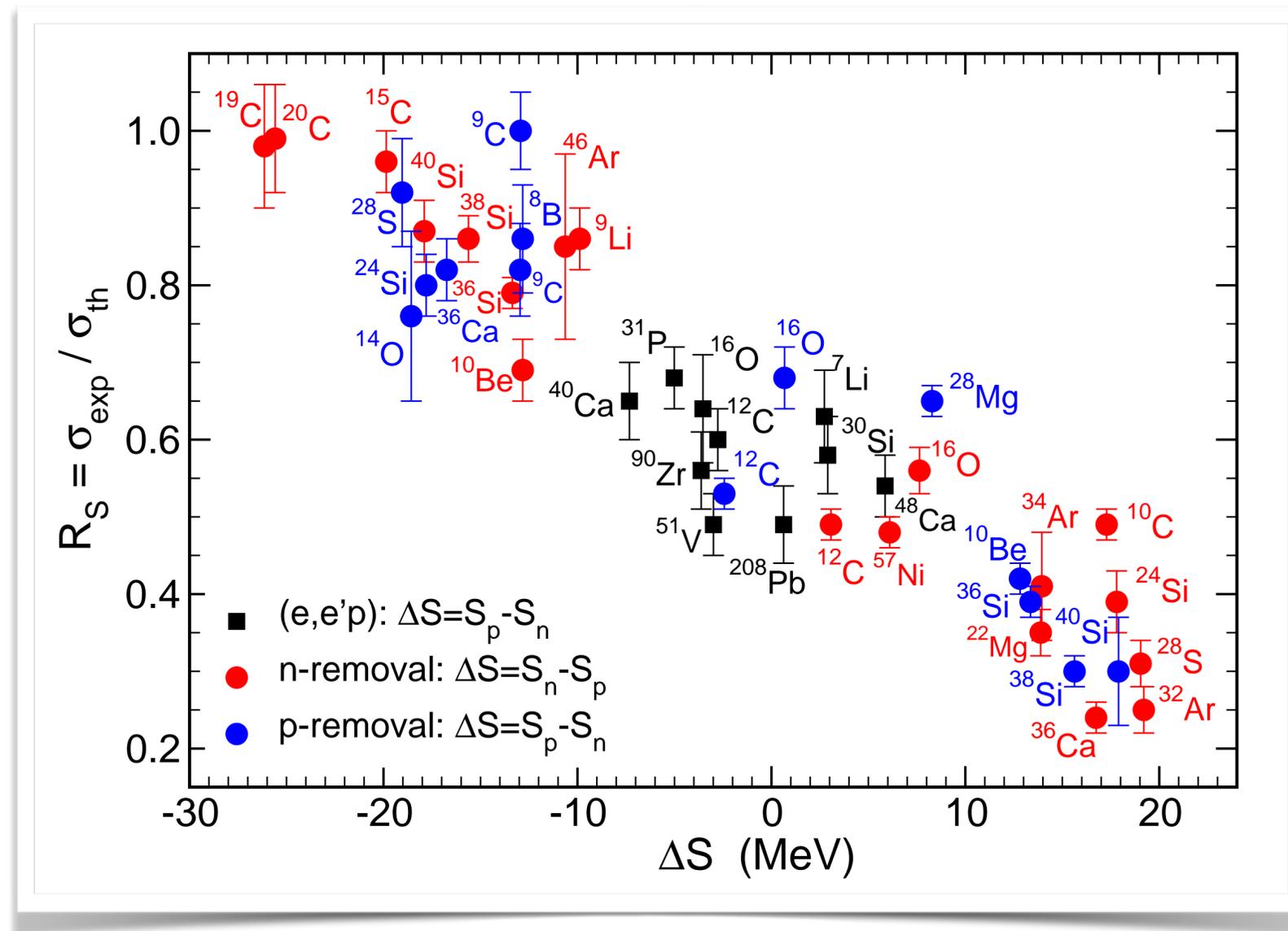
There are a handful of isotopes where reliable experimentally determined cross sections exist from numerous 'equivalent' probes, e.g., proton removal from  $^{12}\text{C}$ .

Same physics results



# Exotic beam reactions bring new puzzles

About 10 years ago it was observed that 'reduction factors' determined from a large body nucleon-knockout cross sections tended to unity for more weakly bound systems and fell as low as  $\sim 0.2$  for the more strongly bound systems.



$\Delta S$  approximates the difference between the proton and neutron Fermi surfaces

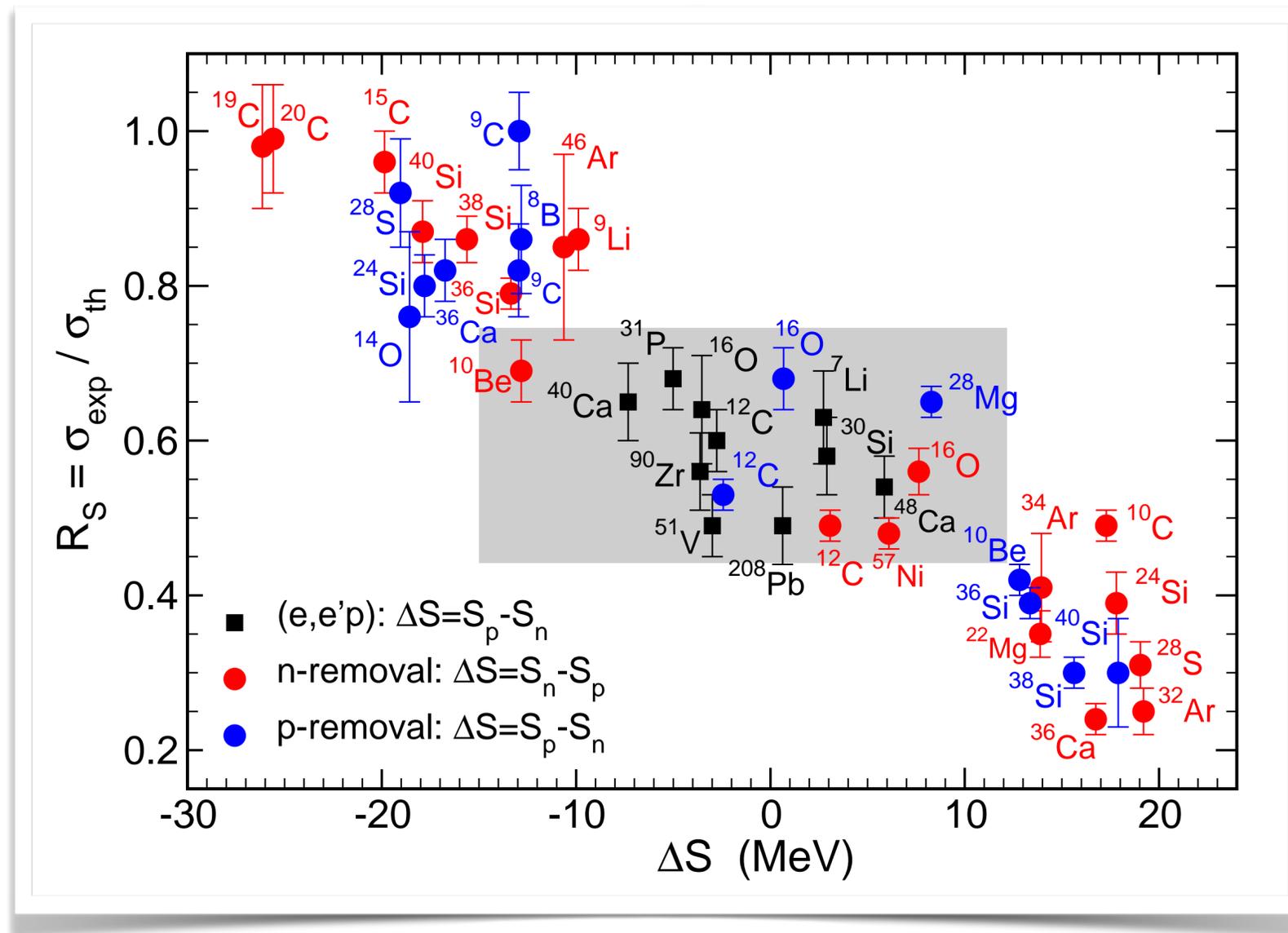
$\Delta S = S_p - S_n$  for proton reactions

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Much work to do ...

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$\Delta S = S_p - S_n$  for proton reactions

$\Delta S = S_p - S_n$  for neutron reactions

Much work to do ...

# Series of experiments

Single-nucleon and two-nucleon transfer on nuclei involved in the  $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ ,  $^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$ ,  $^{130}\text{Te} \rightarrow ^{130}\text{Xe}$ , and  $^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$  decays

Original works, including cross sections and analyzed data:

- S. J. Freeman et al., *Phys. Rev. C* **75**, 051301(R) (**2007**):  $A = 76$  neutron pairing
- J. P. Schiffer et al., *Phys. Rev. Lett.* **100**, 112501 (**2008**):  $A = 76$  neutron occupancies
- B. P. Kay et al., *Phys. Rev. C* **79**, 021301(R) (**2009**):  $A = 76$  proton occupancies
- T. Bloxham et al., *Phys. Rev. C* **82**, 027308 (**2010**):  $A = 130$  neutron (and proton) pairing
- J. S. Thomas et al., *Phys. Rev. C* **86**, 047304 (**2012**):  $A = 100$  neutron pairing
- B. P. Kay et al., *Phys. Rev. C* **87**, 011302(R) (**2013**):  $A = 130$  neutron occupancies
- A. Roberts et al., *Phys. Rev. C* **87**, 051305(R) (**2013**):  $A = 76$  proton pairing
- J. P. Entwisle et al., *Phys. Rev. C* **93**, 064312 (**2016**):  $A = 130$  and  $A = 136$  proton occupancies
- S. V. Szwec et al., *Phys. Rev. C* **94**, 054314 (**2016**):  $A = 136$  neutron occupancies
- S. J. Freeman et al., *Phys. Rev. C* **96**, 054325 (**2017**):  $A = 100$  proton and neutron occupancies

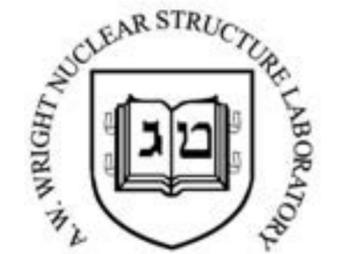
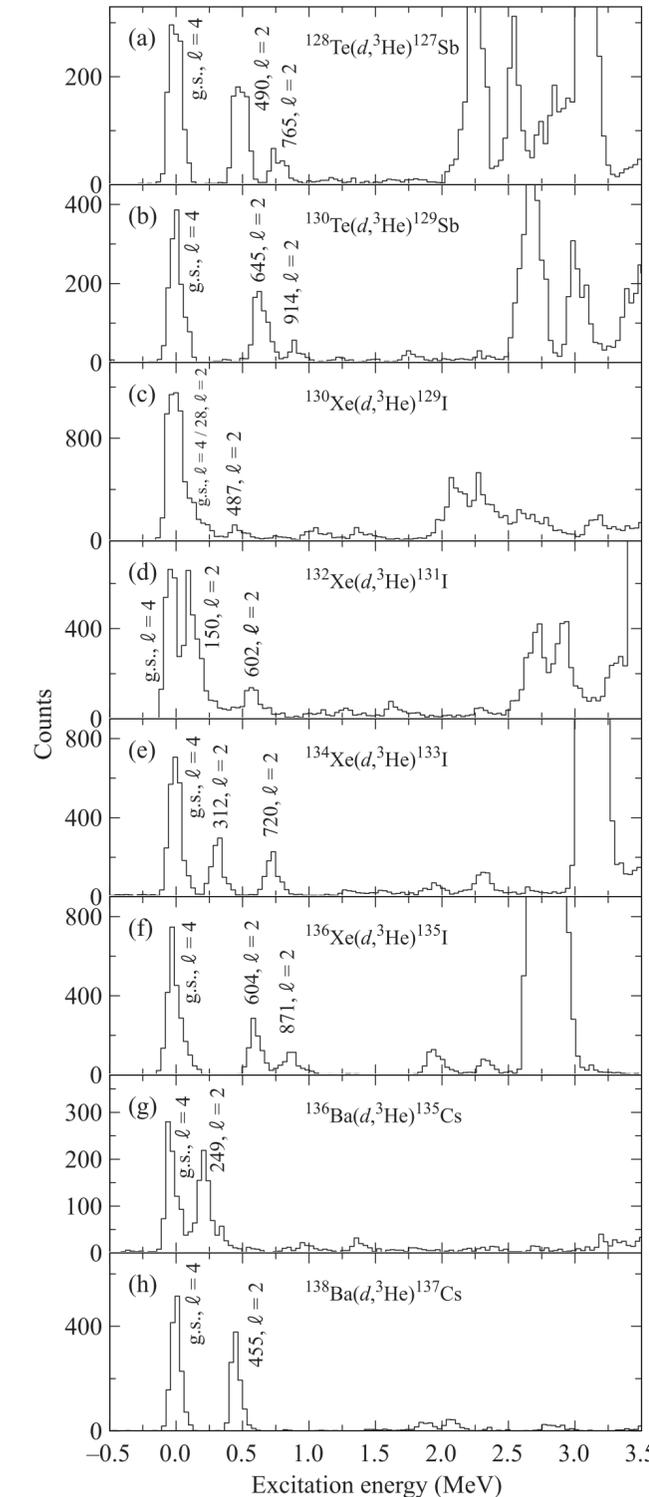
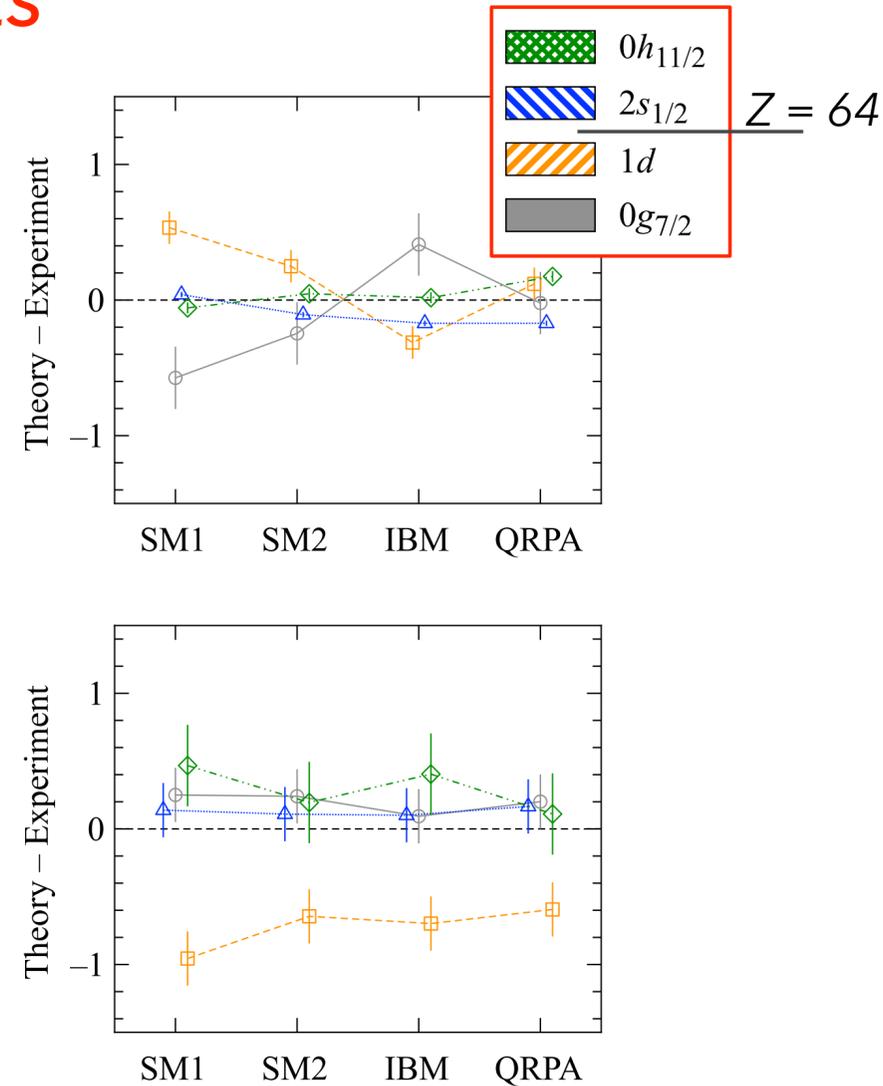
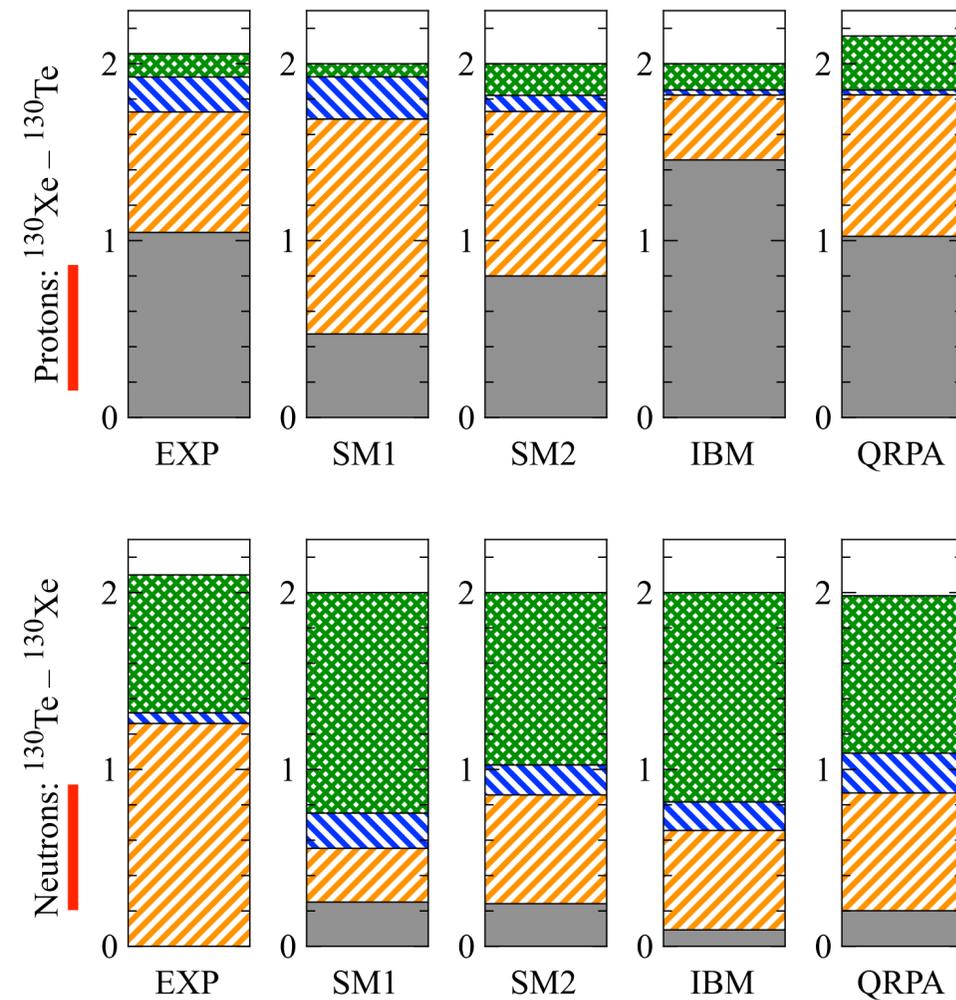
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D. K. Sharp et al., upcoming works on  $A = 116, 124,$  and  $150$  neutron occupancies

# A = 130 occupancies

Cryogenic targets, gas targets

A. Neacsu and M. Horoi, *Phys. Rev. C* **91**, 024309 (2015) [SM1]  
 J. Menéndez et al., *Nucl. Phys. A* **818**, 139 (2009) [SM2]  
 J. Kotila and J. Barea, *Phys. Rev. C* **94**, 034320 (2016) [IBM]  
 J. Suhonen and O. Civitarese, *Nucl. Phys. A* **847**, 207 (2010) [QRPA]



## Valence neutron properties relevant to the neutrinoless double- $\beta$ decay of $^{130}\text{Te}$

B. P. Kay,<sup>1,\*</sup> T. Bloxham,<sup>2</sup> S. A. McAllister,<sup>3</sup> J. A. Clark,<sup>4</sup> C. M. Deibel,<sup>4,5,†</sup> S. J. Freedman,<sup>2</sup> S. J. Freeman,<sup>3</sup> K. Han,<sup>2</sup>  
 A. M. Howard,<sup>3,‡</sup> A. J. Mitchell,<sup>3,§</sup> P. D. Parker,<sup>6</sup> J. P. Schiffer,<sup>4</sup> D. K. Sharp,<sup>5</sup> and J. S. Thomas<sup>5</sup>

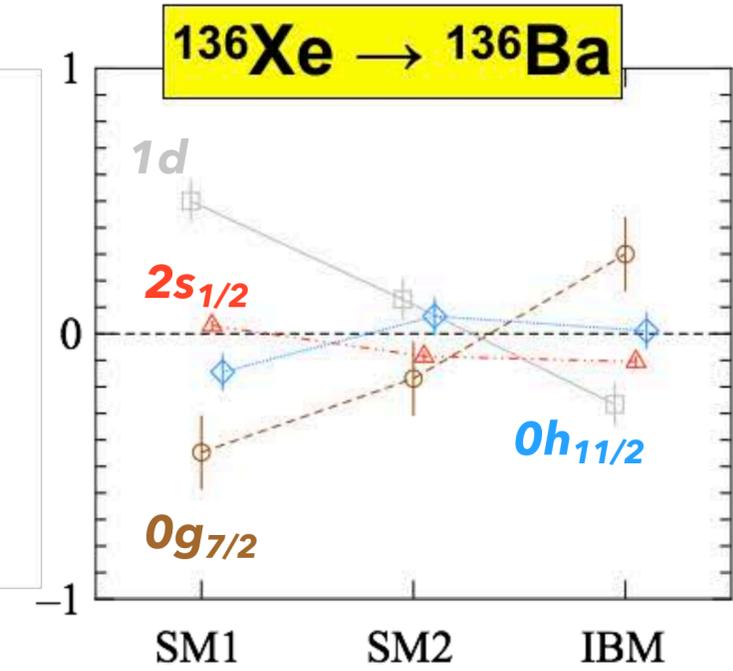
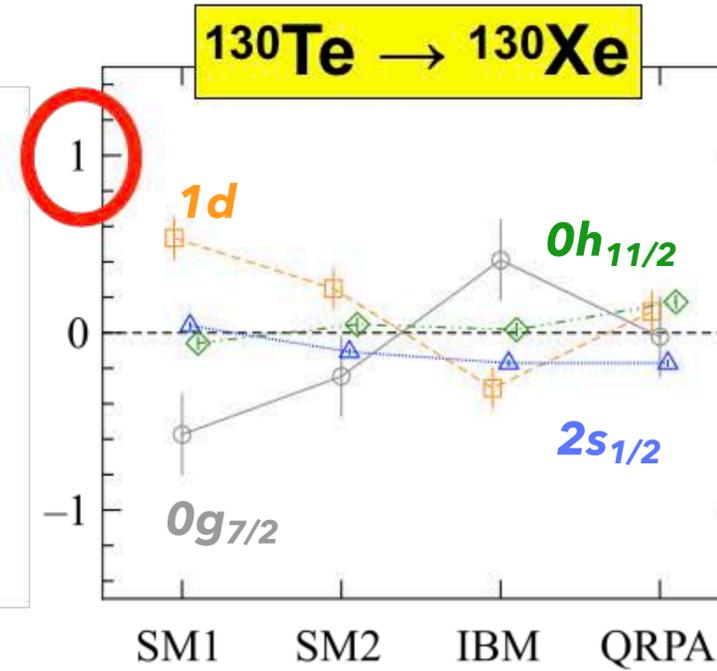
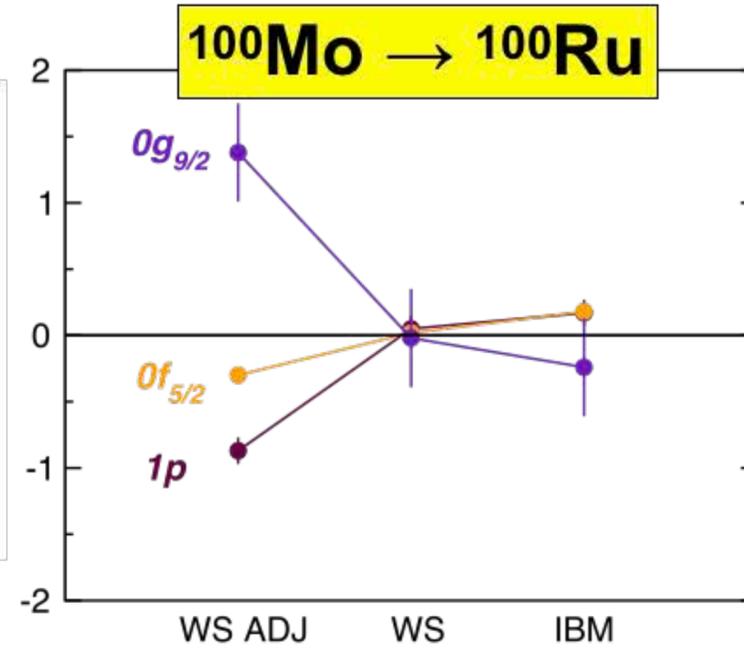
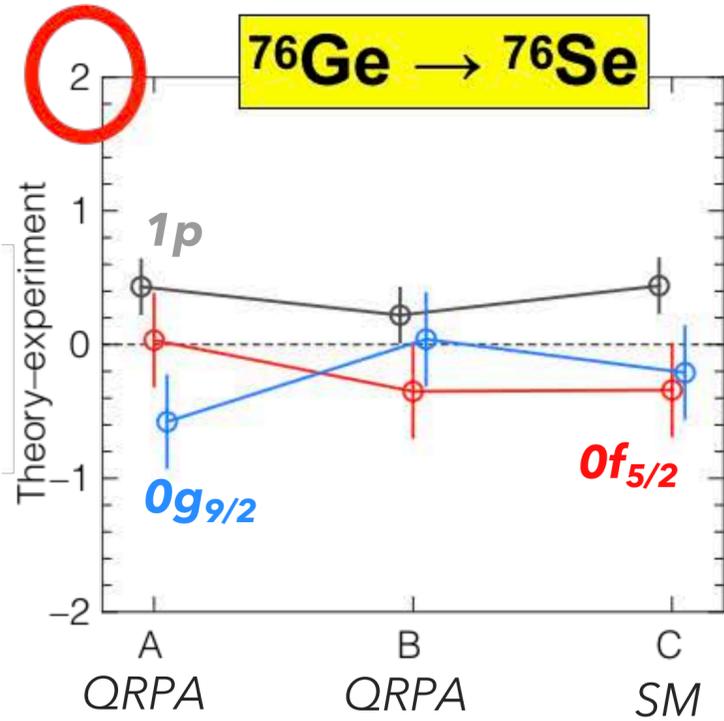
BPK et al., *Phys. Rev. C* **87**, 011302(R) (2013)

J. P. Entwisle et al., *Phys. Rev. C* **93**, 064312 (2016)

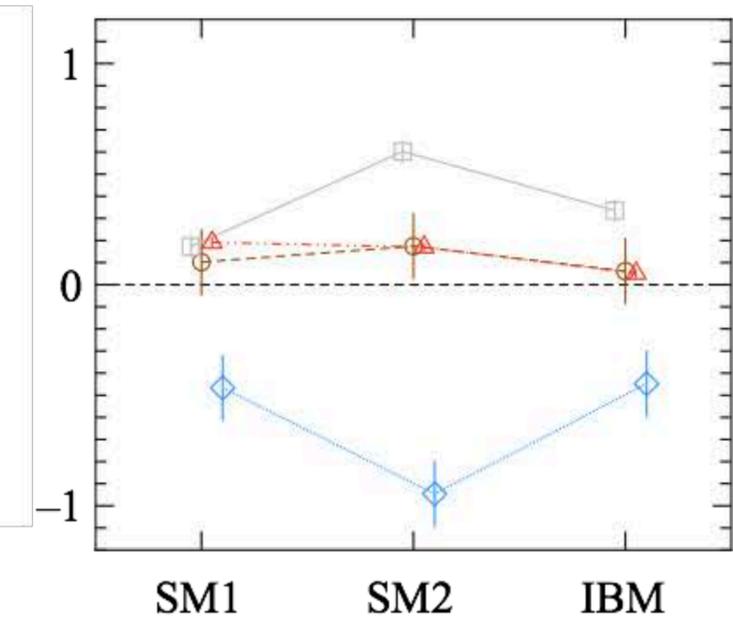
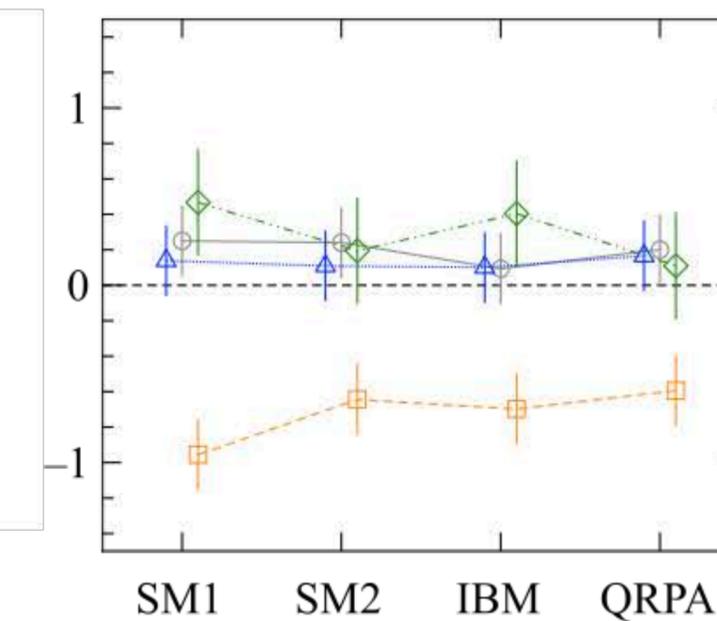
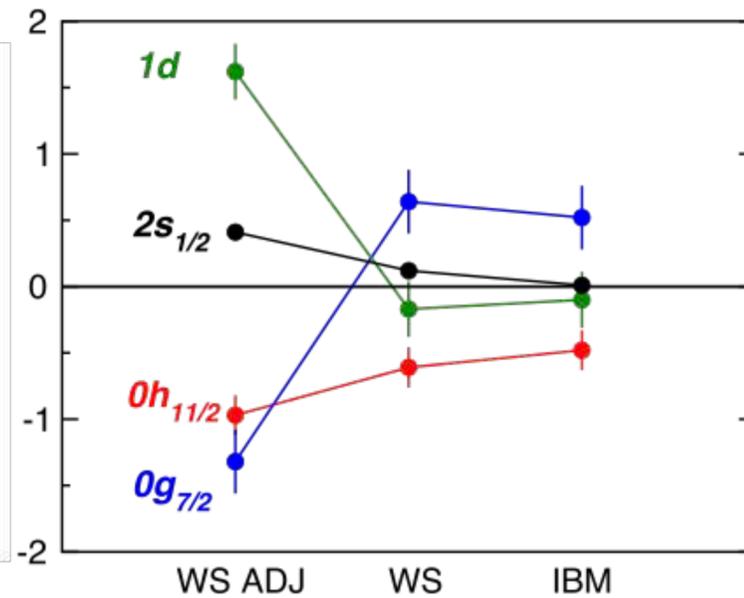
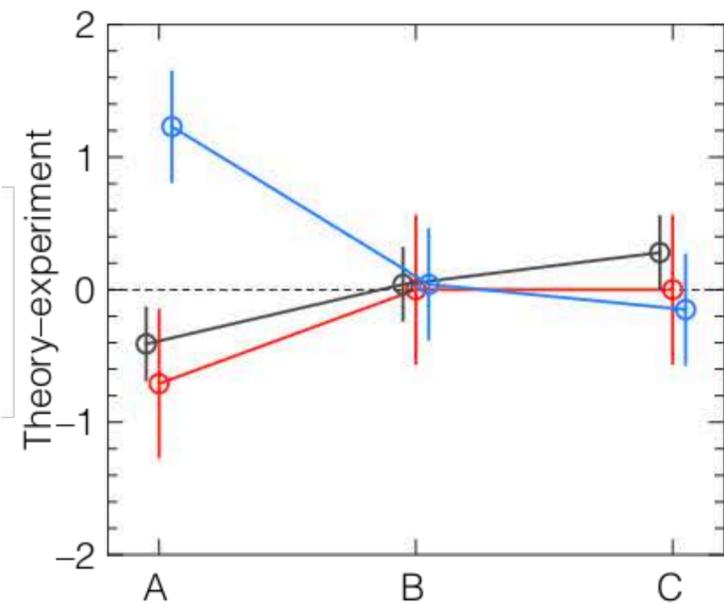
# Overview of all results

Schiffer et al., Phys. Rev. Lett. **100**, 112501 (2008)  
 BPK et al., Phys. Rev. C **79**, 021301(R) (2009)  
 BPK et al., Phys. Rev. C **87**, 011302(R) (2013)  
 Entwisle et al., Phys. Rev. C **93**, 064312 (2016)  
 Szwece et al., Phys. Rev. C **94**, 054314 (2016)  
 Freeman et al., Phys. Rev. C **96**, 054325 (2017)

PROTONS



NEUTRONS

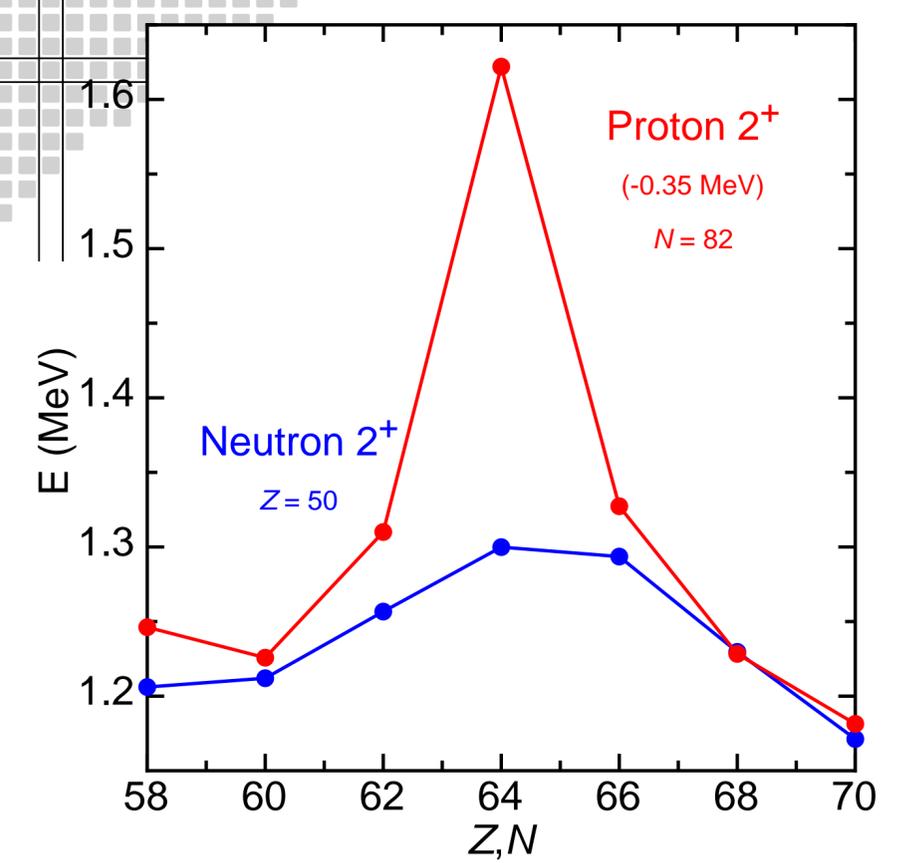
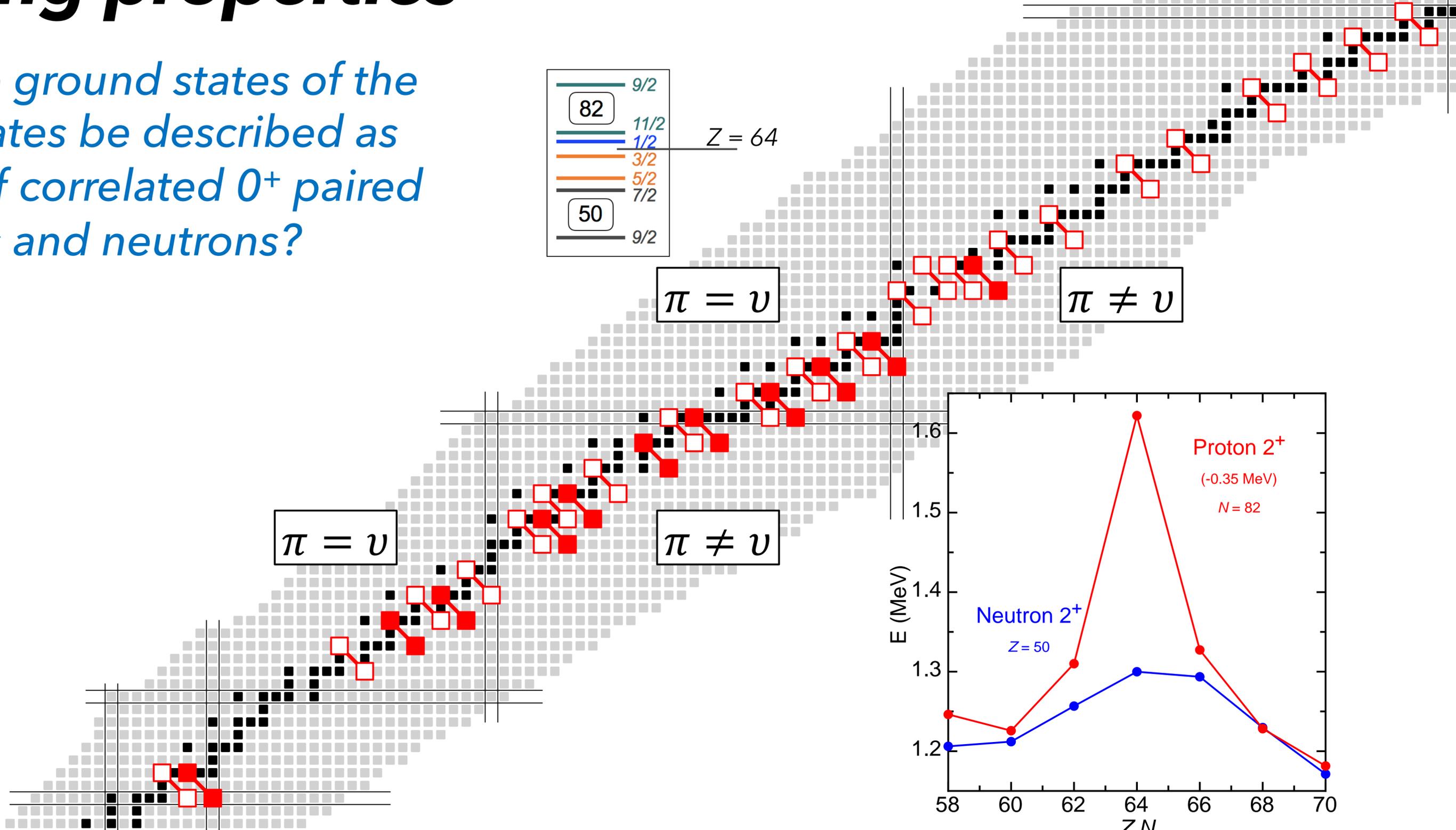


(References to theory work can be found in references above)

# *Pairing, and old data*

# Pairing properties

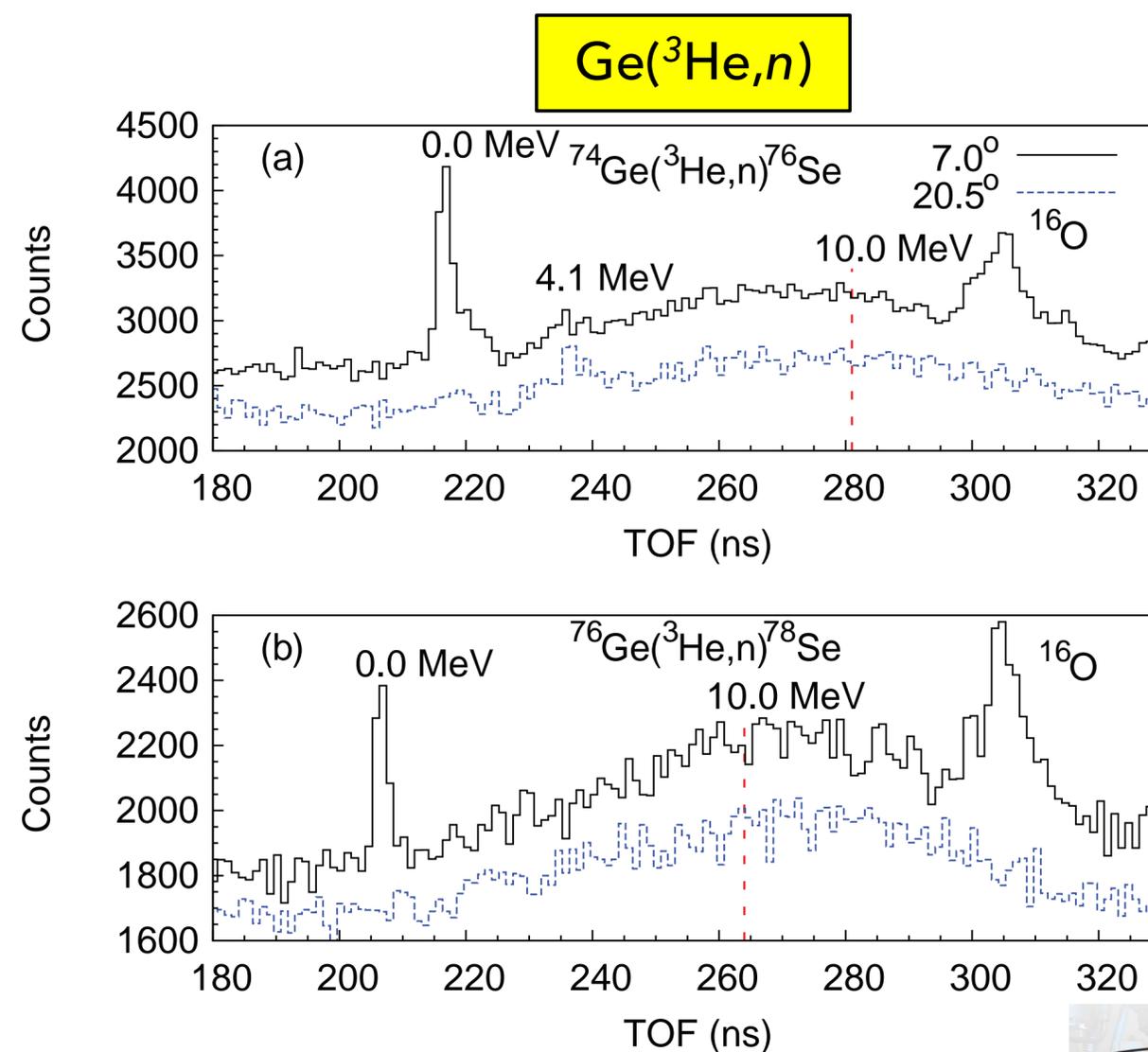
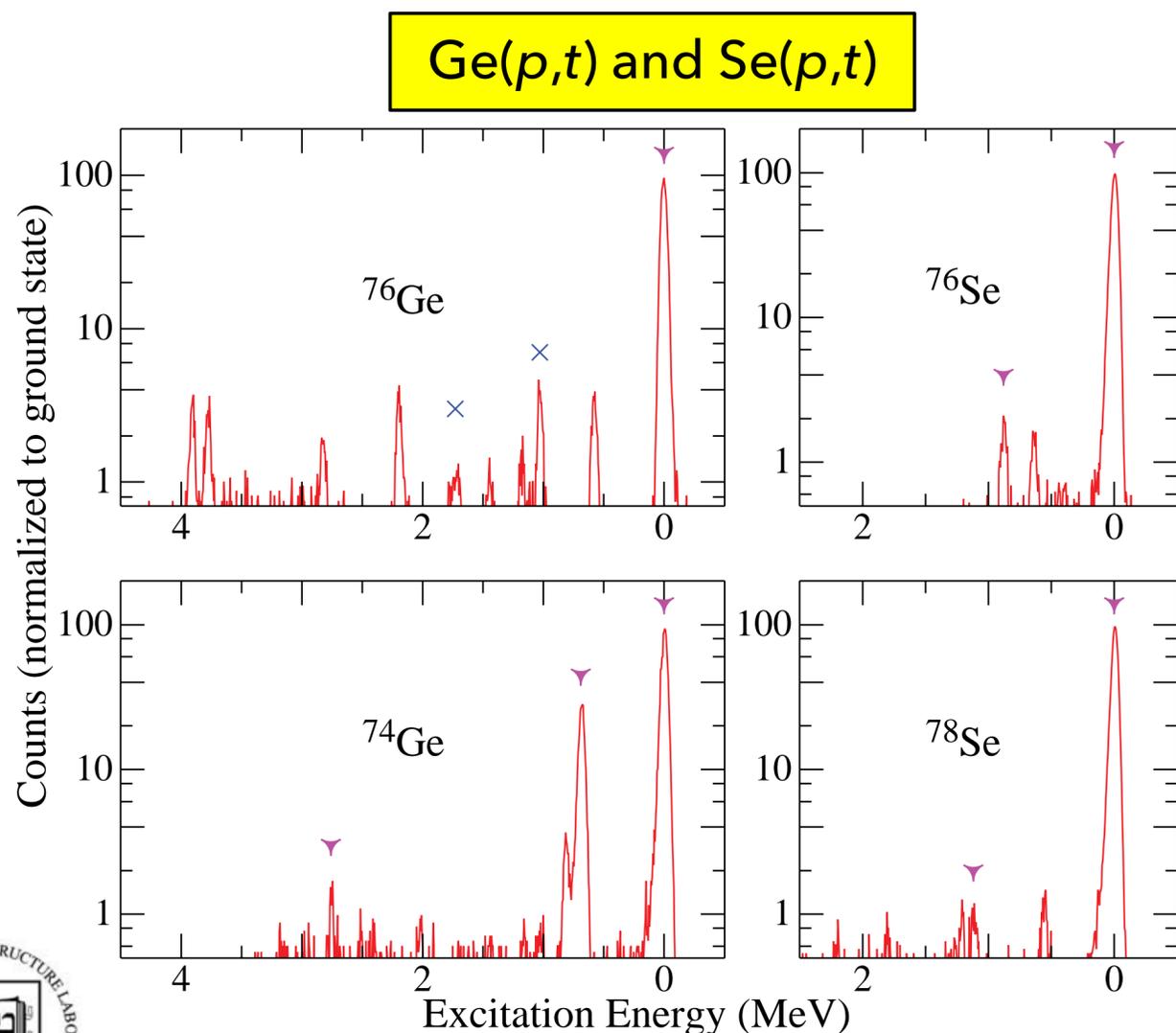
Can the ground states of the candidates be described as 'seas' of correlated  $0^+$  paired protons and neutrons?



e.g. works of Freeman, Bloxham, Thomas, Roberts, etc

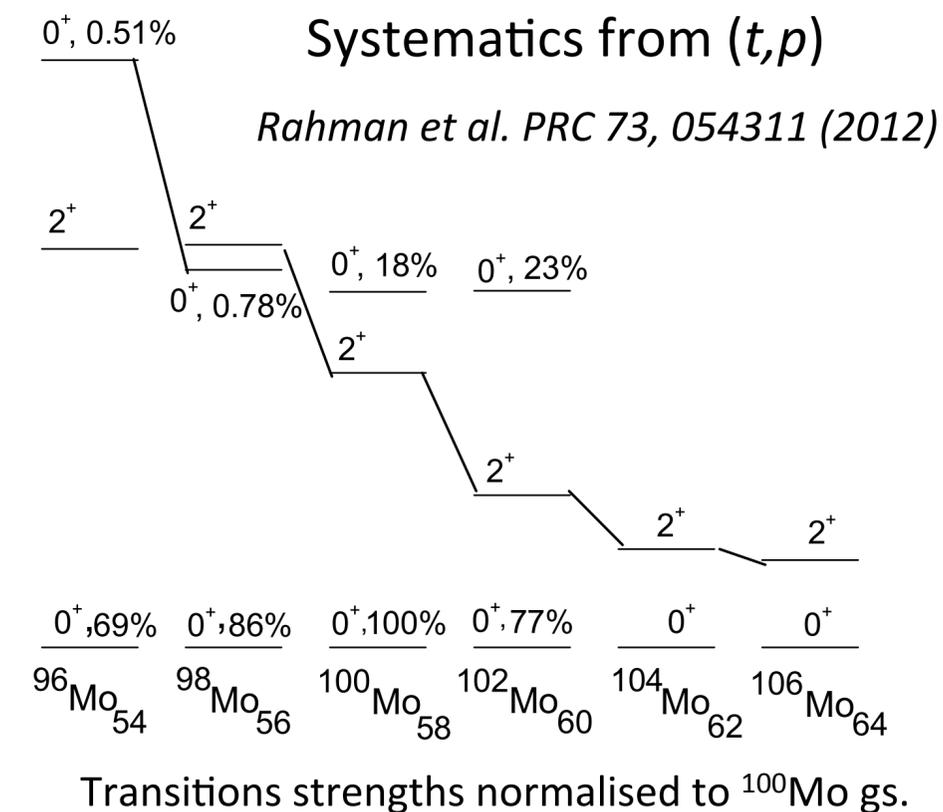
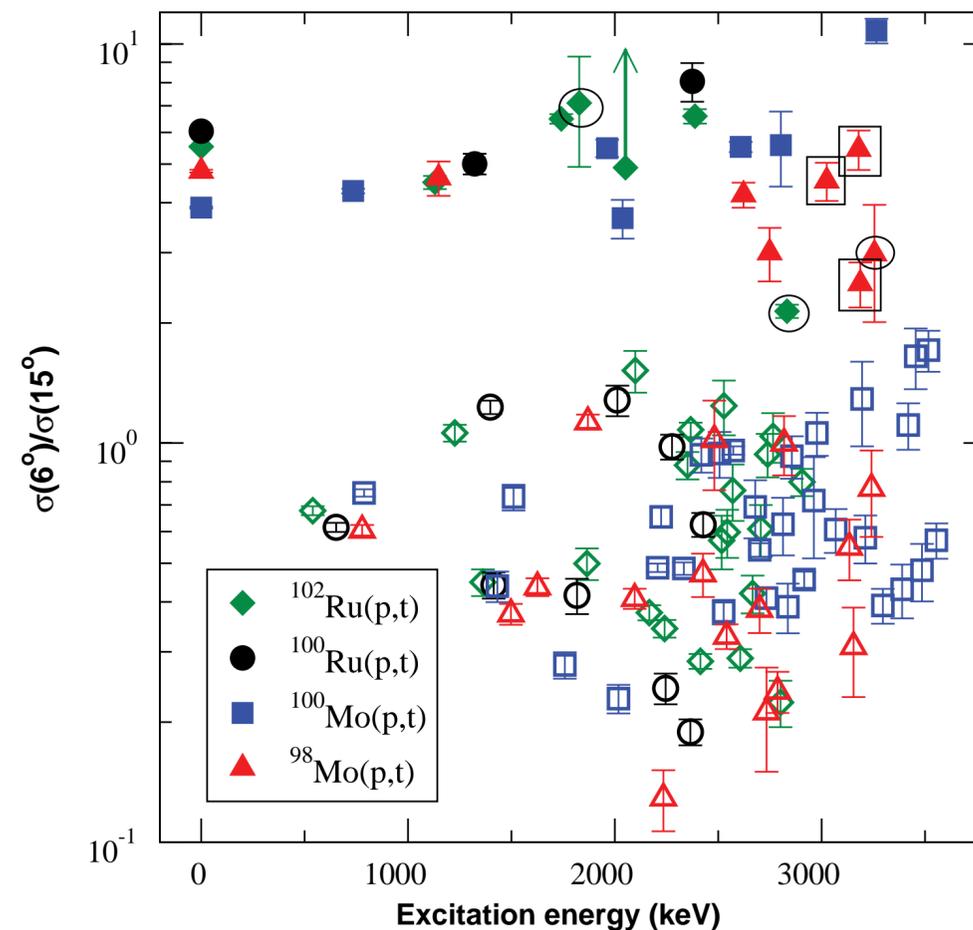
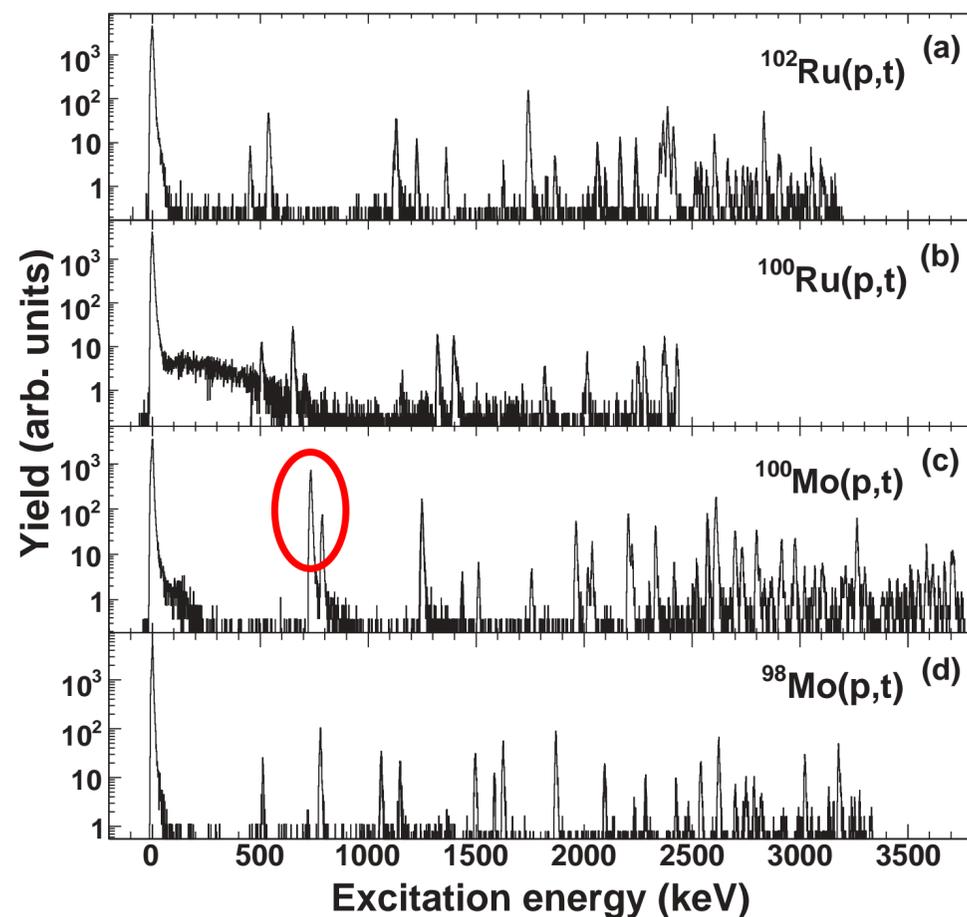
# Pairing around $A \sim 76$

Pair-transfer reactions are a simple and effective probe of pairing correlations  
*No evidence of 'pairing vibrations' in the  $A = 76$  region*



S. J. Freeman et al., *Phys. Rev. C* **75**, 051301(R) (2007) [neutrons]  
A. Roberts et al., *Phys. Rev. C* **87**, 051305(R) (2013) [protons]

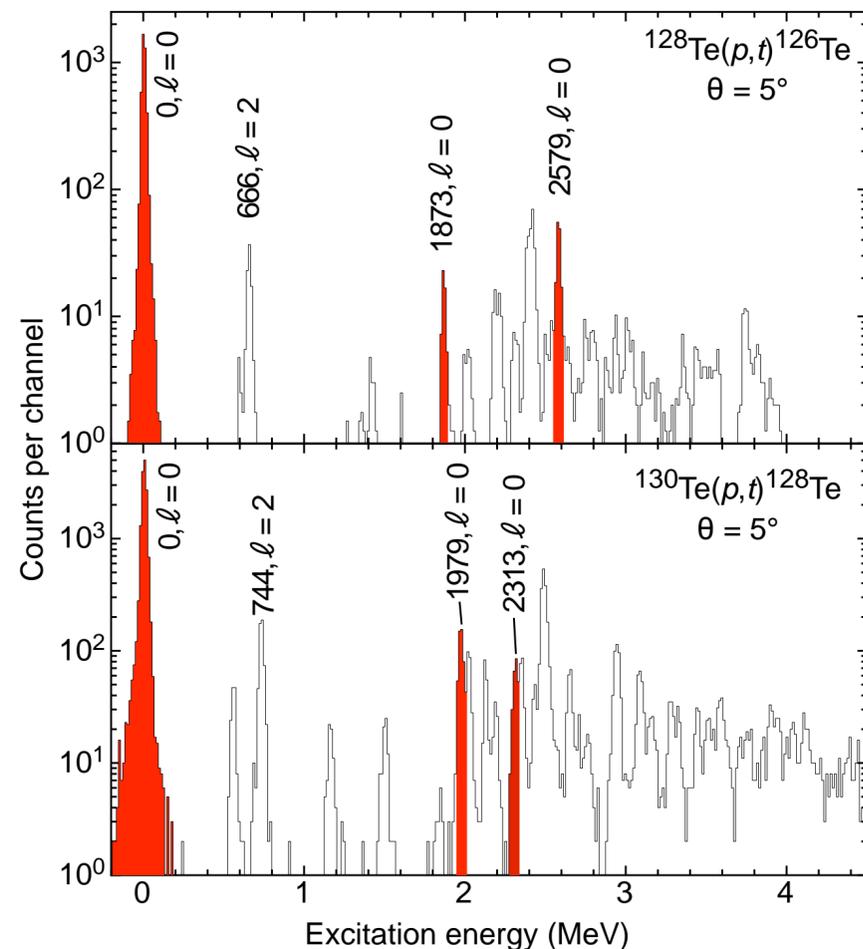
# Pairing around $A \sim 100$



A transitional region with deformation playing a role in the nuclear structure:

- Reactions leading to and from  $^{100}\text{Ru}$  show  $\sim 95\%$  of the  $L=0(p,t)$  strength is in the g.s. (on the spherical side of the transitional region)
- For  $^{100}\text{Mo}$  about 20% of the  $L=0(p,t)$  strength is an excited  $0^+$ , a shape-transitional nucleus
- No evidence for pairing vibrations, but structure is complicated (proton work remains to be done)

# Pairing around $A \sim 130, 136$



Reaction	$E$ (MeV)	$\sigma$ (mb/sr)	Ratio <sup>a</sup>	Normalized strength <sup>b</sup>
$^{128}\text{Te}(p,t)$	0	4.21	90	1.21
	1.873	0.06	20	0.02
	2.579	0.15	21	0.04
$^{130}\text{Te}(p,t)$	0	3.49	89	1.00
	1.979	0.05	50	0.01
	2.313(4) <sup>c</sup>	0.05	>20	0.01
$^{128}\text{Te}(^3\text{He},n)$	0	0.24	—	0.96
	2.13	0.095	—	0.32
$^{130}\text{Te}(^3\text{He},n)$	0	0.26	—	1.00
	1.85	0.098	—	0.34
	2.49	0.062	—	0.21

From the proton-pair adding  $\text{Te}(^3\text{He},n)$  reactions by Alford *et al.*, significant strength is seen in  $\ell=0$  transitions to excited states ...

A **classic case of pair vibration** and likely a consequence of a sub-shell gap at  $Z = 64$   
 Consequences for QRPA? (Does the shell-model include this feature also?)

T. Bloxham *et al.*, *Phys. Rev. C* **82**, 027308 (2010) [neutrons]  
 W. P. Alford *et al.*, *Nucl. Phys. A* **323**, 339 (1979) [protons]

# Recap ...

- Reactions  **$A(a,b)B$**  reveal something about the atomic nucleus
- Single-nucleon transfer (shameful bias in these lectures) can:
  - populate single-particle excitations
  - allow us to deduce spectroscopic factors,  $\ell$
  - ... and thus single-particle energies
  - ... and thus occupancies / vacancies
- I showed ~two topical examples from the last ~decade, where **reactions** have been an **essential tool** in basic nuclear structure and in connection to fundamental symmetries

## ... and next

- **Exotic beams, spectrometers**, ..., bubbles, isomers, ...