

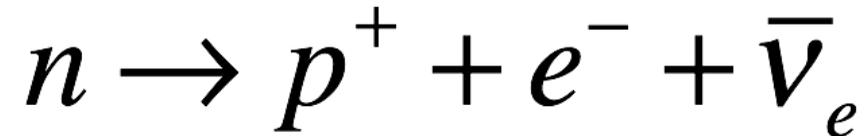
Fundamental Neutron Physics IV

The Neutron Lifetime and Beta Decay

Geoffrey Greene

University of Tennessee / Oak Ridge National Laboratory

A free neutron decays



WHY?

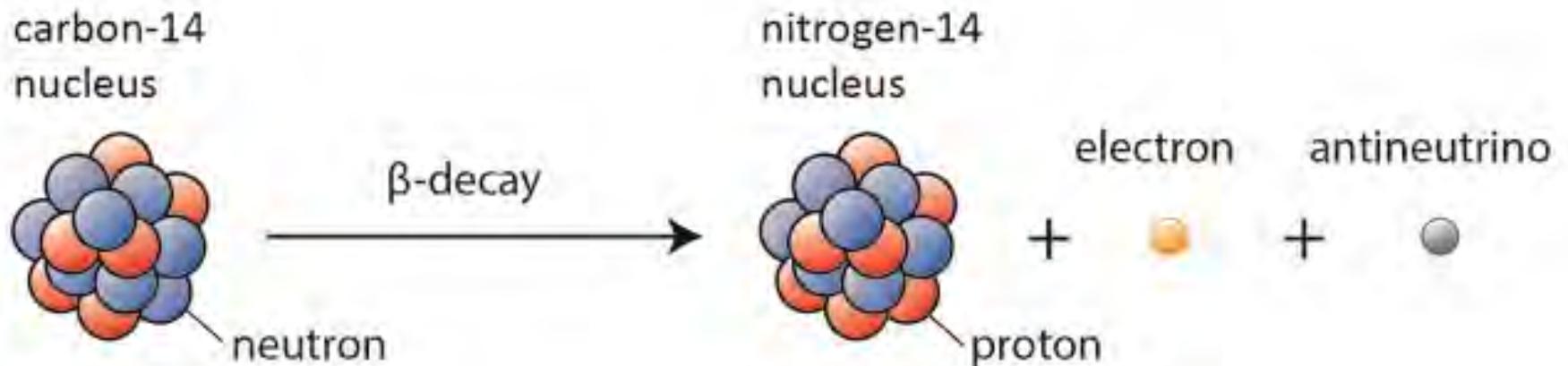
Because It Can!

No conservation laws are violated and,

$$m_n > m_p + m_e$$

Why don't neutrons decay inside nuclei?

Sometimes they do...it's called Nuclear Beta Decay:

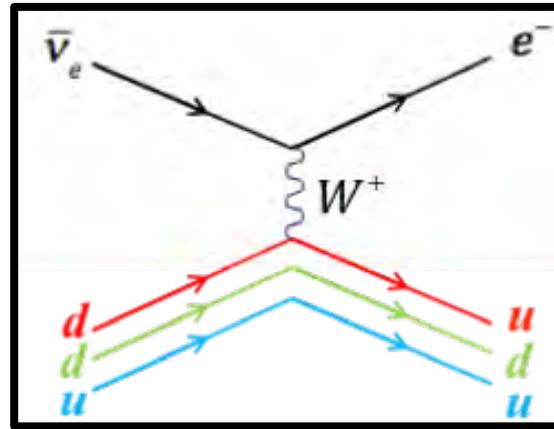


It happens when it is energetically allowed.

Variations in “mother-daughter” binding energies determine stability against beta decay.

Why is the Neutron Lifetime Interesting?

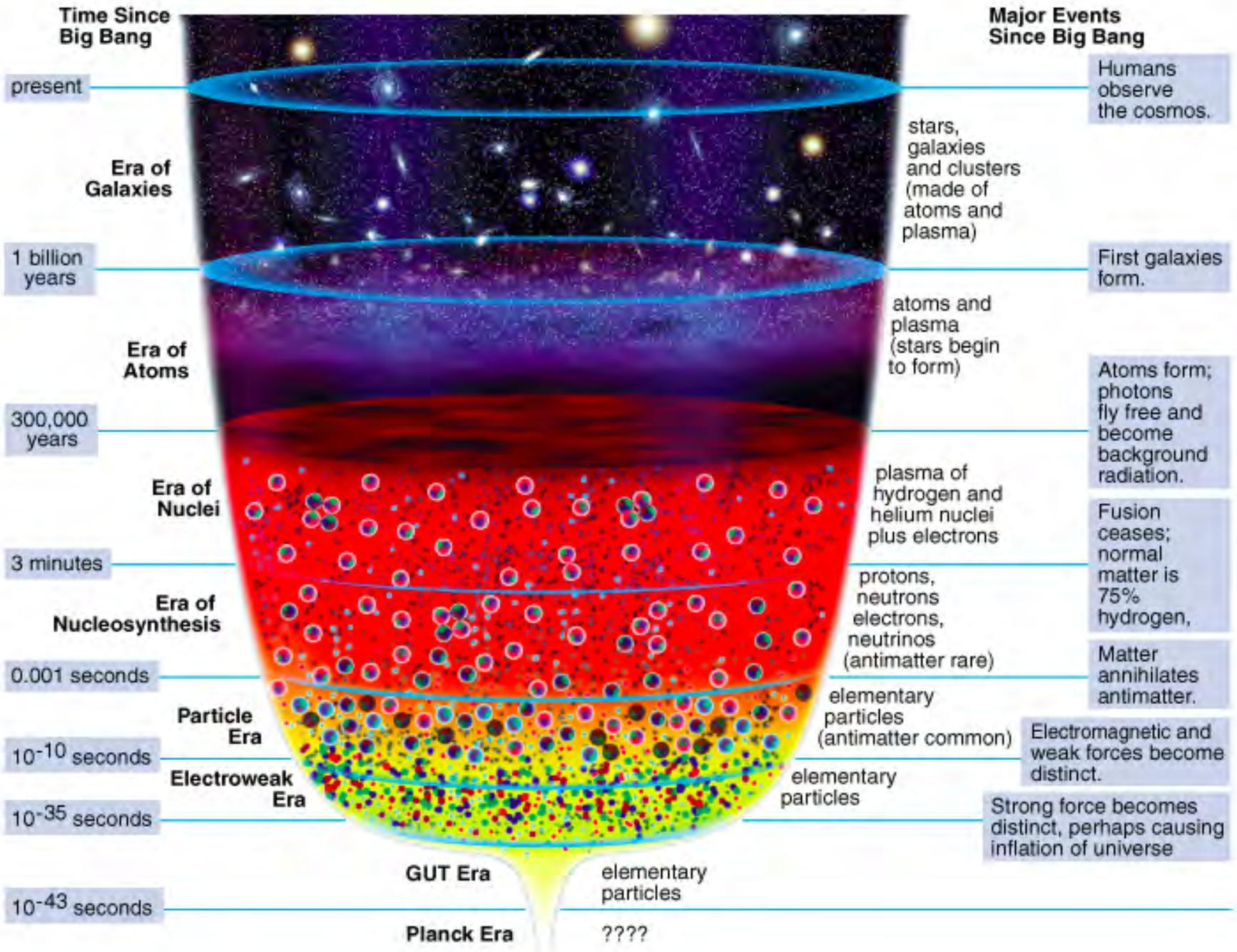
Neutron Decay is the Simplest “Semi-Leptonic” Weak Interaction*



Some Processes with the same Feynman Diagram

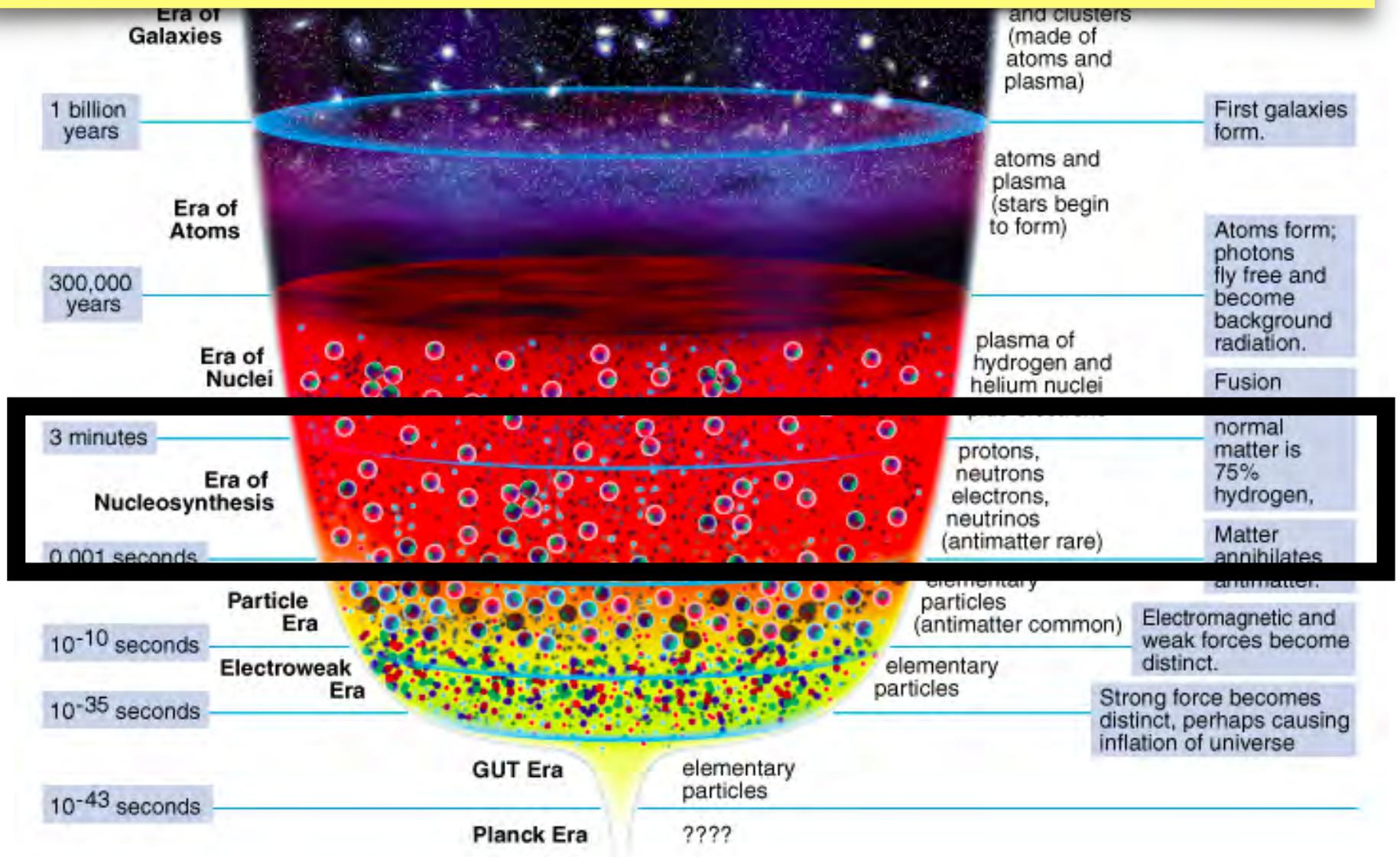
Primordial element formation	$n + e^+ \longleftrightarrow p + \bar{\nu}_e$ $p + e^- \longleftrightarrow n + \nu_e$ $n \longrightarrow p + e^- + \bar{\nu}_e$
Solar cycle	$p + p \longrightarrow {}^2\text{H} + e^+ + \nu_e$ $p + p + e^- \longrightarrow {}^2\text{H} + \nu_e$ etc.
Neutron star formation	$p + e^- \longrightarrow n + \nu_e$
Pion decay	$\pi^- \longrightarrow \pi^0 + e^- + \bar{\nu}_e$
Neutrino detectors	$\nu_e + p \longrightarrow e^+ + n$
Neutrino forward scattering	$\nu_e + n \longrightarrow e^- + p$ etc.

The Neutron Lifetime and the Big-Bang



The Era of Nuclear Physics

0.01 sec - 3 min



The Era of Nuclear Physics

0.01 sec - 3 min

Era of
Galaxies

and clusters
(made of
atoms and
plasma)

1 billion
years

First galaxies
form

Physics is Well Understood

- Only $n, p, e^+, e^-, \nu, \bar{\nu}, \gamma$ remain
- Density is “low” ...only two body interactions
- Particle energies are MeV's
- Cross sections are well known

10^{-43} seconds

GUT Era

elementary
particles

Planck Era

????

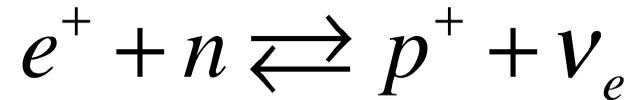
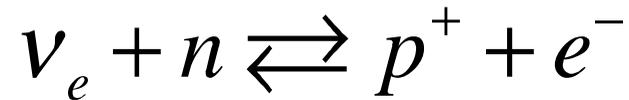
Neutron-Proton Equilibrium

time
0.01s

temperature
~10MeV

Era of Nuclear Physics Begins

Neutrons and Protons are in thermal equilibrium through the processes:



$$\boxed{\frac{N_n}{N_p} = e^{-\frac{(m_n - m_p)}{kT}}}$$

Neutrinos Decouple

time
0.1s

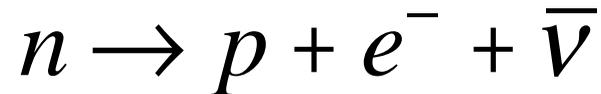
temperature
 $\sim 1\text{MeV}$

Neutrinos Decouple

At this energy neutrino cross sections become so small that thermal equilibrium is no longer maintained.

$$\frac{N_n}{N_p} = \frac{1}{3}$$

If nothing else happened ALL the neutrons would decay via



The universe would end up with only protons (Hydrogen)

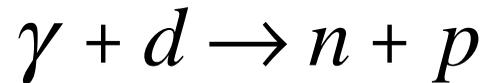
Nuclei Stable Against Photo-disassociation

time
3min

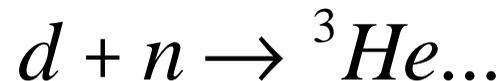
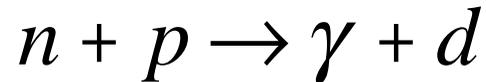
temperature
~100keV

Nucleosynthesis begins

Deuteron is now stable against photo disassociation e.g.



*At this point there are 87% protons and 13% neutrons
Nuclei are quickly formed...*



3½ min

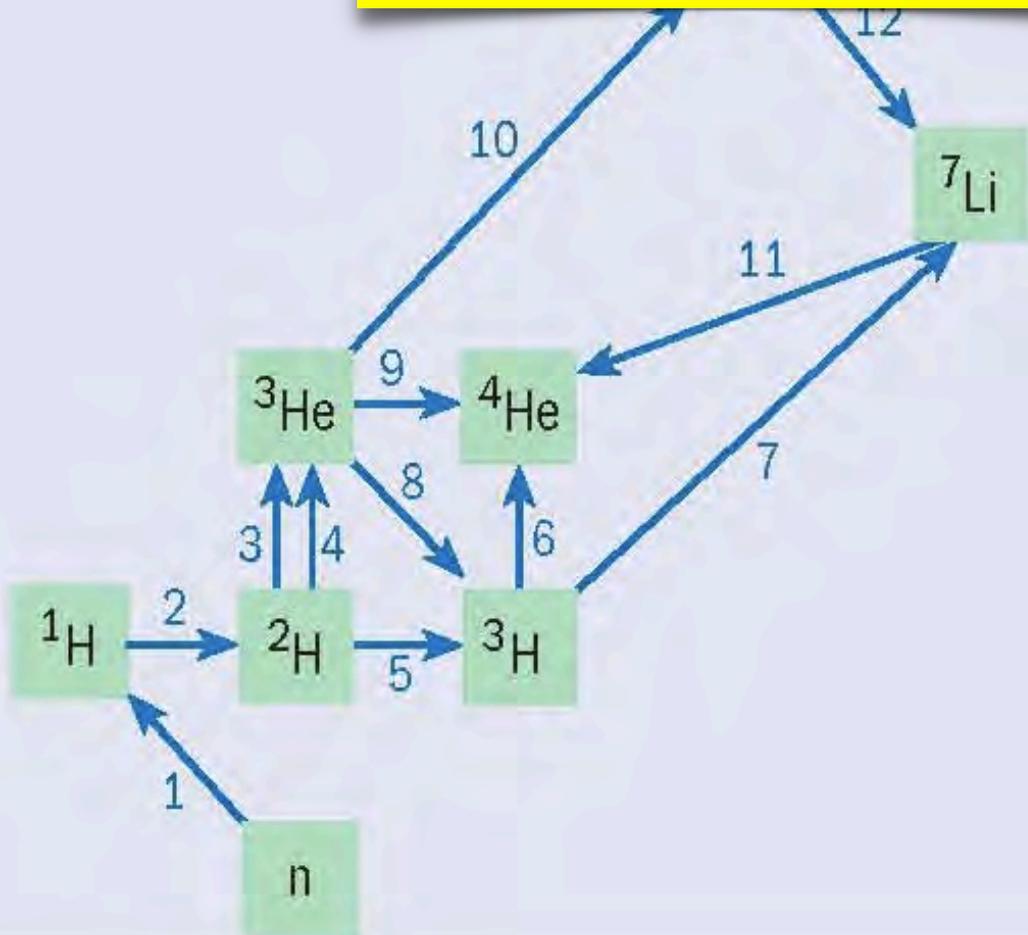
Nucleosynthesis Ends

All neutrons have been “used up” making ${}^4\text{He}$.

Important Reactions in Big Bang Nucleosynthesis

Neutron decay is the "rate determining" step

- 1 $n \rightarrow {}^1\text{H} + e^- + \bar{\nu}$
- 2 ${}^1\text{H} + n \rightarrow {}^2\text{H} + \gamma$
- 3 ${}^2\text{H} + {}^1\text{H} \rightarrow {}^3\text{He} + \gamma$
- 4 ${}^2\text{H} + {}^2\text{H} \rightarrow {}^3\text{He} + n$
- 5 ${}^2\text{H} + {}^2\text{H} \rightarrow {}^3\text{H} + {}^1\text{H}$
- 6 ${}^2\text{H} + {}^3\text{H} \rightarrow {}^4\text{He} + n$
- 7 ${}^3\text{H} + {}^4\text{He} \rightarrow {}^7\text{Li} + \gamma$
- 8 ${}^3\text{He} + n \rightarrow {}^3\text{H} + {}^1\text{H}$
- 9 ${}^3\text{He} + {}^2\text{H} \rightarrow {}^4\text{He} + {}^1\text{H}$
- 10 ${}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be} + \gamma$
- 11 ${}^7\text{Li} + {}^1\text{H} \rightarrow {}^4\text{He} + {}^4\text{He}$
- 12 ${}^7\text{Be} + n \rightarrow {}^7\text{Li} + {}^1\text{H}$



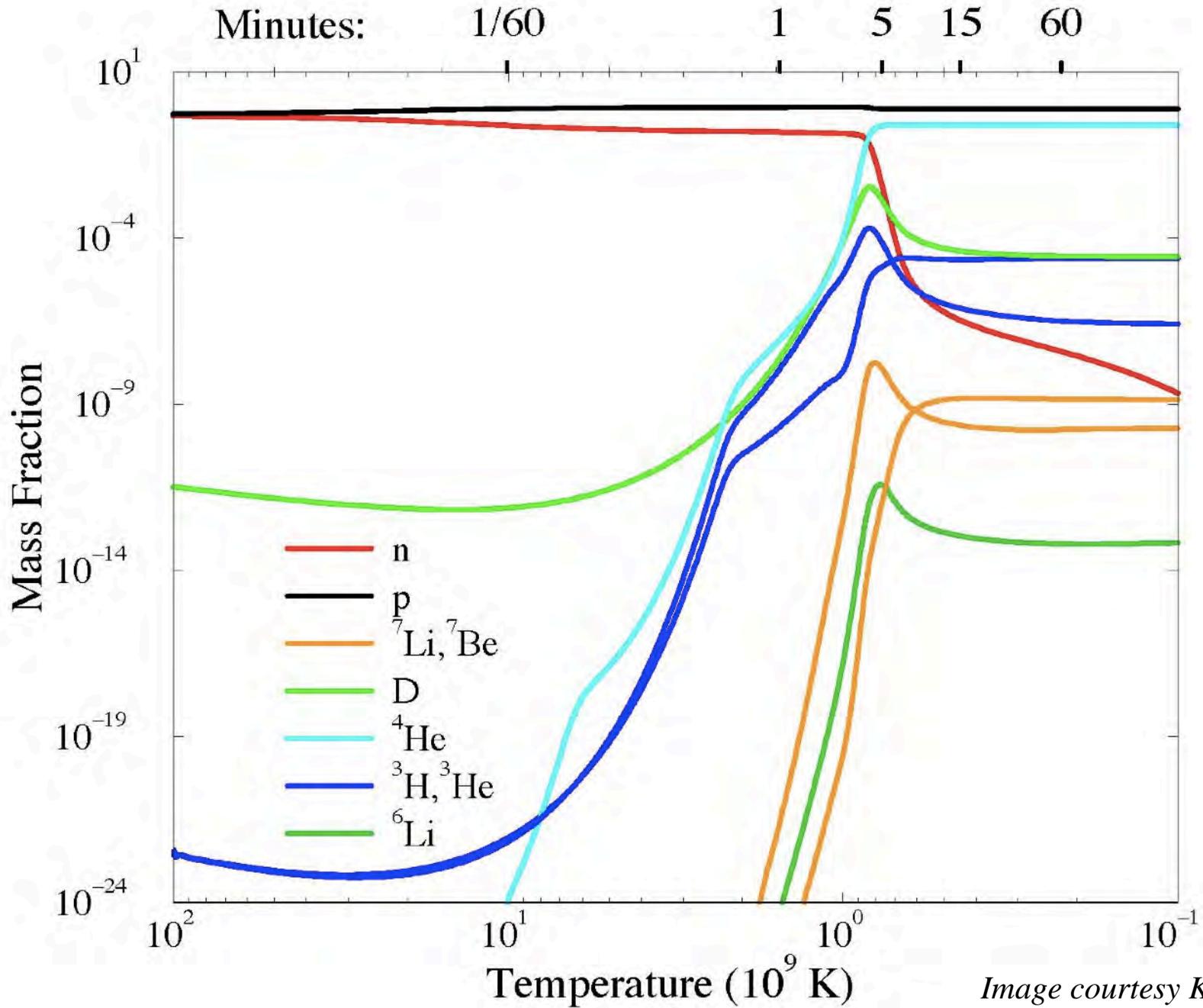


Image courtesy Ken Nollet

*“After 3 minutes, the Helium to Hydrogen ratio was set...
Nothing of interest has happened since.”*

Steven Weinberg

The Cosmic He/H Ratio Depends upon three quantities:

1) *The Cooling rate of the Universe*

Given by the heat capacity of the Universe

Determined mainly by the number of “light particles” ($m \leq 1 \text{ MeV}$)

Includes photons, electrons (positrons), neutrinos (x3)

2) *The Rate at which Neutrons are decaying*

The neutron lifetime

3) *The rate at which nuclear interactions occur*

*Determined by the the logarithm of the density of nucleons (baryons)**

**Because of expansion, the “absolute” baryon density is decreasing with time so the density is scaled as the ratio of matter to photons.*

The Parameters of Big Bang Nucleosynthesis

$$Y_p = 0.264 + 0.023 \log \eta_{10} + 0.018 (\tau_n - 10.28)$$

Cosmic Helium Abundance

Cosmic Baryon Density

Neutron Lifetime in Minutes

We can "invert" this line of reasoning. If we measure the Helium Abundance and the Neutron Lifetime, we can determine the density of "ordinary" matter in the universe.

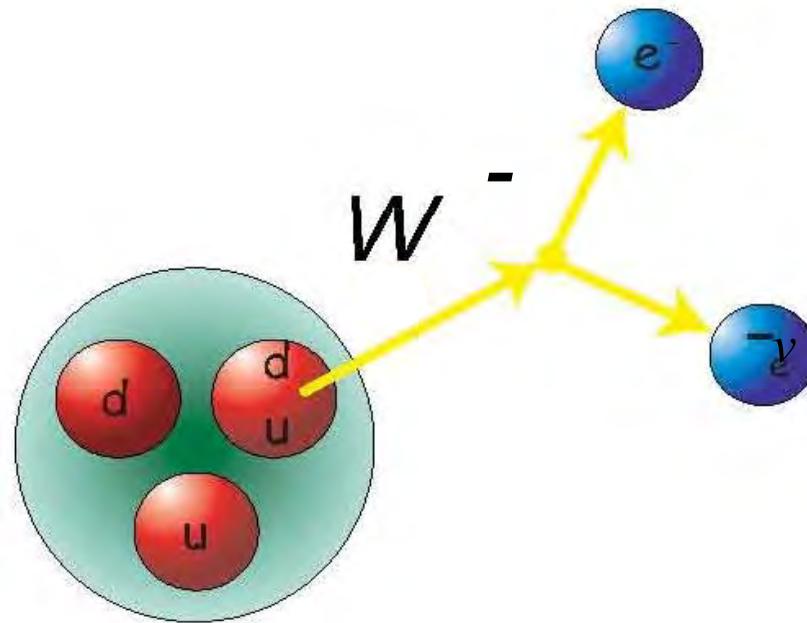
$$\log \eta_{10} = [0.264 - Y_P + 0.018(\tau_n - 10.28)] / 0.023$$

Cosmic Baryon Density

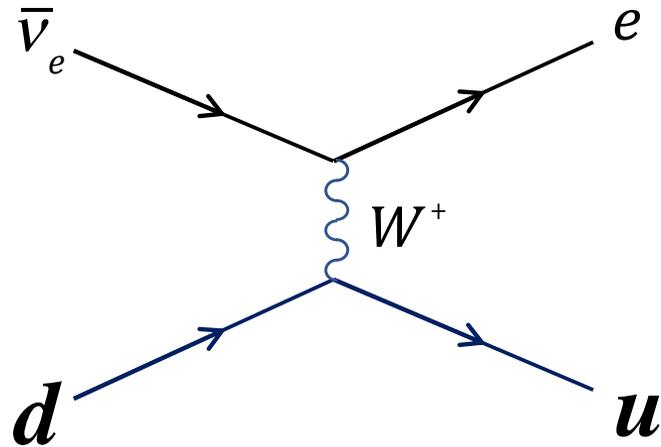
Cosmic Helium Abundance

Neutron Lifetime in Minutes

Introduction to the Theory of Neutron Beta Decay



“Ideal” View of Semi-Leptonic Weak Interaction



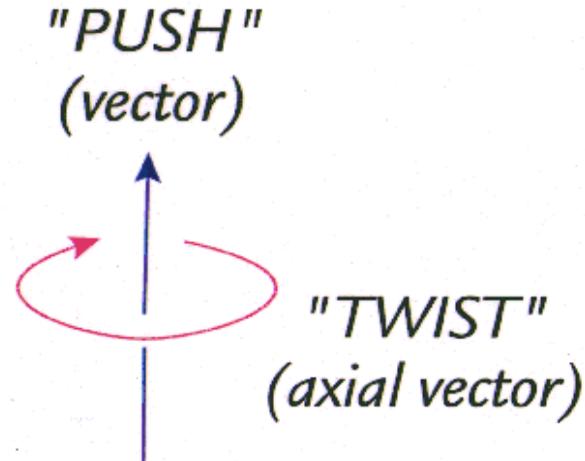
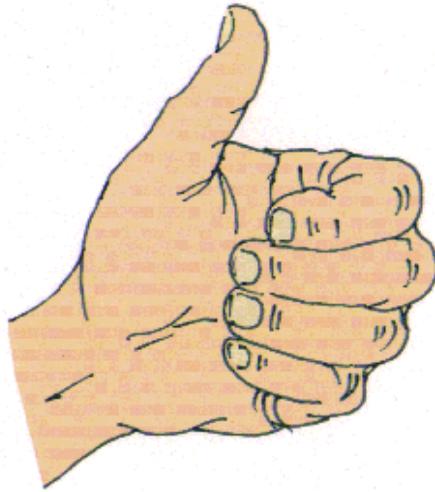
Construct a Current-Current Interaction that couples a down quark to an up quark, and an electron to antineutrino

$$\langle \bar{\nu}_e | \mathbf{H}_{Weak} | e^- \rangle \langle d | \mathbf{H}_{Weak} | u \rangle$$

How to include Parity Violation?



“Handedness”

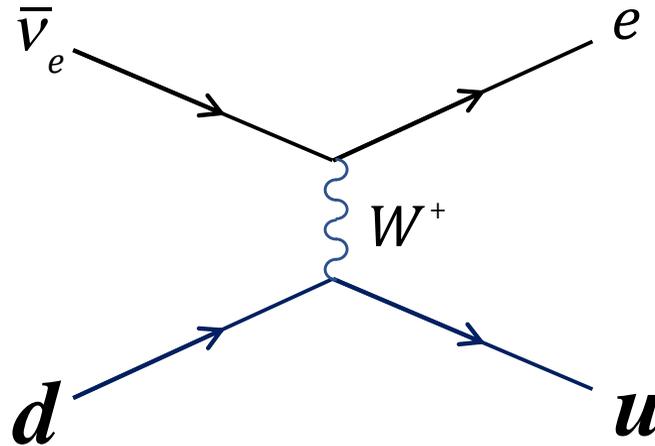


A “Handed” Interaction Results From:

A VECTOR – “V” “Push”
and
An Axial Vector – “A” “Twist”

The relative signs of the vector and axial-vector determines the “handedness

The “Ideal” Standard Model Quark-Lepton Interaction



“vector” operator

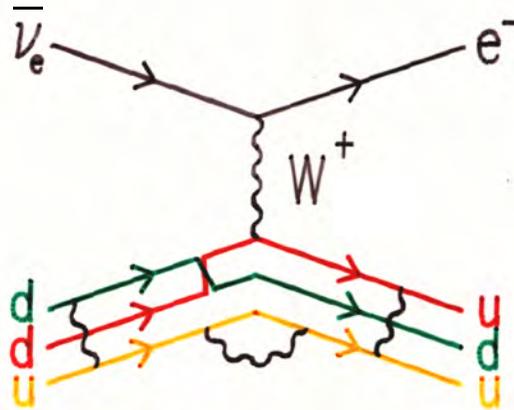
“axial-vector” operator

$$H_{Weak} = G_{Weak} \langle \bar{\nu}_e | \underbrace{\gamma^\mu - \gamma^\mu \gamma^5}_{\text{“V-A”}} | e^- \rangle \langle d | \gamma^\mu - \gamma^\mu \gamma^5 | u \rangle$$

Vector “minus” Axial Vector Means a “Left-Handed” Interaction

Neutron Decay is a Bit More Complicated

Strongly interacting quarks within the neutron modify the relative size of the vector and axial vector couplings



The strong interaction conserves parity.

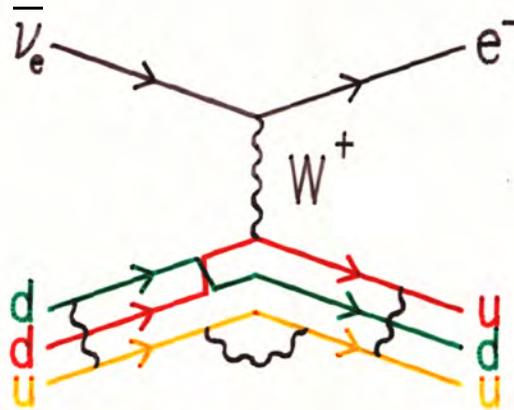
It does not change the relative phase (handedness).

In the low energy limit, the Hamiltonian is still quite simple:

$$H_{Weak} = G_{Weak} \langle \bar{\nu}_e | \gamma^\mu - \gamma^\mu \gamma^5 | e^- \rangle \langle d | g_V \gamma^\mu - g_A \gamma^\mu \gamma^5 | u \rangle$$

Neutron Decay is a Bit More Complicated

Strongly interacting quarks within the neutron modify the relative size of the vector and axial vector couplings



The strong interaction conserves parity.

It does not change the relative phase (handedness).

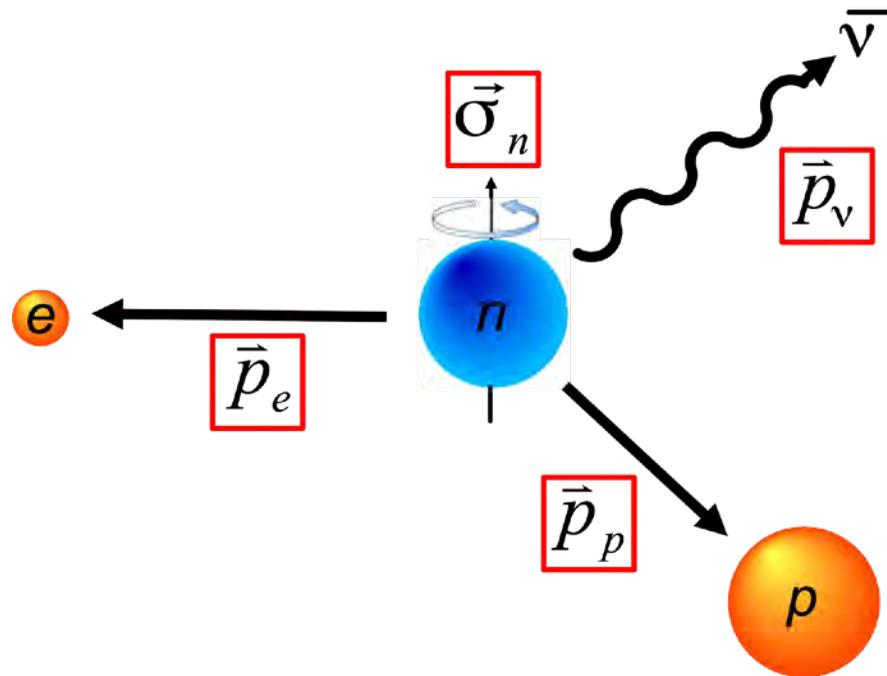
In the low energy limit, the Hamiltonian is still quite simple:

$$H_{Weak} = G_{Weak} \langle \bar{\nu}_e | \gamma^\mu - \gamma^\mu \gamma^5 | e^- \rangle \langle d | g_V \gamma^\mu - g_A \gamma^\mu \gamma^5 | u \rangle$$

**If Neutron Decay is Purely Left-Handed (V-A),
only two parameters completely describe it**

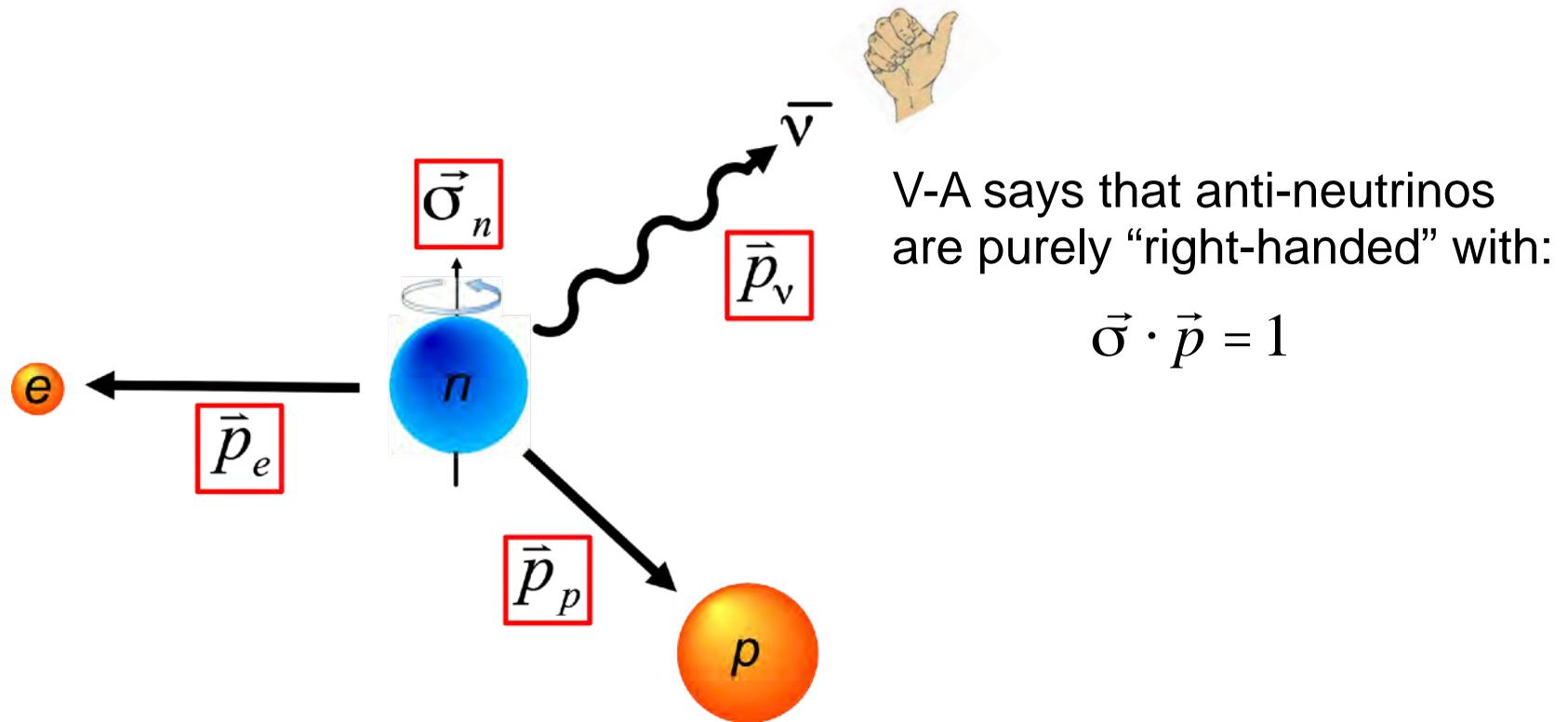
Phenomenology of Neutron Beta Decay

Momentum and Angular Momentum Must Be Conserved



Phenomenology of Neutron Beta Decay

Momentum and Angular Momentum Must Be Conserved



Conservation of linear and angular momentum implies that there are strong correlations between the initial neutron spin and decay particle momenta.

Correlations in Neutron Decay

$$dW \propto \frac{1}{\tau_n} F(E_e) \left[1 + a \frac{\mathbf{p}_e \cdot \mathbf{p}_\nu}{E_e \cdot E_\nu} + b \frac{m_e}{E_e} + A \frac{\boldsymbol{\sigma}_n \cdot \mathbf{p}_e}{E_e} + B \frac{\boldsymbol{\sigma}_n \cdot \mathbf{p}_\nu}{E_\nu} + \dots \right]$$

(Jackson, Treiman, Wyld, 1957)

Correlations in Neutron Decay

Pure V-A, low energy limit

$$dW \propto \frac{1}{\tau_n} F(E_e) \left[1 + a \frac{\mathbf{p}_e \cdot \mathbf{p}_\nu}{E_e \cdot E_\nu} + b \frac{m_e}{E_e} + A \frac{\boldsymbol{\sigma}_n \cdot \mathbf{p}_e}{E_e} + B \frac{\boldsymbol{\sigma}_n \cdot \mathbf{p}_\nu}{E_\nu} + \dots \right]$$

$$\frac{1}{(G_V^2 + 3G_A^2)}$$

$$\frac{1 - \left(\frac{G_A}{G_V}\right)^2}{1 - 3\left(\frac{G_A}{G_V}\right)^2}$$

$$b = 0$$

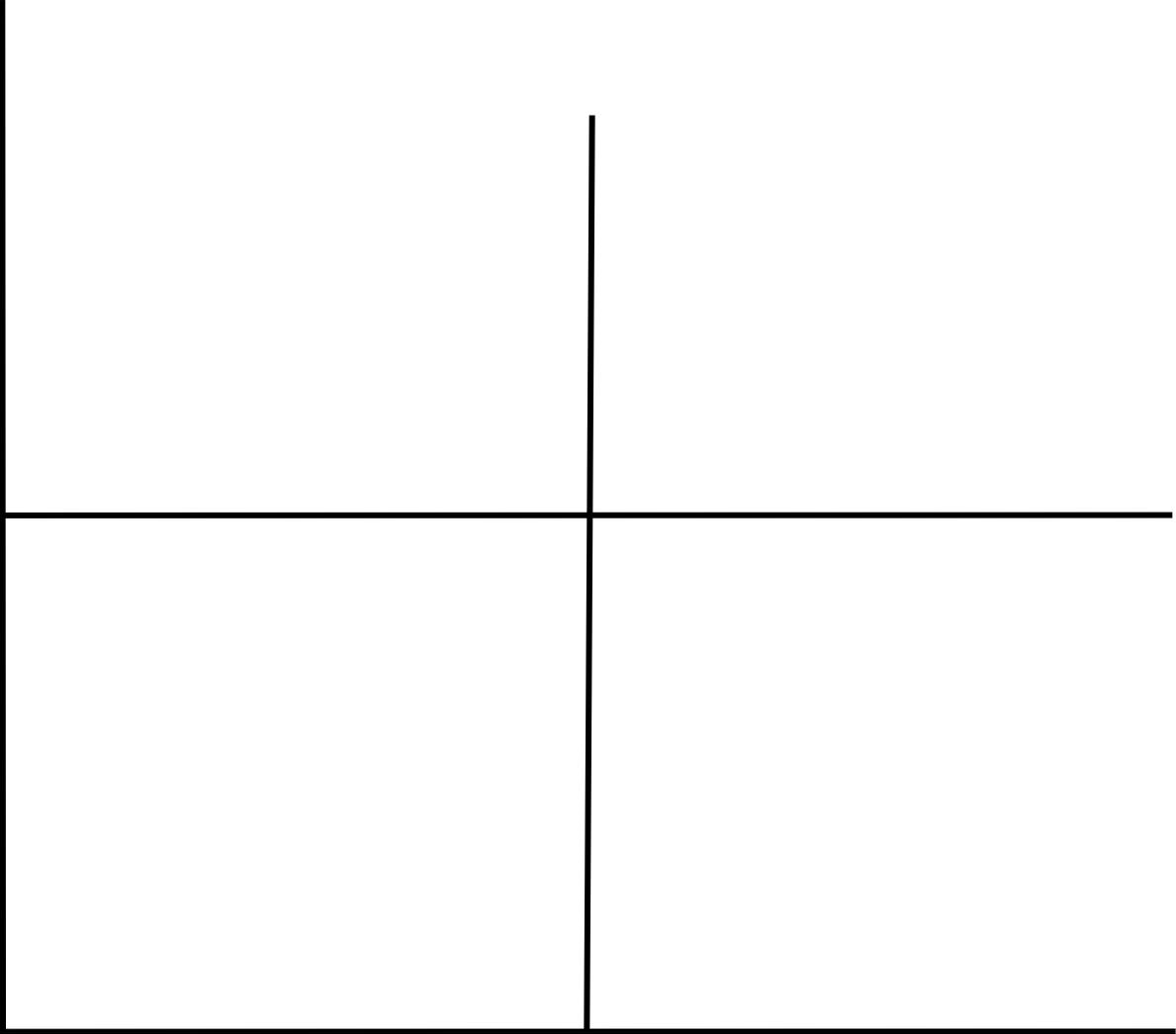
$$\frac{-2 \left[\left(\frac{G_A}{G_V}\right)^2 + \left(\frac{G_A}{G_V}\right) \right]}{1 - 3\left(\frac{G_A}{G_V}\right)^2}$$

$$\frac{-2 \left[\left(\frac{G_A}{G_V}\right)^2 - \left(\frac{G_A}{G_V}\right) \right]}{1 - 3\left(\frac{G_A}{G_V}\right)^2}$$

Neutron beta decay measurements give:

$$\frac{G_A^2 + 3G_V^2}{G_A / G_V}$$

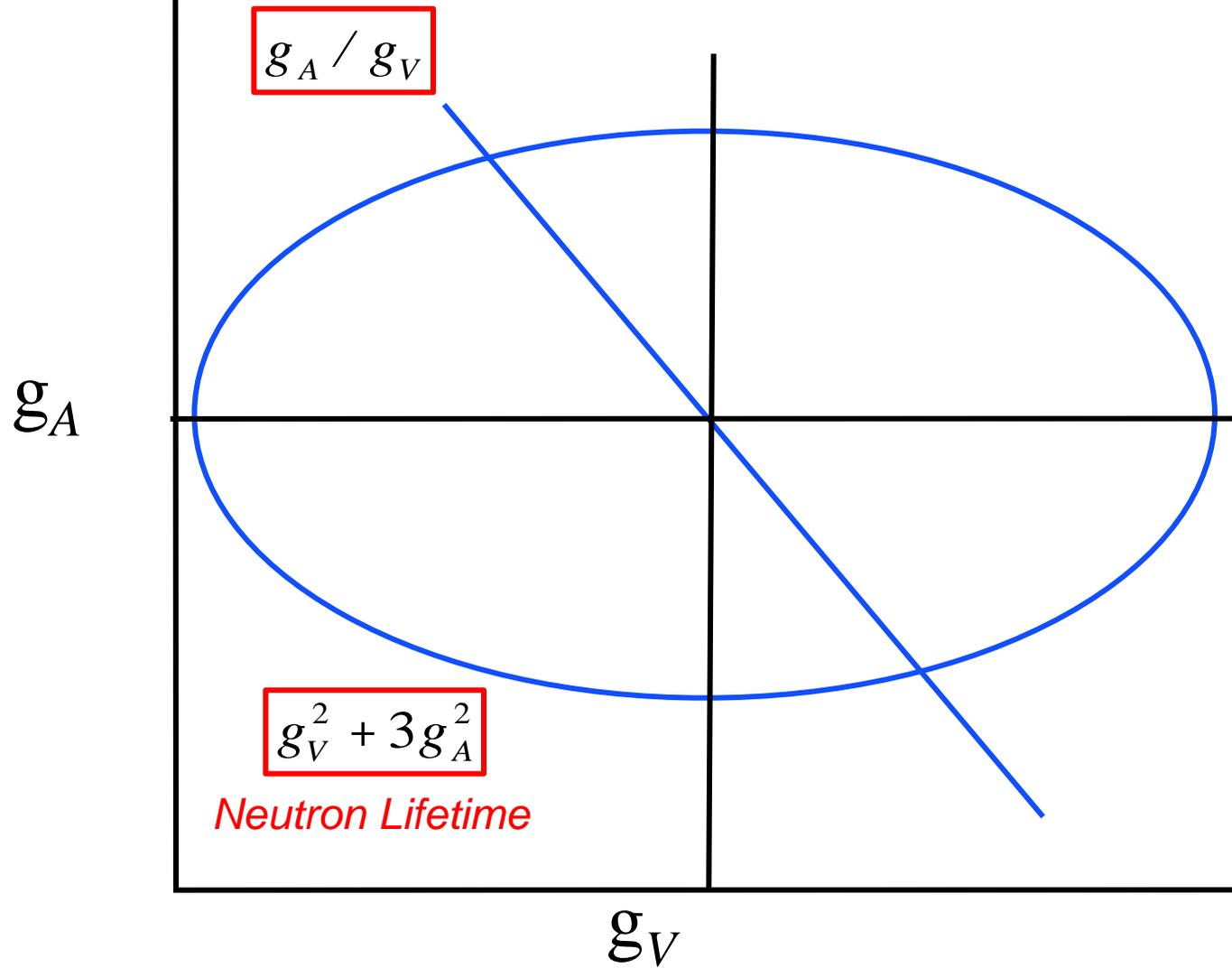
(Jackson, Treiman, Wyld, 1957)



g_A

g_V

Neutron Decay Correlations



The “Conserved Vector Current” - CVC

- *Electric Charge is a conserved quantity*
- *“Electroweak” theory provides a common framework for Electromagnetism and the Weak Interaction*
- *Electric Charge Conservation implies conservation of “something else” in the Weak sector.*
- *That “something else” is the Weak “Vector Current”*

Conservation of the Vector Current implies that the vector coupling constant g_V is same for all nuclear interactions!

n.b. g_A is not similarly constrained but, there are certain nuclear decays ($0^+ \rightarrow 0^+$) where g_A must be identically zero

Sharing the Weak Charge Between Quarks

The eigenstates of the weak interaction are not the quark states

Weak Interaction
Basis States

Cabbibo-Kobayshi-Maskawa
Matrix

Quark States

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

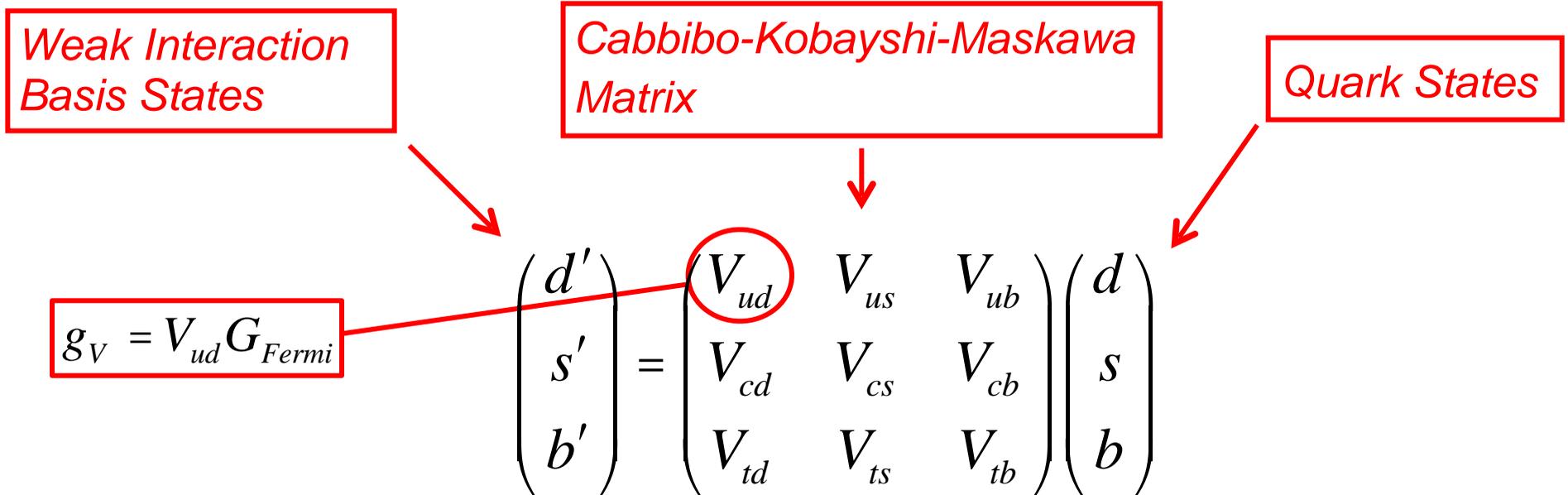
The C-K-M matrix must be unitary or there will be hell to pay!

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1$$

Kaon Physics give V_{us} ($V_{ub} \ll 1$)

Sharing the Weak Charge Between Quarks

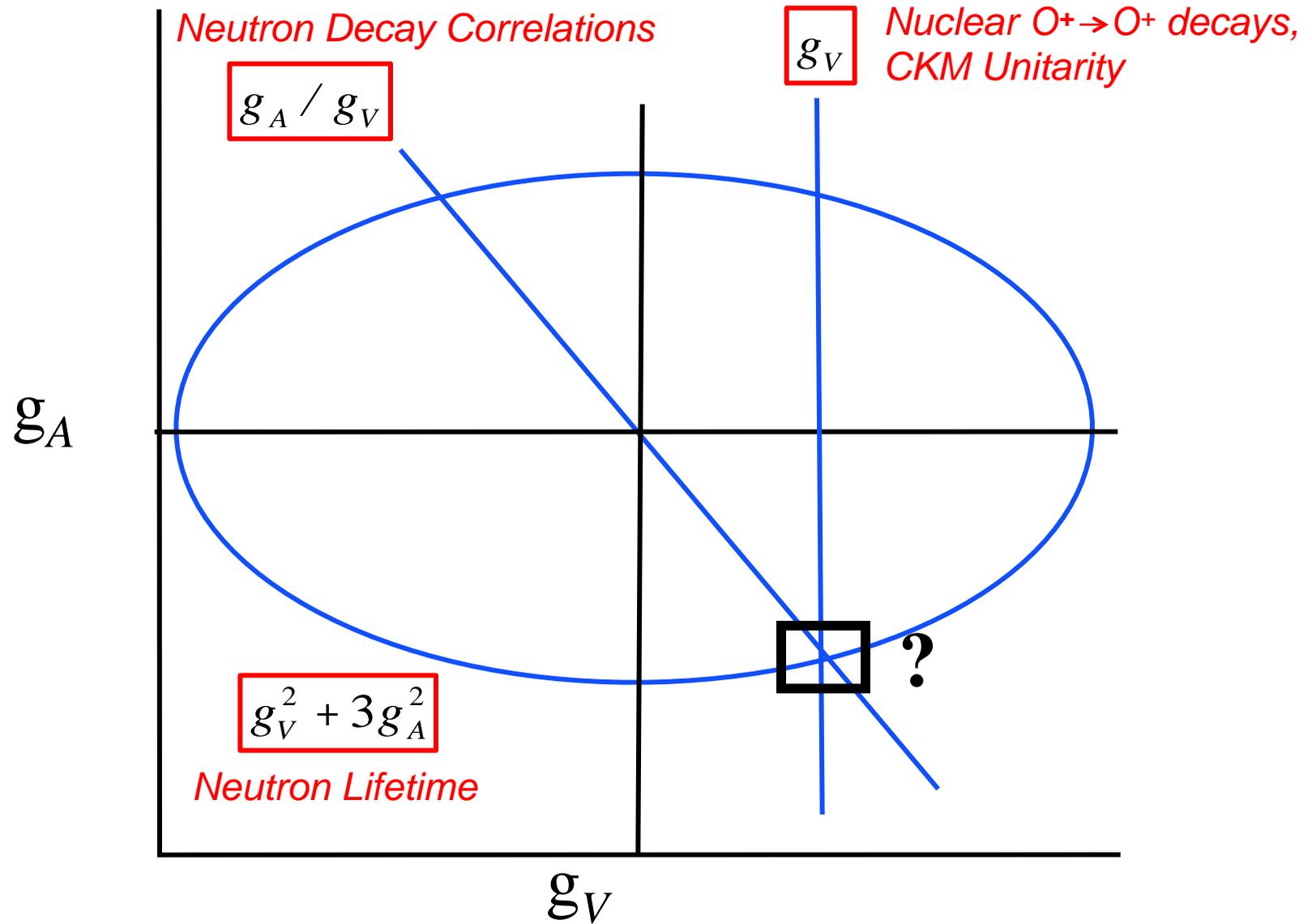
The eigenstates of the weak interaction are not the quark states

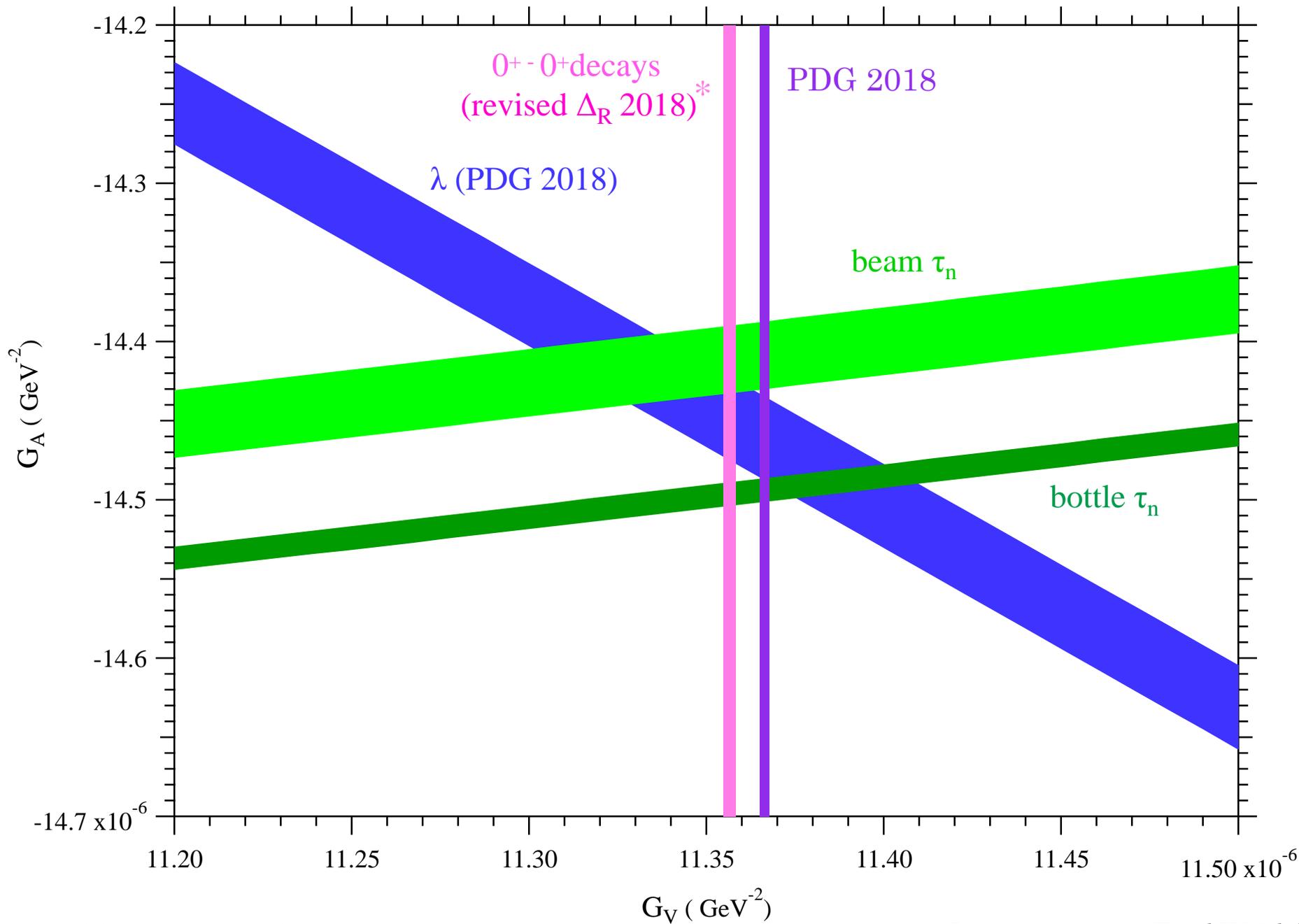


The C-K-M matrix must be unitary or there will be hell to pay!

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1$$

Kaon Physics give V_{us} ($V_{ub} \ll 1$)





*Seng, et al, <https://arxiv.org/abs/1807.10197>

Measuring the Neutron Lifetime

Two Approaches to Measuring a Lifetime

1. Observation time is longer than (or comparable to) the lifetime.

STEP 1: Determine $N(0)$ number unstable nuclei in a sample at $t=0$, and

STEP 2: Determine $N(t)$ number unstable nuclei in a sample at $t=t$.

$$N(t) = N(0)e^{-t/\tau}$$

2. Observation time is much shorter than the lifetime.

STEP 1: Determine N , the number of unstable nuclei in a sample, and

STEP 2: Determine the rate of decays \dot{N} .

$$\frac{dN}{dt} = -\frac{N}{\tau}$$

Two Approaches to Measuring a Lifetime

1. Observation time is longer than (or comparable to) the lifetime.

STEP 1: Determine $N(0)$ number unstable nuclei in a sample at $t=0$, and

STEP 2: Determine $N(t)$ number unstable nuclei in a sample at $t=t$.

$$N(t) = N(0)e^{-t/\tau}$$

Bottle Method

2. Observation time is much shorter than the lifetime.

STEP 1: Determine N , the number of unstable nuclei in a sample, and

STEP 2: Determine the rate of decays \dot{N} .

$$\frac{dN}{dt} = -\frac{N}{\tau}$$

Beam Method

The Bottle Method

The "Bottle" Method

Step 1.

Get one "Neutron Bottle"

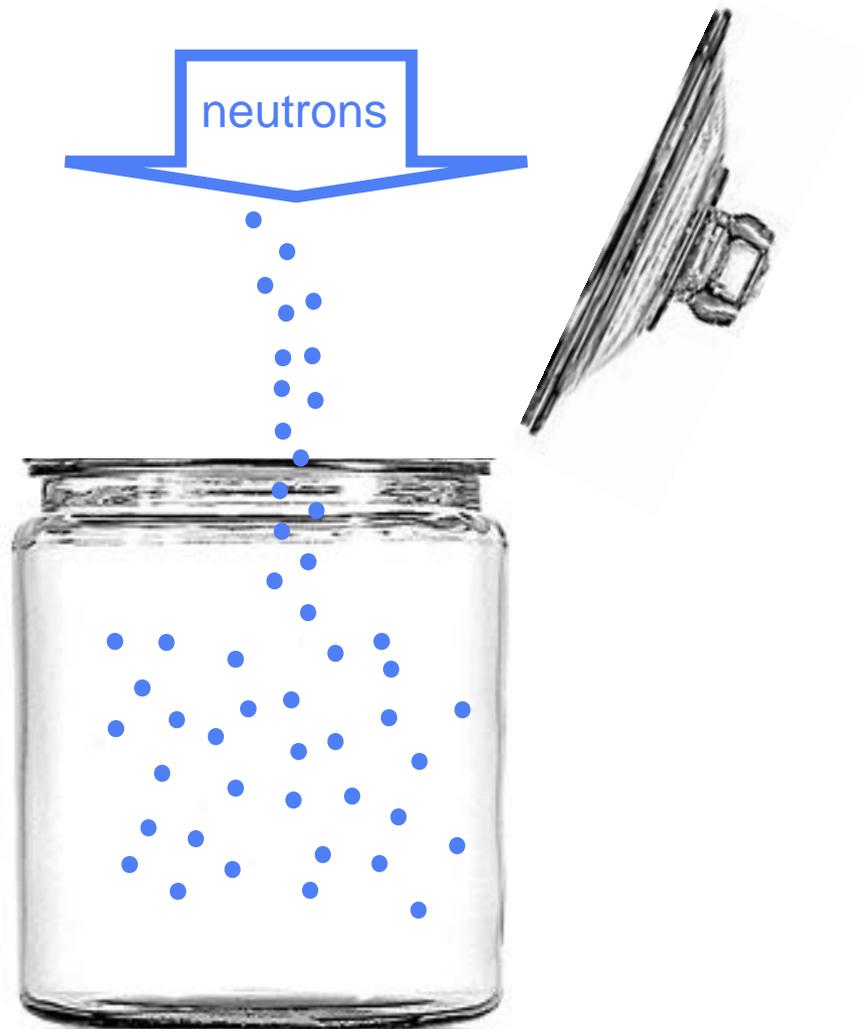
Neutron Bottle



The "Bottle" Method

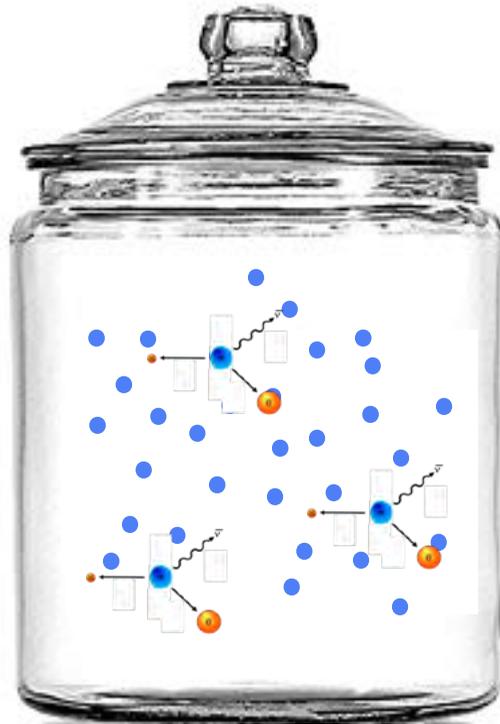
Step 2.

Fill with neutrons



The "Bottle" Method

Step 3. Let neutrons decay for time "t"



The "Bottle" Method

Step 4. Count remaining neutrons



The "Bottle" Method

Step 1.

Get one "Neutron Bottle"

Neutron Bottle



“Ultracold” Neutrons

Neutrons with extremely low energy:

$$E_k \leq 100 \text{ neV} \quad v \leq 5 \text{ m/s}$$

can be confined in material, gravitational, or magnetic “bottles”

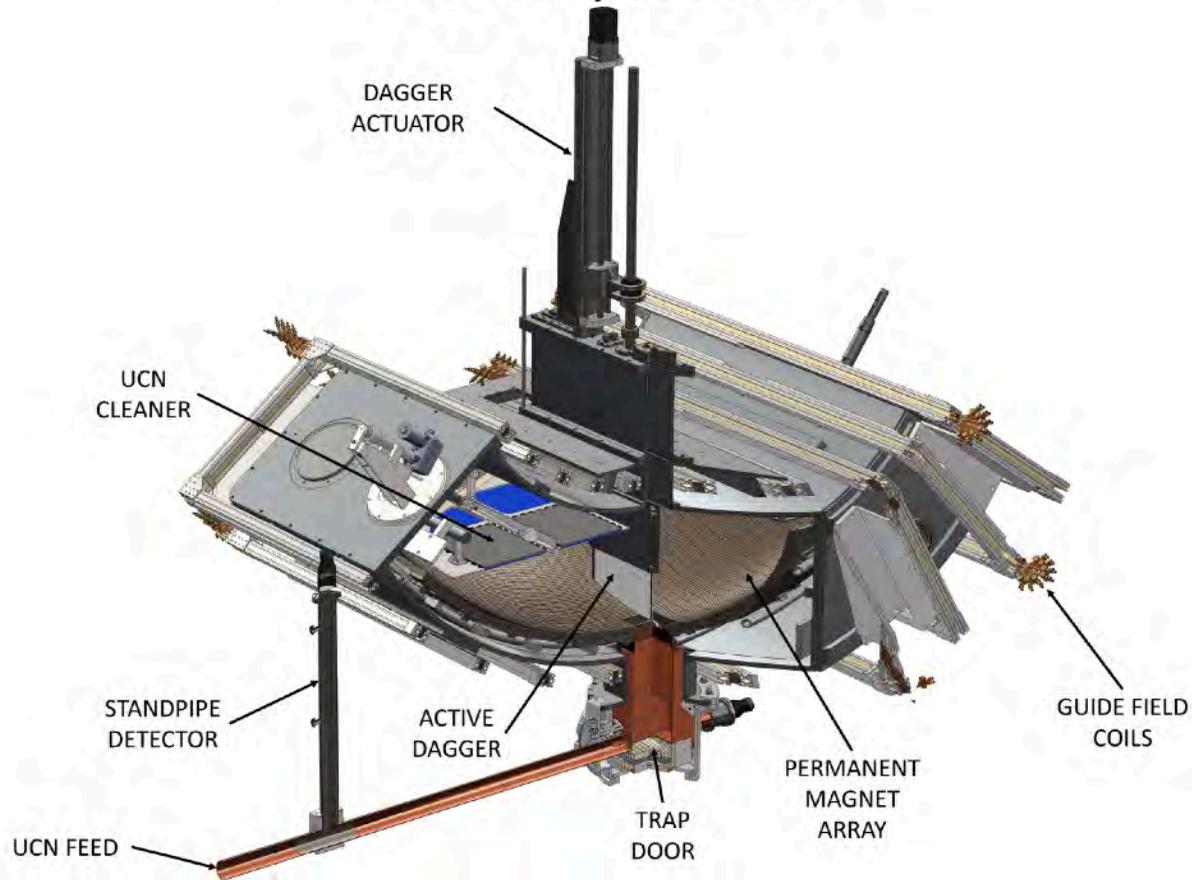
with confinement lifetimes much greater than the beta decay lifetime.

Such neutrons are called “ultracold” with a “temperature” of a few mK

This is far below the energies normally available from a high flux neutron source

The LANL UCN τ Magnetic Bottle Experiment

UCN τ Experiment



877.7 ± 0.7 (stat) $+0.3/-0.1$ (sys) seconds

Pattie, et al, Science, 10.1126/aan8895 (2018)

A Challenge with Bottles

Loss rate in a Bottle can also come from other Mechanisms.

Neutrons can be lost to absorption on walls, up-scattering on walls, escape through gaps in the wall,...

$$\frac{1}{\tau_{\text{exp}}} = \frac{1}{\tau_n} + \frac{1}{\tau_{\text{absorb}}} + \frac{1}{\tau_{\text{escape}}} + \frac{1}{\tau_{\text{up-scatter}}} + \dots^*$$

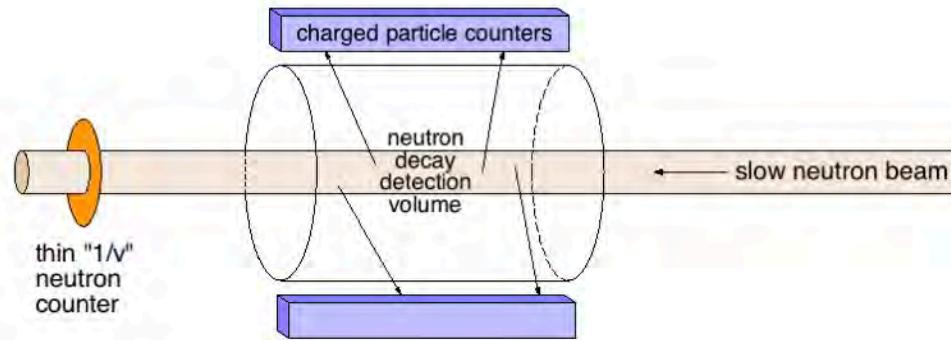
What we want

What we measure

**True if all loss rates are constant... which is not necessarily true.*

*Measuring the Neutron Lifetime
with a Neutron Beam*

In-Beam Neutron Lifetime Determination



Neutron Decay Rate $\Gamma = \frac{N}{\tau}$ $N =$ *average number of neutrons in detector*

Must make several **ABSOLUTE** measurements:

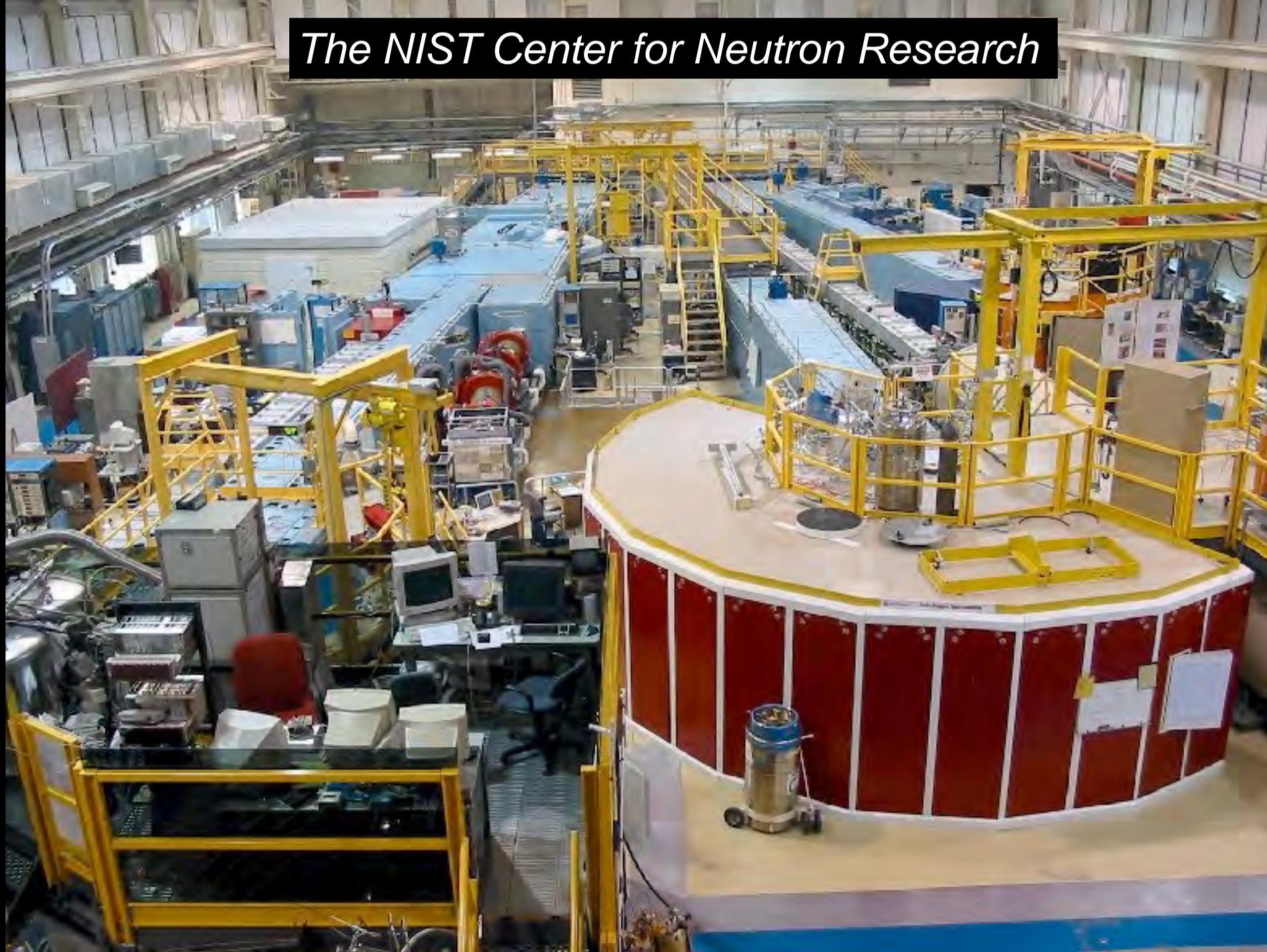
- 1. Decay Rate***
- 2. Detection Fiducial Volume***
- 3. Detector Efficiency***
- 4. Neutron Density in Beam (flux weighted by $1/v$)***

The Beam Method at NIST

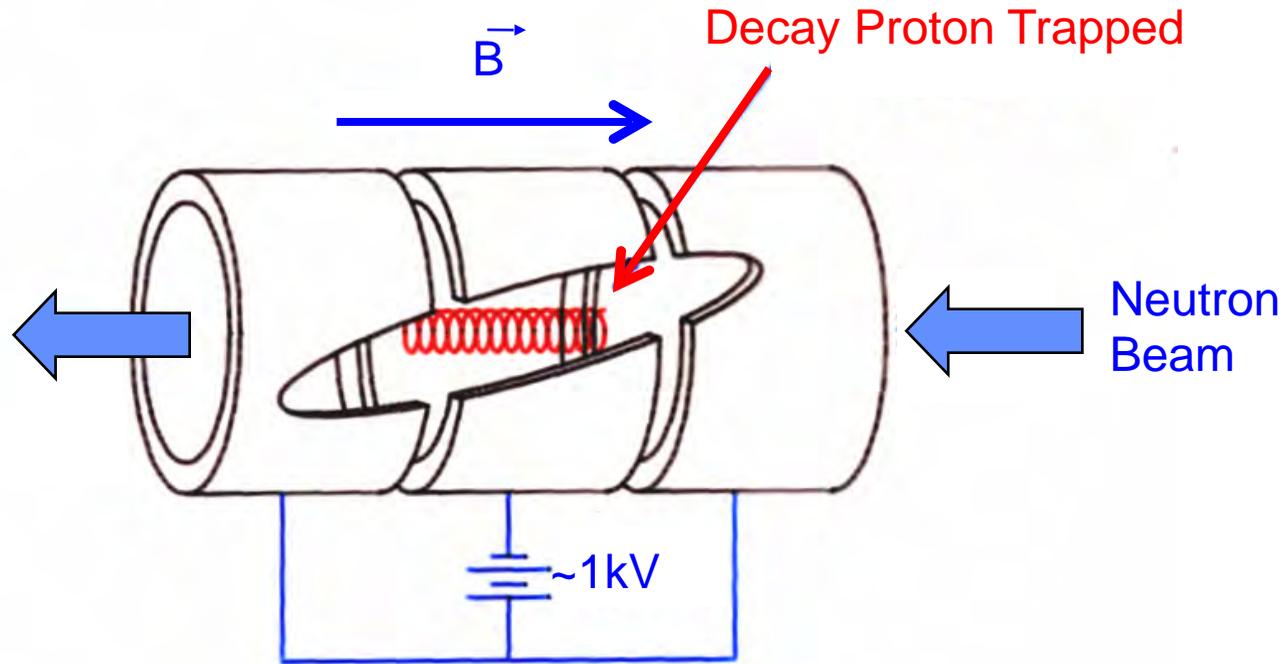
*The NIST Center for Neutron Research
Gaithersburg, Md.*



The NIST Center for Neutron Research



*Decay Protons are Trapped in the
NIST “Beam” Neutron Lifetime Experiment*



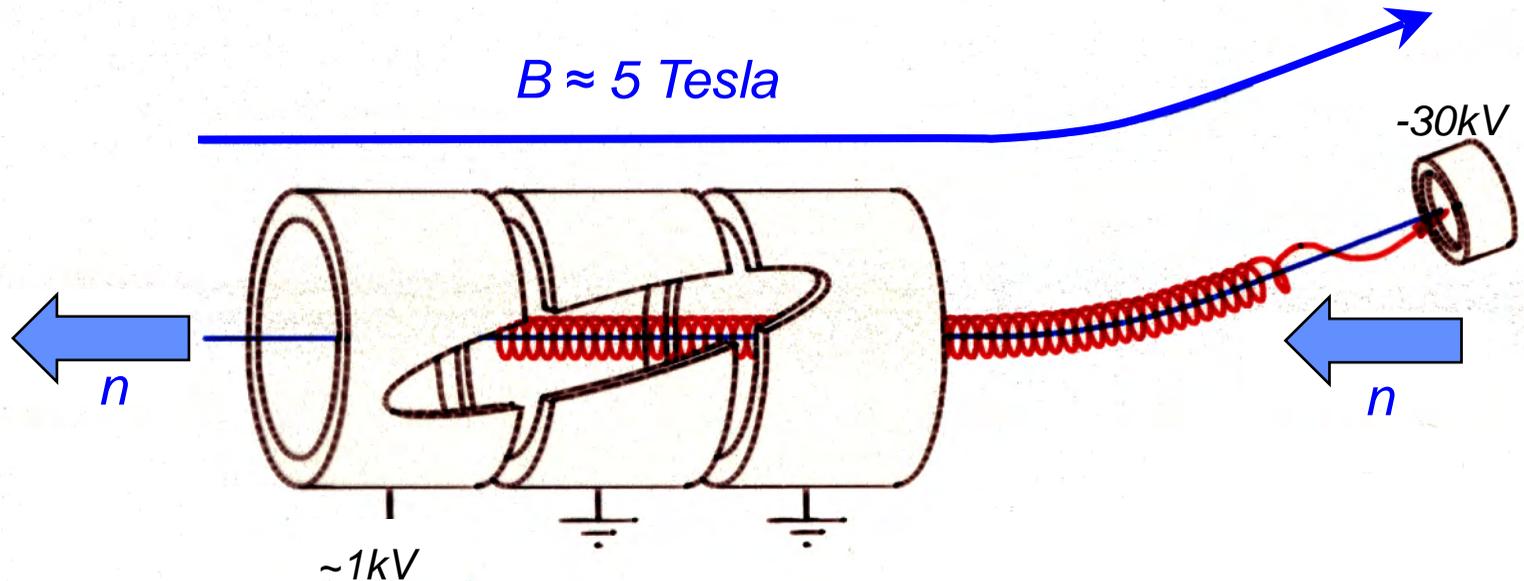
Uncharged neutrons pass through a charged particle “Penning” Trap.

Decay protons have low energy ($E_p < 750\text{eV}$) and are trapped in a combination of electric and magnetic fields.

Neutrons spend a short time in trap so probability of decay is $\sim 10^{-7}$.

Problem: The decay protons have low energy and are stuck in the trap

Solution: The trap is “opened” and the emerging protons are accelerated and detected in off axis detector



Trap Cycle ~10ms

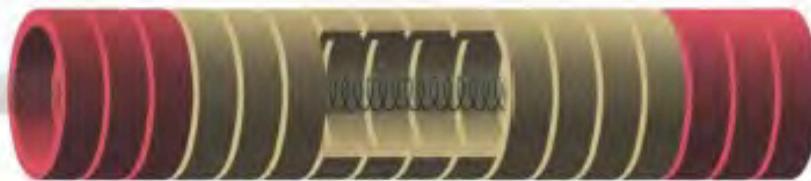
alpha, triton
detector



precision
aperture



${}^6\text{Li}$
deposit



mirror
(+800 V)

central trap electrodes

door closed
(+800 V)

$B = 4.6 \text{ T}$

proton
detector



neutron beam

Count Cycle ~100us

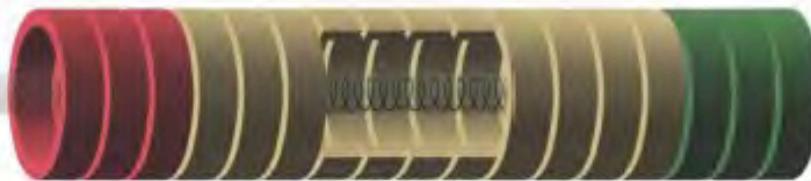
alpha, triton
detector



precision
aperture



^6Li
deposit



mirror
(+800 V)

central trap electrodes

door open
(ground)

$B = 4.6 \text{ T}$

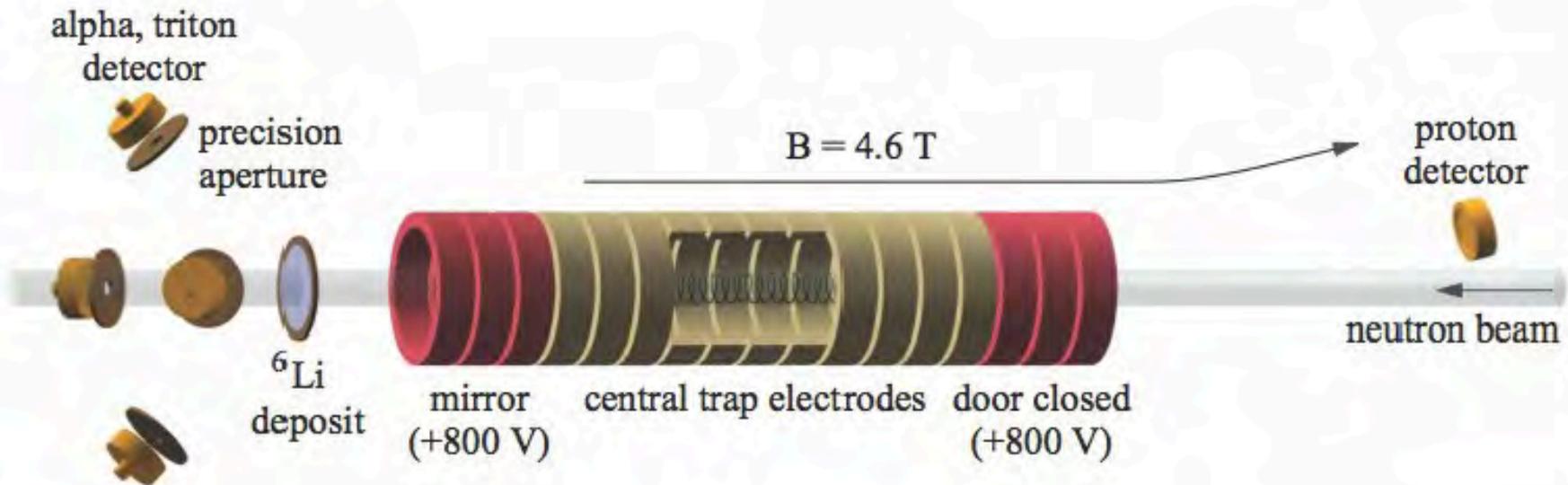
proton
detector



neutron beam

Only count during count cycle
S/N increase by x100 or more

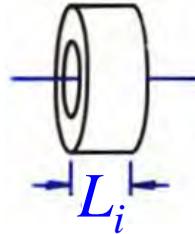
What is the detection volume?



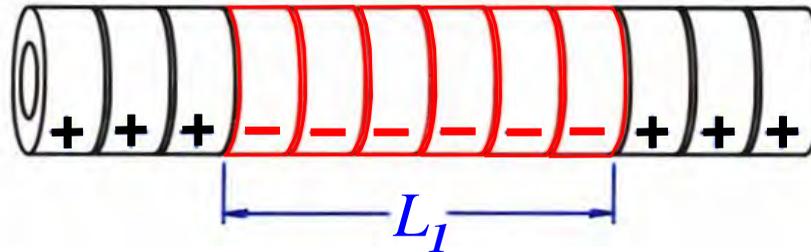
Determination of detection volume (i.e. length)
requires knowledge of trap "end-effects"

Method of "Virtual" Trap

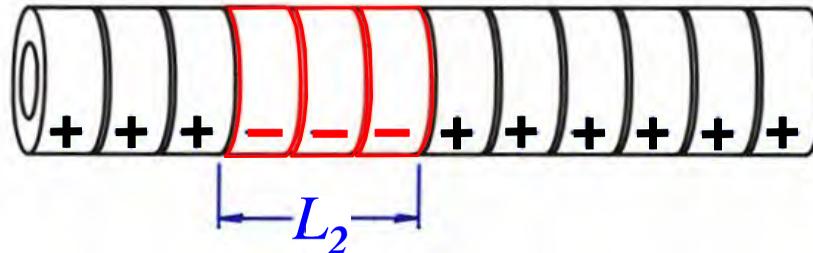
Step 1. Construct trap elements with well known length:



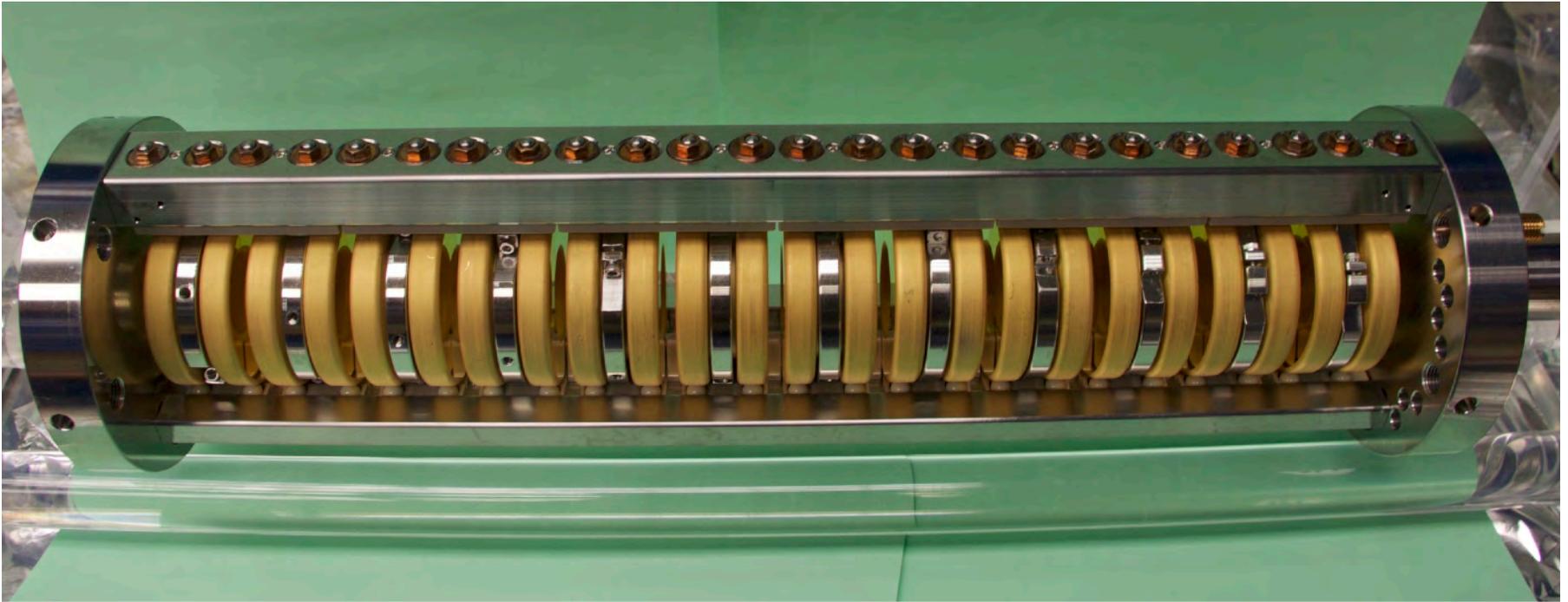
Step 2. Perform decay measurement with a trap length with unknown end effects:

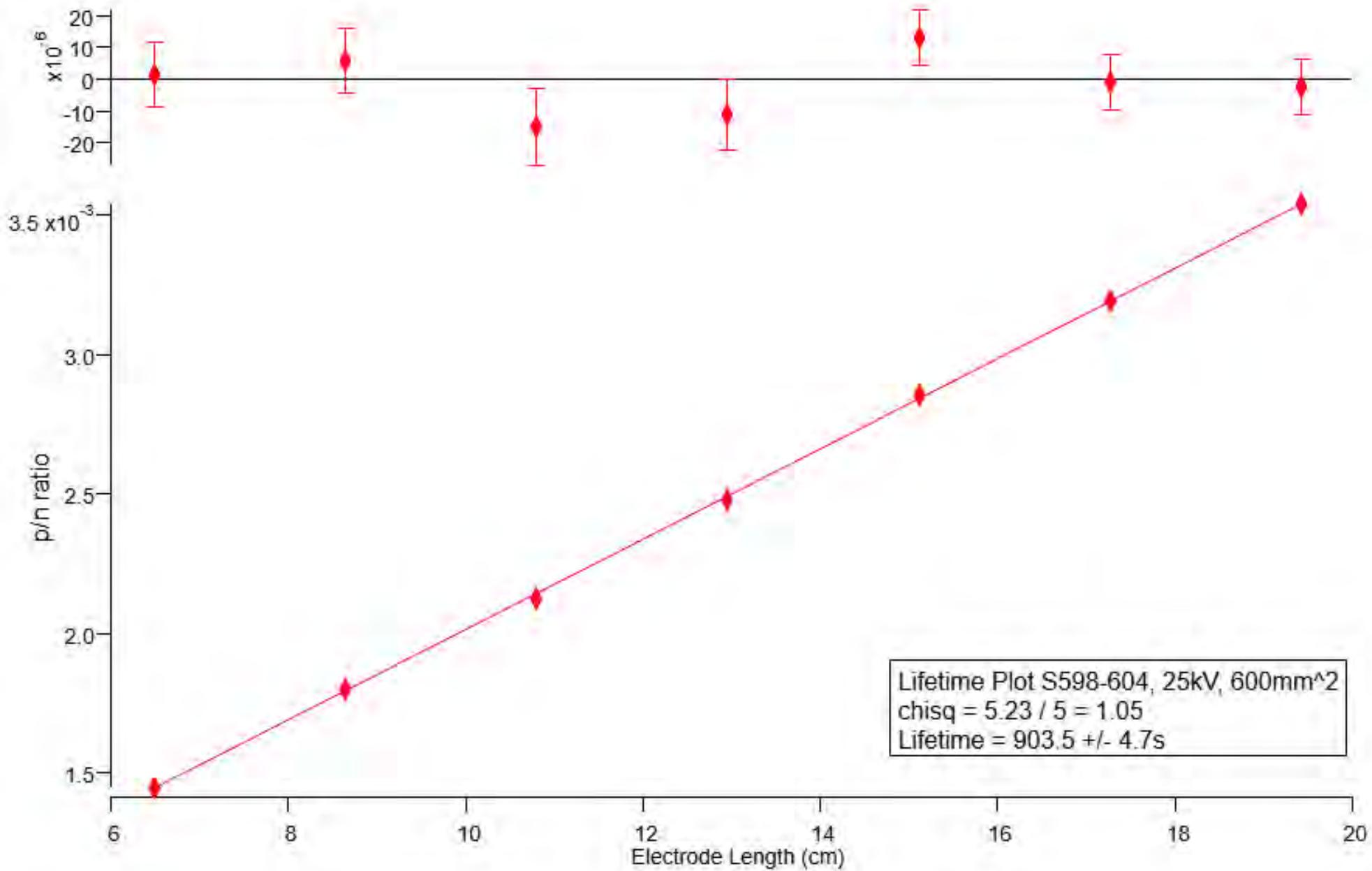


Step 3. Repeat with different length - Length Change is immune to end effects



The NIST Mk III Penning Trap

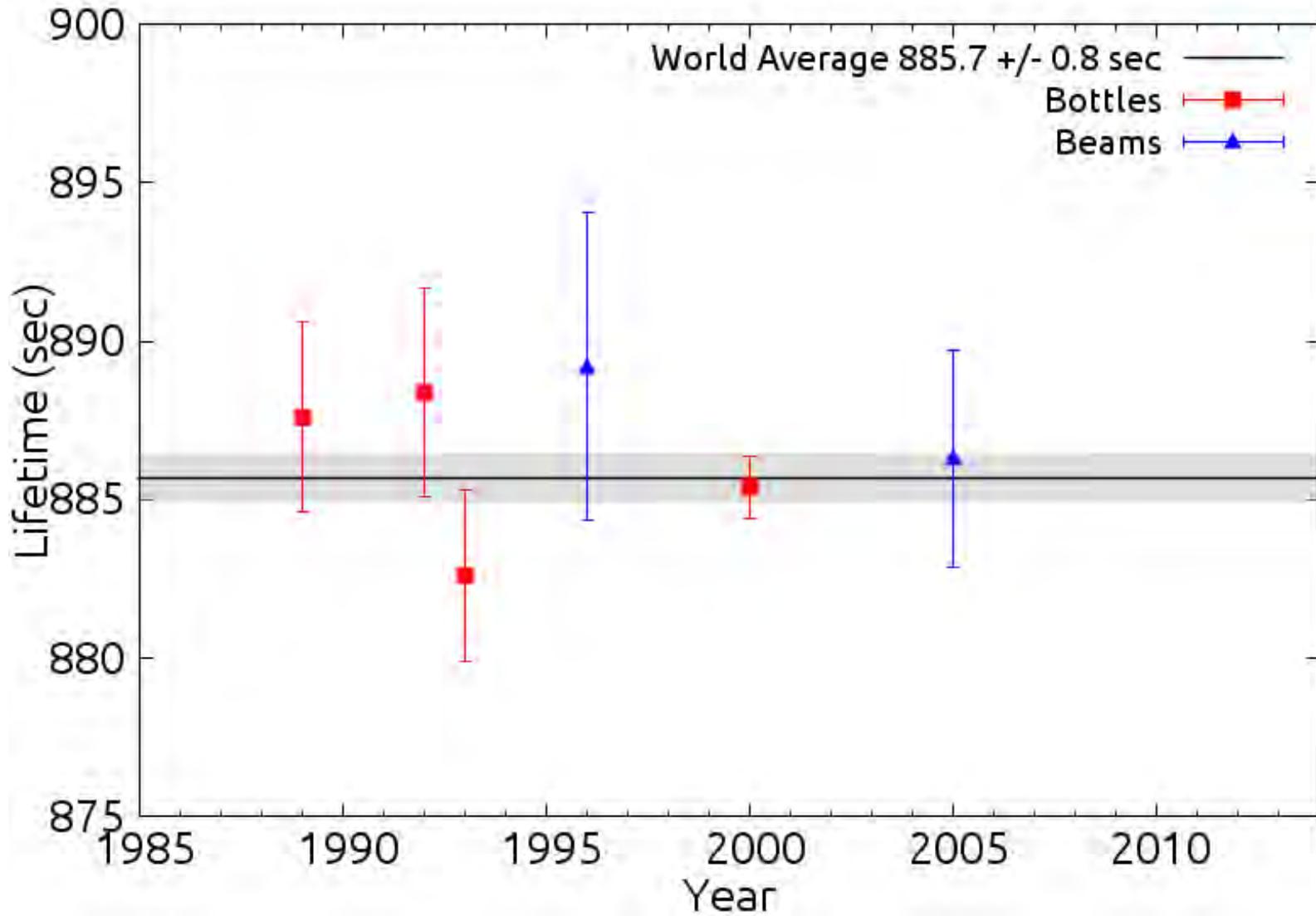




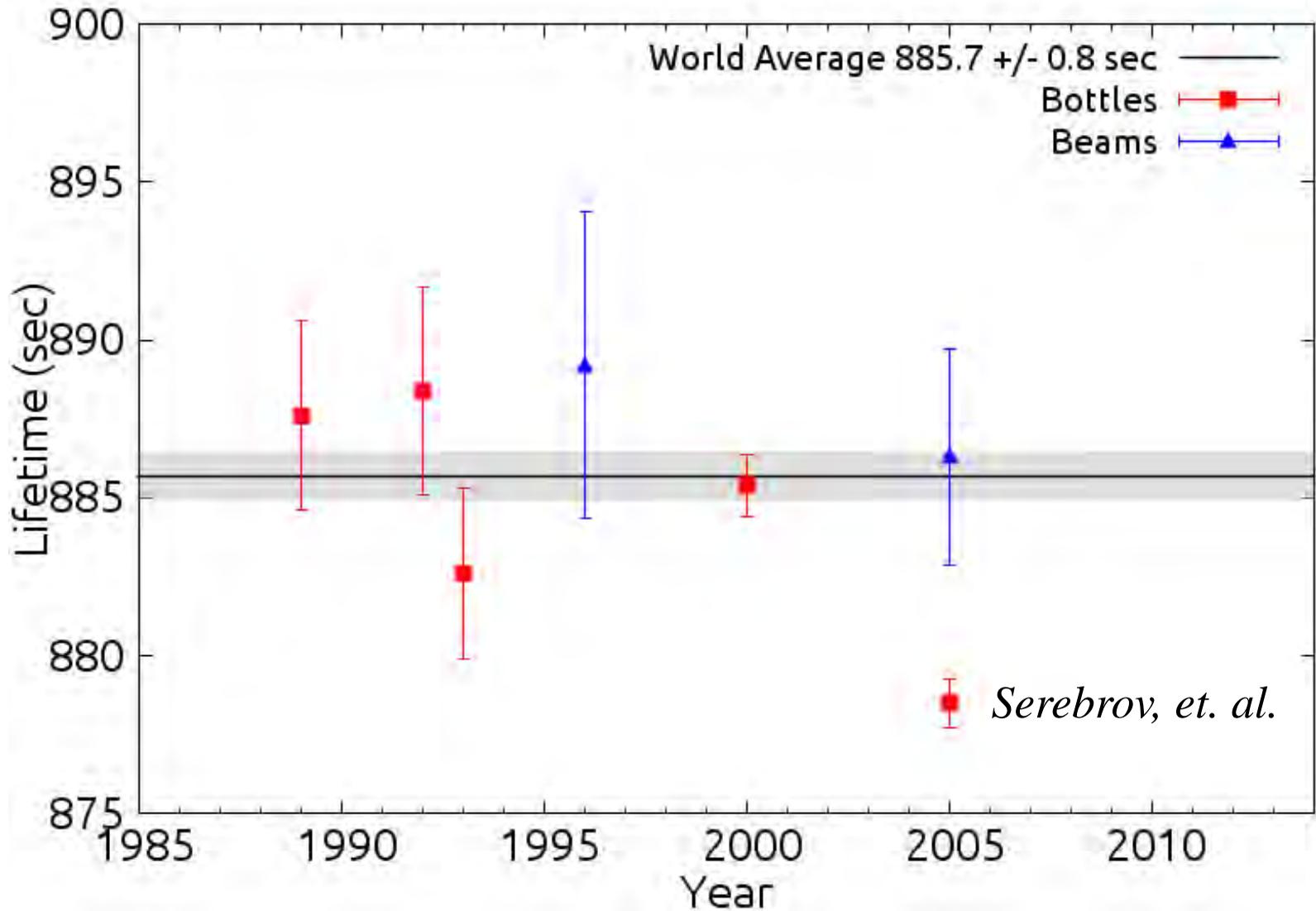
The Neutron Lifetime “Problem”

- A short history -

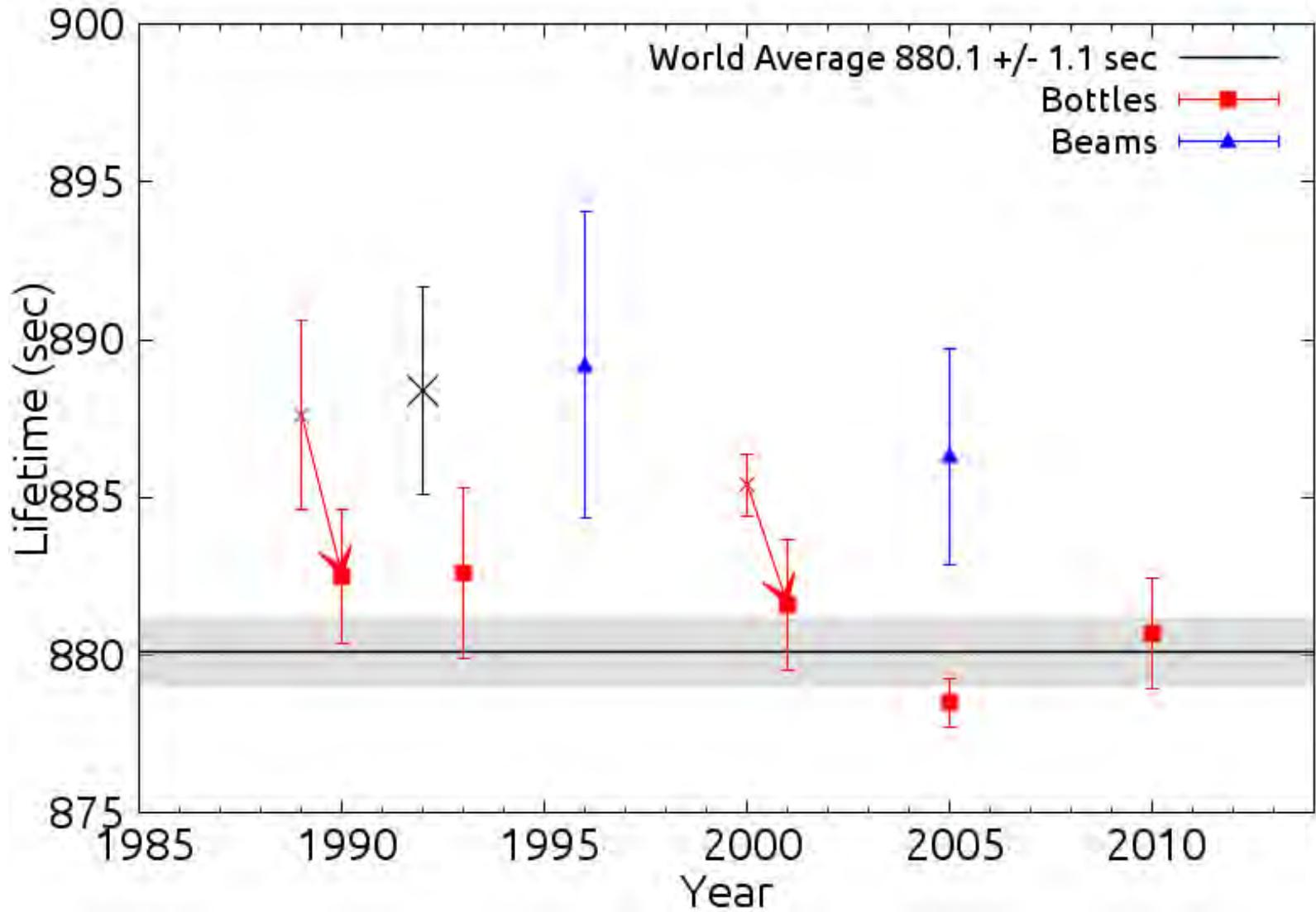
Neutron Lifetime Measurements in 2005



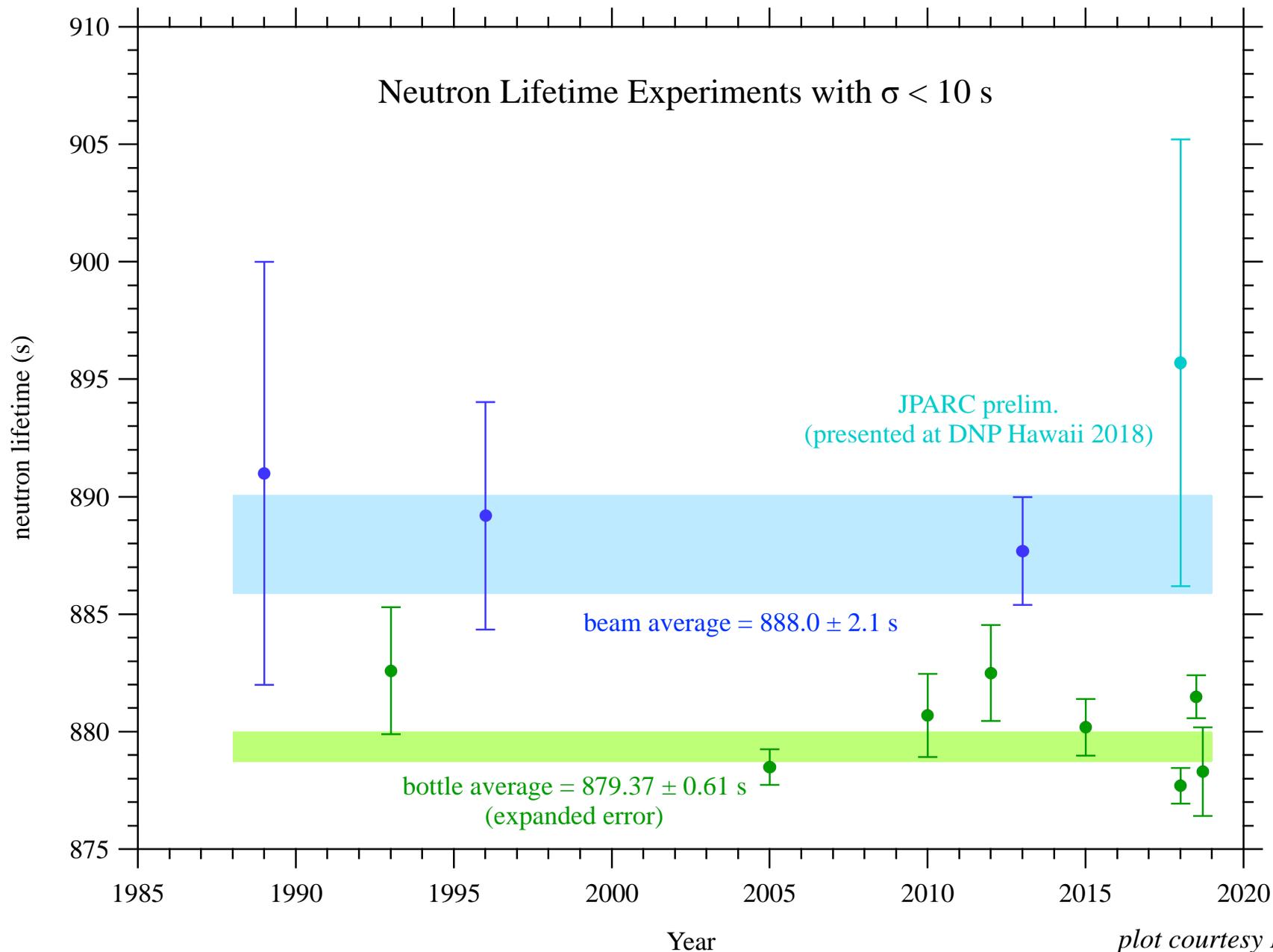
A New Bottle Result



And Then...

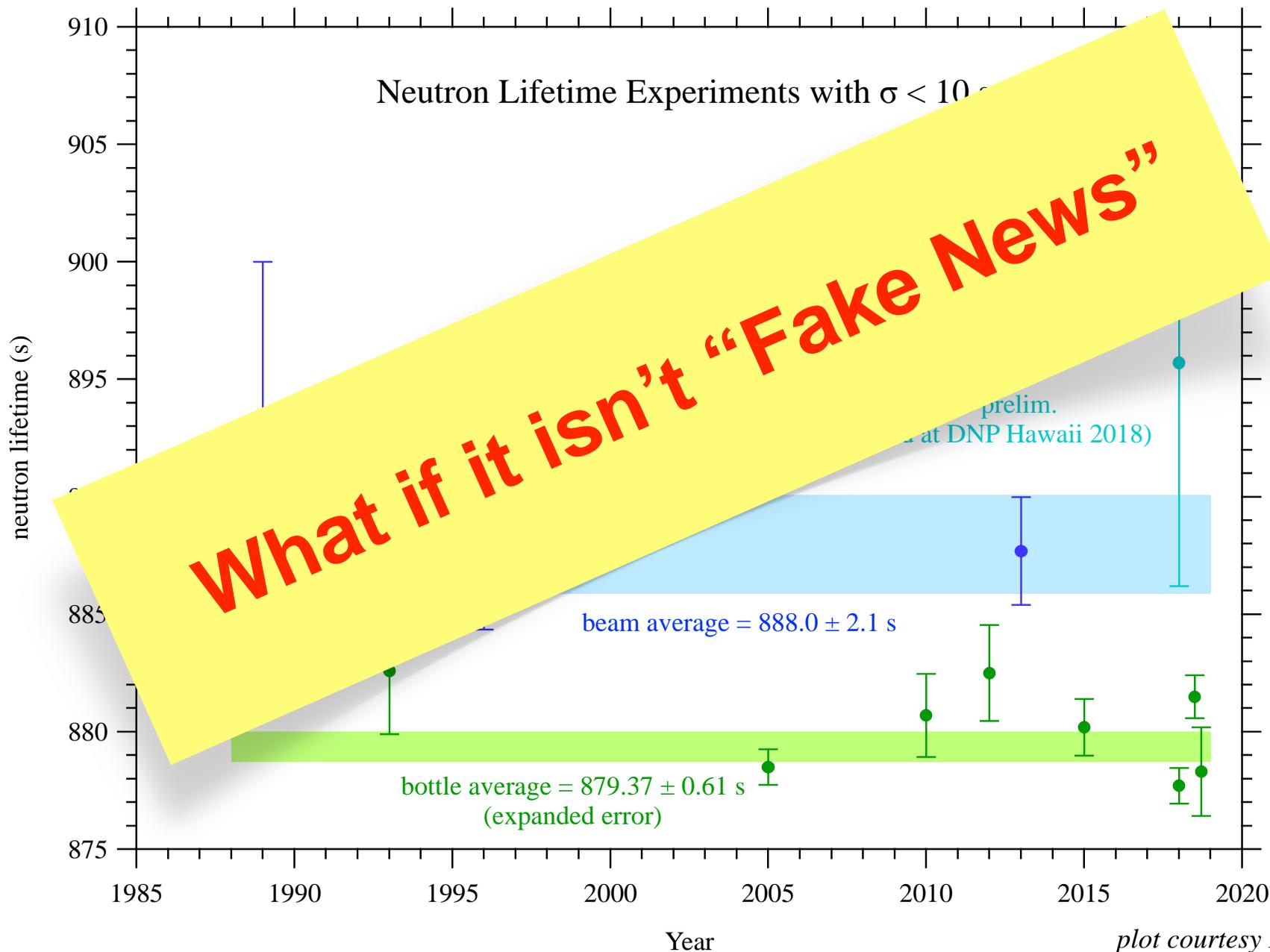


The Situation Today - 2019



plot courtesy F. Weidtfeldt

The Situation Today - 2019



NEW PHYSICS?

Suppose a neutron also decayed via a different channel,



Measurements would give:

$$\tau_{\text{BOTTLE}} < \tau_{\text{BEAM}}$$

Dark Matter Interpretation of the Neutron Decay Anomaly

Bartosz Fornal and Benjamín Grinstein

Department of Physics, University of California, San Diego, 9500 Gilman Drive, La Jolla, California 92093, USA



(Received 19 January 2018; revised manuscript received 3 March 2018; published 9 May 2018)

neutron \longrightarrow **dark particle + photon**

neutron \longrightarrow **dark particle + e^+e^-**

neutron \longrightarrow **two dark particles**

neutron \longrightarrow **...**

Some Recent Related Work

Pfutzner & Riisager, PRC 97, 042501(R) (2018)

McKeen, et. al, PRL 121, 061802 (2018),

Baym, et. al., PRL 121, 061801 (2018),

Motta, Guichon & Thomas, J. Phys. G 45, 05LT01 (2018)

Bringmann, Cline & Cornell, arXiv:1810.08215

Karananas & Kassiteridis, JCAP 09, 036 (2018)

Grinstein, Kouvaris & Nielsen, arXiv:1811.06546

Barducci, Fabbrichesi & Gabrielli, PRD 98, 035049

Berezhiani, arXiv:1807.07906

Berezhiani, LHEP 118, 1 (2019)

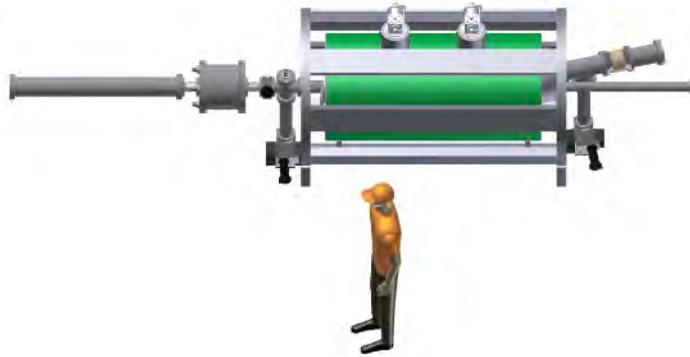
Foral & Grinstein, arXiv:1812.11089 (2018)

Tang et al., PRL 121, 022505 (2018)

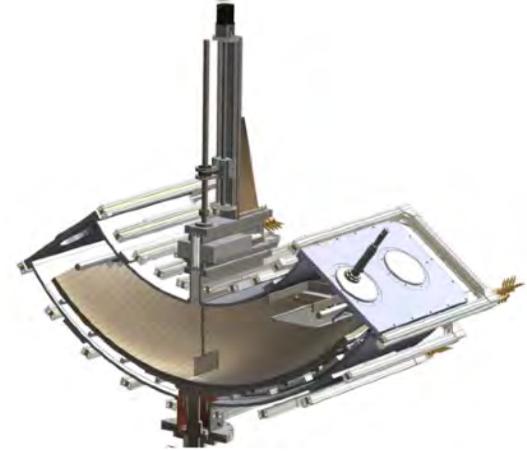
Sun et al., PRC 97, 052501 (2018)

...

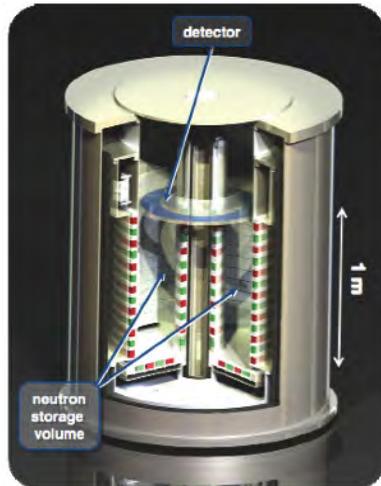
Several New Lifetime Experiments Worldwide



Beam Lifetime 3, NIST
Beam



UCN τ , Los Alamos
Magnetic Bottle



PENELOPE, Munich
Magnetic Bottle



"Big" Gravitational Trap, St. Petersburg
Material Bottle



HOPE, Grenoble
Magnetic Bottle

The neutron lifetime

Fred E. Wietfeldt*

Department of Physics, Tulane University, New Orleans, Louisiana 70118, USA

Geoffrey L. Greene[†]

*Department of Physics, University of Tennessee, Knoxville, Tennessee 37996
and Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA*

The decay of the free neutron into a proton, electron, and antineutrino is the prototype semileptonic weak decay and is the simplest example of nuclear beta decay. It played a key role in the early Universe as it determined the ratio of neutrons to protons during the era of primordial light element nucleosynthesis. Neutron decay is physically related to important processes in solar physics and neutrino detection. The mean neutron lifetime has been the subject of more than 20 major experiments done, using a variety of methods, between 1950 and the present. The most precise recent measurements have stated accuracies approaching 0.1%, but are not in good agreement as they differ by as much as 5σ using quoted uncertainties. The history of neutron lifetime measurements is reviewed and the different methods used are described, giving important examples of each. The discrepancies and some systematic issues in the experiments that may be responsible are discussed, and it is shown by means of global averages that the neutron lifetime is likely to lie in the range of 880–884 s. Plans and prospects for future experiments are considered that will address these systematic issues and improve our knowledge of the neutron lifetime.

*Measuring Decay Correlations
in Neutron Decay*

Correlations in Neutron Decay

Pure V-A, low energy limit

$$dW \propto \frac{1}{\tau_n} F(E_e) \left[1 + a \frac{\mathbf{p}_e \cdot \mathbf{p}_\nu}{E_e \cdot E_\nu} + b \frac{m_e}{E_e} + A \frac{\boldsymbol{\sigma}_n \cdot \mathbf{p}_e}{E_e} + B \frac{\boldsymbol{\sigma}_n \cdot \mathbf{p}_\nu}{E_\nu} + \dots \right]$$

$\frac{1}{\tau_n} \rightarrow \frac{1}{G_V^2 + 3G_A^2}$
 $a \rightarrow \frac{1 - \left(\frac{G_A}{G_V}\right)^2}{1 - 3\left(\frac{G_A}{G_V}\right)^2}$
 $b = 0$
 $A \rightarrow \frac{-2\left(\frac{G_A}{G_V}\right)^2 + \left(\frac{G_A}{G_V}\right)}{1 - 3\left(\frac{G_A}{G_V}\right)^2}$
 $B \rightarrow \frac{-2\left(\frac{G_A}{G_V}\right)^2 - \left(\frac{G_A}{G_V}\right)}{1 - 3\left(\frac{G_A}{G_V}\right)^2}$

Neutron beta decay measurements give:

$$\frac{G_A^2 + 3G_V^2}{G_A / G_V}$$

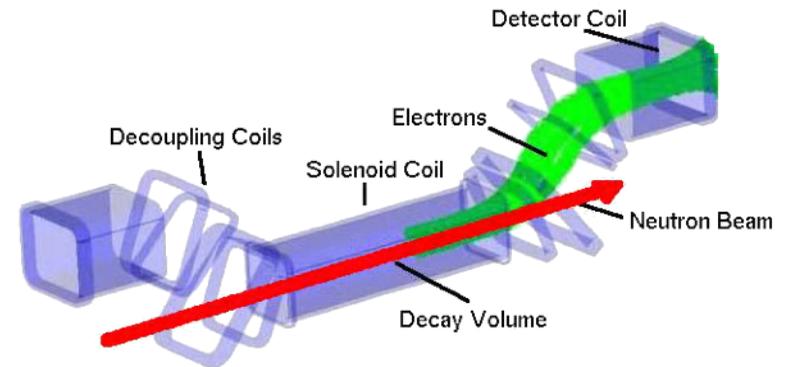
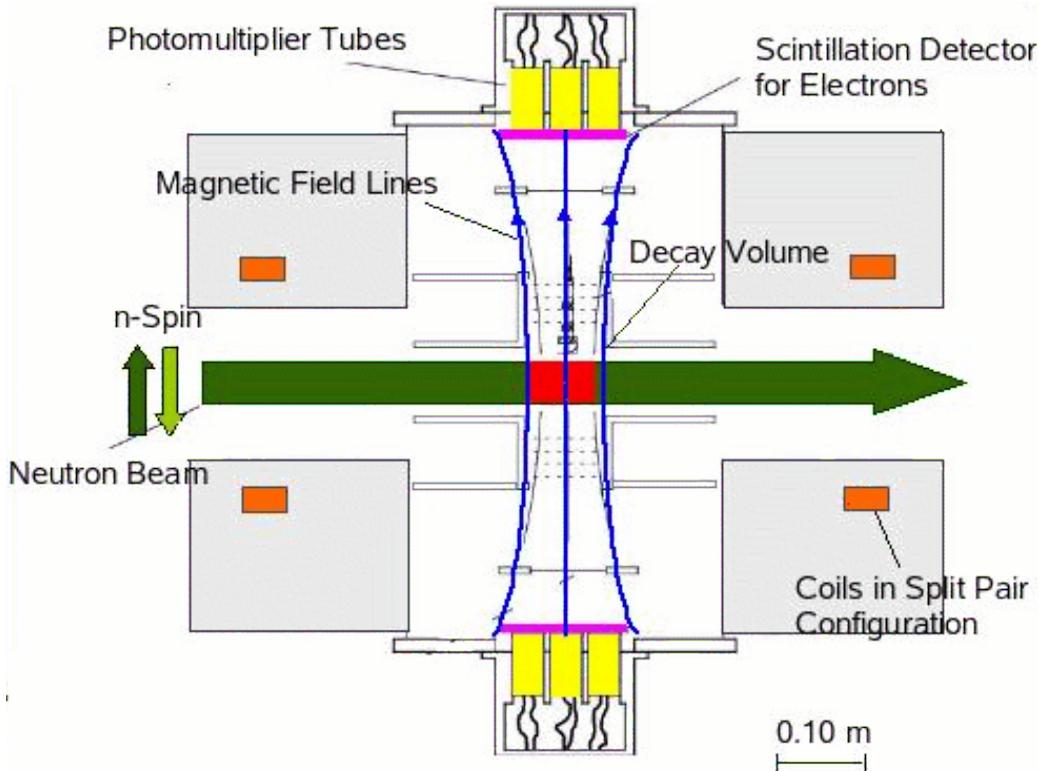
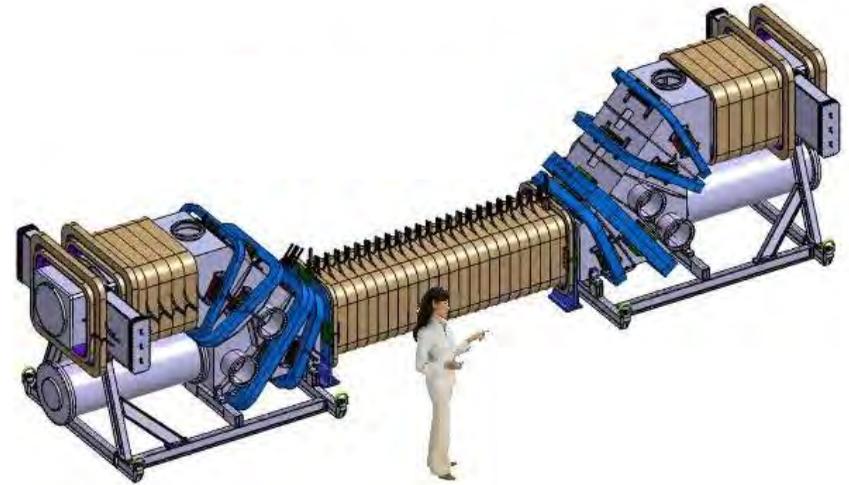
(Jackson, Treiman, Wyld, 1957)

*Some selected recent and ongoing
neutron decay experiments*

PERKEO at ILL, Grenoble

**Polarized neutrons used
to determine “A”**

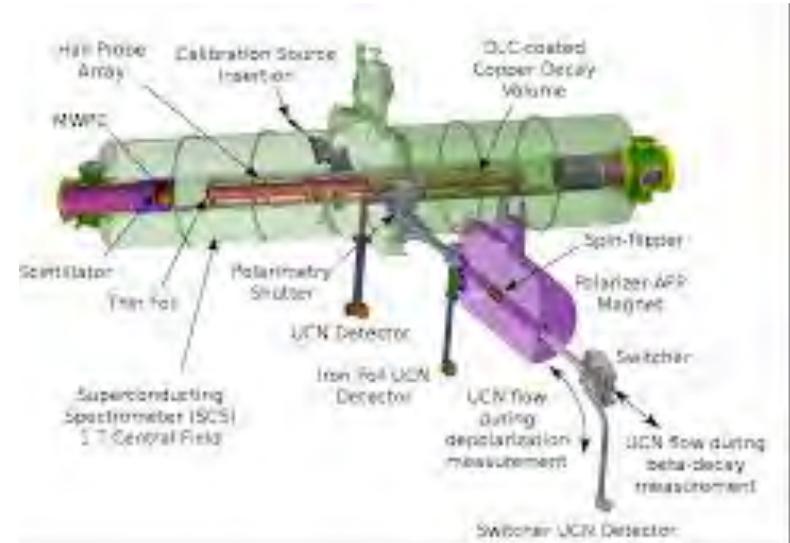
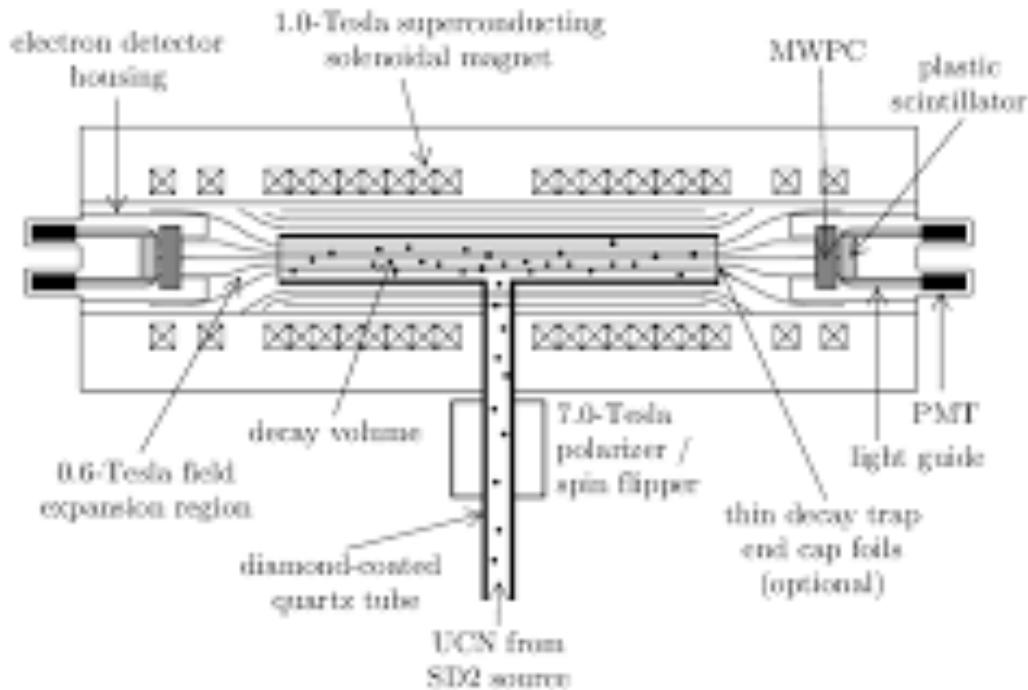
$$A\vec{\sigma}_n \cdot \vec{p}_e$$



UCNA at Los Alamos

**Polarized neutrons used
to determine “A”**

$$A\vec{\sigma}_n \cdot \vec{p}_e$$

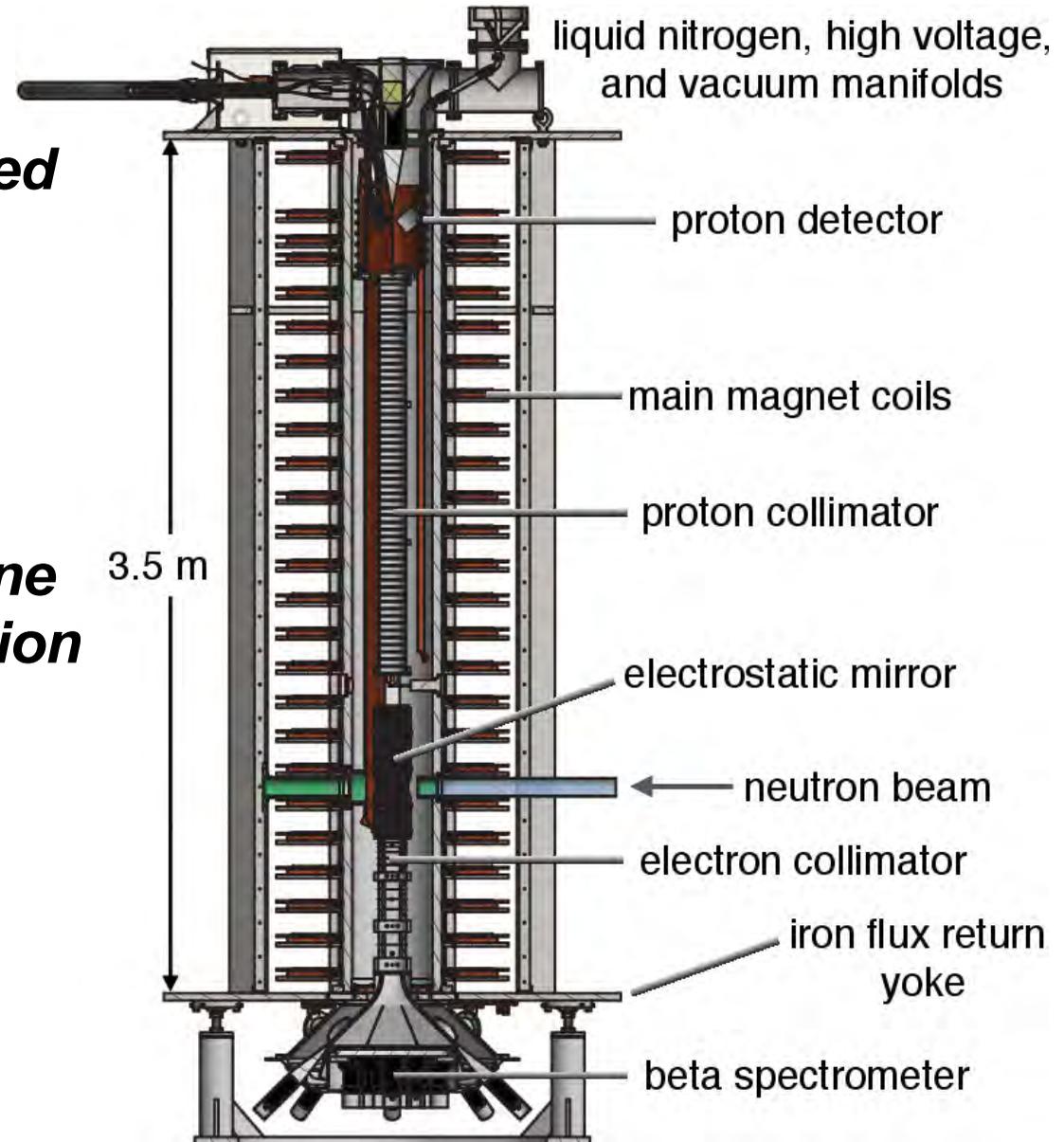


aCORN at NIST

**Unpolarized neutrons used
to determine “a”**

$$a \vec{p}_{\bar{\nu}} \cdot \vec{p}_e$$

**Cannot determine $\vec{p}_{\bar{\nu}}$ so one
deduces this from information
about \vec{p}_e and \vec{p}_p**

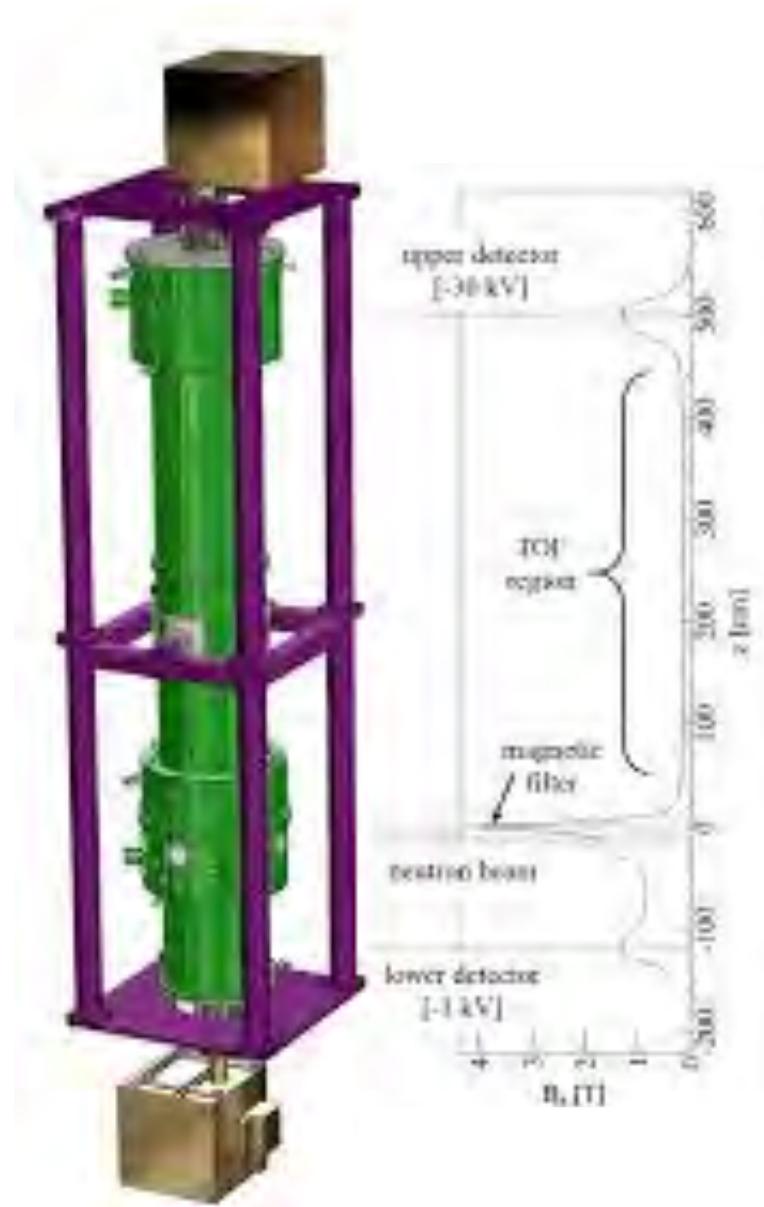


Nab at the Spallation Neutron Source

**Unpolarized neutrons used
to determine “a”**

$$a \vec{p}_{\bar{\nu}} \cdot \vec{p}_e$$

**Cannot determine $\vec{p}_{\bar{\nu}}$ so one
deduces this from information
about \vec{p}_e and \vec{p}_p**



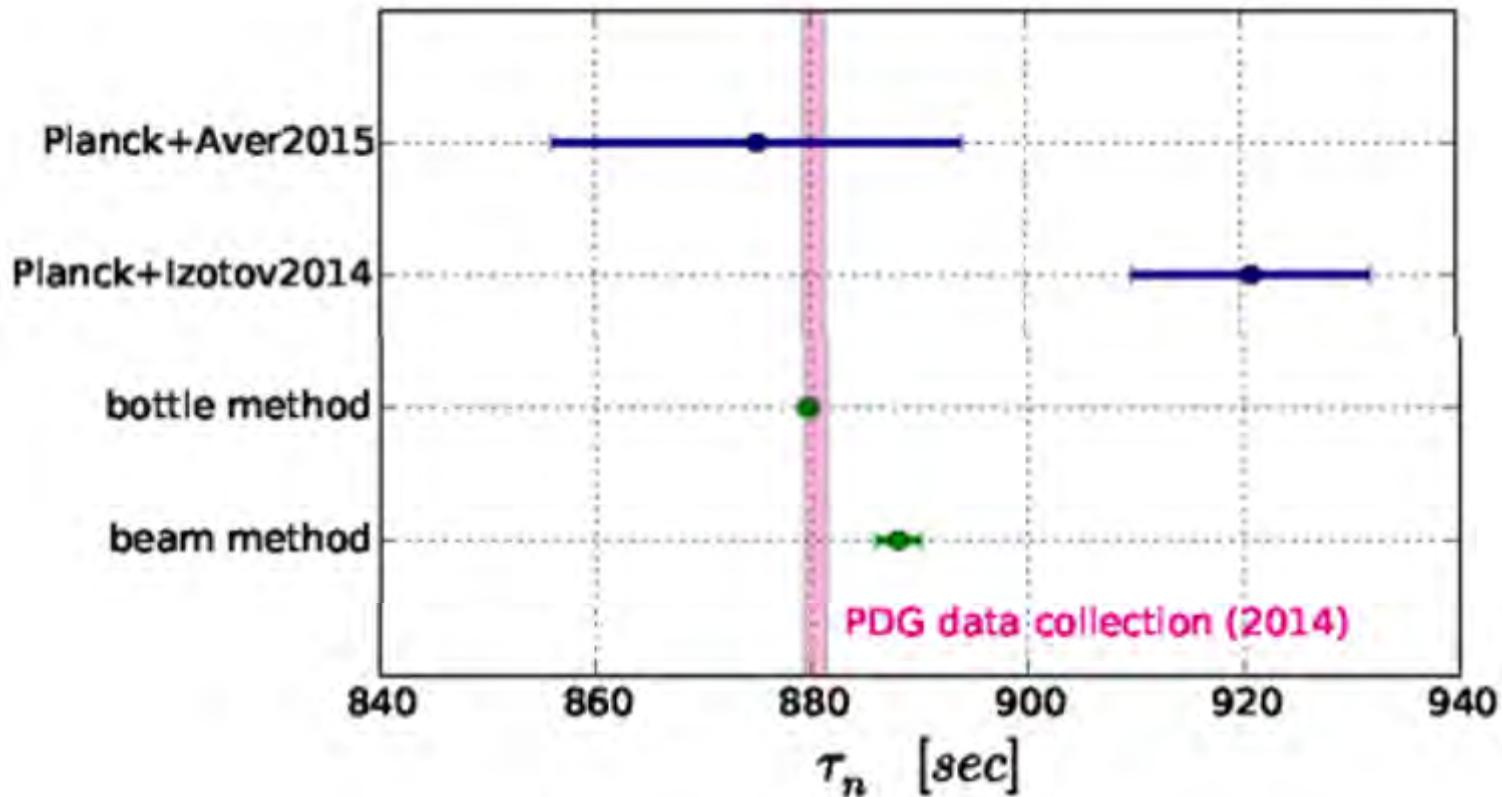
End of Presentation



Cosmological constraints on the neutron lifetime

Laura Salvati, Luca Pagano, Rosa Consiglio, Alessandro Melchiorri

(Submitted on 26 Jul 2015 (v1), last revised 29 Mar 2016 (this version, v3))





The Neutron Lifetime and Axial Coupling Connection

Andrzej Czarnecki, William J. Marciano, Alberto Sirlin

(Submitted on 6 Feb 2018)

$$\tau_n^{favored} = 879.4 \pm 0.6s$$

$$\tau_n^{bottle} = 879.3 \pm 0.75s$$

$$\tau_n^{beam} = 888.0 \pm 2.1s$$

