Introduction to Relativistic Heavy Ion Physics

Lecture 3:

1. Viscosity 101
2. Hydro 101
3. AdS/CFT 101

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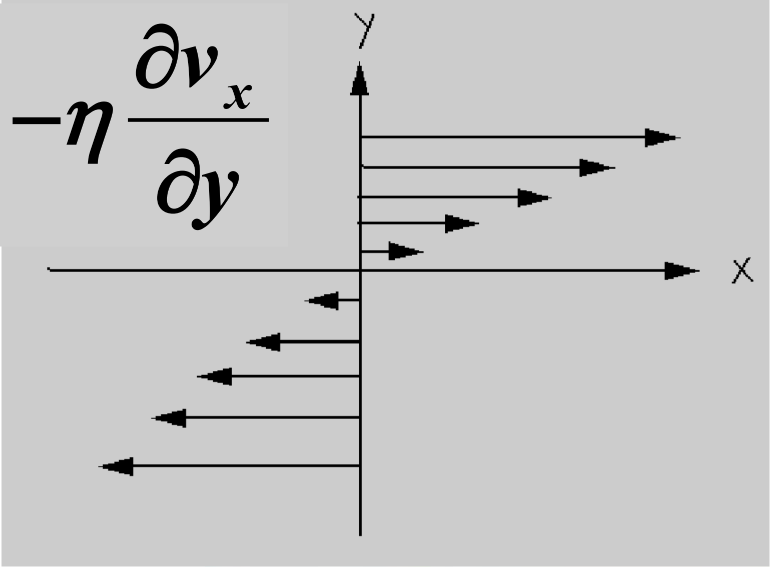
Perfection ←→ (No) Viscosity

- Isotropic in rest frame

➔ No shear stress

➔ no viscosity, $\eta = 0$

$$\frac{F_x}{A} = -\eta \frac{\partial v_x}{\partial y}$$



- Primer:
  - Remove your organic prejudices
  - Viscosity ~ mean free path

$$\eta \sim n\, \bar{p}\, \lambda_{mfp}$$

Exercise 1: Check that this has correct dimensions.

  - *Small* viscosity ➔ *Small* λ_{mfp}
  - *Zero* viscosity ➔ $\lambda_{mfp} = 0$ (!)

RHIC Scientists Serve Up ‘Perfect’ Liquid

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Contacts: Karen McNulty Walsh, (631) 344-8350 or Peter Genzer, (631) 344-3174

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New state of matter more remarkable than predicted — raising many new questions

Monday, April 18, 2005

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"This is fluid motion that is nearly 'perfect,'" Aronson said, meaning it can be explained by equations of hydrodynamics. These equations were developed to describe theoretically "perfect" fluids — those with extremely low viscosity and the ability to reach thermal equilibrium very rapidly due to the high degree of interaction among the particles. While RHIC scientists don't have a direct measure of viscosity, they can infer from the flow pattern that, qualitatively, the viscosity is very low, approaching the quantum mechanical limit.

What Is The Viscosity at RHIC?

- "Perfect fluid" (and/or "ideal hydrodynamics")

  ~ defined as "zero viscosity".

$$\left.\begin{array}{l}\eta_{QGP} \sim 2 \times 10^{11}\ \mathrm{Pa \cdot s} \\ \eta_{H_2O} \sim 1 \times 10^{-3}\ \mathrm{Pa \cdot s}\end{array}\right\} \Rightarrow \frac{\eta_{QGP}}{\eta_{H_2O}} \sim 2 \times 10^{14}$$

$$\eta_{Pitch} \sim 2.3 \times 10^{8}\ \mathrm{Pa \cdot s} \qquad \eta_{Glass(A.P.)} \sim 10^{12}\ \mathrm{Pa \cdot s}$$

?

Numerical Value of RHIC Viscosity

- **Input :** $\eta_{QGP} \sim \frac{\hbar}{4\pi} s_{QGP} \qquad T_{QGP} \sim 200 \text{ MeV} \qquad \hbar c = 200 \quad \text{Mev} \cdot \text{fm}$

- **Estimating s_{QGP} (per degree of freedom):**
  - **Method 1:**

$$s_{QGP} = (\varepsilon + p)/T_{QGP} \sim \frac{4}{3}\frac{\varepsilon}{T_{QGP}} = \frac{4}{3T_{QGP}}\left(\frac{\pi^2}{30}T_{QGP}{}^4\right) = \frac{2\pi^2}{45}T_{QGP}{}^3$$

  - **Method 2 (based on handy rule of thumb):**

$$n \sim \left(\frac{T}{2}\right)^3 \qquad s \sim 3.6n \Rightarrow s \sim \frac{3.6}{2^3}T^3 = 1.03\left(\frac{2\pi^2}{45}T^3\right)$$

- **Number of (assumed massless) degrees of freedom:**

$$n_{d.o.f.} \sim \left\{2_s \cdot 8_g + \frac{7}{8}\cdot 2_s \cdot 2_{a.p.} \cdot 3_c \cdot 3_f\right\} = 47.5 \sim 45$$

$$\Rightarrow \eta_{QGP} \sim \frac{\hbar}{4\pi}s_{QGP} \sim \frac{\hbar}{4\pi}n_{d.o.f.}\left(\frac{2\pi^2}{45}T^3{}_{QGP}\right) \sim \frac{\pi}{2}\frac{\hbar c}{c}T^3{}_{QGP}$$

$$= \frac{\pi \cdot (2\cdot 10^8 \text{ eV} \cdot 10^{-15} \text{ m}) \times (1.6 \cdot 10^{-19} \text{ J/eV})}{2(3\cdot 10^8 \text{ m/s})}(10^{15} \text{ m})^3 = 1.6 \times 10^{11} \frac{\text{J}\cdot\text{s}}{\text{m}^3}$$

A Check Of This Strange Conclusion

- Recall $\eta \sim n\,\bar{p}\,\lambda_{mfp}$
- But $$\lambda_{mfp} = \frac{1}{n\,\sigma} \Rightarrow \eta \sim \frac{\bar{p}}{\sigma}$$
  - Very Important Point !!
  - To get *small* viscosity you need *LARGE* σ
- Using above
- $$\frac{\eta_{QGP}}{\eta_{H_20}} \sim \frac{\bar{p}_{QGP}}{\sigma_{QGP}}\frac{\sigma_{H_20}}{\bar{p}_{H_20}} \sim \frac{T_{QGP}}{\pi\left(\frac{1}{T_{QGP}}\right)^2}\frac{\pi\,a^2{}_{H_20}}{\sqrt{2\pi\,mkT_{H_20}}} \sim 10^{14}$$

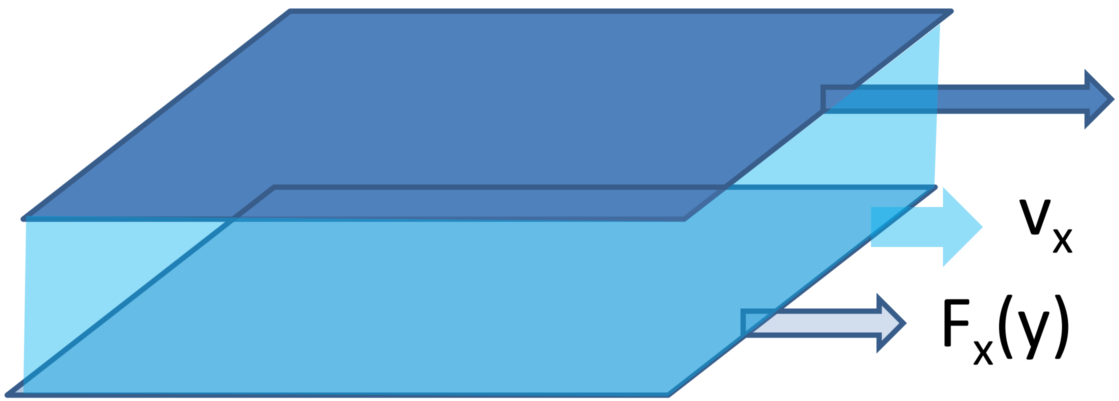
Exercise 2: Check the above value, putting in plausible estimates for the various parameters.

Kinematic Viscosity

- In Reynolds Number: $\mathrm{Re} \equiv \frac{\rho V L}{\eta}$

- Determines relaxation rate

Exercise 3: At what velocity does it feel as if you are 'swimming' in water? (Save yourself some work- kinematic velocities are tabulated.)



$$\frac{\eta}{\rho}\frac{\partial^2 v_x}{\partial y^2} = \frac{\partial v_x}{\partial t}$$

Exercise 4: a) Use F=ma and the definition of viscosity to show that the relaxation of the velocity field $v_x(y)$ follows the diffusion equation.

b) verify solution to same

$$\Rightarrow v_x(y,t) \sim \frac{1}{\sqrt{4\frac{\eta}{\rho}t}} e^{-y^2/(4\frac{\eta}{\rho}t)}$$

Why η/s Matters

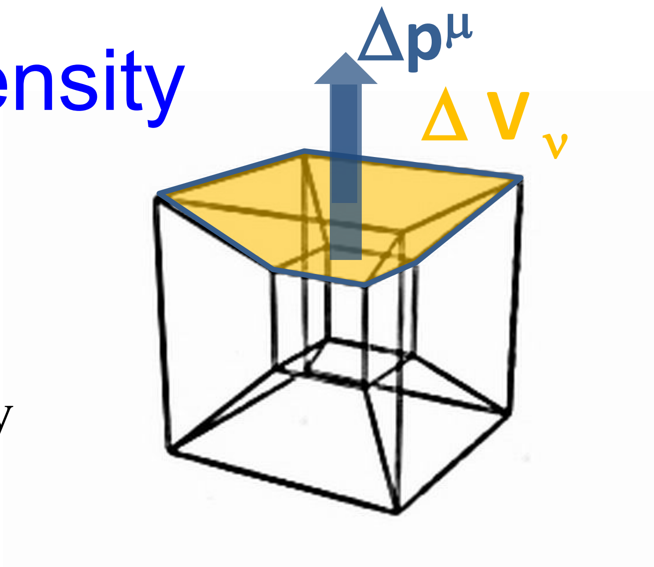
- Any engineer will tell you
  - *Kinematic* viscosity η / ρ ~ [Velocity] x [Length] is what matters (see Landau's remark on Reynolds number)
- Any relativist will tell you
  - $\rho \rightarrow \varepsilon + p$
- Any thermodynamicist will tell you
  - $\varepsilon + p = T\,s$ (at $\mu_B = 0$)
- So
  - $\eta/\rho \rightarrow \eta/(\varepsilon + p) \rightarrow (\eta/sT) = (\eta/s)(1/T)$

    ~ (damping coefficient x thermal time)

Exercise 5: a) Use this and previous statistical mechanics results for massless quanta to find an analytic result for entropy density s.
b) Show that s = 3.6 n
c) Instead of statistical expression for n, *define* n via P = (N/V) T = n T. Show that with this definition s = 4 n for massless quanta. Comment.

Stress-Energy Tensor Reminder

- $T^{\mu\nu}$ $\equiv$ μ-th component of energy-momentum density in ν-th "direction"



- Examples:
  - $T^{00} = \dfrac{\Delta p^0}{(\Delta V)_0} = \dfrac{\Delta E}{\Delta x_1 \Delta x_2 \Delta x_3} = \text{Energy Density}$
  - $T^{11} = \dfrac{\Delta p^1}{(\Delta V)_1} = \dfrac{\Delta p^1}{\Delta x_0 \Delta x_2 \Delta x_3} = \dfrac{\Delta p^1 / \Delta t}{\Delta x_2 \Delta x_3} = \dfrac{F_1}{A_\perp} = \text{Pressure (in "1" direction)}$
  - T^{12} = $(\text{Force})_1$ per unit area in 2 direction $\equiv$ Shear stress

- Energy-momentum conservation: $\partial_\mu T^{\mu\nu} = 0$

Perfect Fluids

- Defined as
  - Isotropic in fluid rest frame
  - Incapable of supporting a shear stress
- So
  - $T^{00} = \varepsilon(x)$ = energy density,
  - $T^{ij} = p(x)\,\delta^{ij}$, p = pressure

$$T^{\mu\nu}(x) = \begin{pmatrix} \epsilon(x) & & & \\ & p(x) & & \\ & & p(x) & \\ & & & p(x) \end{pmatrix}$$

in the fluid rest frame .

- Q. How to write as proper Lorentz tensor ? :
- A. Use fluid four-velocity u^μ to express as

$$T^{\mu\nu} = (\varepsilon+p)u^\mu u^\nu - p\, g^{\mu\nu}$$

Exercise 6: Check this.

Ideal Hydrodynamics

- That is, the hydrodynamics of a perfect fluid:

$$\mathrm{T}^{\mu\nu} = (\varepsilon + P)u^{\mu}u^{\nu} - g^{\mu\nu}P \quad ; \quad \partial_\mu \mathrm{T}^{\mu\nu} = 0$$

$$\mathrm{j}_B{}^{\mu} = n_B u^{\mu} \quad ; \quad \partial_\mu \mathrm{j}_B{}^{\mu} = 0$$

- *Not* enough to solve:
  - ☞ Still need "equation of state"
    - ▸ (could be as simple as P = ε / 3)
  - ❑ Even with E.O.S., still hard without further simplifying assumptions:
    - ♦ Examples:
      - ○ Expansion in 1D only (Landau, Bjorken)
      - ○ 'Symmetric' (2,3)D expansion (Gubser, Csorgo)

Exercise 7: Verify these statements.

Innocent Question

- Why ideal hydrodynamics ?
- (The fluid version of the frictionless plane)

- Answers:
  - It works
  - Non-ideal *very* hard to do relativistically
  - But for *relativistic* fluids, argument from Landau justifying ideal hydrodynamics (to follow)

Landau on Viscosity

1) Use of hydro relies on $R/\lambda >> 1$

2) Negligible viscosity η equivalent to large Reynolds number $\mathcal{Re} \equiv \rho V R / \eta >> 1$

$$\rho V R / \eta \sim V R / v_{th} \lambda$$

but for a *relativistic* system

$$V \sim v_{th}$$

so

$$\mathcal{Re} >> 1 \Rightarrow R / \lambda >> 1 \text{ ; see \#1}$$

and statistical equilibrium is set up.

(2) The second stage of the collision consists in the expansion of the system. Here the hydrodynamic approach must be used, and the expansion may be regarded as the motion of an ideal fluid (zero viscosity and zero thermal con-

† The conditions of applicability of thermodynamics and hydrodynamics are comprised in the requirement $l/L \ll 1$, where l is the "mean free path" and L the least dimension of the system.

† This may be made clear by the following qualitative arguments. If viscosity and thermal conductivity are to be negligible, the Reynolds number LV/lv must be much greater than unity. Here L is the least dimension of the system, V the "macroscopic" velocity, v the "molecular" velocity and l the mean free path. Since V and v are of the order of c, the condition $R \gg 1$ corresponds to $l/L \ll 1$.

ajc

Why (Relativistic) Hydrodynamics is Hard

This innocuous looking equation $\partial_\mu T^{\mu\nu} = 0$

hides a world of non-linear hurt

(even for ideal fluid):

$$\begin{aligned}
\partial_\mu T^{\mu\nu} &= \partial_\mu \left[(\epsilon + p) u^\mu u^\nu - p g^{\mu\nu} \right] = 0 \\
&= \frac{\partial}{\partial x^0} \left[(\epsilon(x) + p(x)) u^\mu(x) u^\nu(x) - p(x) g^{\mu\nu} \right] \\
&+ \frac{\partial}{\partial x^1} \left[(\epsilon(x) + p(x)) u^\mu(x) u^\nu(x) - p(x) g^{\mu\nu} \right] + \cdots
\end{aligned}$$

with $u^\mu(x) \equiv \dfrac{1}{\sqrt{1 - |\vec{v}(x)|^2}} \left(1 \; , \; \vec{v}(x) \right)$

$\Rightarrow$ Must make clever use of symmetries, projectors, etc.

Viscous Relativistic Hydrodynamics

- Why not do a 'real' (that is, Navier-Stokes) hydrodynamic calculation at RHIC?
  - Incorporate non-zero viscosity
  - 'Invert' to determine allowed range for η / s.
- Two little problems:
  - It's wrong
    - Solutions are acausal
    - Needed patch:

$$v_x(y,t) \sim \frac{1}{\sqrt{4\frac{\eta}{\rho}t}} e^{-y^2/(4\frac{\eta}{\rho}t)}$$

$$\frac{\eta}{\rho}\frac{\partial^2 v_x}{\partial y^2} = \frac{\partial v_x}{\partial t} \rightarrow \frac{\eta}{\rho}\frac{\partial^2 v_x}{\partial y^2} = \frac{\partial v_x}{\partial t} + \tau_R \frac{\partial^2 v_x}{\partial t^2}$$

  - It's wrong
    - Solutions are intrinsically unstable
    - No patch, must take *all terms* to 2nd order in gradients

A Partial List of “All Terms”

- Relativistic, Causal, second-order expansion:
  - [Relativistic Fluid Dynamics: Physics for Many Different Scales]
- Neglect various terms at your own risk:
  - Baier et al., [Relativistic viscous hydrodynamics, conformal invariance, and holography]
  - Natsuume and Okamura, [Comment on “Viscous hydrodynamics relaxation time from AdS/CFT correspondence”]

Working out the divergence of the entropy current, and making use of the equations of motion, we arrive at

$$\begin{aligned}\nabla_\mu s^\mu = &-\frac{1}{T}\tau\left[\nabla_\mu u^\mu + \beta_0 u^\mu \nabla_\mu \tau - \alpha_0 \nabla_\mu q^\mu - \gamma_0 T q^\mu \nabla_\mu\left(\frac{\alpha_0}{T}\right) + \frac{\tau T}{2}\nabla_\mu\left(\frac{\beta_0 u^\mu}{T}\right)\right]\\ &-\frac{1}{T}q^\mu\left[\frac{1}{T}\nabla_\mu T + u^\nu \nabla_\nu u_\mu + \beta_1 u^\nu \nabla_\nu q_\mu - \alpha_0 \nabla_\mu \tau - \alpha_1 \nabla_\nu \tau^\nu{}_\mu\right.\\ &\qquad\left.+\frac{T}{2}q_\mu \nabla_\nu\left(\frac{\beta_1 u^\nu}{T}\right) - (1-\gamma_0)\tau T \nabla_\mu\left(\frac{\alpha_0}{T}\right) - (1-\gamma_1) T \tau^\nu{}_\mu \nabla_\nu\left(\frac{\alpha_1}{T}\right)\right]\\ &-\frac{1}{T}\tau^{\mu\nu}\left[\nabla_\mu u_\nu + \beta_2 u^\alpha \nabla_\alpha \tau_{\mu\nu} - \alpha_1 \nabla_\mu q_\nu + \frac{T}{2}\tau_{\mu\nu}\nabla_\alpha\left(\frac{\beta_2 u^\alpha}{T}\right) - \gamma_1 T q_\mu \nabla_\nu\left(\frac{\alpha_1}{T}\right)\right].\end{aligned} \tag{300}$$

In this expression it should be noted that we have introduced (following Lindblom and Hiscock) two further parameters, γ_0 and γ_1. They are needed because without additional assumptions it is not clear how the “mixed” quadratic term should be distributed. A natural way to fix these parameters is to appeal to the Onsager symmetry principle [58], which leads to the mixed terms being distributed “equally” and hence $\gamma_0 = \gamma_1 = 1/2$.

Denoting the comoving derivative by a dot, i.e. using $u^\mu \nabla_\mu \tau = \dot\tau$ etc. we see that the second law of thermodynamics is satisfied if we choose

$$\tau = -\zeta\left[\nabla_\mu u^\mu + \beta_0 \dot\tau - \alpha_0 \nabla_\mu q^\mu - \gamma_0 T q^\mu \nabla_\mu\left(\frac{\alpha_0}{T}\right) + \frac{\tau T}{2}\nabla_\mu\left(\frac{\beta_0 u^\mu}{T}\right)\right], \tag{301}$$

$$\begin{aligned}q^\mu = -\kappa T \perp^{\mu\nu} &\left[\frac{1}{T}\nabla_\nu T + \dot u_\nu + \beta_1 \dot q_\nu - \alpha_0 \nabla_\nu \tau - \alpha_1 \nabla_\alpha \tau^\alpha{}_\nu + \frac{T}{2}q_\nu \nabla_\alpha\left(\frac{\beta_1 u^\alpha}{T}\right)\right.\\ &\left.-(1-\gamma_0)\tau T \nabla_\nu\left(\frac{\alpha_0}{T}\right) - (1-\gamma_1) T \tau^\alpha{}_\nu \nabla_\alpha\left(\frac{\alpha_1}{T}\right) + \gamma_2 \nabla_{[\nu} u_{\alpha]} q^\alpha\right],\end{aligned} \tag{302}$$

$$\tau_{\mu\nu} = -2\eta\left[\beta_2 \dot\tau_{\mu\nu} + \frac{T}{2}\tau_{\mu\nu}\nabla_\alpha\left(\frac{\beta_2 u^\alpha}{T}\right) + \left\langle \nabla_\mu u_\nu - \alpha_1 \nabla_\mu q_\nu - \gamma_1 T q_\mu \nabla_\nu\left(\frac{\alpha_1}{T}\right) + \gamma_3 \nabla_{[\mu} u_{\alpha]} \tau_\nu{}^\alpha \right\rangle\right], \tag{303}$$

where the angular brackets denote symmetrization as before. In these expression we have added yet another two terms, representing the coupling to vorticity. These bring further “free” parameters γ_2 and γ_3. It is easy to see that we are allowed to add these terms since they do not affect the entropy production. In fact, a large number of similar terms may, in principle, be considered (see note added in proof in [53]). The presence of coupling terms of the particular form that we have introduced is suggested by kinetic

Complete Set of Terms

- Daunting:

$$\tau_\Pi \,\dot{\Pi} \;+\; \Pi \;=\; \Pi_{\rm NS}$$

$$\nu$$

$$\tau_q \,\Delta^{\mu\nu}\dot{q}_\nu \;+\; q^\mu \;=\; q^\mu_{\rm NS}$$

$$+\;\hat{\delta}_{1,2}\,\Pi$$

$$\tau_\pi \,\dot{\pi}^{<\mu\nu>} \;+\; \pi$$

$$-\,2\,\tau_\pi\,\pi_\lambda{}^{<\mu}\sigma^{\nu>\lambda} \;-\; 2\,\lambda_{\pi q}\,q^{<\mu}\nabla^{\nu>}\alpha \;+\; 2\,\lambda_{\pi\Pi}\,\Pi\,\sigma^{\mu\nu}$$

$$+\;\hat{\delta}_{2,2}\,\Pi\,\pi^{\mu\nu} \;-\; \hat{\eta}_2\,\pi_\lambda{}^{<\mu}\pi^{\nu>\lambda} \;-\; \hat{\epsilon}_2\,q^{<\mu}q^{\nu>}$$

- And still subject to
  - Poorly constrained initial Conditions
  - Eccentricity fluctuations
  - Poorly constrained equation of state
  - Hadronic rescattering effects
  - Bulk viscosity
  - Numerical viscosity
  - Finite size, core/corona effects

Complete Set of Terms

- Daunting:

$$
\begin{aligned}
\tau_\Pi \dot{\Pi} + \Pi &= \Pi_{\mathrm{NS}} + \tau_{\Pi q}\, q \cdot \dot{u} - \ell_{\Pi q}\, \partial \cdot q - \zeta\, \hat{\delta}_{0,1}\, \Pi\, \theta \\
&\quad + \lambda_{\Pi q}\, q \cdot \nabla \alpha + \lambda_{\Pi \pi}\, \pi^{\mu\nu} \sigma_{\mu\nu} + \hat{\delta}_{0,2}\, \Pi^2 + \hat{\epsilon}_0\, q \cdot q + \hat{\eta}_0\, \pi^{\mu\nu} \pi_{\mu\nu} \\
\tau_q\, \Delta^{\mu\nu} \dot{q}_\nu + q^\mu &= q^\mu_{\mathrm{NS}} - \tau_{q\Pi}\, \Pi\, \dot{u}^\mu - \tau_{q\pi}\, \pi^{\mu\nu}\, \dot{u}_\nu \\
&\quad + \ell_{q\Pi}\, \nabla^\mu \Pi - \ell_{q\pi}\, \Delta^{\mu\nu}\, \partial^\lambda \pi_{\nu\lambda} + \tau_q\, \omega^{\mu\nu}\, q_\nu - \frac{\kappa}{\beta}\, \hat{\delta}_{1,1}\, q^\mu\, \theta \\
&\quad - \lambda_{qq}\, \sigma^{\mu\nu}\, q_\nu + \lambda_{q\Pi}\, \Pi\, \nabla^\mu \alpha + \lambda_{q\pi}\, \pi^{\mu\nu}\, \nabla_\nu \alpha \\
&\quad + \hat{\delta}_{1,2}\, \Pi\, q^\mu + \hat{\eta}_1\, \pi^{\mu\nu}\, q_\nu \\
\tau_\pi\, \dot{\pi}^{<\mu\nu>} + \pi^{\mu\nu} &= \pi^{\mu\nu}_{\mathrm{NS}} + 2\, \tau_{\pi q}\, q^{<\mu} \dot{u}^{\nu>} \\
&\quad + 2\, \ell_{\pi q}\, \nabla^{<\mu} q^{\nu>} + 2\, \tau_\pi\, \pi_\lambda{}^{<\mu} \omega^{\nu>\lambda} - 2\, \eta\, \hat{\delta}_{2,1}\, \pi^{\mu\nu}\, \theta \\
&\quad - 2\, \tau_\pi\, \pi_\lambda{}^{<\mu} \sigma^{\nu>\lambda} - 2\, \lambda_{\pi q}\, q^{<\mu} \nabla^{\nu>} \alpha + 2\, \lambda_{\pi\Pi}\, \Pi\, \sigma^{\mu\nu} \\
&\quad + \hat{\delta}_{2,2}\, \Pi\, \pi^{\mu\nu} - \hat{\eta}_2\, \pi_\lambda{}^{<\mu} \pi^{\nu>\lambda} - \hat{\epsilon}_2\, q^{<\mu} q^{\nu>}
\end{aligned}
$$

- And still subject to
  - Poorly constrained initial Conditions
  - Eccentricity fluctuations
  - Poorly constrained equation of state
  - Hadronic rescattering effects
  - Bulk viscosity
  - Numerical viscosity
  - Finite size, core/corona effects

Putting Viscosity Into Formalism

- The stress-energy tensor now contains off-diagonal terms:
  - $T^{\mu\nu}$ will contain a piece called the shear stress tensor $\pi^{\mu\nu}$:

$$\pi^{\mu\nu} = T^{<\mu\nu>} \equiv \left[\frac{1}{2}\left(\Delta_\alpha{}^\mu \Delta_\beta{}^\nu + \Delta_\alpha{}^\nu \Delta_\beta{}^\mu\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}\right] T^{\alpha\beta}$$

$$\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$$

Exercise 8: Show that Δ projects out the 3-volume orthogonal to four-velocity u .

  - The existence of velocity gradients will *produce* a shear stress
  - Parameterize this via a 'constitutive equation' :

$$\pi^{\mu\nu} = 2\eta\, \nabla^{<\mu} u^{\nu>}$$

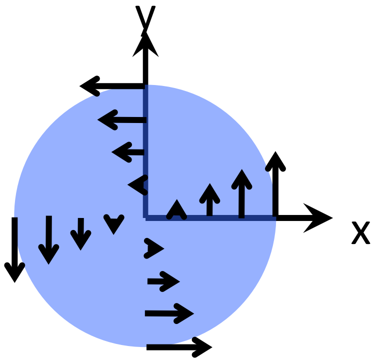
  - Then equation of motion is $\partial_\mu [(T^{\mu\nu}{}_{\substack{perfect\\ fluid}}) + \pi^{\mu\nu}] = 0$

- This is a simplified form of the relativistic Navier-Stokes eq.
  - Ignores heat conduction, bulk viscosity

An Aside on Formalism

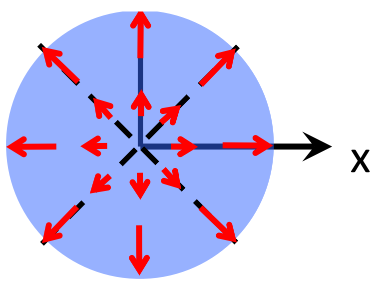
- Q. What are these weird index manipulations ?
- A. They produce a *symmetrized, traceless* ‘gradient’ :
  - Recall we’re interested in velocity gradients: $\frac{F_i}{A_j} = -\eta \frac{\partial v_i}{\partial x_j}$
  - Remove uniform rotation:

$$\frac{\partial v_i}{\partial x_j} = \frac{1}{2}[\partial_j v_i + \partial_i v_j] + \cancel{\frac{1}{2}[\partial_j v_i - \partial_i v_j]}$$



  - Remove uniform (Hubble) expansion:

$$\frac{1}{2}[\partial_j v_i + \partial_i v_j] \rightarrow \frac{1}{2}[\partial_j v_i + \partial_i v_j - \frac{2}{3}\delta_{ij}(\partial_k v_k)]$$



Exercise 6: Check these properties

Why (Relativistic) Hydrodynamics is Hard

This innocuous looking equation $\partial_\mu T^{\mu\nu} = 0$

hides a world of non-linear hurt

(even for ideal fluid):

$$\begin{aligned}\partial_\mu T^{\mu\nu} &= \partial_\mu \left[(\epsilon + p) u^\mu u^\nu - p g^{\mu\nu} \right] = 0 \\ &= \frac{\partial}{\partial x^0} \left[(\epsilon(x) + p(x)) u^\mu(x) u^\nu(x) - p(x) g^{\mu\nu} \right] \\ &+ \frac{\partial}{\partial x^1} \left[(\epsilon(x) + p(x)) u^\mu(x) u^\nu(x) - p(x) g^{\mu\nu} \right] + \cdots\end{aligned}$$

with $u^\mu(x) \equiv \dfrac{1}{\sqrt{1 - |\vec{v}(x)|^2}} \left(1 \ , \ \vec{v}(x) \ \right)$

$\Rightarrow$ Must make clever use of symmetries, projectors, etc.

Bjorken Expansion for Undergraduates

- "make clever use of <u>symmetries</u>, projectors, …"
- Boost invariance ⇒ $\epsilon(\vec{r},t) \to \epsilon(\tau) \quad , \quad p(\vec{r},t) \to p(\tau)$

$$\text{where } (\vec{r},t) \to (\tau, 0, 0, \eta)$$

$$\text{with } \tau^2 \equiv t^2 - z^2 \quad , \quad \eta \equiv \frac{1}{2}\log\left(\frac{t+z}{t-z}\right)$$

- so

$$\partial_t = \cosh\eta\, \partial_\tau - \frac{1}{\tau}\sinh\eta\, \partial_\eta$$

$$\partial_z = -\sinh\eta\, \partial_\tau + \frac{1}{\tau}\cosh\eta\, \partial_\eta$$

Exercise 9: Show this

- Assuming ideal fluid with $v_{fluid} = z/t$ then gives

$$\partial_\tau \epsilon + \frac{\epsilon + p}{\tau} = 0$$

Exercise 10: Show this.
Hint: OK to work at η=0 (why?) *after* taking derivatives.

Somewhat More Generally…

- “make clever use of symmetries, projectors, …”
- Define $D \equiv u^\mu \partial_\mu \sim \dfrac{\partial}{\partial t}$ in local rest frame

$$\nabla^\mu \equiv \Delta^{\mu\nu}\partial_\nu \sim \vec{\nabla} \quad \text{''} \quad \text{''} \quad \text{''} \quad \text{''} \quad \Delta^{\mu\nu} = u^\mu u^\nu - g^{\mu\nu}$$

- Then $\partial_\mu = u_\mu D - \nabla_\mu$ and $u_\mu D u^\mu = 0$
- Use with ideal fluid $T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu - g^{\mu\nu}$
- Result:

$$\begin{aligned} D\epsilon &= -(\epsilon + p)\nabla_\mu u^\mu &\sim\; dU = -p\, dV \\ -\nabla^\mu p &= (\epsilon + p)\; D u^\mu &\sim\; F = ma \end{aligned}$$

Reference: Derek Teaney, *Viscous Hydrodynamics and the Quark-Gluon Plasma*, **arXiv:0905.2433**

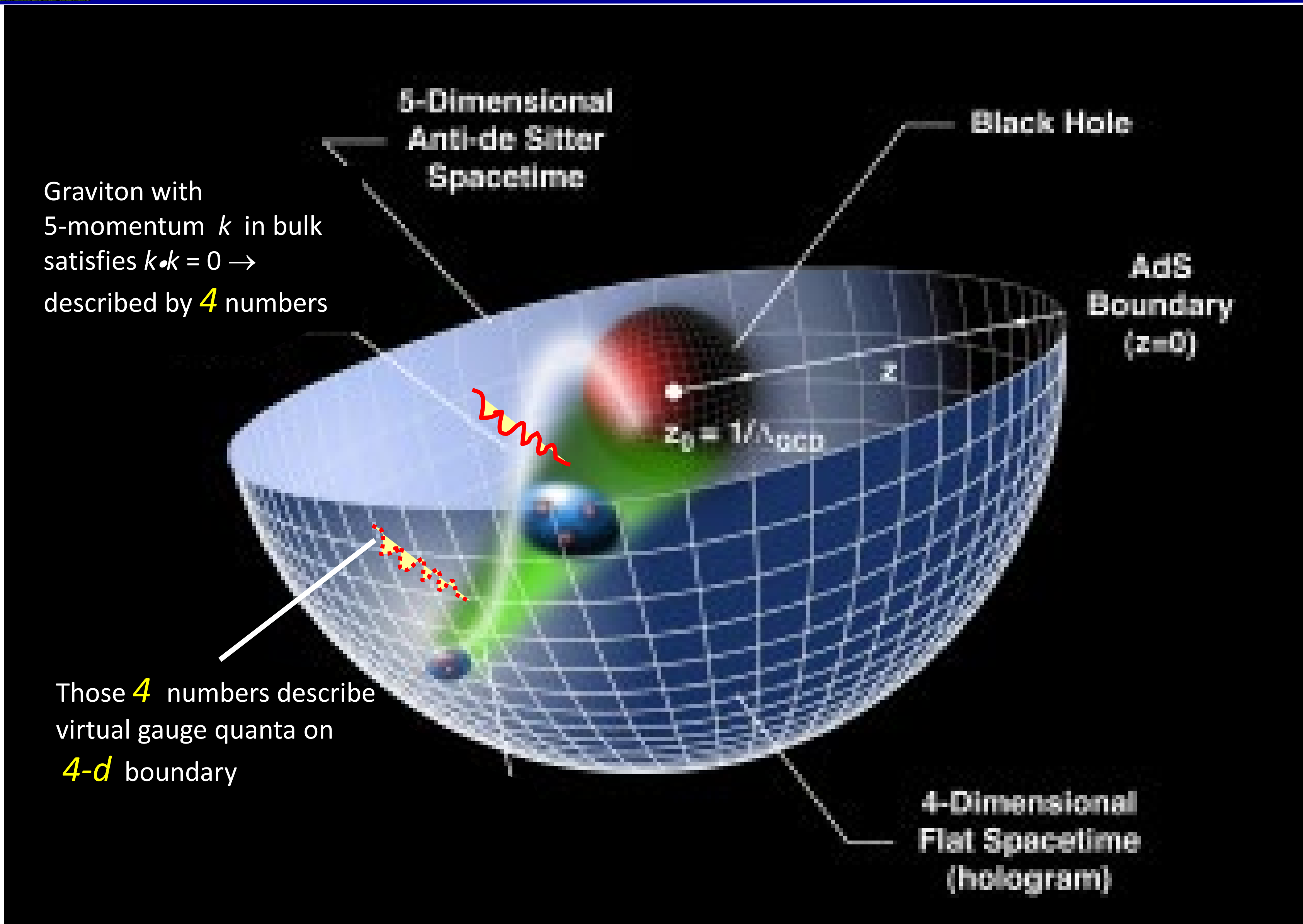
Science Questions

- ✕ Uninteresting question:
  - ❑ What happens when I crash two gold nuclei together?
- ✔ Interesting question:
  - ➡ Are there new states of matter at the highest temperatures and densities?

Science Questions

- Uninteresting question:
  - What happens when I crash two gold nuclei together?
- Interesting question:
  - Are there new states of matter at the highest temperatures and densities?
- $ Compelling question:
  - What fundamental *thermal* properties of our gauge theories of nature can be investigated experimentally?
    - Hint: *Gravity* is a gauge theory…

(Minimal) AdS / CFT $\Rightarrow \dfrac{\eta}{s} \geq \dfrac{1}{4\pi}$



(Adopted from [S. Brodsky figure]())

In Words

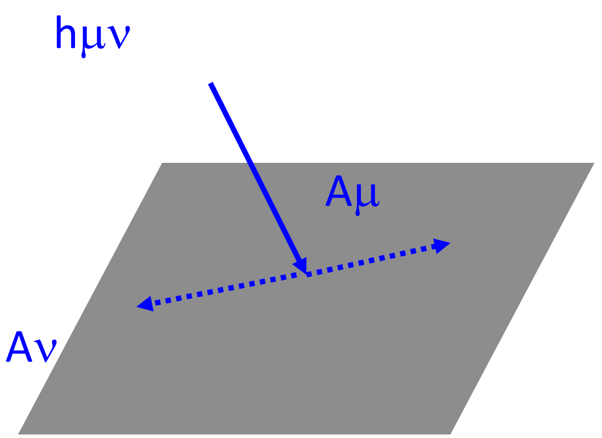
- A stringy theory of gravity in N space-time dimensions (the "bulk" AdS)

  is "dual" (that is, equivalent to)

- A gauge theory without gravity in N-1 space-time dimensions (the "boundary" CFT)
- Notes:
  - AdS ≡ Anti de Sitter space ; CFT ≡ Conformal Field Theory
  - "Equivalent" means "equivalent" – all phenomena in one theory have corresponding "dual" descriptions in the other theory.
  - Maldacena's AdS/CFT correspondence is a realization of hypotheses from both 't Hooft and Susskind that the ultimate limit on the number of degrees of freedom in a spacetime region is proportional to the area of its boundary, *not* its volume (!)

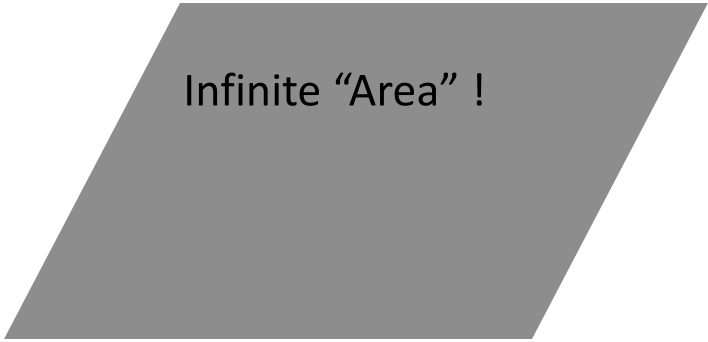
Why Does This Work??

- The easy part:
  - Recall $$\frac{F_x}{A} = -\eta \frac{\partial v_x}{\partial y}$$
  - that is, viscosity ~ x-momentum transport in y-direction ~ T^{xy}
  - There are standard methods (Kubo relations) to calculate such dissipative quantities
$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d\mathbf{x}\, e^{i\omega t} \langle [T_{xy}(t,\mathbf{x}), T_{xy}(0,0)] \rangle$$
- The hard part:
  - This calculation is difficult in a strongly-coupled gauge theory
- The weird part:
  - A (supersymmetric) pseudo-QCD theory can be mapped to a 10-dimensional classical gravity theory on the background of black 3-branes
  - The calculation can be performed there as the absorption of gravitons by the brane
  - THE SHEAR VISCOSITY OF STRONGLY COUPLED N=4 SUPERSYMMETRIC YANG-MILLS PLASMA., G. Policastro, D.T. Son , A.O. Starinets, Phys.Rev.Lett.87:081601,2001 hep-th/0104066



The Result

- Viscosity η = “Area”/16πG



- Normalize by entropy (density) S = “Area”/4G

- Dividing out the infinite “areas” :

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi}\frac{1}{k_B}$$

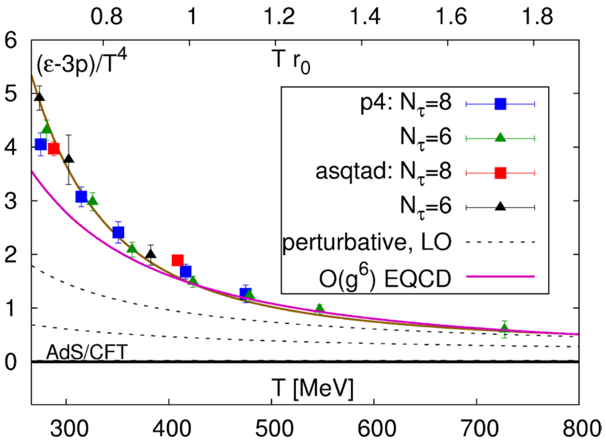
- *Conjectured* to be a lower bound “*for all relativistic quantum field theories at finite temperature and zero chemical potential*”.
- See “*Viscosity in strongly interacting quantum field theories from black hole physics*”, P. Kovtun, D.T. Son, A.O. Starinets, Phys.Rev.Lett.94:111601, 2005, hep-th/0405231

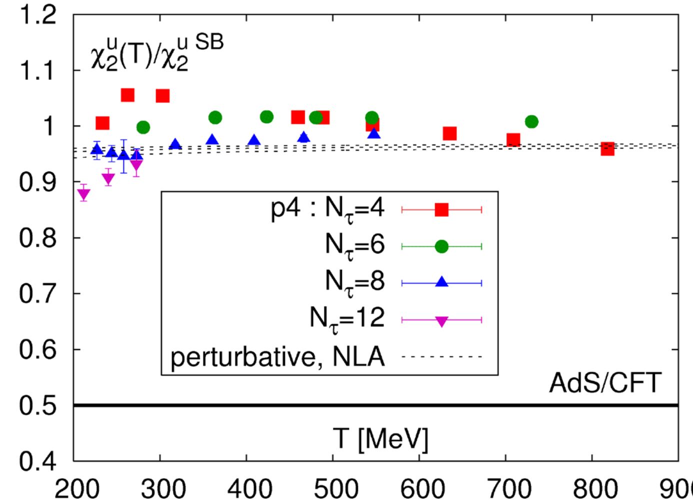
AdS/CFT - Pro

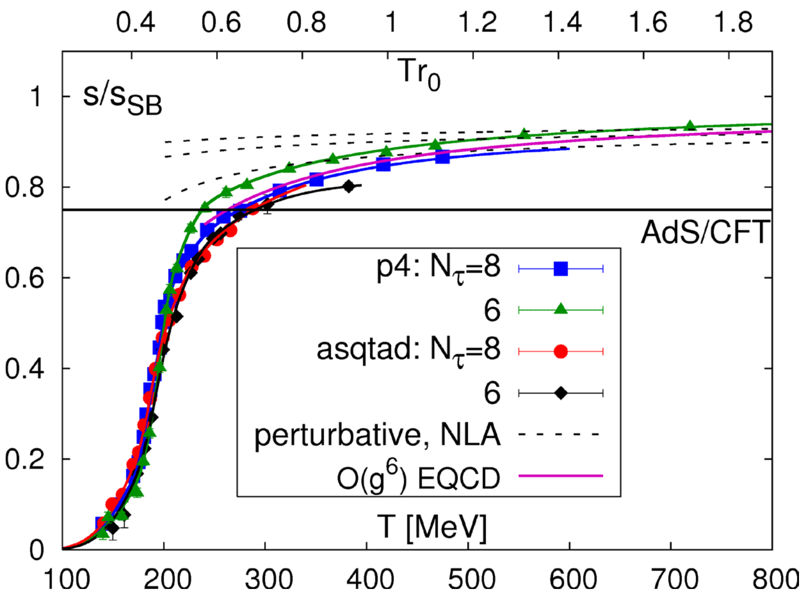
- *Gauge/gravity duality*, G.T. Horowitz and J. Polchinski, gr-qc/0602037)
  - “Hidden within every non-Abelian gauge theory, even within the weak and strong nuclear interactions, is a theory of quantum gravity.”
- *Stringscape*, by Matthew Chalmers, in *Particle World*:
  - “Susskind says that by studying heavy-ion collisions you are also studying quantum gravity that is ‘blown up and slowed down by a factor of 10^{20} ’ ”
- *The Black Hole War*, L. Susskind, ISBN 978-0-316-01640-7 :
  - “…the Holographic Principle is evolving from radical paradigm shift to everyday working tool of – surprisingly – nuclear physics.”

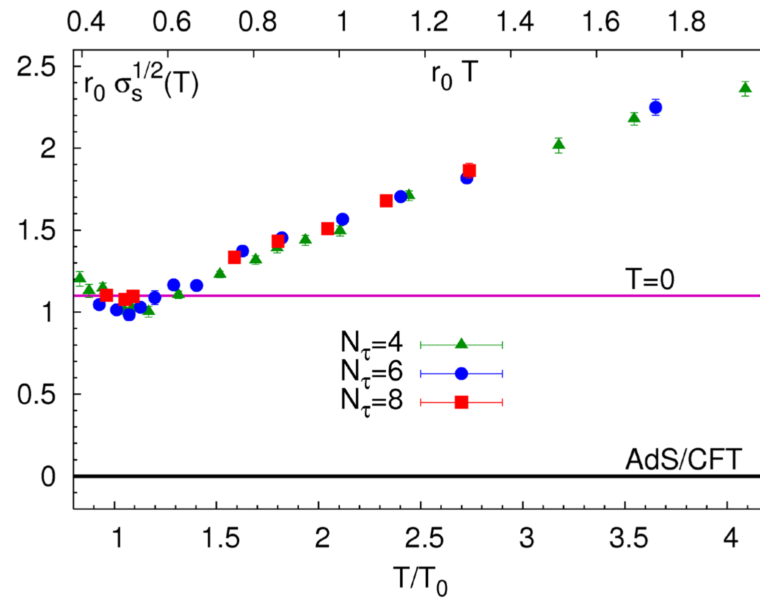
AdS/CFT - Con

- P. Petreczsky, QM09: “AdS/CFT is consistently wrong. ”









Editorial Opinion

- Reality may lie between these two extremes ☺
- At the moment:
  - Hard physics (high p_T energy loss)
    - Fragile predictions
    - Robust extractions
  - Soft physics (hydrodynamics, viscosity)
    - Robust predictions
    - Fragile extractions
- AdS/CFT has led to qualitatively new insights
- Not covered:
  - AdS/QCD (application to confinement, masses)

A Long Time Ago (1985)

- Miklos Gyulassy and Pawel Danielewicz:
  - *Dissipative Phenomena in Quark-Gluon Plasmas*
    P. Danielewicz, M. Gyulassy Phys.Rev. D31, 53,1985.

  noted several restrictions on smallest allowed η :
- Most restrictive:
- **$\lambda > h/<p>$ $\Rightarrow$ $\eta > \sim n / 3$**
- But for the quanta they were considering s = 3.6n
- **$\Rightarrow \eta/s > 1 / (3.6 \times 3) \sim 1 / (4\pi)$!!**

Before estimating λ_i via Eq. (3.2) we note several physical constraints on λ_i. First, the uncertainty principle implies that quanta transporting typical momenta $\langle p \rangle$ cannot be localized to distances smaller than $\langle p \rangle^{-1}$. Hence, it is meaningless to speak about mean free paths smaller than $\langle p \rangle^{-1}$. Requiring $\lambda_i \gtrsim \langle p \rangle_i^{-1}$ leads to the lower bound

$$\eta \gtrsim \tfrac{1}{3} n \;, \tag{3.3}$$

where $n = \sum n_i$ is the total density of quanta. What seems amazing about (3.3) is that it is independent of dynamical details. There is a finite viscosity regardless of how large is the free-space cross section between the quanta. See Refs. 21 and 22 for examples illustrating how the thermalization rate of many-body systems is limited by the uncertainty principle.

Why AdS/CFT Matters…

- All the thermal parts are built upon Bekenstein and Hawking's (unproven) assertion that black holes have entropy:

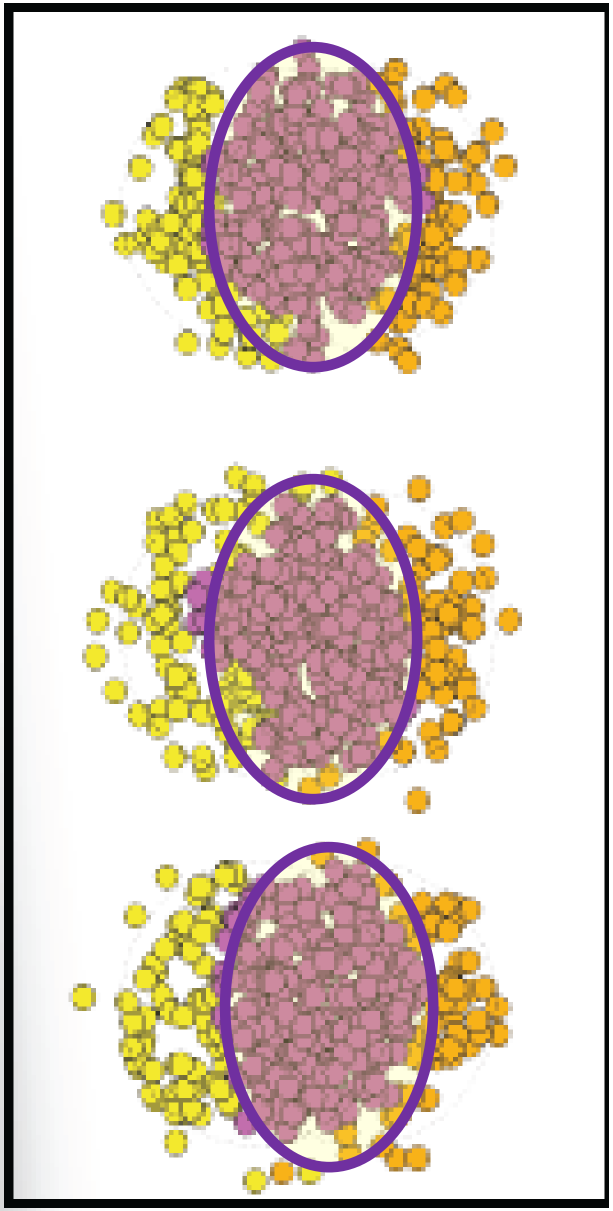
$$S_{BH} = \frac{A}{4G} = \frac{A}{4L_P^{\ 2}}$$

  - □ Black holes have a temperature
  - □ Black holes can radiate
  - □ Black holes don't lose information

- Important to test these very underpinnings

2010: The Noise *Is* The Signal

- Importance of higher harmonics
- $dn/d\phi \sim 1 + 2\, v_2(p_T) \cos(2\,\phi) + \ldots$



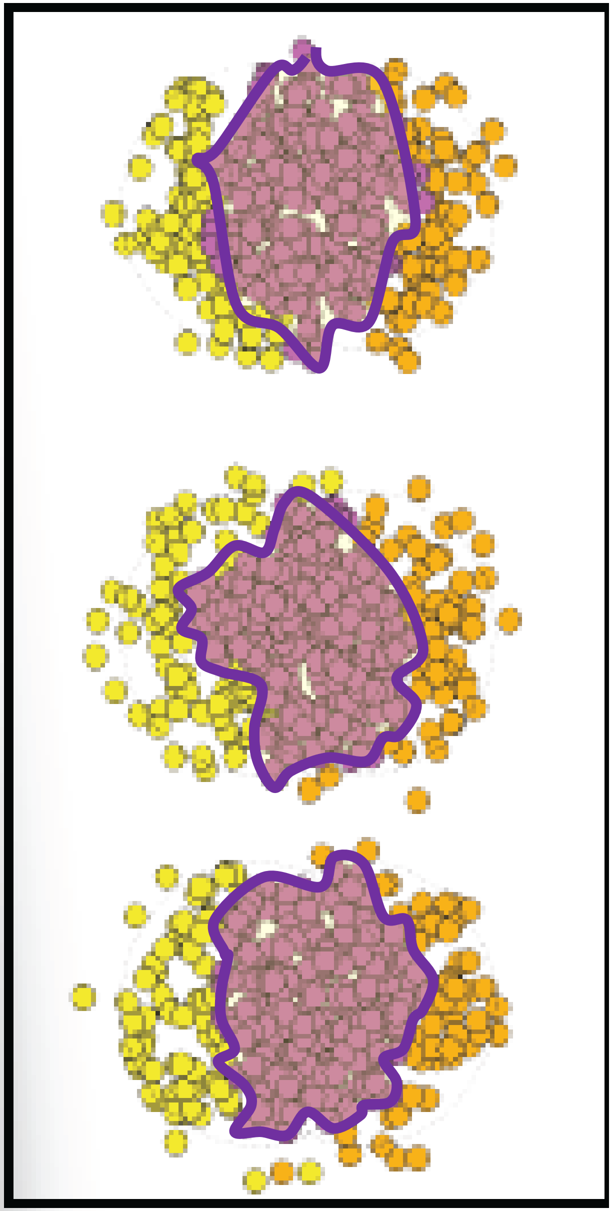
B. Alver and G. Roland, [Phys. Rev. C81, 054905 (2010)]

2010: The Noise *Is* The Signal

- Importance of higher harmonics
- $dn/d\phi \sim 1 + 2\, v_2(p_T) \cos(2\phi)$
  $+ 2\, v_3(p_T) \cos(3\phi)$
  $+ 2\, v_4(p_T) \cos(4\phi) + \ldots$

➢ Fluctuations critical for determining allowed range of η/s .

☛ Persistence of "bumps"➔small η/s !

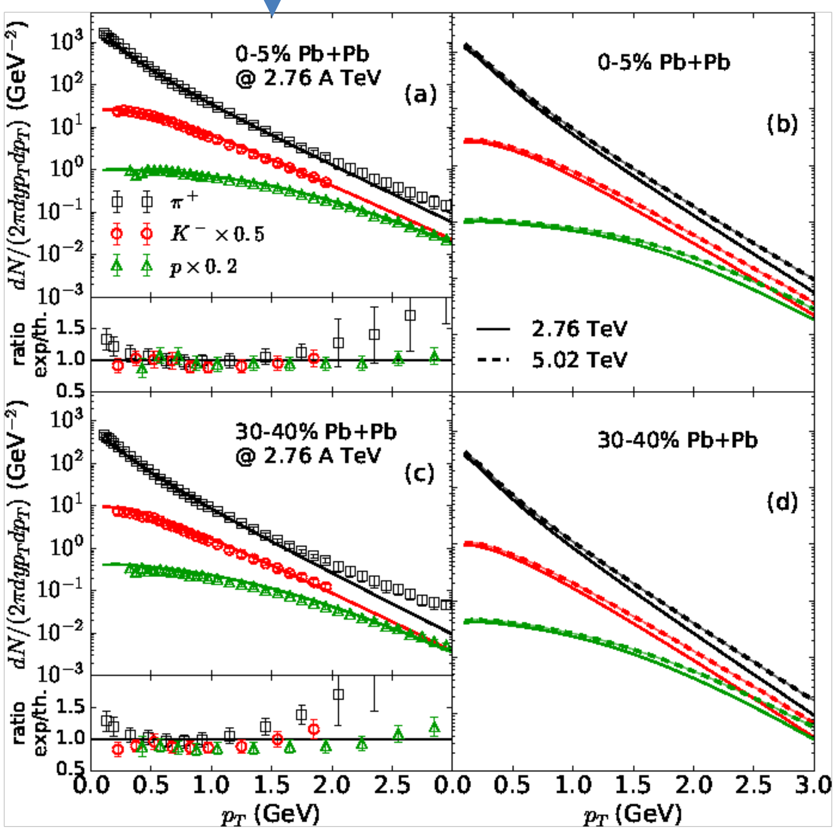


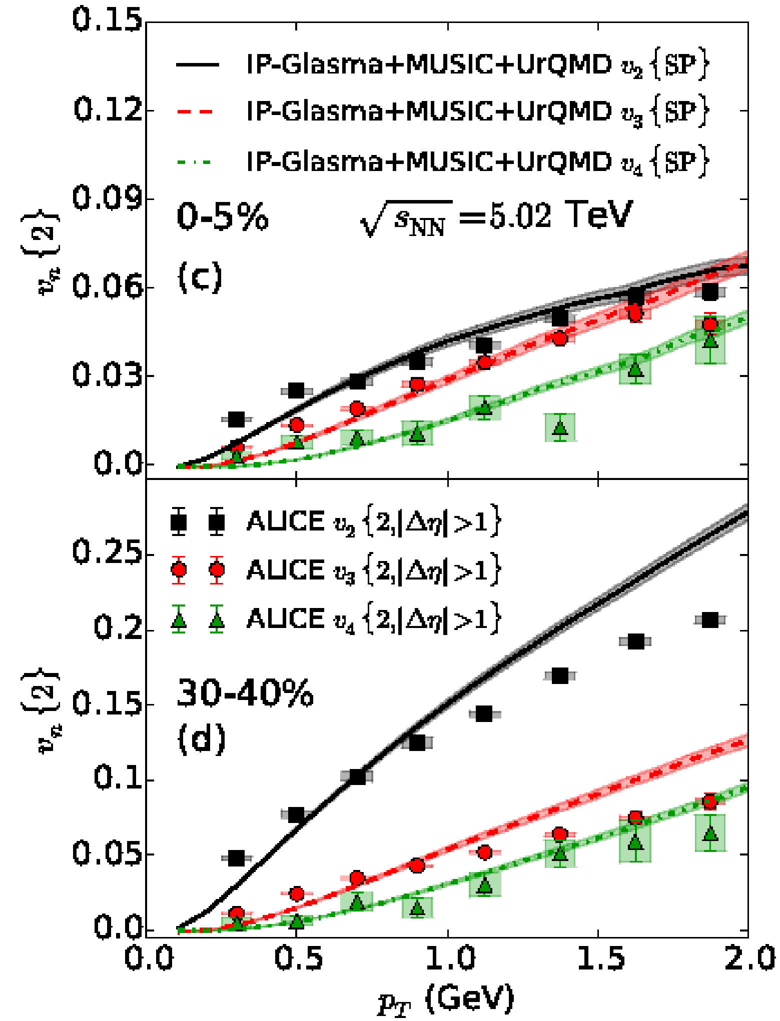
B. Alver and G. Roland, Phys. Rev. C81, 054905 (2010)

Hydro Models Capture These Features

- Describe
  over a broad range both
  spectra and v_n's

$$\frac{\eta}{s} = 0.095 \sim 1.2\ \frac{1}{4\pi} \qquad \frac{\zeta}{s}(T)$$

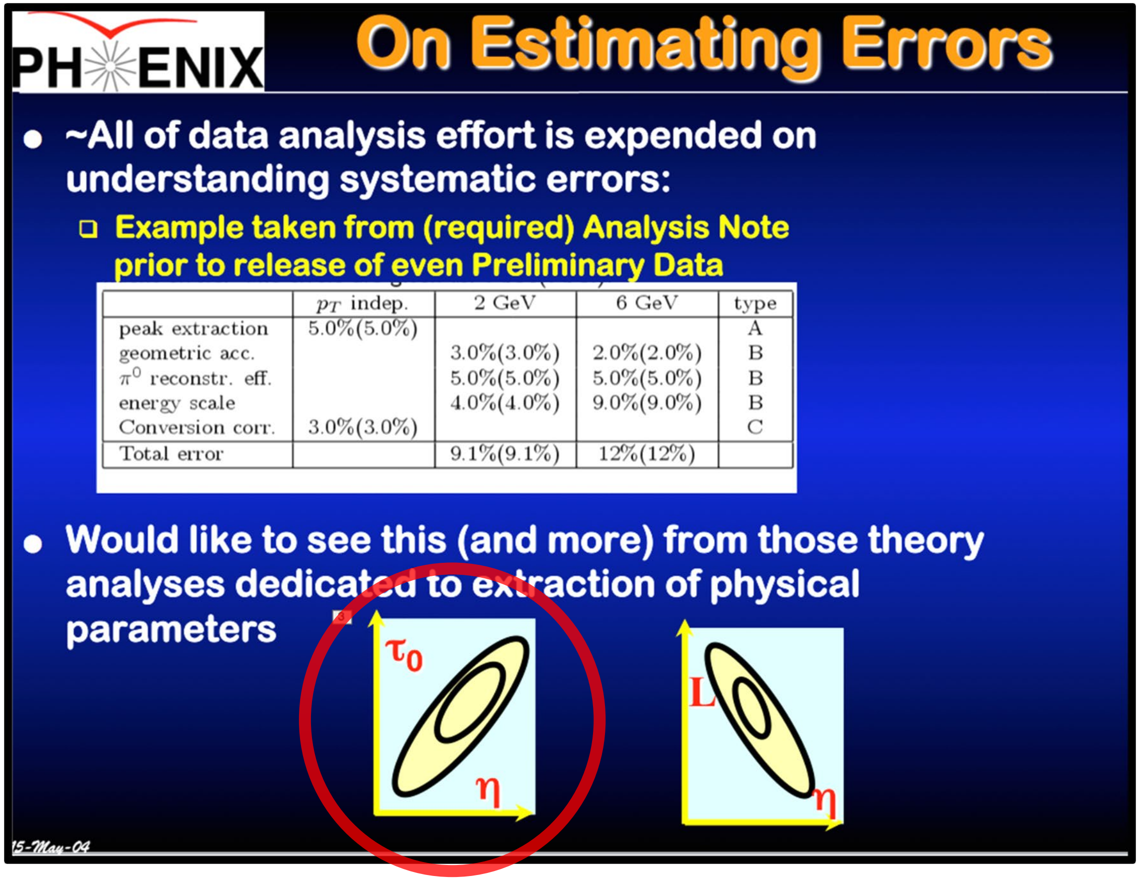




S. McDonald et al., [Phys. Rev. C95, 064913 (2017)]

From Discovery to Precision

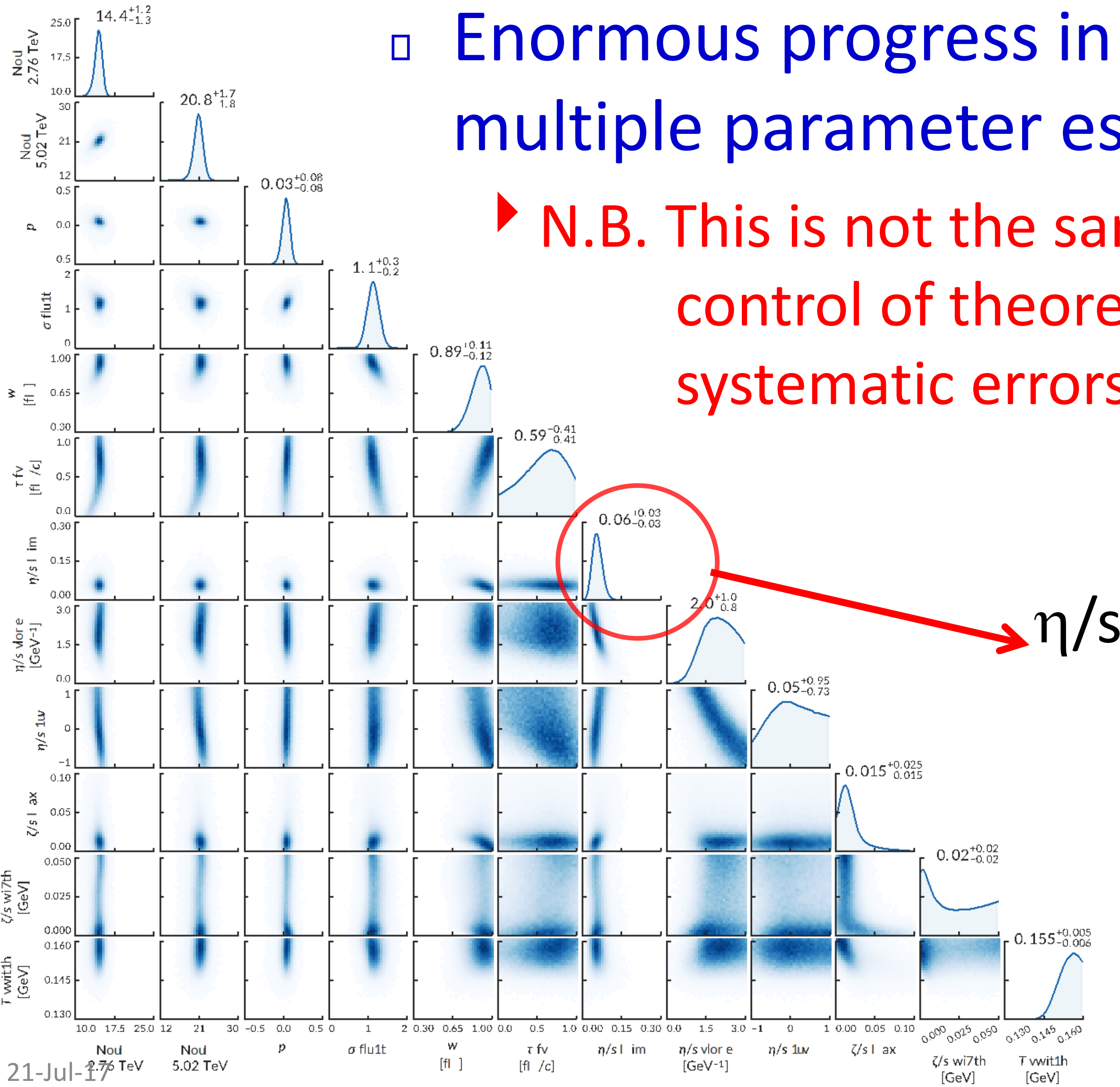
- A consistent theme in our field
  (and in all good science):
  increasing reliability, robustness and rigor.
- From a talk
  at the
  May 15, 2004
  RBRC
  Workshop
  “New Discoveries
  at RHIC”

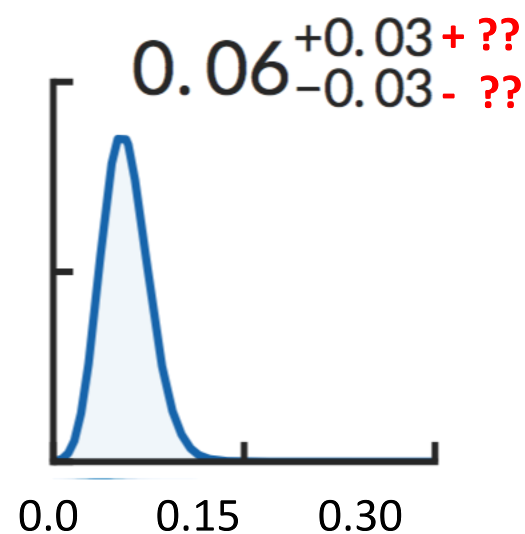


	p_T indep.	2 GeV	6 GeV	type
peak extraction	5.0%(5.0%)			A
geometric acc.		3.0%(3.0%)	2.0%(2.0%)	B
π^0 reconstr. eff.		5.0%(5.0%)	5.0%(5.0%)	B
energy scale		4.0%(4.0%)	9.0%(9.0%)	B
Conversion corr.	3.0%(3.0%)			C
Total error		9.1%(9.1%)	12%(12%)	

2017: Wish (Partially) Granted

- Enormous progress in multiple parameter estimation
  - N.B. This is not the same as control of theoretical systematic errors !





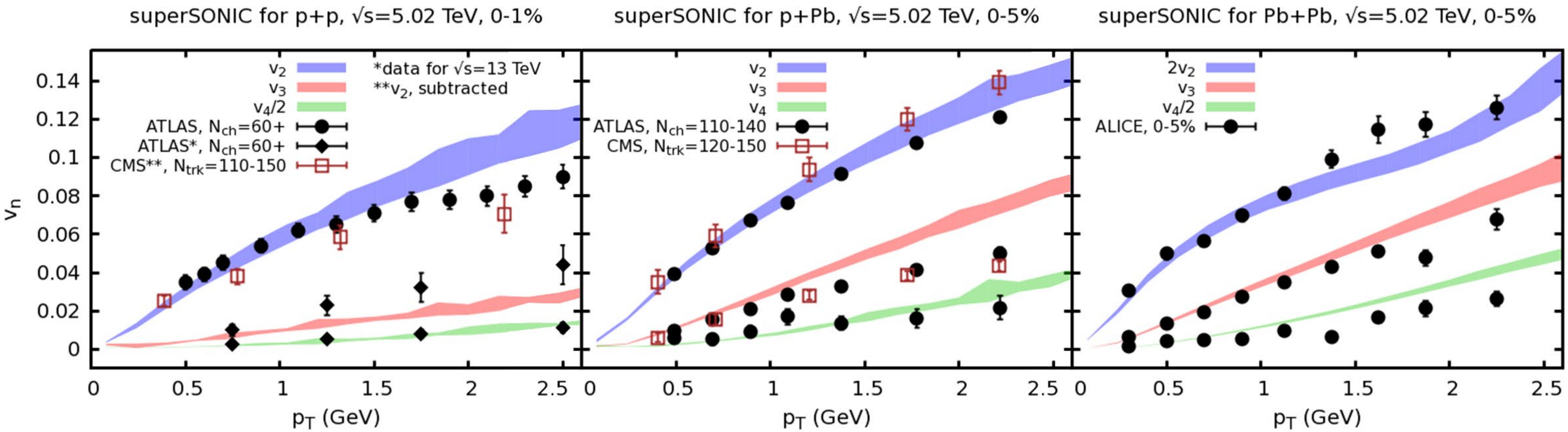
[S. Bass: Determination of QGP Parameters...]

[J. Bernhard, Characterization of...]

A Spectacular New Development

- *Single* hydro description provides "reasonable" *simultaneous* description of $v_2(p_T)$, $v_3(p_T)$ and $v_4(p_T)$ in p+p, p+Pb, and Pb+Pb:

P. Romatschke: Do nuclear collisions create a locally equilibrated quark-gluon plasma?



[1701.07145, see poster H8 by R. Weller]

- "One fluid to rule them all…"
- N.B.: *Geometry initialized at constituent quark level*
- But following Feynman's dictum… (next slide)

A Gratifying Development at QM17

- Explicit examples of this credo:

Details that could throw doubt on your interpretation must be given, if you know them. You must do the best you can--if you know anything at all wrong, or possibly wrong--to explain it. If you make a theory, for example, and advertise it, or put it out, then you must also put down all the facts that disagree with it, as well as those that agree with it....

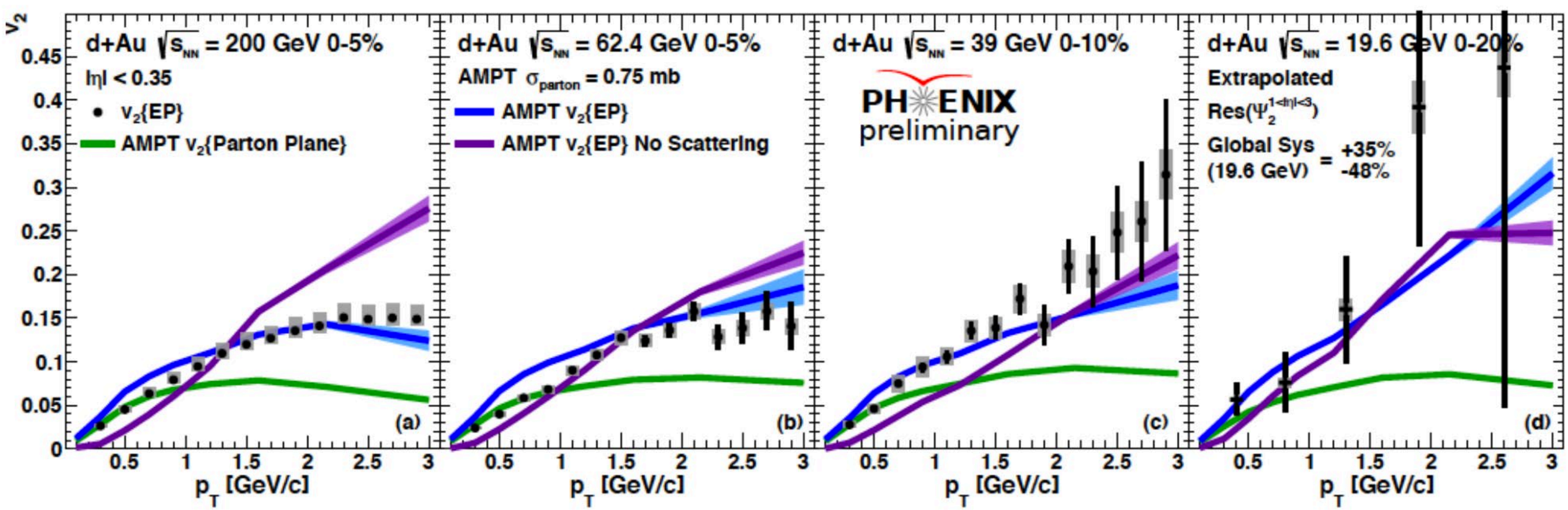
In summary, the idea is to give all of the information to help others to judge the value of your contribution; not just the information that leads to judgment in one particular direction
or another.

R.P. Feynman, Cargo Cult Science,
1974 Caltech Commencement Address

The Unreasonable Effectiveness…

of multiphase transport codes such as AMPT in describing “hydrodynamic” phenomena

200 GeV **62 GeV** **39 GeV** **20 GeV**



J. Velkovska: PHENIX results on elliptic and triangular flow in d+Au collisions…

The Current Situation

- The ”unreasonable effectiveness” of (viscous) hydrodynamics in small systems
- is both a stimulating challenge to,
  and a fundamental probe of,
  our understanding of thermal QCD.

Final Exam: Use what you’ve learned to derive these expressions for the mean free path and cross section (assuming well-defined quasi-particles. Comment on implications.

$$\lambda \sim 2 \left(\frac{T_0}{T}\right)^3 \left(\frac{\sigma_1}{\sigma}\right) \quad \text{with } T_0 = 200 \text{ Mev and } \sigma_1 = 1 \text{ mb}$$

$$\frac{\eta}{s} = \frac{1}{4\pi} \quad \Rightarrow \quad \sigma \sim (20 \text{ mb}) \left(\frac{T_0}{T}\right)^2$$

- The challenge: to understand how this near-perfect fluidity emerges in QCD

Sincere Thanks to

Emily Flanagan

Edward Kinney
Jamie Nagle
Dennis Perepelitsa
Paul Romatschke

All of *you* for your attention and many excellent questions