Hadron Structure

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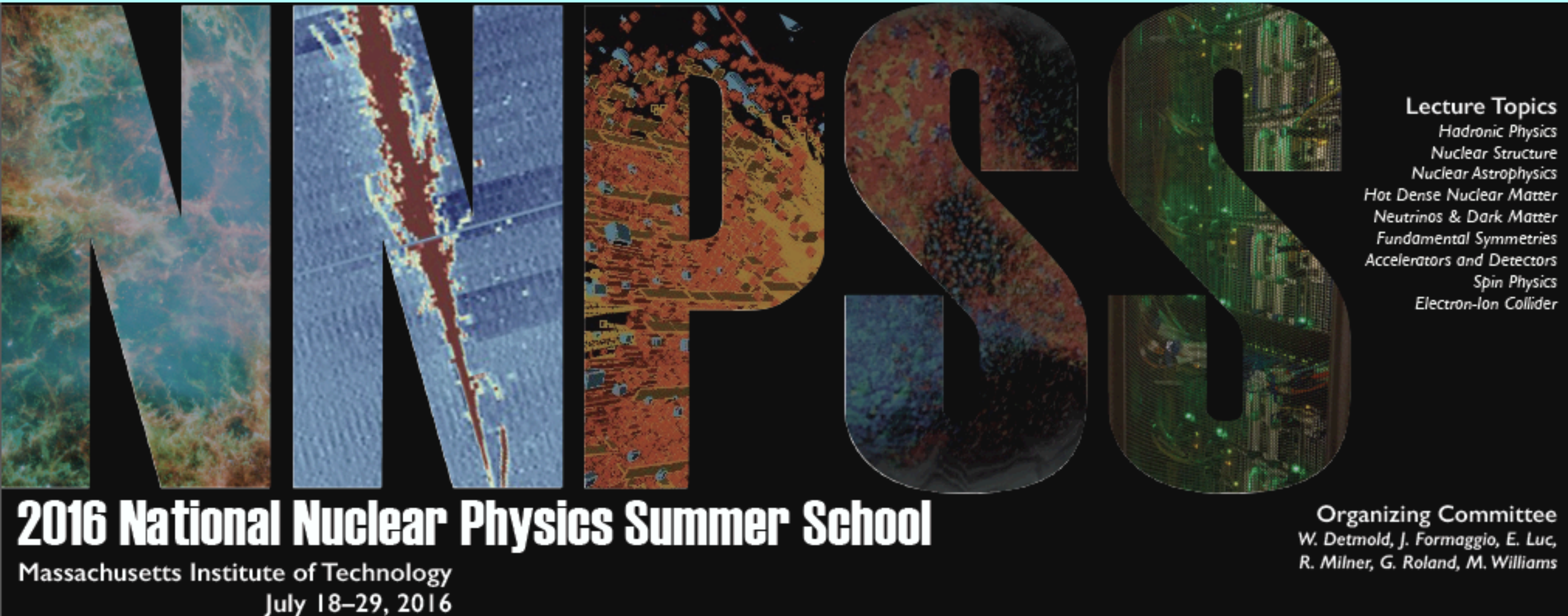

Lecture Topics
Hadronic Physics
Nuclear Structure
Nuclear Astrophysics
Hot Dense Nuclear Matter
Neutrinos & Dark Matter
Fundamental Symmetries
Accelerators and Detectors
Spin Physics
Electron-Ion Collider

NNPSS

2016 National Nuclear Physics Summer School
Massachusetts Institute of Technology
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Organizing Committee
W. Detmold, J. Formaggio, E. Luc,
R. Milner, G. Roland, M. Williams

The plan for my three lectures

- ☐ The Goal:

  To understand the hadron structure in terms of QCD and its hadronic matrix elements of quark-gluon field operators, to connect these matrix elements to physical observables, and to calculate them from QCD (lattice QCD, inspired models, …)

- ☐ The outline:

  Hadrons, partons (quarks and gluons), and probes of hadron structure

  One lecture

  Parton Distribution Functions (PDFs) and Transverse Momentum Dependent PDFs (TMDs)

  One lecture

  Generalized PDFs (GPDs) and multi-parton correlation functions

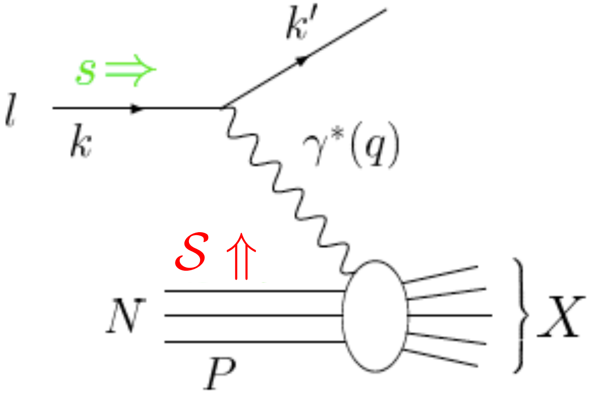
  One lecture

See also
lectures by Shepard on
“Hadron Spectroscopy”,
and
lectures by Deshpande on
“Electron-Ion Collider”
and
lectures by Gandolfi on
“Nuclear Structure”
and
lectures by Aschenauer on
“Accelerators & detectors”

Transverse single-spin asymmetry (TSSA)

- ☐ 50 years ago, Profs. Christ and Lee proposed to use A_N of inclusive DIS to test the Time-Reversal invariance

N. Christ and T.D. Lee, Phys. Rev. 143, 1310 (1966)



They predicted:

In the approximation of one-photon exchange, A_N of inclusive DIS vanishes if Time-Reversal is invariant for EM and Strong interactions

A_N for inclusive DIS

- **DIS cross section:** $\sigma(\vec{s}_\perp) \propto L^{\mu\nu}\, W_{\mu\nu}(\vec{s}_\perp)$

- **Leptionic tensor is symmetric:**

$$L^{\mu\nu} = L^{\nu\mu}$$

- **Hadronic tensor:**

$$W_{\mu\nu}(\vec{s}_\perp) \propto \langle P, \vec{s}_\perp |\, j^\dagger_\mu(0)\, j_\nu(y)\, | P, \vec{s}_\perp \rangle$$

- **Polarized cross section:**

$$\Delta\sigma(\vec{s}_\perp) \propto L^{\mu\nu}\, \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp) \right]$$

- **Vanishing single spin asymmetry:**

$$\boxed{\begin{aligned} A_N = 0 \quad \Leftrightarrow \quad & \langle P, \vec{s}_\perp |\, j^\dagger_\mu(0)\, j_\nu(y)\, | P, \vec{s}_\perp \rangle \\ & \overset{?}{=} \langle P, -\vec{s}_\perp |\, j^\dagger_\nu(0)\, j_\mu(y)\, | P, -\vec{s}_\perp \rangle \end{aligned}}$$

A_N for inclusive DIS

- ☐ **Define two quantum states:**

$$\langle\beta| \equiv \langle P, \vec{s}_\perp | \, j^\dagger_\mu(0)\, j_\nu(y) \qquad\qquad |\alpha\rangle \equiv |P, \vec{s}_\perp\rangle$$

- ☐ **Time-reversed states:**

$$|\alpha_T\rangle = V_T\, |P, \vec{s}_\perp\rangle = |-P, -\vec{s}_\perp\rangle$$

$$\begin{aligned}|\beta_T\rangle &= V_T\, \left[j^\dagger_\mu(0)\, j_\nu(y)\right]^\dagger |P, \vec{s}_\perp\rangle \\ &= \left(V_T j^\dagger_\nu(y) V_T^{-1}\right) \left(V_T j_\mu(0) V_T^{-1}\right) |-P, -\vec{s}_\perp\rangle\end{aligned}$$

- ☐ **Time-reversal invariance:**

$$\langle\alpha_T|\beta_T\rangle = \langle\alpha|\, V_T^\dagger\, V_T\, |\beta\rangle = \langle\alpha|\beta\rangle^* = \langle\beta|\alpha\rangle$$

$$\boxed{\begin{aligned}\Longrightarrow\ & \langle -P, -\vec{s}_\perp|\, \left(V_T j^\dagger_\nu(y) V_T^{-1}\right) \left(V_T j_\mu(0) V_T^{-1}\right) |-P, -\vec{s}_\perp\rangle \\ &= \langle P, \vec{s}_\perp |\, j^\dagger_\mu(0)\, j_\nu(y)\, |P, \vec{s}_\perp\rangle\end{aligned}}$$

A_N for inclusive DIS

- **Parity invariance:**

$$\boxed{1 = U_P^{-1} U_P = U_P^{\dagger} U_P}$$

$$\langle -P, -\vec{s}_\perp | \left(V_T j_\nu^\dagger(y) V_T^{-1} \right) \left(V_T j_\mu(0) V_T^{-1} \right) | -P, -\vec{s}_\perp \rangle$$

$$\Downarrow$$

$$\langle P, -\vec{s}_\perp | \left(U_P V_T j_\nu^\dagger(y) V_T^{-1} U_P^{-1} \right) \left(U_P V_T j_\mu(0) V_T^{-1} U_P^{-1} \right) | P, -\vec{s}_\perp \rangle$$

$$\Downarrow$$

$$\langle P, -\vec{s}_\perp | \, j_\nu^\dagger(-y) \, j_\mu(0) \, | P, -\vec{s}_\perp \rangle$$

Translation invariance:

$$\Downarrow$$

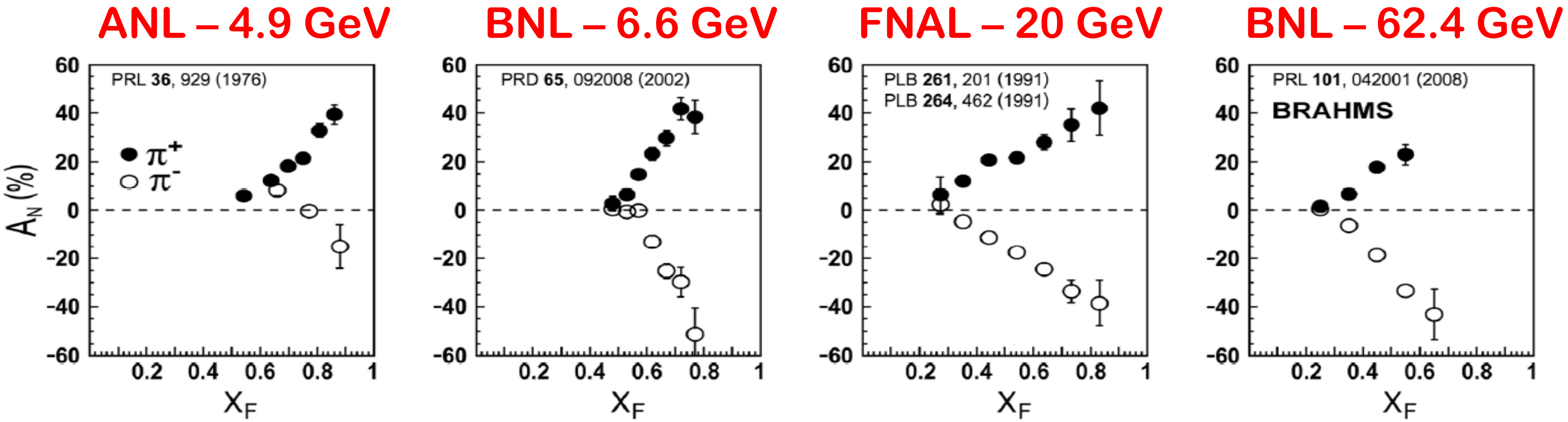
$$\boxed{\begin{aligned} &\langle P, -\vec{s}_\perp | \, j_\nu^\dagger(0) \, j_\mu(y) \, | P, -\vec{s}_\perp \rangle \\ &= \langle P, \vec{s}_\perp | \, j_\mu^\dagger(0) \, j_\nu(y) \, | P, \vec{s}_\perp \rangle \end{aligned}}$$

- **Polarized cross section:**

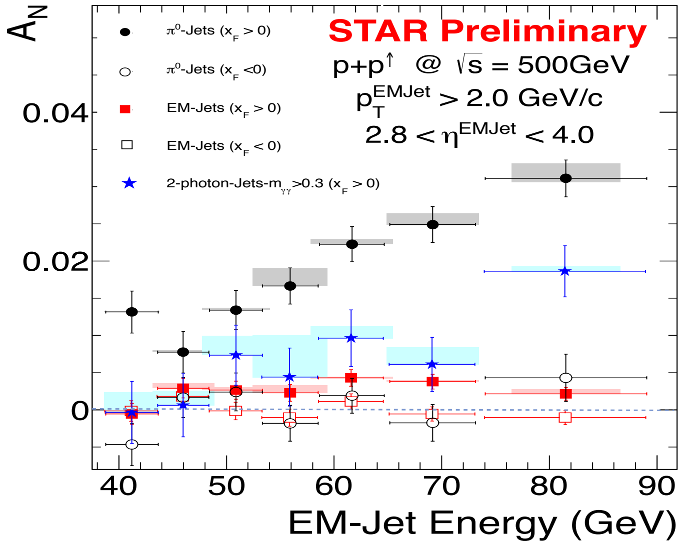
$$\boxed{\begin{aligned} \Delta\sigma(\vec{s}_\perp) &\propto L^{\mu\nu} \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp) \right] \\ &= L^{\mu\nu} \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\nu\mu}(\vec{s}_\perp) \right] = 0 \end{aligned}}$$

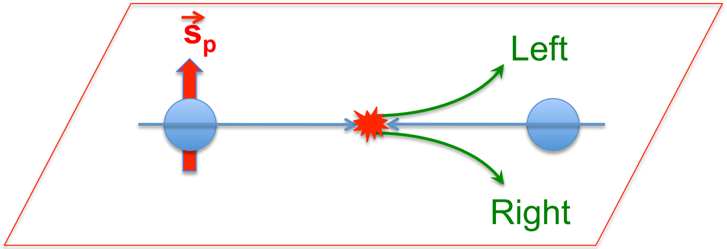
A_N in hadronic collisions

- ❑ A_N - consistently observed for over 35 years!



- ❑ Survived the highest RHIC energy:





$$A_N \equiv \frac{\Delta\sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})}$$

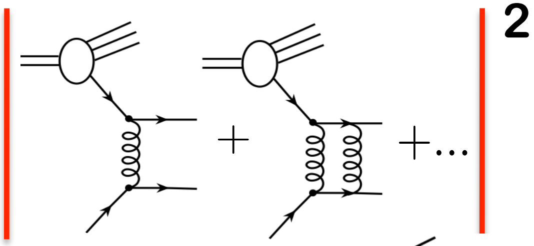
Do we understand this?

Do we understand it?

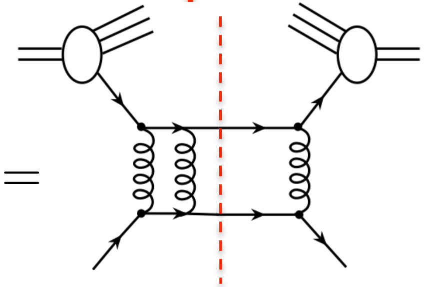
Kane, Pumplin, Repko, PRL, 1978

☐ Early attempt:

Cross section: $\sigma_{AB}(p_T, \vec{s}) \propto$



Asymmetry: $\sigma_{AB}(p_T, \vec{s}) - \sigma_{AB}(p_T, -\vec{s}) =$



$\propto \alpha_s \frac{m_q}{p_T}$

Too small to explain available data!

☐ What do we need?

$$A_N \propto i\vec{s}_p \cdot (\vec{p}_h \times \vec{p}_T) \Rightarrow i\epsilon^{\mu\nu\alpha\beta} p_{h\mu} s_\nu p_\alpha p'_{h\beta}$$

Need a phase, a spin flip, enough vectors

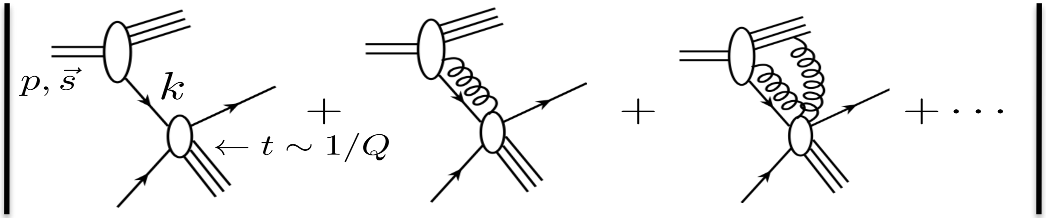
☐ Vanish without parton’s transverse motion:

→ A direct probe for parton’s transverse motion,

Spin-orbital correlation, QCD quantum interference

How collinear factorization generates TSSA?

❑ Collinear factorization beyond leading power:



$$\sigma(Q,\vec{s}) \propto \left| \;\cdots\; + \;\cdots\; + \;\cdots\; + \cdots \right|^2 \left(\frac{\langle k_\perp \rangle}{Q}\right)^n$$ – Expansion

$$\sigma(Q,s_T) = H_0 \otimes f_2 \otimes f_2 + (1/Q)\, H_1 \otimes f_2 \otimes f_3 + \mathcal{O}(1/Q^2)$$

Too large to compete!

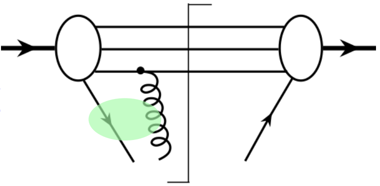
Three-parton correlation

❑ Single transverse spin asymmetry:

Efremov, Teryaev, 82;
Qiu, Sterman, 91, etc.

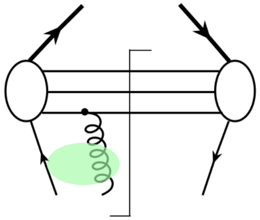
$$\Delta\sigma(s_T) \propto T^{(3)}(x,x) \otimes \hat{\sigma}_T \otimes D(z) + \delta q(x) \otimes \hat{\sigma}_D \otimes D^{(3)}(z,z) + \ldots$$

$T^{(3)}(x,x) \propto$



Qiu, Sterman, 1991, …

$D^{(3)}(z,z) \propto$



Kang, Yuan, Zhou, 2010

Integrated information on parton's transverse motion!

Needed Phase: Integration of "dx" using unpinched poles

Twist-3 distributions relevant to A_N

❑ Twist-2 distributions:

- **Unpolarized PDFs:**

$$q(x) \propto \langle P|\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y)|P\rangle$$

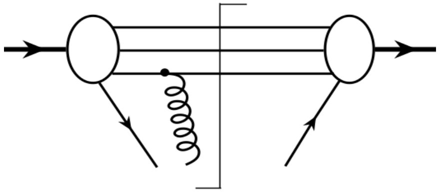
$$G(x) \propto \langle P|F^{+\mu}(0)F^{+\nu}(y)|P\rangle(-g_{\mu\nu})$$

- **Polarized PDFs:**

$$\Delta q(x) \propto \langle P,S_\parallel|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\psi_q(y)|P,S_\parallel\rangle$$

$$\Delta G(x) \propto \langle P,S_\parallel|F^{+\mu}(0)F^{+\nu}(y)|P,S_\parallel\rangle(i\epsilon_{\perp\mu\nu})$$

❑ Two-sets Twist-3 correlation functions:



No probability interpretation!

$$\widetilde{\mathcal{T}}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \langle P,s_T|\overline{\psi}_q(0)\,\frac{\gamma^+}{2}\left[\epsilon^{s_T\sigma n\bar{n}}F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P,s_T\rangle$$

Kang, Qiu, 2009

$$\widetilde{\mathcal{T}}_{G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \frac{1}{P+}\langle P,s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}}F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P,s_T\rangle(-g_{\rho\lambda})$$

$$\widetilde{\mathcal{T}}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \langle P,s_T|\overline{\psi}_q(0)\,\frac{\gamma^+\gamma^5}{2}\left[i\,s_T^\sigma\,F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P,s_T\rangle$$

$$\widetilde{\mathcal{T}}_{\Delta G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \frac{1}{P+}\langle P,s_T|F^{+\rho}(0)\left[i\,s_T^\sigma\,F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P,s_T\rangle\,(i\epsilon_{\perp\rho\lambda})$$

Role of color magnetic force!

❑ Twist-3 fragmentation functions:

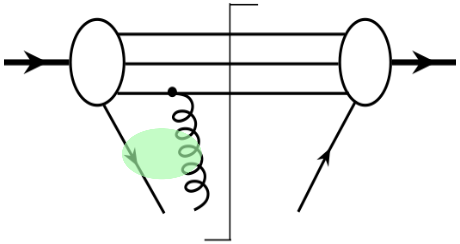
See Kang, Yuan, Zhou, 2010, Kang 2010

“Interpretation” of twist-3 correlation functions

❑ Measurement of direct QCD quantum interference:

Qiu, Sterman, 1991, …

$$T^{(3)}(x, x, S_\perp) \ \propto$$



Interference between a single active parton state and an active two-parton composite state

❑ “Expectation value” of QCD operators:

$$\langle P, s|\overline{\psi}(0)\gamma^+\psi(y^-)|P, s\rangle \longrightarrow \langle P, s|\overline{\psi}(0)\gamma^+ \left[\epsilon_\perp^{\alpha\beta} s_{T\alpha} \int dy_2^- F_\beta{}^+(y_2^-)\right] \psi(y^-)|P, s\rangle$$

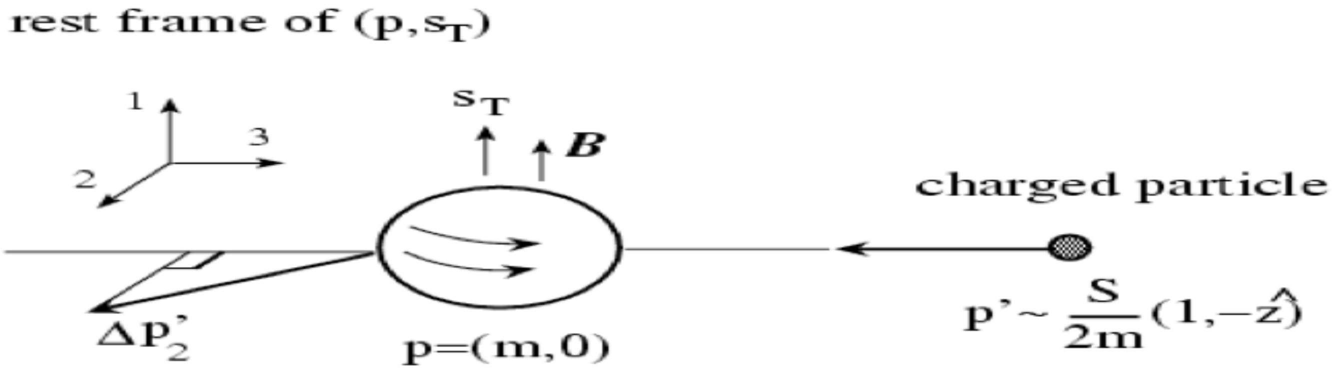
$$\langle P, s|\overline{\psi}(0)\gamma^+\gamma_5\psi(y^-)|P, s\rangle \longrightarrow \langle P, s|\overline{\psi}(0)\gamma^+ \left[i\, g_\perp^{\alpha\beta} s_{T\alpha} \int dy_2^- F_\beta{}^+(y_2^-)\right] \psi(y^-)|P, s\rangle$$

How to interpret the “expectation value” of the operators in **RED**?

A simple example

- The operator in Red – a classical Abelian case:

Qiu, Sterman, 1998



- Change of transverse momentum:

$$\frac{d}{dt}p_2' = e(\vec{v}' \times \vec{B})_2 = -ev_3B_1 = ev_3F_{23}$$

- In the c.m. frame:

$$(m,\vec{0}) \to \bar{n} = (1,0,0_T), \quad (1,-\hat{z}) \to n = (0,1,0_T)$$

$$\Longrightarrow \quad \frac{d}{dt}p_2' = e\,\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma^{\ +}$$

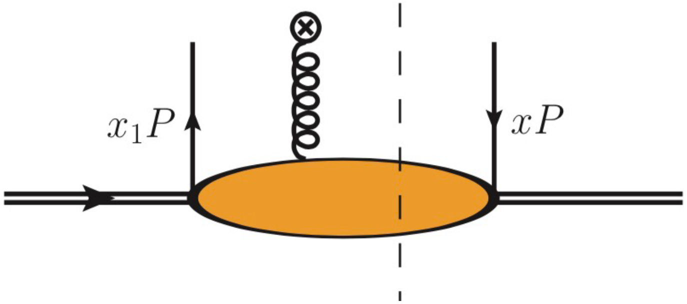
- The total change:

$$\boxed{\Delta p_2' = e\int dy^-\,\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma^{\ +}(y^-)}$$

Net quark transverse momentum imbalance caused by color Lorentz force inside a transversely polarized proton

Collinear twist-3 contribution to A_N

$$\begin{aligned} d\Delta\sigma(s_T) &\equiv d\sigma(s_T) - d\sigma(-s_T) \\ &= H \otimes \boxed{f_{a/A^\uparrow(3)}} \otimes f_{b/B(2)} \otimes D_{c/C(2)} \\ &+ H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} \\ &+ H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)} \end{aligned}$$



SGP

$T_{FT}(x,x)$

↓

Sivers-type function

SFP

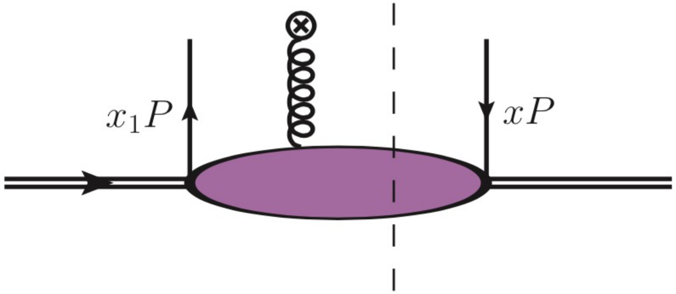
$T_{FT}(0,x), \ldots$

$G_{FT}(0,x), \ldots$

Also tri-gluon correlators at SG

Collinear twist-3 contribution to A_N

$$\begin{aligned} d\Delta\sigma(s_T) &\equiv d\sigma(s_T) - d\sigma(-s_T) \\ &= H \otimes f_{a/A^\uparrow(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} \\ &+ H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} \\ &+ H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)} \end{aligned}$$



SGP	**SFP**
$T_{FT}(x,x)$	$T_{FT}(0,x), \ldots$
$H_{FU}(x,x)$	$H_{FU}(0,x), \ldots$

↓

Boer-Mulders-type function

Collinear twist-3 contribution to A_N

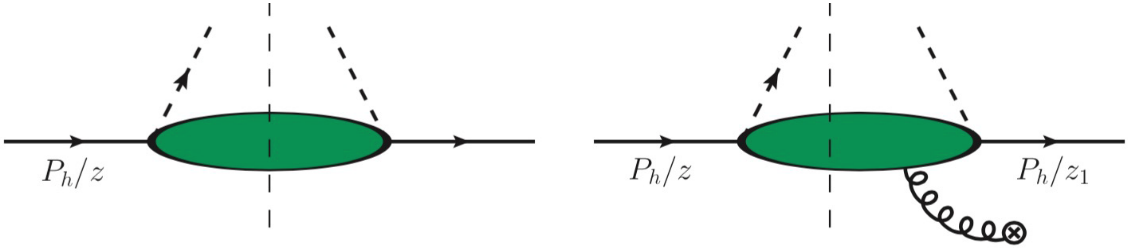
$$
\begin{aligned}
d\Delta\sigma(s_T) &\equiv d\sigma(s_T) - d\sigma(-s_T) \\
&= H \otimes f_{a/A^{\uparrow}(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} \\
&+ H' \otimes f_{a/A^{\uparrow}(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} \\
&+ H'' \otimes f_{a/A^{\uparrow}(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}
\end{aligned}
$$

SGP	**SFP**
$T_{FT}(x,x)$	$T_{FT}(0,x),\ldots$
$H_{FU}(x,x)$	$H_{FU}(0,x),\ldots$

$\hat{H}(z), H(z), \hat{H}_{FU}(z,z_1),\ldots$

↓

Collins-type function



Collinear twist-3 contribution to A_N

$$\begin{aligned} d\Delta\sigma(s_T) &\equiv d\sigma(s_T) - d\sigma(-s_T) \\ &= H \otimes \boxed{f_{a/A^\uparrow(3)}} \otimes f_{b/B(2)} \otimes D_{c/C(2)} \\ &+ H' \otimes f_{a/A^\uparrow(2)} \otimes \boxed{f_{b/B(3)}} \otimes D_{c/C(2)} \\ &+ H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes \boxed{D_{c/C(3)}} \end{aligned}$$

SGP	**SFP**
$T_{FT}(x,x)$	$T_{FT}(0,x),\ldots$
$H_{FU}(x,x)$	$H_{FU}(0,x),\ldots$
$\hat{H}(z), H(z), \hat{H}_{FU}(z,z_1),\ldots$	

❑ Early work (before 2013):

Assumed that SGP (Sivers-type) dominates the twist-3 contribution to TSSAs in:

$$p^\uparrow + p \to \pi(x_F, p_T) + X$$

Qiu, Sterman (1991, 98)

$$E_\ell \frac{d^3\Delta\sigma(\vec{s}_T)}{d^3\ell} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int_{z_{\min}}^{1} \frac{dz}{z^2} D_{c\to h}(z) \int_{x'_{\min}}^{1} \frac{dx'}{x'} \frac{1}{x'S + T/z} \phi_{b/B}(x')$$

$$\times \sqrt{4\pi\alpha_s} \left(\frac{\epsilon^{\ell s_T n \bar{n}}}{z\hat{u}}\right) \frac{1}{x} \left[T_{a,F}(x,x) - x\left(\frac{d}{dx} T_{a,F}(x,x)\right)\right] H_{ab\to c}(\hat{s},\hat{t},\hat{u})$$

- ✧ Growth in x_F
- ✧ Slow fall off in p_T

Collinear twist-3 contribution to A_N

$$
\begin{aligned}
d\Delta\sigma(s_T) \equiv d\sigma(s_T) - d\sigma(-s_T)& \\
&= H \otimes f_{a/A^{\uparrow}(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} \\
&\boxed{+\, H' \otimes f_{a/A^{\uparrow}(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)}} \\
&+\, H'' \otimes f_{a/A^{\uparrow}(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}
\end{aligned}
$$

→ Negligible
Kanazawa & Koike (2000)

Collinear twist-3 contribution to A_N

$$d\Delta\sigma(s_T) \equiv d\sigma(s_T) - d\sigma(-s_T)$$

$$= H \otimes f_{a/A^\uparrow(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)}$$

$$\boxed{+ H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)}}$$

→ **Negligible**

Kanazawa & Koike (2000)

$$\boxed{+ H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}}$$

→ **Important**

Metz & Pitonyak (2013)

❑ Twist-3 fragmentation contribution:

$$\frac{P_h^0\, d\sigma_{pol}}{d^3\vec{P}_h} = -\frac{2\alpha_s^2 M_h}{S}\,\epsilon_{\perp\mu\nu}\,S_\perp^\mu P_{h\perp}^\nu \sum_i \sum_{a,b,c} \int_{z_{min}}^1 \frac{dz}{z^3} \int_{x'_{min}}^1 \frac{dx'}{x'}\,\frac{1}{x'S + T/z}\,\frac{1}{-x\hat{u} - x'\hat{t}}$$

$$\times\, \frac{1}{x}\,h_1^a(x)\,f_1^b(x') \left\{ \left(\hat{H}^{C/c}(z) - z\frac{d\hat{H}^{C/c}(z)}{dz} \right) S_{\hat{H}}^i + \frac{1}{z}\,H^{C/c}(z)\,S_H^i \right.$$

$$\left. + 2z^2 \int \frac{dz_1}{z_1^2}\,PV\frac{1}{\frac{1}{z} - \frac{1}{z_1}}\,\hat{H}_{FU}^{C/c,\Im}(z,z_1)\,\frac{1}{\xi}\,S_{\hat{H}_{FU}}^i \right\}$$

$$2z^3 \int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z} - \frac{1}{z_1}}\,\hat{H}_{FU}^{\Im}(z,z_1) = H(z) + 2z\hat{H}(z)$$

3-parton correlator

$$\hat{H}(z) = H_1^{\perp(1)}(z)$$

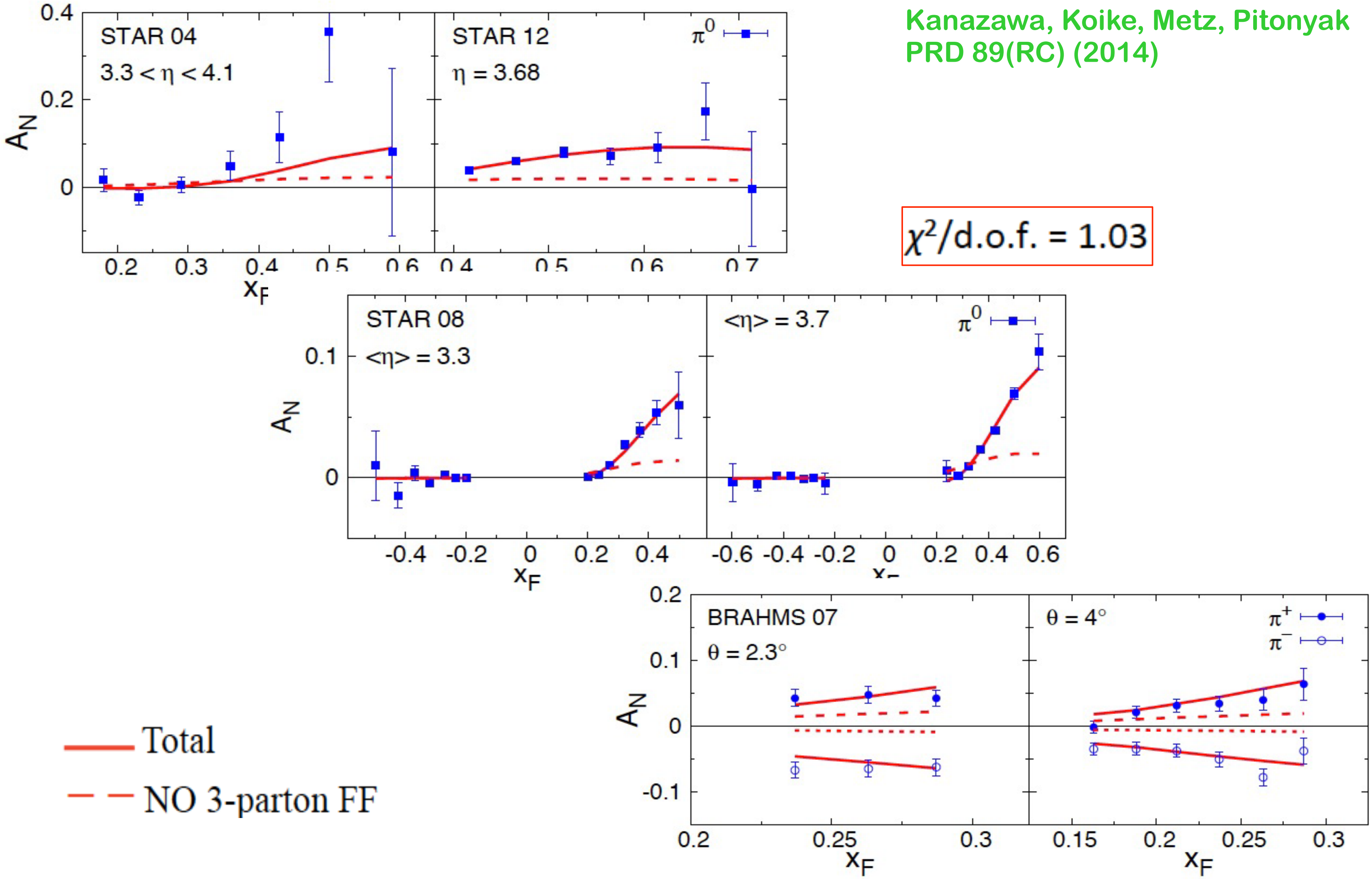
Collins-type function

Collinear twist-3 contribution to A_N

❑ **Fragmentation + QS (fix through Sivers function):**

Kanazawa, Koike, Metz, Pitonyak
PRD 89(RC) (2014)

χ^2/d.o.f. = 1.03



Multi-gluon correlation functions

- ☐ **Diagonal tri-gluon correlations:**

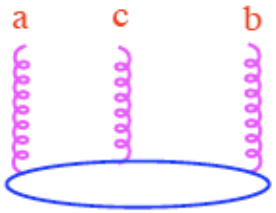
Ji, PLB289 (1992)

$$T_G(x,x) = \int \frac{dy_1^- dy_2^-}{2\pi} e^{ixP^+y_1^-} \times \frac{1}{xP^+} \langle P, s_\perp | F^+{}_\alpha(0) \left[\epsilon^{s_\perp \sigma n \bar{n}} F_\sigma{}^+(y_2^-)\right] F^{\alpha +}(y_1^-) | P, s_\perp \rangle$$

- ☐ **Two tri-gluon correlation functions – color contraction:**

$$T_G^{(f)}(x,x) \propto i f^{ABC} F^A F^C F^B = F^A F^C (\mathcal{T}^C)^{AB} F^B$$

$$T_G^{(d)}(x,x) \propto d^{ABC} F^A F^C F^B = F^A F^C (\mathcal{D}^C)^{AB} F^B$$



Quark-gluon correlation: $T_F(x,x) \propto \overline{\psi}_i F^C (T^C)_{ij} \psi_j$

- ☐ **D-meson production at EIC:**
  - ✧ Clean probe for gluonic twist-3 correlation functions
  - ✧ $T_G^{(f)}(x,x)$ could be connected to the gluonic Sivers function

Test QCD at twist-3 level

Kang, Qiu, 2009

❑ Scaling violation – "DGLAP" evolution:

$$\mu_F^2\frac{\partial}{\partial\mu_F^2}\underbrace{\begin{bmatrix}\widetilde{\mathcal{T}}_{q,F}\\ \widetilde{\mathcal{T}}_{\Delta q,F}\\ \widetilde{\mathcal{T}}^{(f)}_{G,F}\\ \widetilde{\mathcal{T}}^{(d)}_{G,F}\\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F}\\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F}\end{bmatrix}}_{(x,\,x+x_2,\,\mu,\,s_T)} = \underbrace{\begin{pmatrix} K_{qq} & K_{q\Delta q} & K^{(f)}_{qG} & K^{(d)}_{qG} & K^{(f)}_{q\Delta G} & K^{(d)}_{q\Delta G}\\ K_{\Delta qq} & K_{\Delta q\Delta q} & K^{(f)}_{\Delta qG} & K^{(d)}_{\Delta qG} & K^{(f)}_{\Delta q\Delta G} & K^{(d)}_{\Delta q\Delta G}\\ K^{(f)}_{Gq} & K^{(f)}_{G\Delta q} & K^{(ff)}_{GG} & K^{(fd)}_{GG} & K^{(ff)}_{G\Delta G} & K^{(fd)}_{G\Delta G}\\ K^{(d)}_{Gq} & K^{(d)}_{G\Delta q} & K^{(df)}_{GG} & K^{(dd)}_{GG} & K^{(df)}_{G\Delta G} & K^{(dd)}_{G\Delta G}\\ K^{(f)}_{\Delta Gq} & K^{(f)}_{\Delta G\Delta q} & K^{(ff)}_{\Delta GG} & K^{(fd)}_{\Delta GG} & K^{(ff)}_{\Delta G\Delta G} & K^{(fd)}_{\Delta G\Delta G}\\ K^{(d)}_{\Delta Gq} & K^{(d)}_{\Delta G\Delta q} & K^{(df)}_{\Delta GG} & K^{(dd)}_{\Delta GG} & K^{(df)}_{\Delta G\Delta G} & K^{(dd)}_{\Delta G\Delta G}\end{pmatrix}}_{(\xi,\,\xi+\xi_2;\,x,\,x+x_2,\,\alpha_s)} \otimes \underbrace{\begin{bmatrix}\widetilde{\mathcal{T}}_{q,F}\\ \widetilde{\mathcal{T}}_{\Delta q,F}\\ \widetilde{\mathcal{T}}^{(f)}_{G,F}\\ \widetilde{\mathcal{T}}^{(d)}_{G,F}\\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F}\\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F}\end{bmatrix}}_{\int d\xi \int d\xi_2}$$

❑ Evolution equation – consequence of factorization:

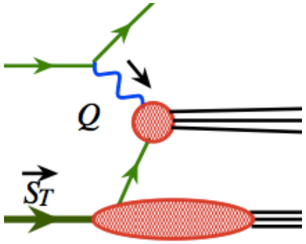
Factorization: $\Delta\sigma(Q, s_T) = (1/Q)H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F)$

DGLAP for f_2: $\frac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F).$

Evolution for f_3: $\frac{\partial}{\partial \ln(\mu_F)} f_3 = \left(\frac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)}\right) \otimes f_3$

Current understanding of TSSAs

- Symmetry plays important role:

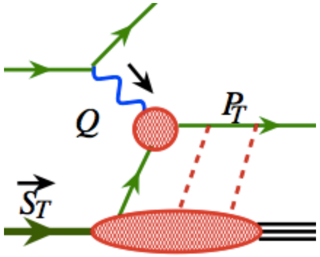


Inclusive DIS
Single scale
Q

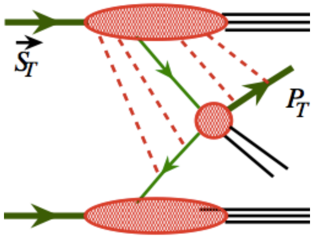
Parity
Time-reversal

$A_N = 0$

- One scale observables – $Q >> \Lambda_{QCD}$:



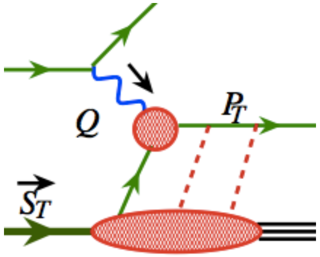
SIDIS: $Q \sim P_T$



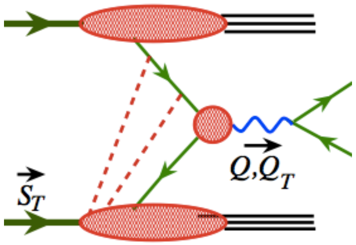
DY: $Q \sim P_T$; Jet, Particle: P_T

Collinear factorization
Twist-3 distributions

- Two scales observables – $Q_1 >> Q_2 \sim \Lambda_{QCD}$:



SIDIS: $Q>>P_T$

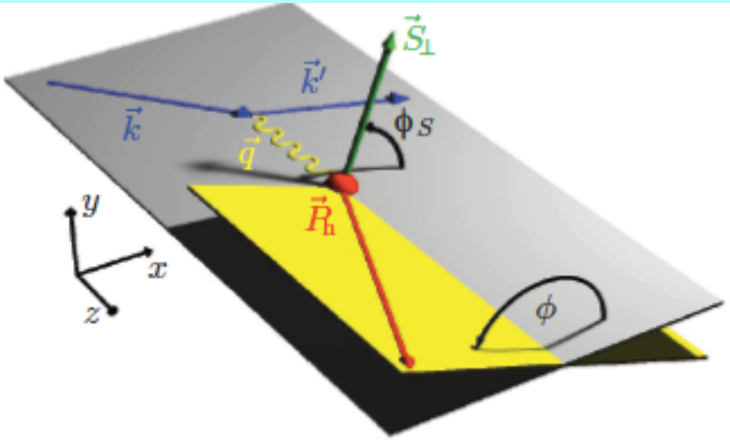


DY: $Q>>P_T$ or $Q<<P_T$

TMD factorization
TMD distributions

Brodsky et al. explicit
calculation with m_q=\=0

Semi-inclusive DIS (SIDIS)



☐ **Process:**

$$e(k) + N(p) \longrightarrow e'(k') + h(P_h) + X$$

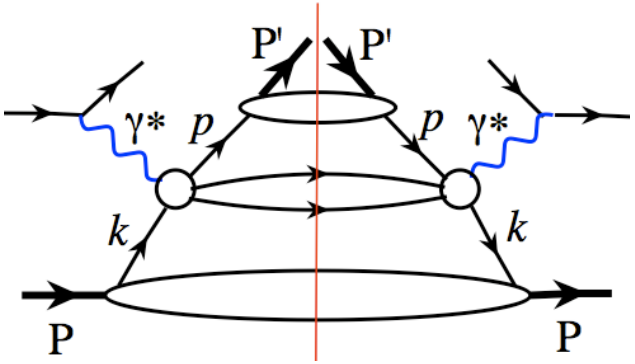
☐ **Natural event structure:**

In the photon-hadron frame: $P_{h_T} \approx 0$

Semi-Inclusive DIS is a natural observable with TWO very different scales

$Q \gg P_{h_T} \gtrsim \Lambda_{\rm QCD}$ **Localized probe sensitive to parton's transverse motion**

☐ **Collinear QCD factorization holds if P_{hT} integrated:**



Single hard scale!

$$d\sigma_{\gamma^* h\to h'} \propto \phi_{f/h} \otimes d\hat{\sigma}_{\gamma^* f\to f'} \otimes D_{f'\to h'}(z)$$

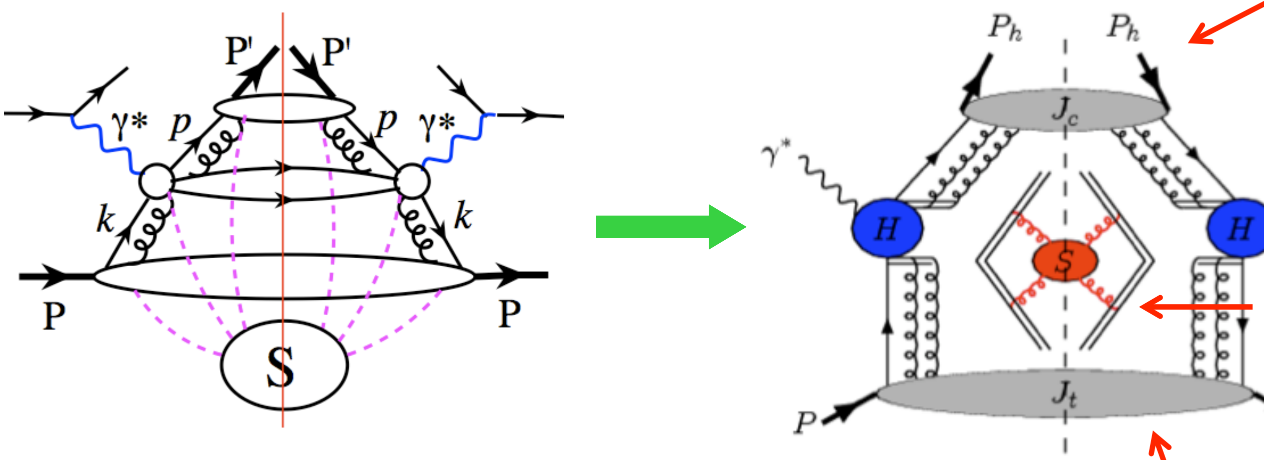
$$z = \frac{P_h \cdot p}{q \cdot p} \qquad y = \frac{q \cdot p}{k \cdot p}$$

☐ **"Total c.m. energy":** $s_{\gamma^* p} = (p+q)^2 \approx Q^2 \left[\frac{1-x_B}{x_B}\right] \approx \frac{Q^2}{x_B}$

Definitions of TMDs

❑ **Perturbative definition – in terms of TMD factorization:**

SIDIS as an example:



TMD fragmentation

$$+\,\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2},\frac{\langle p^2\rangle}{Q^2}\right)$$

Soft factors

TMD parton distribution

❑ **Low P_{hT} – TMD factorization:**

$$\sigma_{\rm SIDIS}(Q,P_{h\perp},x_B,z_h)=\hat{H}(Q)\otimes\Phi_f(x,k_\perp)\otimes\mathcal{D}_{f\to h}(z,p_\perp)\otimes\mathcal{S}(k_{s\perp})+\mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

❑ **High P_{hT} – Collinear factorization:**

$$\sigma_{\rm SIDIS}(Q,P_{h\perp},x_B,z_h)=\hat{H}(Q,P_{h\perp},\alpha_s)\otimes\phi_f\otimes D_{f\to h}+\mathcal{O}\left(\frac{1}{P_{h\perp}},\frac{1}{Q}\right)$$

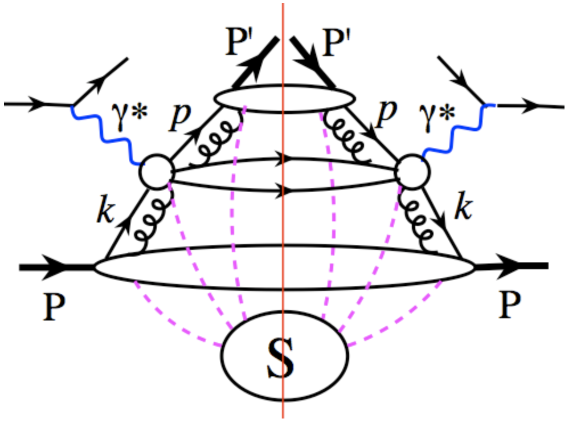
❑ **P_{hT} Integrated - Collinear factorization:**

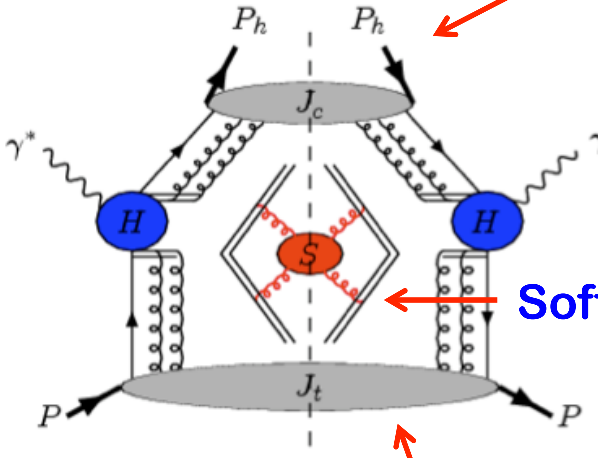
$$\sigma_{\rm SIDIS}(Q,x_B,z_h)=\tilde{H}(Q,\alpha_s)\otimes\phi_f\otimes D_{f\to h}+\mathcal{O}\left(\frac{1}{Q}\right)$$

Definitions of TMDs

- **Perturbative definition – in terms of TMD factorization:**

SIDIS as an example:





TMD fragmentation

$$+\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2},\frac{\langle p^2\rangle}{Q^2}\right)$$

Soft factors

TMD parton distribution

- **Extraction of TMDs:**

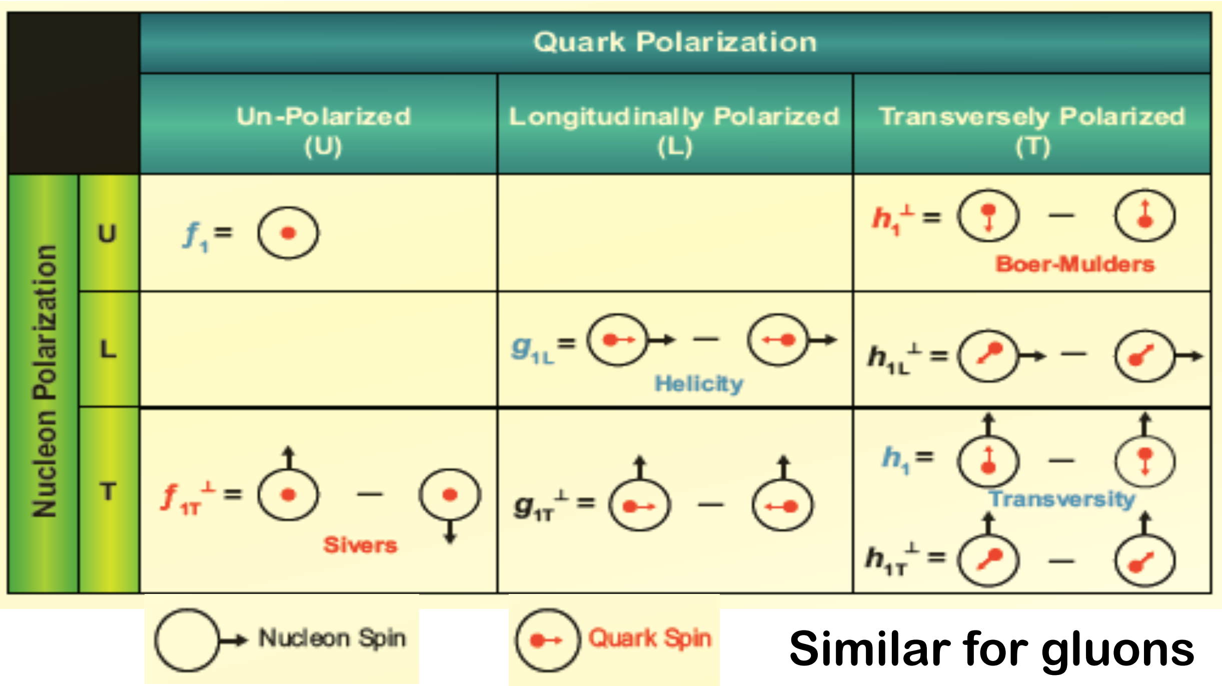
$$\sigma_{\text{SIDIS}}(Q,P_{h\perp},x_B,z_h)=\hat{H}(Q)\otimes\Phi_f(x,k_\perp)\otimes\mathcal{D}_{f\to h}(z,p_\perp)\otimes\mathcal{S}(k_{s\perp})+\mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

TMDs are extracted by fitting DATA using the factorization formula (approximation) and the perturbatively calculated $\hat{H}(Q;\mu)$.

→ Extracted TMDs are valid only when the <p^2> << Q^2

The Present: TMDs

- **Power of spin – many more correlations:**

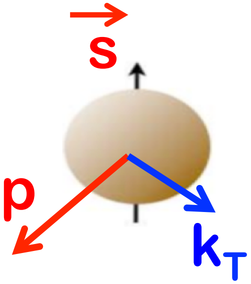


		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 =		$h_1^\perp$ = − Boer-Mulders
	L		g_{1L} = − Helicity	$h_{1L}^\perp$ = −
	T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = −	h_1 = − Transversity $h_{1T}^\perp$ = −

Nucleon Spin

Quark Spin

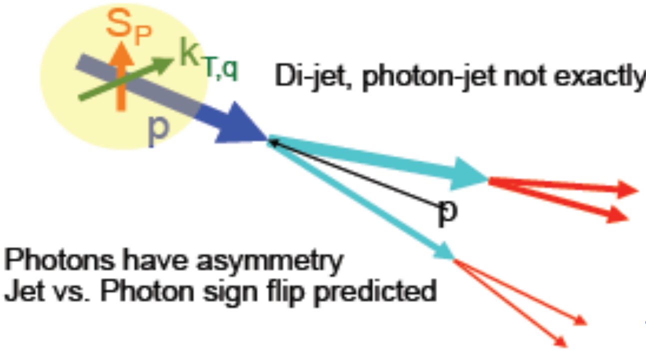
Similar for gluons



Require two Physical scales

More than one TMD contribute to the same observable!

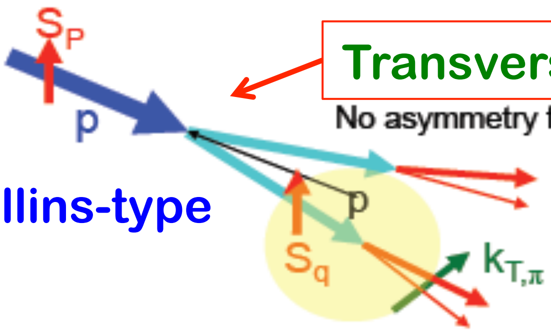
- **A_N – single hadron production:**



Di-jet, photon-jet not exactly back to back

Photons have asymmetry
Jet vs. Photon sign flip predicted

Sivers-type



Transversity

No asymmetry for the jet axis

Collins-type

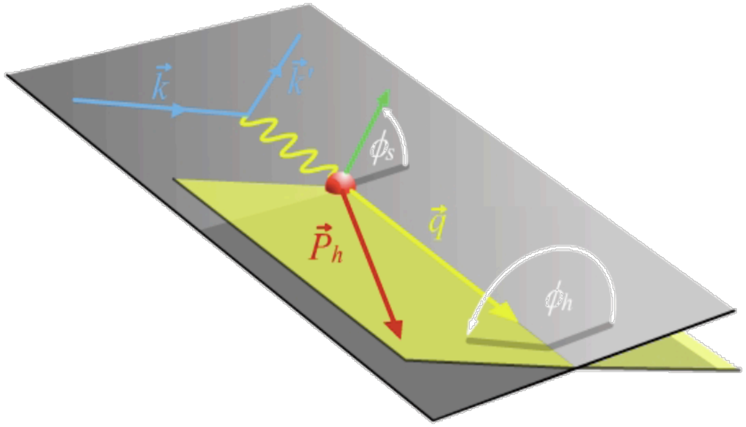
SIDIS is the best for probing TMDs

☐ **Naturally, two scales & two planes:**

$$A_{UT}(\varphi_h^l,\varphi_S^l) = \frac{1}{P}\frac{N^\uparrow - N^\downarrow}{N^\uparrow + N^\downarrow}$$

$$= A_{UT}^{Collins}\sin(\phi_h+\phi_S) + A_{UT}^{Sivers}\sin(\phi_h-\phi_S)$$

$$+ A_{UT}^{Pretzelosity}\sin(3\phi_h-\phi_S)$$

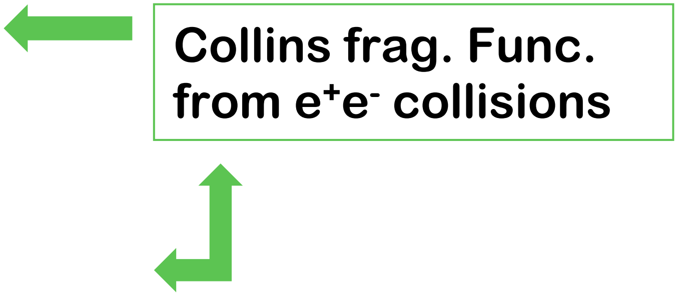


☐ **Separation of TMDs:**

$$A_{UT}^{Collins} \propto \langle\sin(\phi_h+\phi_S)\rangle_{UT} \propto h_1 \otimes H_1^\perp$$

$$A_{UT}^{Sivers} \propto \langle\sin(\phi_h-\phi_S)\rangle_{UT} \propto f_{1T}^\perp \otimes D_1$$

$$A_{UT}^{Pretzelosity} \propto \langle\sin(3\phi_h-\phi_S)\rangle_{UT} \propto h_{1T}^\perp \otimes H_1^\perp$$



Collins frag. Func. from e^+e^- collisions

Hard, if not impossible, to separate TMDs in hadronic collisions

Using a combination of different observables (not the same observable): jet, identified hadron, photon, …

Evolution equations for TMDs

J.C. Collins, in his book on QCD

❑ TMDs in the b-space:

$$\tilde{F}_{f/P^{\uparrow}}(x, \mathrm{b_T}, S; \mu; \zeta_F) = \tilde{F}^{\mathrm{unsub}}_{f/P^{\uparrow}}(x, \mathrm{b_T}, S; \mu; y_P - (-\infty))\sqrt{\frac{\tilde{S}_{(0)}(\mathrm{b_T}; +\infty, y_s)}{\tilde{S}_{(0)}(\mathrm{b_T}; +\infty, -\infty)\tilde{S}_{(0)}(\mathrm{b_T}; y_s, -\infty)}} Z_F \, Z_2$$

❑ Collins-Soper equation:

Renormalization of the soft-factor

$$\frac{\partial \tilde{F}_{f/P^{\uparrow}}(x, \mathrm{b_T}, S; \mu; \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(b_T; \mu)\tilde{F}_{f/P^{\uparrow}}(x, \mathrm{b_T}, S; \mu; \zeta_F)$$

$$\zeta_F = M_P^2 x^2 e^{2(y_P - y_s)}$$

$$\tilde{K}(b_T; \mu) = \frac{1}{2}\frac{\partial}{\partial y_s} \ln\left(\frac{\tilde{S}(b_T; y_s, -\infty)}{\tilde{S}(b_T; +\infty, y_s)}\right)$$

Introduced to regulate the rapidity divergence of TMDs

❑ RG equations:

Wave function Renormalization

$$\frac{d\tilde{K}(b_T; \mu)}{d \ln \mu} = -\gamma_K(g(\mu))$$

$$\frac{d\tilde{F}_{f/P^{\uparrow}}(x, \mathrm{b_T}, S; \mu; \zeta_F)}{d \ln \mu} = \gamma_F(g(\mu); \zeta_F/\mu^2)\tilde{F}_{f/P^{\uparrow}}(x, \mathrm{b_T}, S; \mu; \zeta_F).$$

Evolution equations are only valid when $b_T << 1/\Lambda_{QCD}$!

❑ Momentum space TMDs:

Need information at large b_T for all scale μ !

$$F_{f/P^{\uparrow}}(x, \mathrm{k_T}, S; \mu, \zeta_F) = \frac{1}{(2\pi)^2}\int d^2\mathrm{b}_T \, e^{i\mathrm{k}_T \cdot \mathrm{b}_T} \, \tilde{F}_{f/P^{\uparrow}}(x, \mathrm{b_T}, S; \mu, \zeta_F)$$

Evolution equations for Sivers function

Aybat, Collins, Qiu, Rogers, 2011

☐ Sivers function:

$$F_{f/P^\uparrow}(x,k_T,S;\mu,\zeta_F) = F_{f/P}(x,k_T;\mu,\zeta_F) - F_{1T}^{\perp f}(x,k_T;\mu,\zeta_F)\frac{\epsilon_{ij}k_T^i S^j}{M_p}$$

☐ Collins-Soper equation:

Its derivative obeys the CS equation

$$\frac{\partial \ln \tilde{F}'^{\perp f}_{1T}(x,b_T;\mu,\zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(b_T;\mu)$$

$$\tilde{F}'^{\perp f}_{1T}(x,b_T;\mu,\zeta_F) \equiv \frac{\partial \tilde{F}^{\perp f}_{1T}(x,b_T;\mu,\zeta_F)}{\partial b_T}$$

☐ RG equations:

$$\frac{d\tilde{F}'^{\perp f}_{1T}(x,b_T;\mu,\zeta_F)}{d\ln\mu} = \gamma_F(g(\mu);\zeta_F/\mu^2)\tilde{F}'^{\perp f}_{1T}(x,b_T;\mu,\zeta_F)$$

$$\frac{d\tilde{K}(b_T;\mu)}{d\ln\mu} = -\gamma_K(g(\mu)) \quad \Longrightarrow \quad \frac{\partial\gamma_F(g(\mu);\zeta_F/\mu^2)}{\partial\ln\sqrt{\zeta_F}} = -\gamma_K(g(\mu)),$$

☐ Sivers function in momentum space:

$$F_{1T}^{\perp f}(x,k_T;\mu,\zeta_F) = \frac{-1}{2\pi k_T}\int_0^\infty db_T\, b_T\, J_1(k_T b_T)\tilde{F}'^{\perp f}_{1T}(x,b_T;\mu,\zeta_F)$$

JI, Ma, Yuan, 2004
Idilbi, et al, 2004,
Boer, 2001, 2009,
Kang, Xiao, Yuan, 2011
Aybat, Prokudin, Rogers, 2012
Idilbi, et al, 2012,
Sun, Yuan 2013, …

Extrapolation to large b_T

❑ CSS b*-prescription:

Aybat and Rogers, arXiv:1101.5057
Collins and Rogers, arXiv:1412.3820

$$\tilde{F}_{f/P}(x, b_T; Q, Q^2) = \overbrace{\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b)}^{\text{AA}}$$

$$\times \overbrace{\times \exp\left\{ \ln\frac{Q}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln\frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\}}^{\text{BB}}$$

$$\times \boxed{\overbrace{\exp\left\{ g_{f/P}(x, b_T) + g_K(b_T) \ln\frac{Q}{Q_0} \right\}}^{\text{CC}}}$$

Nonperturbative “form factor”

$$b_* = \frac{b_\mathrm{T}}{\sqrt{1 + b_\mathrm{T}^2 / b_\mathrm{max}^2}} \qquad \text{with } b_\mathrm{max} \sim 1/2 \ \mathrm{GeV}^{-1}$$

❑ Nonperturbative fitting functions

Various fits correspond to different choices for $g_{f/P}(x, b_T)$ **and** $g_K(b_T)$
e.g.

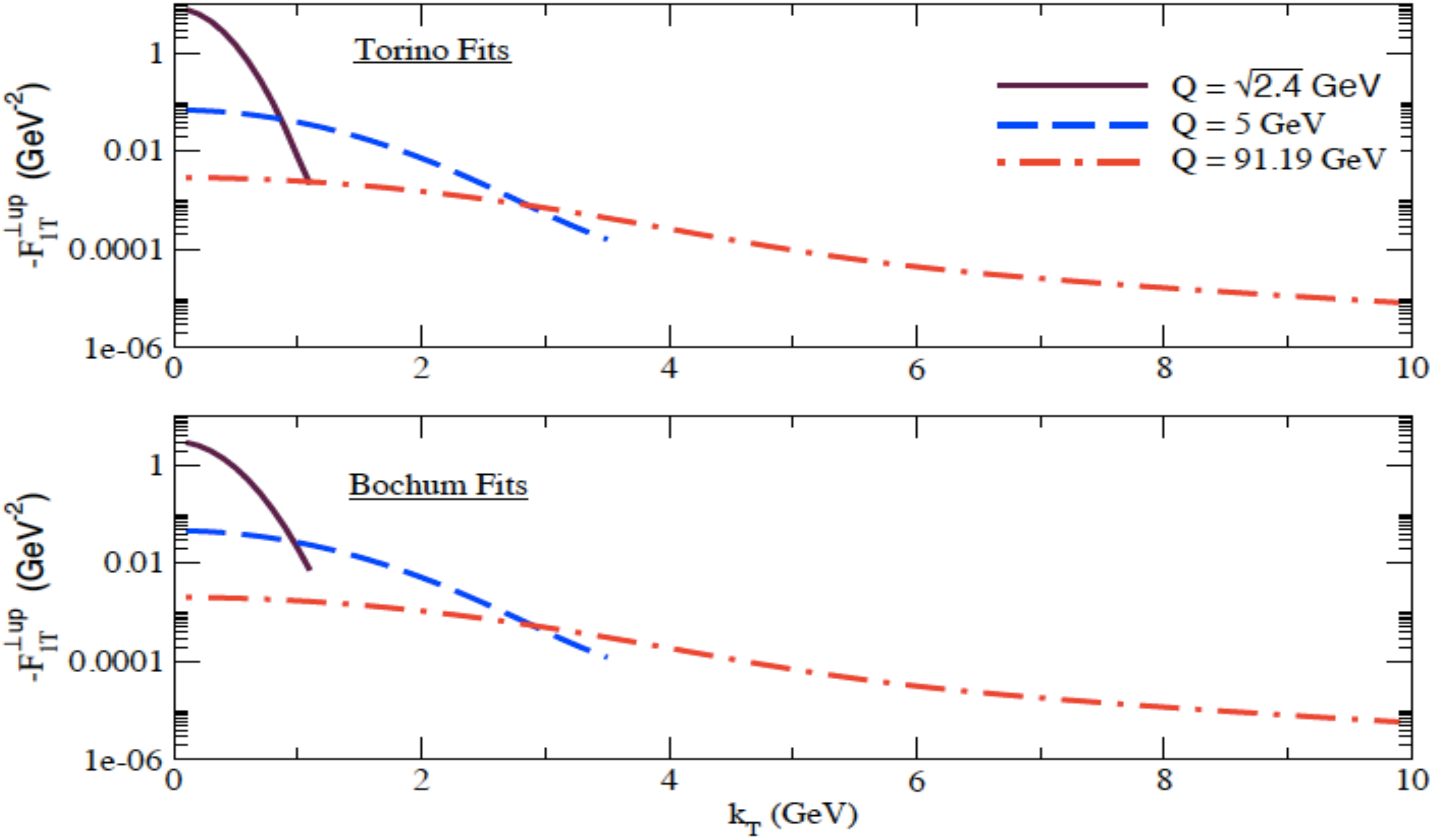
$$g_{f/P}(x, b_T) + g_K(b_T) \ln\frac{Q}{Q_0} \equiv -\left[g_1 + g_2 \ln\frac{Q}{2Q_0} + g_1 g_3 \ln(10x) \right] b_T^2$$

Different choice of g2 & b_* could lead to different over all Q-dependence!

Evolution of Sivers function

Aybat, Collins, Qiu, Rogers, 2011

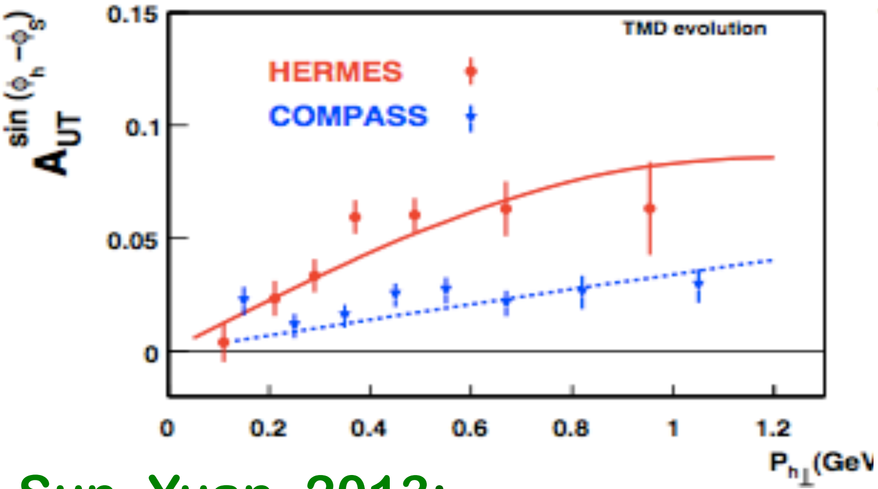
- ❑ **Up quark Sivers function:**

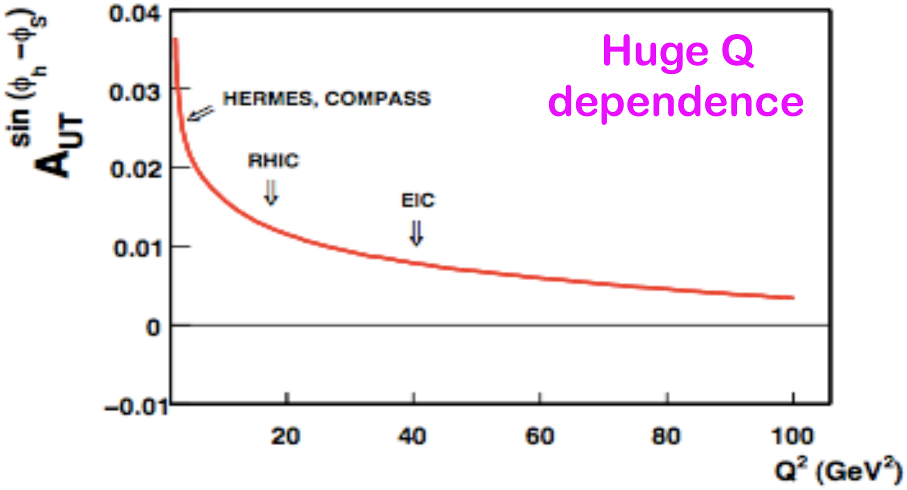


Very significant growth in the width of transverse momentum

Different fits – different Q-dependence

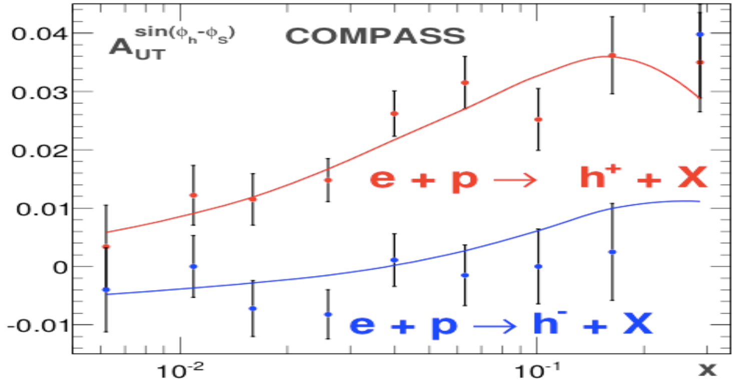
❑ Aybat, Prokudin, Rogers, 2012:

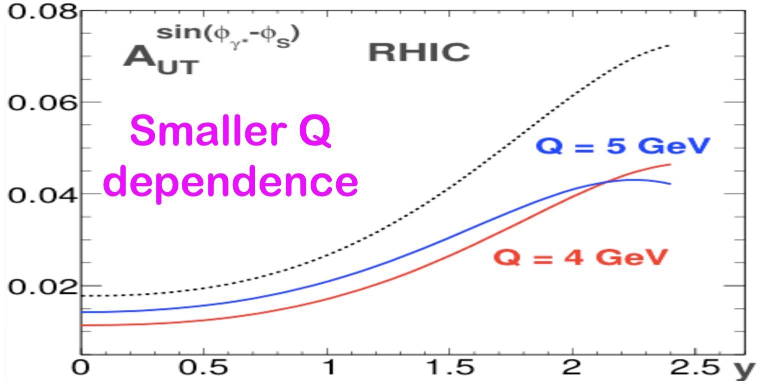




Huge Q dependence

❑ Sun, Yuan, 2013:





Smaller Q dependence

No disagreement on evolution equations!

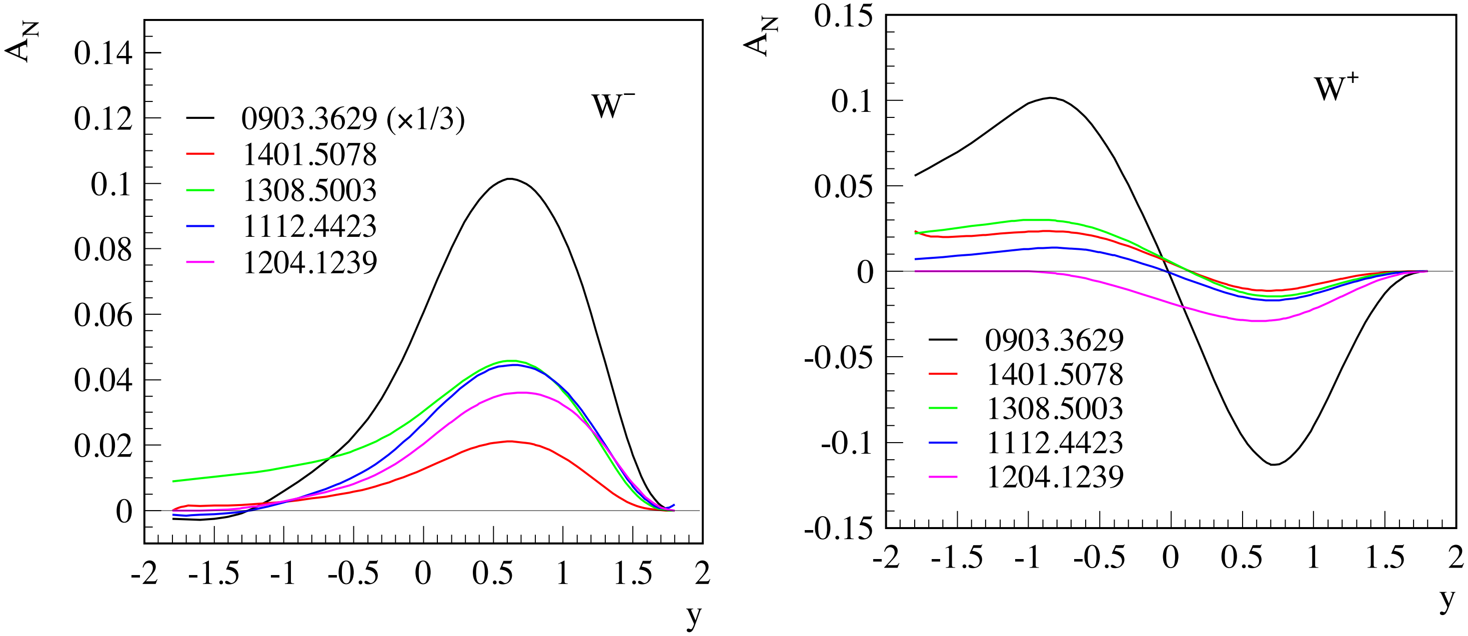
Issues: Extrapolation to non-perturbative large b-region
Choice of the Q-dependent "form factor"

"Predictions" for A_N of W-production at RHIC?

❑ **Sivers Effect:**

- ✧ Quantum correlation between the spin direction of colliding hadron and the preference of motion direction of its confined partons
- ✧ QCD Prediction: Sign change of Sivers function from SIDIS and DY

❑ **Current "prediction" and uncertainty of QCD evolution:**



TMD collaboration proposal: Lattice, theory & Phenomenology

RHIC is the excellent and unique facility to test this (W/Z – DY)!

What happened?

- ❑ **Sivers function:**

Differ from PDFs!

$$F_{1T}^{\perp f}(x,k_T;\mu,\zeta_F) = \frac{-1}{2\pi k_T}\int_0^\infty db_T\, b_T\, J_1(k_T b_T)\,\tilde{F}_{1T}^{\prime\perp f}(x,b_T;\mu,\zeta_F)$$

Need non-perturbative large b_T information for any value of Q! ***$Q = \mu$***

- ❑ **What is the “correct” Q-dependence of the large b_T tail?**

$$\tilde{F}_{f/P}(x,\mathrm{b}_T;Q,Q^2) = \overbrace{\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}}\tilde{C}_{f/j}(x/\hat{x},b_*;\mu_b^2,\mu_b,g(\mu_b))\,f_{j/P}(\hat{x},\mu_b)}^{\mathrm{AA}}$$

$$\times \overbrace{\times\exp\left\{\ln\frac{Q}{\mu_b}\tilde{K}(b_*;\mu_b)+\int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[\gamma_F(g(\mu');1)-\ln\frac{Q}{\mu'}\gamma_K(g(\mu'))\right]\right\}}^{\mathrm{BB}}$$

$$\times \boxed{\overbrace{\exp\left\{g_{f/P}(x,b_T)+g_K(b_T)\ln\frac{Q}{Q_0}\right\}}^{\mathrm{CC}}}$$

Nonperturbative “form factor”

$$g_{f/P}(x,b_T)+g_K(b_T)\ln\frac{Q}{Q_0} \equiv -\left[g_1+g_2\ln\frac{Q}{2Q_0}+g_1g_3\ln(10x)\right]b_T^2$$

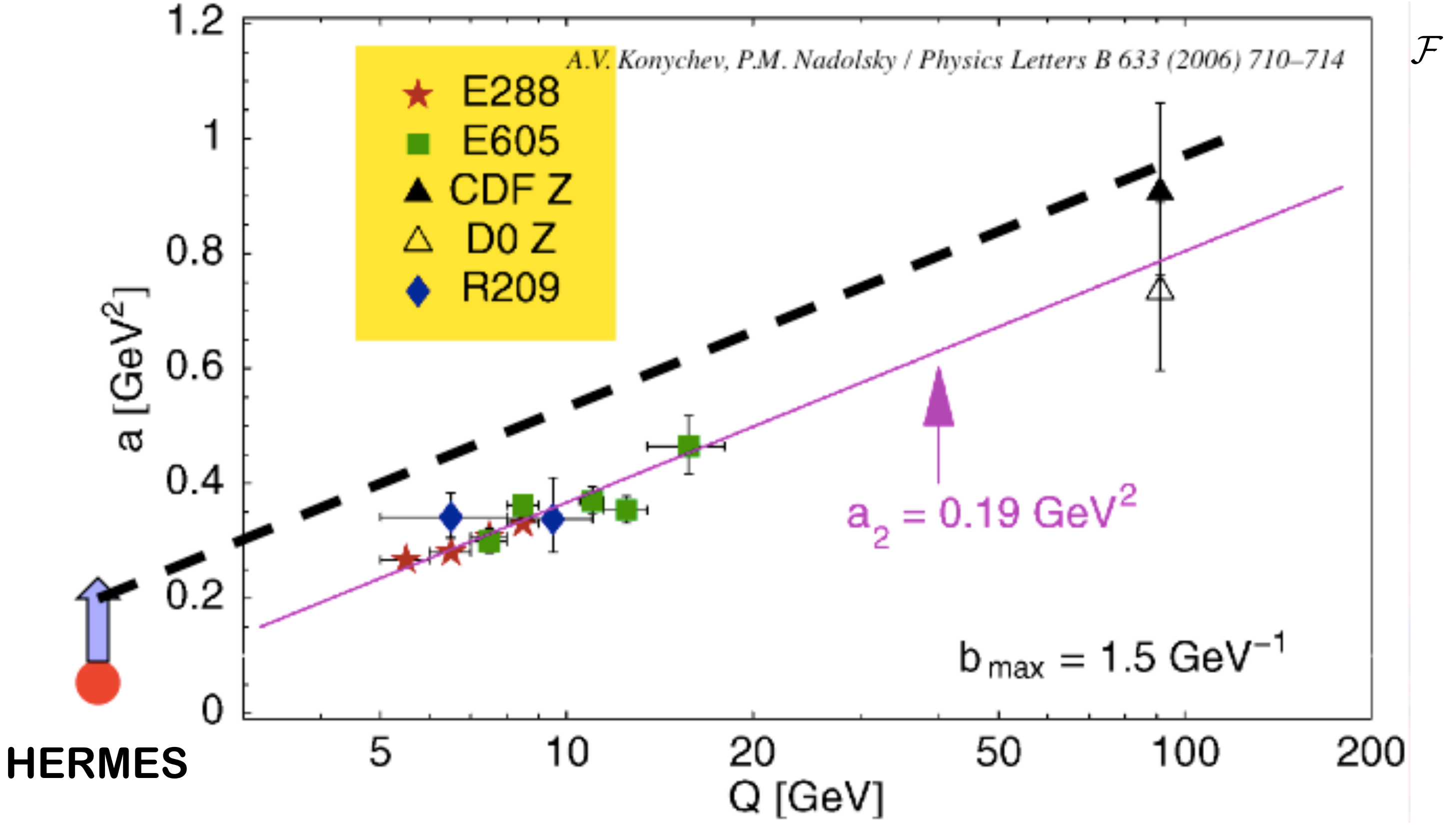
Is the log(Q) dependence sufficient? Choice of g_2 & b_* affects Q-dep.

The “form factor” and b_* change perturbative results at small b_T!

Q-dependence of the “form” factor

Konychev, Nadolsky, 2006

- ☐ **Q-dependence of the “form factor” :**



$$\mathcal{F}^{\rm NP}(b,Q) = a(Q^2)\, b^2$$

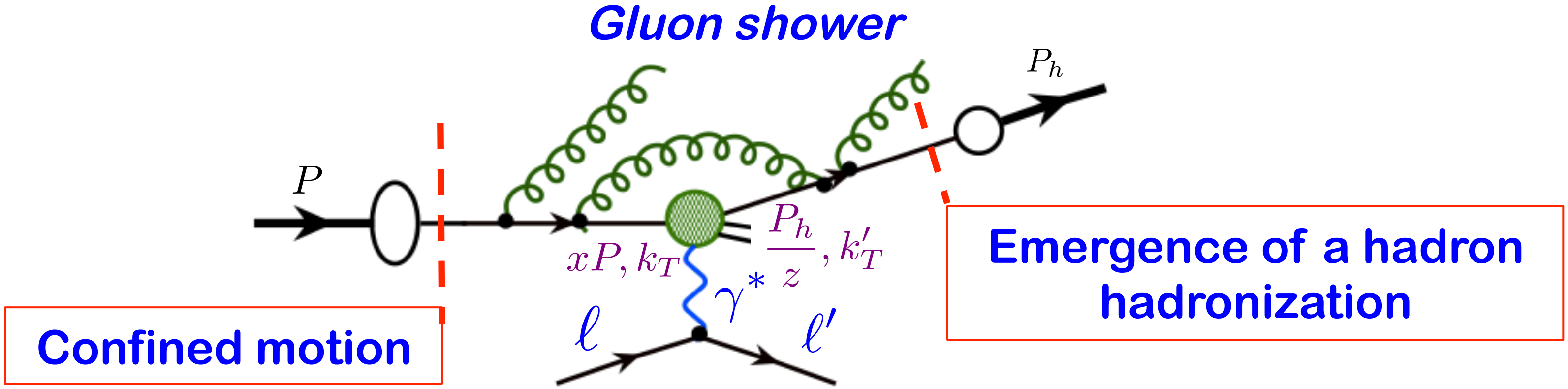
At Q ~ 1 GeV, ln(Q/Q$_0$) term may not be the dominant one!

$$\mathcal{F}^{\rm NP} \approx b^2(a_1 + a_2 \ln(Q/Q_0) + a_3 \ln(x_A x_B) + \ldots) + \ldots$$

Power correction? ***($Q_0/Q)^n$-term?*** ***Better fits for HERMES data?***

Parton k_T at the hard collision

- ☐ **Sources of parton k_T at the hard collision:**



- ☐ **Large k_T generated by the shower (caused by the collision):**
  - ✧ Q^2-dependence – linear evolution equation of TMDs in b-space
  - ✧ The evolution kernels are perturbative at small b, but, not large b

➡ The nonperturbative inputs at large b could impact TMDs at all Q^2

- ☐ **Challenge: to extract the "true" parton's confined motion:**
  - ✧ Separation of perturbative shower contribution from nonperturbative hadron structure – not as simple as PDFs

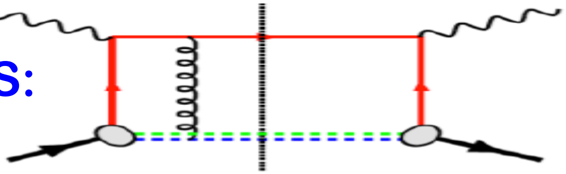
Broken universality for TMDs

❑ **Definition:**

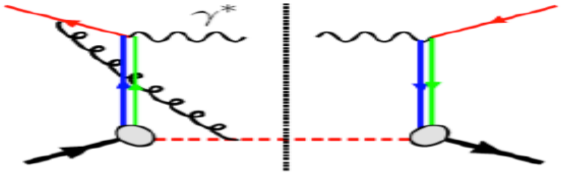
$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3}\, e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp)\boxed{\text{Gauge link}}\frac{\gamma^+}{2}\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$

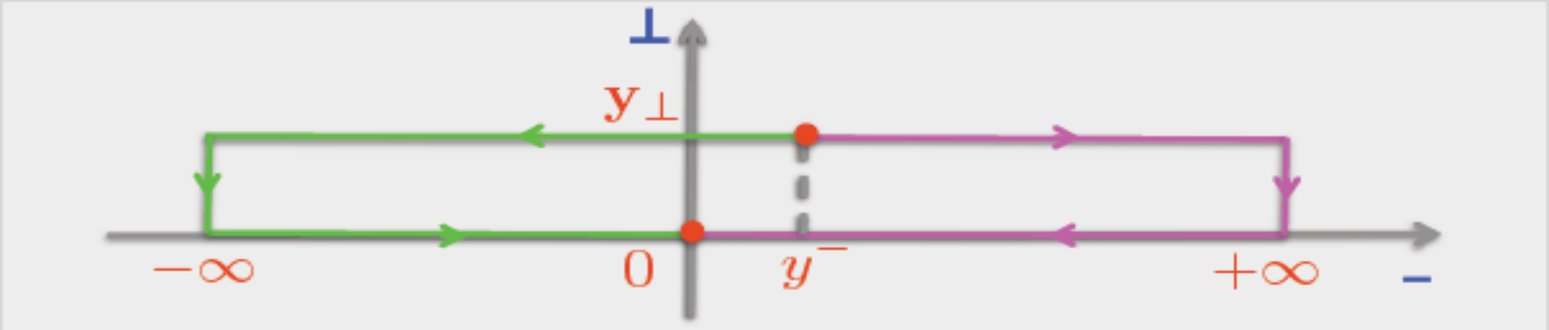
❑ **Gauge links:**

SIDIS:



DY:





❑ **Process dependence:**

$$f^{\text{SIDIS}}_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \neq f^{\text{DY}}_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S})$$

Collinear factorized PDFs are process independent

Critical test of TMD factorization

- ☐ **Parity – Time reversal invariance:**

$$f_{q/h^\uparrow}^{\rm SIDIS}(x,\mathbf{k}_\perp,\vec{S}) = f_{q/h^\uparrow}^{\rm DY}(x,\mathbf{k}_\perp,-\vec{S})$$

- ☐ **Definition of Sivers function:**

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \equiv f_{q/h}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/h^\uparrow}(x,k_\perp)\,\vec{S}\cdot\hat{p}\times\hat{\mathbf{k}}_\perp$$

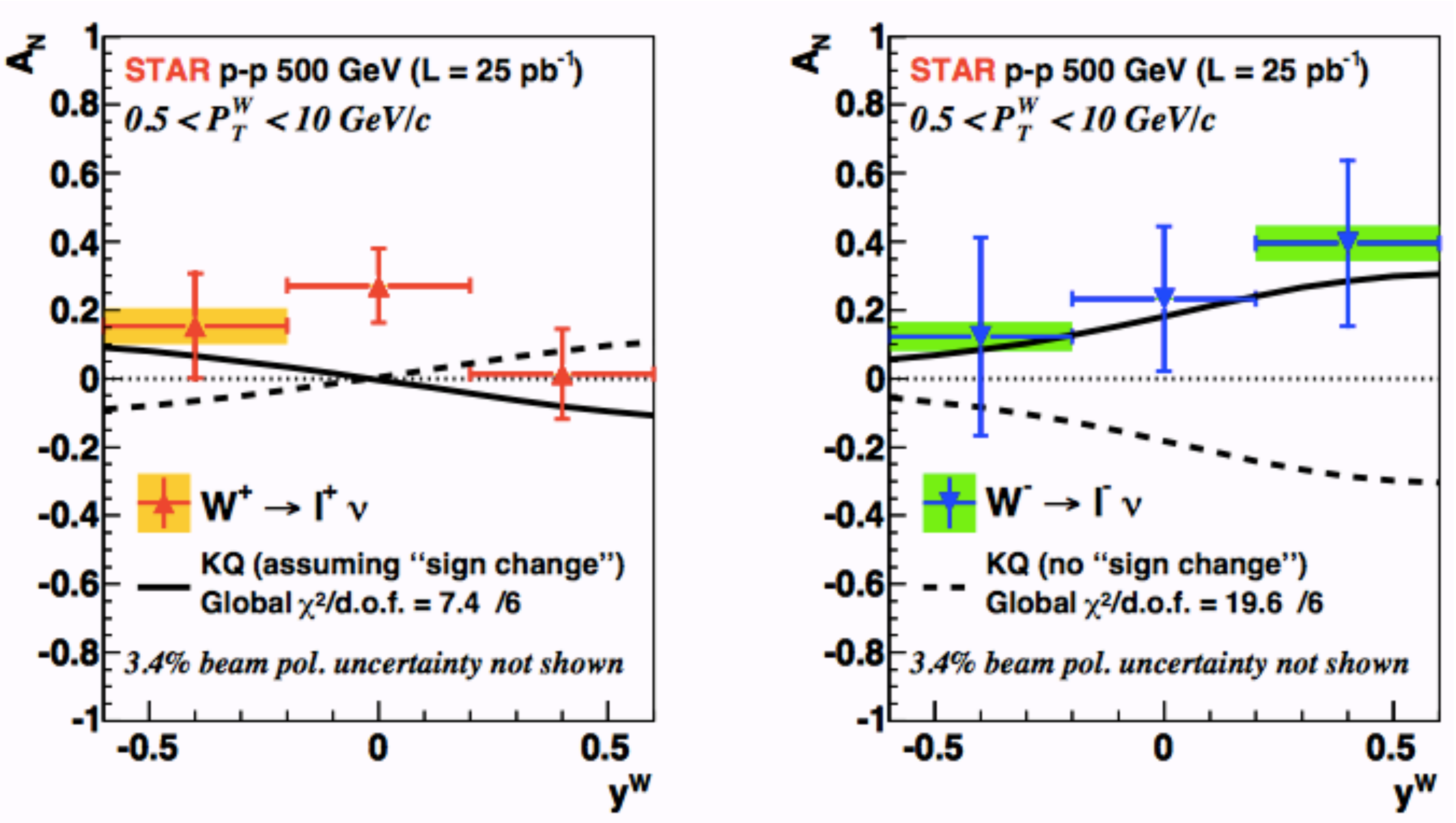
- ☐ **Modified universality:**

$$\Delta^N f_{q/h^\uparrow}^{\rm SIDIS}(x,k_\perp) = -\Delta^N f_{q/h^\uparrow}^{\rm DY}(x,k_\perp)$$

Same applies to TMD gluon distribution

Spin-averaged TMD is process independent

A_N for W production at RHIC



Data from STAR collaboration on A_N for W-production are consistent with a sign change between SIDIS and DY

STAR Collab. Phys. Rev. Lett. 116, 132301 (2016)

Boosted 3D nucleon structure

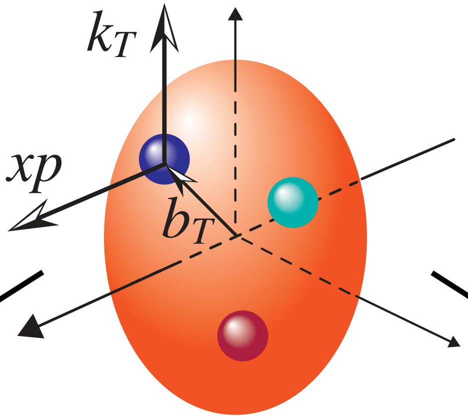
❑ **High energy probes "see" the boosted partonic structure:**

Momentum Space

TMDs

Coordinate Space

GPDs



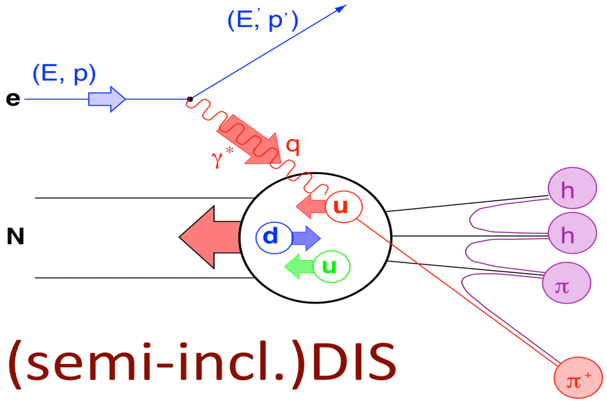
$\int d^2b_T$

$\int d^2k_T$

$f(x,k_T)$

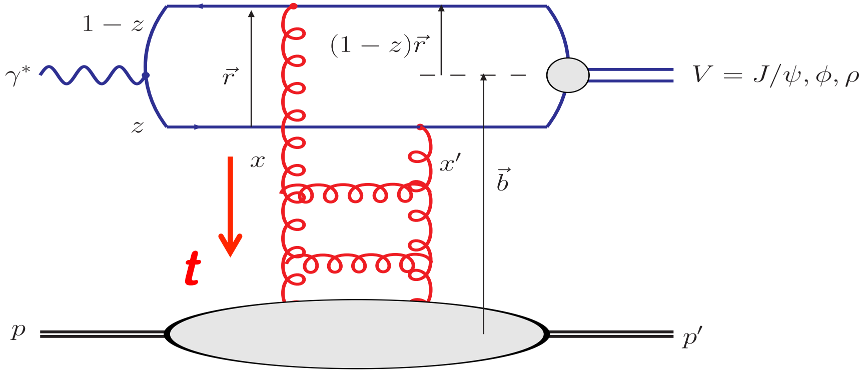
$f(x,b_T)$

3D momentum space images



(semi-incl.)DIS

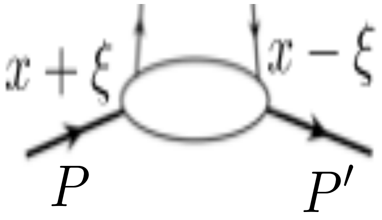
2+1D coordinate space images



Major parts of JLab12's physics program – large x

GPDs – its role in solving the spin puzzle

❑ Quark "form factor":



$$F_q(x,\xi,t,\mu^2) = \int \frac{d\lambda}{2\pi} e^{-ix\lambda} \langle P'|\bar{\psi}_q(\lambda/2)\frac{\gamma\cdot n}{2P\cdot n}\psi_q(-\lambda/2)|P\rangle$$

$$\equiv \; H_q(x,\xi,t,\mu^2)\,[\bar{\mathcal{U}}(P')\gamma^\mu\mathcal{U}(P)]\,\frac{n_\mu}{2P\cdot n}$$

$$+ \; E_q(x,\xi,t,\mu^2)\left[\bar{\mathcal{U}}(P')\frac{i\sigma^{\mu\nu}(P'-P)_\nu}{2M}\mathcal{U}(P)\right]\frac{n_\mu}{2P\cdot n}$$

with $\xi = (P'-P)\cdot n/2 \;\; \text{and} \;\; t = (P'-P)^2 \;\Rightarrow\; -\Delta_\perp^2 \;\; \text{if} \;\; \xi \to 0$

$\tilde{H}_q(x,\xi,t,Q)$, $\tilde{E}_q(x,\xi,t,Q)$ **Different quark spin projection**

❑ Total quark's orbital contribution to proton's spin:

Ji, PRL78, 1997

$$J_q = \frac{1}{2}\lim_{t\to 0}\int dx\, x\,[H_q(x,\xi,t) + E_q(x,\xi,t)]$$

$$= \frac{1}{2}\Delta q + L_q$$

❑ Connection to normal quark distribution:

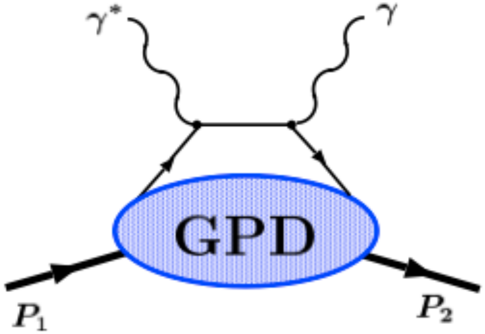
$H_q(x,0,0,\mu^2) = q(x,\mu^2)$ **The limit when** $\xi \to 0$

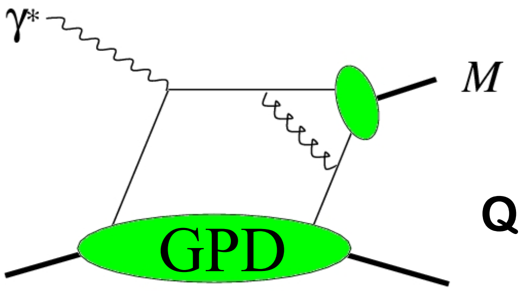
Exclusive DIS: Hunting for GPDs

Mueller et al., 94;
Ji, 96;
Radyushkin, 96

- **Experimental access to GPDs:**
  - **Diffractive exclusive processes – high luminosity:**

DVCS: Deeply virtual Compton Scattering
DVEM: Deeply virtual exclusive meson production





Require

$Q^2 >> (-t), \Lambda^2_{QCD}, M^2$

  - **No factorization for hadronic diffractive processes – EIC is ideal**

- **Much more complicated – (x, ξ, t) variables:**

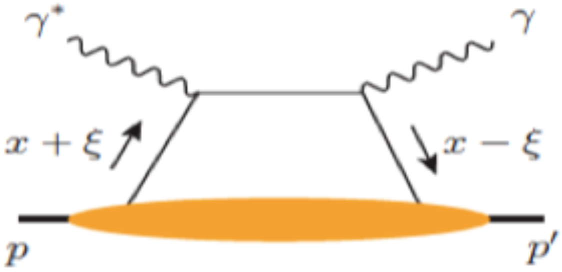
  Challenge to derive GPDs from data

- **Great experimental effort:**

  HERA, HERMES, COMPASS, JLab → JLab12, COMPASS-II, EIC

Deep virtual Compton scattering

❑ **The LO diagram:**



$$\xi \;=\; Q^2/(2\bar{P}\cdot q)$$
$$P' = P + \Delta$$

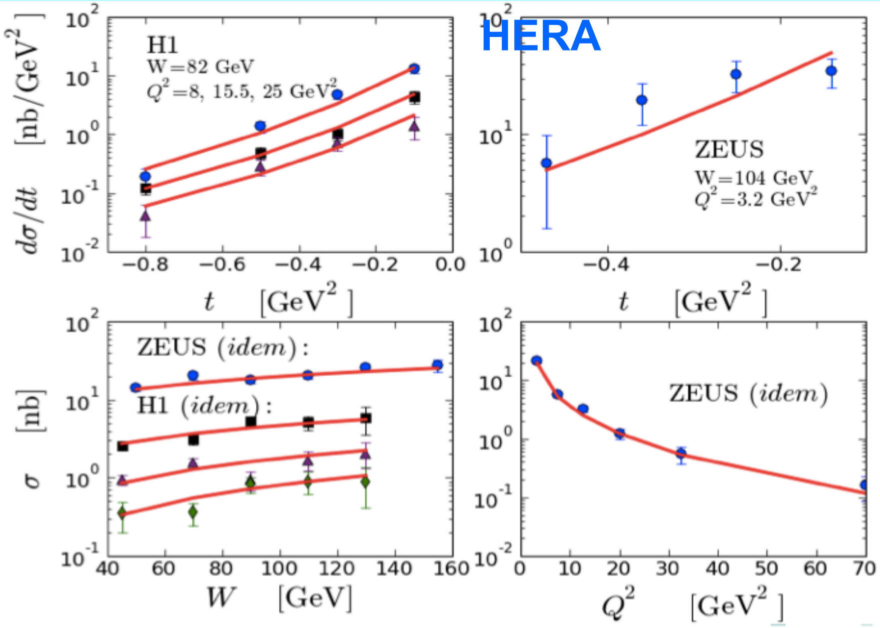
❑ **Scattering amplitude:**

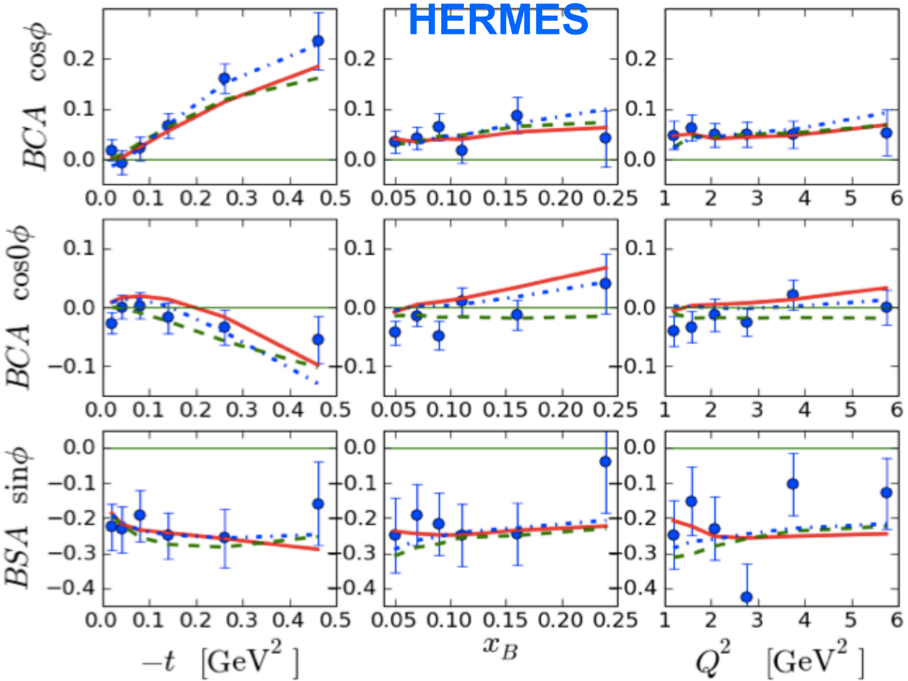
$$T^{\mu\nu}(P,q,\Delta) = -\frac{1}{2}(p^\mu n^\nu + p^\nu n^\mu - g^{\mu\nu})\int dx\left(\frac{1}{x-\xi/2+i\epsilon}+\frac{1}{x+\xi/2+i\epsilon}\right)$$
$$\times\left[H(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\not{n}U(P)+E(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{i\sigma^{\alpha\beta}n_\alpha\Delta_\beta}{2M}U(P)\right]$$
$$-\frac{i}{2}\epsilon^{\mu\nu\alpha\beta}p_\alpha n_\beta\int dx\left(\frac{1}{x-\xi/2+i\epsilon}-\frac{1}{x+\xi/2+i\epsilon}\right)$$
$$\times\left[\tilde{H}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\not{n}\gamma_5U(P)+\tilde{E}(x,\Delta^2,\Delta\cdot n)\frac{\Delta\cdot n}{2M}\bar{U}(P')\gamma_5U(P)\right]$$

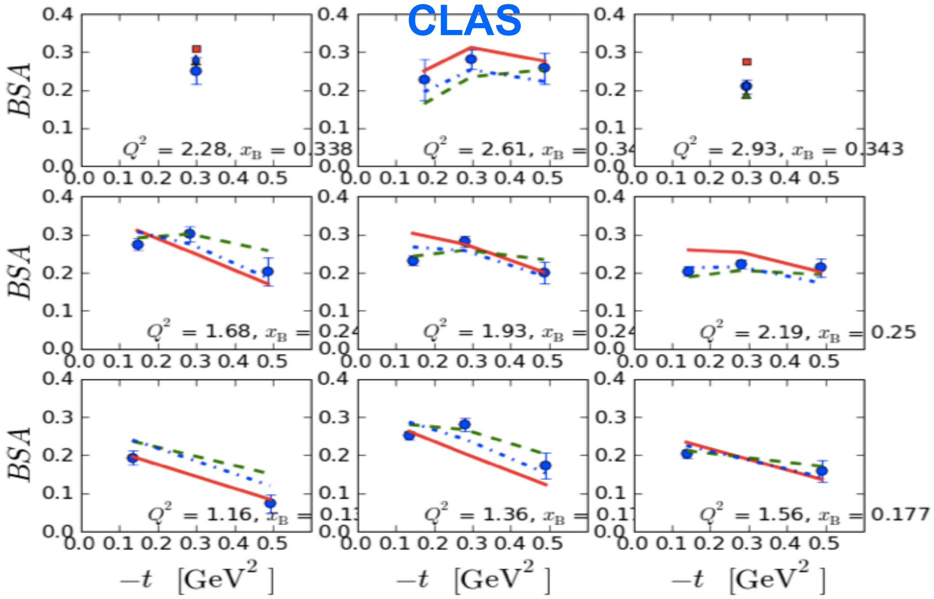
❑ **GPDs:**

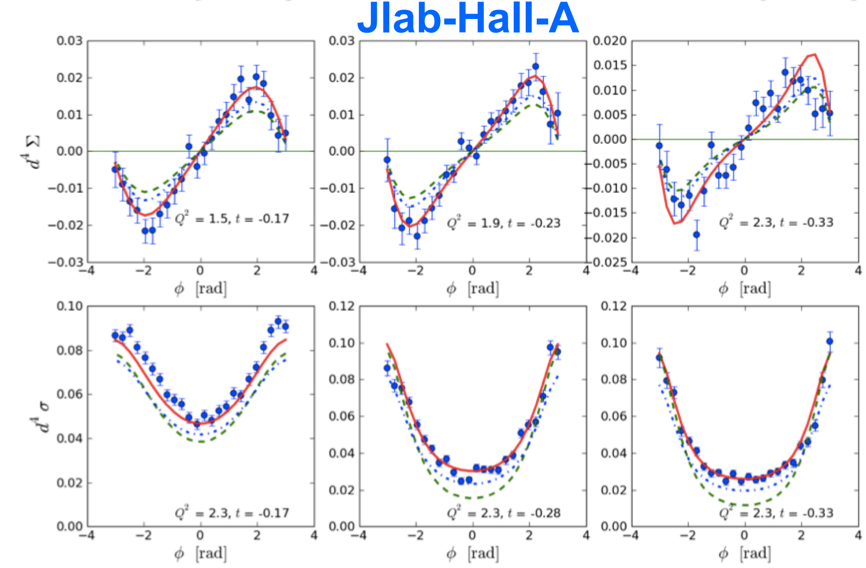
$$\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P'|\bar{\psi}(-\lambda n/2)\gamma^\mu\psi(\lambda n/2)|P\rangle = H(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\gamma^\mu U(P)$$
$$+E(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}U(P)+\ldots$$
$$\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P'|\bar{\psi}(-\lambda n/2)\gamma^\mu\gamma_5\psi(\lambda n/2)|P\rangle = \tilde{H}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\gamma^\mu\gamma_5U(P)$$
$$+\tilde{E}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{\gamma_5\Delta^\mu}{2M}U(P)+\ldots$$

GPDs: just the beginning


HERA

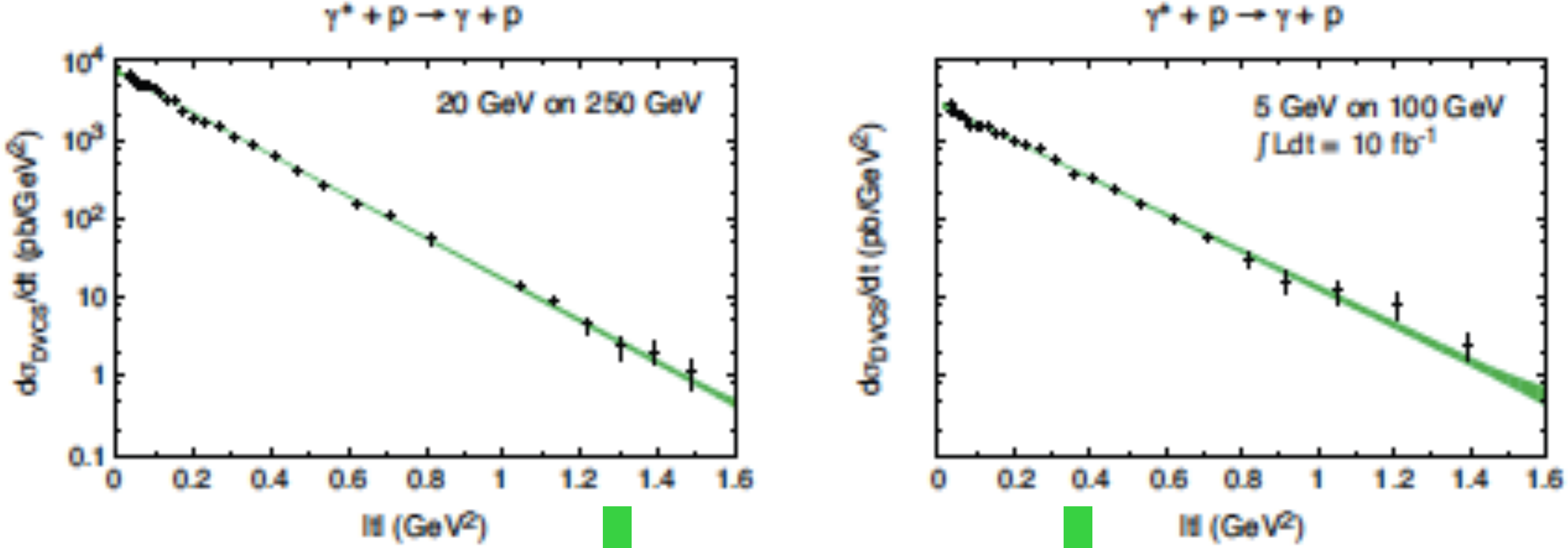

HERMES


CLAS

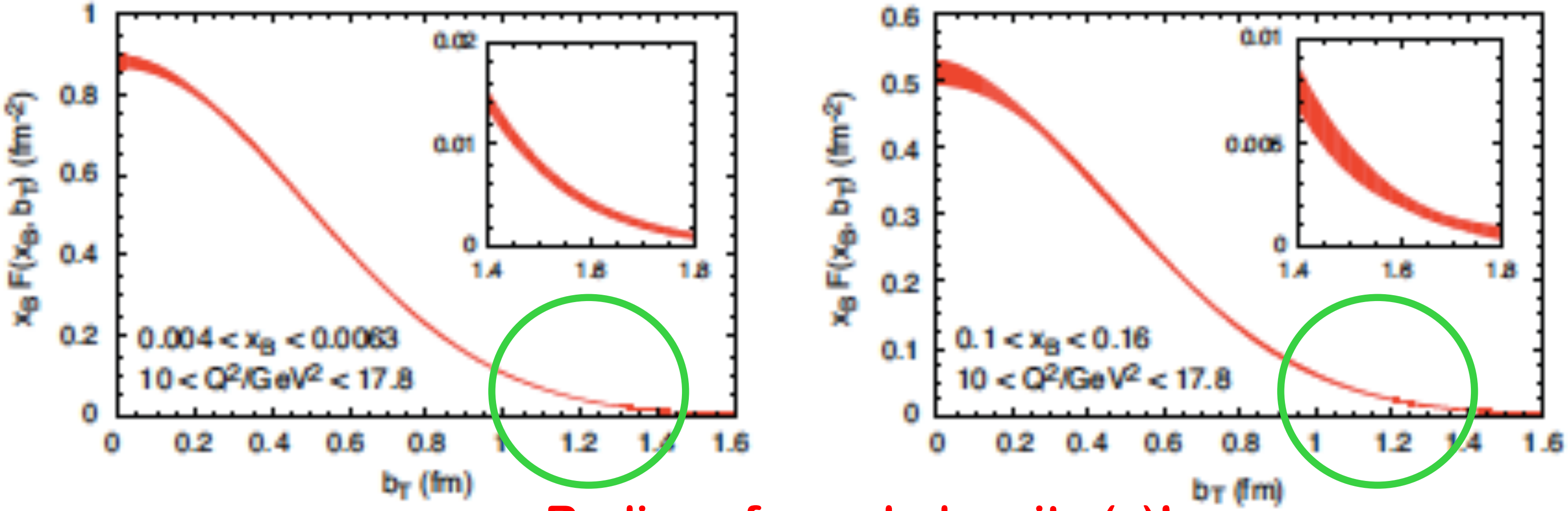

Jlab-Hall-A

DVCS @ EIC

- ☐ **Cross Sections:**



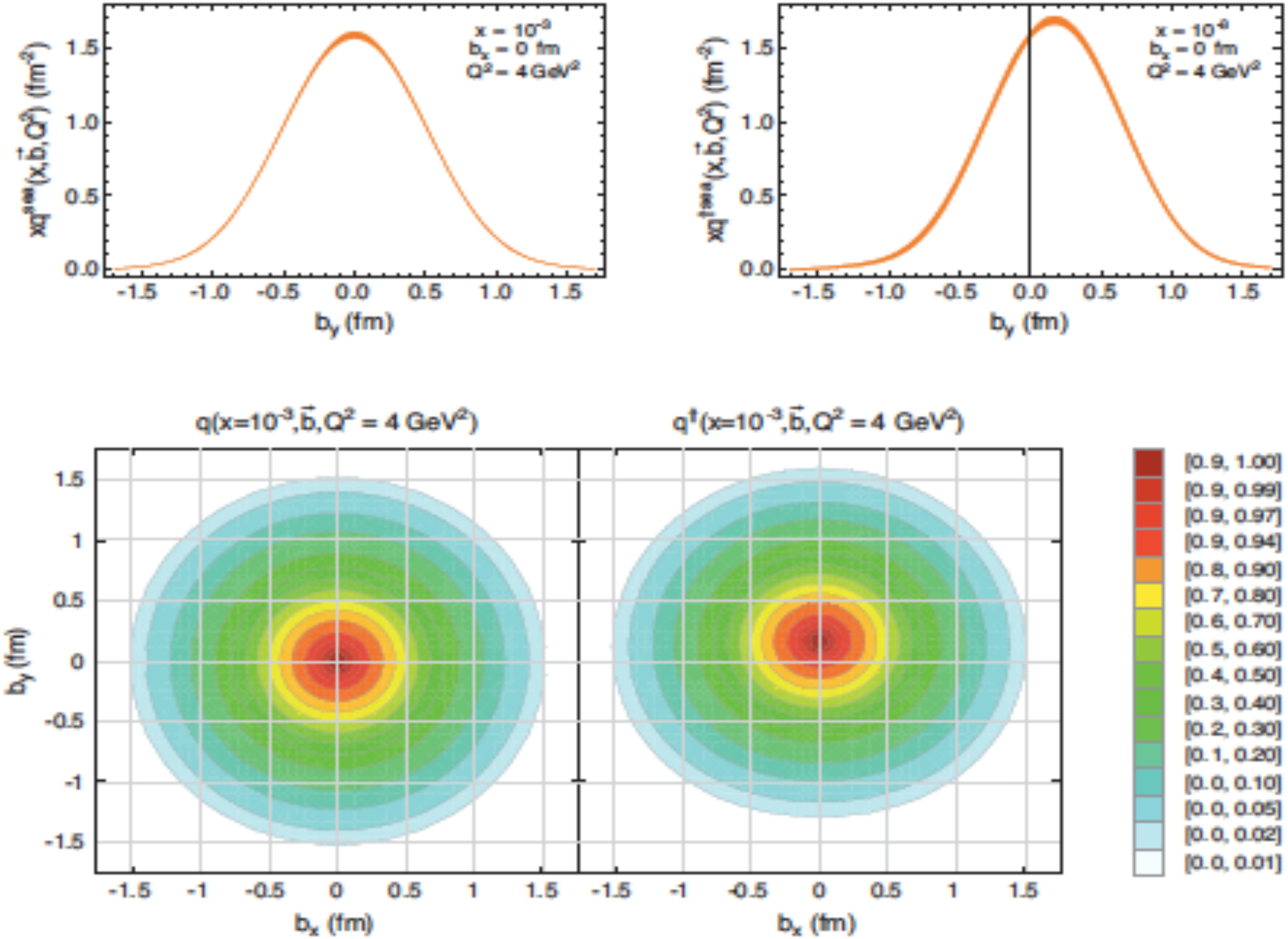
- ☐ **Spatial distributions:**



Radius of quark density (x)!

Polarized DVCS @ EIC

- ❑ **Spin-motion correlation:**

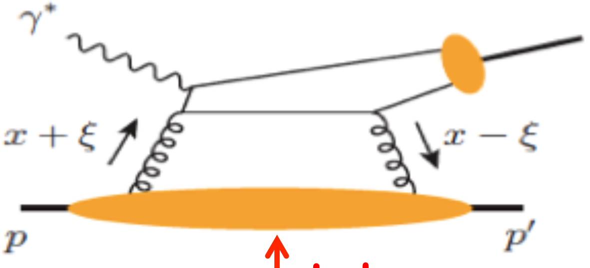


Spatial distribution of gluons

❑ Exclusive vector meson production:

$$\frac{d\sigma}{dx_B dQ^2 dt}$$

EIC-WhitePaper



J/Ψ, Φ, …

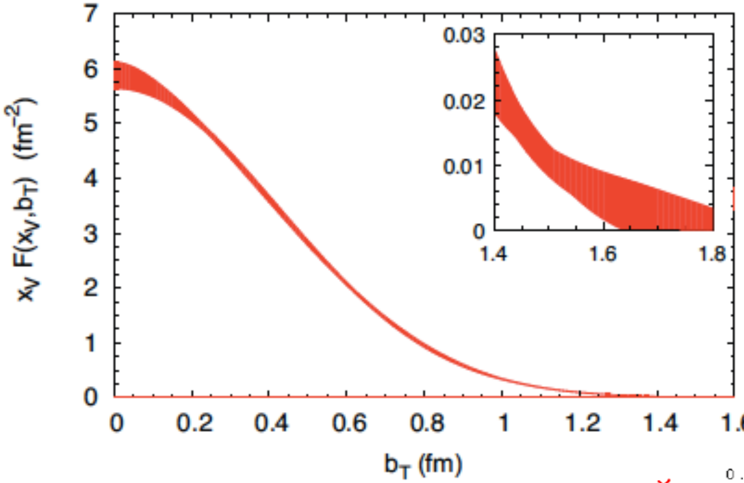
t-dep

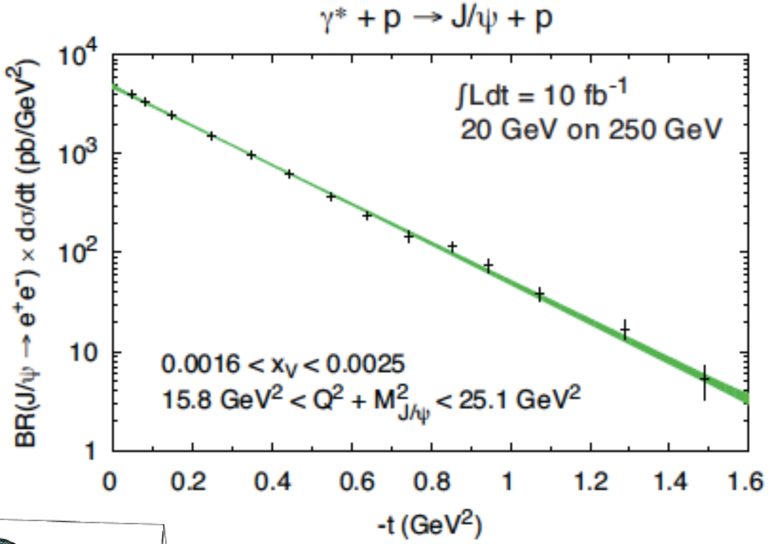
✧ Fourier transform of the t-dep

Spatial imaging of glue density

✧ Resolution ~ 1/Q or $1/M_Q$

❑ Gluon imaging from simulation:

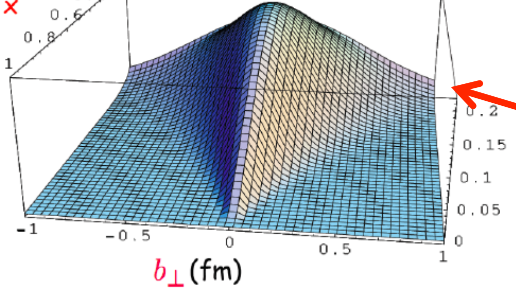




Only possible at the EIC

Gluon radius?

Gluon radius (x)!



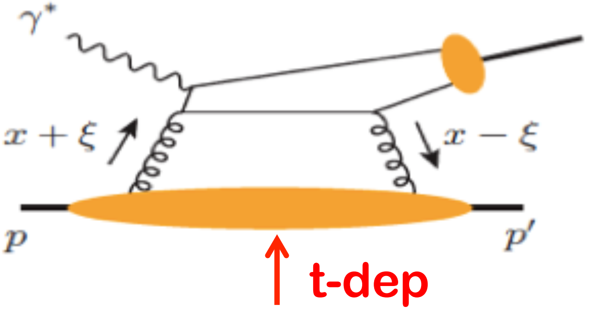
How spread
at small-x?
Color confinement

Spatial distribution of gluons

❑ Exclusive vector meson production:

$$\frac{d\sigma}{dx_B dQ^2 dt}$$

EIC-WhitePaper



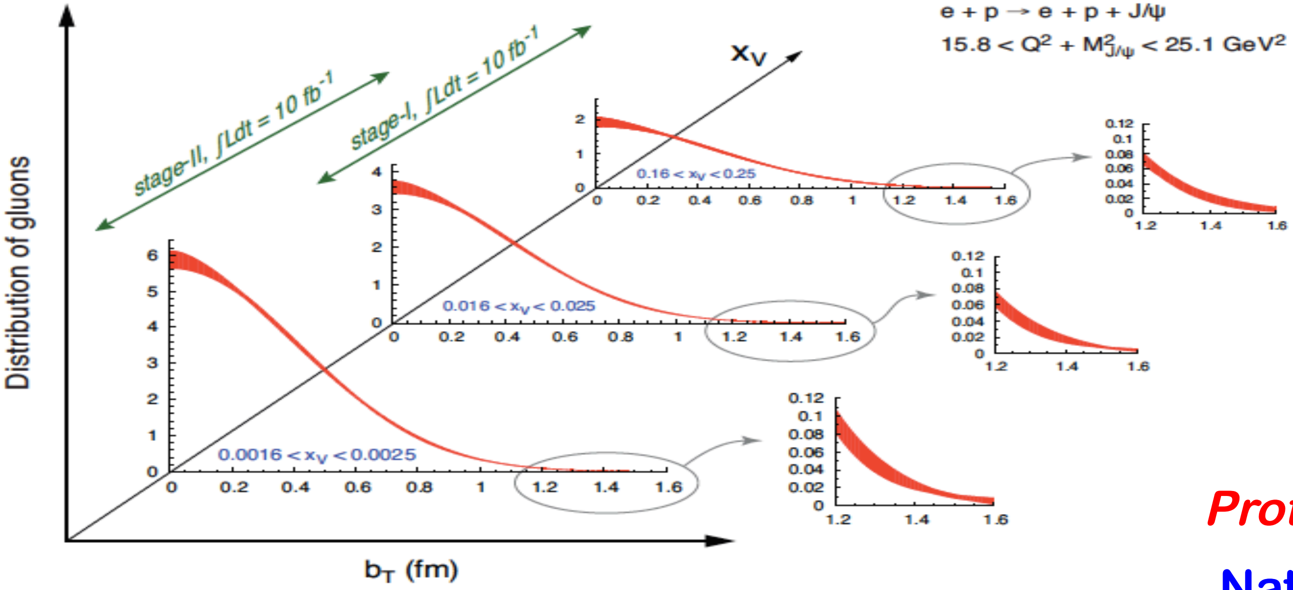
J/Ψ, Φ, …

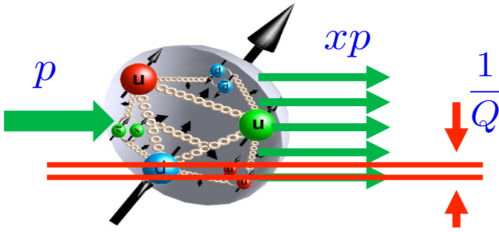
- ✧ Fourier transform of the t-dep

➡ Spatial imaging of glue density

- ✧ Resolution ~ 1/Q or 1/M_Q

❑ Gluon imaging from simulation:





Images of gluons from exclusive J/ψ production

Proton's "gluon radius"

Nature of pion cloud?

Model dependence – parameterization?

EIC simulation

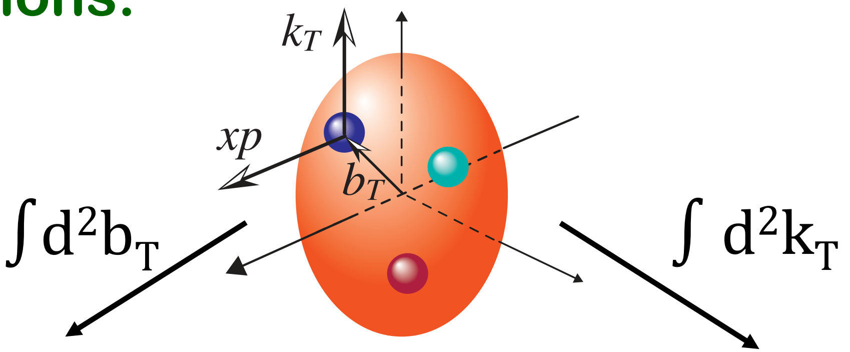
Unified view of nucleon structure

- **Wigner distributions:**

Momentum Space

TMDs

Confined motion



$\int d^2b_T$

$\int d^2k_T$

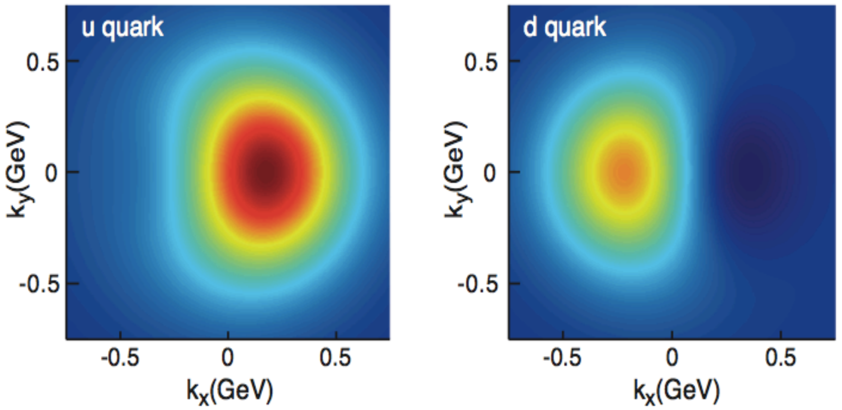
$f(x,k_T)$ **Two-scales observables** $f(x,b_T)$

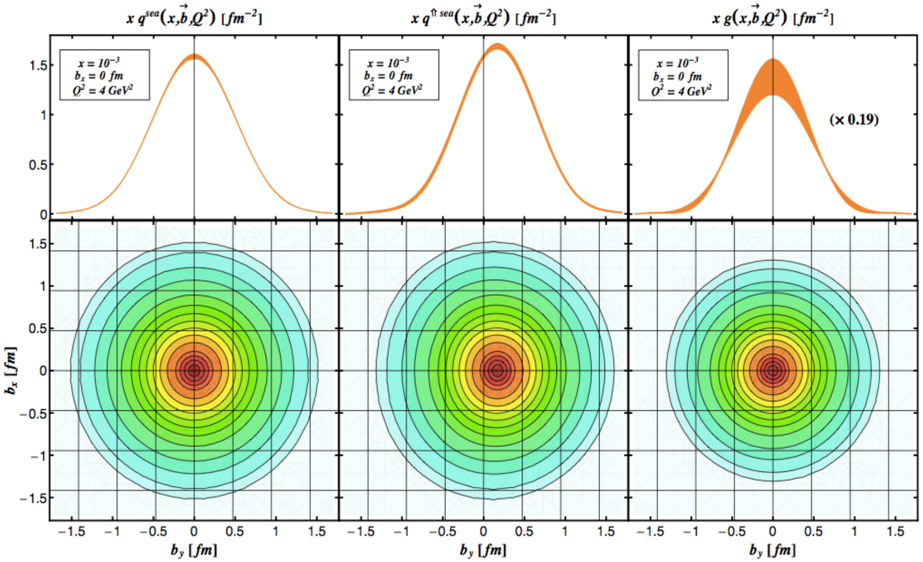
Coordinate Space

GPDs

Spatial distribution

Sivers Functions

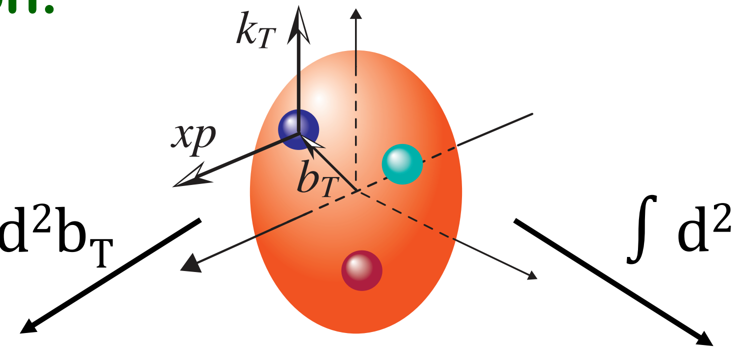




Position *r* x Momentum *p* → Orbital Motion of Partons

Unified view of nucleon structure

❑ Wigner distribution:



Momentum Space

TMDs

Confined motion

$\int d^2b_T$

$f(x,k_T)$

Two-scales observables

$\int d^2k_T$

$f(x,b_T)$

Coordinate Space

GPDs

Spatial distribution

❑ Note:

- ✧ Partons' confined motion and their spatial distribution are unique – the consequence of QCD
- ✧ But, the TMDs and GPDs that represent them are not unique!
  - – Depending on the definition of the Wigner distribution and QCD factorization to link them to physical observables

Position *r* x Momentum *p* → Orbital Motion of Partons

Orbital angular momentum

OAM: Correlation between parton’s position and its motion
– in an averaged (or probability) sense

❑ **Jaffe-Manohar’s quark OAM density:**

$$\mathcal{L}_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{\partial})\right]^3 \psi_q$$

❑ **Ji’s quark OAM density:**

$$L_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{D})\right]^3 \psi_q$$

❑ **Difference between them:**

Hatta, Lorce, Pasquini, …

✧ **compensated by difference between gluon OAM density**

✧ **represented by different choice of gauge link for OAM Wagner distribution**

$$\mathcal{L}_q^3 \left\{L_q^3\right\} = \int dx\, d^2b\, d^2k_T \left[\vec{b} \times \vec{k}_T\right]^3 \mathcal{W}_q(x,\vec{b},\vec{k}_T) \left\{W_q(x,\vec{b},\vec{k}_T)\right\}$$

with

$$\mathcal{W}_q \left\{W_q\right\}(x,\vec{b},\vec{k}_T) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{i\vec{\Delta}_T\cdot\vec{b}} \int \frac{dy^- d^2y_T}{(2\pi)^3} e^{i(xP^+y^- - \vec{k}_T\cdot\vec{y}_T)}$$
$$\times \langle P'|\,\overline{\psi}_q(0)\,\frac{\gamma^+}{2}\,\underbrace{\Phi^{\mathrm{JM}\{\mathrm{Ji}\}}(0,y)}_{\text{Gauge link}}\,\psi(y)\,|P\rangle_{y^+=0}$$

JM: “staple” gauge link
Ji: straight gauge link

between 0 and y=(y^+=0,y^-,y_T)

Orbital angular momentum

OAM: Correlation between parton’s position and its motion – in an averaged (or probability) sense

❑ **Jaffe-Manohar’s quark OAM density:**

$$\mathcal{L}_q^3 = \psi_q^\dagger \left[\vec{x}\times(-i\vec{\partial})\right]^3 \psi_q$$

❑ **Ji’s quark OAM density:**

$$L_q^3 = \psi_q^\dagger \left[\vec{x}\times(-i\vec{D})\right]^3 \psi_q$$

❑ **Difference between them:**

Hatta, Yoshida, Burkardt, Meissner, Metz, Schlegel, …

✧ generated by a “torque” of color Lorentz force

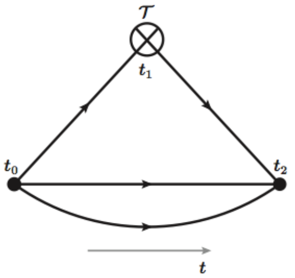
$$\mathcal{L}_q^3 - L_q^3 \propto \int \frac{dy^- d^2y_T}{(2\pi)^3} \langle P'|\overline{\psi}_q(0)\frac{\gamma^+}{2}\int_{y^-}^{\infty} dz^- \Phi(0,z^-)$$
$$\times \underbrace{\sum_{i,j=1,2}\left[\epsilon^{3ij} y_T^i F^{+j}(z^-)\right]}_{\text{“Chromodynamic torque”}} \Phi(z^-,y)\psi(y)|P\rangle_{y^+=0}$$

Similar color Lorentz force generates the single transverse-spin asymmetry (Qiu-Sterman function), and is also responsible for the twist-3 part of g_2

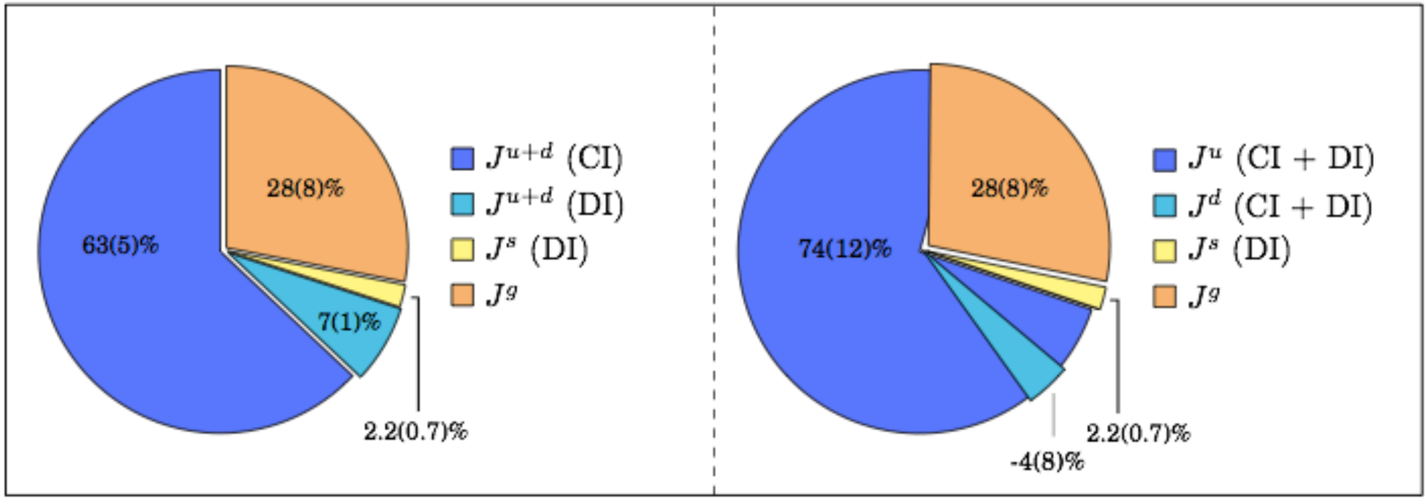
Nucleon spin and OAM from lattice QCD

❑ χQCD Collaboration:

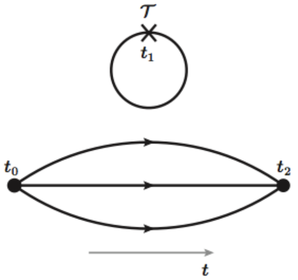
[Deka *et al.* arXiv:1312.4816]



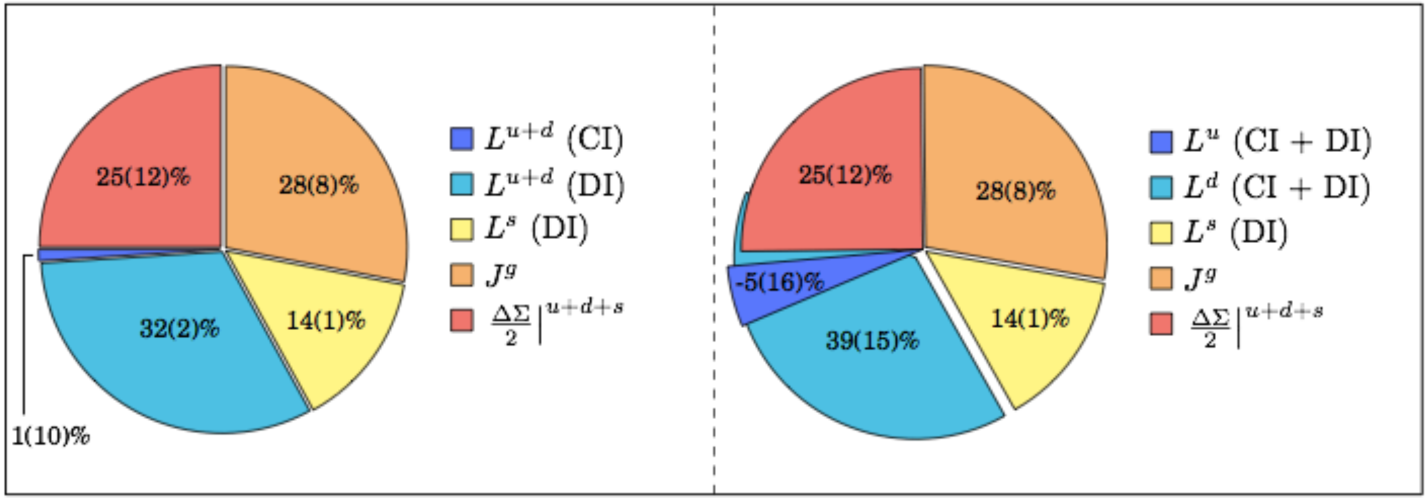
Connected Interaction (CI)



(b)



Disconnected Interaction (DI)

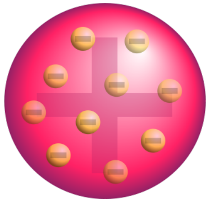


(c)

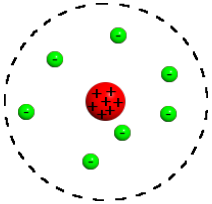
Why 3D hadron structure?

❑ **Rutherford's experiment – atomic structure (100 years ago):**

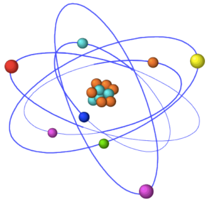
Atom:



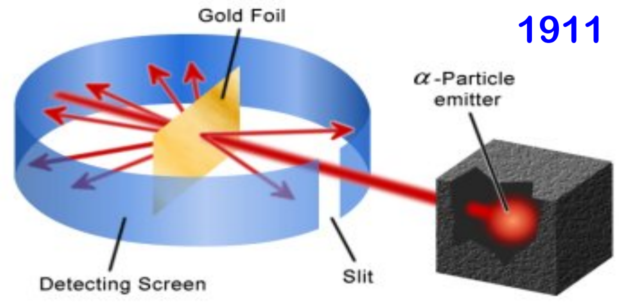
J.J. Thomson's
plum-pudding model



Rutherford's
planetary model



Modern model
Quantum orbitals



1911

Discovery of nucleus
A localized
charge/force center
A vast
"open" space

Discovery of
Quantum Mechanics,
and
the Quantum World!

❑ **Completely changed our "view" of the visible world:**

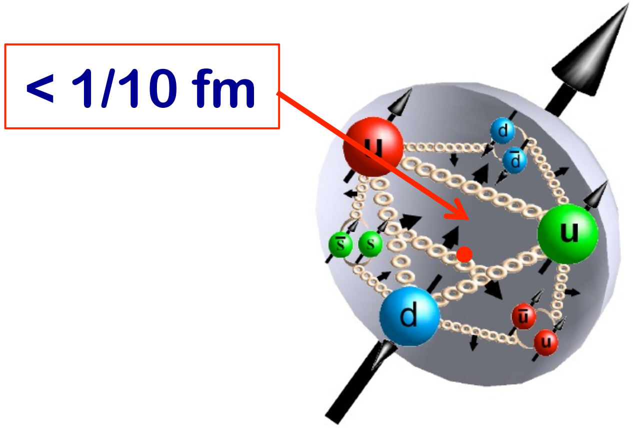
- ✧ Mass by "tiny" nuclei – *less than 1 trillionth in volume of an atom*
- ✧ Motion by quantum probability – *the quantum world!*
- ✧ Provided infinite opportunities to improve things around us, …

What would we learn from the hadron structure in QCD, …?

Summary

☐ QCD has been extremely successful in interpreting and predicting high energy experimental data!

☐ But, we still do not know much about hadron structure – work just started!



☐ Cross sections with large momentum transfer(s) and identified hadron(s) are the source of structure information

☐ QCD factorization is necessary for any controllable "probe" for hadron's quark-gluon structure!

☐ But, EIC is a ultimate QCD machine, and will provide answers to many of our questions on hadron structure, in particular, the confined transverse motions (TMDs), spatial distributions (GPDs), and multi-parton correlations, …

Thank you!

Backup slides

Parity and Time-reversal invariance

- ☐ In quantum field theory, physical observables are given by matrix elements of quantum field operators

- ☐ Consider two quantum states: $|\alpha\rangle \quad |\beta\rangle$

- ☐ Parity transformation:

$$|\alpha_P\rangle \equiv U_P\,|\alpha\rangle \qquad\qquad |\beta_P\rangle \equiv U_P\,|\beta\rangle$$

$$\langle\alpha_P|\beta_P\rangle = \langle\alpha|\,U_P^\dagger\,U_P\,|\beta\rangle = \langle\alpha|\beta\rangle$$

- ☐ Time-reversal transformation:

$$|\alpha_T\rangle \equiv V_T\,|\alpha\rangle \qquad\qquad |\beta_T\rangle \equiv V_T\,|\beta\rangle$$

$$\langle\alpha_T|\beta_T\rangle = \langle\alpha|\,V_T^\dagger\,V_T\,|\beta\rangle = \langle\alpha|\beta\rangle^* = \langle\beta|\alpha\rangle$$

Parity and Time-reversal invariance

- ☐ **Parton fields under P and T transformation:**

$$U_P\,\psi(y_0,\vec{y})\,U_P^{-1} = \gamma^0\,\psi(y_0,-\vec{y})$$

$$V_T\,\psi(y_0,\vec{y})\,V_T^{-1} = (i\gamma^1\gamma^3)\,\psi(-y_0,\vec{y}) \qquad \boxed{\mathcal{J} = i\,\gamma^1\,\gamma^3}$$

→

$$\boxed{\begin{aligned}&\langle P,\vec{s}_\perp|\,\bar{\psi}(0)\,\Gamma_i\,\psi(y^-)\,|P,\vec{s}_\perp\rangle\\ &= \langle P,-\vec{s}_\perp|\,\bar{\psi}(0)\left[\mathcal{J}\left(\Gamma_i^\dagger\right)^*\mathcal{J}^\dagger\right]\psi(y^-)\,|P,-\vec{s}_\perp\rangle\end{aligned}}$$

- ☐ **Quark correlations contribute to polarized X-sections:**

$$T_i(x;\vec{s}_\perp) = -T_i(x;-\vec{s}_\perp) \quad \longrightarrow \quad \mathcal{J}\left(\Gamma_i^\dagger\right)^*\mathcal{J}^\dagger = -\Gamma_i$$

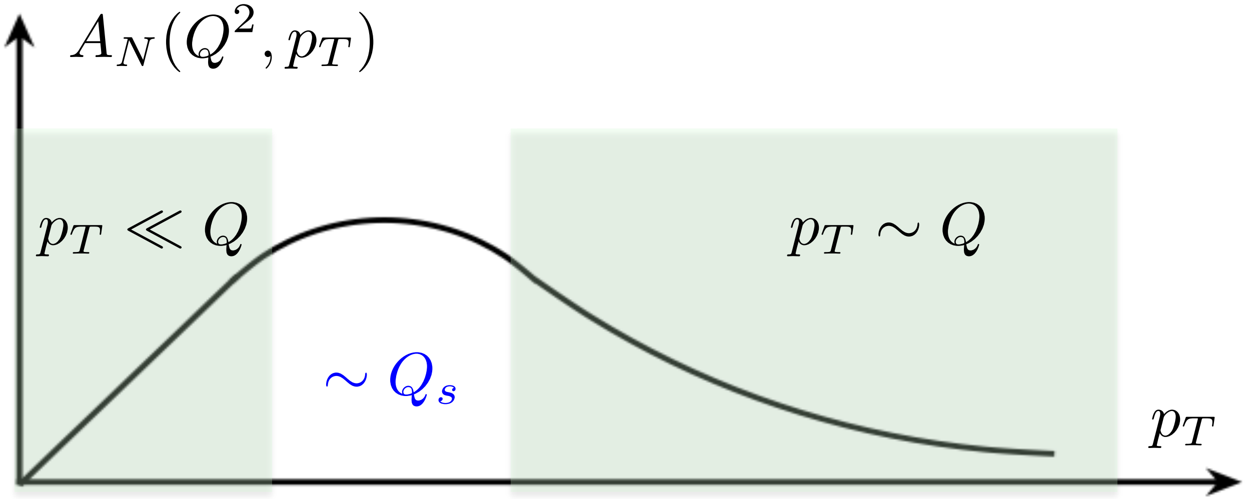
$$\boxed{\Gamma_i \;=\; \gamma^\mu\gamma_5, \quad \sigma^{\mu\nu} \quad \text{(or} \quad \sigma^{\mu\nu}\,(i\gamma_5)}$$

$$\Gamma_i \;=\; I, \quad i\gamma_5, \quad \gamma^\mu$$

contribute to spin-avg X-sections:

Transition from low p_T to high p_T

- ❑ **Two-scale becomes one-scale:**



TMD **Collinear Factorization**

- ❑ **TMD factorization to collinear factorization:**

Ji,Qiu,Vogelsang,Yuan,
Koike, Vogelsang, Yuan

Two factorization are consistent in the overlap region: $\Lambda_{\rm QCD} \ll p_T \ll Q$

A_N finite – requires correlation of multiple collinear partons

No probability interpretation! New opportunities!