Relativistic Heavy Ion Collisions and the Quark Gluon Plasma

I. Idealized Partonic Matter (1st lecture)

II. Modeling Heavy Ion Collisions and connecting QGP properties to experiment (2nd and 3rd lectures)

III. Quantifying our knowledge of the QGP (3rd lecture)

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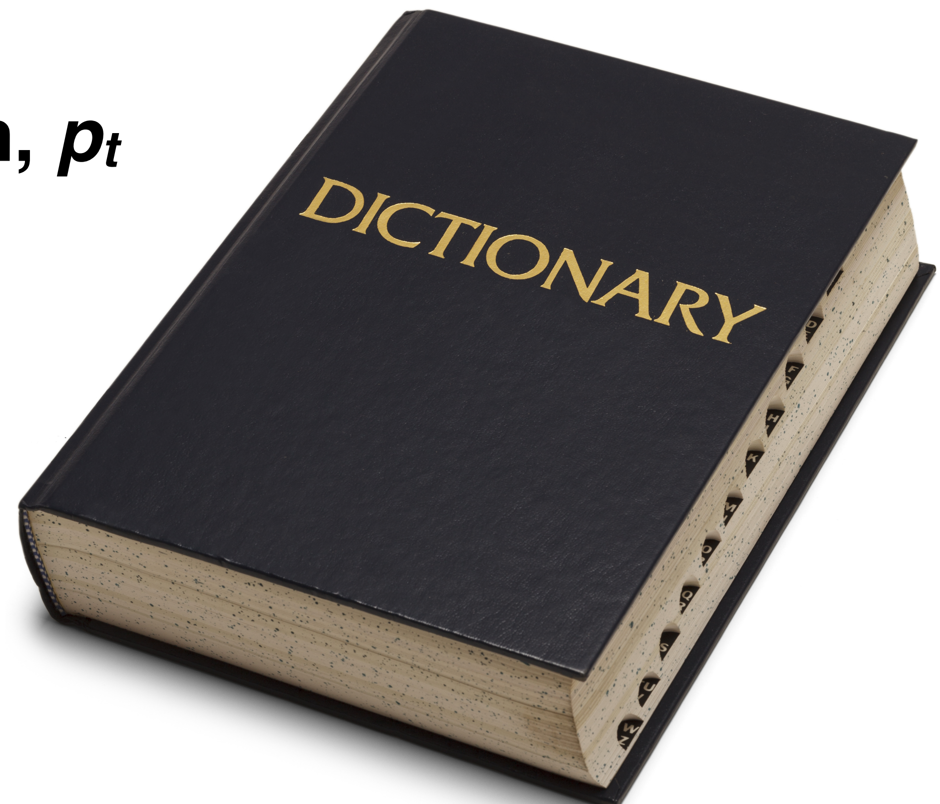
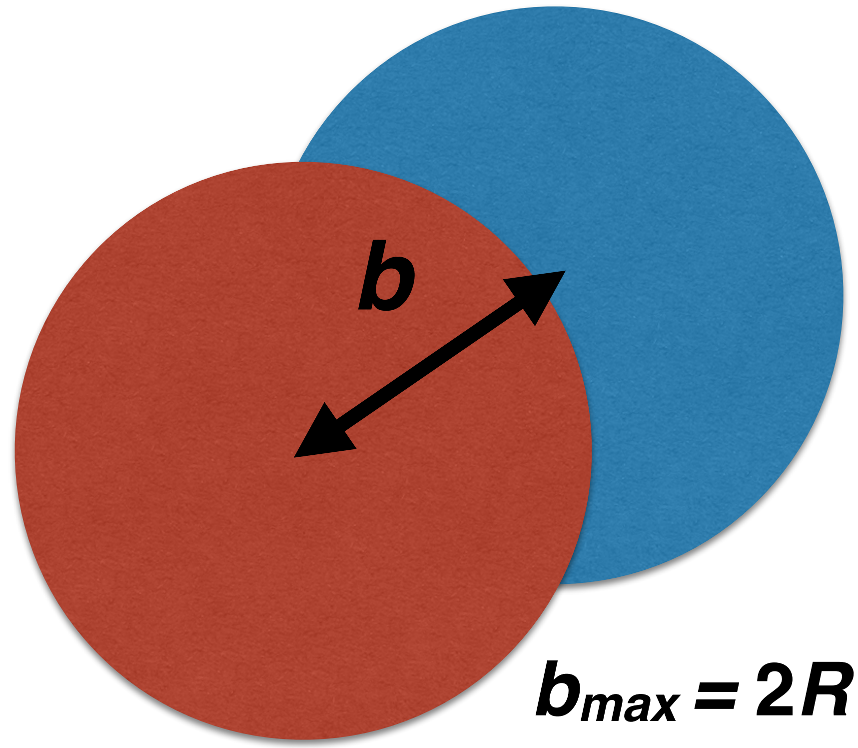
Kinematics

Vocabulary:

- **centrality**
- **rapidity, y**
- **pseudo-rapidity, η**
- **spatial-rapidity, η_s**
- **proper time, τ**
- **transverse momentum, p_t**



Kinematics — Centrality



$$\%\,\text{centrality} = \frac{\pi b^2}{\pi b_{\max}^2}$$

$$= \%\,\text{of events with higher multiplicity}$$

Can extract impact parameter

Kinematics (Rapidity)

measure of longitudinal (along beam) velocity

$$y = \frac{1}{2}\ln\frac{1+v_z}{1-v_z}$$

$$\sinh y = \gamma_z v_z$$

$$\cosh y = \gamma_z$$

$$\tanh y = v$$

For small *v*

$$y \approx v_z$$

rapidities add for longitudinal boosts

$$u'_z = \gamma u_z + \gamma v_z u_0$$

$$\gamma = \cosh y_1, \quad \gamma v_z = \sinh y_1$$

$$u_0 = \cosh y_2, \; u_z = \sinh y_2$$

$$u'_z = \cosh y_1 \sinh y_2 + \sinh y_1 \cosh y_2 = \sinh(y_1 + y_2)$$

Kinematics (Rapidity)

AT RHIC (Au) / LHC (Pb)

$$y_{\text{beam}} = 5.37 \;\; / \;\; 8.69$$

Used to express spectra

$$\frac{E_p dN}{d^3 p} = \frac{dN}{2\pi p_t dp_t dy}$$

Kinematics (Pseudorapidity)

Simply a measure of polar angle

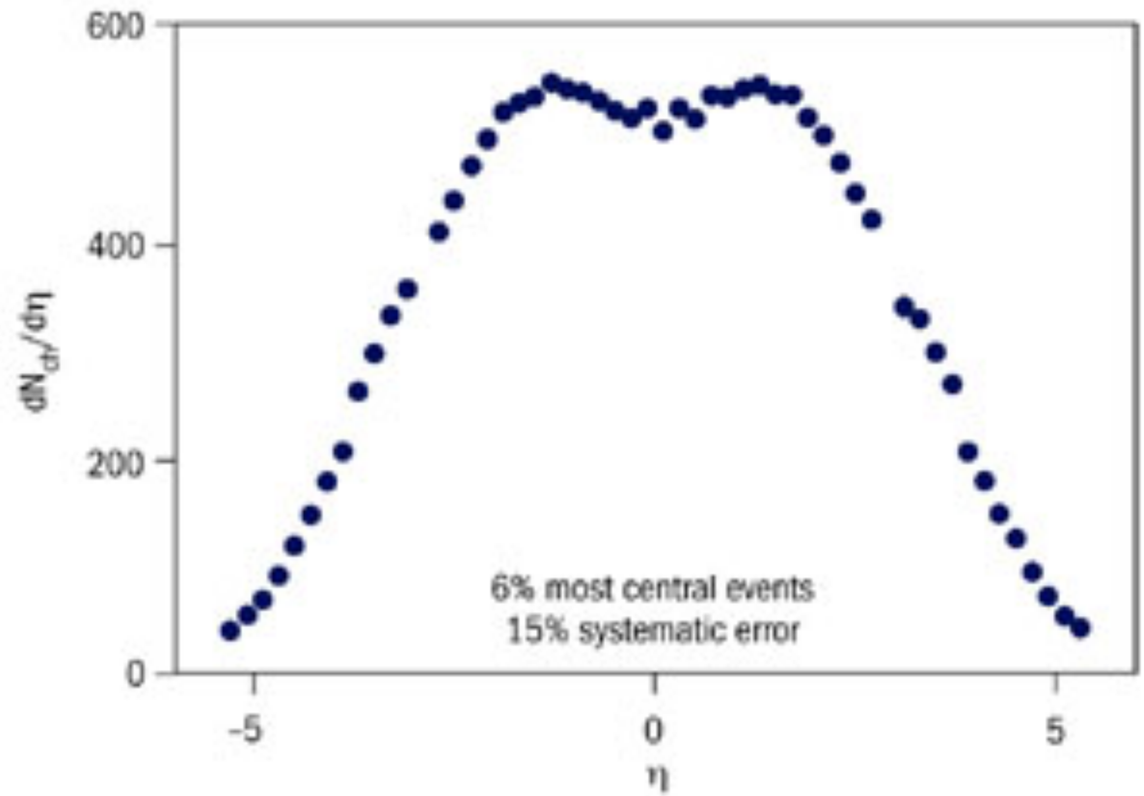
$$\eta = \frac{1}{2}\ln\frac{1+\cos\theta}{1-\cos\theta} = \tanh^{-1}(\sin\theta)$$

For *m=0*, η=*y*

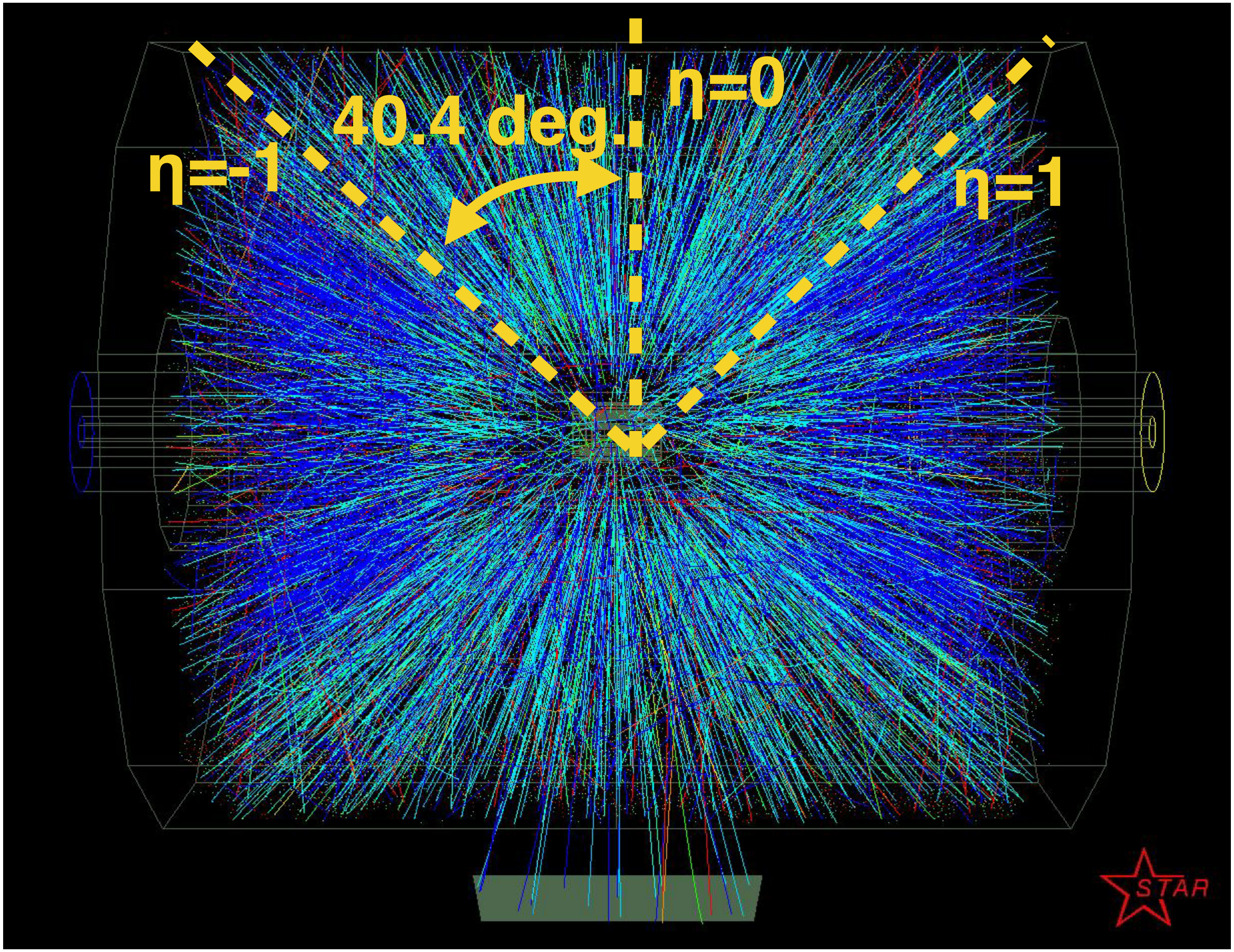
Phobos, 2001

$$E\,dy = p\,d\eta$$

$$\frac{dN}{d\eta} = \frac{dN}{dy}\frac{p}{E}$$



Kinematics (Pseudorapidity)



Kinematics (Bjorken expansion, η_s and τ)

Bjorken (PRC '83) Hydro
Let matter "coast" from z=t=0

$$z = v_z t, \; v_z = z/t$$

Coasting expected if boost invariant,
η_s refers to rapidity of local matter

$$\eta_s = y = \tanh^{-1}(v_z) = \tanh^{-1}(z/t)$$

τ measured by observer starting at origin

$$\tau = \frac{t}{\gamma} = t\sqrt{1 - z^2/t^2} = \sqrt{t^2 - z^2}$$

Kinematics (η_s and τ)

$$t = \tau \cosh(\eta_s)$$

$$z = \tau \sinh(\eta_s)$$

If boost-invariant

- **physics would depend only on τ**
- **independent of η_s**
- **s ~ 1/τ**

Bjorken estimate of energy density at time τ

$$\epsilon \approx 1.5 \times \frac{dE_t / dy}{\pi R^2 \tau}$$

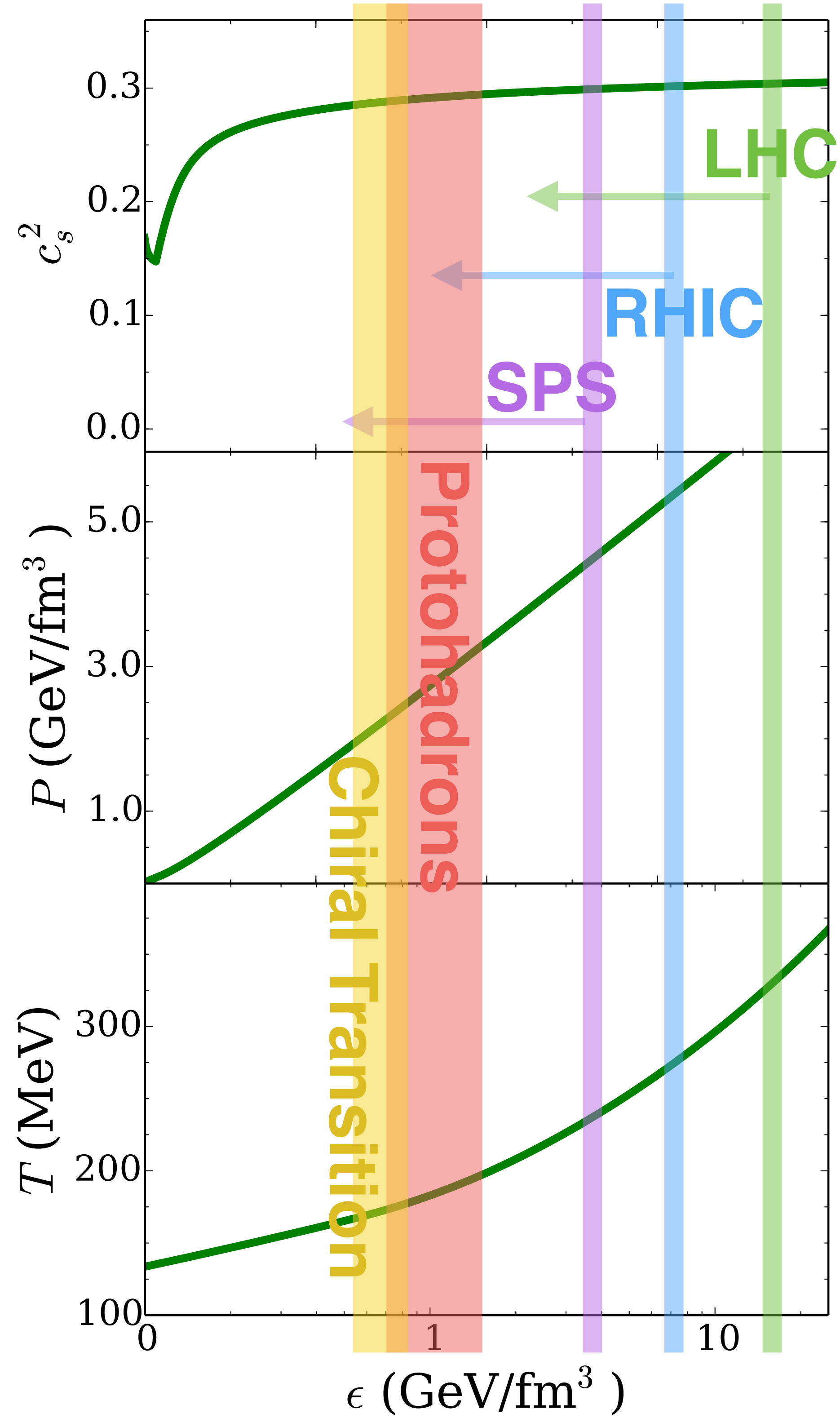
At τ=1.0 fm/c

SPS ~ 4 GeV/fm^3
RHIC ~ 7 GeV/fm^3
LHC ~ 15 GeV/fm^3

Energy Densities..

Time above T_c (175 MeV)
SPS: ~ 2.5 fm/c
RHIC: ~ 5 fm/c
LHC: ~ 8 fm/c

Collisions last ~ 15-25 fm/c



Kinematics (η_s and τ)

Exercise 4: Consider a particle of momentum p, at z=0 at time t_0=τ_0. The particle moves without scattering until it is at new position where the new proper time is τ'.

Find the momenta, p'_x, p'_y, p'_z as determined by an observer moving with the rest frame of the fluid at the new position. Give answer in terms of p_x, p_y, p_z and τ'/τ_0.

Linking matter properties to measurement

Properties to discuss:

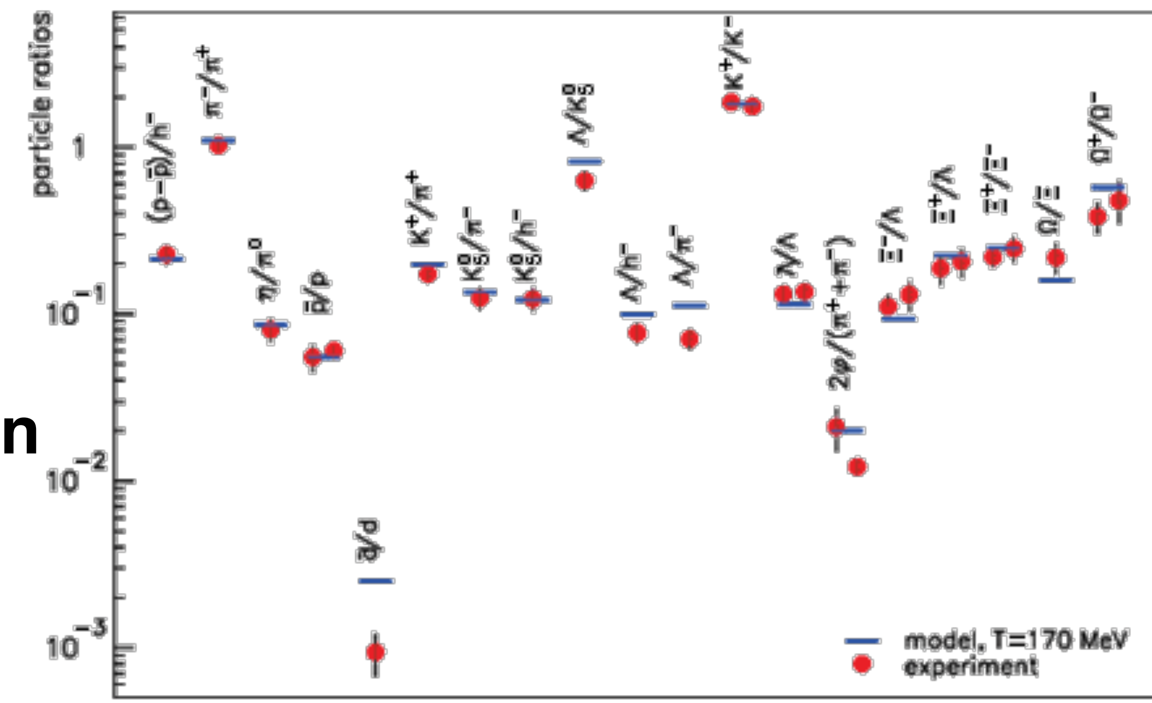
0. ***Did the matter equilibrate?***
1. **Eq. of State**
2. **Chemistry**
3. **Chiral Symmetry**
4. **Color screening***
5. **Viscosity**
6. **Diffusion Constant***
7. **Jet damping***
8. **Stopping and Thermalization**

***will skip**

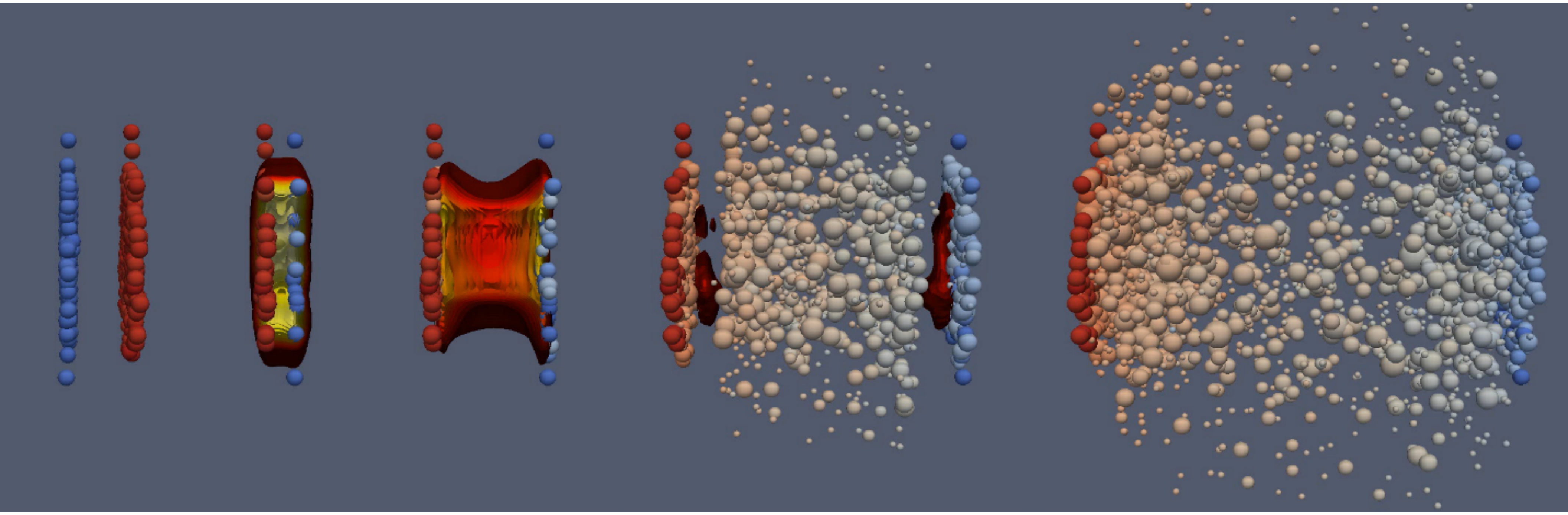
0. Does Matter Equilibrate

I. **Local kinetic equilibrium**
— **fairly easy to attain**
— **strong collective provides strong evidence**

II. **Chemical equilibrium**
— **more difficult to attain**
— **appears to last until chemical freeze out, T ~ 165 MeV**
— **some baryon annihilation afterward**



P.Braun-Munzinger,K. Redlich, J. Stachel, 2004



MODELING

I. Pre-Equilibrium ($0<\tau<0.6$ fm/c)
parametric forms, parton cascades, Yang-Mills fields…

II. Viscous Hydrodynamics ($0.6<\tau<6$ fm/c, T>160 MeV)

III. Hadronic Cascade

Afterburners: jets, femtoscopic correlations, heavy-quark observables, photons, dileptons

Is Hydrodynamics Valid?

$$\partial_\mu T^{\mu\nu} = 0,$$

energy-momentum conservation

$$T_{ij} = P\delta_{ij} + \pi_{ij}$$

viscous correction

$$\frac{d}{d\tau}\left(\frac{\pi_{ij}}{\alpha}\right) = -\frac{1}{\tau}\left\{\left(\frac{\pi_{ij} - \eta(\partial_i v_j + \partial_j v_i - (2/3)\nabla \cdot v)}{\alpha}\right)\right\}$$

$$\alpha^2 = \frac{\eta T s}{\tau}$$

relax toward Navier-Stokes

Validity requires:

- **within eyesight of N.S.**
- **all matter moves with one velocity**

1a. Discerning the EoS — $\langle p_t \rangle$

1a. p_t vs. beam energy or multiplicity (Van Hove 1982)

Fig. 1

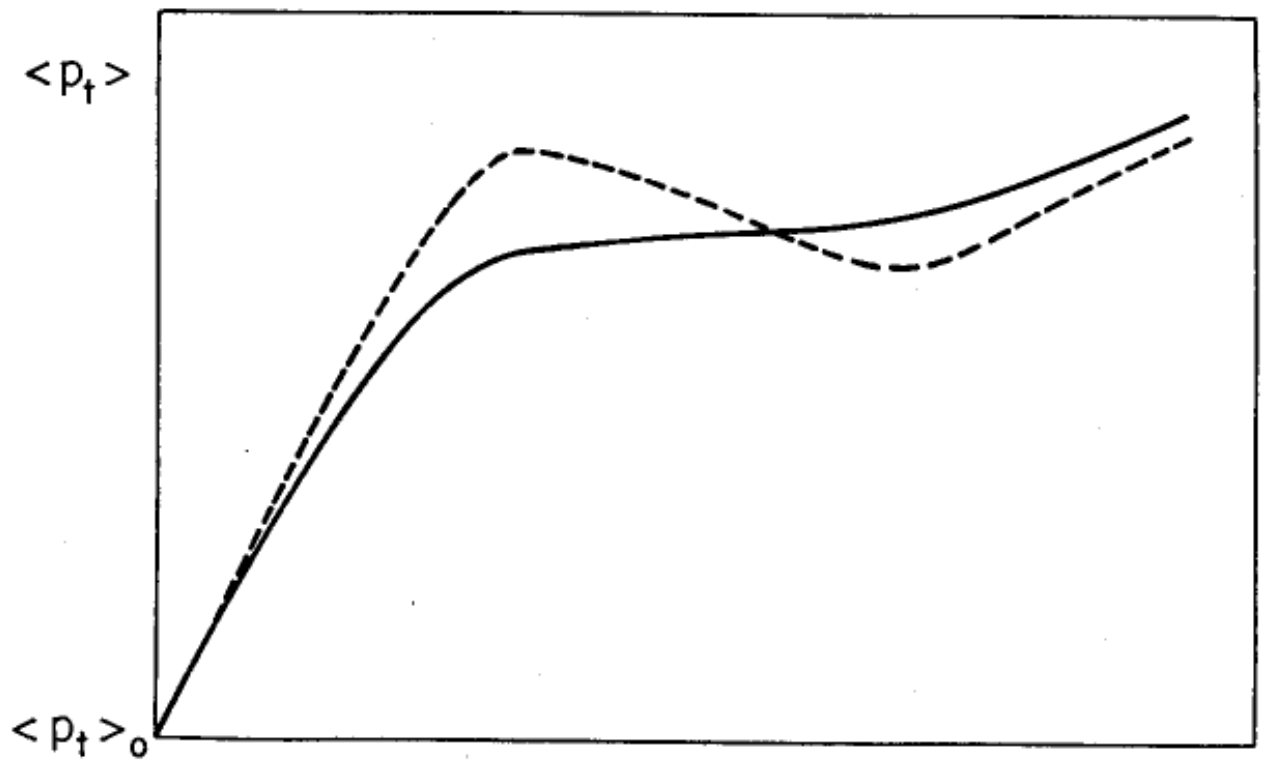


Fig. 2

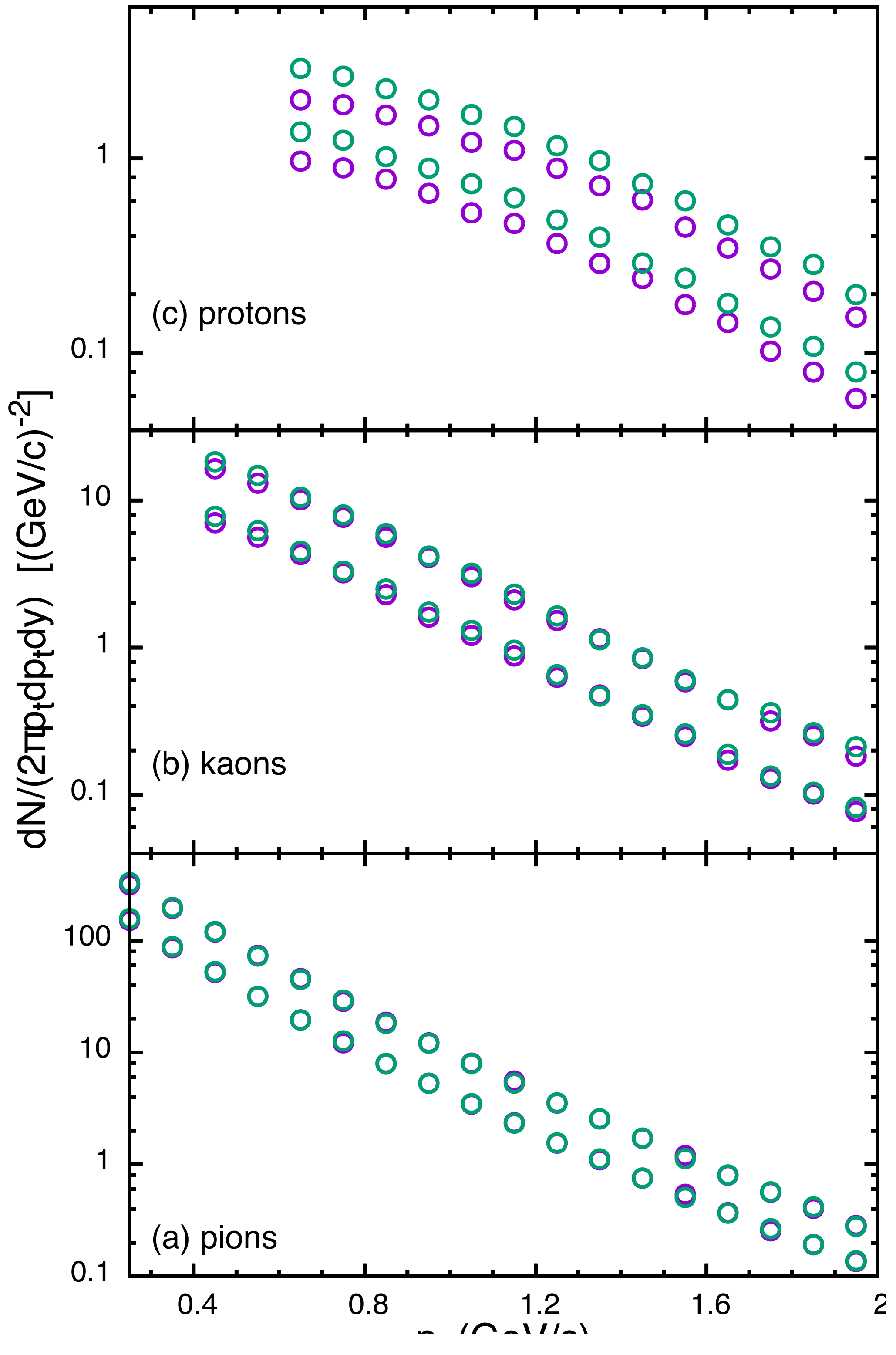
good signal if 1st order PT

1b. Discerning the EoS Collective Radial Flow

More pressure, more flow

$$\langle KE_t \rangle = T + \frac{1}{2} M\, v_{\rm coll}^2$$

More mass, more $\langle E_t \rangle$



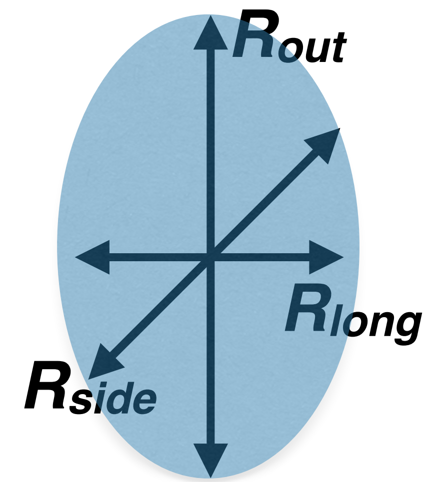
1. Discerning the EoS — F

1c. Femtoscopic Correlations

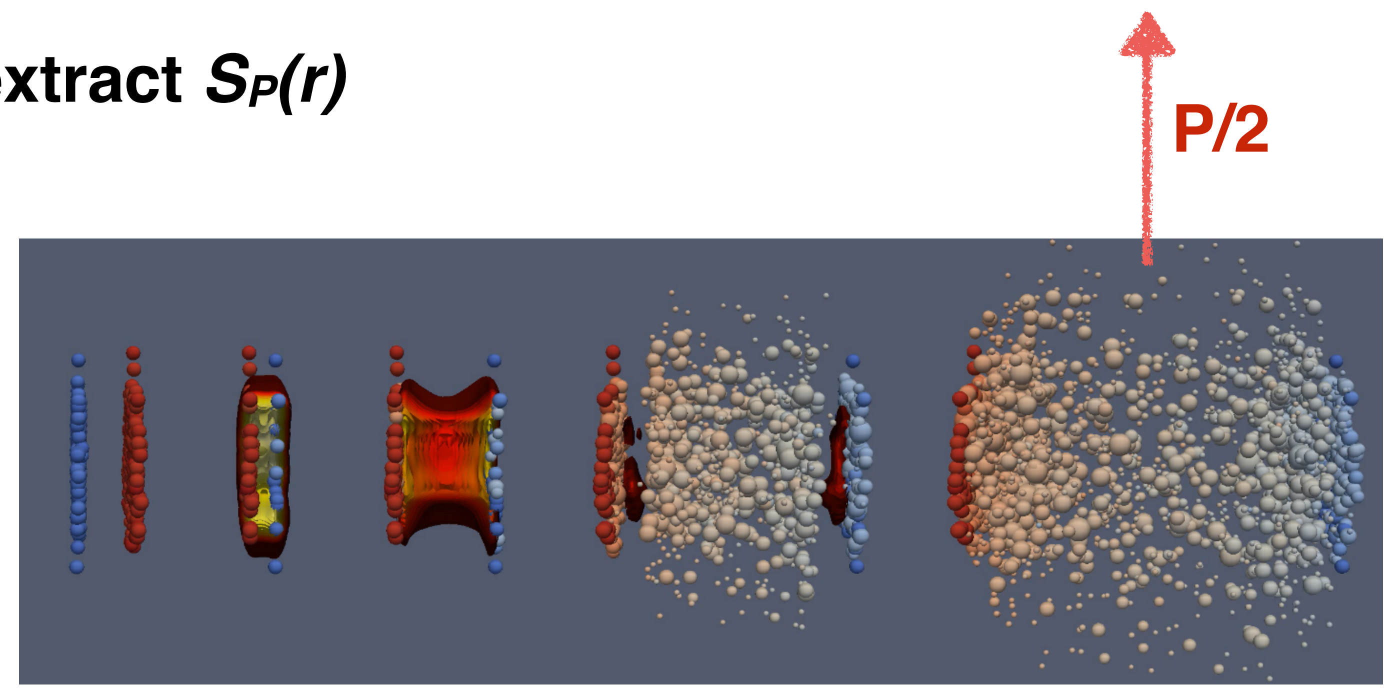
Determine size shape of *f(p,r,t→∞)*

$$S_P(\vec{r}) = \frac{\int d^3r_1 d^3r_2\, f(\vec{P}/2,\vec{r}_1,t)\, f(\vec{P}/2,\vec{r}_2,t)\, \delta^3(\vec{r}-(\vec{r}_1-\vec{r}_2))}{\int d^3r_1 d^3r_2\, f(\vec{P}/2,\vec{r}_1,t)\, f(\vec{P}/2,\vec{r}_2,t)}$$

$$C(\vec{P},\vec{q}) = \int d^3r \left|\phi_{\vec{q}}(\vec{r})\right|^2 S_{\vec{P}}(\vec{r})$$



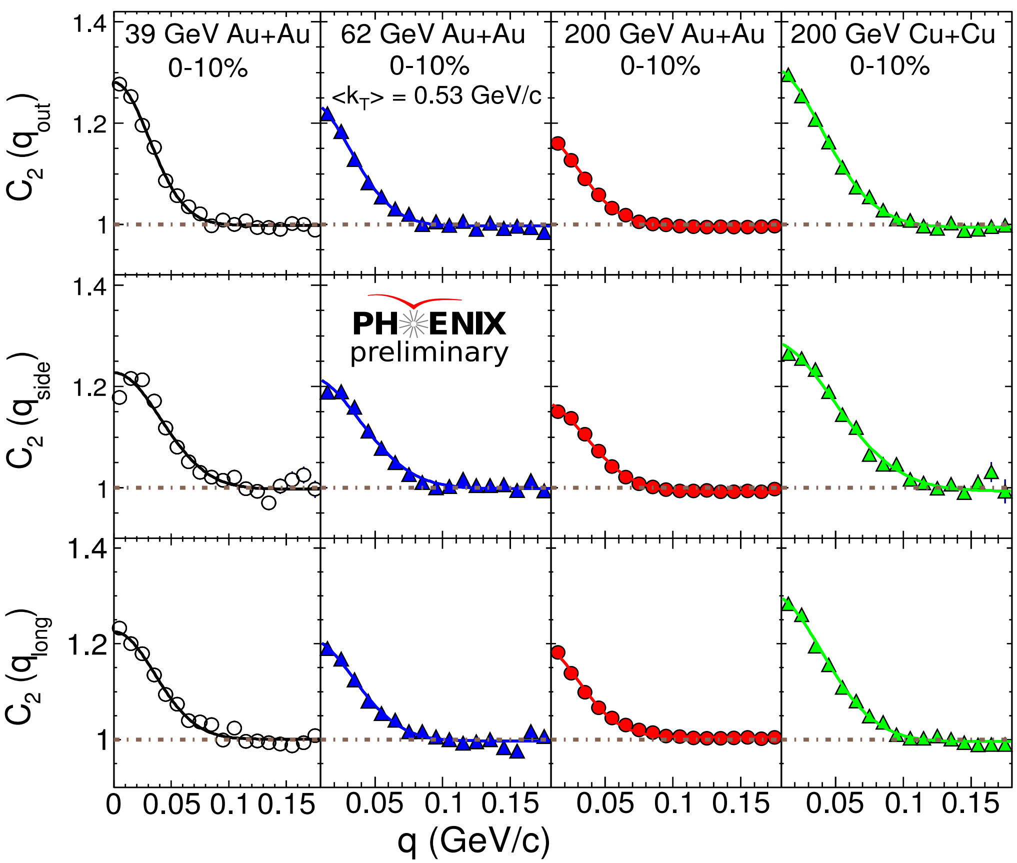
Measure *C(P,q)* →extract *S_P(r)*

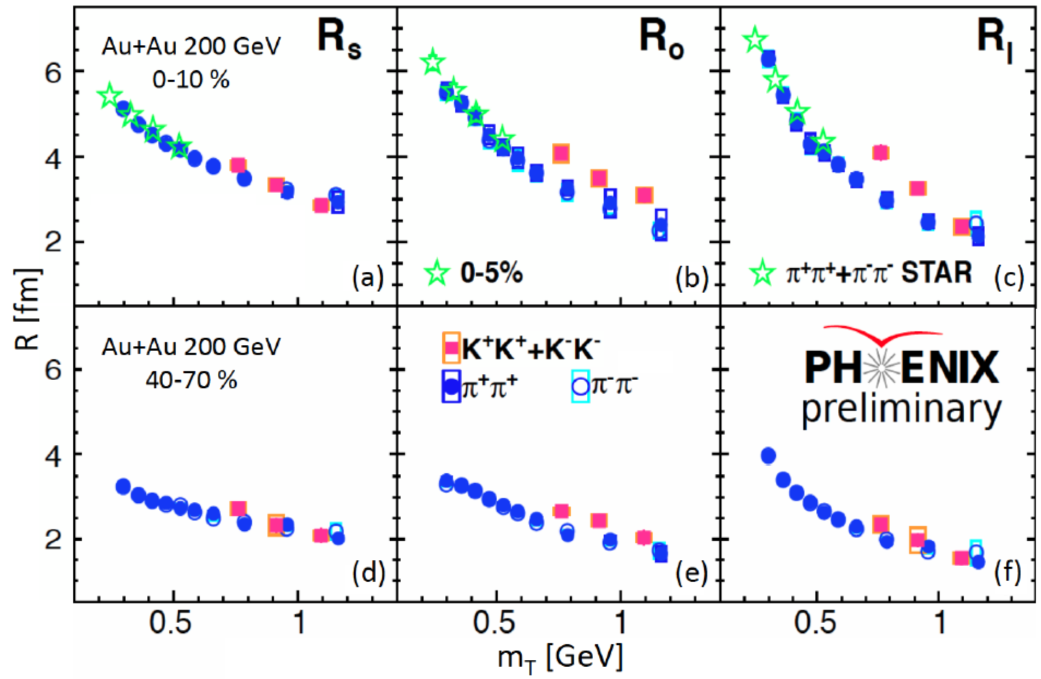


1c. Discerning the EoS — Femtoscopy

For identical bosons, $\left|\phi_{\vec{q}}(\vec{r})\right|^2 = 1 + \cos(2\vec{q}\cdot\vec{r})$

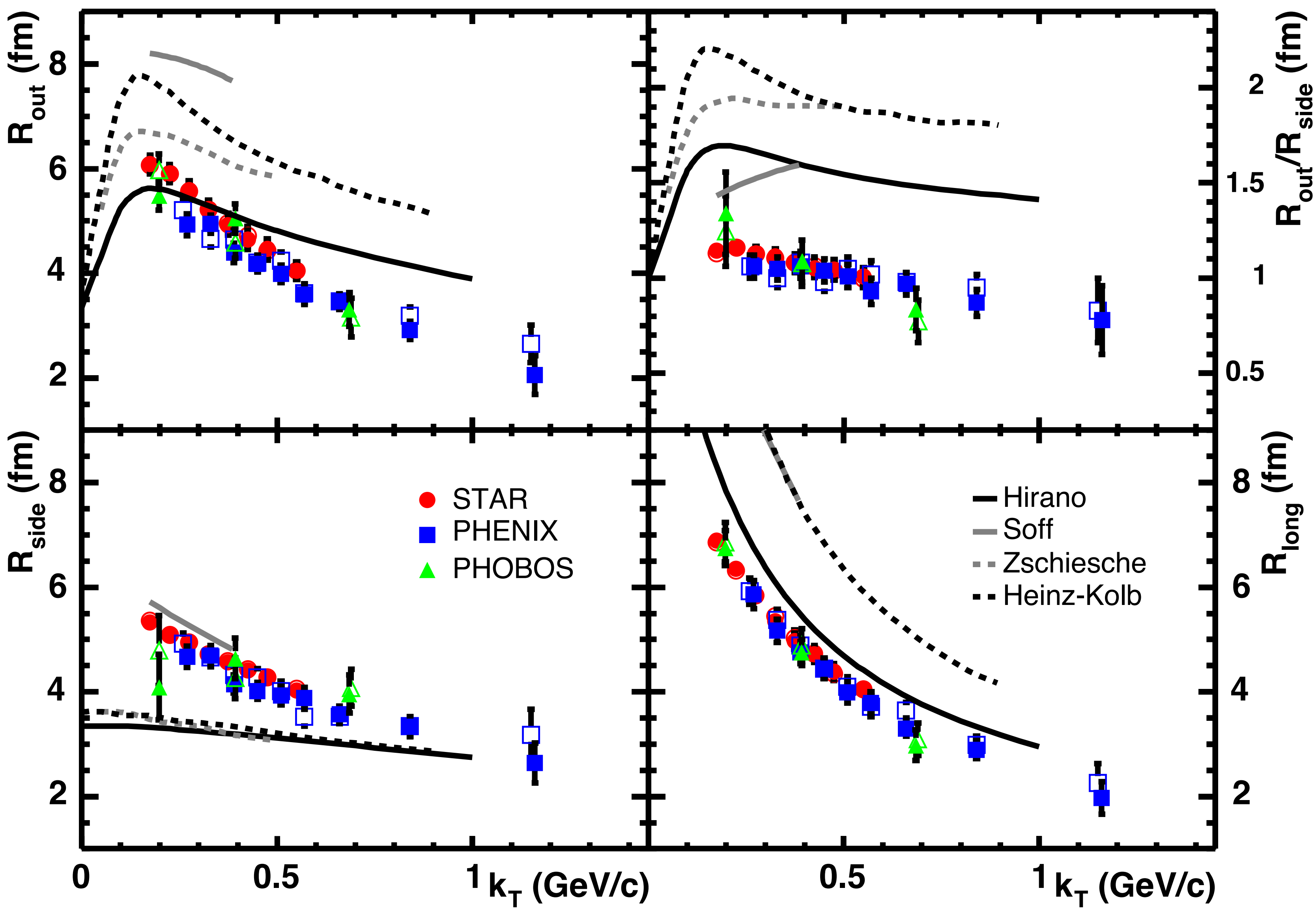
Trickier with strong/Coulomb added





1c. Discerning the EoS — Femtoscopy

Sensitive to EoS (*R_{out}/R_{side}*)

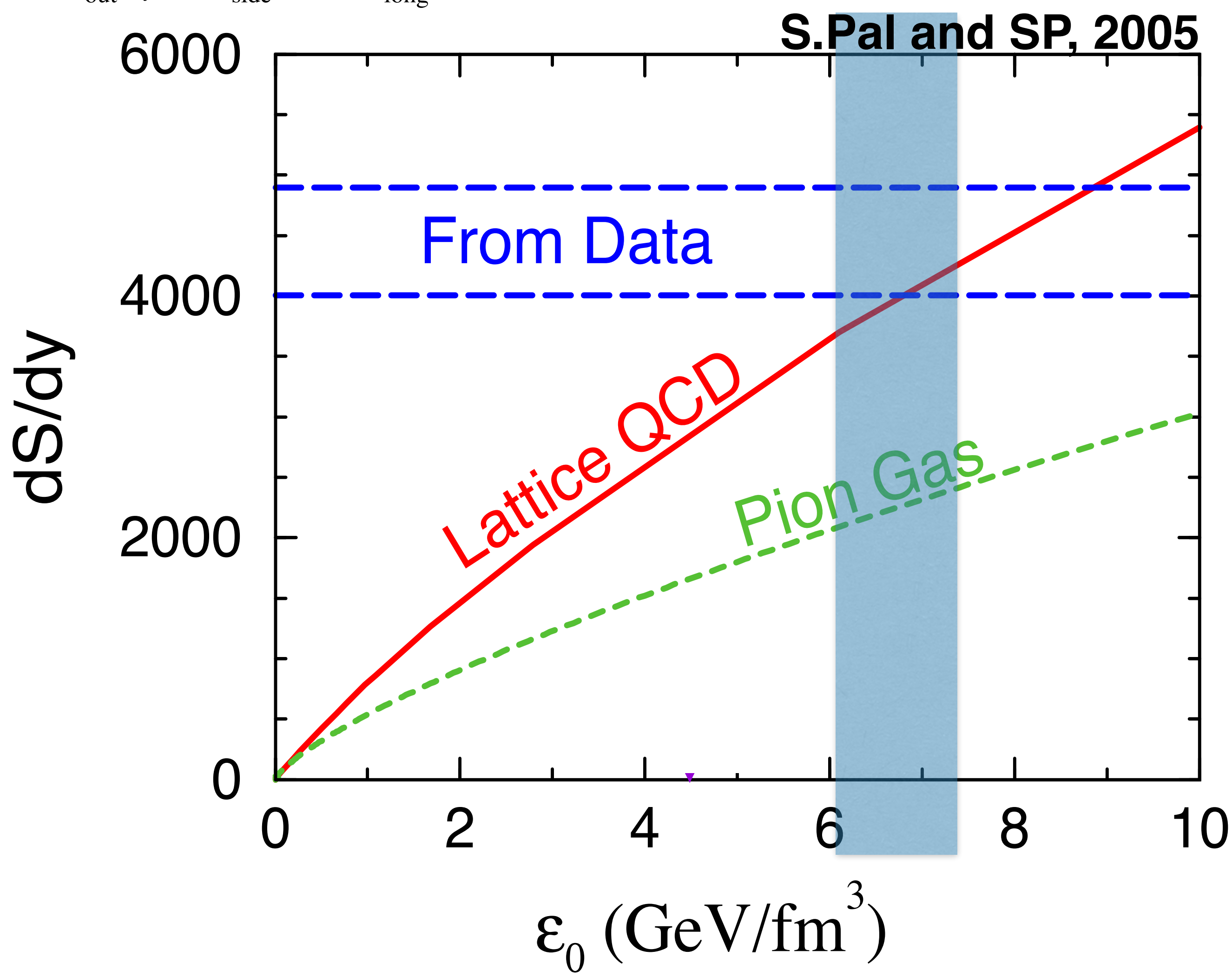


Lisa et al, 2005

1d. Discerning the EoS — Entropy

$$S = \sum_{\alpha} (2S_{\alpha}+1) \int \frac{d^3p\, d^3r}{(2\pi)^3} \{-f \ln f \pm (1 \pm f) \ln(1 \pm f)\}$$

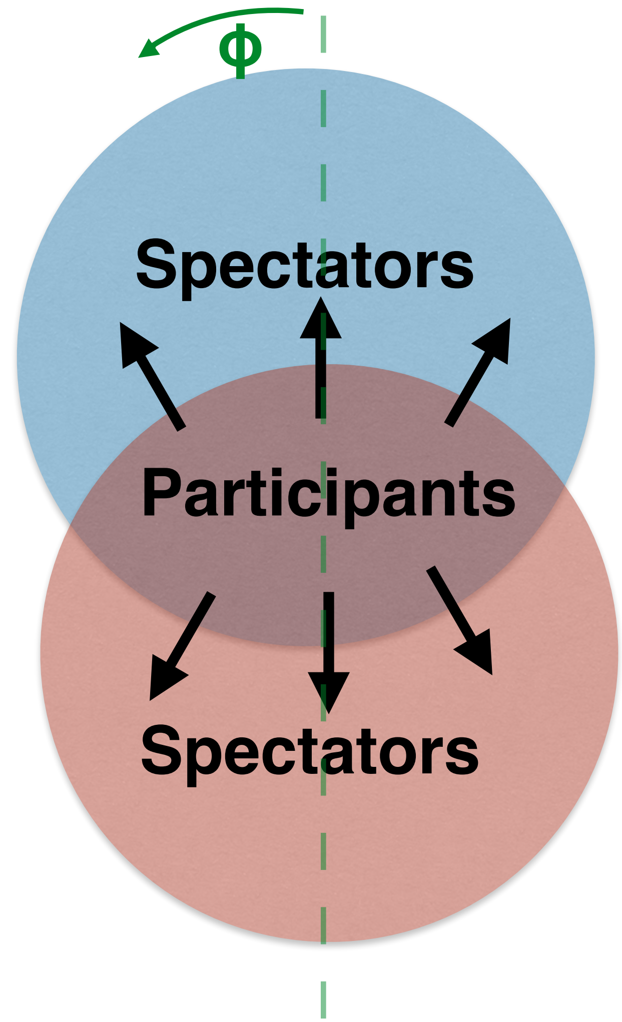
$$f = \frac{(2\pi)^{3/2}}{(2S+1)} \frac{dN / d^3p}{R_{\mathrm{out}} R_{\mathrm{side}} R_{\mathrm{long}}} e^{-x^2/2R_{\mathrm{out}}^2 - y^2/2R_{\mathrm{side}}^2 - z^2/2R_{\mathrm{long}}^2}$$



5. Viscosity — Elliptic Flow

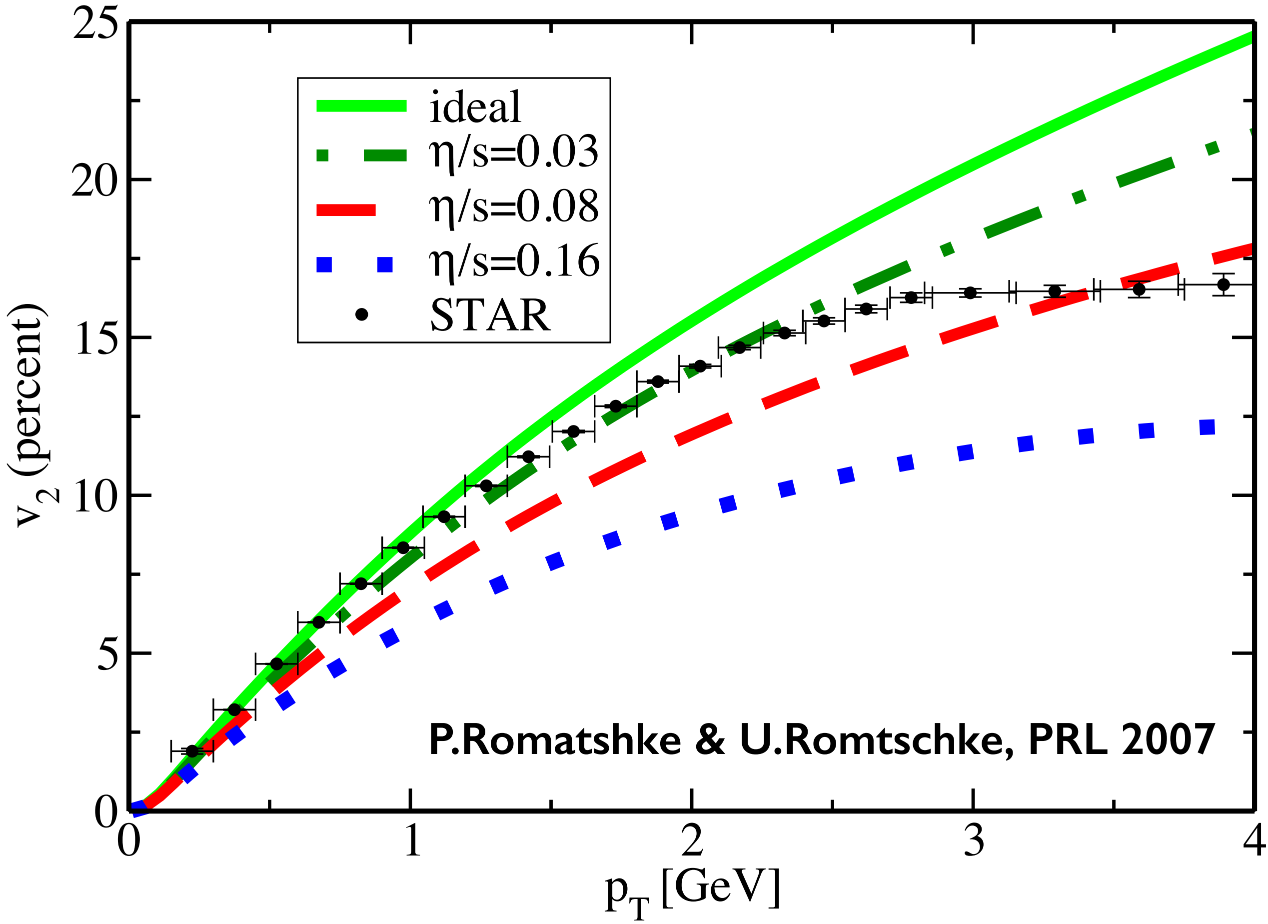
$$\frac{dN}{d\phi} = N_0(1 + 2v_2\cos(2\phi))$$

$$v_2 = \langle \cos(2\phi) \rangle$$



No collisions (no viscosity) → No v_2

5. Viscosity — Elliptic Flow



In 2007 viscosity appeared anomalously low

2. Chemistry

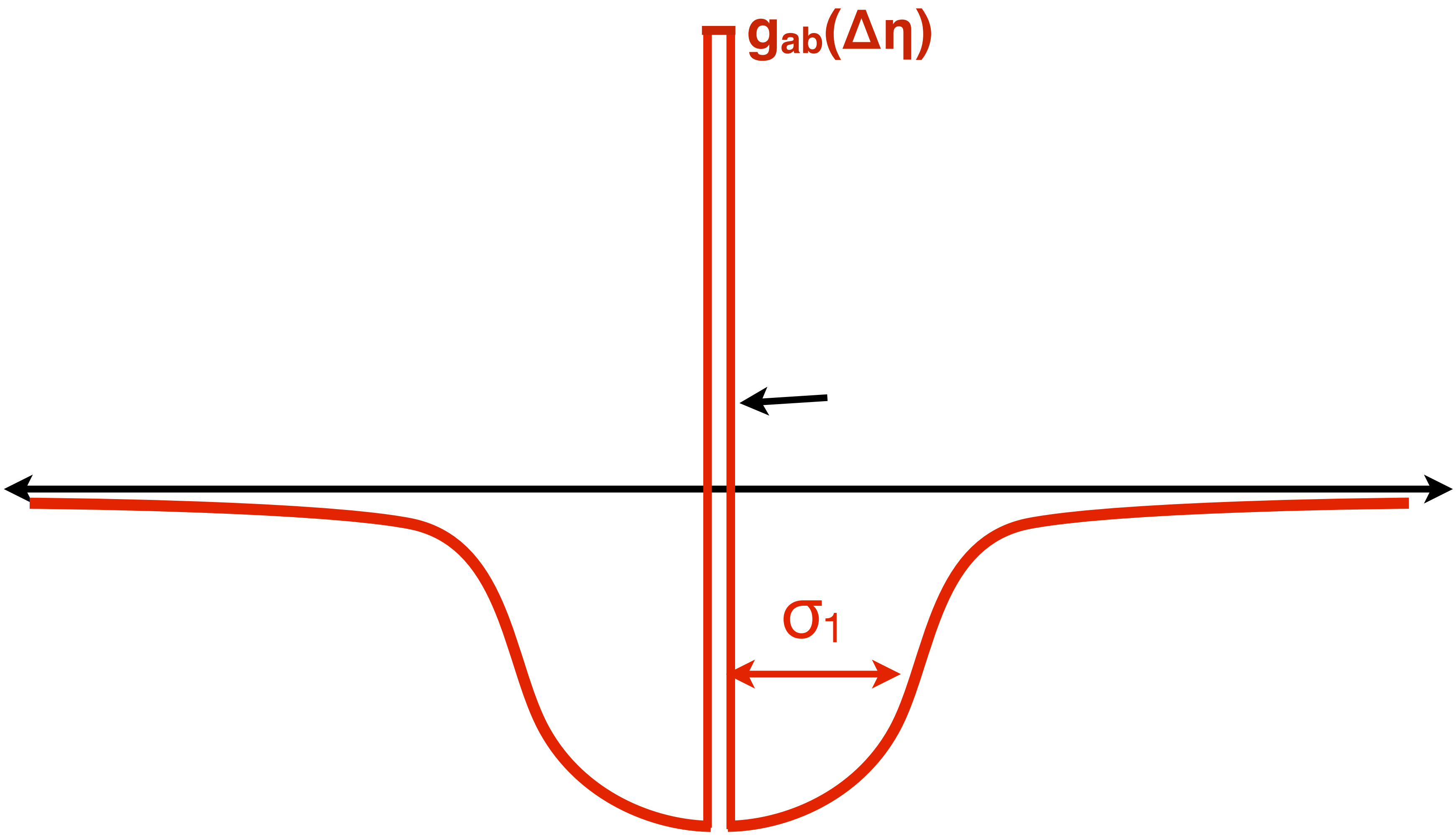
Goal: Determine $\chi_{ab}=\langle Q_a Q_b \rangle / V$
Challenge: CHARGE DOESN'T FLUCTUATE IN FINITE SYSTEM!

$$g_{ab}(\Delta\eta_s) = \langle \rho_a(0)\rho_b(\Delta\eta_s)\rangle, \ \langle \rho_a \rangle = 0$$
$$= \chi_{ab}\left(\delta(\Delta\eta) - \frac{1}{(2\pi\sigma^2)^{1/2}} e^{-\Delta\eta^2/2\sigma^2}\right)$$

integrates to zero

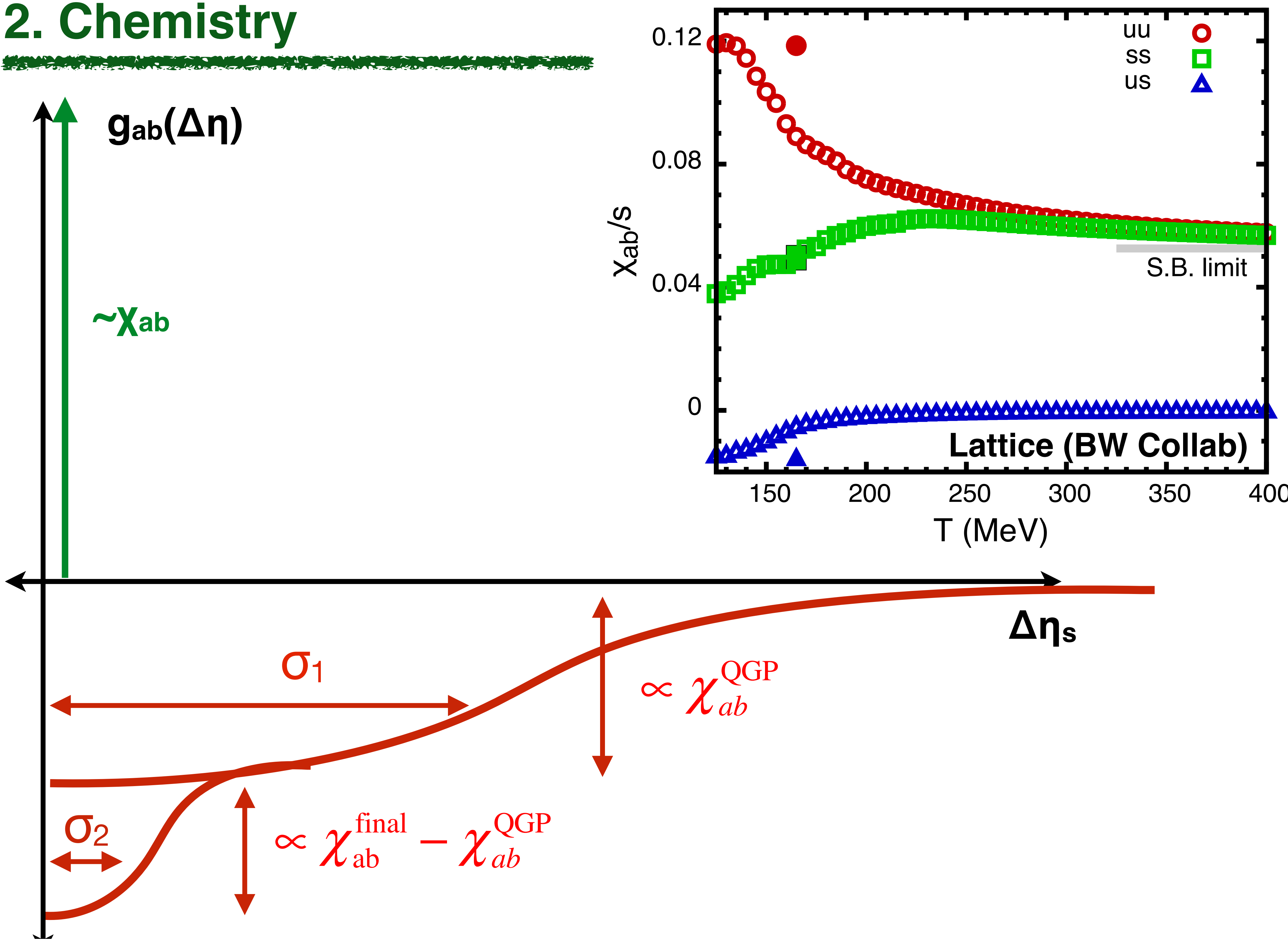
***R* increases over time**

2. Chemistry



Must integrate to zero

2. Chemistry



gab(Δη)
~χab
Δηs
σ1
σ2
∝ χ_{ab}^{QGP}
∝ $\chi_{ab}^{final} - \chi_{ab}^{QGP}$
0.12
0.08
0.04
0
uu
ss
us
S.B. limit
Lattice (BW Collab)
χab/s
150
200
250
300
350
400
T (MeV)

2. Chemistry

g_{ab} causes correlations in hadrons

$$\delta n_\alpha = \langle n_\alpha \rangle \sum_b q_{\alpha,a} \chi_{ab}^{\mathrm{final},-1} \delta\rho_b$$

$$g_{ab}(\Delta\eta_s) \to G_{\alpha\beta}(\Delta\eta_s) \equiv \langle (n_\alpha(0) - n_{\bar{\alpha}}(0))(n_\beta(\Delta\eta_s) - n_{\bar{\beta}}(\Delta\eta_s)) \rangle$$

+collective flow & thermal motion

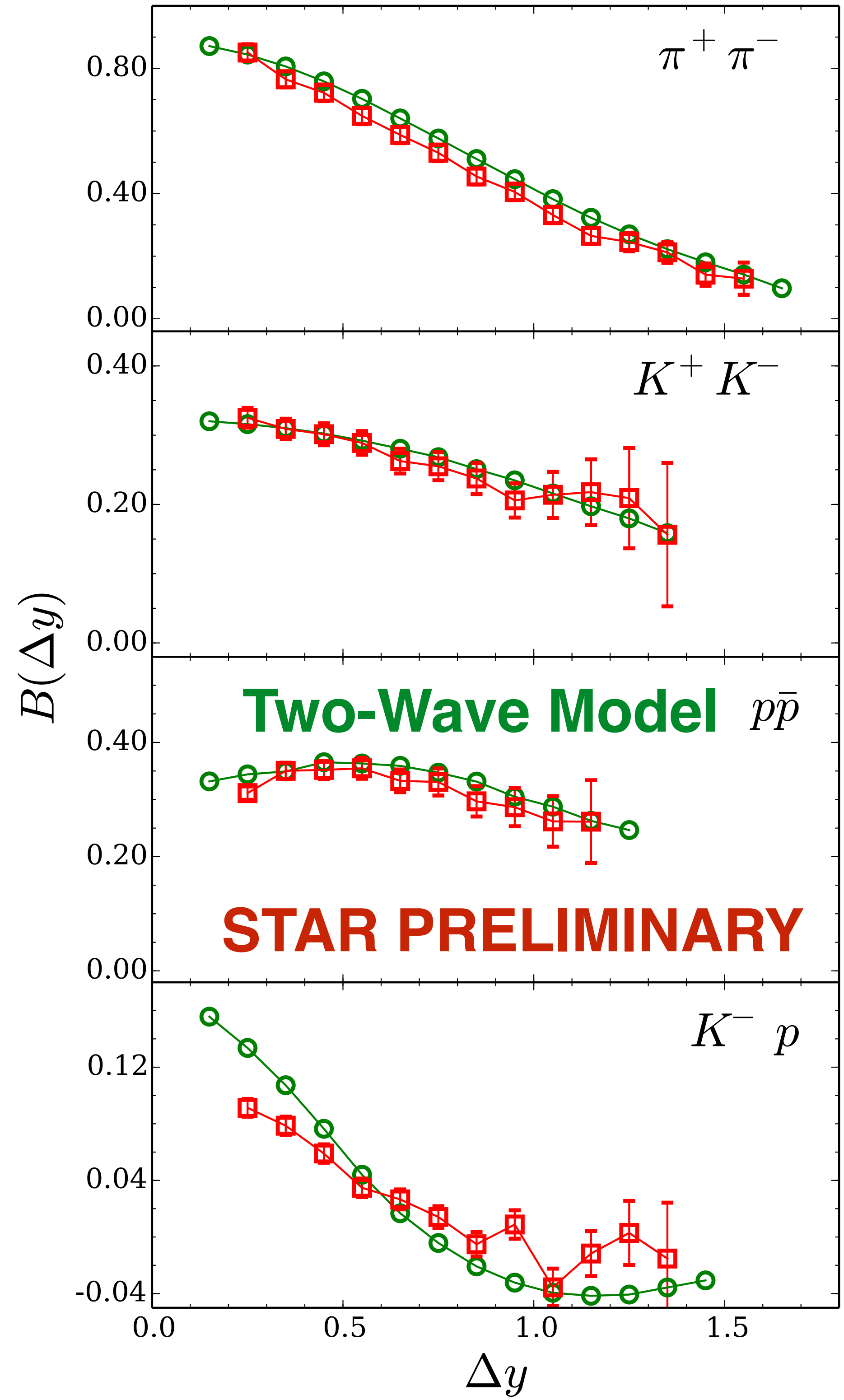
$$\to B_{\alpha\beta}(\Delta y) \equiv \frac{G_{\alpha\beta}(\Delta y)}{\langle n_\alpha \rangle}$$

measured

2. Chemistry — Charge Balance Functions

$$B_{\alpha\beta}(\Delta y) \equiv \frac{\langle (n_\alpha(0) - n_{\bar{\alpha}}(0))(n_\beta(\Delta y) - n_{\bar{\beta}}(\Delta y)) \rangle}{\langle n_\alpha + n_{\bar{\alpha}} \rangle}$$

Fits best with $\sigma_1 \sim 1.0$, $\sigma_2 \sim 0.2$
$\chi_1 \sim \chi$ from lattice!!!
Can't fit with "one-wave"



3. Chiral Symmetry

For 160 < T < 200, chiral symmetry with hadrons
Hadron mass evolution is not understood

- **Do hadrons become light?** $M_{\text{hadron}} \sim \langle \sigma \rangle$

  OR

- **Do masses merge? e.g.** $M_{a2} \rightarrow M_{\rho}$

Difficult due to limited decays during window and collision broadening. STAR and PHENIX disagree.

NEAR FUTURE: Experimental results should be clarified (PHENIX HBD)

8. Stopping and Thermalization

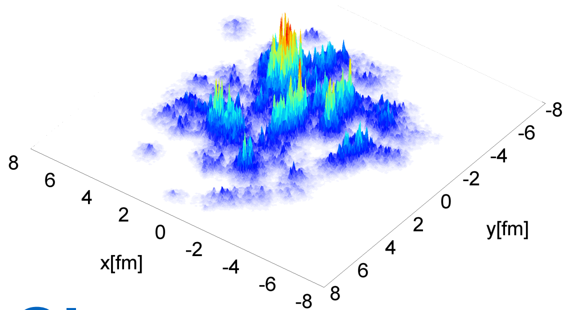
Microscopic approaches:

- **CGC based on classical Yang-Mills Fields**
- **Parton cascades**

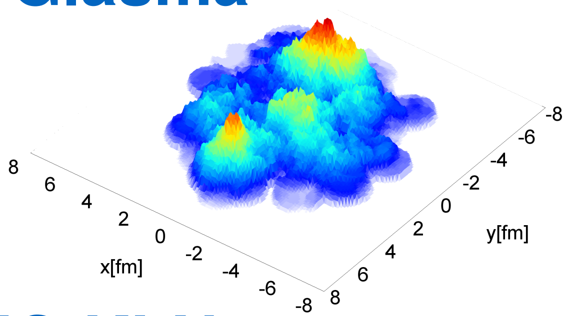
New focus driven by new data

- **Fluctuations: v_3, v_4, …**
- **Distributions of v_2**
- ***pA* collisions**
- **Looking away from mid rapidity**
- **Long-range rapidity correlations**

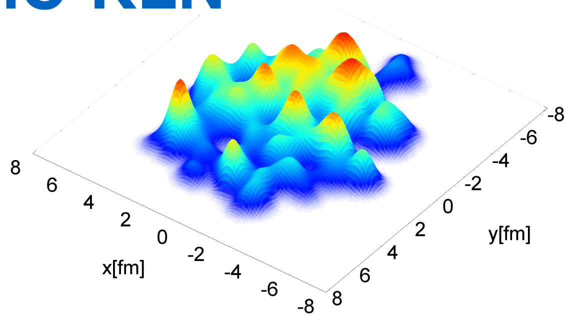
8. Stopping and Thermalization (Fluctuations)



IP-Glasma

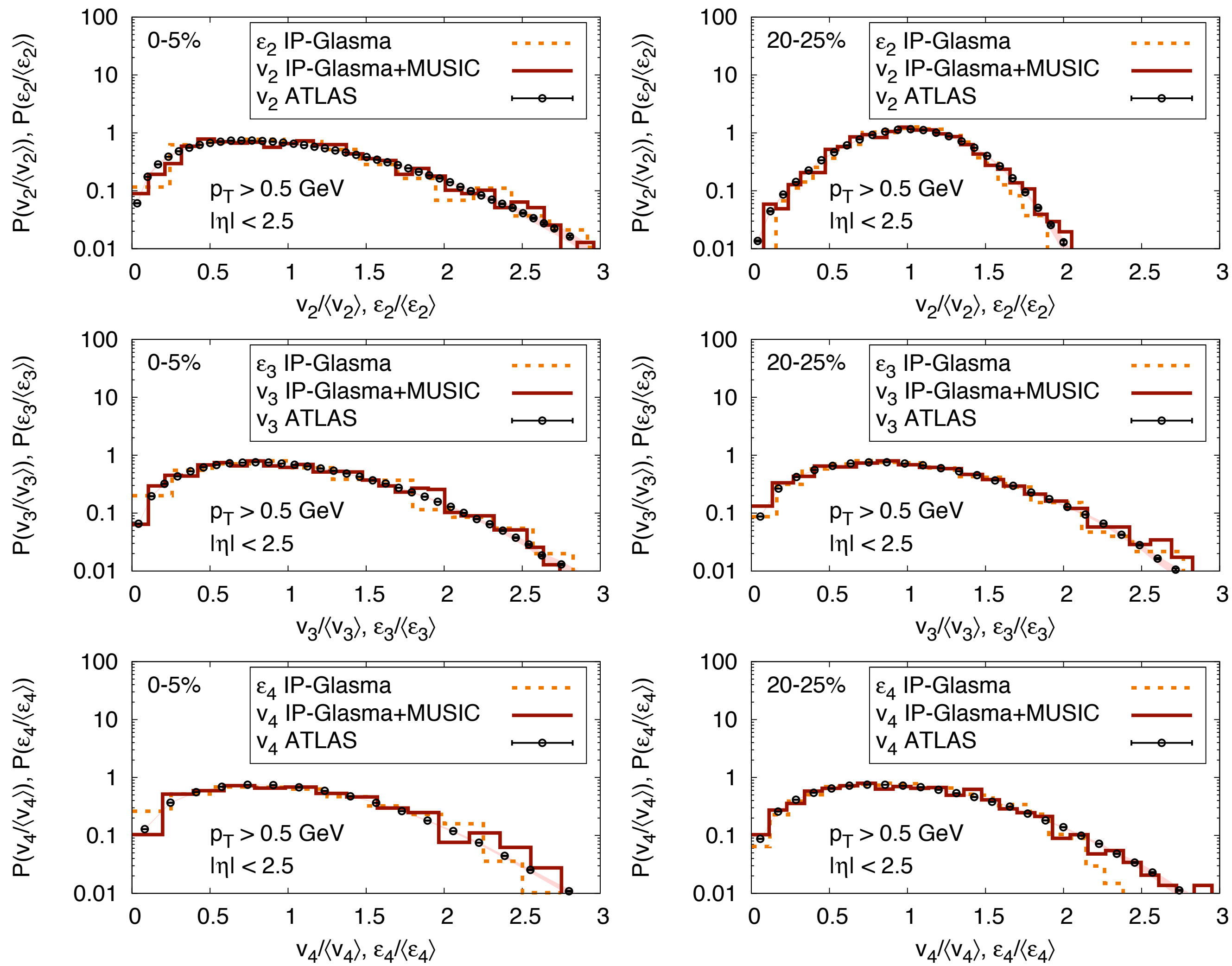


MC-KLN



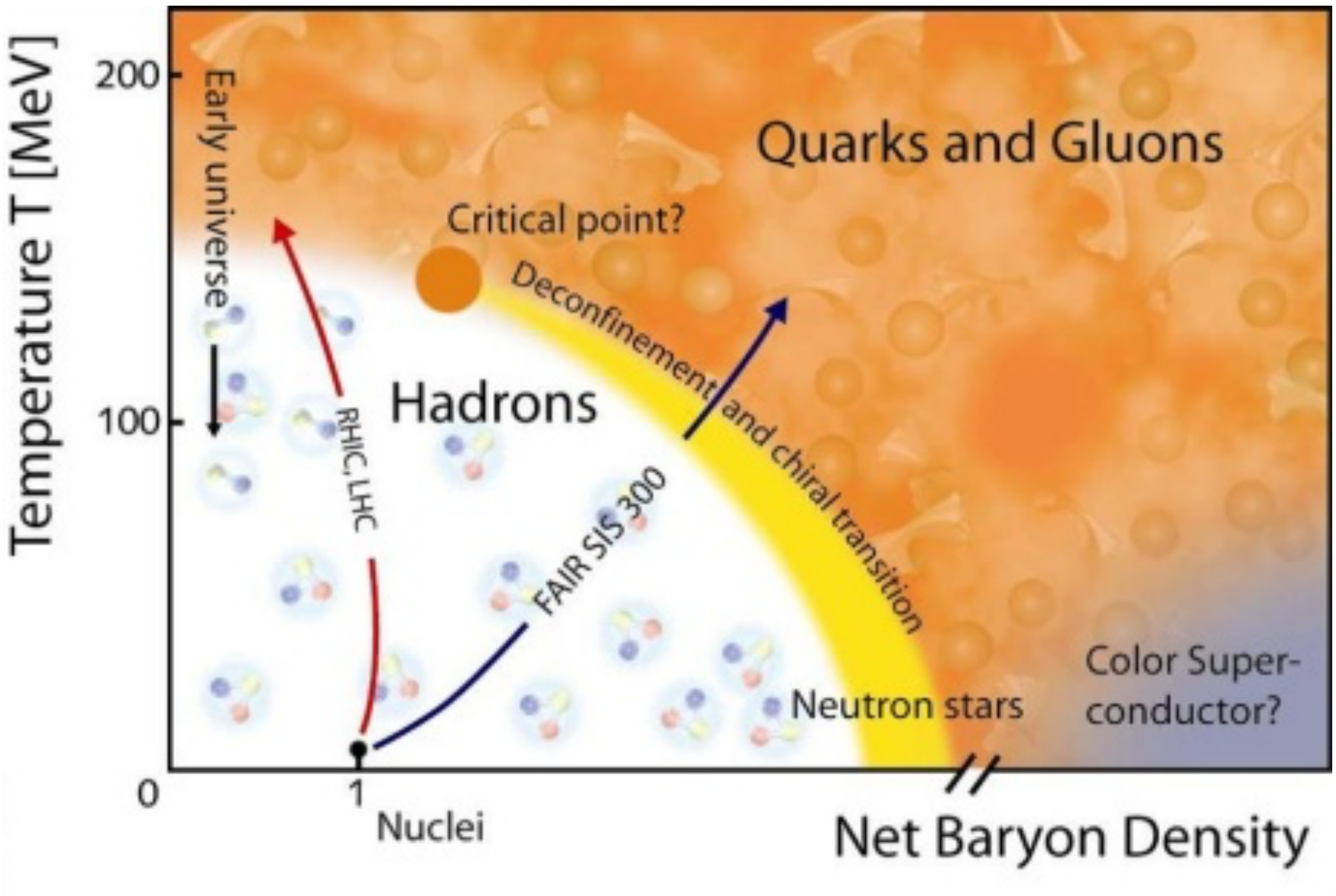
MC-Glauber

IP-Glasma



Gale, Jeon, Scheke, Tribidy & Venugopalan, 2014

1. EoS at Finite Baryon Density



First-order phase transition and critical point?

- **no evidence in neutron star observations or heavy ion physics**
- **fluctuations may be ephemeral**
- **requires great care in comparing theory to experiment**