Part II

the QCD toolbox

asymptotic freedom, IR safety,
QCD final state, factorization

dichotomy of QCD

the gauge principle is elegant and powerful but any theory must ultimately stand (or fall) by its success (or failure)

QCD is the theory of **strong** interactions

- how can we make use of **perturbative** methods?

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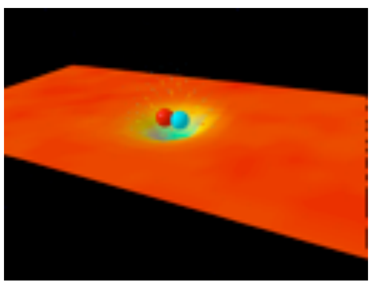
confinement



D. Leinweber

non-perturbative structure of hadrons

e.g. through **lattice QCD**

asymptotic freedom

hard scattering cross sections and renormalization group

with **perturbative methods**

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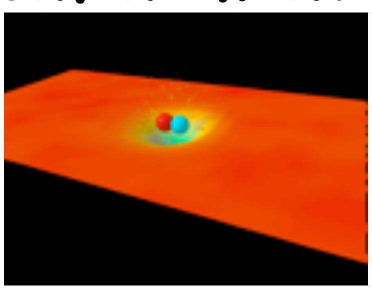

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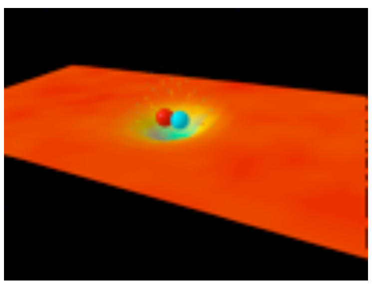

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interplay

probing hadronic structure with
weakly interacting quanta of asymptotic freedom

asymptotic freedom

Gross, Wilczek; Politzer ('73/'74)
Nobel prize 2004

value of strong coupling $\alpha_s = g^2/4\pi$ depends on distance r (i.e., on energy Q)

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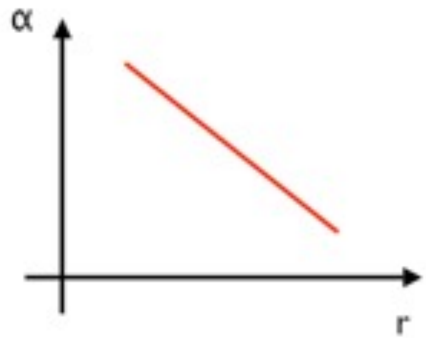

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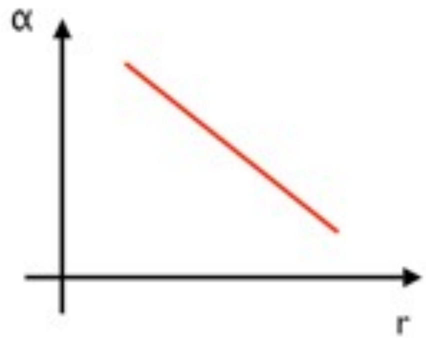

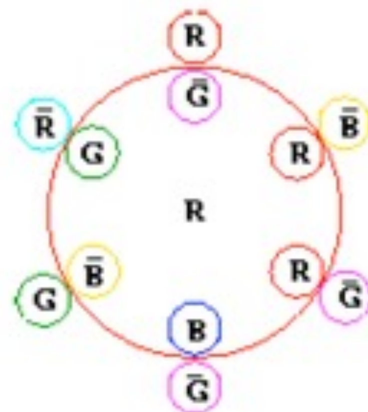

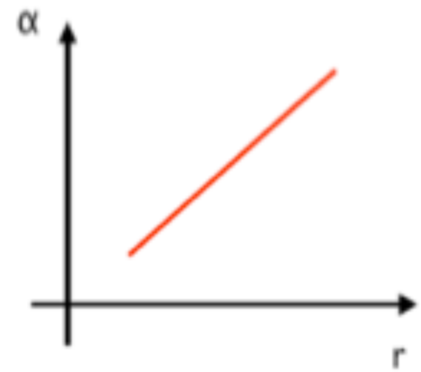

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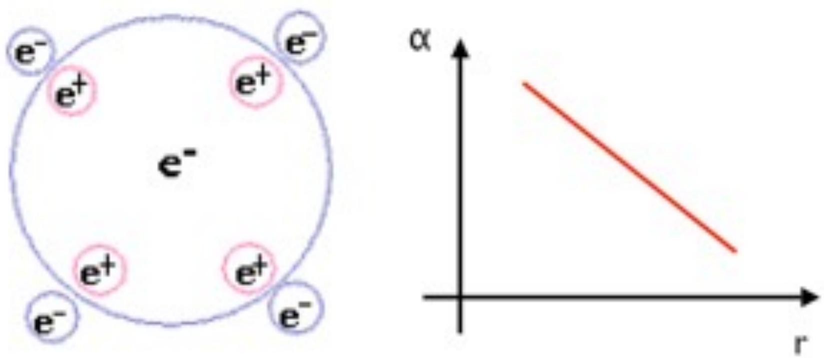

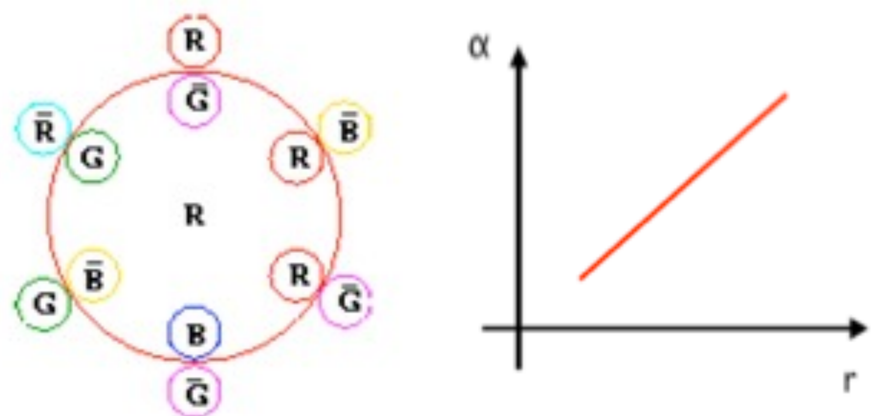

who wins ?

$$\alpha_s(Q^2) \approx \frac{4\pi}{(\frac{11}{3}C_A - \frac{4}{3}T_F N_f)\ln(Q^2/\Lambda^2)}$$

$Q \sim 1/r$

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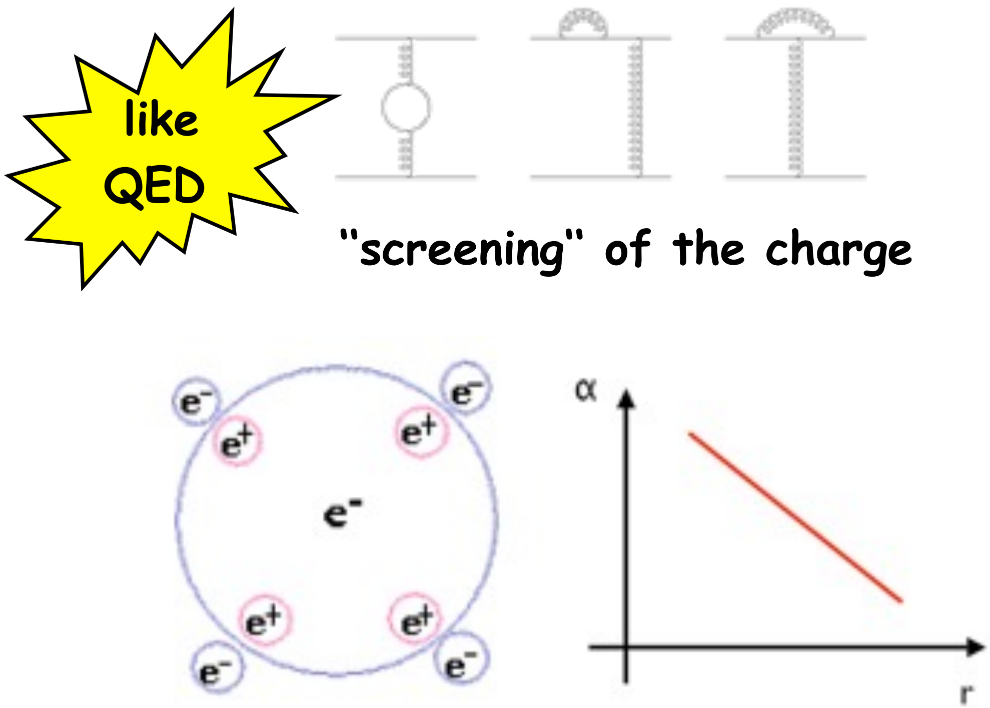

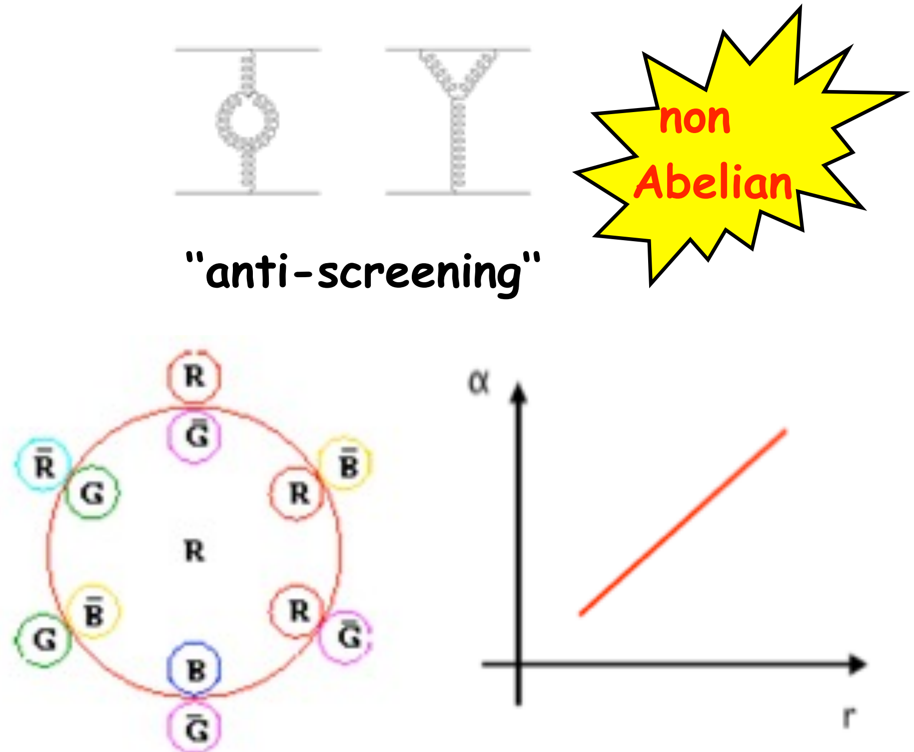

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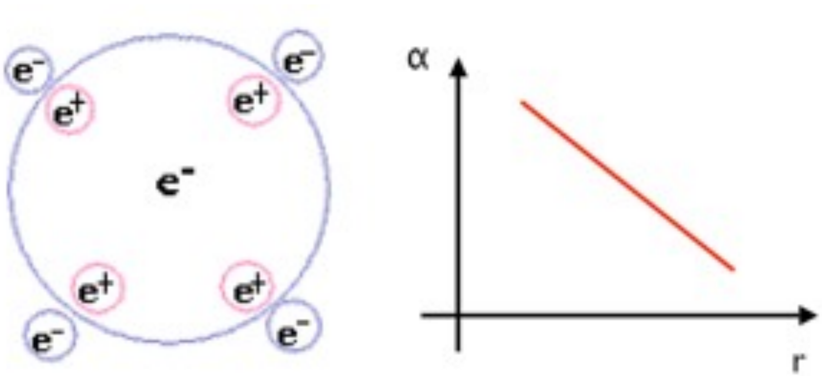

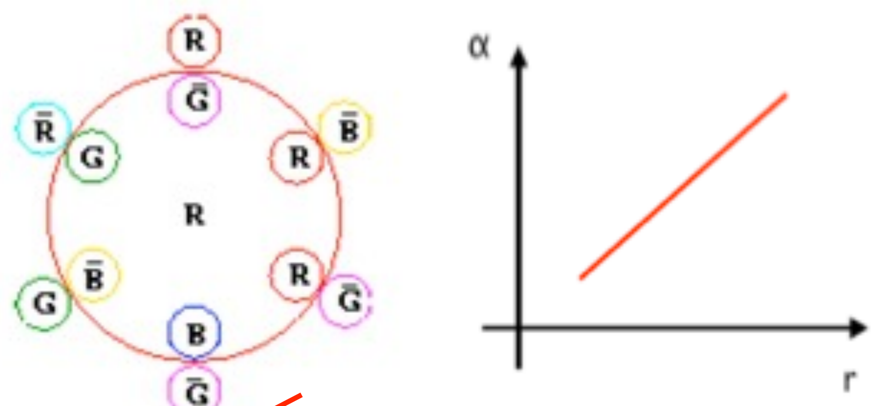

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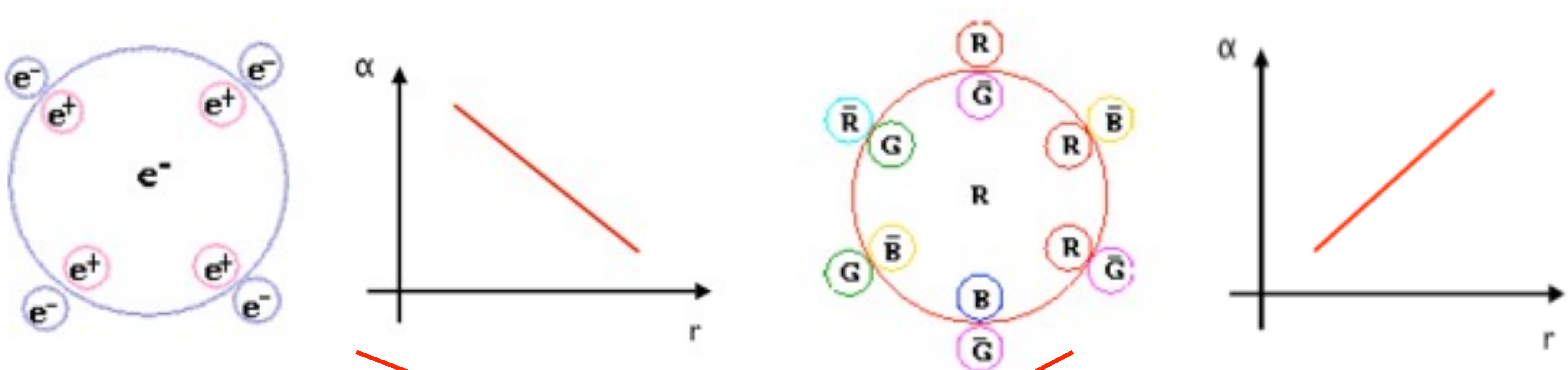

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typical hadronic scale O(200 MeV)
Λ depends on N_f, pert. order and scheme

more formally: the QCD beta function

van Ritbergen, Vermaseren, Larin

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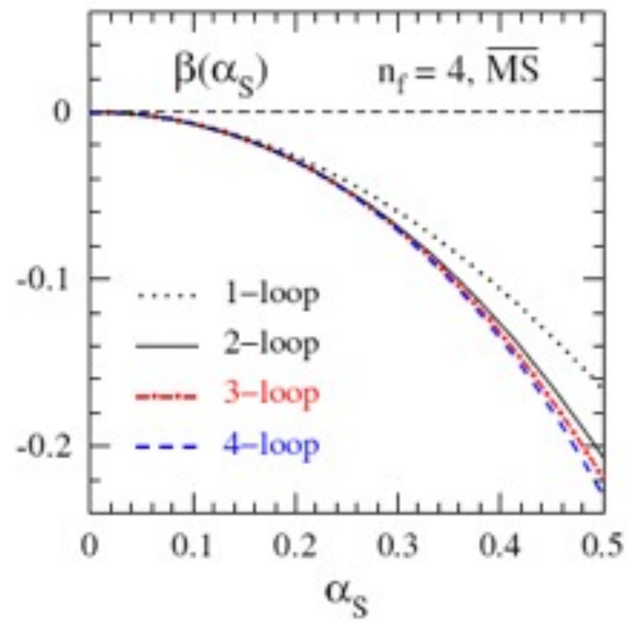
	$\beta_0 a_s^2$	$\beta_1 a_s^3$	$\beta_2 a_s^4$	$\beta_3 a_s^5$
Year	('71), '73	'74	'80	'97
Order	LO	NLO	NNLO	N³LO



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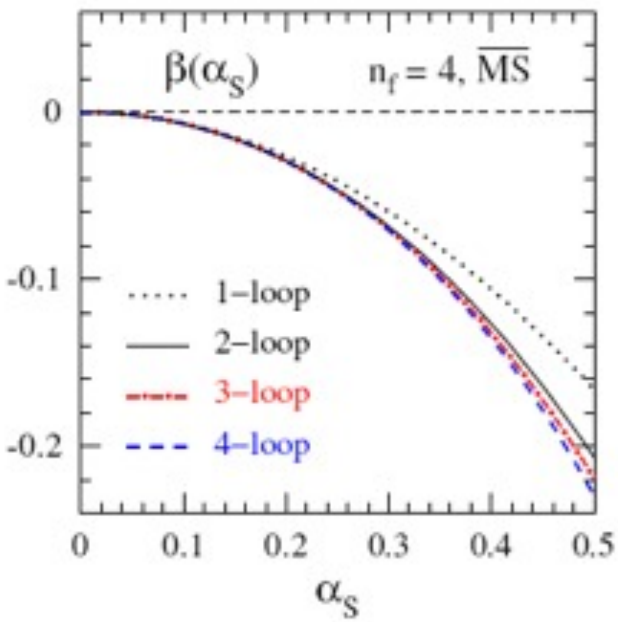

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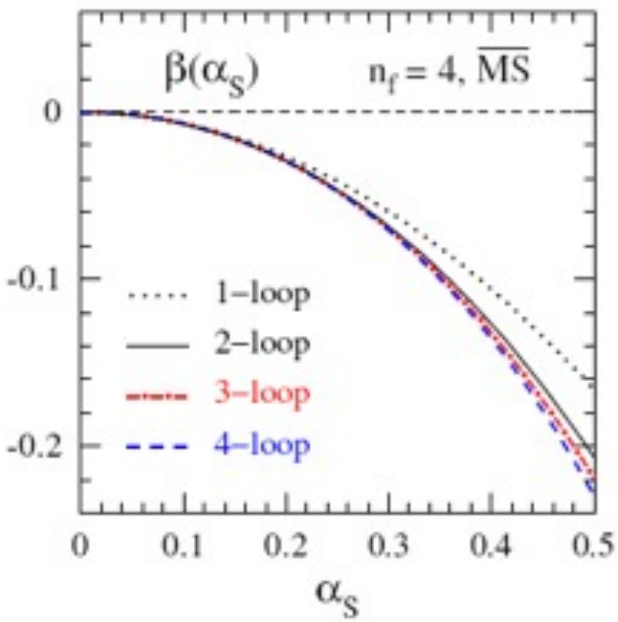

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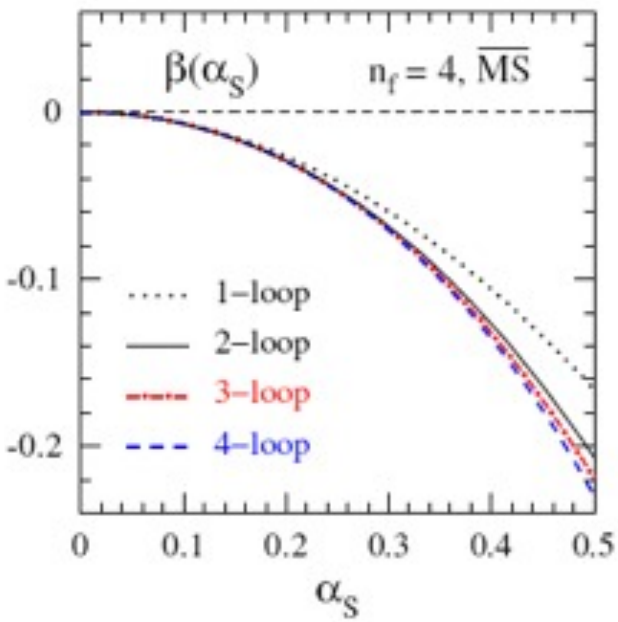

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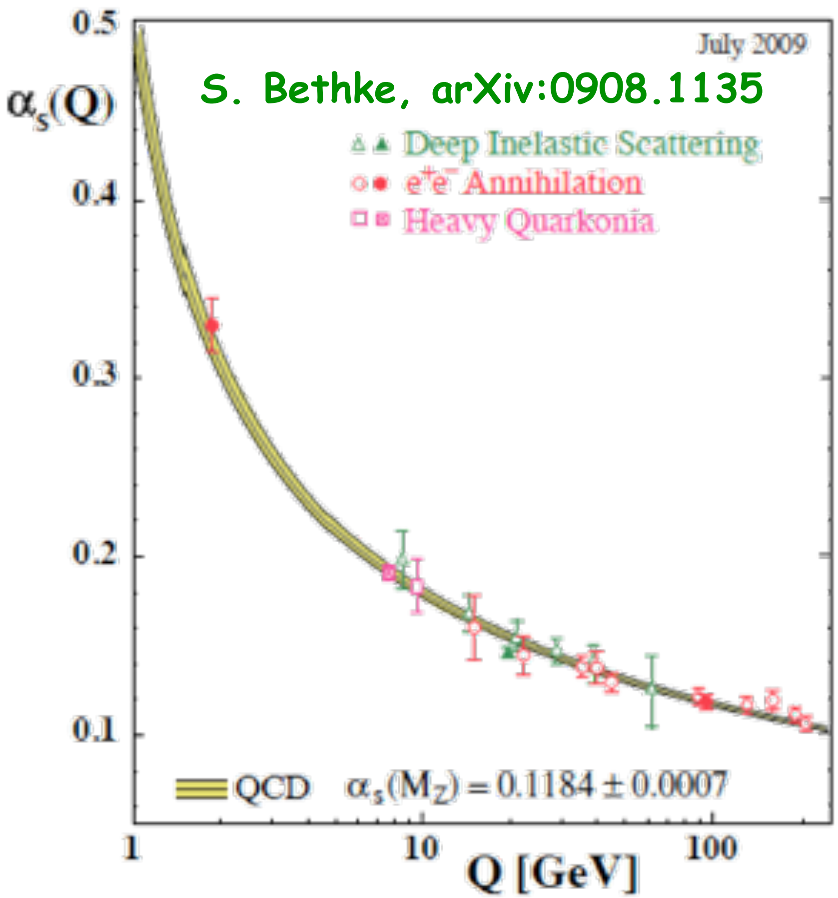
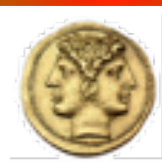
tells us how α_s varies with scale but not its absolute value at μ_0

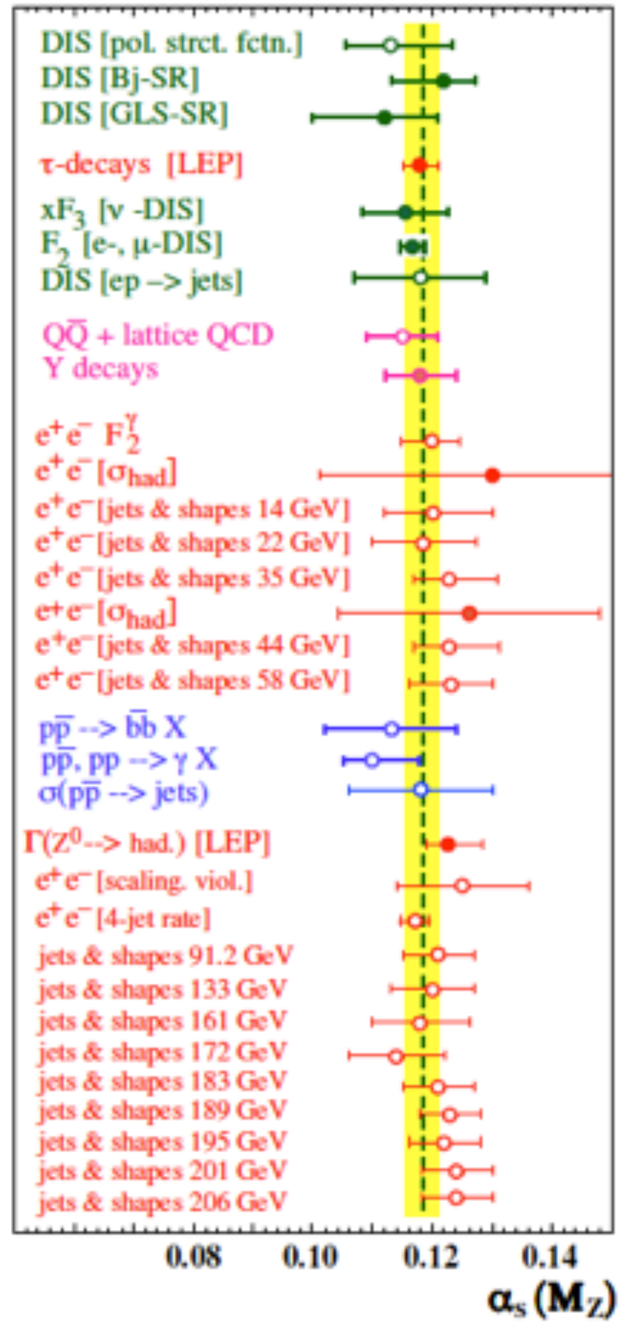
1st example of a renormalization group equation

consistent picture from many observables



confinement  asymp. freedom

exp. evidence for $\log(Q^2)$ fall-off is persuasive



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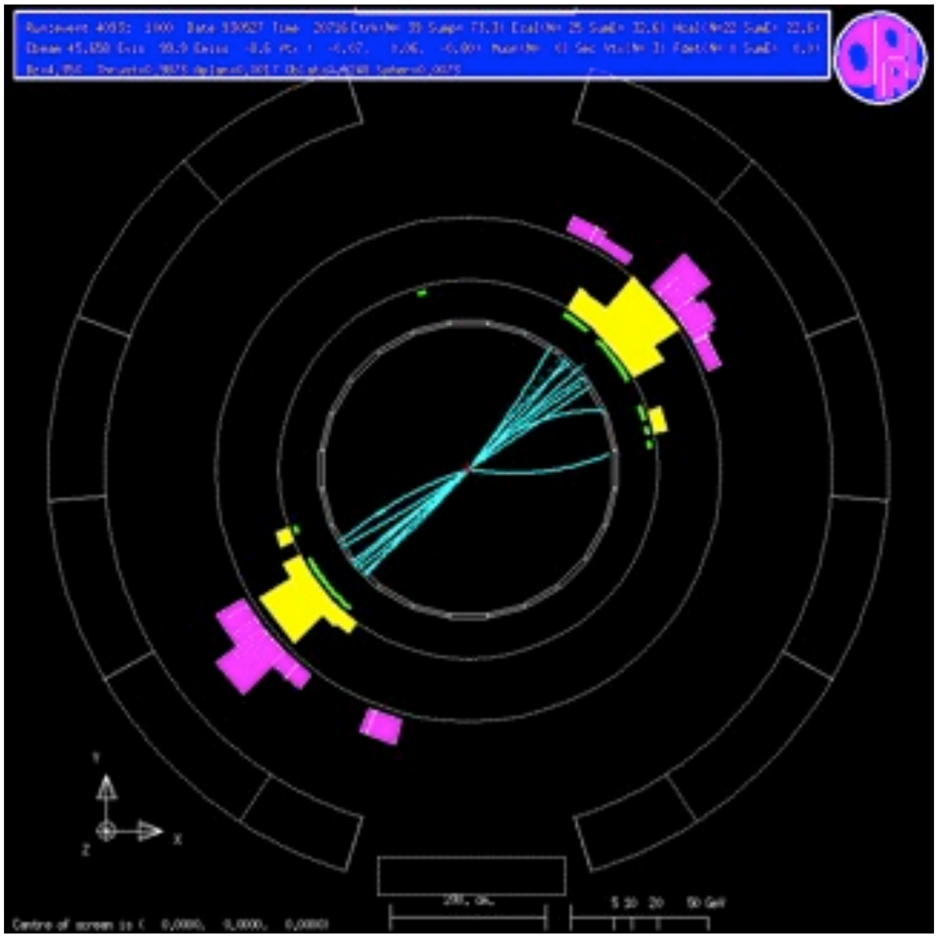
let's study electron-positron annihilation to see what this is all about ...

e^+e^- annihilation: the QCD guinea pig

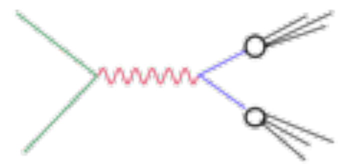
1989-2000

most of the hadronic events at CERN-LEP had **two back-to-back jets**

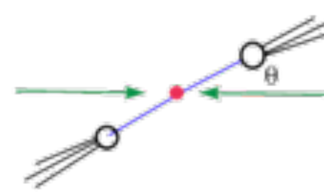


jet: pencil-like collection of hadrons

- jets resemble features of underlying 2->2 hard process $e^+e^- \to q\bar{q}$



- angular distribution of jet axis w.r.t. beam axis as predicted for **spin-$\frac{1}{2}$ quarks**



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1989-2000

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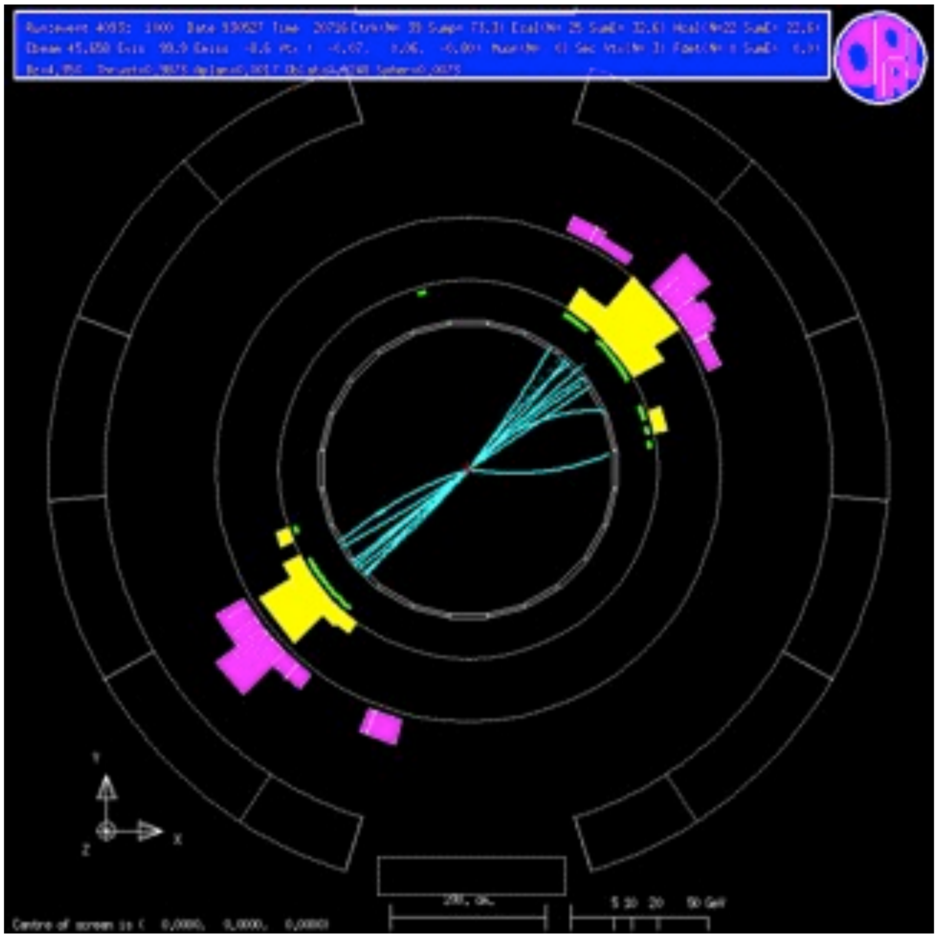
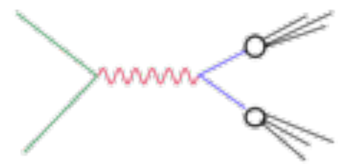
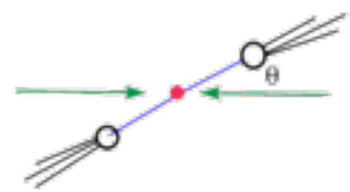


jet: pencil-like collection of hadrons

- jets resemble features of underlying 2->2 hard process $e^+e^- \to q\bar{q}$



- angular distribution of jet axis w.r.t. beam axis as predicted for **spin-$\frac{1}{2}$ quarks**

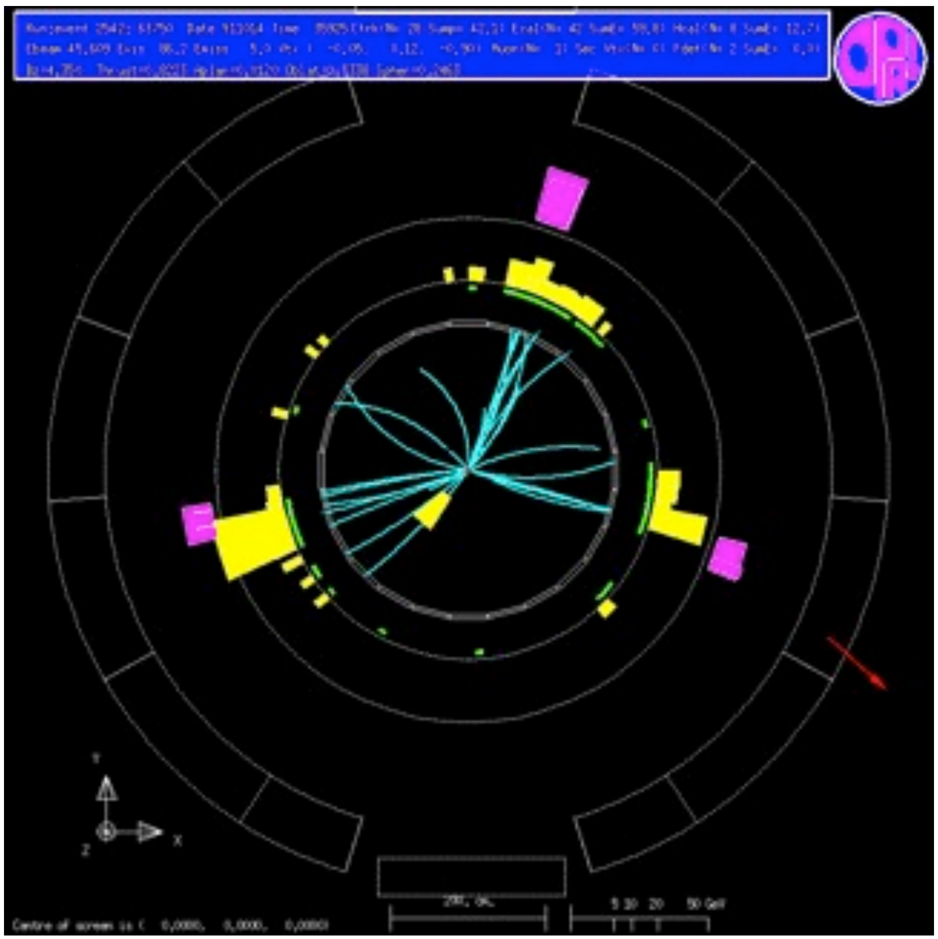


jets play major role in hadron-hadron collisions at TeVatron, RHIC, LHC

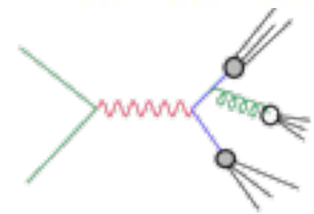
e^+e^- annihilation: three-jet events

about 10% of the events had a **third jet**

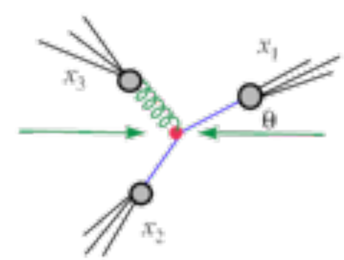
first discovered at DESY-PETRA in 1979



- jets resemble features of underlying 2->3 hard process $e^+e^- \to q\bar{q}g$



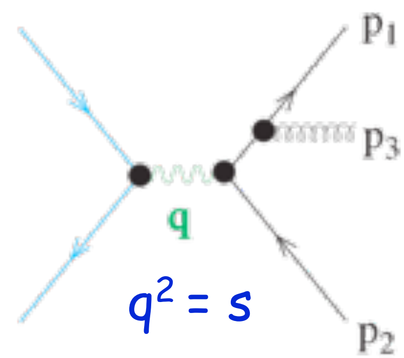
- 10% rate consistent with $\alpha_s \simeq 0.1$ (**determination of α_s**)
- angular distribution of jets w.r.t. beam axis as expected for **spin-1 gluons**



exploring the QCD final-state: $e^+e^- \rightarrow$ 3 partons

simplest process in pQCD: $e^+e^- \rightarrow q\bar{q}g$

(all partons massless)



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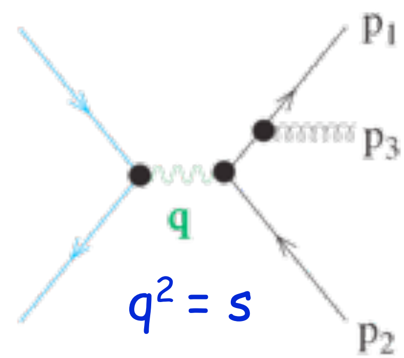

some kinematics first:

- energy fractions & conservation: $x_i \equiv \frac{2p_i \cdot q}{s} = \frac{E_i}{\sqrt{s}/2}$ $\quad \sum x_i = \frac{2(\sum p_i) \cdot q}{s} = 2$

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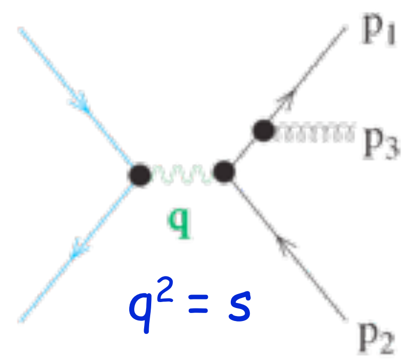

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(other angles by cycl. permutation)

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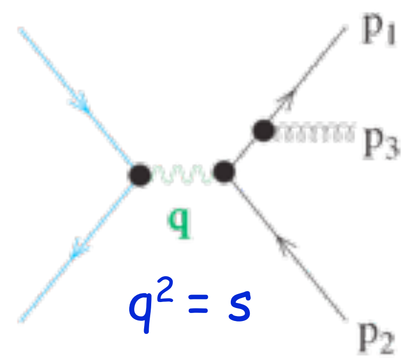

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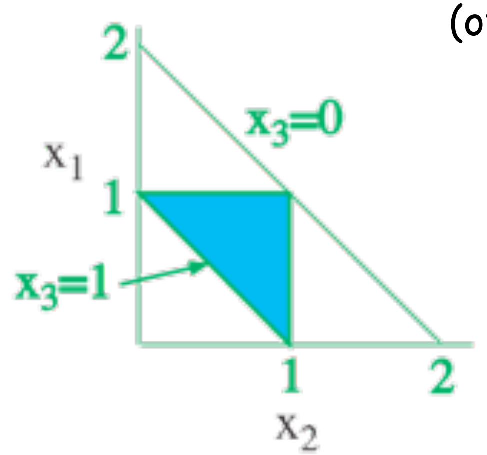
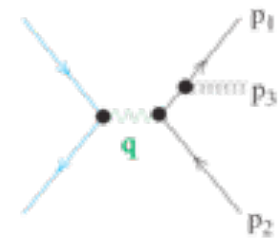
$\Rightarrow$ $0 \leq x_i \leq 1$

allowed values for x_i lie within a triangle



massless "Dalitz plot"

collinear and soft configurations



at the boundaries of phase space we encounter
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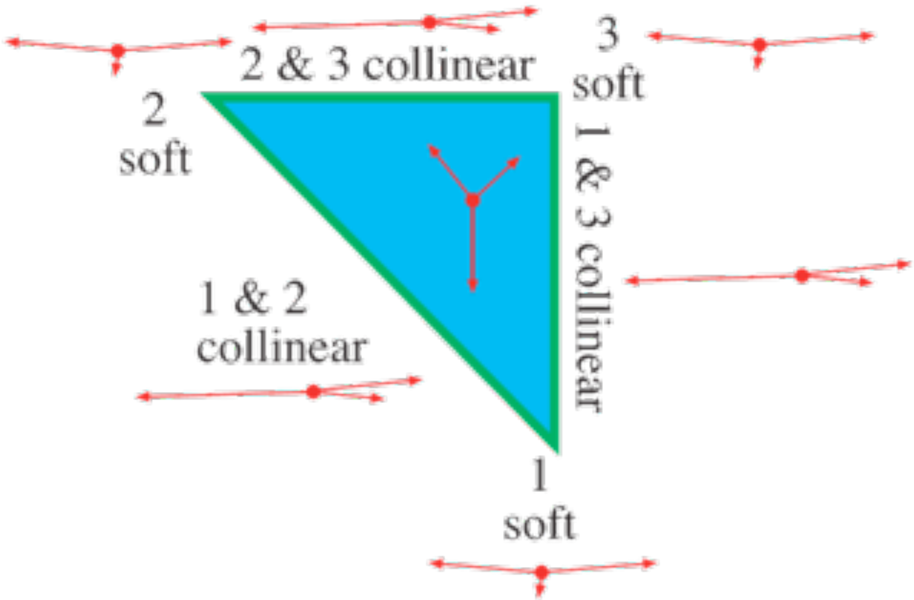

- "edges": **two partons collinear**

  e.g. $\theta_{13} \to 0 \Leftrightarrow x_2 \to 1$

- "corners": **one parton soft**

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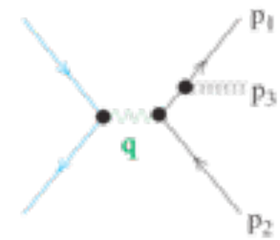

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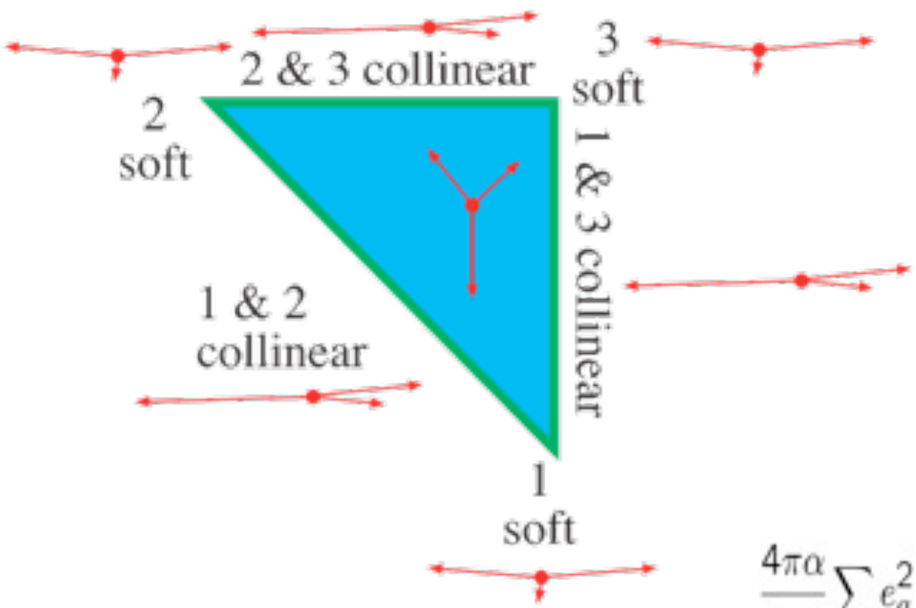

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in the **cross section**:

$$\frac{4\pi\alpha}{s}\sum e_q^2$$

$$\frac{1}{\sigma_0}\frac{d\sigma}{dx_1 dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

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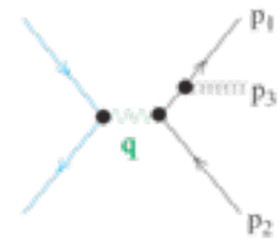

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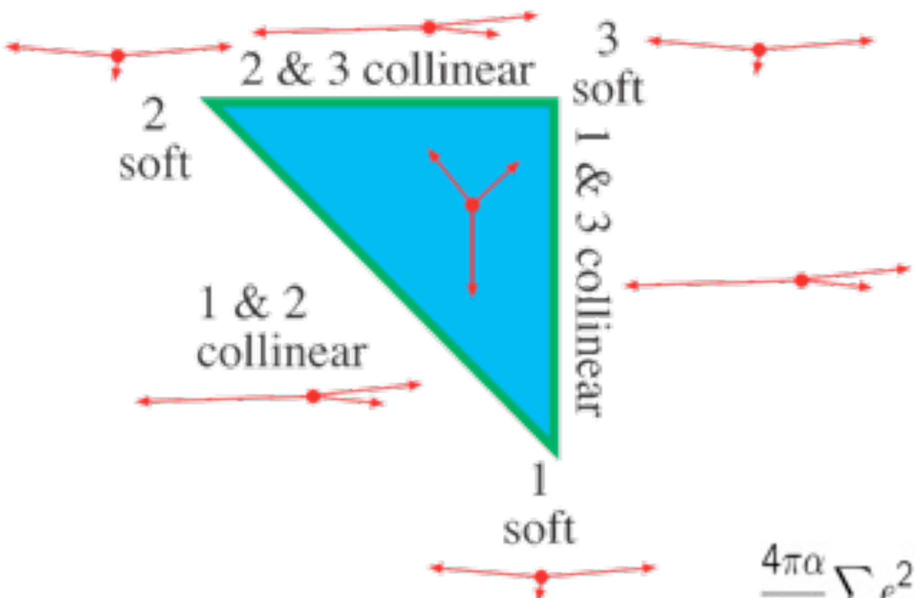

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collinear singularities:

$x_1 \to 1$: gluon ∥ antiquark

$x_2 \to 1$: gluon ∥ quark

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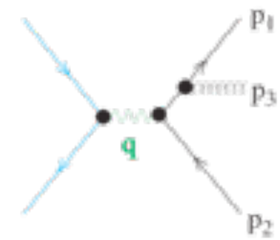

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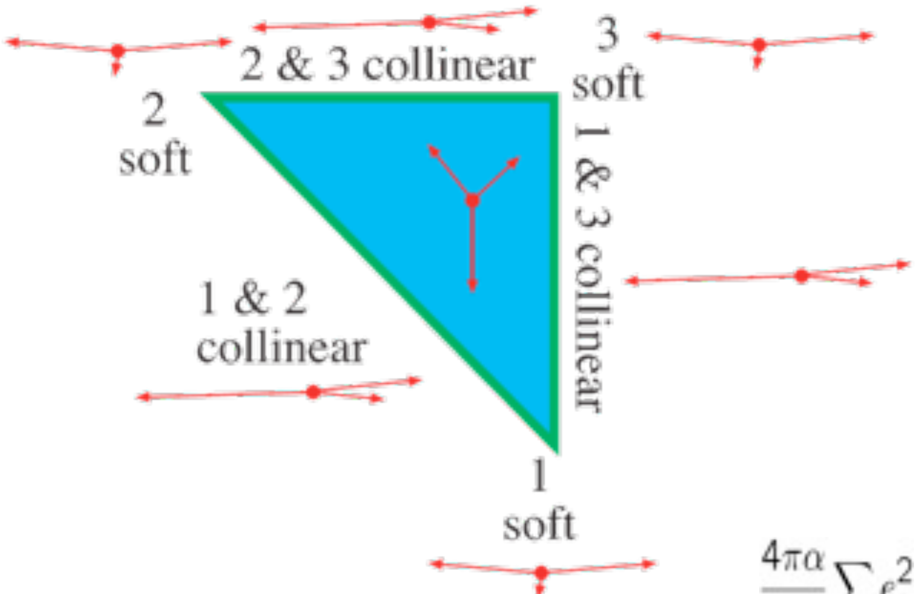

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soft gluon singularity:

$x_3 \to 0$: $p_3 \to 0$

$\leftrightarrow x_1 \to 1$ & $x_2 \to 1$

collinear singularities:

$x_1 \to 1$: gluon || antiquark

$x_2 \to 1$: gluon || quark

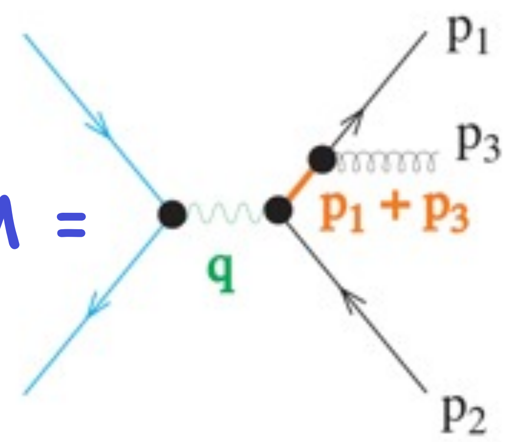
general nature of these singularities

soft/collinear limit:

internal propagator goes on-shell

here: $$\frac{1}{(p_1+p_3)^2} = \frac{1}{2E_1E_3(1-\cos\theta_{13})}$$

M =



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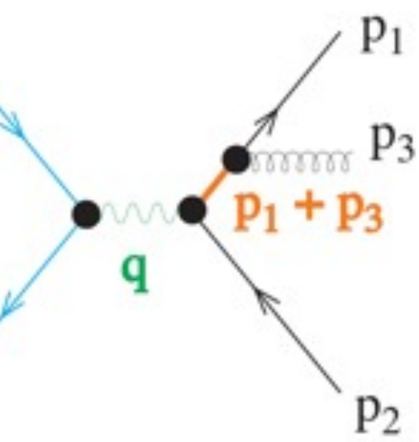

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explicit calculation yields:

$$d\sigma \propto \int E_3 dE_3 d\theta_{13}^2 \left[\frac{\theta_{13}}{E_3\theta_{13}^2}\right]^2 = \int \frac{dE_3}{E_3}\frac{d\theta_{13}^2}{\theta_{13}^2}$$ logarithmically divergent

phase space factor — from $|M|^2$

note: "soft quarks" (here $E_1 \to 0$) never lead to singularities (canceled by numerator)

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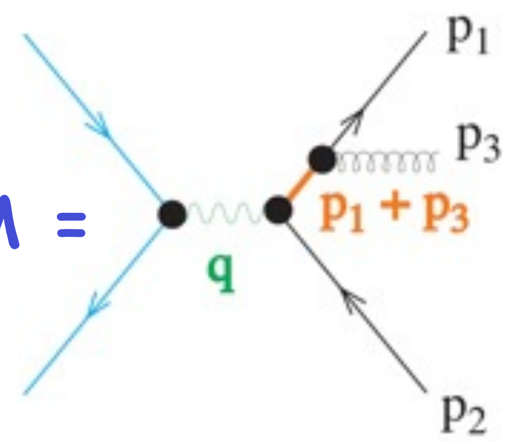
$M =$ 

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logarithmically divergent

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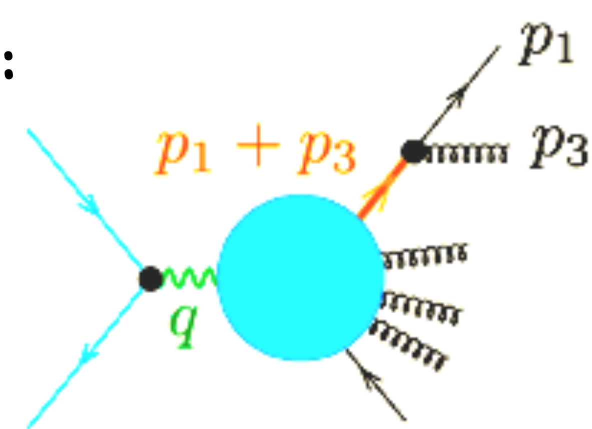
this structure is generic for QCD tree graphs:

$$\mathcal{M}_{n+1} \sim [\mathcal{M}_n]\Big|_{1,3\,\text{on-shell}} \frac{\text{spinors}}{(p1+p3)^2}$$



basis for parton-shower MC codes
like PYTHIA, HERWIG, SHERPA, ...

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NO! Perturbative QCD only tries to tell us that we are not doing the right thing! Our cross section is not defined properly, it is **not infrared safe**!

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the lesson is:

whenever the 2->(n+1) kinematics collapses to an effective 2->n parton kinematics due to

- the emission of a soft gluon
- a collinear splitting of a parton into two partons

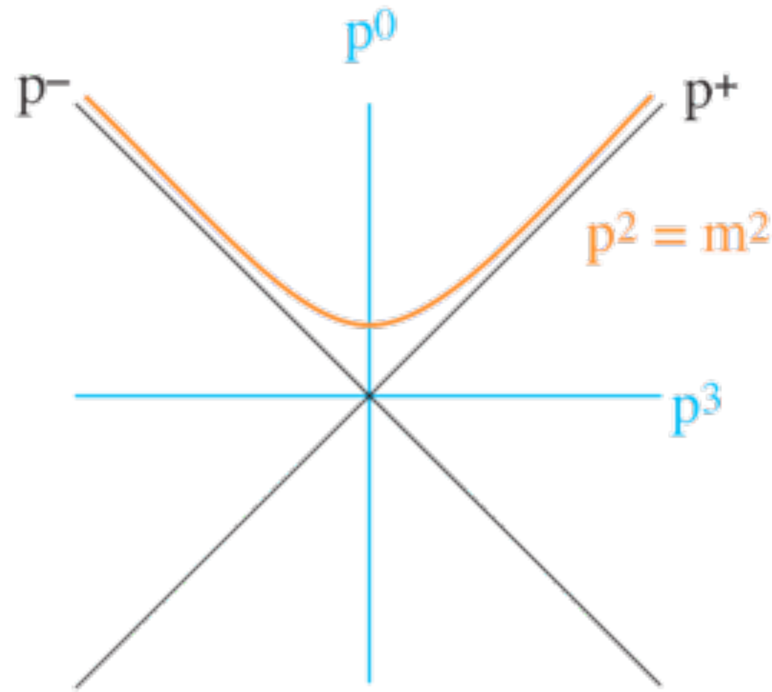
we have to be much more careful and work a bit harder!

this applies to all pQCD calculations

towards a space-time picture of the singularities

interlude: light-cone coordinates

$$p^{\pm} \equiv (p^0 \pm p^3)/\sqrt{2}$$
$$p^2 = 2p^+p^- - \vec{p}_T^{\,2}$$
$$p^- = (p_T^2 + m^2)/2p^+$$



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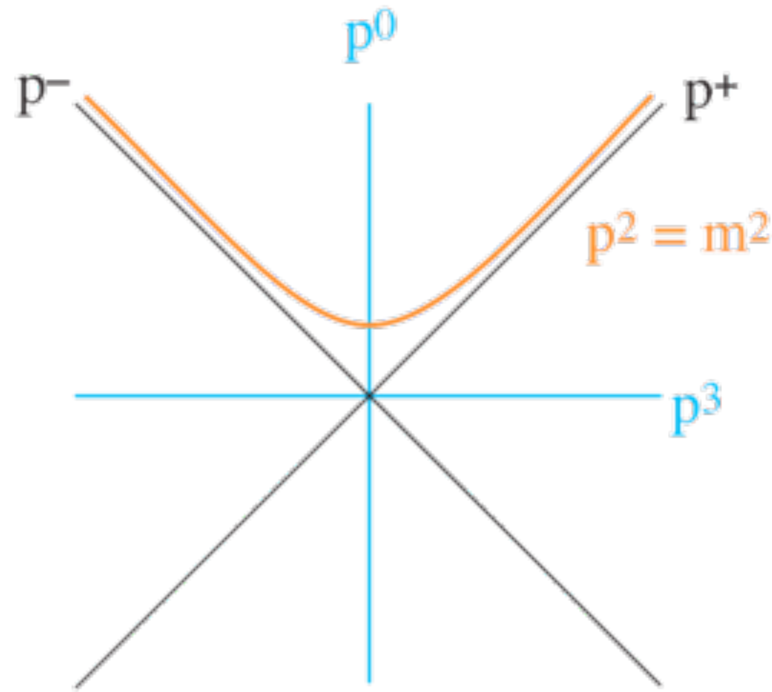

➡ particle with large momentum in $+p^3$ direction has large p^+ and small p^-

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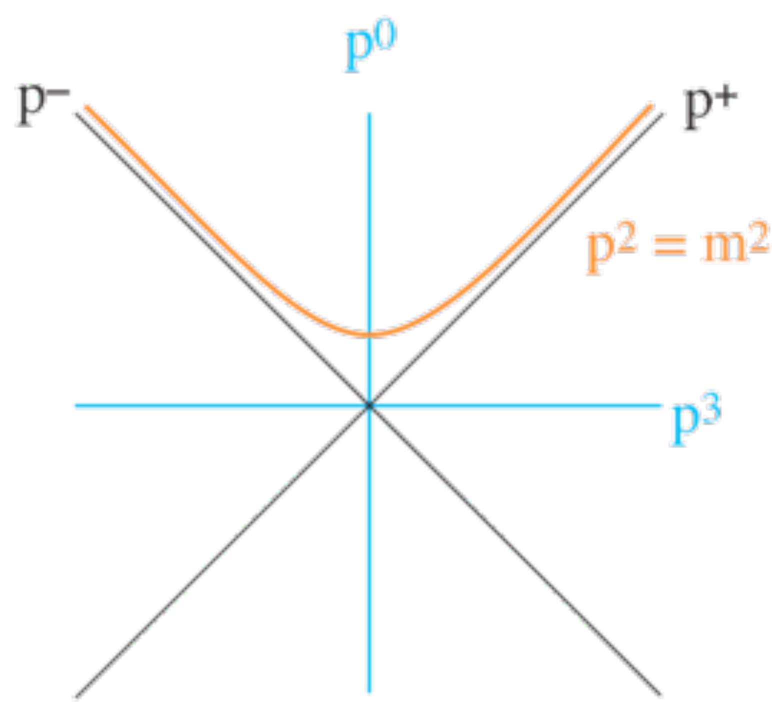

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momentum space ⟷ coordinate space (Fourier transform, $e^{ip\cdot x}$)

$$p \cdot x = p^+x^- + p^-x^+ - \vec{p}_T \cdot \vec{x}_T$$

--> x^- is conjugate to p^+ and x^+ is conjugate to p^-

space-time picture of the singularities

What does this imply for our propagator going on-shell?

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- define $k \equiv p_1 + p_3$
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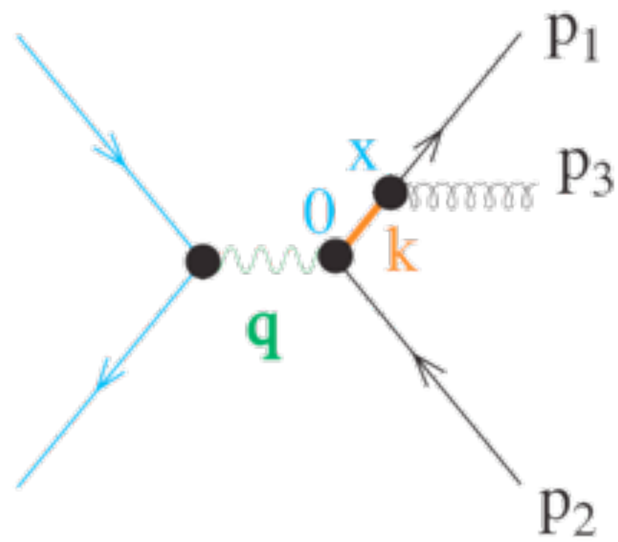

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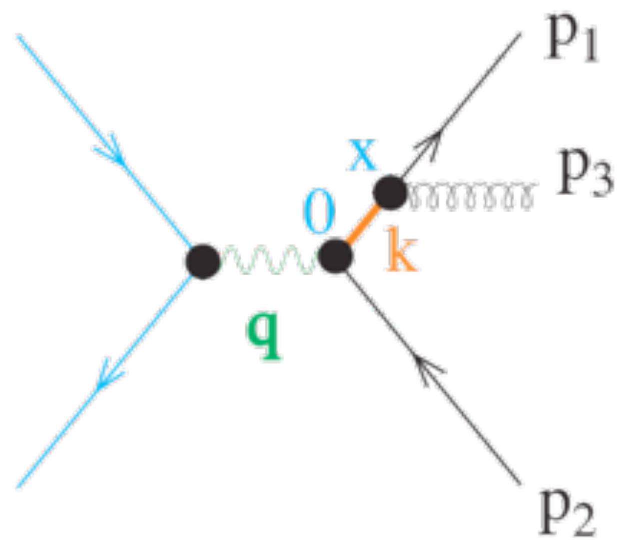

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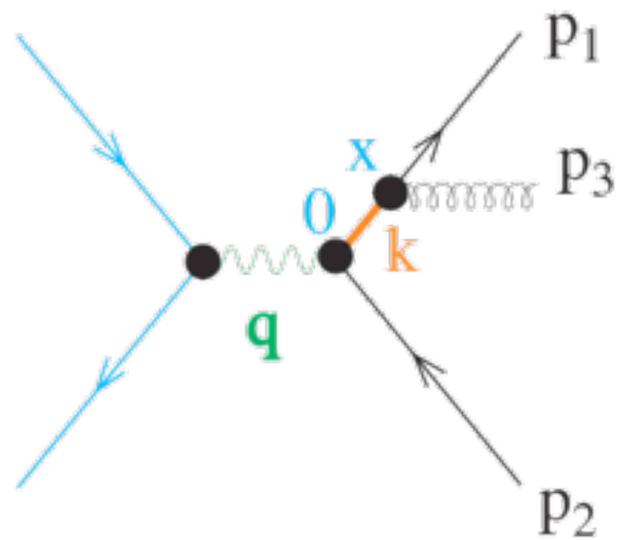

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$$k^+ \simeq \sqrt{s}/2 \quad \text{large}$$

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$\downarrow$ Fourier

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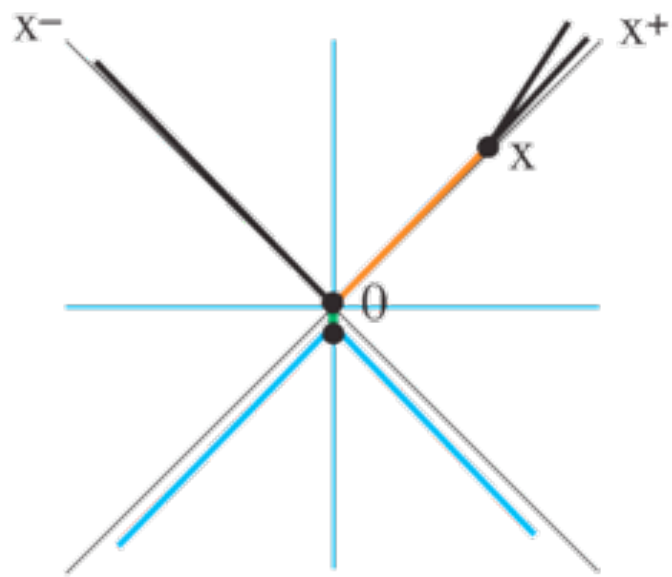

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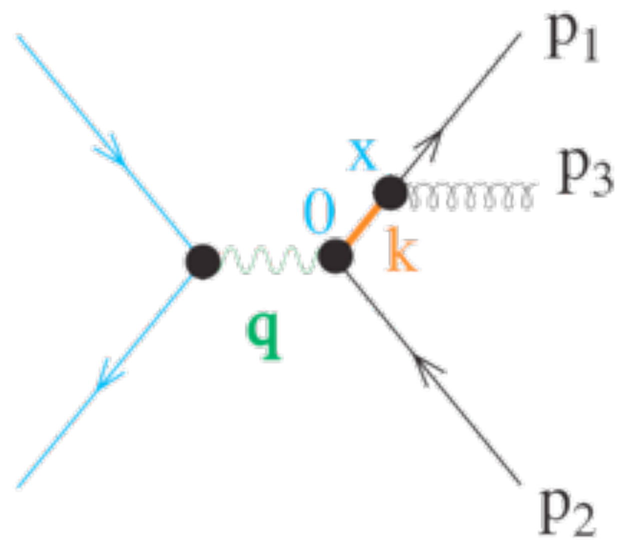

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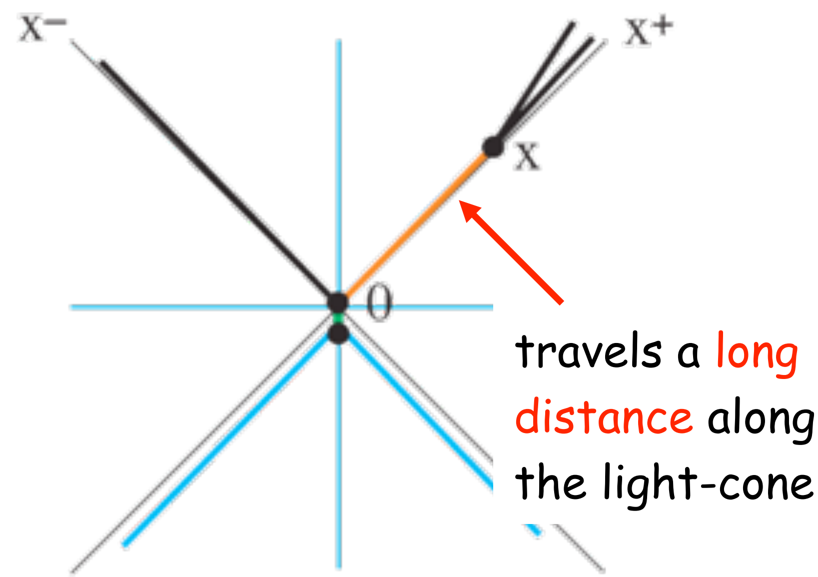

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to answer this, we have to formulate the

concept of infrared safety

infrared-safe observables

formal definition of infrared safety:

Kunszt, Soper

study inclusive observables which do not distinguish between (n+1) partons and n partons in the soft/collinear limit, i.e., are insensitive to what happens at long-distance

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$$\begin{aligned} \mathcal{I} &= \frac{1}{2!}\int d\Omega_2 \frac{d\sigma[2]}{d\Omega_2} \mathcal{S}_2(p_1,p_2) \\ &+ \frac{1}{3!}\int d\omega_2 dE_3 d\Omega_3 \frac{d\sigma[3]}{d\Omega_2 dE_3 d\Omega_3} \mathcal{S}_3(p_1,p_2,p_3) \\ &+ \ldots \end{aligned}$$

measurement fcts.
(define your observable)

infrared-safe observables

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measurement fcts.
(define your observable)

infrared safe **iff** [for $\lambda=0$ (soft) and $0 < \lambda < 1$ (collinear)]

$$\mathcal{S}_{n+1}(p_1,\ldots,(1-\lambda)p_n,\lambda p_n) = \mathcal{S}_n(p_1,\ldots,p_n)$$

physics behind formal IR safety requirement

cannot resolve soft and collinear partons experimentally

$\rightarrow$ intuitively reasonable that a theoretical calculation can be infrared safe as long as it is insensitive to long-distance physics (not a priori guaranteed though)

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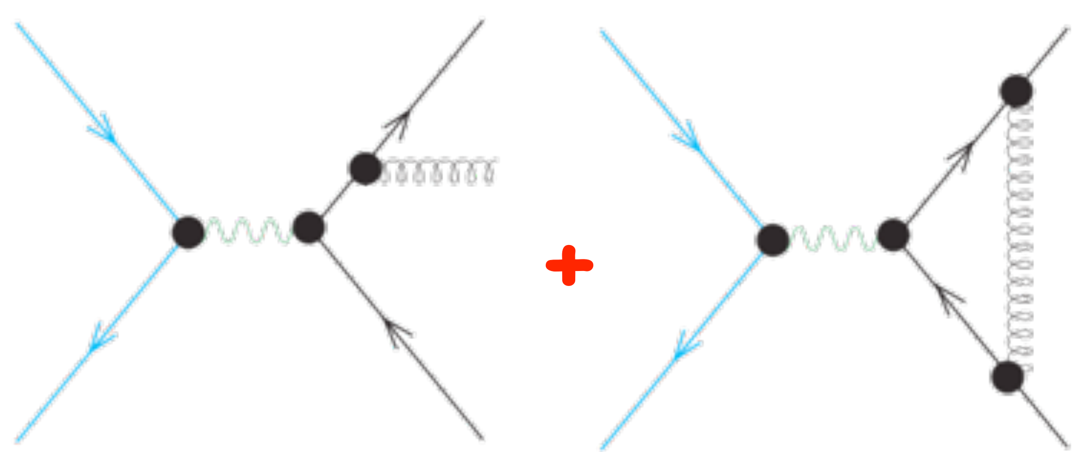
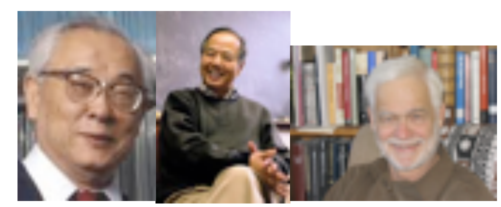
at a level of a pQCD calculation (e.g. e^+e^- at $O(\alpha_s)$, i.e., n=2)

$$S_{n+1}(p_1, \ldots, (1-\lambda)p_n, \lambda p_n) = S_n(p_1, \ldots, p_n)$$

→ singularities of real gluon emission and virtual corrections cancel in the sum





extension of famous theorems by Kinoshita-Lee-Nauenberg and Bloch-Nordsieck

example I: total cross section $e^+e^- \to$ hadrons

simplest case: $$S_n(p_1, \ldots, p_n) = 1$$

fully inclusive quantity $\longleftrightarrow$ we don't care what happens at long-distance

- the produced partons will all hadronize with probability one
- we do not observe a specific type of hadron
  (i.e. sum over a complete set of states)
- we sum over all degenerate kinematic regions

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  (i.e. sum over a complete set of states)
- we sum over all degenerate kinematic regions

infrared safe by definition

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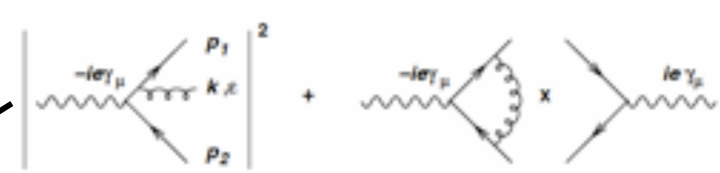
$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum e_q^2 (1 + \Delta_{QCD})$$



need to add up real and virtual corrections

example I: total cross section $e^+e^- \to$ hadrons

simplest case: $S_n(p_1, \ldots, p_n) = 1$

fully inclusive quantity ⟷ we don't care what happens at long-distance

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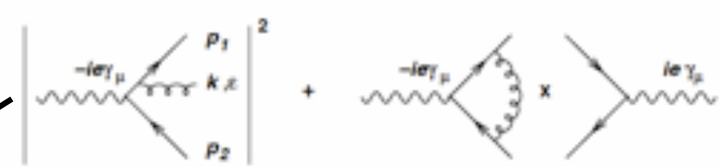

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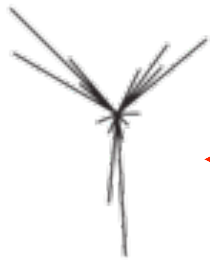
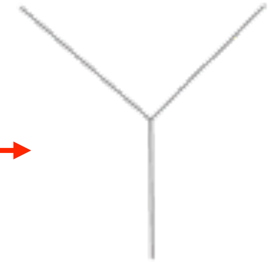
not IR safe:

- energy of hardest gluon in event
- multiplicity of gluons or 1-gluon cross section

example II: n-jet cross section

experiment | **QCD theory**





real physical event with 3 hadron-jets

theor. jet event with 3 parton-jets

jets are the central link between theory and experiment

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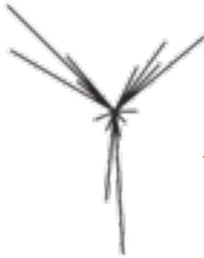
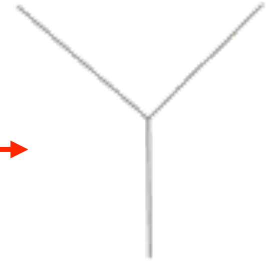


approx. equivalent ↔ infrared safety



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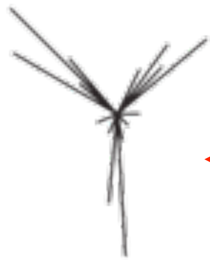

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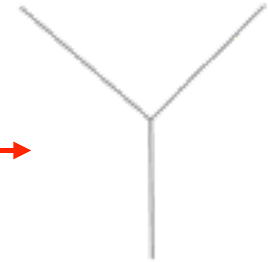

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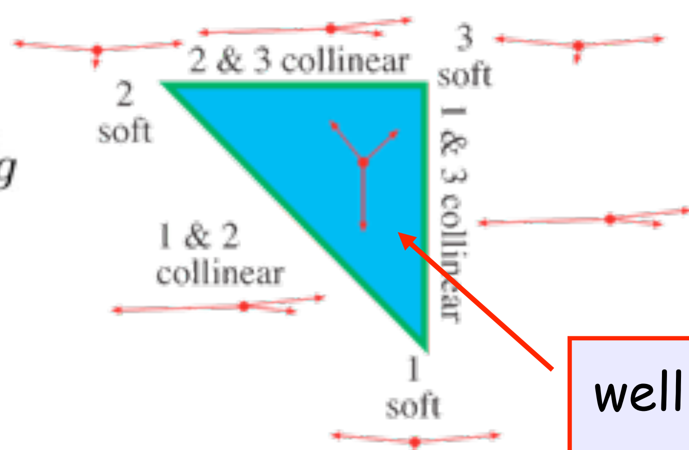

well inside: 3-jets
near edges: 2-jets

jet "measure"/"algorithm":
classify the final-state of hadrons (exp.) or partons (th.) according to the number of jets

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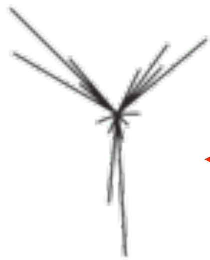

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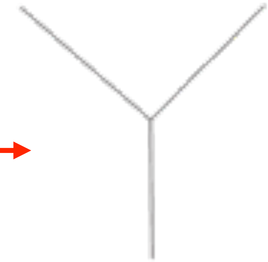

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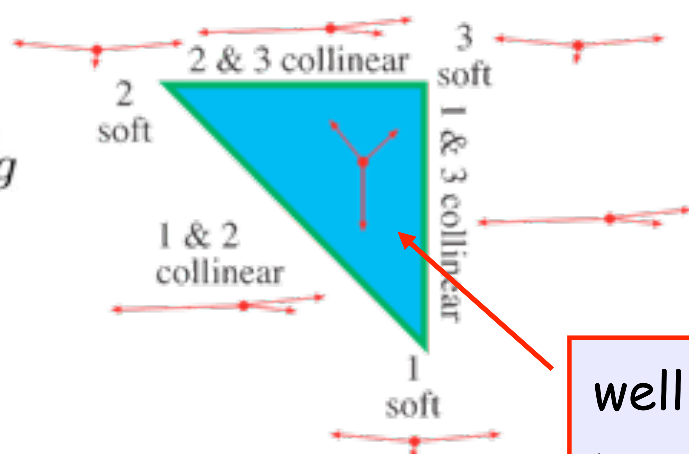

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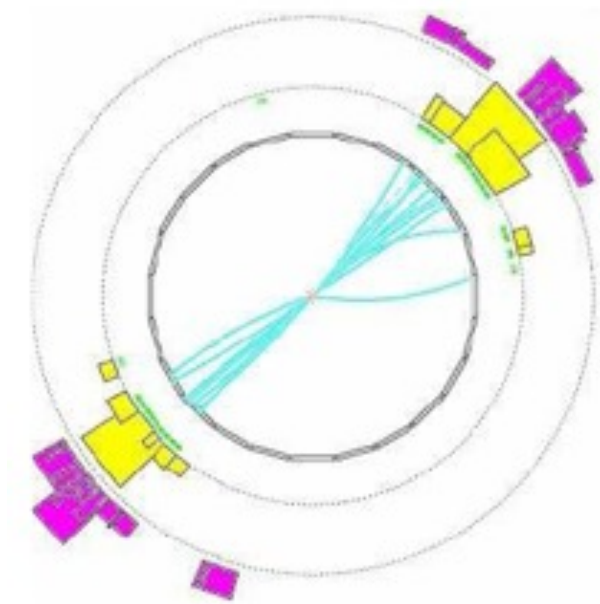
"2 or 3" depends on algorithm

seeing vs. defining jets



clearly (?) a 2-jet event

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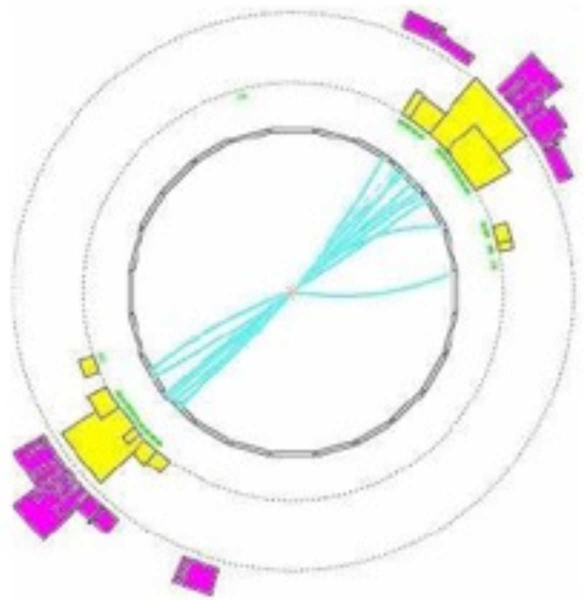

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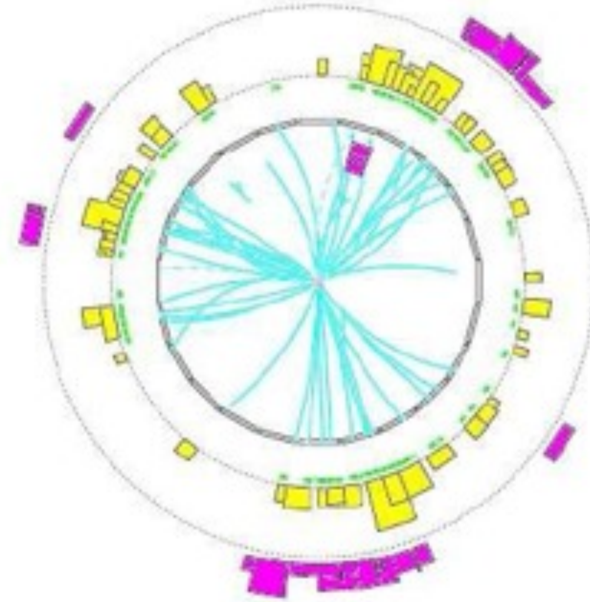
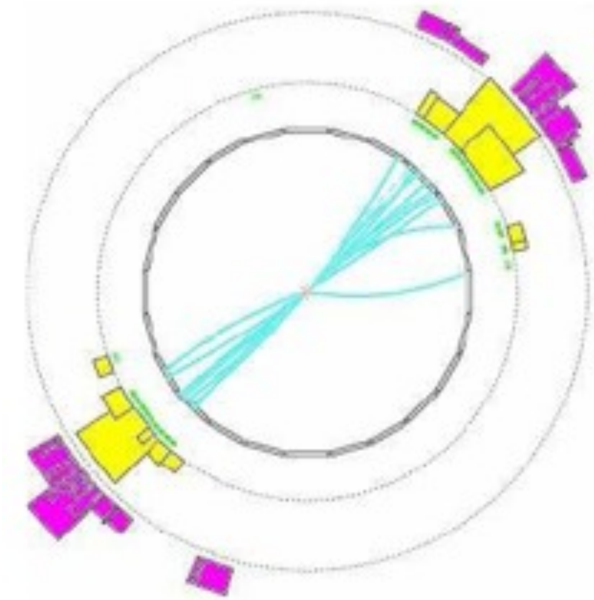


how many jets do you count?

seeing vs. defining jets



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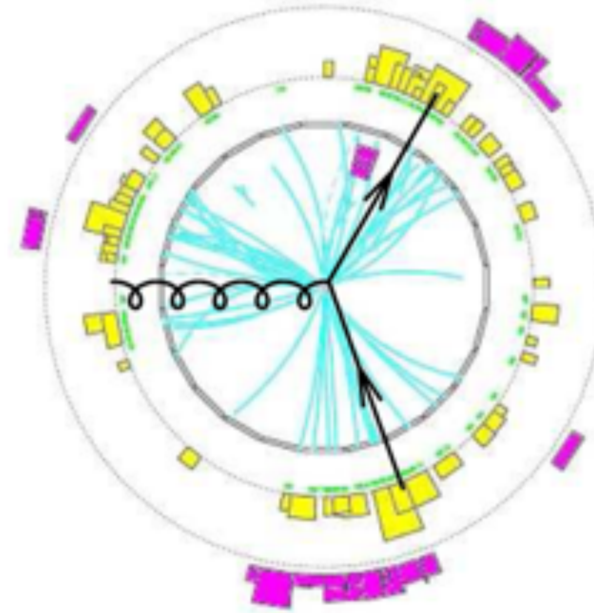

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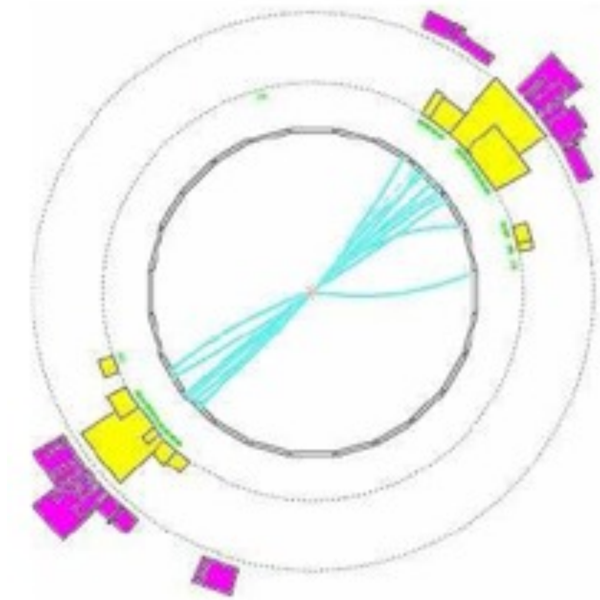

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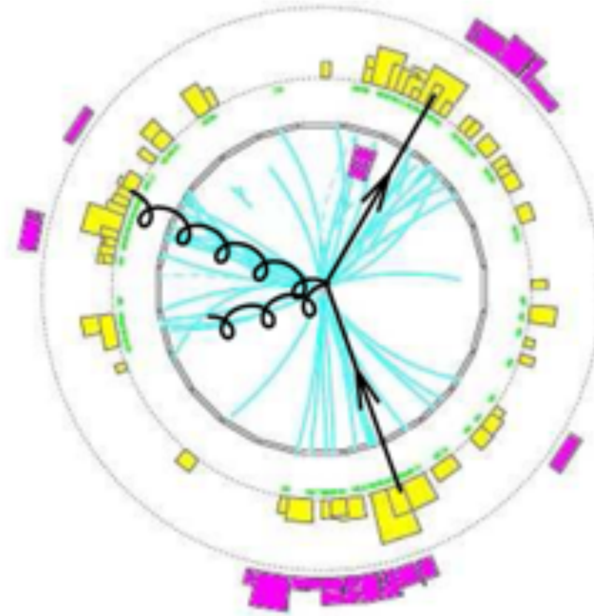

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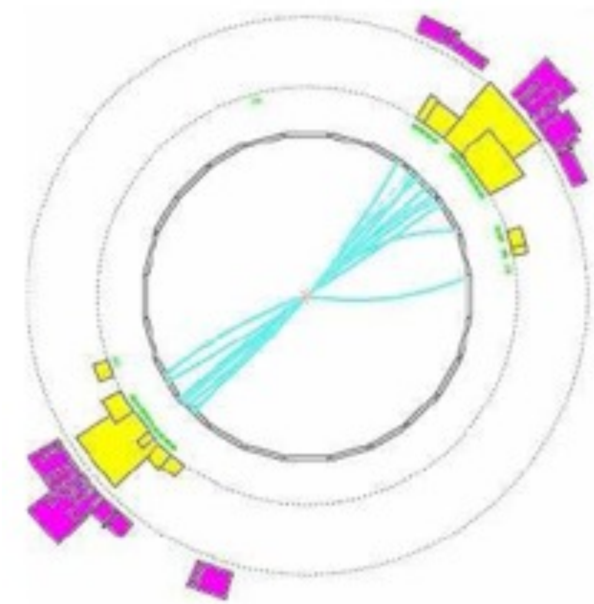

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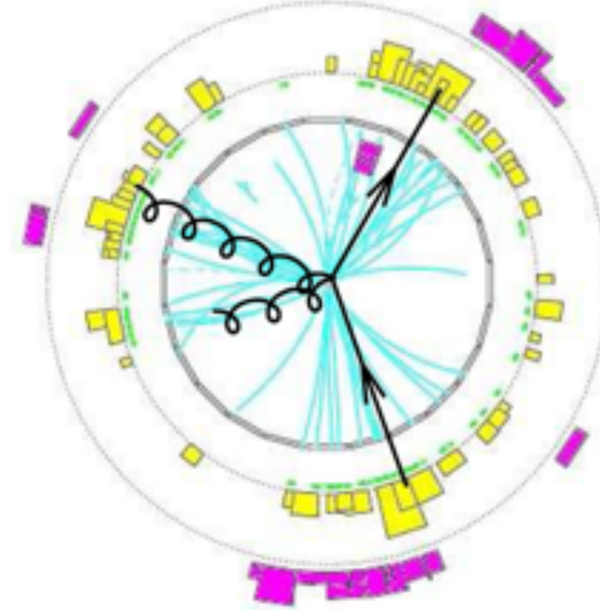

how many jets do you count?

the "best" jet definition does not exist – construction is unavoidably ambiguous

basically two issues:

- which particles/partons get put together in a jet → **jet algorithm**
- how to combine their momenta → **recombination scheme**

basic requirements for a jet definition

projection to jets should be resilient to QCD & detector effects

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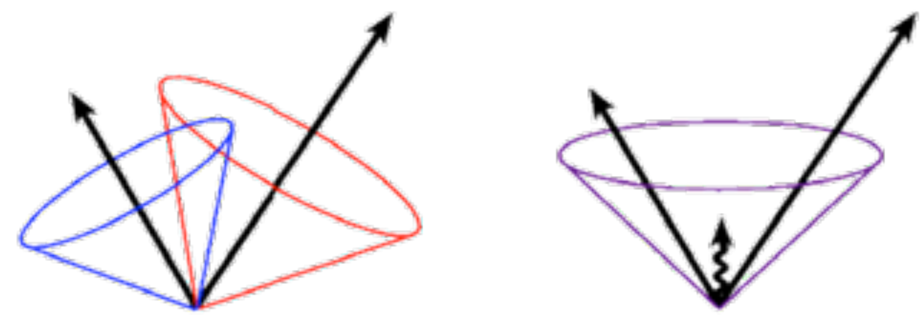

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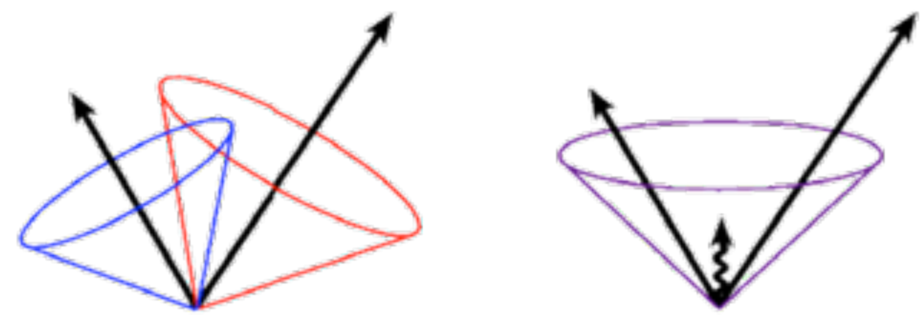

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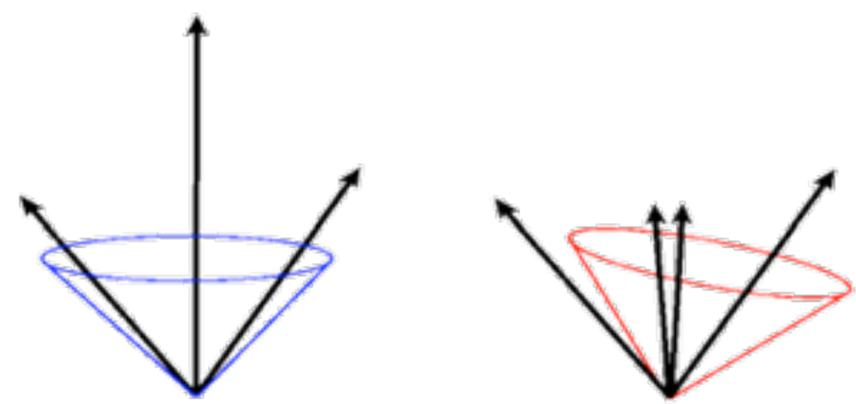

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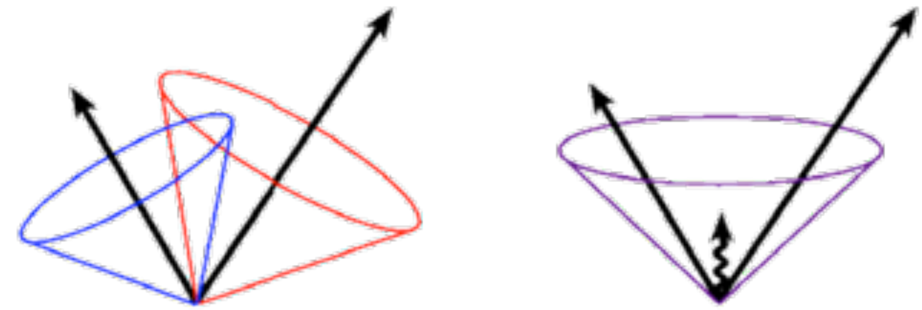

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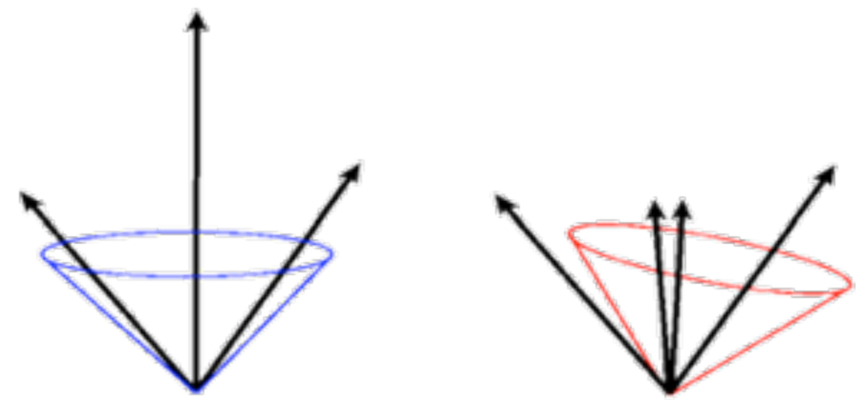

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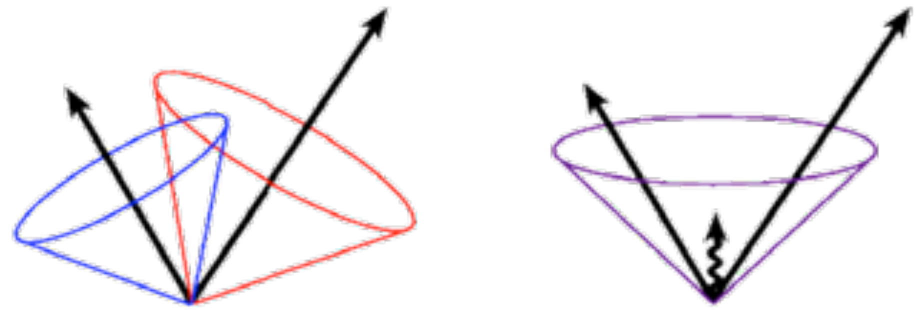

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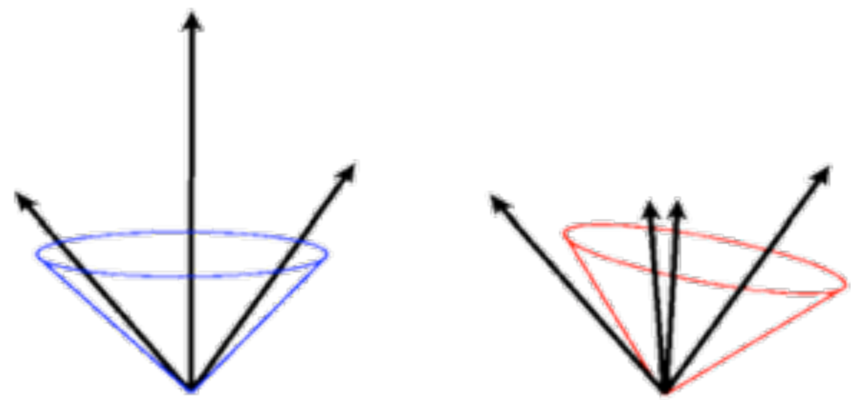

(anti-) k_T algorithms are the method of choice these days

Cacciari, Salam, Soyez (FastJet tool)

summary so far

pQCD cannot give all the answers
but it does cover a lot of ground
despite the “long-distance problem”

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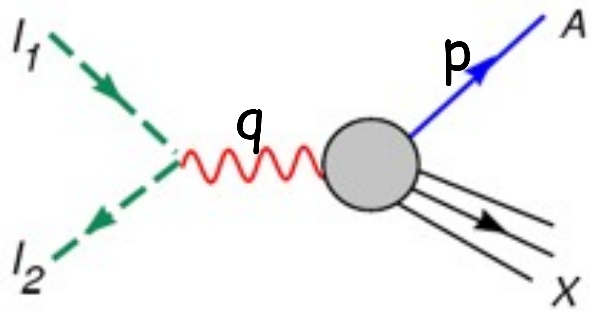
despite the “long-distance problem”

the **concept of factorization** will allow us to compute cross sections for a much wider class of processes than considered so far
(involving hadrons in the initial and/or final state)

HERA, TeVatron, JLab, RHIC, LHC, ..., EIC

hadrons: a new “long distance problem”

consider the one-particle inclusive cross section:



identified hadron
e.g. ($A = \pi$)

not measured

$$\frac{d\sigma(e^+e^- \to \pi + X)}{dE_\pi}$$

not infrared safe by itself!

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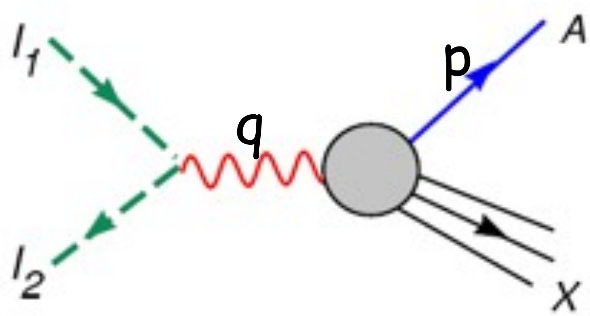

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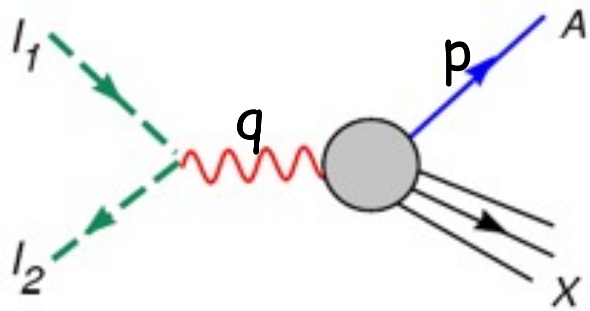

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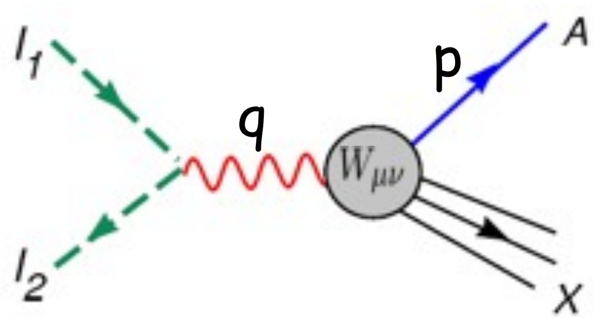
general feature of QCD processes with observed (=identified) **hadrons in the initial and/or final state**

factorization

strategy: try to factorize the physical observable into a calculable infrared safe and a non-calculable but universal piece

how does it work?



$$d\sigma = \frac{4\alpha^2}{sQ^2}\frac{d^3\vec{p}}{2|\vec{p}|}L^{\mu\nu}W_{\mu\nu}$$

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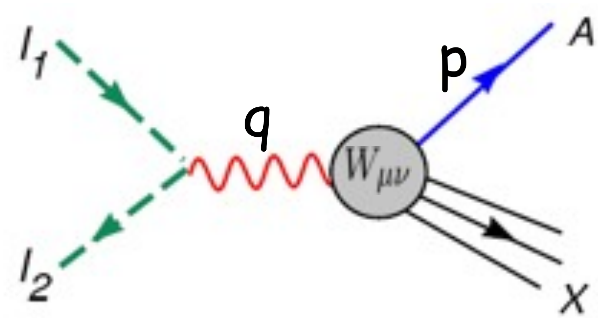

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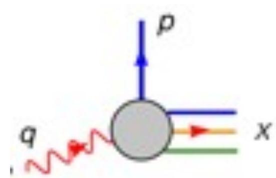
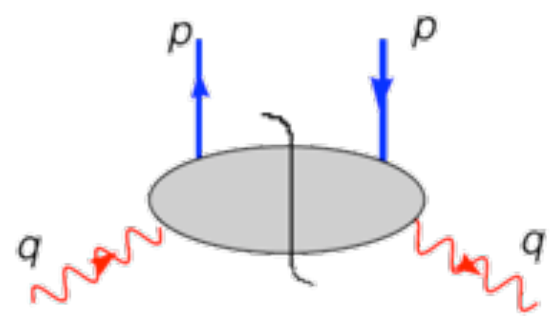


square of the hadronic scattering amplitude
summed over all final-states X except A(p)



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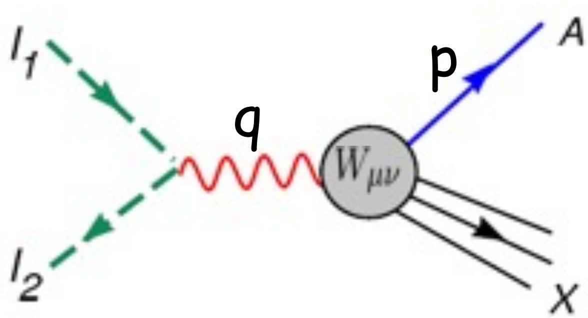

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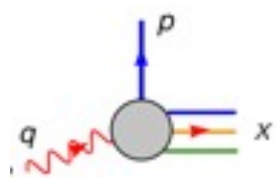

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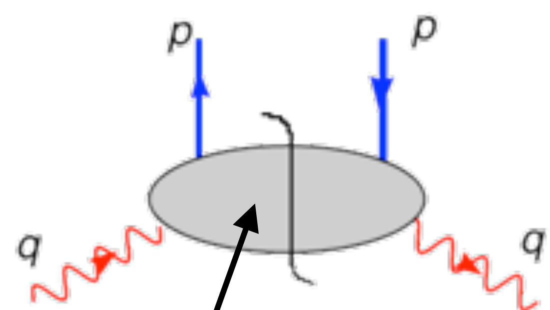

need to factorize long-distance physics

concept of factorization - pictorial sketch

factorization = isolating and absorbing infrared singularities accompanying observed hadrons

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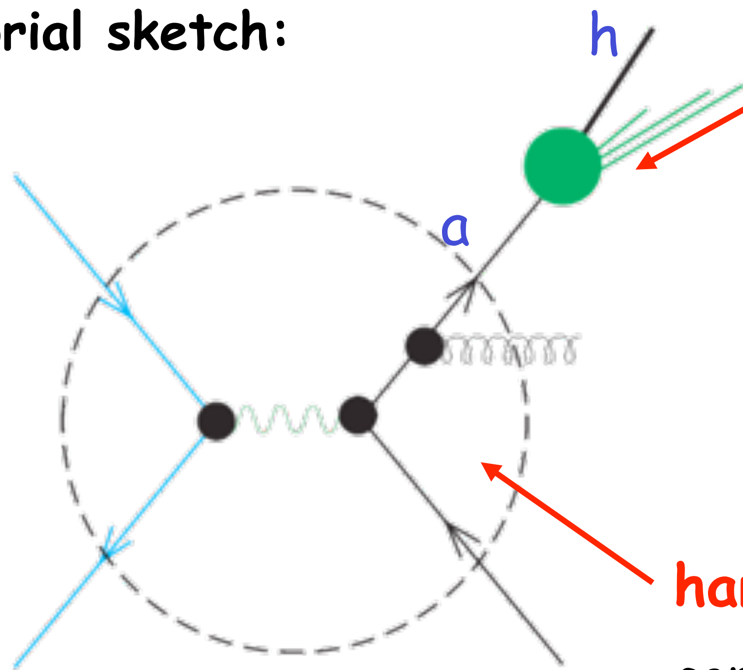
pictorial sketch:



fragmentation functions D_a^h

contains all **long-distance** interactions hence not calculable but universal

physical interpretation:

probability to find a hadron carrying a certain momentum of parent parton

hard scattering $\hat{F}_a$

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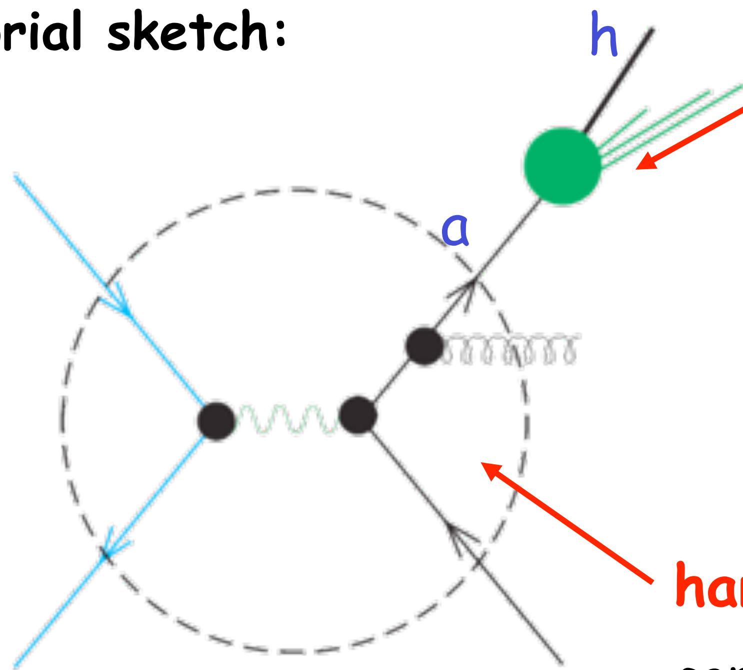

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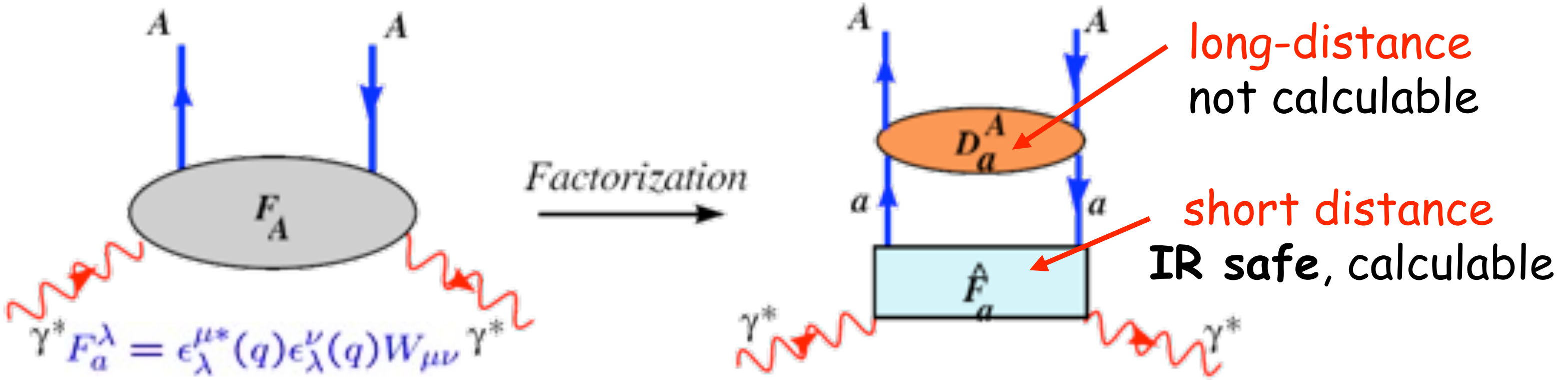
aside: fragmentation fcts. play an important role in learning about nucleon (spin) structure from semi-inclusive DIS data by COMPASS & HERMES or from hadron production at RHIC

factorization - detailed picture

more explicitly



$F_a^\lambda = \epsilon_\lambda^{\mu *}(q)\epsilon_\lambda^{\nu}(q)W_{\mu\nu}$

λ=L,T (pol. of γ^*)

$$\frac{d\sigma}{dz\,d\cos\theta} = \frac{\pi\alpha^2}{2s}\left[F_A^T(z,Q)(1+\cos^2\theta) + F_A^L(z,Q)\sin^2\theta\right]$$

where

$$F_A^{T,L}(z,Q) = \sum_a \hat{F}_a^{T,L}(z,\frac{Q}{\mu_f}) \otimes D_a^h(z,\mu_f)$$

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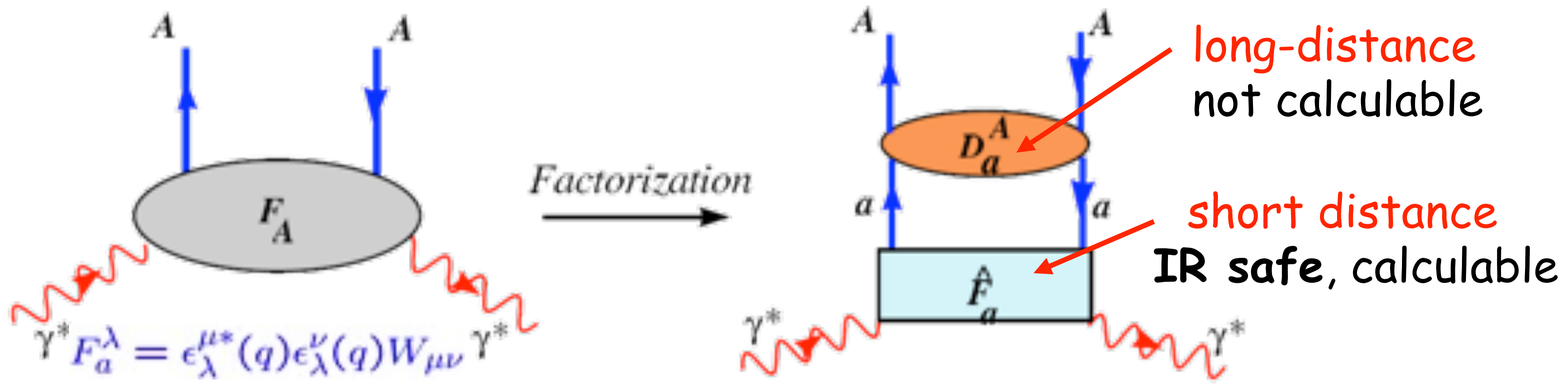

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factorization scale (arbitrary!)

characterizes the boundary between short and long-distance physics

physics indep. of $\mu_f \rightarrow$ renormalization group

factorization - detailed picture

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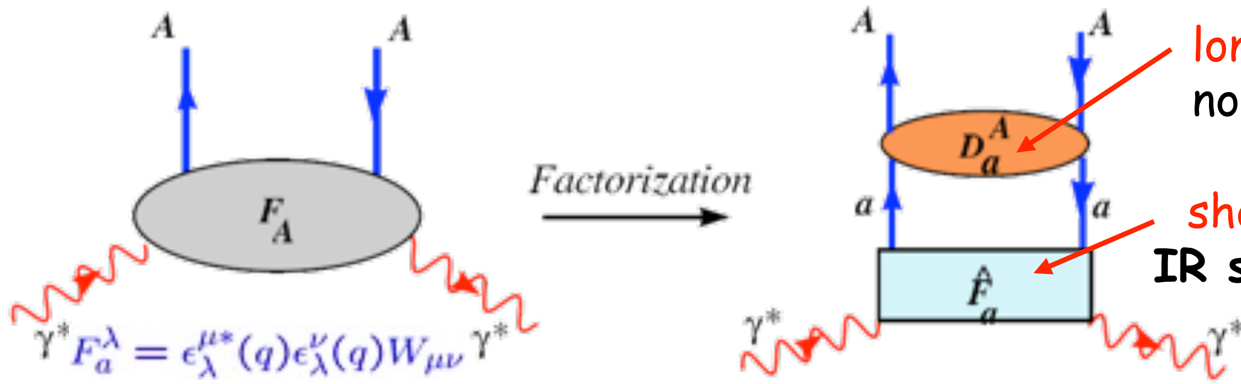

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"convolution"

$$f(x)\otimes g(x) \equiv \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) g(y)$$

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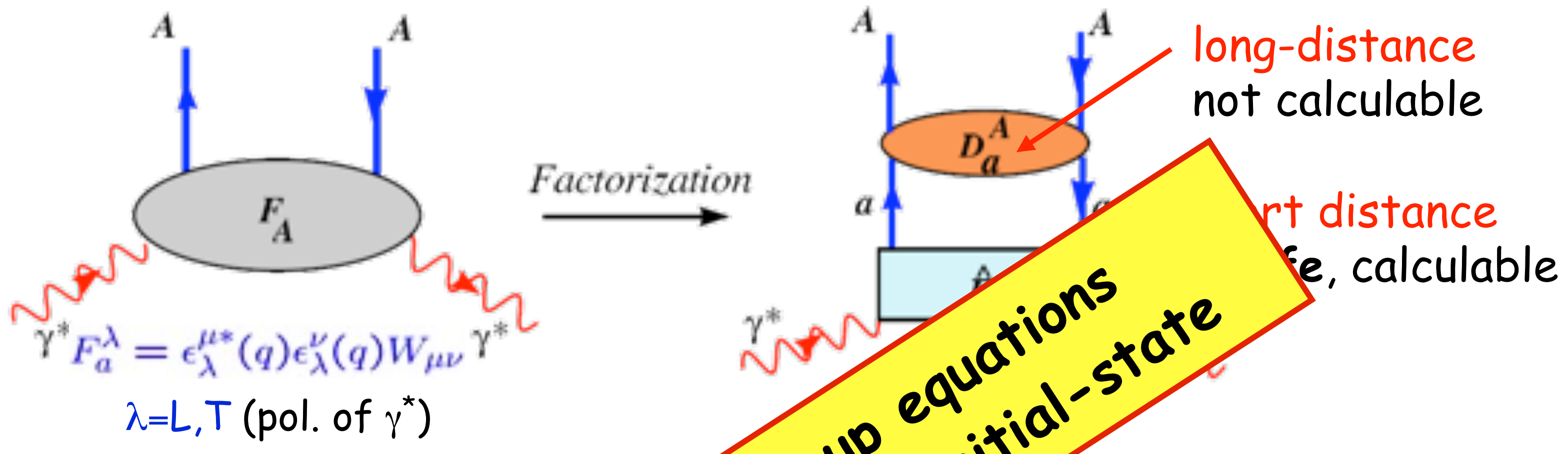

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$$F_A^{T,L}(\ldots \sum_a F_a^{T,L}(z,\frac{Q}{\mu_f}) \otimes D_a^h(z,\mu_f)$$

"conv

$$f(x)\otimes g(x) \equiv \int_x \frac{dy}{y} f\left(\frac{x}{y}\right) g(y)$$

before studying renormalization group equations let's first introduce hadrons also in the initial-state

factorization scale (arbitrary!)

characterizes the boundary between short and long-distance physics

physics indep. of μ_f → renormalization group

take home message for part II
the QCD toolbox

- QCD is a non-Abelian gauge theory: gluons are self-interacting $\rightarrow$ asymptotic freedom (large Q), confinement (small Q)
- QCD calculations are singular when any two partons become collinear or a gluon becomes soft; basis for parton shower MCs
- choose infrared/collinear safe observables for comparison between experiment and perturbative QCD
- jets (= cluster of partons): best link between theory and exp.; needs a proper IR safe jet definition in theory and experiment
- factorization allows to deal with hadronic processes introduces arbitrary scale -> leads to RGEs