Lattice QCD at non-zero temperature and density

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Introduction: QGP in theory and experiment

A new state of matter – Quark-Gluon Plasma (QGP) – is expected in QCD due to the asymptotic freedom, and has been observed in heavy-ion collision experiments.

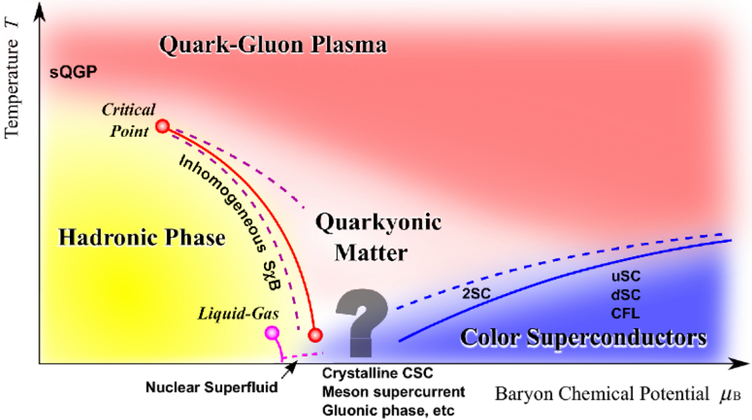
Experiment (RHIC, LHC):

- Particle spectra.
- Heavy-quark bound states.
- Thermal photons and dileptons.

Theory (Lattice QCD):

- Properties of the transition region.
- Fluctuations and correlations of conserved charges.
- The QCD equation of state.
- Spectral functions, transport properties.

Introduction: conjectured phase diagram

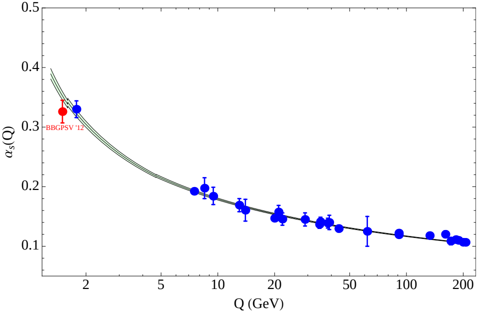


Introduction: Quantum Chromodynamics

The Lagrangian:

$$\mathcal{L} = -\frac{1}{4}F^c_{\mu\nu}F^{\mu\nu,c} + \sum_{\alpha=1}^{n_f} \bar{\psi}_\alpha(i\gamma^\mu D_\mu - m_\alpha)\psi_\alpha$$

- α_s is small at large energy scale (asymptotic freedom), and large at low energies (where we live), – perturbation theory breaks down.
- Recent lattice QCD determination of α_s, Bazavov et al. PRD86 (2012) 114031, arXiv:1205.6155 [hep-ph].



Introduction: Lattice QCD

- Quantum field theory (QCD) in path-integral formulation in Euclidean (imaginary time) formalism:

$$
\begin{aligned}
\langle \mathcal{O} \rangle &= \frac{1}{Z} \int D\bar{\psi} D\psi DU \; \mathcal{O} \exp(-S), \\
Z &= \int D\bar{\psi} D\psi DU \exp(-S), \qquad S = \int d^4x \mathcal{L}_E,
\end{aligned}
$$

- Discrete space-time: 4D hypercubic lattice $N_s^3 \times N_\tau$, lattice spacing a serves as a cutoff (momenta restricted to π/a).
- Temperature is set by compactified temporal dimension: $T = 1/(N_\tau a)$, lattice spacing a is varied at fixed N_τ, or N_τ at fixed a (fixed scale approach).
- Evaluate QCD path integrals stochastically, using Monte Carlo techniques. (S needs to be real!)
- Physics is recovered in the continuum limit (cutoff effects are the major source of systematic uncertainties).

Introduction: Lattice QCD

- Lattice action

$$S = S_{gauge} + S_{fermion}, \qquad S_{fermion} = \sum_{x,y} \bar{\psi}_x M_{x,y} \psi_y$$

  ($M_{x,y}$ is the fermion matrix) preserves the gauge symmetry, but there is the infamous fermion doubling problem – 16 species of fermions in 4D.

- Quarks live on sites and gluons on links as $SU(3)$ matrices

$$U_{x,\mu} = \mathcal{P} \exp \left\{ ig \int_x^{x+a\hat{\mu}} dy_\nu \, A_\nu(y) \right\}.$$

- Fermions are challenging due to the fermion doubling problem and also due to non-locality of the action when the Grasmann variables are integrated out:

$$Z = \int DU \det M[U] \exp(-S_{gauge}).$$

  (If $\det M[U]$ is neglected, this is called quenched approximation.)

Introduction: Lattice QCD

- Various fermion discretization schemes, at fixed lattice spacing:
  - Staggered – preserve a part of the chiral symmetry, computationally cheap, require taking 4-th root of the Dirac operator.
  - Wilson – no chiral symmetry.
  - Domain-wall – amount of symmetry breaking is controlled by the fifth dimension L_s, exact in $L_s \to \infty$ limit.
  - Overlap – exact chiral symmetry.

Introduction: what if S is not real?

- "Sign" problem – Monte Carlo sampling breaks down, because the integrand loses probabilistic meaning.
- This happens at non-zero chemical potential!
- Indirect way to explore the phase diagram at small μ is to Taylor expand in μ/T. Computationally feasable for first few terms.
- Attempts to get around the sign problem in various models:
  - alter the action – find a formulation where there is no sign problem (Grabowska, Kaplan and Nicholson, PRD87 (2013) 014504, arXiv:1208.5760 [hep-lat]; Chandrasekharan and Li, PRD85 (2012) 091502, arXiv:1202.6572 [hep-lat]),
  - alter the integral – integration on orbits (Bloch, PRD86 (2012) 074505, arXiv:1205.5500 [hep-lat]) or along a certain trajectory in extended phase space (Cristoforetti, Di Renzo, Scorzato, PRD86 (2012) 074506, arXiv:1205.3996 [hep-lat]),
  - alter the sampling procedure – complex Langevin dynamics (Aarts et al., JHEP 1303 (2013) 073, arXiv:1212.5231 [hep-lat]).
- No solution for QCD so far...

Finite-temperature transition in QCD: restoration of the chiral symmetry and deconfinement

Chiral condensate and susceptibility

Chiral condensate:

$$\langle\bar{\psi}\psi\rangle_{q,x} = \frac{1}{4}\frac{1}{N_\sigma^3 N_\tau}\mathrm{Tr}\langle M_q^{-1}\rangle, \quad q = l, s, \quad x = 0, \tau.$$

The susceptibility:

$$\chi_{m,q}(T) \quad = \quad \frac{\partial\langle\bar{\psi}\psi\rangle_l}{\partial m_q} = 2\chi_{q,disc} + \chi_{q,con} \ ,$$

$$\chi_{q,disc} \quad = \quad \frac{1}{16 N_\sigma^3 N_\tau}\left\{\langle\left(\mathrm{Tr} M_q^{-1}\right)^2\rangle - \langle \mathrm{Tr} M_q^{-1}\rangle^2\right\} \ ,$$

and

$$\chi_{q,con} \quad = \quad \frac{1}{4}\mathrm{Tr}\sum_x \langle \, M_q^{-1}(x,0) M_q^{-1}(0,x) \, \rangle \ , \quad q = l, s.$$

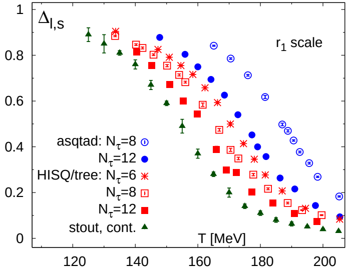
The renormalized condensate:

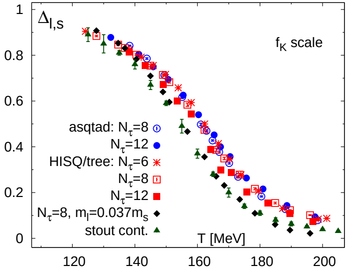
$$\Delta_{l,s}(T) = \frac{\langle\bar{\psi}\psi\rangle_{l,\tau} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,\tau}}{\langle\bar{\psi}\psi\rangle_{l,0} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,0}}$$

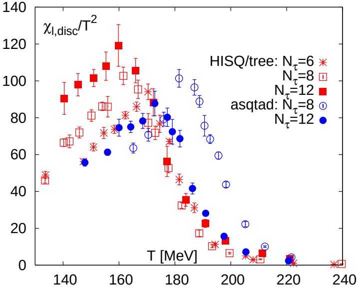
or

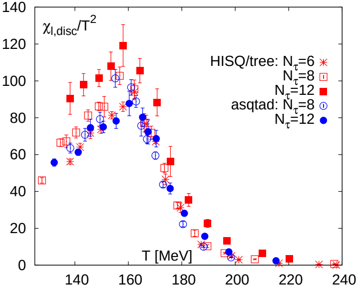
$$\Delta_l^R = d + m_s r_0^4(\langle\bar{\psi}\psi\rangle_{l,\tau} - \langle\bar{\psi}\psi\rangle_{l,0}).$$

Chiral condensate and susceptibility





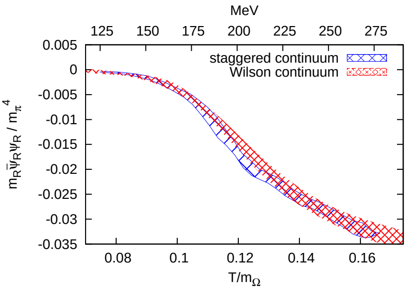


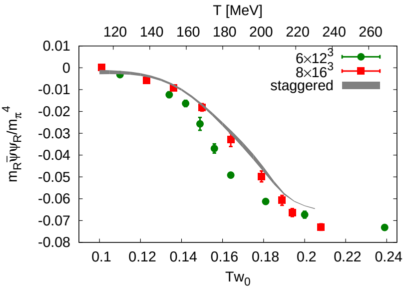


Pseudo-critical temperature, T_c

- At the physical values of light quark masses there is no genuine phase transition in QCD, but a crossover.
- Define a pseudo-transition temperature associated with restoration of chiral symmetry as a peak position in the disconnected chiral susceptibility. (Which diverges in the chiral limit.)
- Agreement on T_c between the groups using staggered fermions, in the continuum limit at the physical light quark masses:
  - BW, stout action, $m_\pi = 140$ MeV, $T_c = 147(4) - 155(4)$ MeV, JHEP09 (2010) 073, arXiv:1005.3508 [hep-lat]
  - HotQCD, HISQ/tree action, $m_\pi = 160$ MeV, extrapolated to $m_\pi = 140$ MeV, $T_c = 154(9)$ MeV, PRD85 (2012) 054503, arXiv:1111.1710 [hep-lat]
- Crosschecks between staggered and other fermion discretization schemes:
  - BW, overlap fermions, $m_\pi = 350$ MeV, arXiv:1204.4089 [hep-lat]
  - BW, Wilson fermions, $m_\pi = 540$ MeV, arXiv:1205.0440 [hep-lat]
  - HotQCD, domain wall fermions, $m_\pi = 200$ MeV, arXiv:1205.3535 [hep-lat]

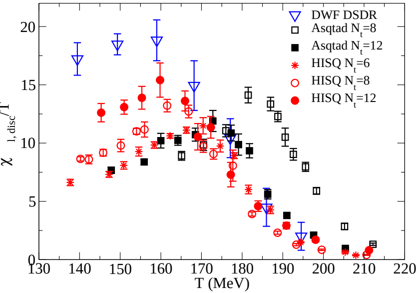
Chiral symmetry restoration





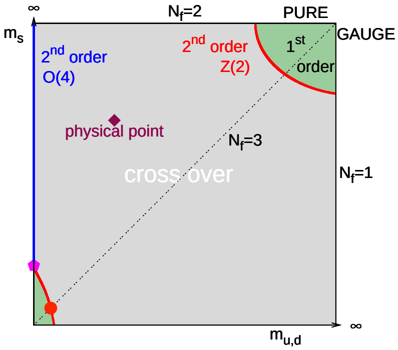
- Left: Renormalized chiral condensate, staggered vs. Wilson.
- Right: Renormalized chiral condensate, staggered vs. overlap.

Chiral symmetry restoration



- The disconnected chiral susceptibility, staggered vs. domain wall.
- At fixed lattice spacing, but peak location agrees, difference in height – presumably finite-volume effect.

Chiral symmetry restoration



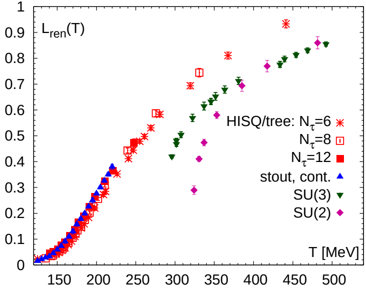
- In the chiral limit the critical behavior is governed by the $O(4)$ universality class.
- At the physical light quark mass scaling behavior with non-universal corrections still applies.
- Search for the first-order region along $m_l = m_s$ line, staggered fermions, Ding et al., arXiv:1111.0185 [hep-lat].
- Current bound on the first-order region $m_\pi = 75$ MeV.

Deconfinement

The Polyakov loop:

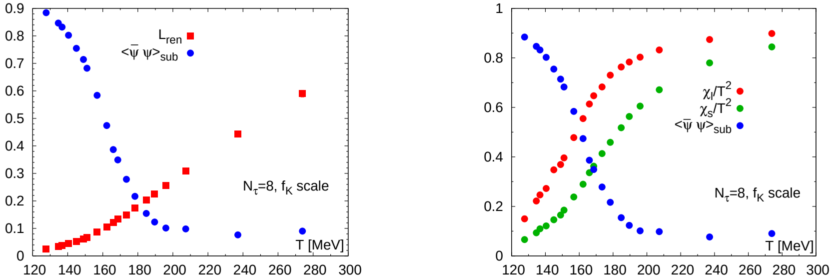
$$L_{ren}(T) = z(\beta)^{N_\tau} L_{bare}(\beta), \qquad L_{bare}(\beta) = \left\langle \frac{1}{3}\mathrm{Tr} \prod_{x_0=0}^{N_\tau - 1} U_0(x_0, \vec{x}) \right\rangle$$

- Related to the free energy of a static quark anti-quark pair $L_{ren}(T) = \exp(-F_\infty(T)/(2T))$



- The increase of $L_{ren}(T)$ (and decrease of $F_\infty(T)$) is related to the onset of screening at higher temperatures.
- The order parameter in pure gauge theory but not in full QCD, the behavior in $SU(2)$, $SU(3)$ and 2+1 flavor QCD is quite different!

Deconfinement



The renormalized chiral condensate plotted together with the renormalized Polyakov loop (left) and the light and strange quark number susceptibility (defined on next slides) (right) for $N_\tau = 8$ lattice, the HISQ/tree action.

Deconfinement happens gradually, no unique transition temperature can be associated with it in full QCD.

Fluctuations

Deconfinement: fluctuations

- Fluctuations and correlations of conserved charges:

$$\frac{\chi_i(T)}{T^2} = \frac{1}{T^3 V} \frac{\partial^2 \ln Z(T, \mu_i)}{\partial(\mu_i/T)^2}\bigg|_{\mu_i=0},$$
$$\frac{\chi_{11}^{ij}(T)}{T^2} = \frac{1}{T^3 V} \frac{\partial^2 \ln Z(T, \mu_i, \mu_j)}{\partial(\mu_i/T)\partial(\mu_j/T)}\bigg|_{\mu_i=\mu_j=0}.$$

- Consider light and strange quark number susceptibility.
- At low temperatures they are carried by massive hadrons and their fluctuations are suppressed.
- At high temperatures they are carried by quarks and therefore can signal deconfiment.

Hadron Resonance Gas model

- Following Hagedorn's picture, the Hadron Resonance Gas model approximates the spectrum with currently known states from PDG

$$p^{HRG}/T^4 = \frac{1}{VT^3} \sum_{i \in \, mesons} \ln \mathcal{Z}^M_{m_i}(T, V, \mu_{X^a}) + \frac{1}{VT^3} \sum_{i \in \, baryons} \ln \mathcal{Z}^B_{m_i}(T, V, \mu_{X^a}),$$

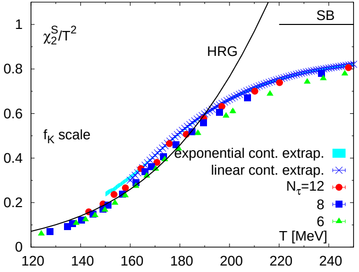
where

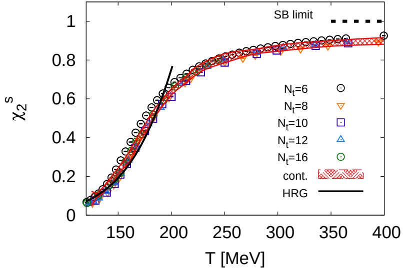
$$\ln \mathcal{Z}^{M/B}_{m_i} = \mp \frac{Vd_i}{2\pi^2} \int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\varepsilon_i/T}) \quad ,$$

with energies $\varepsilon_i = \sqrt{k^2 + m_i^2}$, degeneracy factors d_i and fugacities

$$\ln z_i = \sum_a X_i^a \mu_{X^a}/T \ .$$

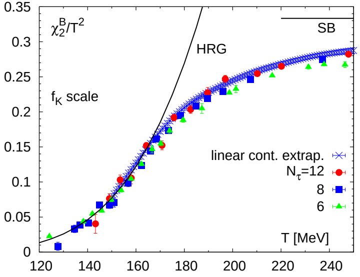
Fluctuations: strangeness

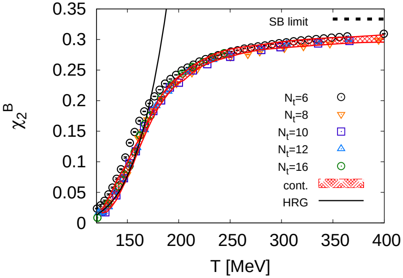




- Left: HotQCD, HISQ/tree action, $m_\pi = 160$ MeV, PRD86 (2012) 034509, arXiv:1203.0784 [hep-lat].
- Right: BW, stout action, $m_\pi = 140$ MeV, JHEP 1201 (2012) 138, arXiv:1112.4416 [hep-lat].

Fluctuations: baryon number





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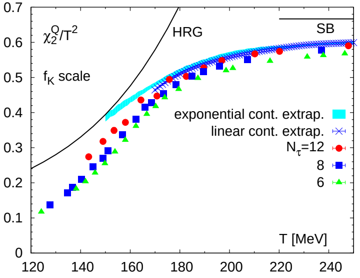
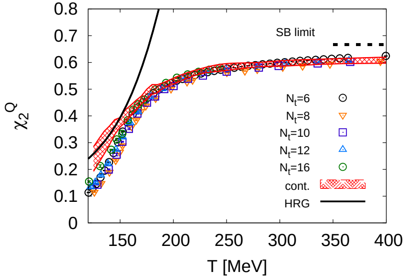
Fluctuations: electric charge





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Freeze-out parameters

Freeze-out parameters

- Consider[1] mean (M_X), variance (σ_X^2) and skewness (S_X) of corresponding charge distribution $X = B, Q, S$.
- Net strangeness and electric charge can be constrained to

$$M_S = 0, \;\; M_Q = rM_B, \;\; (r \simeq 0.4 \text{ in gold-gold and lead-lead collisions}),$$

and the chemical potentials related to the baryon chemical potential:

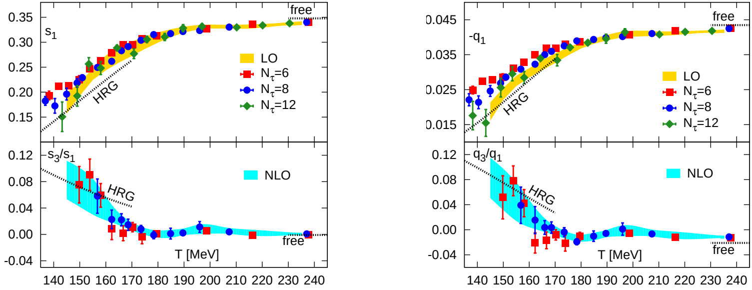
$$\frac{\mu_Q}{T} = q_1 \, \frac{\mu_B}{T} + q_3 \, \left(\frac{\mu_B}{T}\right)^3, \quad \frac{\mu_S}{T} = s_1 \, \frac{\mu_B}{T} + s_3 \, \left(\frac{\mu_B}{T}\right)^3,$$

$$q_1 = \frac{r\left(\chi_2^B\chi_2^S - \chi_{11}^{BS}\chi_{11}^{BS}\right) - \left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}{\left(\chi_2^Q\chi_2^S - \chi_{11}^{QS}\chi_{11}^{QS}\right) - r\left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)},$$

$$s_1 = -\frac{\chi_{11}^{BS}}{\chi_2^S} - \frac{\chi_{11}^{QS}}{\chi_2^S} \, q_1 \, .$$

[1]BNL-Bielefeld (Bazavov et al.), PRL109 (2012) 192302, arXiv:1208.1220 [hep-lat]

Freeze-out parameters



- Leading order (top half) and next-to-leading order (bottom half) expressions.
- NLO are within 10% in the transition region (up to $T \sim 160$ MeV) and rapidly decrease at higher temperature.

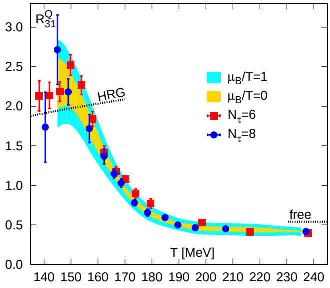
Freeze-out parameters

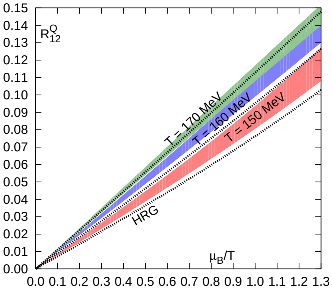
- Once μ_Q and μ_S satisfying the constraints are fixed, consider ratios of cumulants, for instance:

$$R_{12}^X \equiv \frac{M_X}{\sigma_X^2} = \frac{\mu_B}{T}\left(R_{12}^{X,1} + R_{12}^{X,3}\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}((\mu_B/T)^4)\right) ,$$

$$R_{31}^X \equiv \frac{S_X\sigma_X^3}{M_X} = R_{31}^{X,0} + R_{31}^{X,2}\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}((\mu_B/T)^4) .$$

Freeze-out parameters





- If R^Q_{31} is determined from experiment, this determines the freeze-out temperature T_f (left).
- R^Q_{12} and T_f determine the freeze-out chemical potential μ^f_B (right).

Equation of state

Trace anomaly

- The trace anomaly

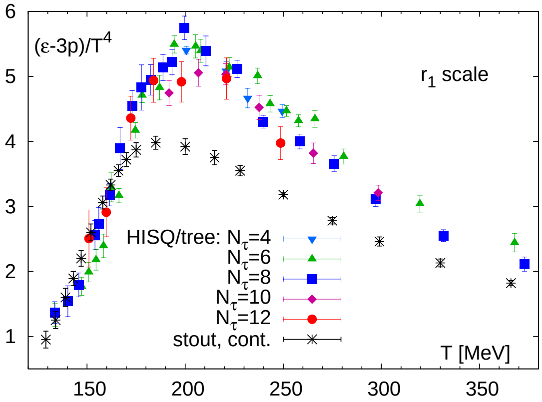
$$\varepsilon - 3p = -\frac{T}{V}\frac{d\ln Z}{d\ln a} \quad\Rightarrow\quad \frac{p}{T^4} - \frac{p_0}{T_0^4} = \int_{T_0}^{T} dT' \frac{\varepsilon - 3p}{T'^5}$$

- Requires subtraction of UV divergencies (take difference of zero- and finite-temperature quantities evaluated at the same values of the gauge coupling):

$$\begin{aligned}\frac{\varepsilon - 3p}{T^4} &= R_\beta[\langle S_g\rangle_0 - \langle S_g\rangle_T] \\ &- R_\beta R_m[2m_l(\langle\bar{l}l\rangle_0 - \langle\bar{l}l\rangle_T) + m_s(\langle\bar{s}s\rangle_0 - \langle\bar{s}s\rangle_T)]\end{aligned}$$

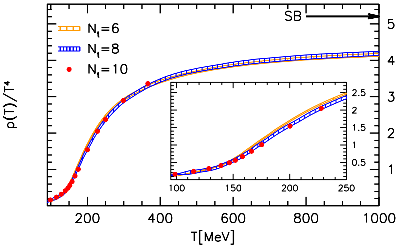
$$R_\beta(\beta) = -a\frac{d\beta}{da}, \quad R_m(\beta) = \frac{1}{m}\frac{dm}{d\beta}, \quad \beta = \frac{10}{g^2}$$

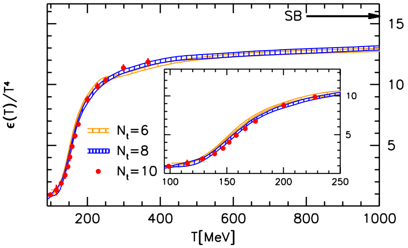
Trace anomaly



- HotQCD, HISQ/tree action, $m_\pi = 160$ MeV, at fixed N_τ, QM2012, preliminary.
- BW, stout action, $m_\pi = 140$ MeV, JHEP 1011:077,2010, arXiv:1007.2580v2 [hep-lat].
- Need better control over the continuum limit.

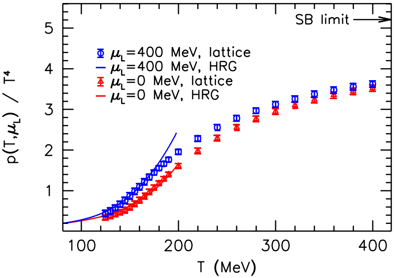
Equation of state

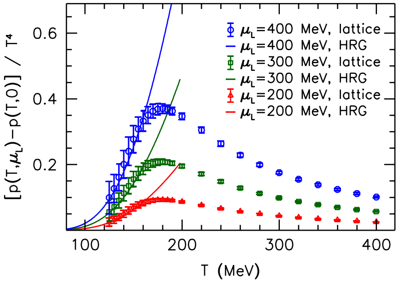


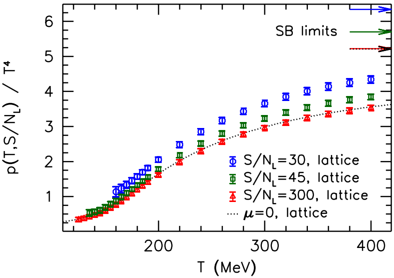


- BW, stout action, $m_\pi = 140$ MeV, JHEP 1011:077,2010, arXiv:1007.2580v2 [hep-lat].
- Pressure (left) and the energy density (right).

Equation of state at $\mu \neq 0$







- BW, stout action, $m_\pi = 140$ MeV, arXiv:1204.6710 [hep-lat].
- Pressure (left) and difference from $\mu = 0$ (right).
- Isentropic EoS (bottom).

Conclusion

- Lattice QCD provides means to study QCD at finite temperature and (to a limited extent) finite density.
- Pseudo-transition temperature associated with the chiral symmetry restoration is established in the continuum limit at the physical light quark mass. Agreement between staggered studies, crosschecks with Wilson, domain-wall and overlap, but at higher pion mass.
- Deconfinement is a gradual phenomenon, no unique transition temperature can be associated with it in full QCD.
- Fluctuations are useful tools in studying deconfinement, some continuum results are available, agreement between staggered calculations.
- (Under extra assumptions) fluctuations of conserved charges can provide the freeze-out parameters from first principles.
- Equation of state: work towards the continuum limit, need to study higher temperatures to connect to perturbative regime.