II: Symmetry and Weak Interactions

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Lessons from yesterday:

i) Symmetry is omnipresent in physics though usually broken in one of three ways: explicit, spontaneous, anomalous

ii) Symmetry combined with effective field theory is a powerful tool

Today look at specific examples, but first big picture: Consider weak interaction

$$\mathcal{H}_w \sim \frac{g^2}{M_W^2 - q^2} J^\mu J_\mu^\dagger \stackrel{q^2 << M_W^2}{\longrightarrow} \frac{G}{\sqrt{2}} J^\mu J_\mu^\dagger$$

where $G \simeq 10^{-5}/m_p^2$. Hence can infer size of M_W via $M_W \sim 1/\sqrt{G} \sim 300 GeV$ by LOW energy experiments.

Now consider possible BSM weak effects such as possible scalar interaction

$$H_S \sim \frac{g_S^2}{M_S^2 - q^2} J J^\dagger \stackrel{q^2 << M_S^2}{\longrightarrow} \frac{K}{\sqrt{2}} J J^\dagger$$

If can experimentally limit $K < 0.01G$ then conclude that $M_S \geq 10 M_W$. Hence LOW energy limits shed light on HIGH energy physics.

Weak beta decay interactions described by standard model:

$$\mathcal{H}_w^{semileptonic} \simeq \frac{g_w^2}{8M_W^2} V_{ud}\bar{u}\gamma_\alpha(1-\gamma_5)d\bar{e}\gamma^\alpha(1-\gamma_5)\nu_e$$

Here $\frac{g_w^2}{8M_W^2} \equiv G_F/\sqrt{2}$ where $G_F \simeq 10^{-5}$ GeV^{-2} is Fermi constant.

Universality: $\bar{e}\gamma^{\mu}(1-\gamma_5)\nu_e \rightarrow$

$$\bar{e}\gamma^{\mu}(1-\gamma_5)\nu_e + \bar{\mu}\gamma^{\mu}(1-\gamma_5)\nu_{\mu} + \bar{\tau}\gamma^{\mu}(1-\gamma_5)\nu_{\tau}$$

How to measure G_F—Muon Decay. Universality gives

$$\mathcal{H}_w^{leptonic} \simeq G_F/\sqrt{2}\bar{\nu}_{\mu}\gamma_{\alpha}(1-\gamma_5)\mu\bar{e}\gamma^{\alpha}(1-\gamma_5)\nu_e$$

and leads to

$$\Gamma_{\mu} = \frac{G_F^2 m_{\mu}^5}{192\pi^3}$$

Inclusion of radiative and electron mass corrections yields

$$G_F = (1.16639 \pm 0.00001) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

How to check universality—$\pi \to e\nu_e/\pi \to \mu\nu_\mu$. Define

$$< 0|A_\mu^-|\pi_{p_1}^+ > \equiv \sqrt{2}F_\pi p_{1\mu}$$

Then

$$\Gamma_\pi = \frac{G_F^2 F_\pi^2 V_{ud}^2}{256\pi m_\pi^3} m_\ell^2 (m_\pi^2 - m_\ell^2)^2$$

yields $F_\pi = (92.4 \pm 0.3)$ MeV.

Then

$$R_\pi \equiv \left(\frac{\Gamma(\pi^+ \to e^+ \nu_e)}{\Gamma(\pi^= \to \mu^+ \nu_\mu)} \right)^{theo}$$

$$= \frac{m_e^2 (m_\pi^2 - m_e^2)^2}{m_\mu^2 (m_\pi^2 - m_\mu^2)^2} = 1.28 \times 10^{-4}$$

Small because of helicity suppression.

Electromagnetic corrections change to

$$R_\pi^{theo} = (1.2353 \pm 0.0001) \times 10^{-4}$$

Compare to

$$R_\pi^{exp} = (1.230 \pm 0.004) \times 10^{-4}$$

i) Pure $V_\mu - A_\mu$ structure—no scalar, pseudoscalar, or tensor interactions;

ii) Time reversal invariance;

iii) G-parity: Defining $G = C\exp(-i\pi I_2)$ the weak currents satisfy

$$G(V_\mu - A_\mu)G^{-1} = V_\mu + A_\mu$$

This requirement is generally called *no second class currents*;

iv) CVC: The weak vector current is related to the electromagnetic current via a simple isotopic spin rotation

$$V_\mu^\pm = \mp[I_\pm, V_\mu^{em}]$$

where $I_\pm = I_1 \pm iI_2$ are isospin raising/lowering lowering operators. This condition is termed the "conserved vector current" or CVC hypothesis;

v) PCAC: As we have seen, the axial current would also be conserved were this symmetry not broken spontaneously. However, due to this breaking and because the axial divergence is a pseudoscalar, it can be used as an interpolating field for the pion

$$\partial^\mu A_\mu = F_\pi m_\pi^2 \phi_\pi + \mathcal{O}(\phi_\pi^3)$$

This requirement is called the "partially conserved axial vector current" or PCAC hypothesis and is closely tied in to chiral symmetry;

vi) Unitarity: The weak coupling constant $G_F^\beta V_{ud}$ responsible for nuclear beta decay is identical to that in nuclear muon capture and is related to that responsible for muon decay—G_F^μ—via

$$G_F^\beta = G_F^\mu$$

where V_{ud} is related by unitarity to other mixing angles via

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

vi) ....

Use "allowed" decays—$\Delta J = 0, \pm 1, \; \Delta \Pi = \mathrm{no.}$
Example is neutron decay

$$< p_{p'} | V_\mu | n_p >$$

$$= \bar{u}_p(p') \left(\gamma_\mu f_1(q^2) - i \sigma_{\mu\nu} q^\nu \frac{f_2(q^2)}{2M} + q_\mu \frac{f_3(q^2)}{2M} \right) u_n(p)$$

$$< p_{p'} | A_\mu | n_p >$$

$$= \bar{u}_p(p') \left(\gamma_\mu g_1(q^2) - i \sigma_{\mu\nu} q^\nu \frac{g_2(q^2)}{2M} + q_\mu \frac{g_3(q^2)}{2M} \right) \gamma_5 u_n(p)$$

f_1, g_1: Vector, axial couplings
f_2, g_2: Weak magnetism, weak electricity couplings
f_3, g_3: Induced scalar, pseudoscalar couplings

Consider arbitrary allowed transition—

$$J^{\pm} \rightarrow J^{\pm}, J \pm 1^{\pm}$$

to NLO in recoil—$q/m_N, q^2R^2$.

$$\begin{aligned}
\ell^{\mu} < \beta|V_{\mu}|\alpha > &= \left(a(q^2)\frac{P\cdot\ell}{2M} + e(q^2)\frac{q\cdot\ell}{2M}\right)\delta_{JJ'}\delta_{MM'} \\
&+ i\frac{b(q^2)}{2M}C^{M'k;M}_{J'1;J}(\vec{q}\times\vec{\ell})_k \\
&+ C^{M'k;M}_{J'2;J}\left[\frac{f(q^2)}{2M}C^{nn';k}_{11;2}\ell_n q_{n'}\right. \\
&+ \left.\frac{g(q^2)}{(2M)^3}P\cdot\ell\sqrt{\frac{4\pi}{5}}Y^k_2(\hat{q})\vec{q}^2 + \ldots\right]
\end{aligned}$$

$$
\begin{aligned}
& \ell^{\mu} < \beta | A_{\mu} | \alpha > \\
= \quad & C^{M'k;M}_{J'1;J} \epsilon_{ijk} \epsilon_{ij\lambda\eta} \frac{1}{4M} \Big[c(q^2) \ell^{\lambda} P^{\eta} - d(q^2) \ell^{\lambda} q^{\eta} \\
+ \quad & \frac{1}{(2M)^2} h(q^2) q^{\lambda} P^{\eta} q \cdot \ell \Big] \\
+ \quad & C^{M'k;M}_{J'2;J} C^{nn';k}_{12;2} \ell_n \sqrt{\frac{4\pi}{5}} Y^{n'}_2(\hat{q}) \frac{\vec{q}^2}{(2M)^2} j_2(q^2) \\
+ \quad & C^{M'k;M}_{J'3;J} C^{nn';k}_{12;3} \ell_n \sqrt{\frac{4\pi}{5}} Y^{n'}_2(\hat{q}) \frac{\vec{q}^2}{(2M)^2} j_3(q^2) + \ldots
\end{aligned}
$$

$$
\begin{array}{ll}
a \to f_1, & c \to g_1 \\
b \to f_2, & d \to g_2 \\
e \to f_3, & h \to g_3
\end{array}
$$

No neutron analog for f, g, j_2, j_3 since $\Delta J = 2, 3$

Note there are impulse approximation (one-body) predictions for each form factor. In leading case

$$a(0) = f_1(0) M_F \quad \text{and} \quad c(0) = g_1(0) M_{GT}$$

where $f_1(0) = 1$ is the neutron vector coupling and $M_F =< \beta || \sum_n \tau_n^{\pm} || \alpha >$ is Fermi matrix element and $g_1(0) = -1.275$ is the neutron axial coupling and $M_{GT} =< \beta || \sum_n \tau_n^{\pm} \vec{\sigma}_n ||| \alpha >$ is the Gamow-Teller matrix element. Here M_F vanishes unless $|\alpha >, |\beta >$ are isotopic analogs such as $^{10}C, ^{10}B$ or $^{14}O, ^{14}N$, etc. in which case $M_F = \sqrt{2}$.

For $0^+ - 0^+$ transitions such as these $M_{GT} = 0$ so if define phase space factor

$$f(Z, R, E_0) = \int_{m_e}^{E_0} dE_e F(Z, R, E_e)(E_0 - E_e)^2 E_e p_e$$

then

$$ft_{\frac{1}{2}} = \frac{\pi^3 \log 2}{G_F^2 V_{ud}}|^2$$

and should be same for all such transitions. Find

	E_0(KeV)	f^Rt(sec)
^{10}C	886	3076.7±4.6
^{14}O	1809	3071.5±3.3
^{26}Al^m	3211	3072.4±1.4
^{34}Cl	4470	3070.6±2.1
^{38}K^m	5023	3072.5±2.4
^{42}Sc	5402	3072.4±2.7
^{46}V	6029	3073.3±2.7
^{50}Mn	6610	3070.9±2.8
^{54}Co	7220	3069.9±3.3

Can now measure V_{ud} using $ft^{av}_{\frac{1}{2}} = 3071.8 \pm 0.8$ sec, yielding

$$V_{ud} = 0.97425 \pm 0.00022$$

Using $V_{us} = 0.2253 \pm 0.0009$ from $K_{\ell 3}$ decay and $V_{ub} = 0.00339 \pm 0.00044$ from PDG unitarity test gives

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.99990 \pm 0.00060$$

Another CVC test—Gell-Mann suggested use of $A=12$ isotriplet— $^{12}B, {}^{12}C(15.11 \text{ MeV}, 1^+), {}^{12}N$

Define shape factor $S(E)$ via

$$\frac{d\Gamma}{dE} = \text{Phase Space} \cdot S(E)$$

where

$$\text{Phase Space} \sim F(\pm Z, E)pE(E_0 - E)^2$$

Then

$$S(E) = 1 \pm \frac{4E}{3M}\frac{b}{c} \quad \text{for} \quad e^{\mp} \text{ decay}$$

CVC prediction for b from 15.11 MeV ^{12}C radiative width

$$b = \sqrt{\frac{6M^2\Gamma_{M1}}{\alpha E_0^3}}$$

yields

$$\frac{b}{Ac}_{\rm nuclear} \sim \frac{b}{c}_{\rm nucleon} \sim 4.7$$

and

$$\frac{dS}{dE} \simeq \pm \frac{4}{3m_N}\frac{b}{Ac} \sim \pm 0.5\%/{\rm MeV}$$

Experiment:

		Exp.	%/MeV Thy
$\frac{dS}{dE}^-$	Lee et al.	0.48± 0.10	0.43
$\frac{dS}{dE}^+$	Lee et al.	-0.52± 0.06	-0.50

Now look at second class currents. Suppose we have both currents such as $\bar{q}\gamma_\mu\gamma_5 q$, $\bar{q}\gamma_\mu q$ which are "first class"—$GV^I_\mu G^{-1} = V^I_\mu$, $GA^I_\mu G^{-1} = -A^I_\mu$ *and* BSM "second class currents"—$GV^{II}_\mu G^{-1} = -V^{II}_\mu$, $GA^{II}_\mu G^{-1} = A^{II}_\mu$.

Then

$$\exp(-i\pi I_2) J^{x\pm}_\mu \exp(i\pi I_2) = \epsilon_x J^{x\pm}_\mu$$

with

$$\epsilon_I = \mp 1 \text{ if } T J^I_\mu T^{-1} = \pm J^{I\,\mu\dagger}$$

$$\epsilon_{II} = \pm 1 \text{ if } T J^{II}_\mu T^{-1} = \pm J^{II\,\mu\dagger}$$

For transitions within a common isotopic multiplet

$$< I, I_3 \pm 1; \vec{p}', J, M' | J^x_\mu | I, I_3; \vec{p}, J, M >$$

$$= -\epsilon_x < I, -I_3; \vec{p}, J, M | J^x_\mu | I, -I_3 \pm 1; \vec{p}', J, M' >^*$$

so

$$0 = a^{II}(q^2) = b^{II}(q^2) = c^{II}(q^2)$$
$$= g^{II}(q^2) = h^{II}(q^2) = j_3^{II}(q^2)$$

$$0 = d^I(q^2) = e^I(q^2) = f^I(q^2) = j_2^I(q^2)$$

and no second class currents says $d, e, f, j_2 = 0$ for analog transitions.

For transitions *not* within a common isotopic multiplet

$$< I', I_3 \pm 1; \vec{p}', J', M' | J^{x\pm}_\mu | I, I_3; \vec{p}, J, M >$$

$$= (-)^{I-I'+1} \epsilon_x < I', -I_3 \pm 1; \vec{p}', J', M' | J^{x\mp}_\mu | I, -I_3 \pm 1; \vec{p}, J, M >^*$$

Thus for first class

$$F_I(q^2; I_3 \to I_3 \pm 1) = (-)^{I-I'} F_I^*(q^2; -I_3 \to -I_3 \mp 1)$$

while for second class

$$F_{II}(q^2; I_3 \to I_3 \pm 1) = -(-)^{I-I'} F_{II}^*(q^2; -I_3 \to -I_3 \mp 1)$$

How to check? Consider A=12 system and produce alignment

$$\Lambda = 1 - \frac{3}{2} < J_z^2 >$$

and measure

$$\frac{d^5\Gamma}{dE_e d\Omega_e} \propto 12\Lambda\delta(E)\left(\left(\frac{\vec{p}_e \cdot \hat{n}}{E_e}\right)^2 - \frac{1}{3}\frac{p_e^2}{E_e^2}\right)$$

for both ^{12}N, ^{12}B decays. Since

$$\delta^{\pm}(E) = \frac{E}{2M}\left(\pm\frac{b + d^{II}}{c} + \frac{d^{I}}{c}\right)$$

find

$$\delta^{-}(E) - \delta^{+}(E) = -\frac{E}{M}\frac{b + d^{II}}{c}$$

Subtraction of CVC weak magnetism value yields d^{II}.

Has also been done for A=8 and A=20 systems using $\beta - \alpha$ and $\beta - \gamma$ correlations to eliminate higher order form factors. Results are

	d^{II}/Ac	d^{II}/b
A=8	-0.24 ± 0.31	-0.03 ± 0.04
A=12	-0.15 ± 0.17	-0.04 ± 0.04
A=20	0.18 ± 0.48	0.02 ± 0.06

Now look at right-handed currents. Standard model based on $SU(2)_L \otimes U(1)$. Why not $SU(2)_L \otimes SU(2)_R \otimes U(1)$? Would then be two sets of gauge bosons—$W^{\pm}_{L\mu}$ and $W^{\pm}_{R\mu}$—with $M_{W_R} >> M_{W_L}$ so that $G_{FL} >> G_{FR} = G_{FL}\frac{M^2_{WL}}{M^2_{WR}}$. Since mass W_1, W_2 and chiral W_L, W_R eigenstates need not be the same define

$$W_1 = \cos\chi W_L - \sin\chi W_R$$

$$W_2 = \sin\chi W_L - \cos\chi W_R$$

and $\sigma = M_1^2/M_2^2$. Then standard model is $\sigma\chi = 0$.

But in more general case

$$\mathcal{H}_w \sim \frac{G}{\sqrt{2}}[\gamma_\mu(1-\gamma_5) \otimes \gamma^\mu(1+\epsilon\gamma_5)$$

$$+\gamma_\mu(1+\gamma_5) \times (\gamma^\mu(x - y\epsilon\gamma_5)]$$

with

$$x \approx \sigma - \chi, \quad y = \sigma + \chi, \quad \epsilon = \frac{1-x}{1-y}$$

How to detect x, y? Can compare positron helicities for Fermi vs. Gamow-Teller decays—

$$\frac{P_L^F}{P_L^{GT}} \simeq 1 - 2x^2 + 2y^2 = 1 + 8\sigma\chi$$

Experimentally

$$\frac{P_L^F}{P_L^{GT}} = 1.003 \pm 0.004$$

Compare ft and $asymmetries$ for ^{19}Ne—

$$\frac{ft^{\rm Fermi}}{ft^{Ne}} = \frac{a^2 + c^2 + x^2a^2 + y^2c^2 + T_3}{a_F^2(1+x^2)}$$

and

$$A = \frac{\frac{2}{\sqrt{3}}c(a + \frac{c}{\sqrt{3}}) - 2y\frac{c}{\sqrt{3}}(xa + y\frac{c}{\sqrt{3}}) + T_1}{a^2 + c^2 + x^2a^2 + y^2c^2 + T_2}$$

where

$$\frac{d\Gamma_\beta}{d\Omega_e} \sim 1 + AP\hat{J} \cdot \vec{p}_\beta / E_\beta + \ldots$$

Why ^{19}Ne? Value

$$\left(\frac{c}{a}\right)^{^{19}Ne} \simeq \sqrt{\frac{2\,f\,t^{\rm Fermi}}{f\,t^{^{19}Ne}} - 1} \simeq -1.60$$

is very near the value $c/a = -\sqrt{3}$ at which pure left-handed asymmetry would vanish, so very sensitive to RH effects.

Look at Muon decay spectrum

$$\frac{d^2\Gamma_\mu}{s^2 ds\, d(\cos\theta)} \sim 3-2s+\left(\frac{4}{3}\rho - 1\right)(4s-3)+12\frac{m_e}{s m_\mu}(1-s)\eta$$

$$+\xi P_\mu \cos\theta \left[\left(\frac{4}{3}\delta - 1\right)(4s-3) + 2s - 1\right]$$

with $s = E_e/E_{\rm max}$ and Michel parameters

$$\rho \simeq \frac{3}{4}\left(1 - \frac{1}{2}(x-y)^2\right)$$

$$\xi \simeq 1 - x^2 - y^2$$

$$\delta \approx \frac{3}{4}$$

$$\eta \approx 0$$

Results from TRIUMF

$$\frac{\xi P_\mu \delta}{\rho} > 0.9959 \quad \text{at } 90\% \; C.L.$$

yields

$$\frac{\xi P_\mu \delta}{\rho} \simeq 1 - 2(\sigma + \chi)^2 - 2\sigma^2$$

and yield the constraints shown. From PSI

$$P_\mu \xi = 1.0027 \pm 0.0084$$

Testing PCAC—two direct predictions in weak interactions.

One is Goldberger-Treiman relation

$$\frac{1}{2}(m_p + m_n) g_A(0) = F_\pi g_{\pi NN}(0)$$

Here $\frac{1}{2}(m_p + m_n) = 939$ MeV, F_π=92.4 MeV, and $g_A(0) = 1.275$ are well determined, but not so $g_{\pi NN}(0)$. Karlsruhe dispersive analysis gives $g_{\pi NN}(m_\pi^2) = 13.45$ but VPI gives 13.1. Basically works well to per cent level.

Second prediction is for induced pseudoscalar form factor—$g_P(q^2)$. Expect

$$g_P(q^2) = \frac{4MF_\pi}{m_\pi^2 - q^2} g_{\pi NN}(q^2) \simeq \frac{4MF_\pi}{m_\pi^2 - q^2} g_{\pi NN}(q^2) - \frac{2M^2}{3} g_A(0) r_A^2$$

Use combination

$$r_P = \frac{m_\mu}{2Mg_A(0)} g_P(q^2 = -0.9m_\mu^2) = 6.7$$

relevant for muon capture. Need muon capture since in beta decay effects are $\mathcal{O}(r_P m_e^2/(2Mm_\mu)) \sim 10^{-5}$ but in muon capture $\mathcal{O}(r_P m_\mu/2M) \sim 0.35$.

However, capture rate is one number, so must assume CVC, no second class currents, q^2-dependence of form factors is known. Then can determine r_P. Current results are

$$r_P = \begin{cases} 7.3 \pm 1.1 & H \\ 6.9 \pm 0.2 & {}^3He \\ 9.0 \pm 1.7 & {}^{12}C \end{cases}$$

$$\frac{d^3\Gamma}{dE_e d\Omega_e d\Omega_\nu} \propto F(Z, E_e) p_e E_e (E_0 - E_e)^2 \times \left(f_1(E_e) + a_{\beta\nu}(E_e) \frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + b_{\text{Fierz}}(E_e) \frac{m_e}{E_e} \right)$$

$$a_{\beta\nu} = \big(|M_F|^2 (|C_V|^2 + |C'_V|^2 - |C_S|^2 - |C'_S|^2) - \tfrac{1}{3} |M_{\text{GT}}|^2 (|C_A|^2 + |C'_A|^2 - |C_T|^2 - |C'_T|^2) \big) \xi^{-1}, \quad (2)$$

where

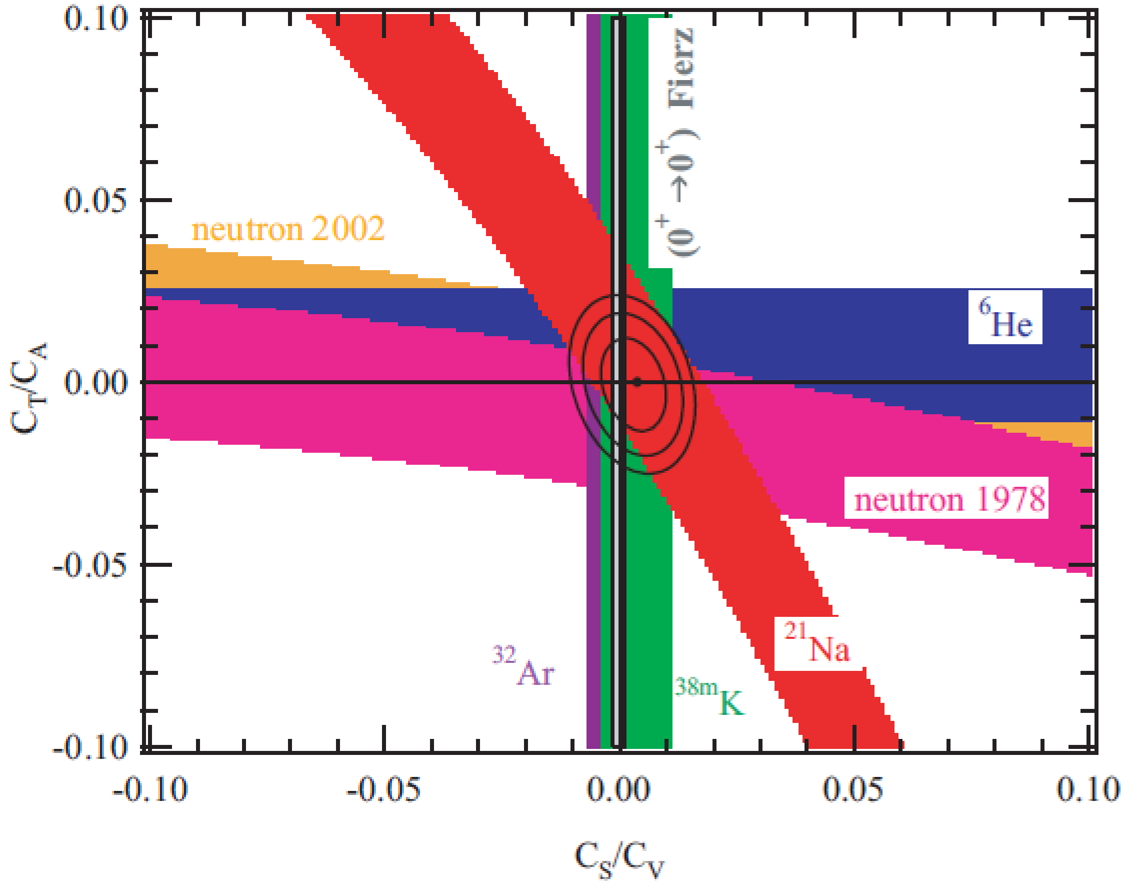
$$\xi = |M_F|^2 (|C_V|^2 + |C'_V|^2 + |C_S|^2 + |C'_S|^2) + |M_{\text{GT}}|^2 (|C_A|^2 + |C'_A|^2 + |C_T|^2 + |C'_T|^2) \quad (3)$$

$$b_{\text{Fierz}} = \pm 2\sqrt{1 - (Z\alpha)^2} \, \text{Re}[|M_F|^2 (C_S C_V^* + C'_S C'^*_V) + |M_{\text{GT}}|^2 (C_T C_A^* + C'_T C'^*_A)] \xi^{-1}.$$

Recent ^{21}Na trapping measurements, yielded

$$a^{exp} = 0.5502(60) \quad \text{vs.} \quad a^{the-SM} = 0.553(2)$$

puts limits on possible tensor,scalar couplings.



0.10
0.05
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C_T/C_A
C_S/C_V
neutron 2002
$(0^+ \rightarrow 0^+)$ Fierz
^{6}He
neutron 1978
^{21}Na
^{32}Ar
^{38m}K