

Neutron Stars and Their Crusts



Introduction: **neutron rich matter.**
Lec. 1: Neutron stars and their crusts.

Lec. 2: Supernovae.

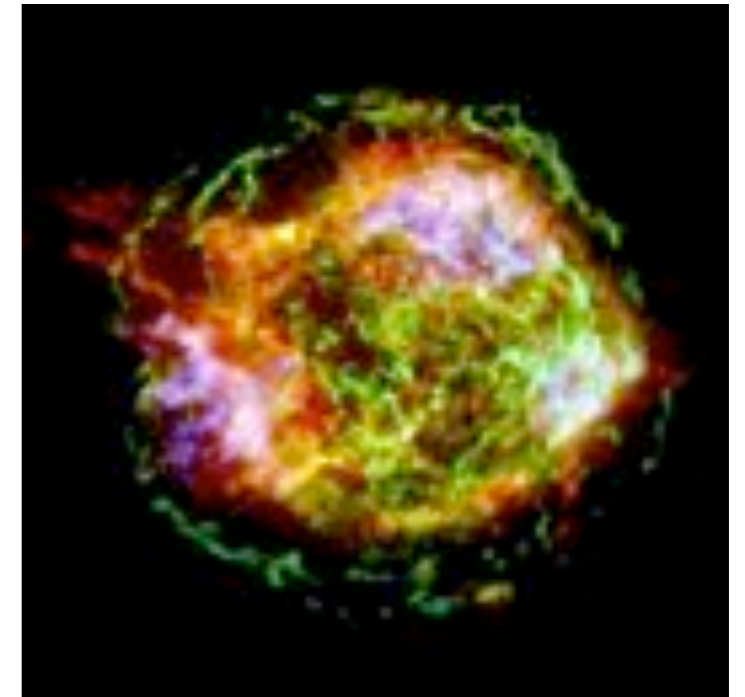
Lec. 3: Nucleosynthesis and gravitational waves.

Questions: horowitz@indiana.edu

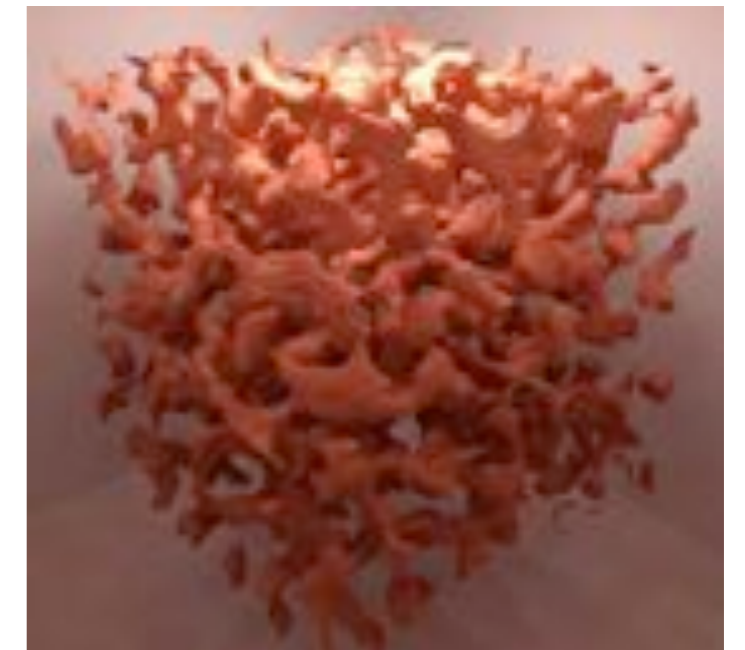
C. J. Horowitz, Indiana University, National Nuclear Physics Summer School, Santa Fe, July 2012.

Neutron Rich Matter

- Compress almost anything to $10^{11}+$ g/cm³ and electrons react with protons to make neutron rich matter. This material is at the heart of many fundamental questions in nuclear physics and astrophysics.
 - What are the high density phases of QCD?
 - Where did the chemical elements come from?
 - What is the structure of many compact and energetic objects in the heavens, and what determines their electromagnetic, neutrino, and gravitational-wave radiations?
- Interested in neutron rich matter over a tremendous range of density and temperature were it can be a *gas, liquid, solid, plasma, liquid crystal (nuclear pasta), superconductor ($T_c=10^{10}$ K!), superfluid, color superconductor...*
- *Focus on simpler liquid, solid, and gas phases.*



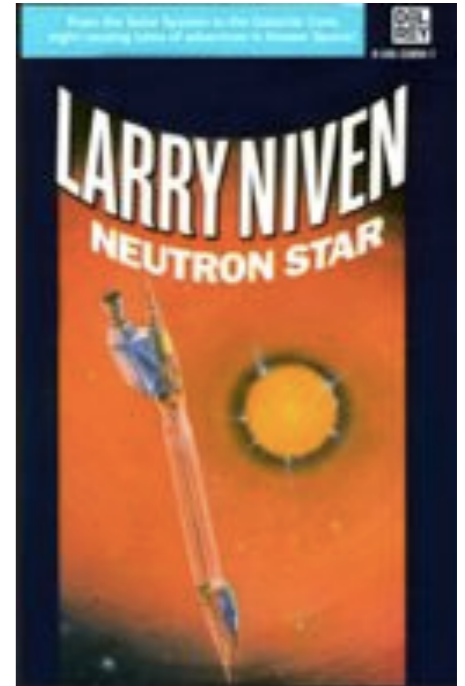
Supernova remanent
Cassiopea A in X-rays



MD simulation of Nuclear
Pasta with 100,000 nucleons

Neutronium

- In Larry Niven's Known Space fictional universe (1964), neutronium is actual neutron star core material. Niven imagines that small blobs of it could be artificially created under pressure, and their instability overcome by containing them in Slaver stasis fields.
- In the Star Trek universe, neutronium is an extremely hard and durable substance, often used as armor, which conventional weapons cannot penetrate or even dent.
- Our large scale MD simulations find neutron star crust to be the strongest material known 10^{10} x stronger than steel ---PRL**102**, 191102.



Probes of Neutron Rich Matter

- **Multi-Messenger Astronomy:** “seeing” the same event with very different probes should lead to fundamental advances. Often **photons** from *solid* neutron star crust, supernova **neutrinos** from low density *gas*, and **gravitational waves** from energetic motions of *liquid* interior of neutron stars.
- **Laboratory:** Nuclei are liquid drops so most experiments probe liquid n rich matter. However one can also study vapor phase by evaporating nucleons.
 - Electroweak measurements, Heavy ion collisions, Radioactive beams of neutron rich nuclei...
- **Computational:** Important theoretical and computational advances aid in study of n rich matter.
 - Chiral effective field theory depends on important and poorly known *three neutron forces*.
 - Large scale computations: molecular dynamics, monte carlo, no core shell model, coupled cluster,

Review article: Multi-messenger observations of neutron rich matter, Int. J. Mod. Phys. E **20** (2011) 1.

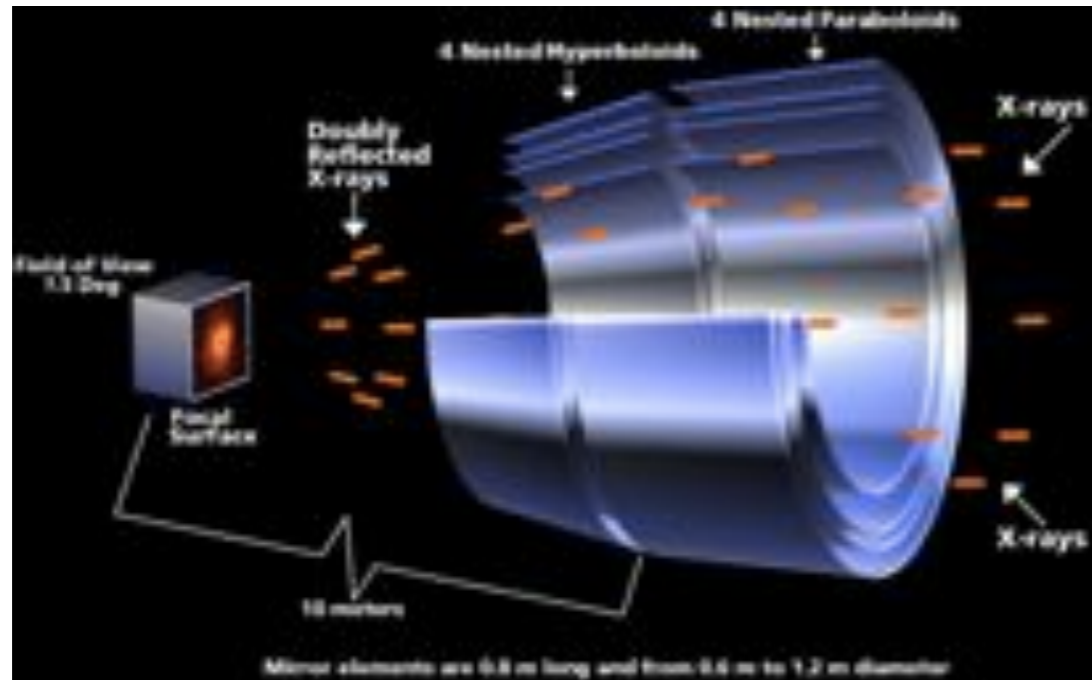


Limits on computations for cold dense matter

- Lattice QCD at finite T but finite density is presently very hard.
- Chiral pert theory only converges near nuclear density and below. Higher order contributions may be large at higher densities.
- Can do GFMC or other microscopic calculations at high densities. However, there may be important ambiguities in your hamiltonian.
- **Can't calculate properties of dense neutron rich matter, must observe!**

X-ray observatories Chandra and XMM Newton

Chandra has good spatial resolution, XMM Newton has larger collecting area.



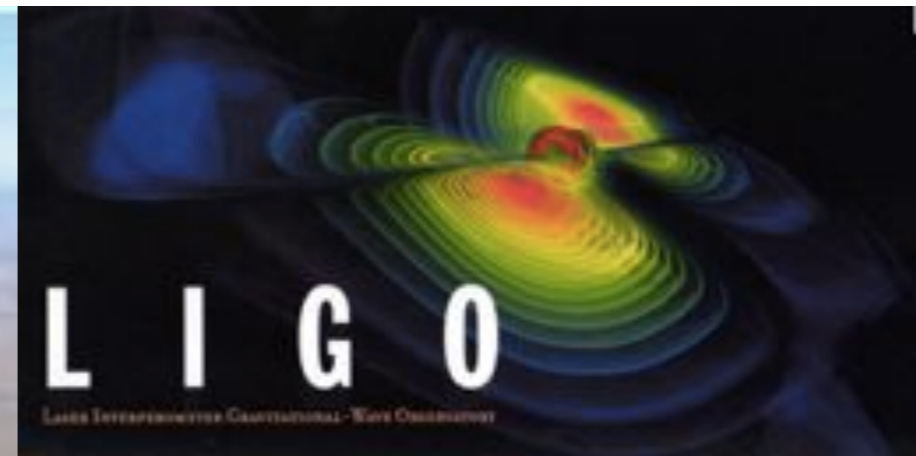
A number of smaller X-ray telescopes with wide fields and or good timing and gamma ray observatories such as Fermi. However no large scale X-ray observatory to replace aging Chandra and XMM Newton.

Radio Telescopes

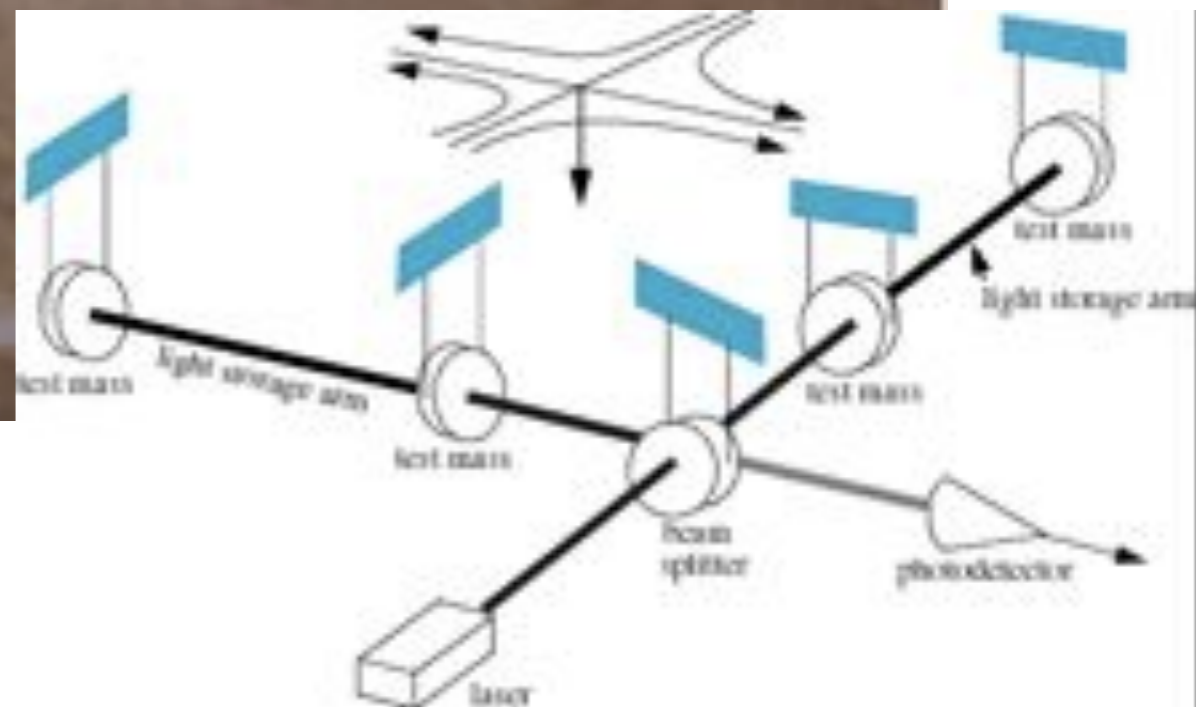


Very Large Array

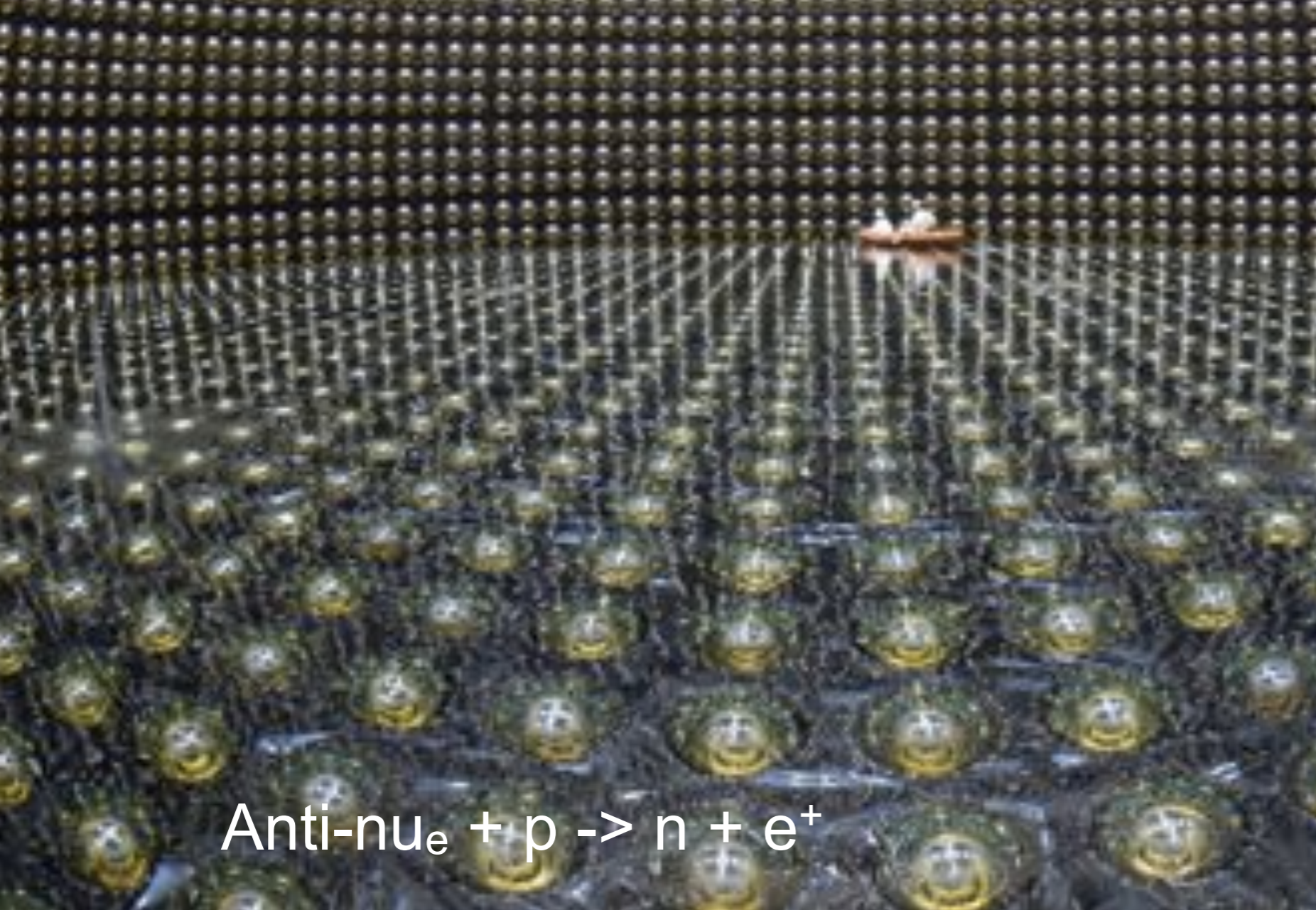
- New radio telescopes will combine power of many small detectors.
- Increased computer power greatly aids searches for radio pulsars.
- New large scale surveys will find many more pulsars.



Large scale gravitational wave detectors in Washington State, Louisiana, Italy (VIRGO) ...



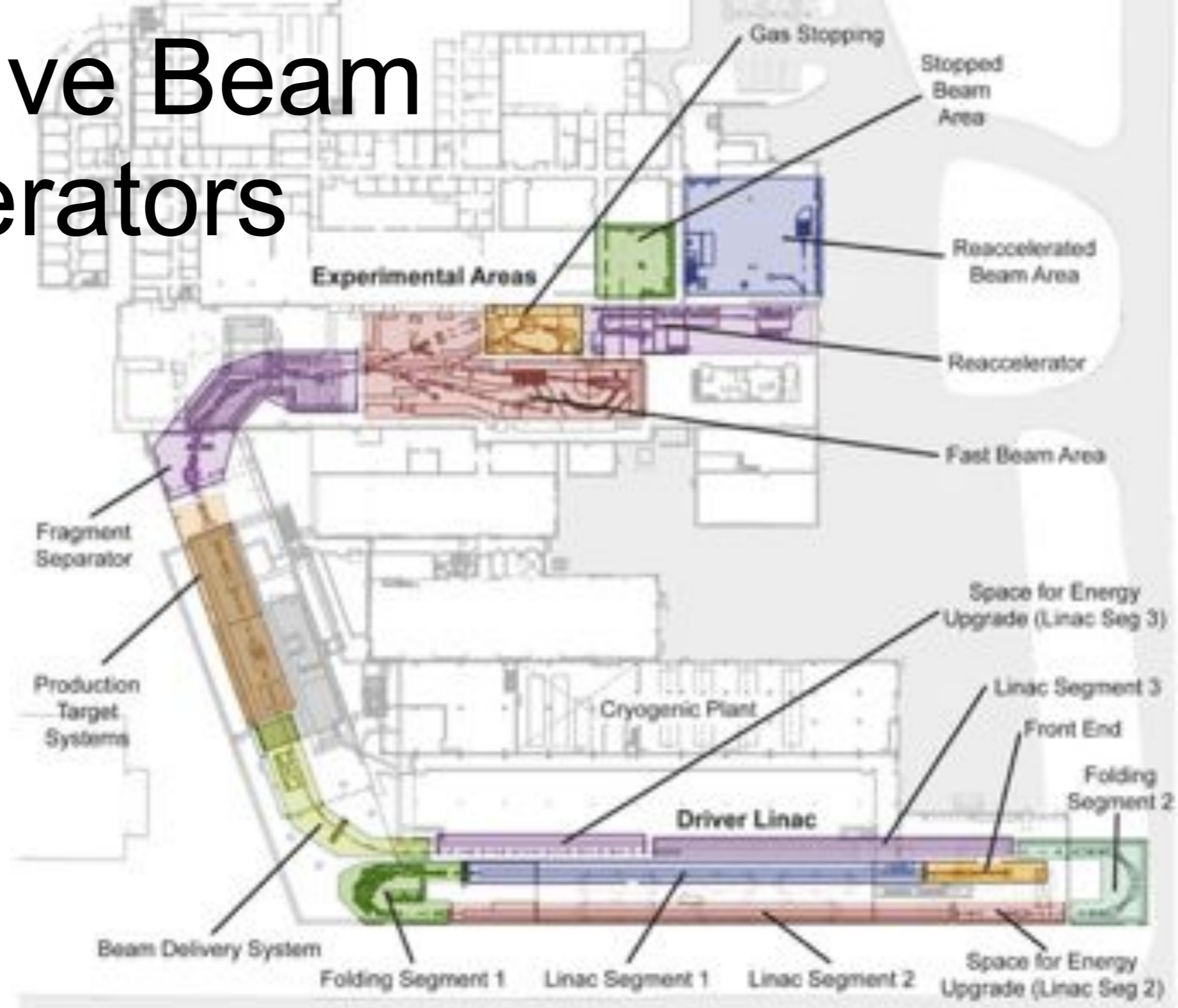
Supernova Neutrinos



Super Kamikande

- A variety of large neutrino detectors should see tens of thousands of events from the next galactic supernova.

Radioactive Beam Accelerators



- It is remarkable that we can directly produce beams of neutron rich nuclei in the laboratory and quickly study them before they decay.

It's the same "stuff" in the laboratory and in the heavens



- For Newton the stuff was mass. In 19th century it was spectral lines. Gave birth to astrophysics and the very name of the 2nd element helium.
- Today the stuff is neutron rich matter. In astrophysics and in the laboratory it is the same neutrons, the same strong interactions, and the same equation of state.
- A measurement in one domain (astronomy or the lab) has important implications in the other domain.

Stability of Hydrogen

- Pressure ionization: compress molecular hydrogen gas and it undergoes a phase transition to **metallic hydrogen**.
 - Instead of electrons being bound into individual H₂ molecules they now form a nearly free Fermi gas.
 - Electron Fermi energy becomes much larger than electrostatic binding E.
- At a density (exercise 1) of 1.3×10^7 g/cm³ hydrogen becomes unstable to electron capture: $e + p \rightarrow n + \nu$.
- $E_F + m_p = m_n$, with $E_F = (k_F^2 + m_e^2)^{1/2}$, $n_e = k_F^3 / 3\pi^2$

Exercise 1

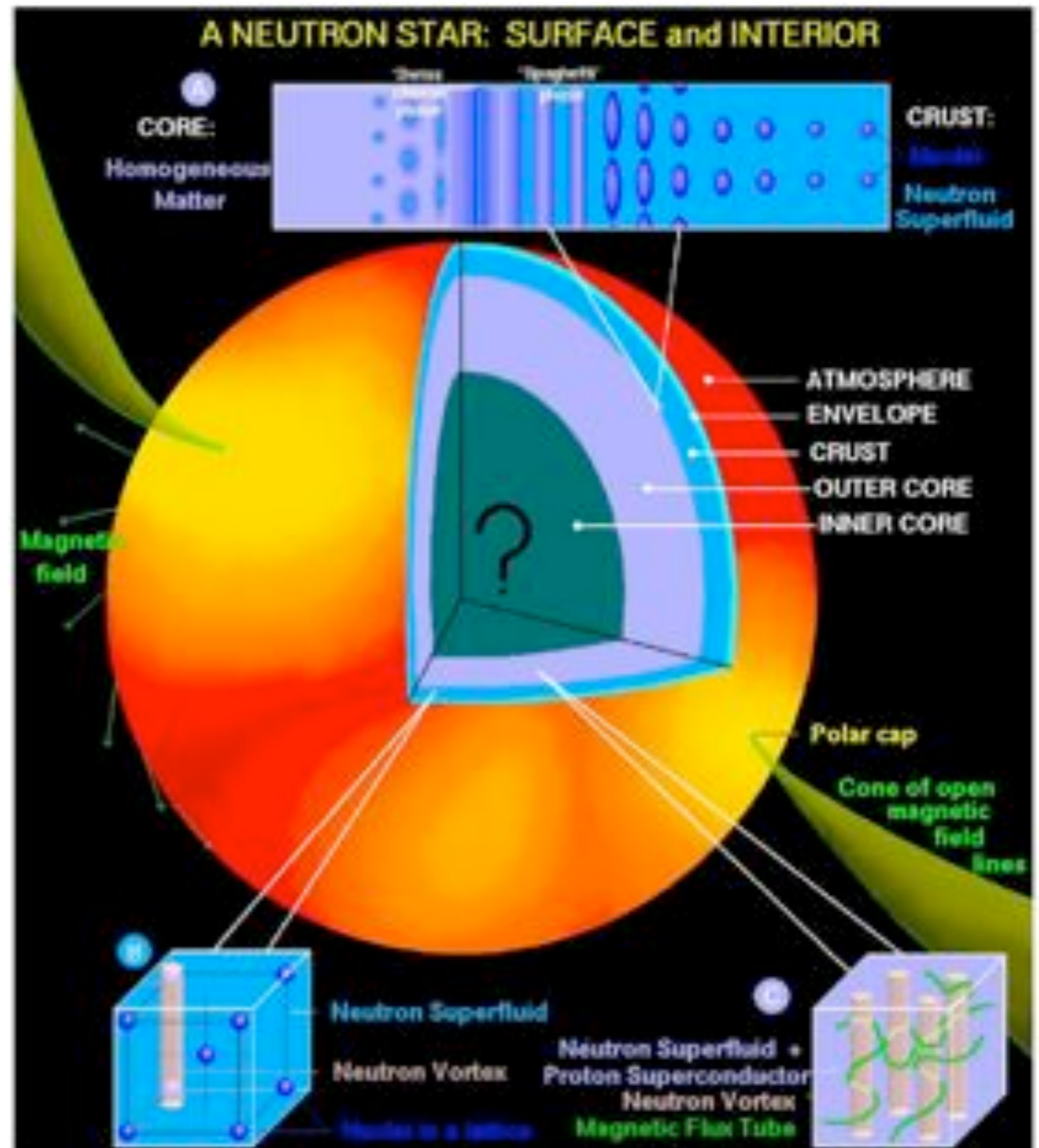
- $m_n = m_n c^2 = 939.57 \text{ MeV}$, $m_p = 938.27 \text{ MeV}$,
or $m_p = 1.67 \times 10^{-24} \text{ g}$.
- $e + p \rightarrow n + \nu$ so $k_F^2 + m_e^2 = (m_n - m_p)^2$.
- $k_F = 1.20 \text{ MeV} = 0.0061 \text{ fm}^{-1}$, $\rho_e = k_F^3 / 3\pi^2$
 $= 7.5 \times 10^{30} \text{ cm}^{-3}$, $\rho_e m_p = 1.3 \times 10^7 \text{ g/cm}^3$.

Units

$$\hbar = c = 1, \quad \hbar c = 197.33 \text{ MeV fm}$$

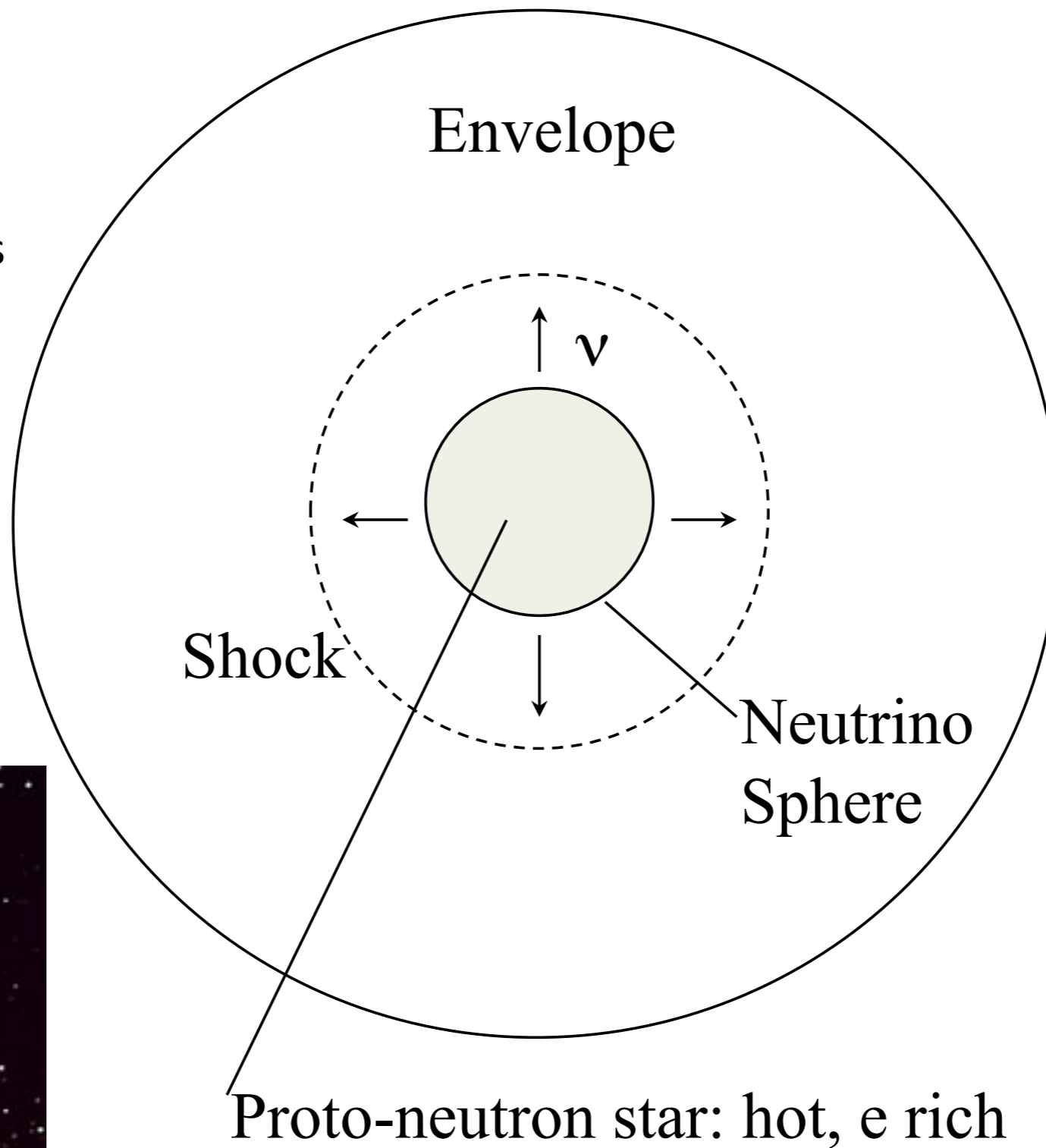
Neutron stars

- Neutron stars are formed from the collapse of a massive star in a supernova explosion.
- Mass $\sim 1.4 M_{\text{sun}}$, Radius ~ 10 km
- Solid crust ~ 1 km thick ($\sim 10^{12}$ g/cm³) over liquid mantle (outer core) of neutron rich matter ($\sim 10^{15}$ g/cm³).
- Possible exotic phase in center: de-confined quark matter, strange matter, meson condensates, color superconductor...
- Structure determined by Equation of State (pressure vs density) of n rich matter.
- Figure: **Dany Page**, UNAM



NS Born in Core Collapse Supernovae

Core of massive star collapses to form proto-neutron star. ν vs form neutron star energizes shock that ejects outer 90% of star.

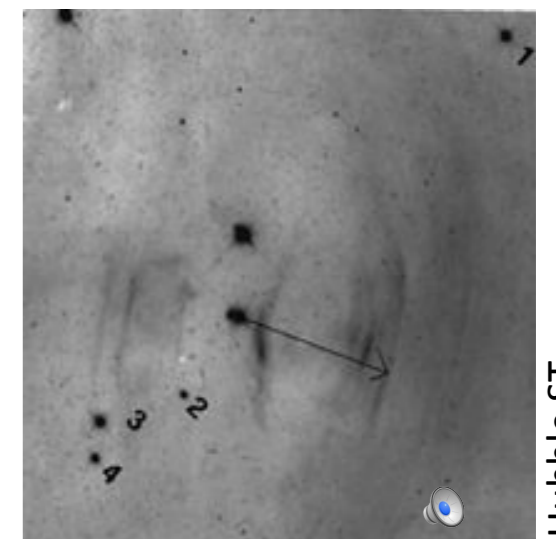


July 5, 1054

Crab nebula



Crab Pulsar



Proto-neutron star: hot, e rich

Why study neutron stars?

- Unique laboratory for extreme physics.
 - Strongest known magnetic fields (to 10^{15} G).
 - Strongest gravity (short of black hole).
 - Densest matter in the Universe.
 - Study all four forces: **E+M** (X-ray, radio telescopes...), **Weak** (SN neutrinos), **Strong** (lab. accelerators), and **Gravity** (gravitational waves).
- Only way to study cold dense matter and the phases of dense QCD. Can form warm/ hot dense matter in heavy ion collisions. However no way to get entropy out to look at cold dense matter in the laboratory.

Equation of state

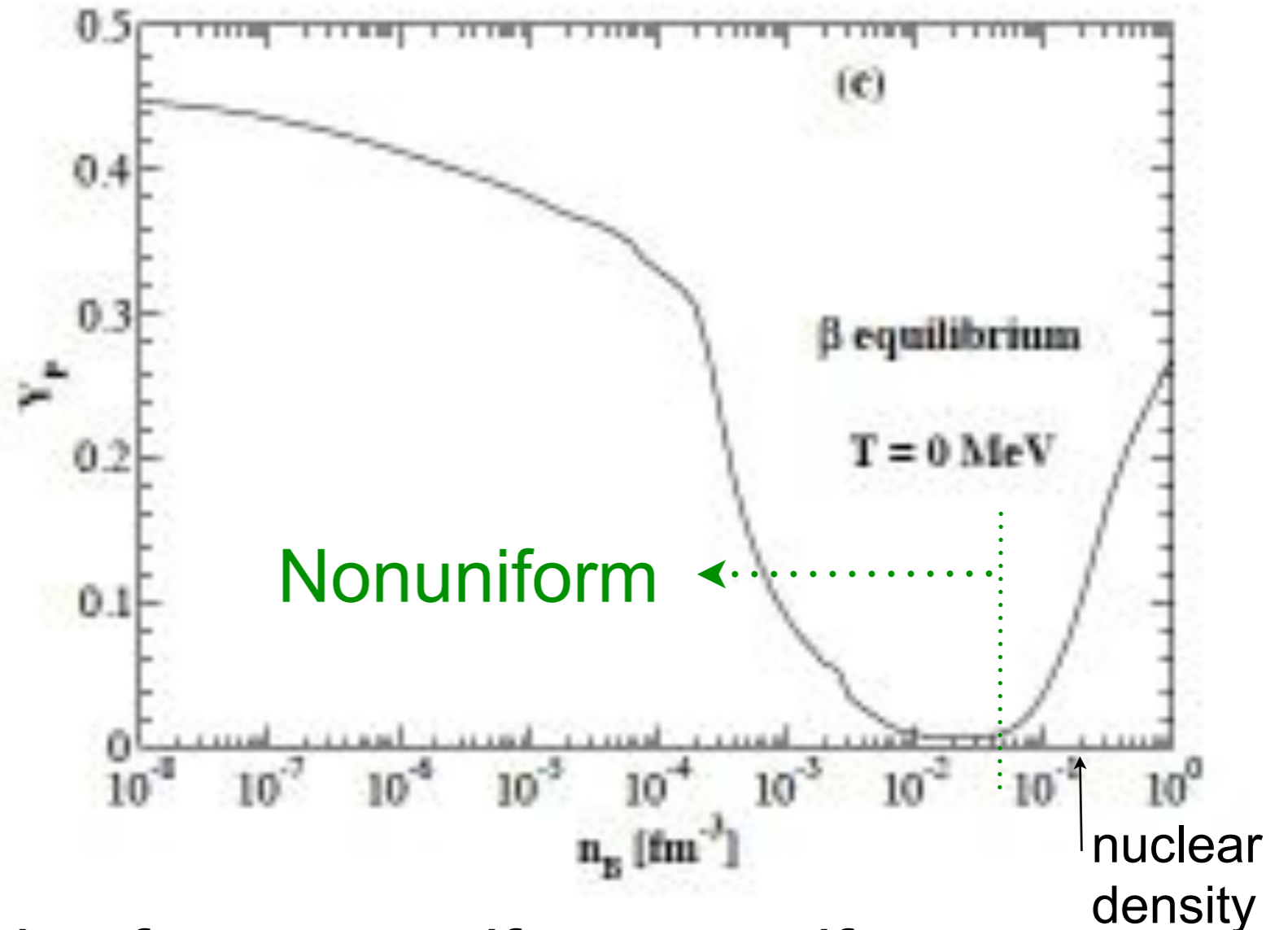
- Neutron star structure only depends on the equation of state: pressure as a function of energy density.
- $P = -dE/dV = -d(E/A)/d(V/A) = n^2 d(E/A)/dn$, V =volume.
- Here the **baryon density** $n=(A/V)$. [particles/fm³]
- The **energy density** is $\epsilon=E/V=(E/A)n$. [MeV/fm³]
- In general $E/A(n, Y_p)$ where the proton fraction Y_p is the number of protons per baryon $Y_p=Z/A$. This is determined by **beta equilibrium that gives** $Y_p=Y_p(n)$.
- Calculate both $P=P[E/A(n, Y_p(n))]$ and $\epsilon(n, Y_p(n))$ as a function of n giving $P(n)$, $\epsilon(n)$ and tabulate **$P=P(\epsilon)$ which is the equation of state.**
- Note that E includes rest mass mc^2 . So the **mass density** is $\rho=\epsilon/c^2$. [g/cm³] (in nonrel. limit $\rho=mn$)

Beta Equilibrium

- Consider $e + p \leftrightarrow n + \nu_e$
- Chemical potentials balance when in equilibrium
 $\mu_e + \mu_p = \mu_n + \mu_\nu$, with $\mu_\nu=0$. (nu escape)
- At zero T, $\mu_i = E_F^i$, charge neutrality $k_F^e = k_F^p$.
- **Exercise 2:** calculate Y_p as a function of baryon density n for uniform matter in beta equilibrium assuming free fermi gases of p, n and e. Note for free Fermi gas, $E_F^i = [(k_F^i)^2 + m_i^2]^{1/2}$.
- Hint: calculate k_F^n as a function of k_F^p and then calculate Y_p and n and make a table k_F^p, Y_p, n . 18

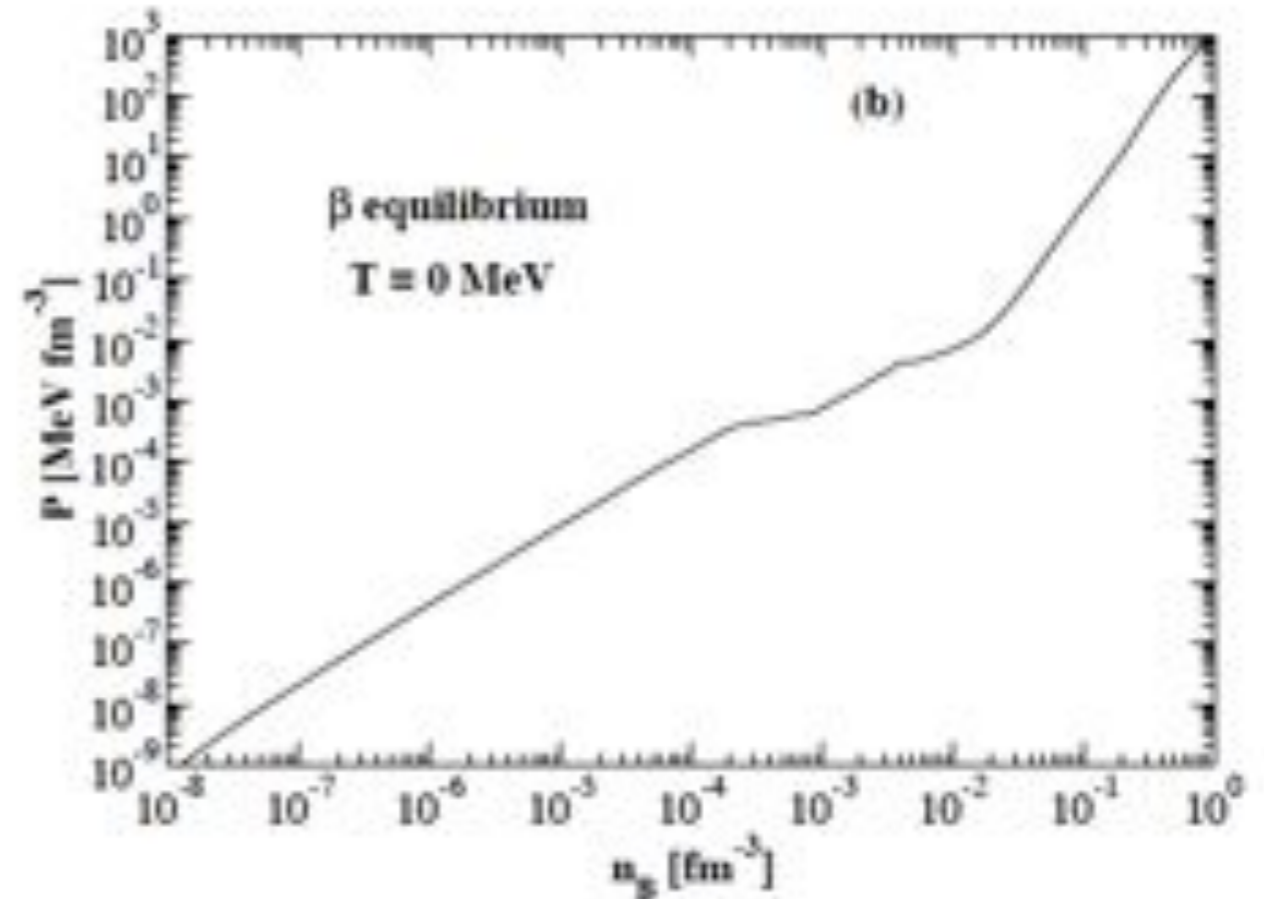
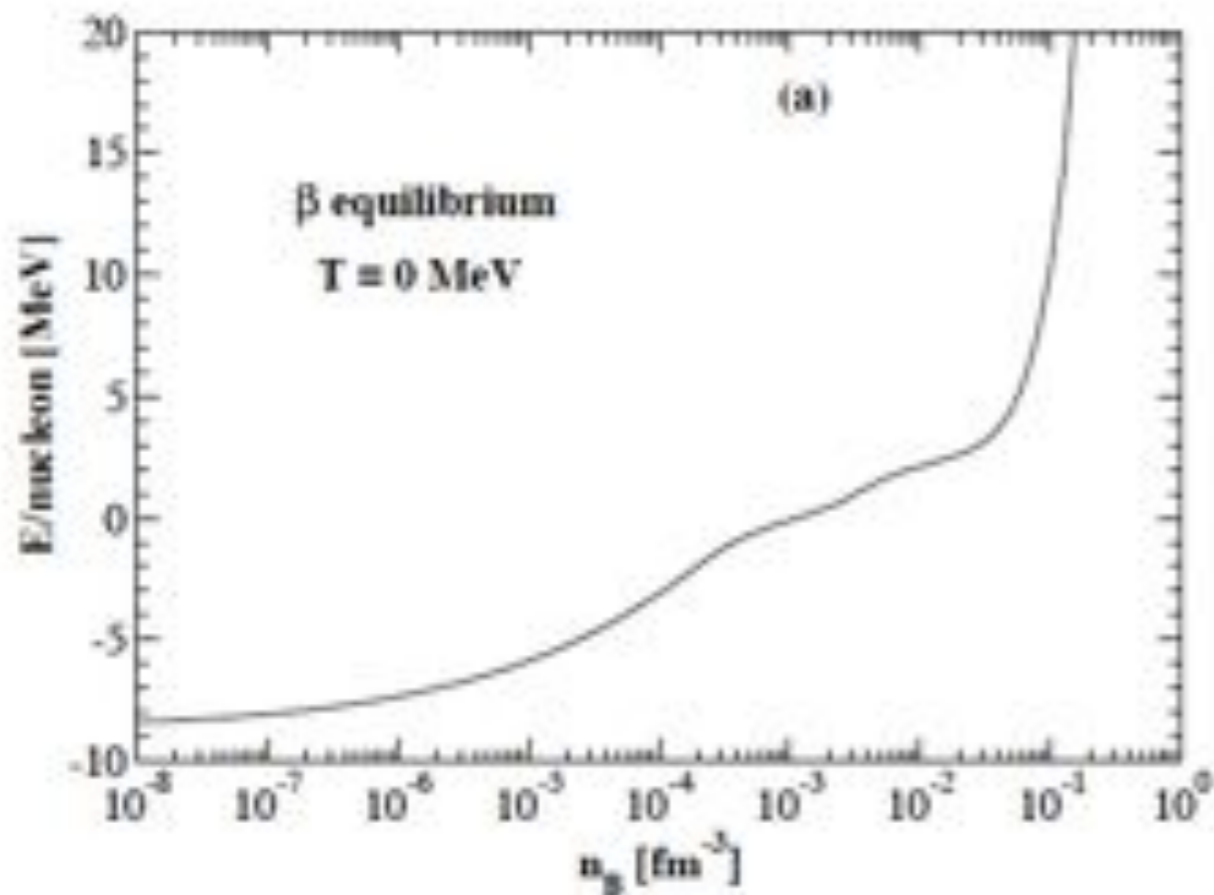
Beta Eq. for NL3 RMFT inter.

- At very low density $Y_p \sim 0.5$ for stable nuclei.
- This drops with increasing density as electron Fermi energy drives e capture.



- Near $n=0.05 \text{ fm}^{-3}$ transition from nonuniform to uniform matter (Y_p is minimum). Then Y_p slowly rises with increasing density as large neutron Fermi E favors n beta decay.
- Neutron stars are approx. 90% n , 10% p and e (unless core is exotic hyperions, quarks...)

NL3 EOS in Beta Eq.

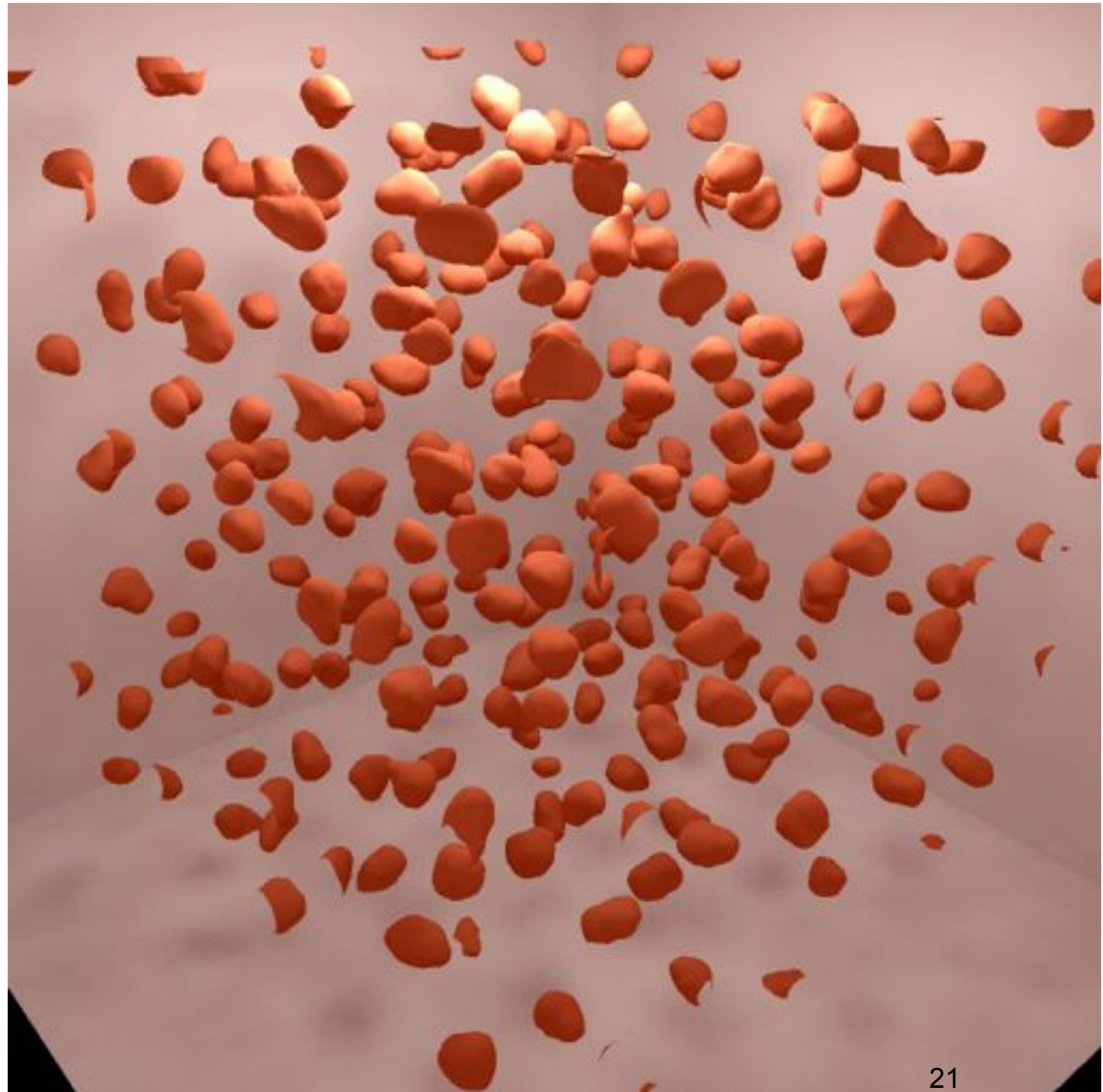


- Near nuclear density, E/A rises abruptly (large nuclear incompressibility).
- P at low density dominated by electrons, at high density, nuclear incompressibility.

$Y_p=0.2$
 $T=1$ MeV

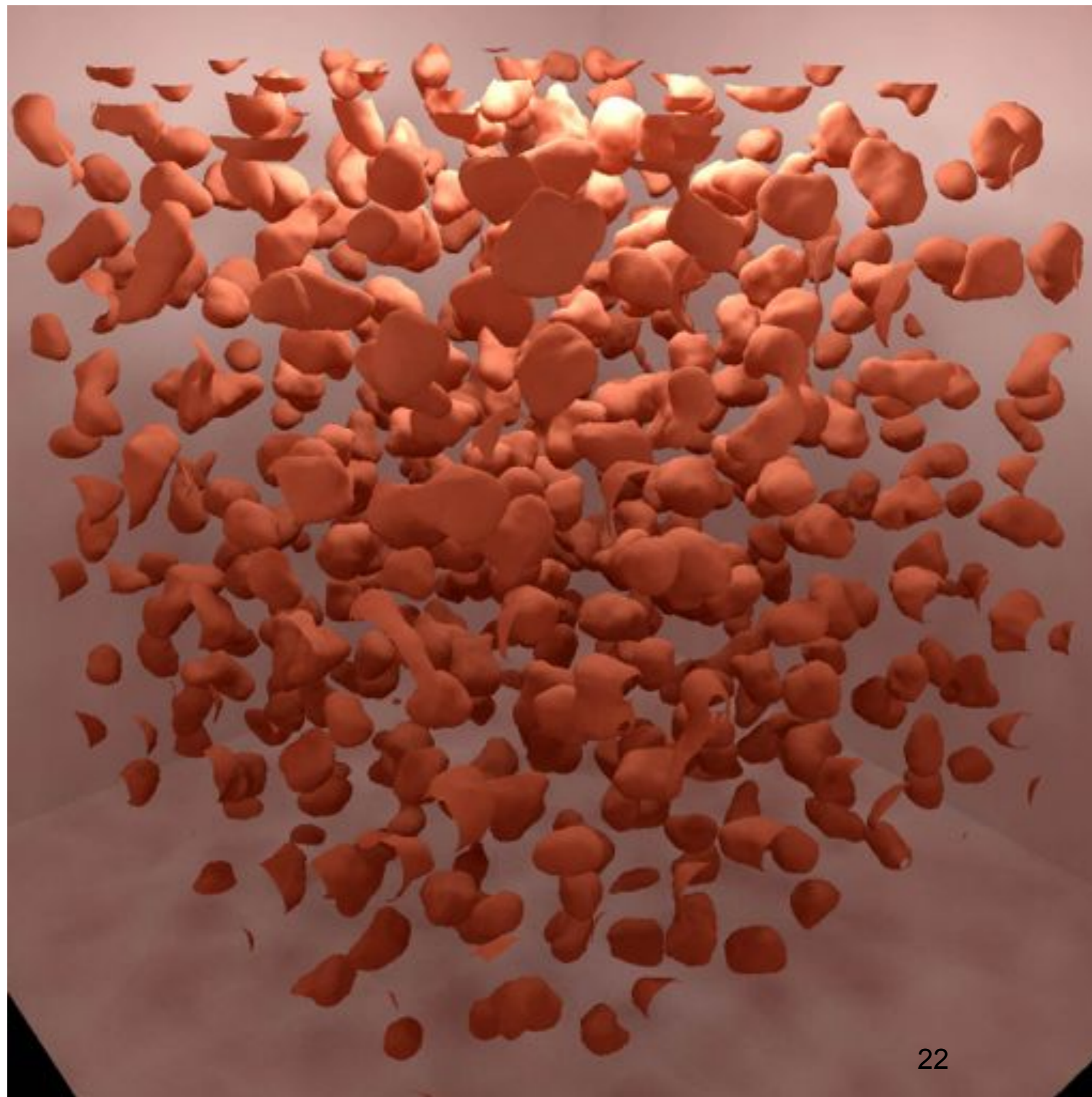
Semi-
classical MD
simulation
with 40,000
to 100,000
nucleons.

0.03 fm^{-3}
surface of the
proton density
at $\rho=0.01 \text{ fm}^{-3}$



$Y_p=0.2$
 $T=1$ MeV

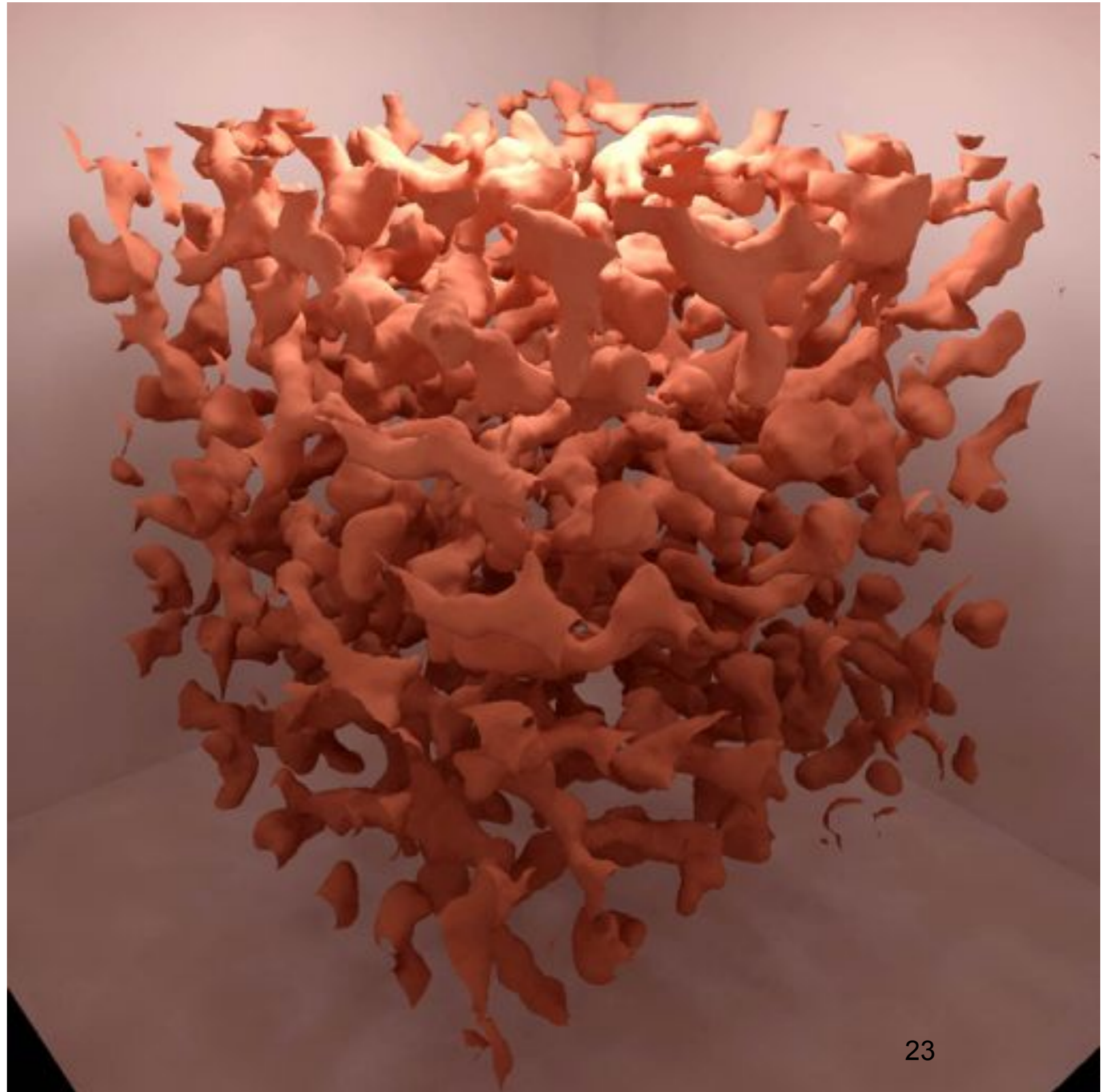
Proton density
at $\rho=0.025$ fm⁻³



$Y_p=0.2$
 $T=1 \text{ MeV}$

Complex
shapes
are
known as
nuclear
pasta

Proton density
at $\rho=0.05 \text{ fm}^{-3}$



NS hydrostatic equilibrium

- As you go deeper into a NS pressure p rises because of weight of material above you.

$$\frac{dp}{dr} = -\frac{G\rho(r)\mathcal{M}(r)}{r^2} = -\frac{G\epsilon(r)\mathcal{M}(r)}{c^2 r^2}$$
$$\frac{d\mathcal{M}}{dr} = 4\pi r^2 \rho(r) = \frac{4\pi r^2 \epsilon(r)}{c^2}$$
$$\mathcal{M}(r) = 4\pi \int_0^r r'^2 dr' \rho(r') = 4\pi \int_0^r r'^2 dr' \epsilon(r') / c^2$$

- Above assumes Newtonian gravity. See NS for undergraduates R. R. Silbar, S. Reddy, Am. J. Phys. **72** (2004) 892

GR hydrostatic equilibrium

$$\frac{dp}{dr} = -\frac{G\epsilon(r)\mathcal{M}(r)}{c^2r^2} \left[1 + \frac{p(r)}{\epsilon(r)}\right] \left[1 + \frac{4\pi r^3 p(r)}{\mathcal{M}(r)c^2}\right] \left[1 - \frac{2G\mathcal{M}(r)}{c^2r}\right]^{-1}$$

- First two terms in []s are special relativistic corrections of order v^2/c^2 . Note $p \sim$ kinetic $E = mv^2/2$ while $\epsilon \sim mc^2$.
- Last term is GR correction associated with curvature of space. It is of order GM_{sun}/r with $GM_{\text{sun}} \sim 1.5 \text{ km}$.
- All three terms act to increase p in star. Effects are nonlinear, eventually gravity wins and star will collapse to black hole. --> Maximum mass for NS that depend on EOS.

Causality and maximum mass

- Speed of sound $s = (dp/d\epsilon)^{1/2} \leq c$.
- Therefore causality demands any EOS satisfy $p < \epsilon$.
- As the mass of a NS increases, gravity makes p rise. However there is a limit to the p because EOS must be causal.
- Of course ϵ can also rise. However ϵ acts as source for gravity and this makes p need to rise even faster.
- Nonlinear GR gravity will always eventually win and star collapses into a black hole.
- For any EOS there is a maximum possible NS mass. Note this is not true in Newtonian gravity.

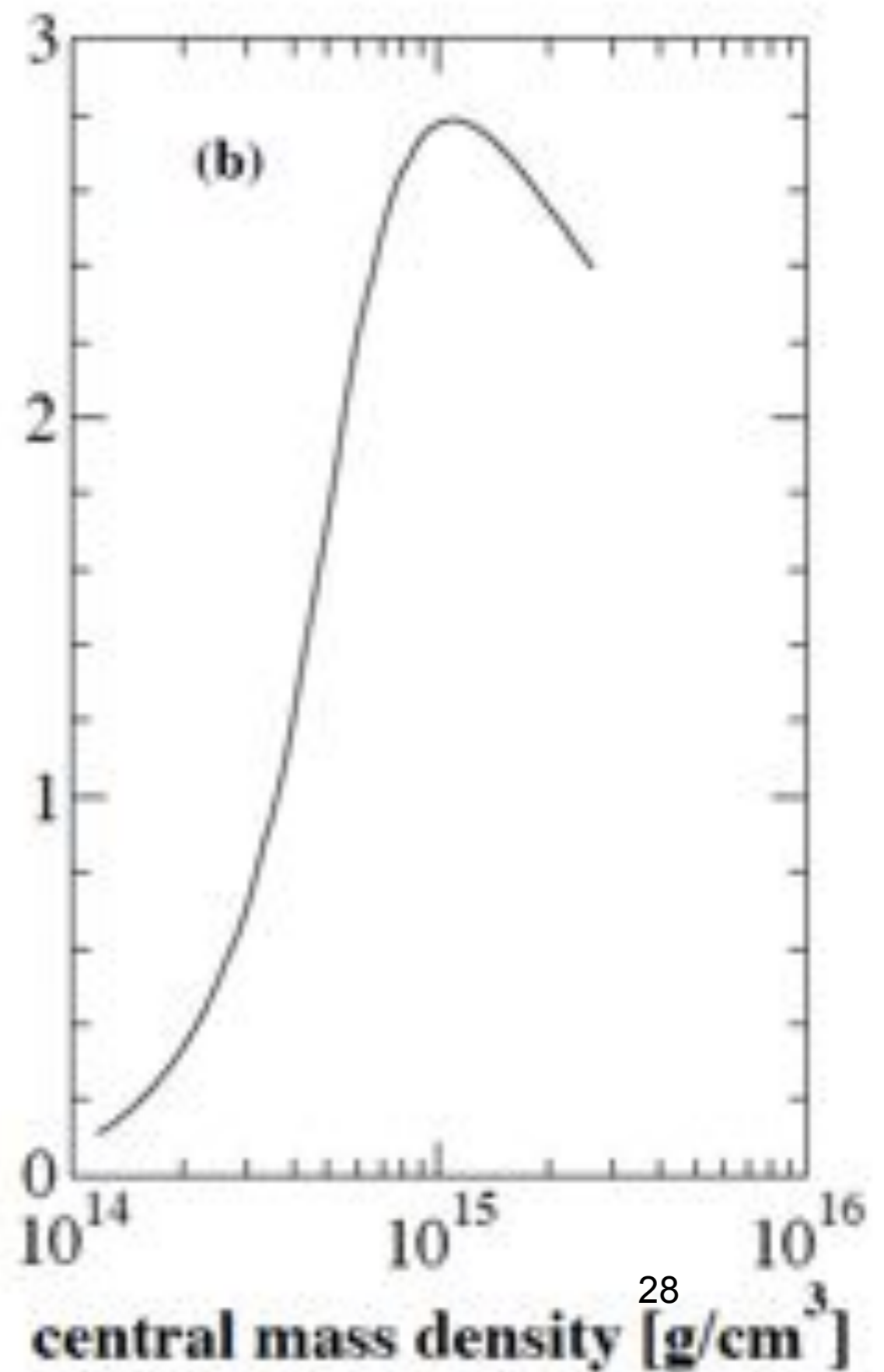
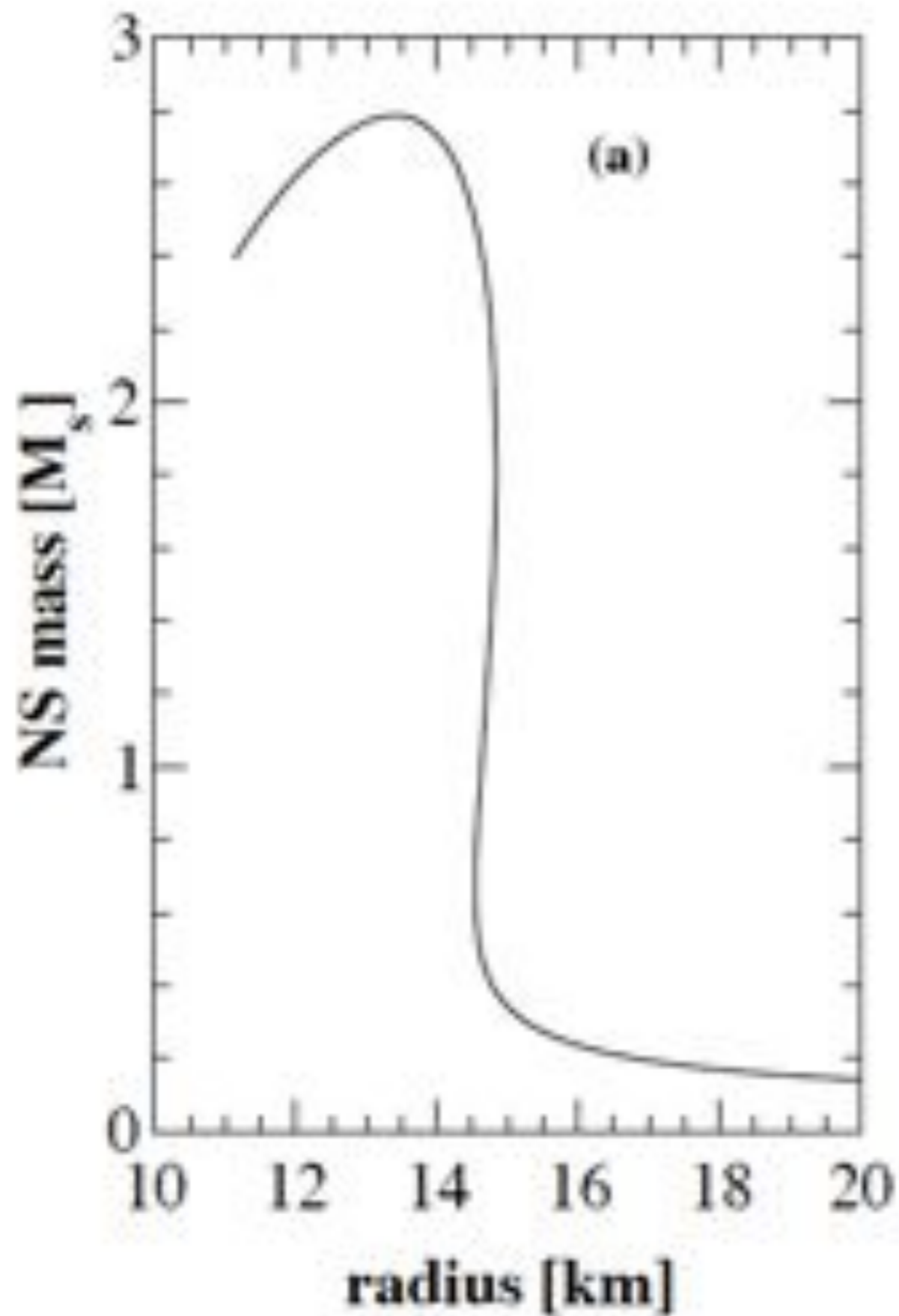
TOV Equations

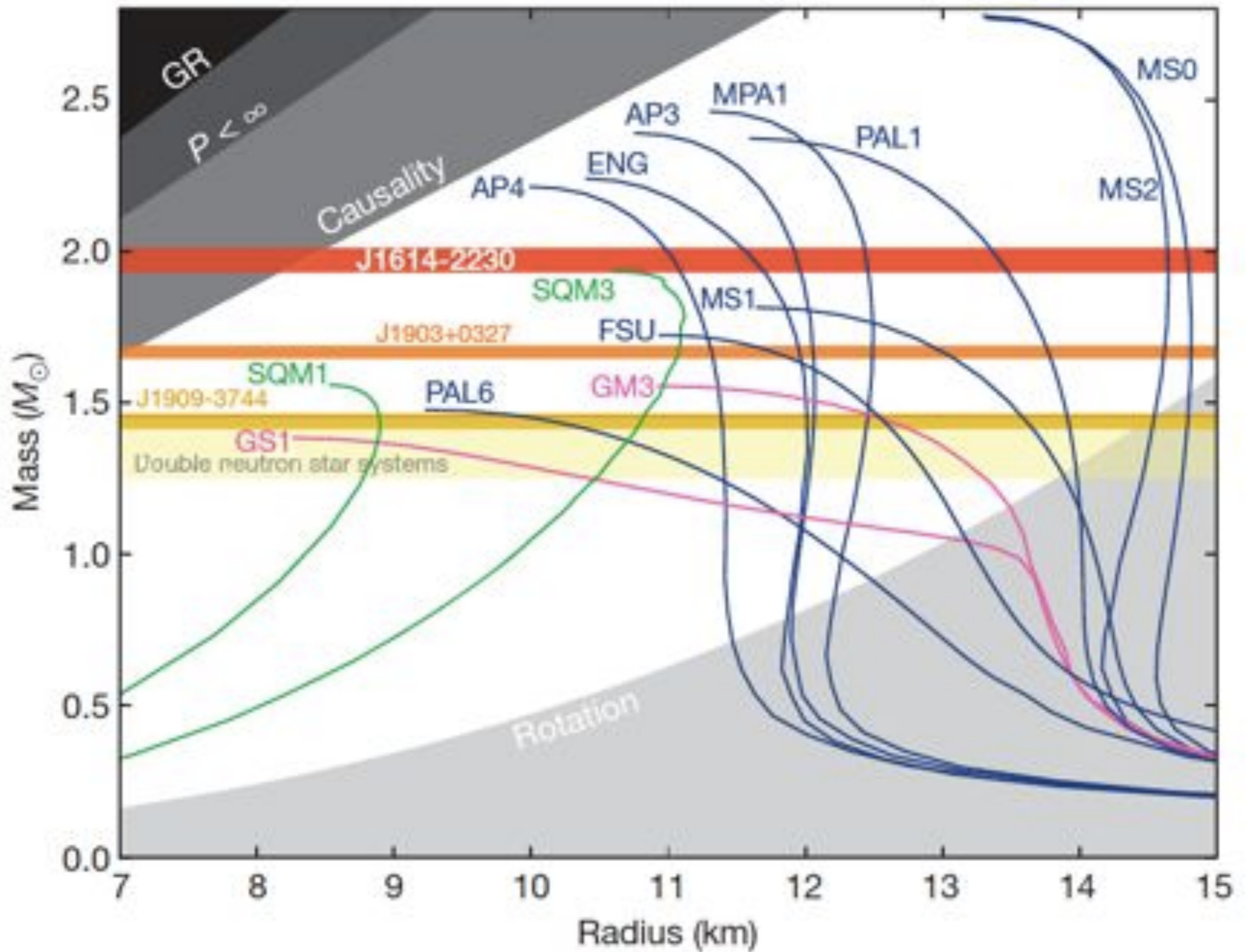
$$\frac{dp}{dr} = -\frac{G\epsilon(r)\mathcal{M}(r)}{c^2 r^2} \left[1 + \frac{p(r)}{\epsilon(r)} \right] \left[1 + \frac{4\pi r^3 p(r)}{\mathcal{M}(r)c^2} \right] \left[1 - \frac{2G\mathcal{M}(r)}{c^2 r} \right]^{-1}$$

$$\mathcal{M}(r) = 4\pi \int_0^r r'^2 dr' \rho(r') = 4\pi \int_0^r r'^2 dr' \epsilon(r') / c^2$$

- For given EOS $p=p(\epsilon)$ make family of NS models. Guess central p . EOS \rightarrow central density. Integrate p , and enclosed mass equations (TOV eqs 1939) outward from $r=0$ until $p=0$. This gives radius and mass of star. Repeat for different central p to determine mass of NS as a function of radius.
- A “stiff” EOS with a larger p will produce stars with larger radii and give a larger maximum mass.

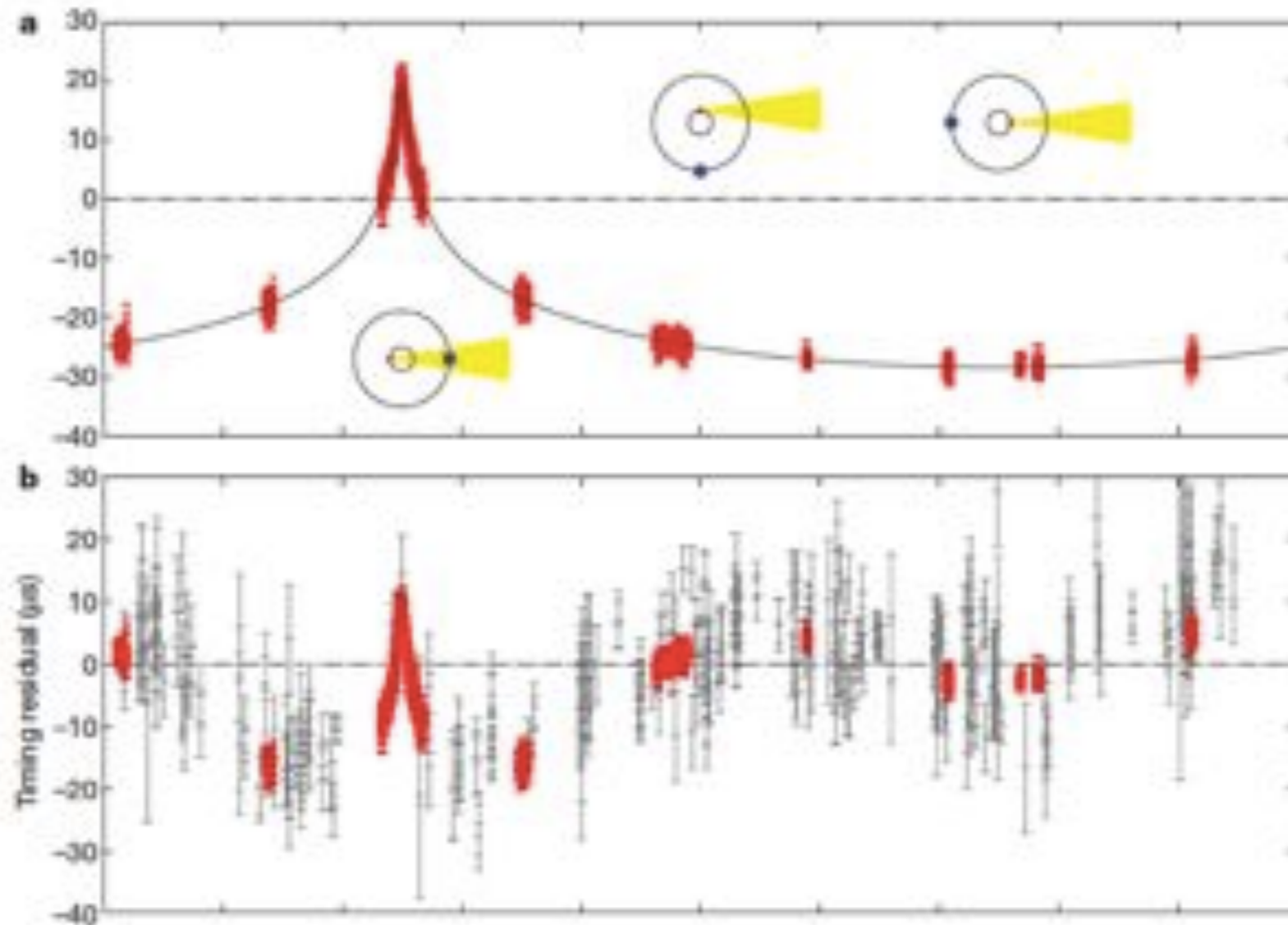
NL3 EOS





NS mass versus radius for different EOS. Demorest et al.

2 Solar mass NS measured using Shapiro delay.



Demorest et al. find PSR J1614-2230 has $1.97 \pm 0.04 M_{\text{sun}}$.

Kepler mass function:
$$f(m_p, m_c) = \frac{4\pi^2}{G} \frac{(a_p \sin i)^3}{P_b^2} = \frac{(m_c \sin i)^3}{(m_p + m_c)^2},$$

Implications of $2M_{\text{sun}}$ NS

- The observed mass of J1614-2230 is a *LOWER LIMIT* on the maximum NS mass that is supported by the true EOS.
- The EOS at high densities must be stiff enough (have a high enough p) to support at least this mass against collapse to a black hole. *All soft EOS are immediately ruled out.*
- If true maximum mass is exactly $2M_{\text{sun}}$ --> unlikely to find NS exactly at maximum mass. Strongly suggest true maximum mass somewhat greater than 2 say $2.2M_{\text{sun}}$.