The 2010 NNPSS-TSI
A. Richter 2nd Lecture

Giant Resonances – Wavelets, Scales and Level Densities

- Giant resonances
- Damping mechanisms, time and energy scales
- Fine structure
- Wavelets and characteristic scales
- Application: GQR
- Many-body nuclear models and damping mechanisms
- Relevance of scales: GTR
- Level densities of $J^\pi = 1^+, 2^+, 2^-$ states

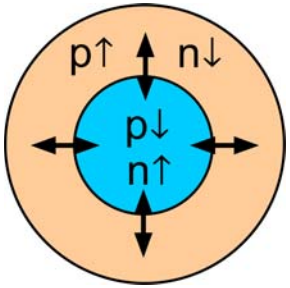
Supported by DFG under SFB 634, 446 JAP 113 / 267 / 0-1 und 445 SUA-113 / 6 / 0-1

Key References for 2nd Lecture

- A. Shevchenko et al., Phys. Rev. Lett. **93**, 122501 (2004)
- Y. Kalmykov et al., Phys. Rev. Lett. **96**, 012502 (2006)
- Y. Kalmykov, C. Özen, K. Langanke, G. Martínez-Pinedo, P. von Neumann-Cosel and A. Richter, Phys. Rev. Lett. **99**, 202502 (2007)
- A. Shevchenko et al., Phys. Rev. C **77**, 024302 (2008)
- A. Shevchenko et al., Phys. Rev. C **79**, 044305 (2009)
- I. Petermann, K. Langanke, G. Martinez-Pinedo, P. von Neumann-Cosel, F. Nowacki and A. Richter, Phys. Rev. C **81**, 014308 (2010)

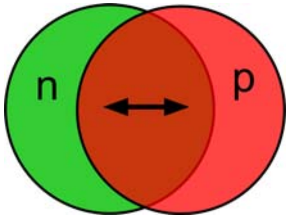
Giant Resonances

Gamow-Teller

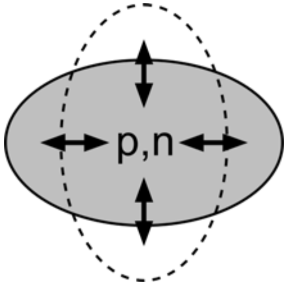


Monopole
$\Delta L = 0$

Dipole
$\Delta L = 1$



Quadrupole
$\Delta L = 2$



$\Delta T = 0$	$\Delta T = 1$	$\Delta T = 0$	$\Delta T = 1$
$\Delta S = 0$	$\Delta S = 0$	$\Delta S = 1$	$\Delta S = 1$

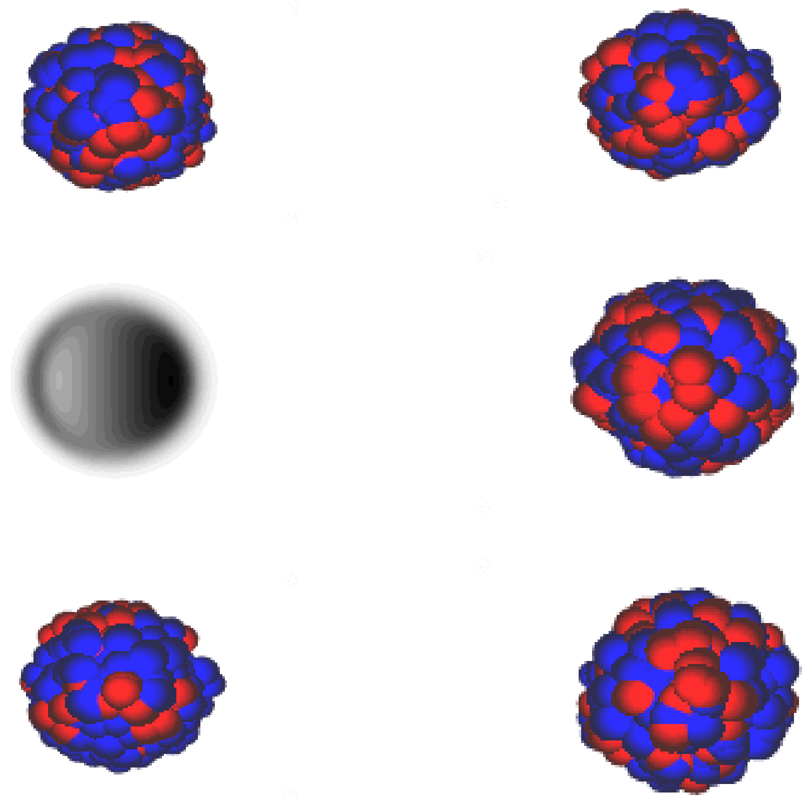
Giant Resonances

Isoscalar | Isovector

Monopole
$\Delta L = 0$

Dipole
$\Delta L = 1$

Quadrupole
$\Delta L = 2$



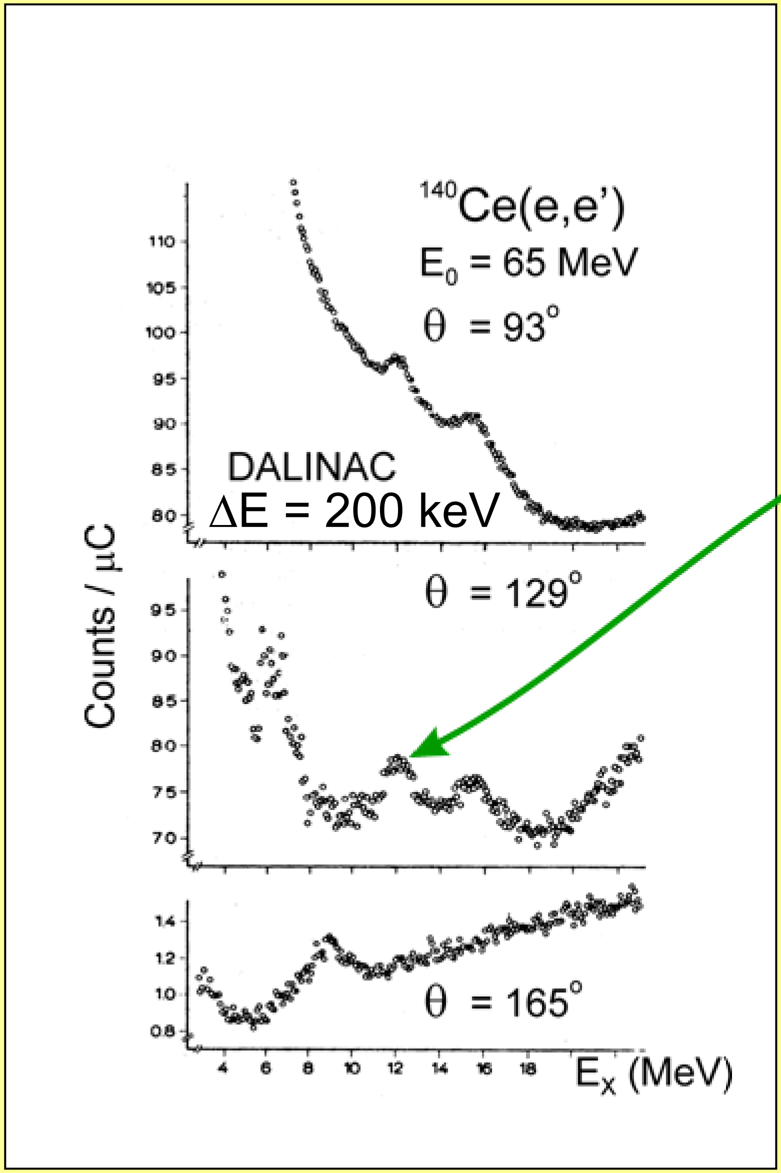
$\Delta T = 0$
$\Delta S = 0$

$\Delta T = 1$
$\Delta S = 0$

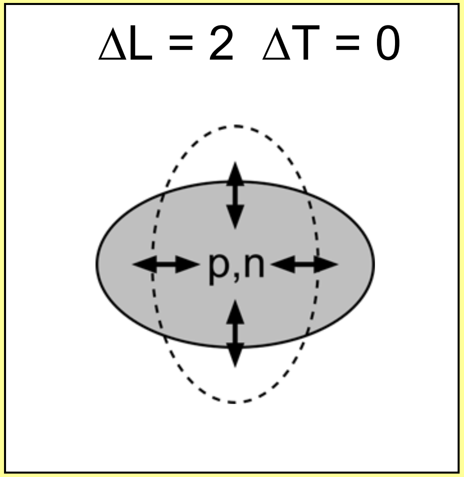
→ interplay between collective and s.p. motion

Courtesy of P. Adrich

Isoscalar Quadrupole Mode

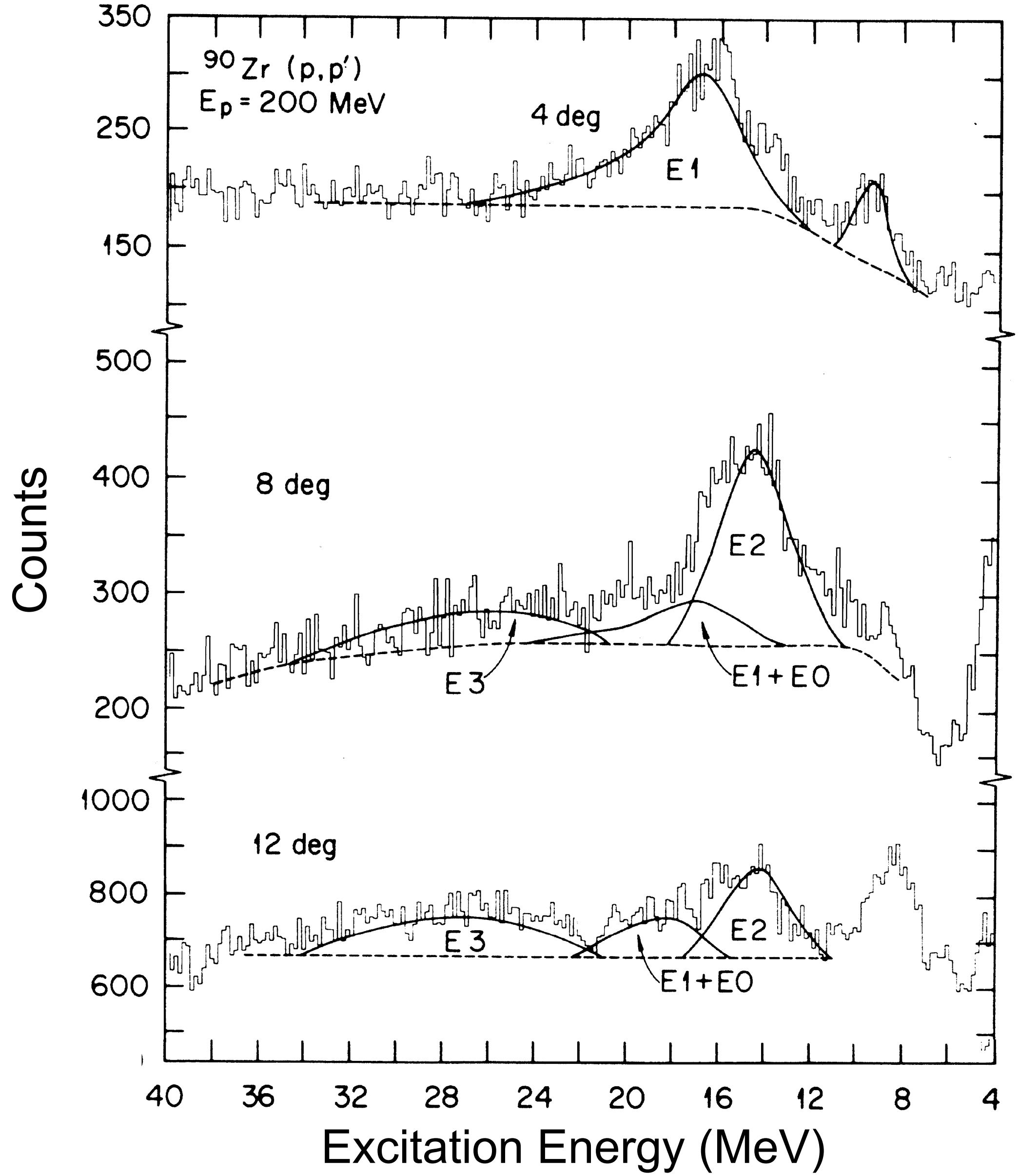


- Pitthan and Walcher (Darmstadt, 1972)



- Centroid energy: $E_X \sim 63\ A^{-1/3}$ MeV
- Width
- Damping mechanisms

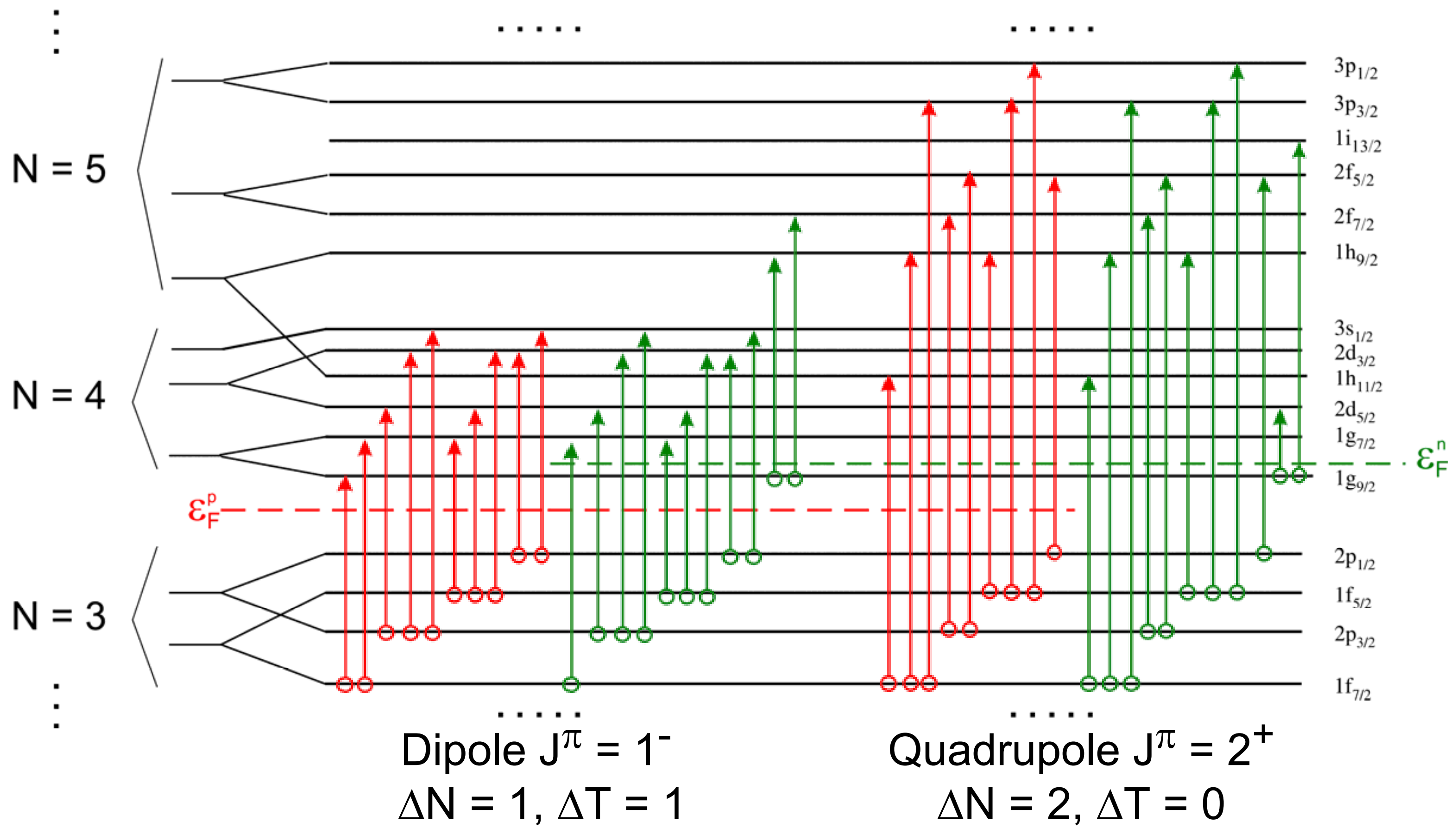
Early Work on GR's: ^{90}Zr



Bertrand et al. (1981)

ΔE (FWHM) ~ 1 MeV

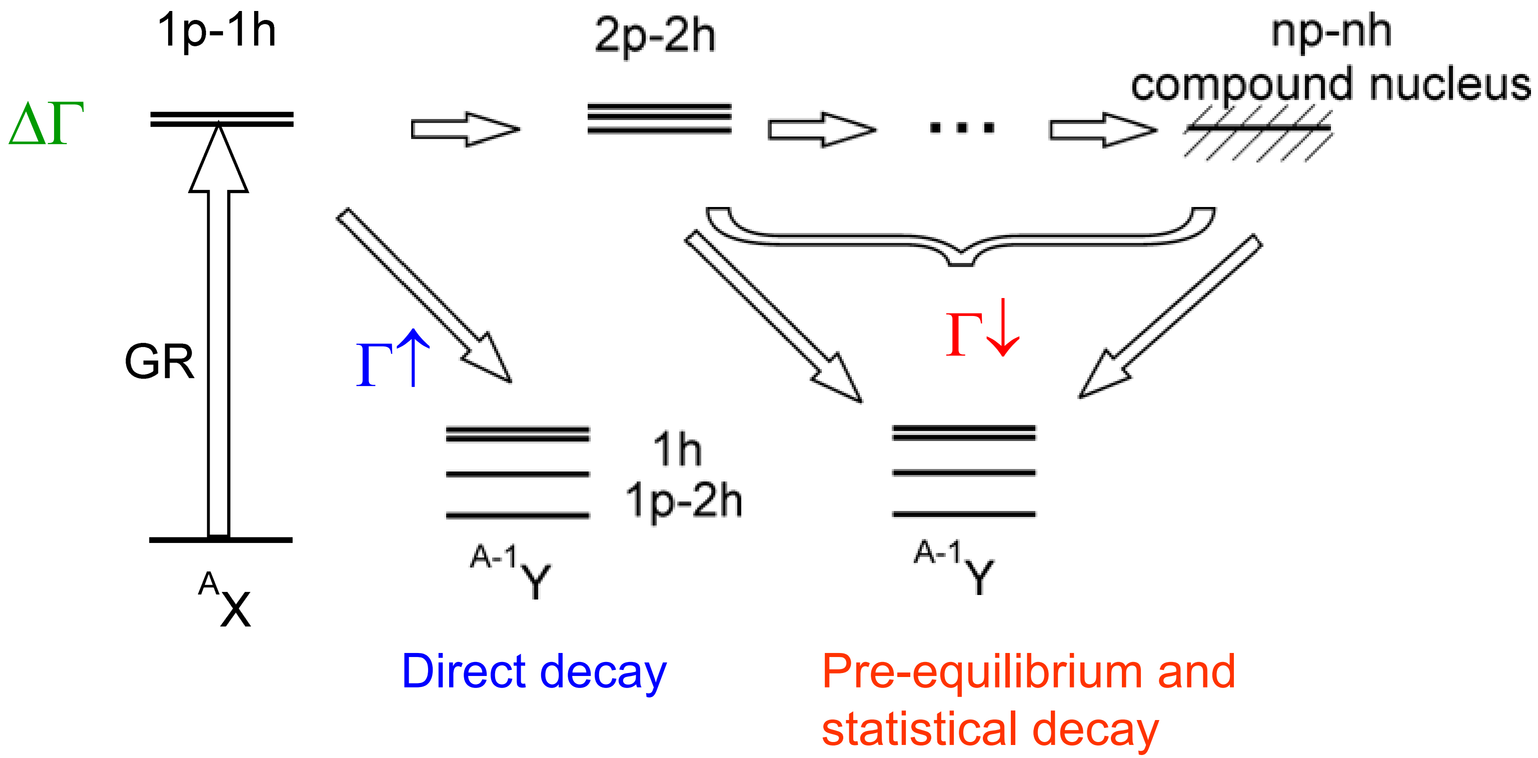
Microscopic Picture of Giant Resonances: ^{90}Zr



Giant vibration ≙ coherent superposition of elementary p-h excitations

$$\Psi^{J\pi} = \sum_\alpha c_\alpha | (p-h)^{J\pi}_\alpha \rangle \qquad H(1,..A) = \underbrace{H_0(1,..A)}_{IPM} + \underbrace{\sum_{i,k}^{A} V(i,k)}_{?}$$

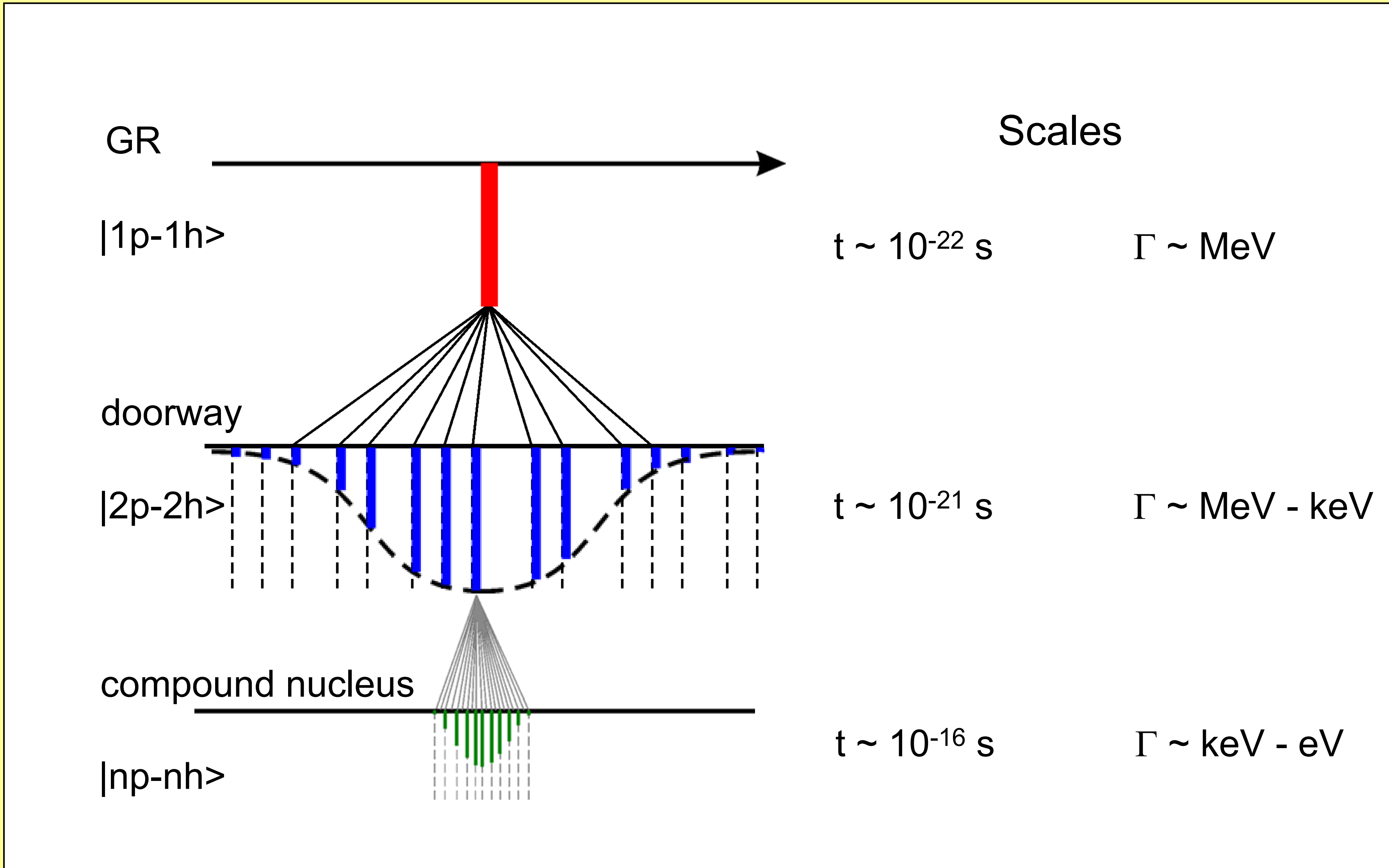
Excitation and Decay of Giant Resonances



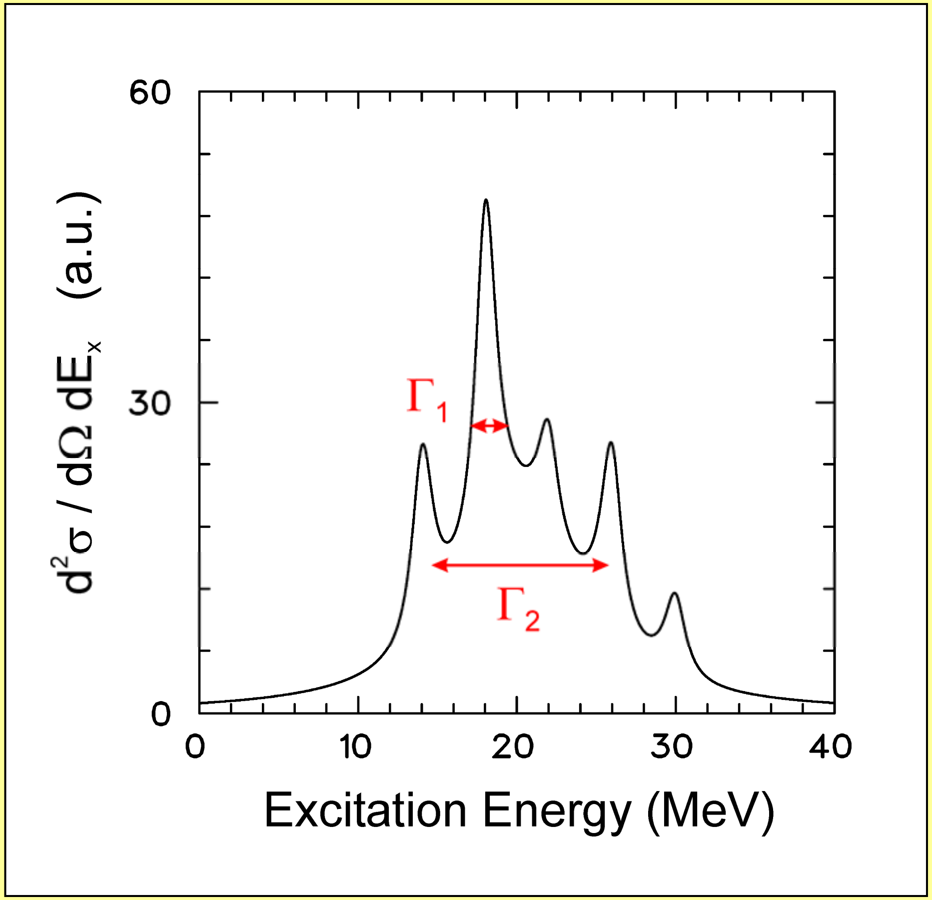
$$\Gamma = \Delta\Gamma + \Gamma\uparrow + \Gamma\downarrow$$

Resonance width	Landau damping	Escape width	Spreading width

Doorway State Model

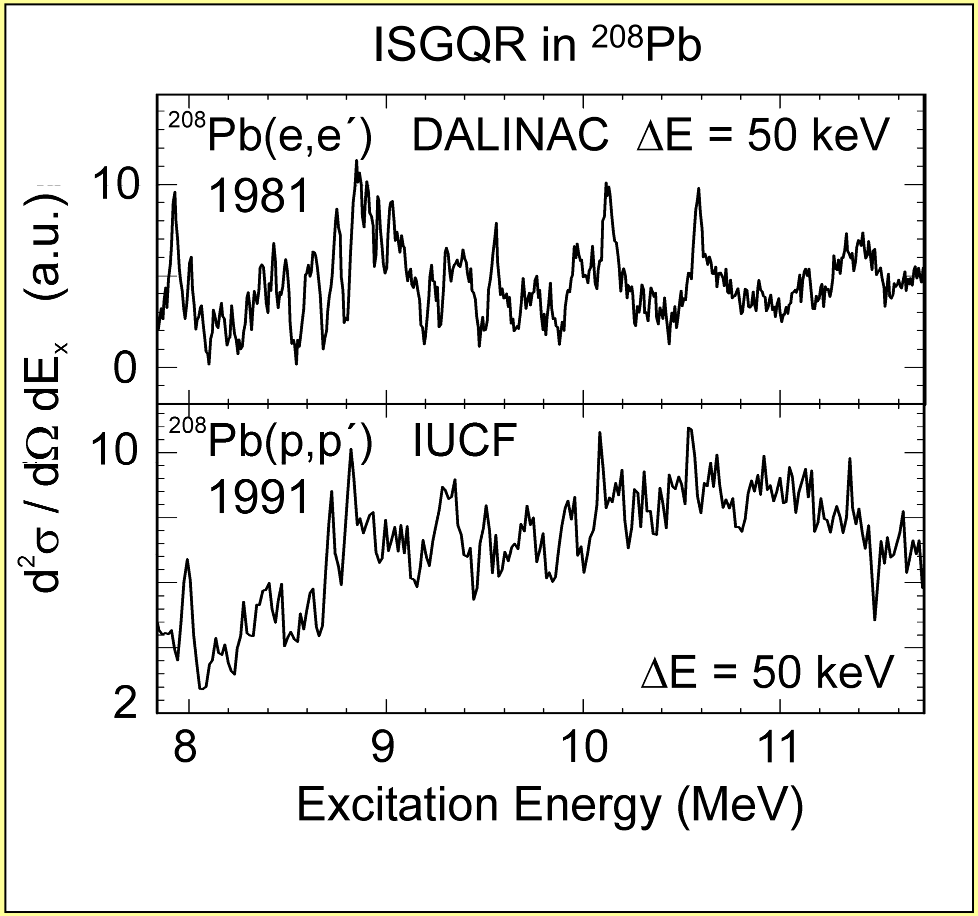


Fine Structure of Giant Resonances



- High resolution is crucial
- Possible probes: electrons and hadrons

Fine Structure of Giant Resonances



- Different probes but similar structures

→ physical information content is the same

Fine Structure of Giant Resonances

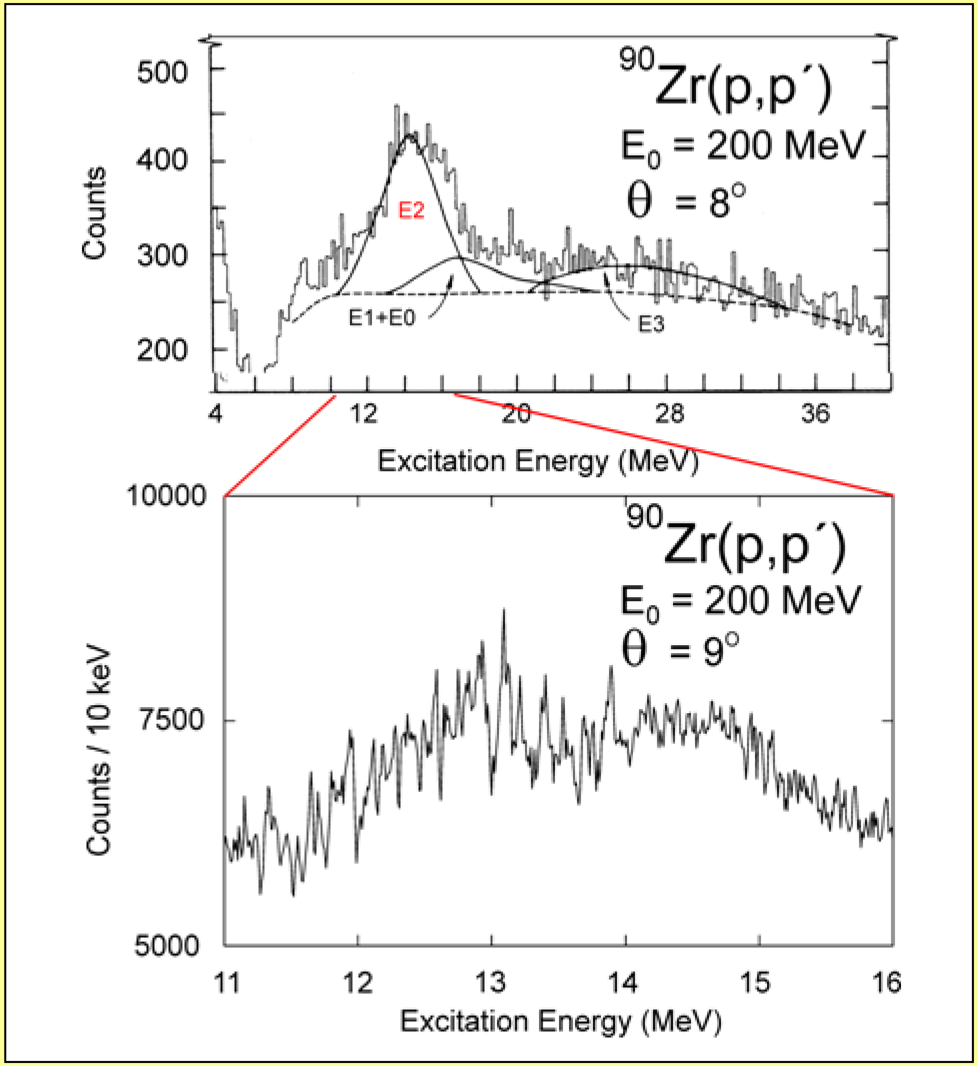
- Global phenomenon?
  - Other nuclei
  - Other resonances
- Methods for characterization of fine structure
- Goal: dominant damping mechanisms

Recent Experiments

Place:	iThemba LABS, South Africa	RCNP, Osaka
Reaction:	ISGQR from (p,p´)	GT from (^{3}He,t)
Nuclei:	^{58}Ni (28, 30), ^{89}Y (39, 50), ^{90}Zr (40, 50), ^{120}Sn (50, 70), ^{142}Nd (60, 82), ^{166}Er (68, 98), ^{208}Pb (82, 126)	^{90}Zr (40, 50), ^{168}Er (68, 100)
Beam energy:	200 MeV	140 MeV/u
Scattering angles:	8° – 10° (ΔL = 2)	0° (ΔL = 0)
Energy resolution: (FWHM)	ΔE = 35 - 50 keV	ΔE = 50 keV

New data: ^{27}Al, ^{28}Si, ^{40}Ca presently analysed

Fine Structure of the ISGQR

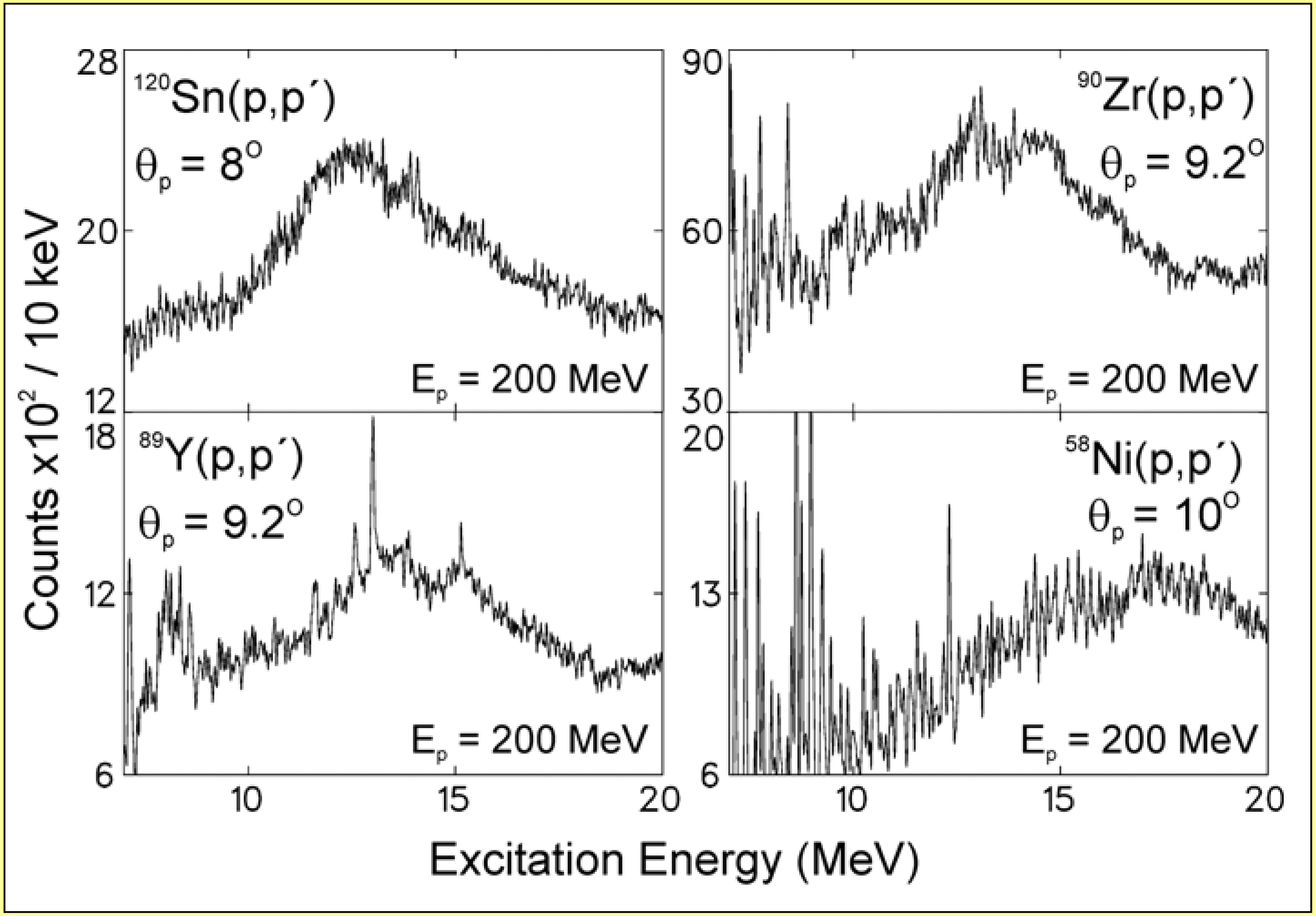


ΔE ≈ 1 MeV
TRIUMF (1981)

ΔE ≈ 40 keV
iThemba (2004)

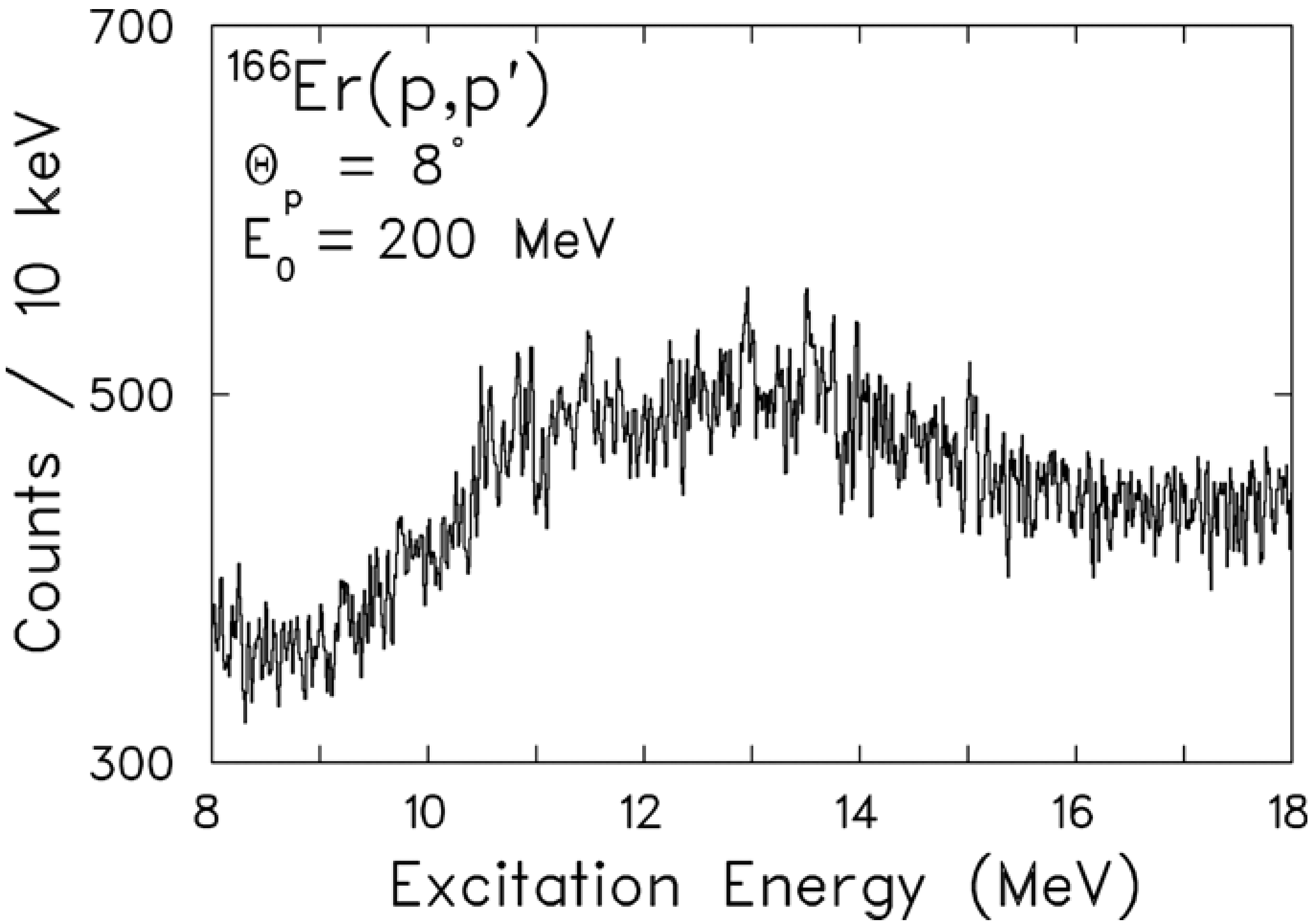
- Fluctuations of different strengths and scales
- Not a Lorentzian

Fine Structure of the ISGQR in Other Nuclei



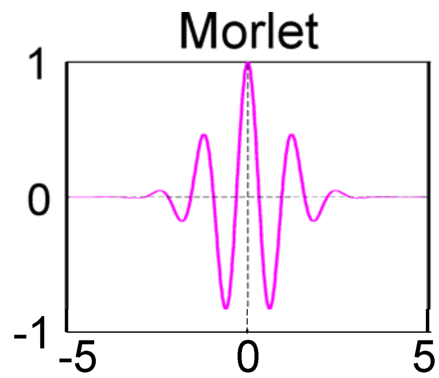
- Fine structure of the ISGQR is a global phenomenon

Fine Structure in Deformed Nuclei?



Wavelet Analysis

$$\int_{-\infty}^{\infty} \Psi^*(x)\, dx = 0 \qquad \text{and} \qquad \int_{-\infty}^{\infty} \left|\Psi^*(x)\right|^2 dx < \infty$$



$$\Psi(x) = \cos(2\pi\omega x)\, e^{-x^2/2}$$

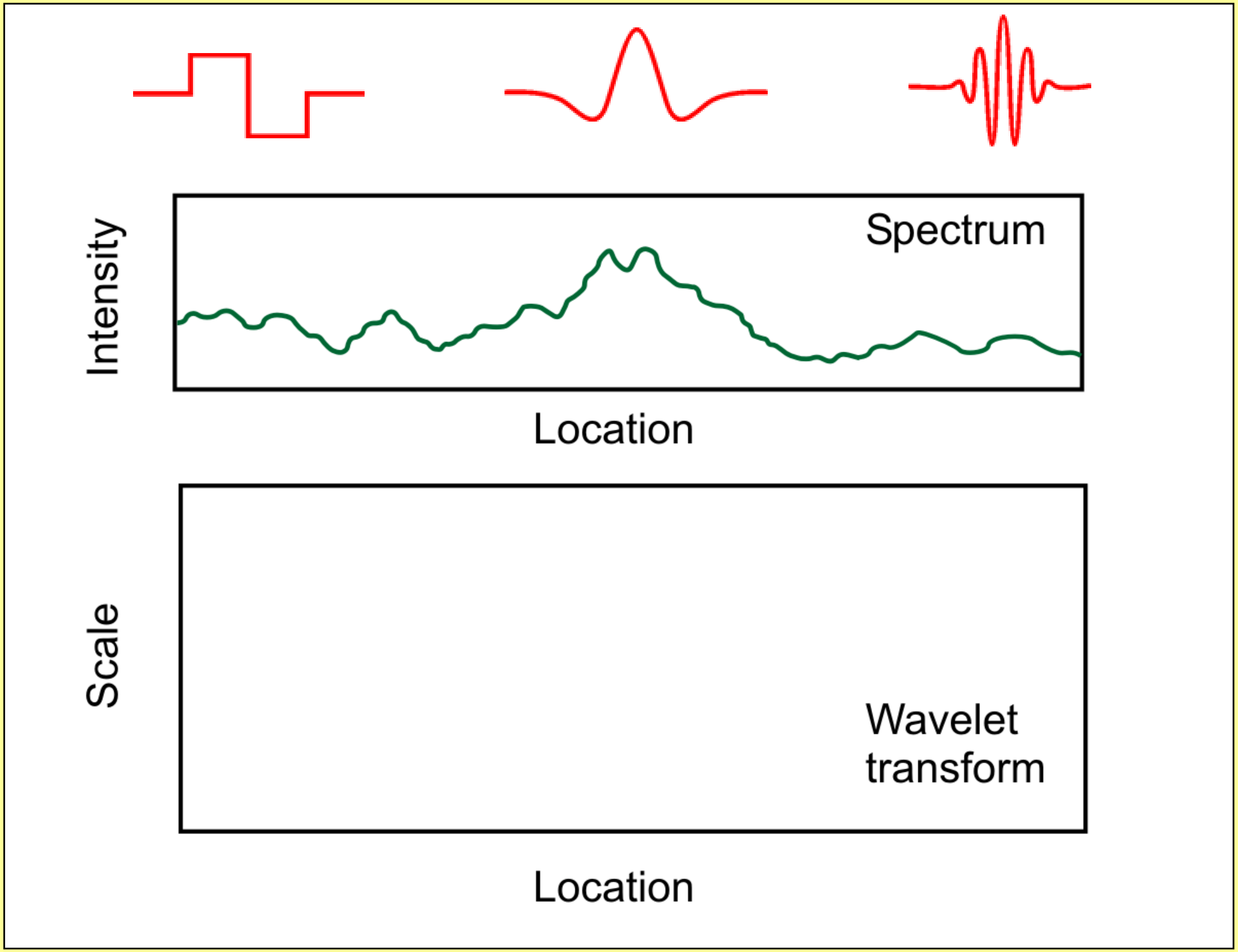
Wavelet coefficients:

$$C(\delta E, E_x) = \frac{1}{\sqrt{\delta E}} \int \sigma(E)\, \Psi^*\left(\frac{E_x - E}{\delta E}\right) dE$$

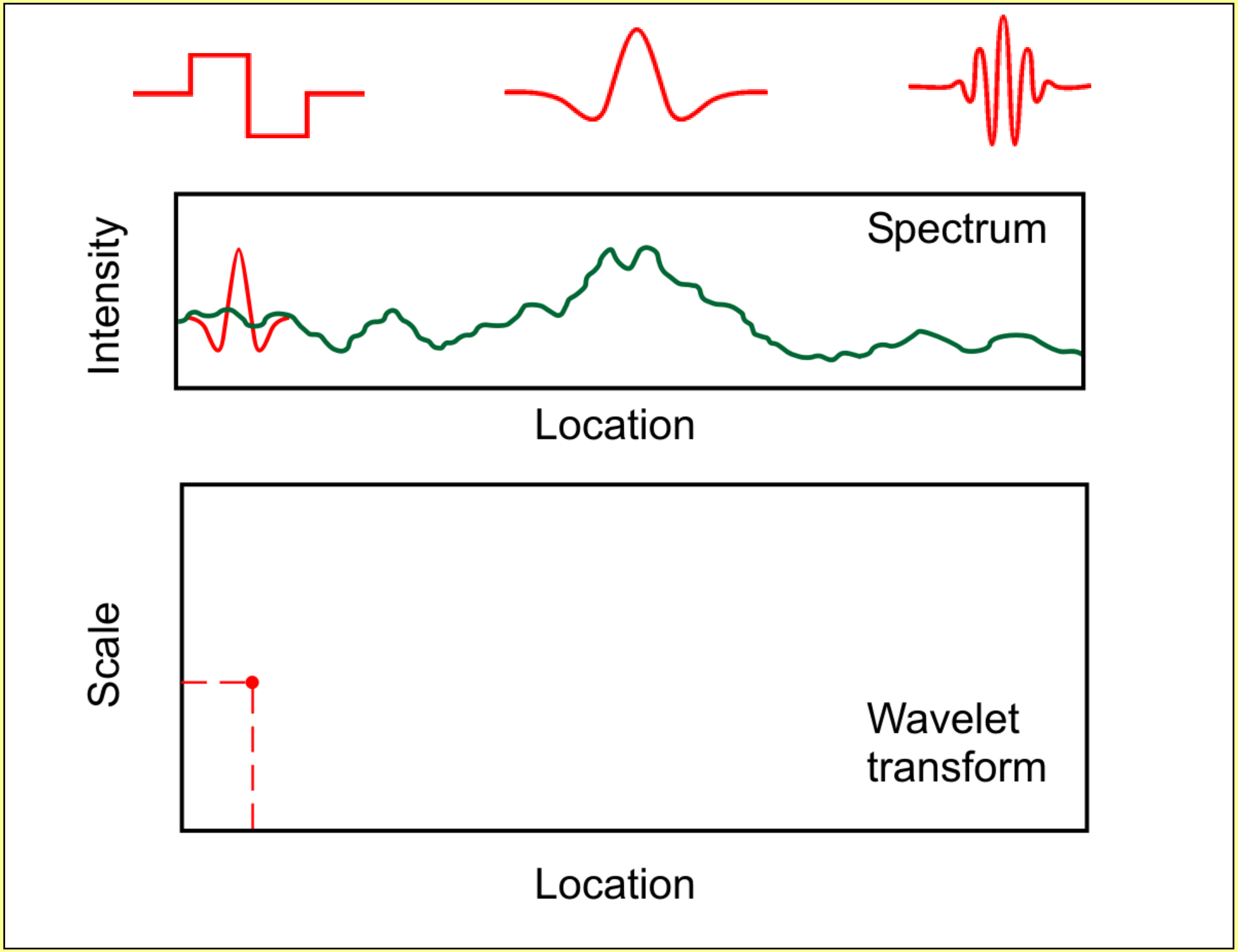
scale — position — spectrum — wavelet

Continuous: δE, E_x are varied continuously

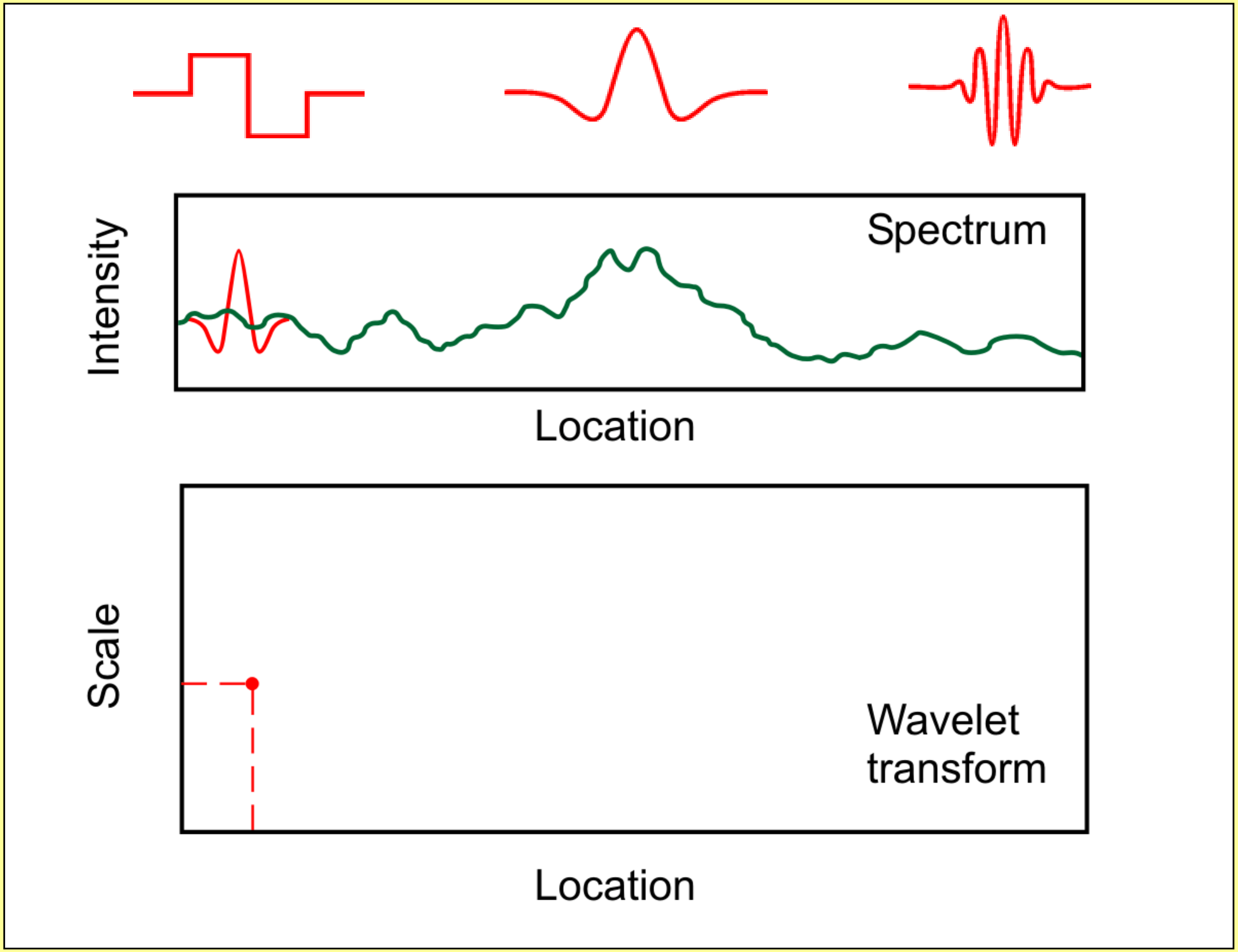
Wavelet Analysis



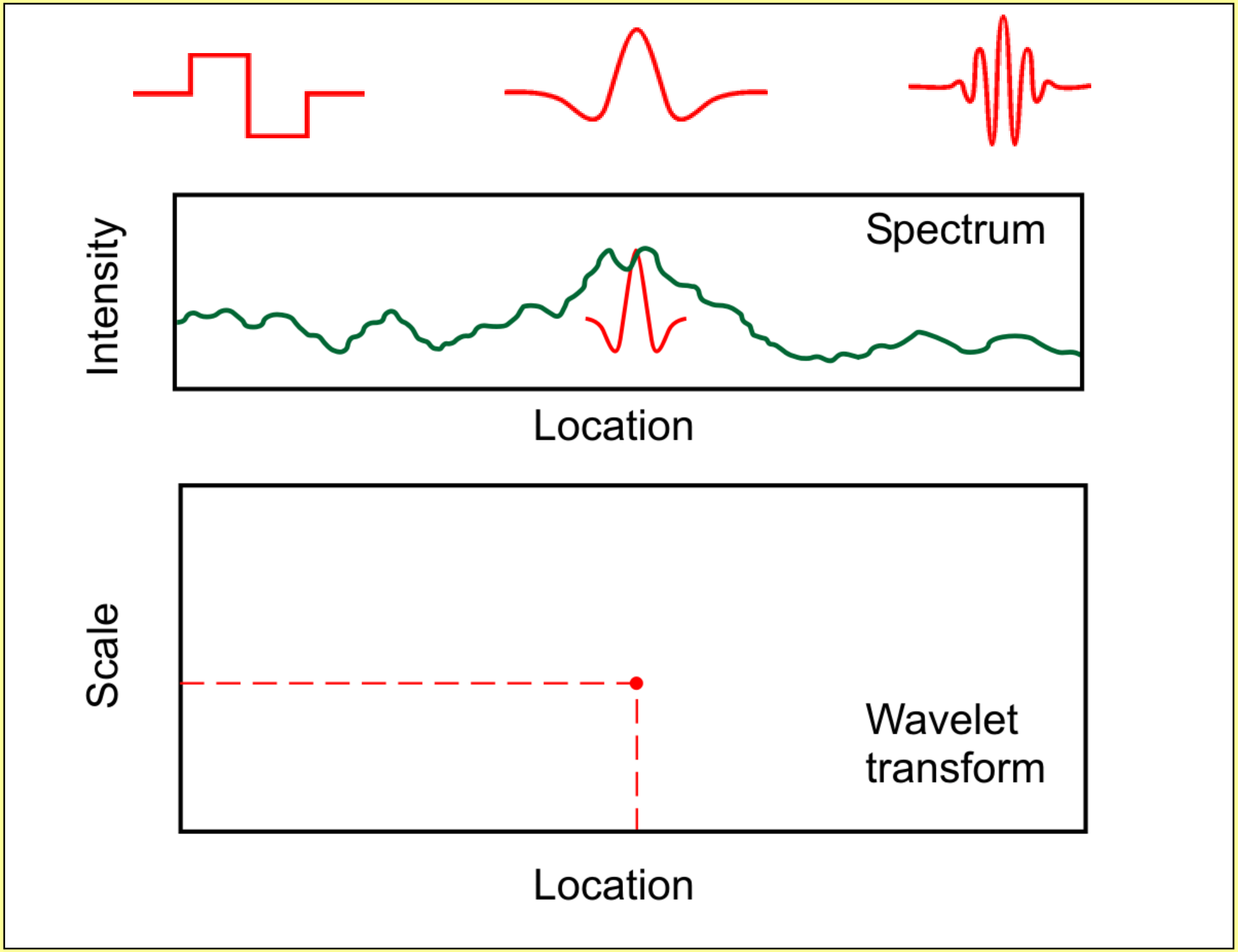
Wavelet Analysis



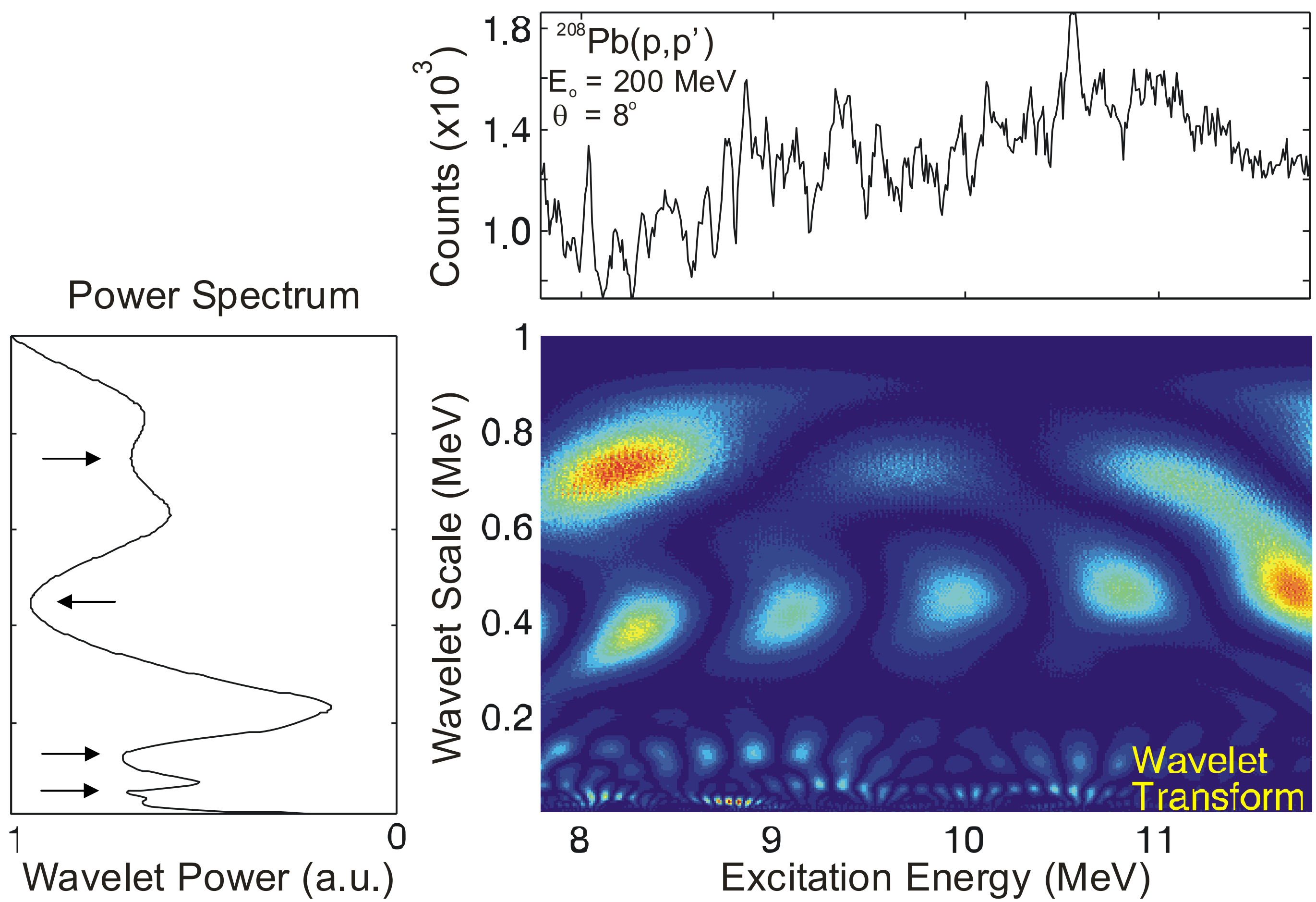
Wavelet Analysis



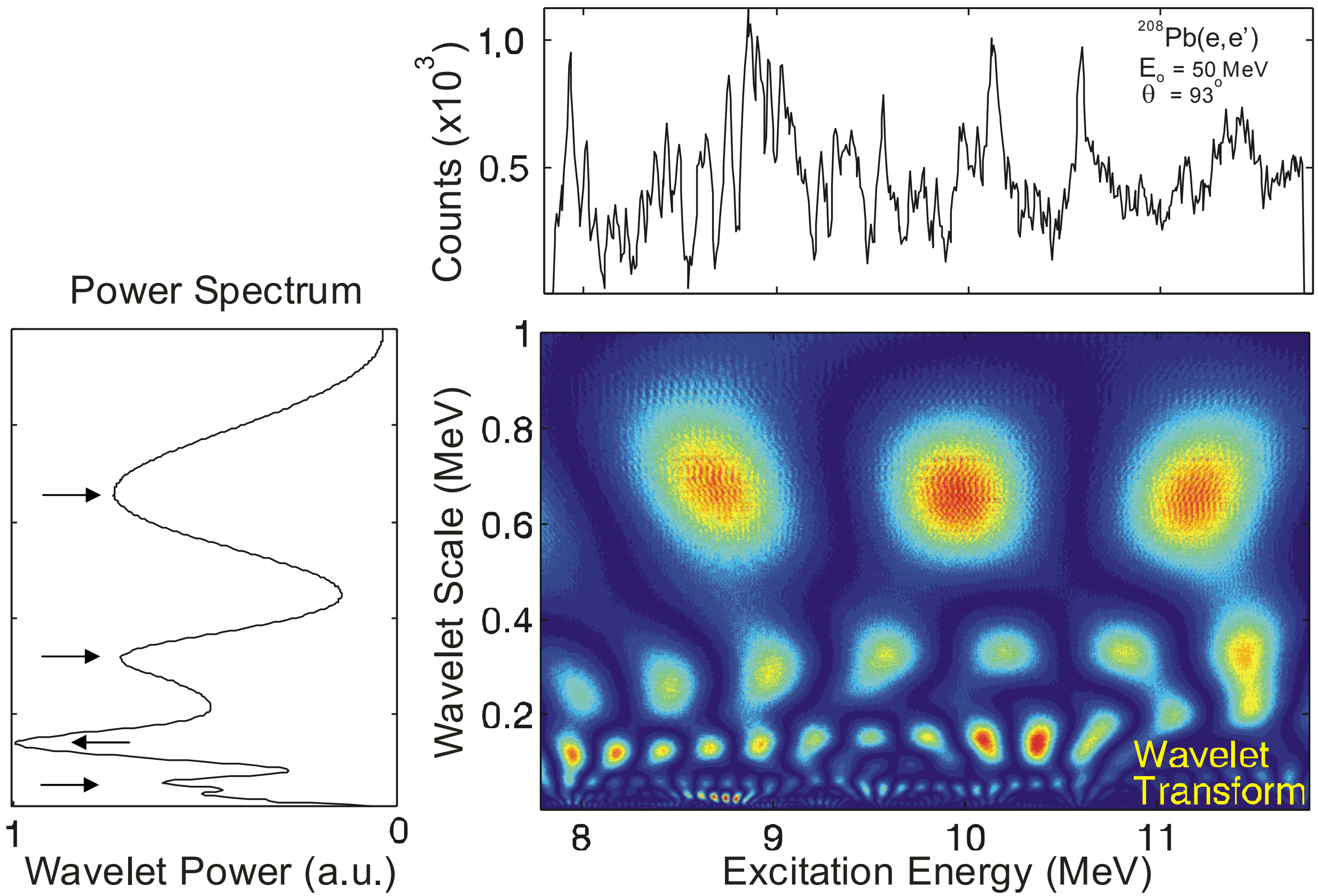
Wavelet Analysis



$^{208}Pb(p,p')$ at iThemba LABS



^{208}Pb(e,e´) at DALINAC



Summary of Scales

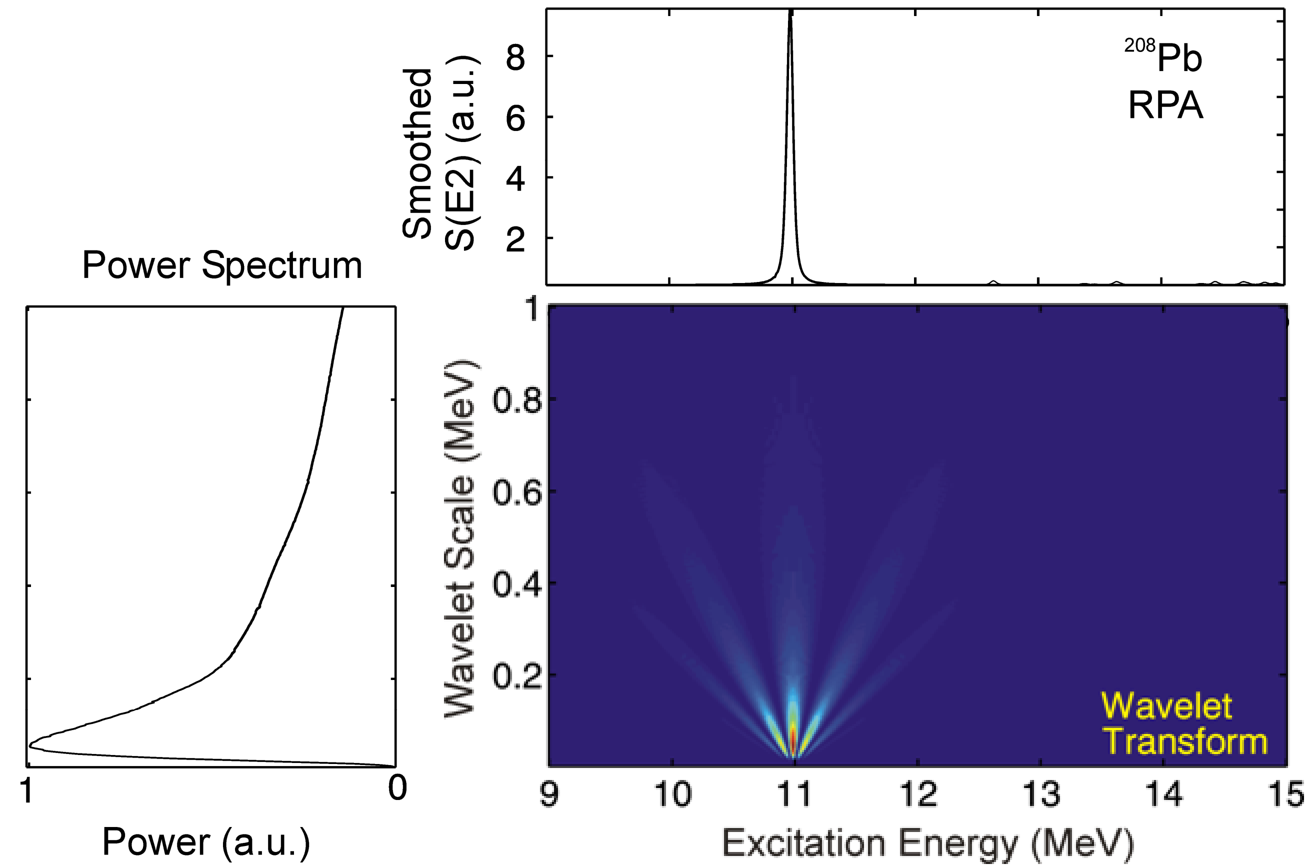
- Scales are found in all nuclei
- Three classes

I :	~ 100 keV	all nuclei
II :	~ 200 – 900 keV	changes with mass number
III :	~ 1.2 – 4.7 MeV	gross width

Interpretation of the Scales - Models

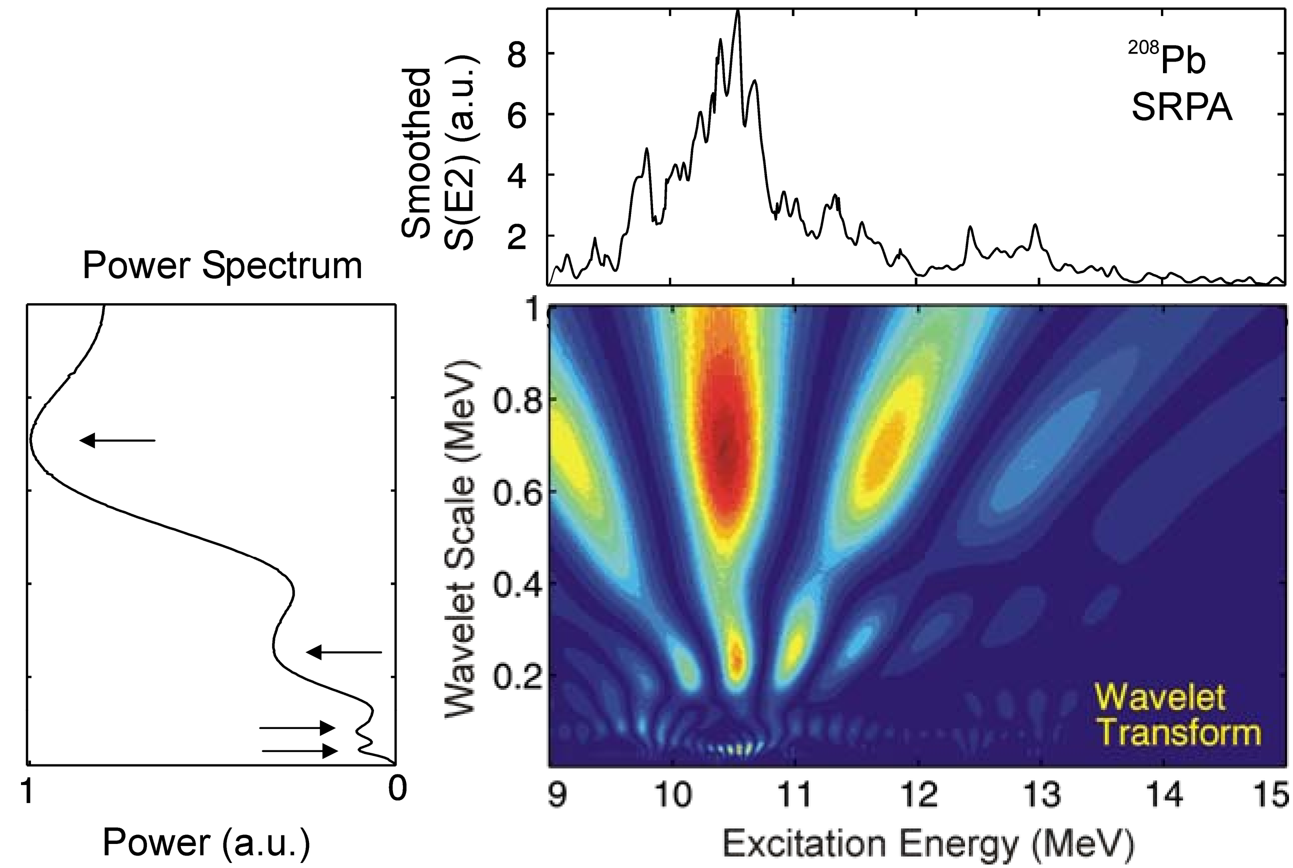
RPA	Wambach et al.	(2000)	⇦
SRPA	Wambach et al.	(2000)	⇦
QPM	Ponomarev	(2003)	⇦
ETDHF	Lacroix et al.	(1997)	
$1p-1h \otimes$ phonon ETFFS	Kamerdziev et al.	(1997)	

^{208}Pb RPA



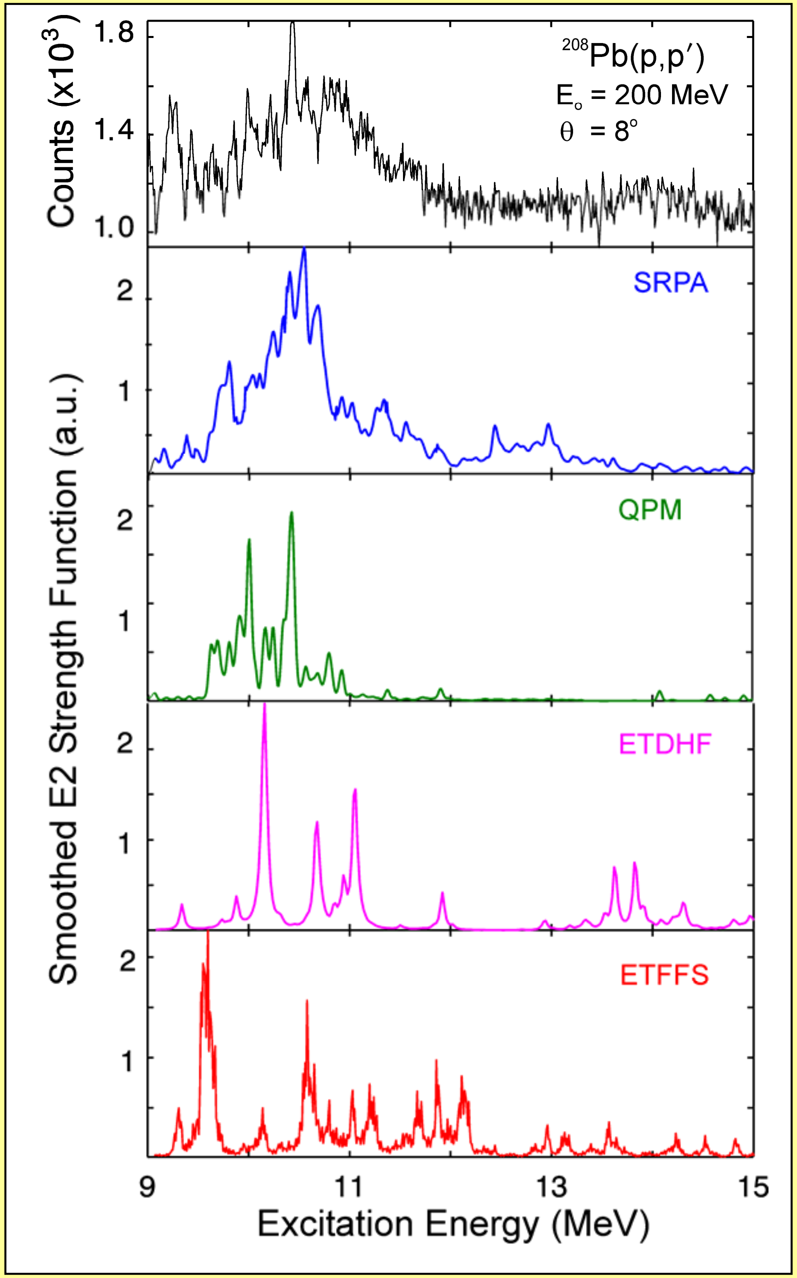
- No scales from 1p-1h states

^{208}Pb SRPA



- Coupling to 2p-2h generates fine structure and scales

Microscopic Models: Case of ^{208}Pb



- Large differences between model predictions
- No a priori judgement possible which model should be preferred
- Use wavelet analysis for a quantitative measure in comparison with the experimental observations

Experiment vs. Model Predictions

	Scales (keV)		
	I	II	III
Exp / keV	110	550	1500 2600
Models / keV			
SRPA	80	250 800	2100
QPM	110	770	1400
ETDHF	120	230	1000
ETFFS	130	310 570	2500

- Three classes of scales as in the experiment on a qualitative level
- But strong variations of class II and class III scales
- Take QPM for semi-quantitative analysis of damping mechanisms

Semi-Quantitative Attempt of Interpretation: ^{208}Pb as Example

Two types of dissipation mechanisms:

collective damping

⬇

low-lying surface vibrations

⬇

$1p-1h \otimes$ phonon

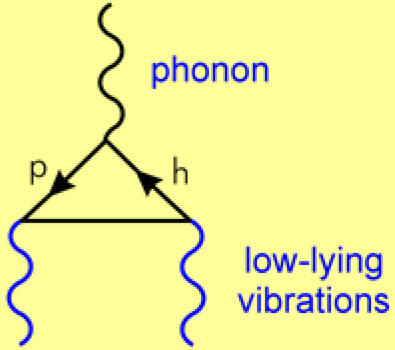
non-collective damping

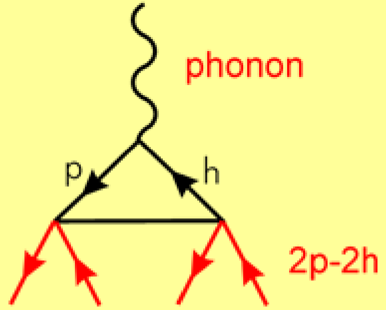
⬇

background states

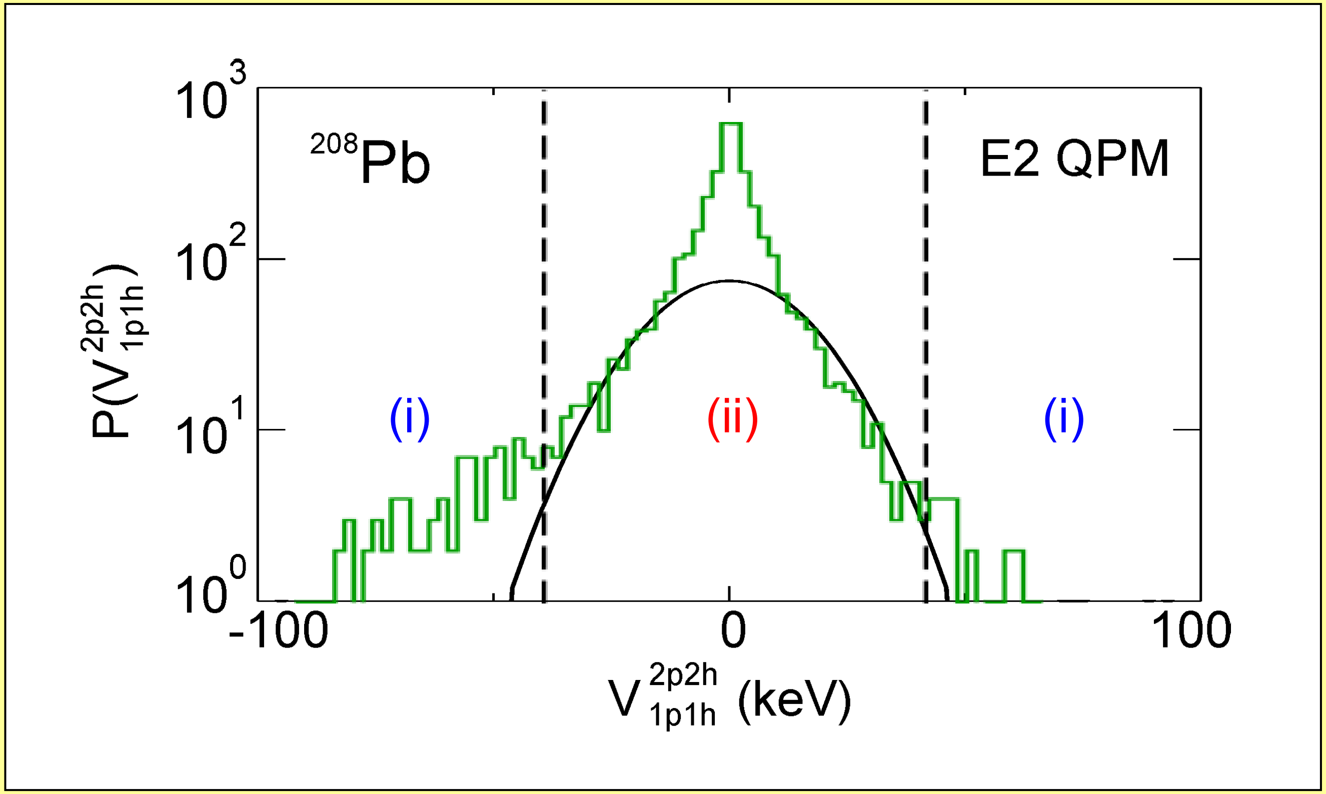
⬇

coupling to $2p-2h$ states



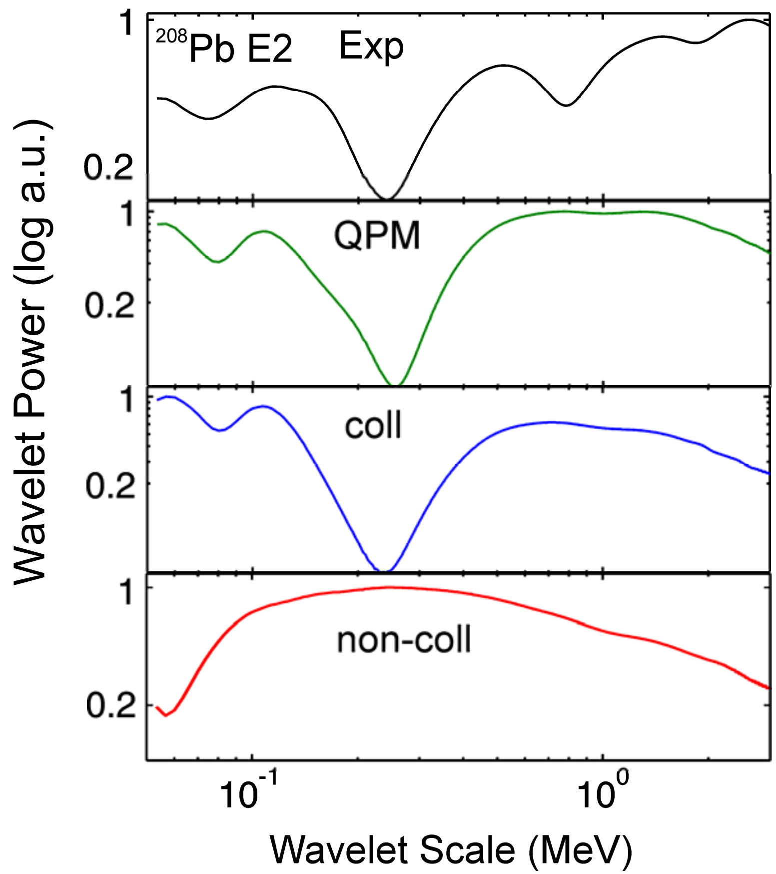


How Can the Two Mechanisms Be Separated: Distribution of the Coupling Matrix Elements



- QPM: distribution for $\langle 1p1h \mid V_{1p1h}^{2p2h} \mid 2p2h \rangle$
- RMT: deviations at large and at small m.e. → **4nd Lecture**
- Large m.e. define the collective damping mechanism
- Small m.e. are responsible for the non-collective damping

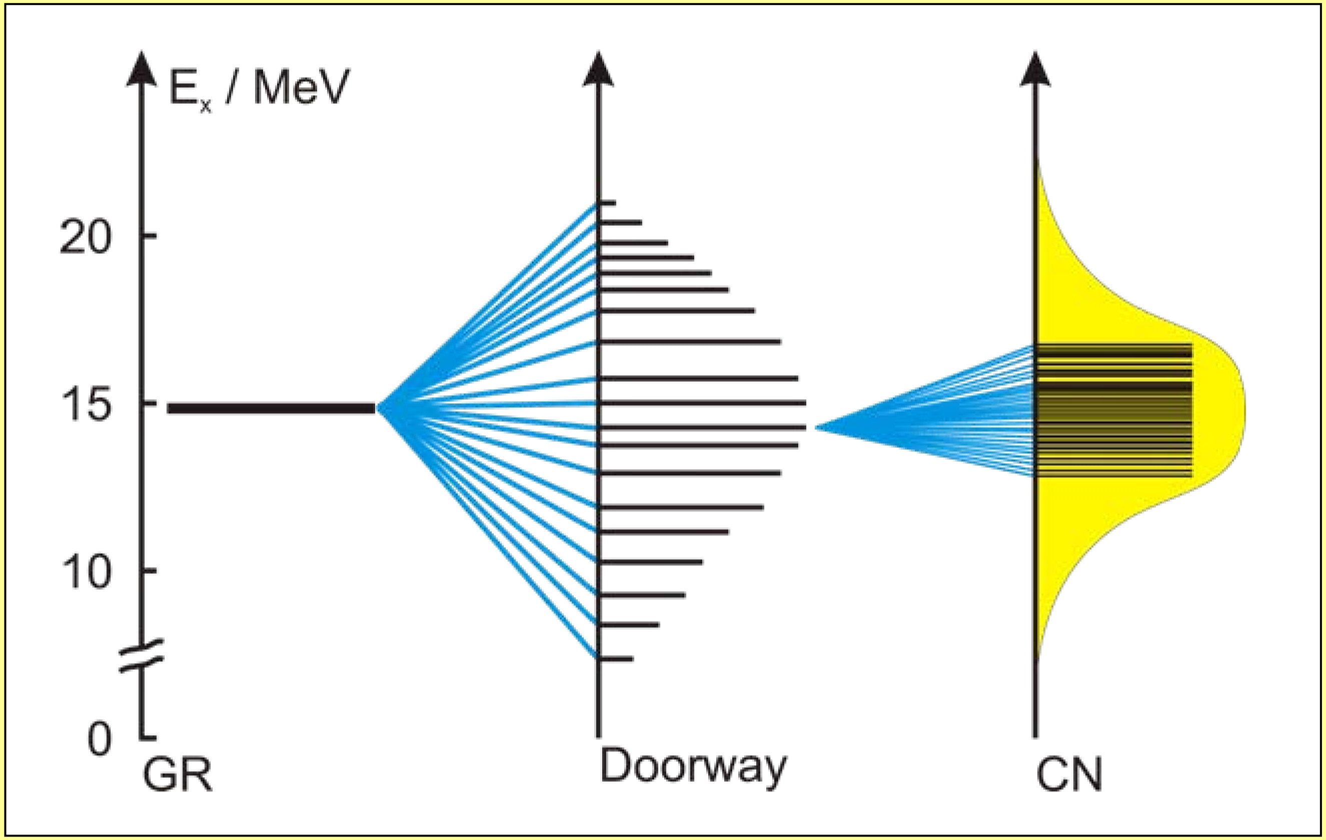
Collective vs. Non-Collective Damping in ^{208}Pb



- Collective part: all scales
- Non-collective part: no prominent scales

→ Stochastic coupling

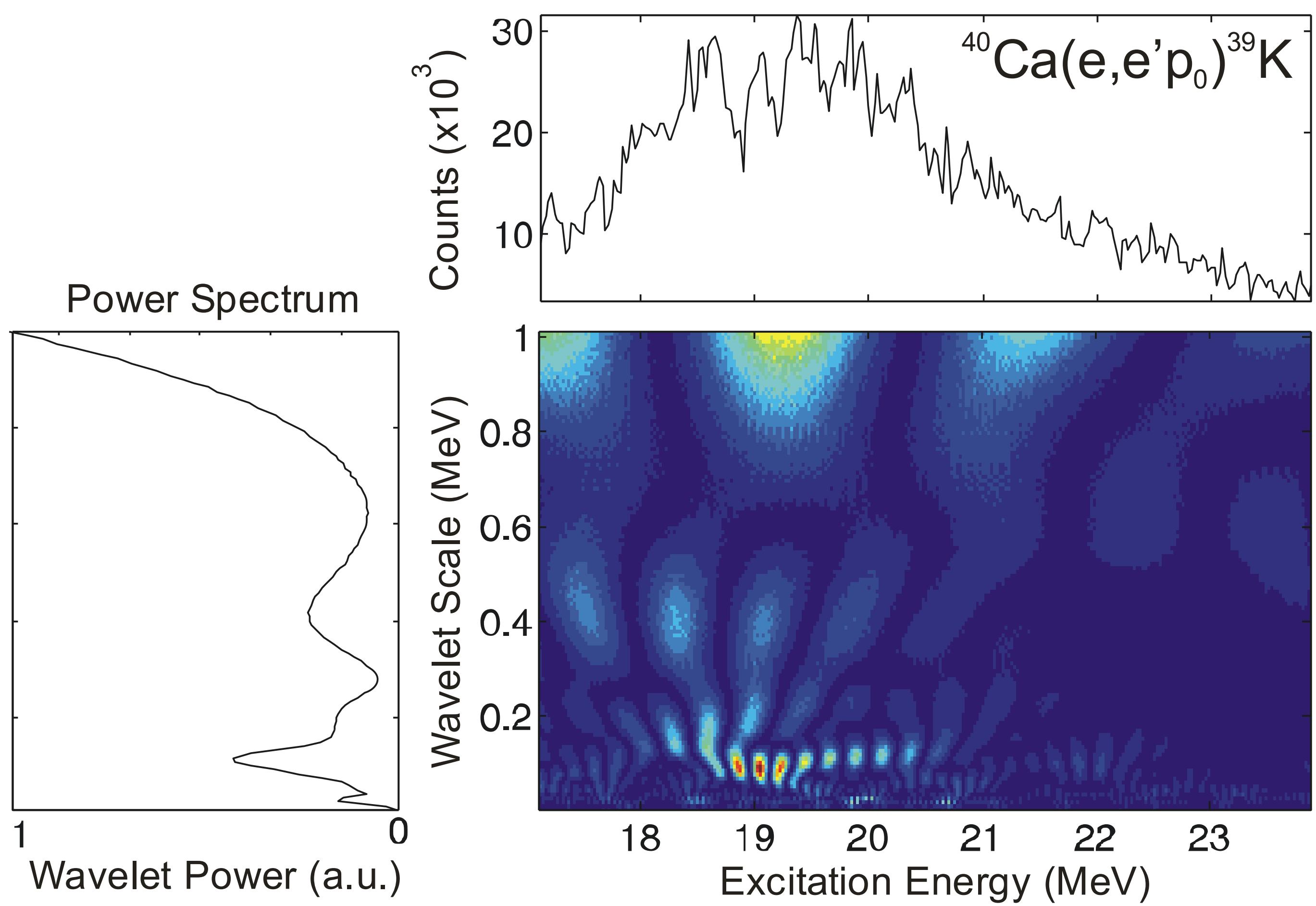
Spreading of a GR due to the Coupling to Doorway States and Decay into Compound Nucleus States



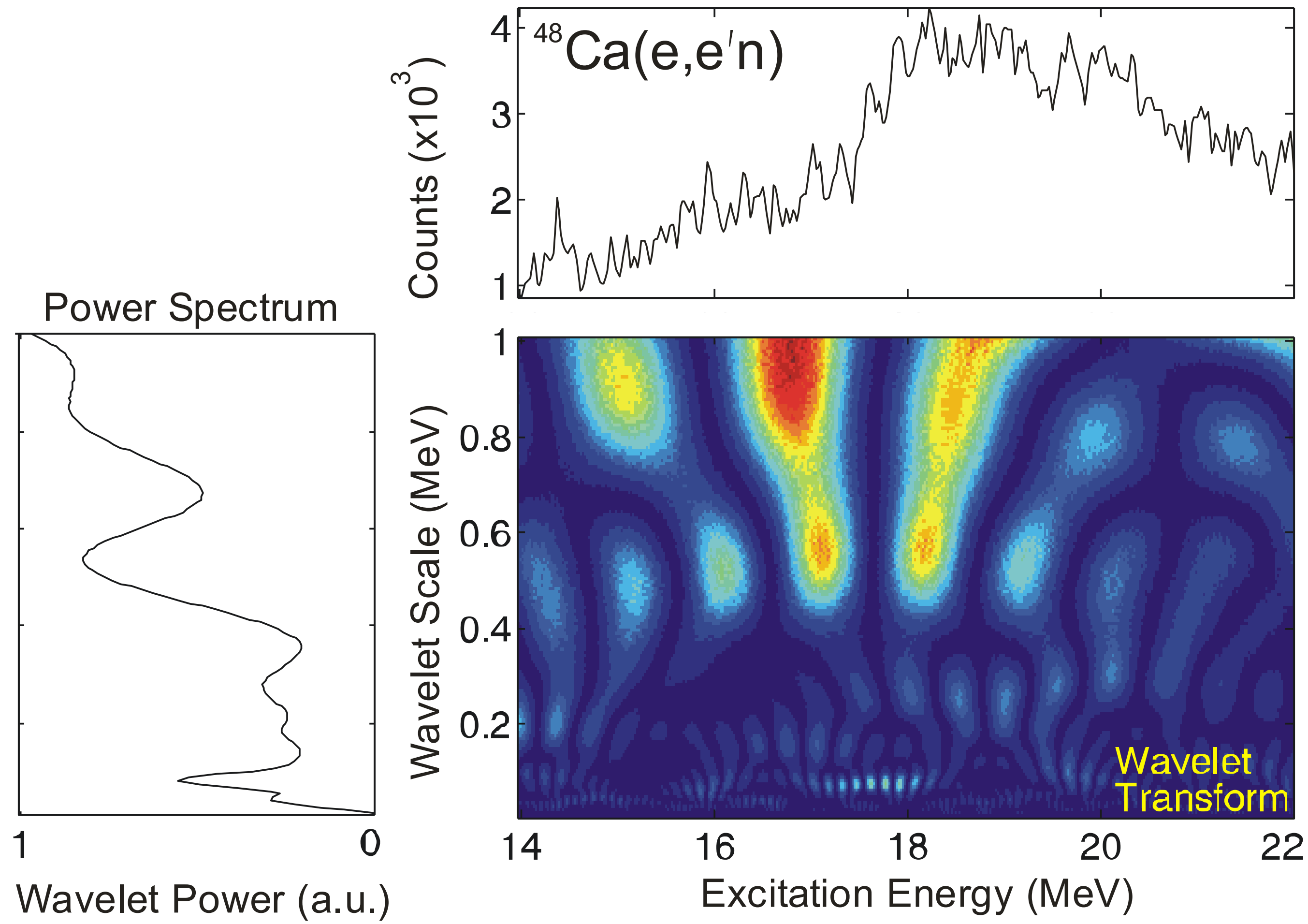
- First step of coupling hierarchy 1p-1h $\rightarrow$ 2p-2h has been tested
- Further steps require improvement in resolution

IVGDR in Exclusive Electron Scattering

$^{40}Ca(e,e'p_0)^{39}K$



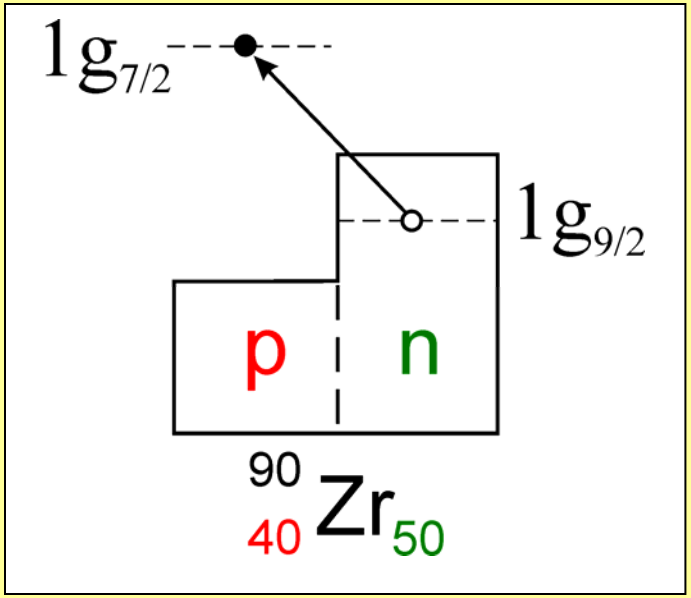
$^{48}Ca(e,e'n)$



Power Spectrum
1
0
Wavelet Power (a.u.)
Counts (x10^3)
4
3
2
1
48Ca(e,e'n)
Wavelet Scale (MeV)
1
0.8
0.6
0.4
0.2
Wavelet
Transform
14
16
18
20
22
Excitation Energy (MeV)

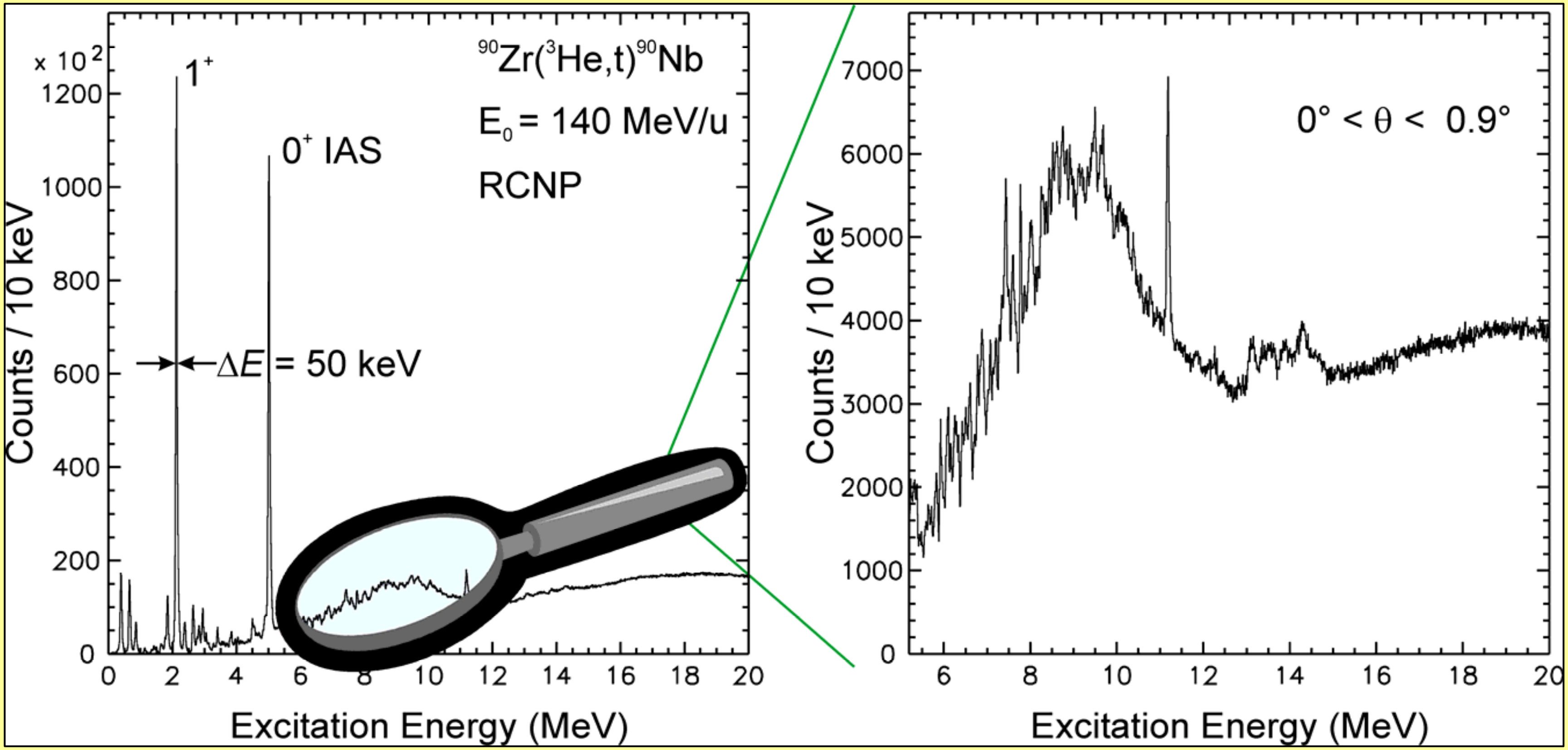
Are All Scales Equally Relevant for the Fine Structure?

Example: Gamow-Teller Giant Resonance



$\Delta L = 0, \ \Delta T = 1, \ \Delta S = 1$

Fine Structure of the Spin-Flip GTR



- High energy resolution
- Asymmetric fluctuations

Discrete Wavelet Analysis *

Wavelet coefficients:

$$C\left(\delta E, E_x\right) = \frac{1}{\sqrt{\delta E}} \int \sigma(E)\ \Psi^*\left(\frac{E_x - E}{\delta E}\right) dE$$

(scale: δE; position: E_x; spectrum: $\sigma(E)$; wavelet: Ψ^*)

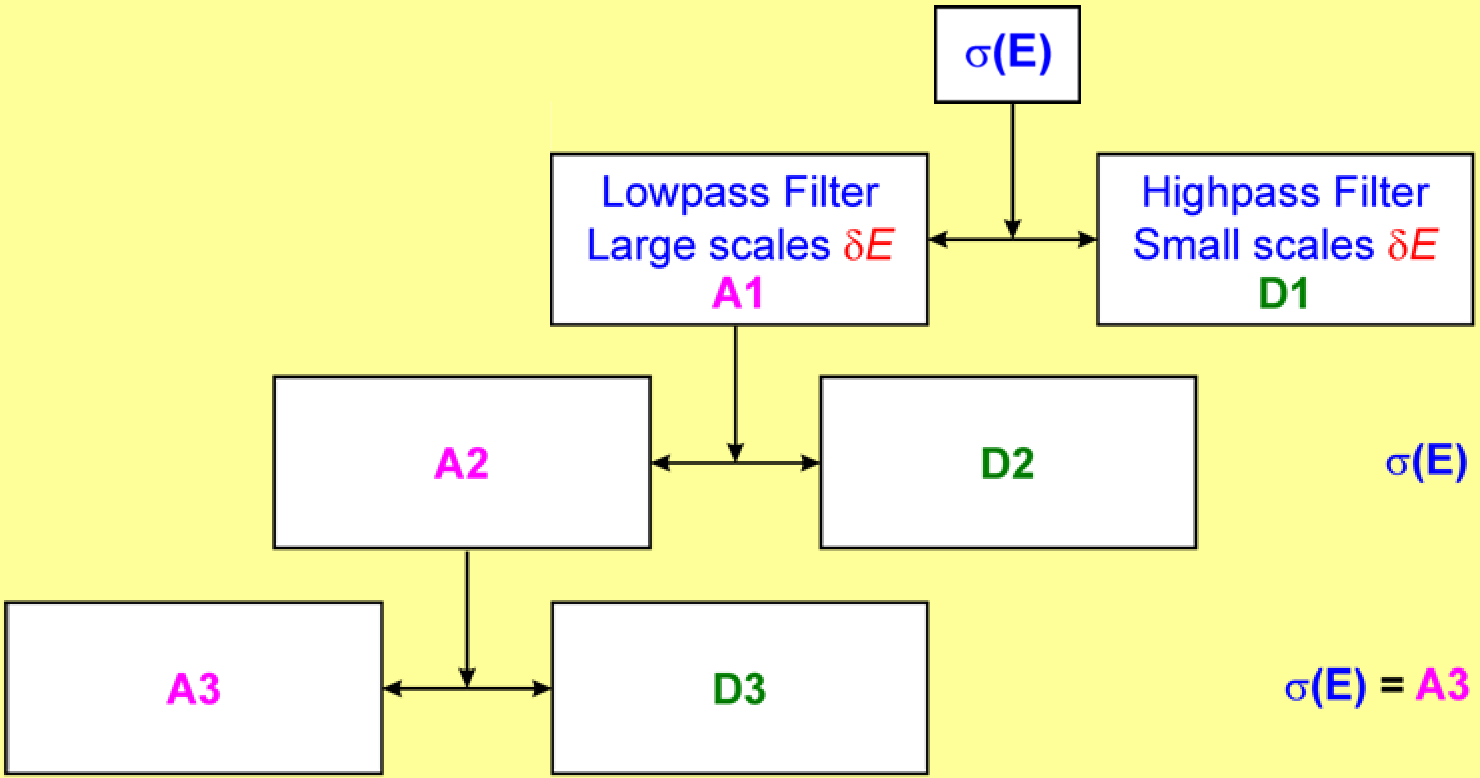
Discrete: $\delta E = 2^j$ and $E_x = k\,\delta E$ with $j, k = 1, 2, 3, \ldots$

- Orthogonal basis of wavelet functions (e.g. biorthogonal form)
- Exact reconstruction of the spectrum is possible and it is fast
- Relevance of scales
- $$\int_{-\infty}^{+\infty} E^n\, \Psi^*\left(\frac{E_x - E}{\delta E}\right) dE = 0, \qquad n = 0, 1 \ldots m-1$$ vanishing moments

  this defines the shape and magnitude of the background

* http://www.mathworks.com/products/wavelet/

Decomposition



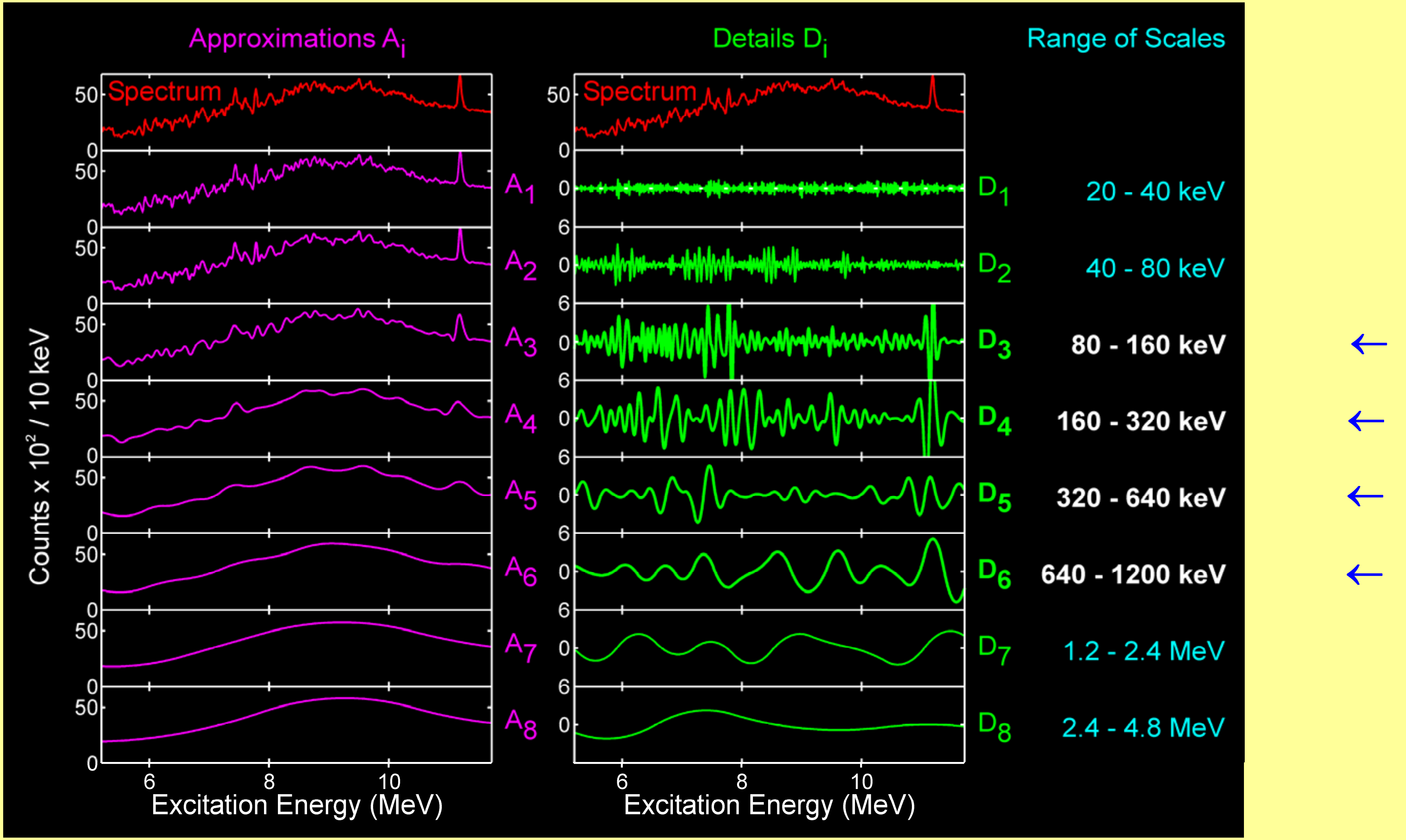
$\sigma(E)$ = A1 + D1

$\sigma(E)$ = A2 + D2 + D1

$\sigma(E)$ = A3 + D3 + D2 + D1

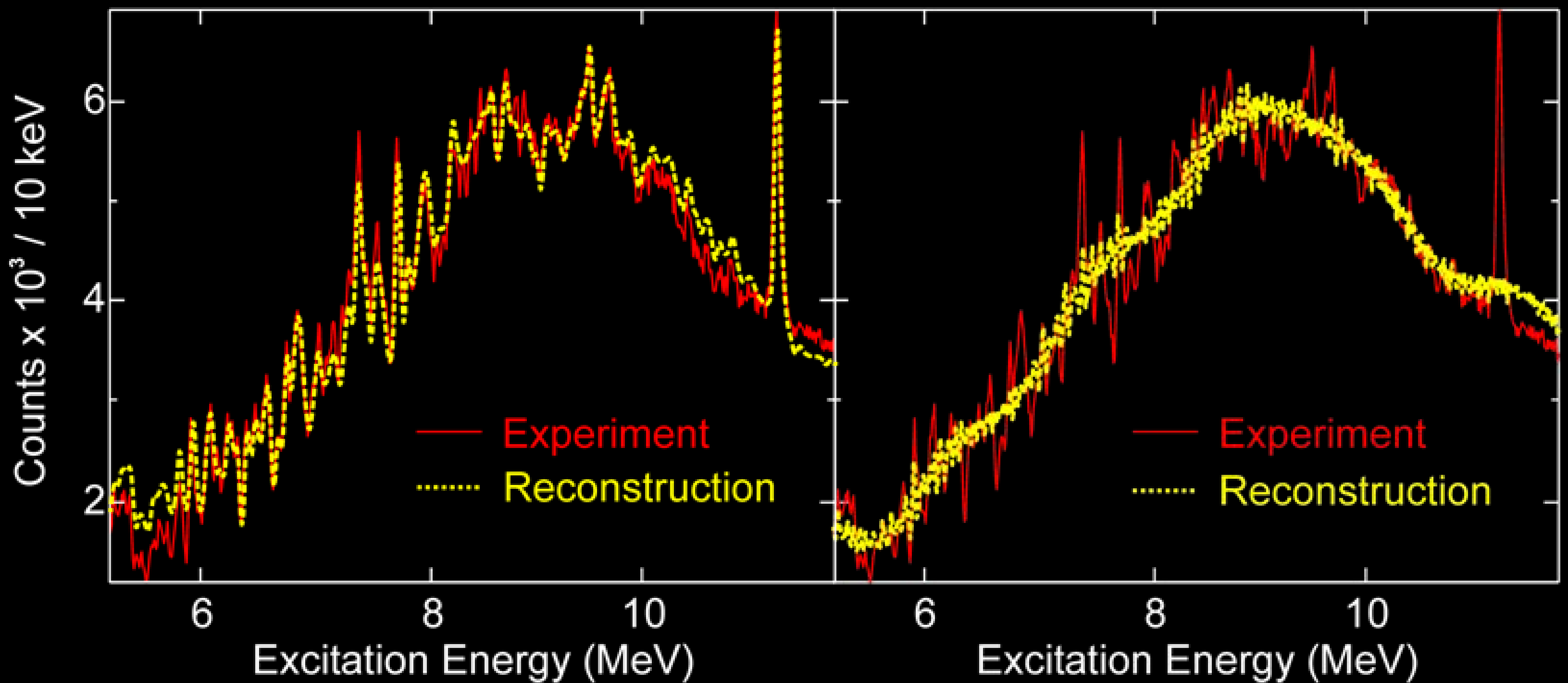
•
•
•

Decomposition of $^{90}Zr(^{3}He,t)^{90}Nb$ Spectrum



- Reconstruct the spectrum using important scales
- Model-independent background determination + fluctuations → level densities

Discrete Wavelet Transform: Reconstructed Spectra



$\sigma_R(E) = A_8 + D_6 + D_5 + D_4 + D_3$

$\sigma_R(E) = A_8 + D_8 + D_7 + D_2 + D_1$

Level Densities

- Astrophysical network calculations

- Back-shifted Fermi gas model
  - semiempirical approach
  - shell and pairing effects

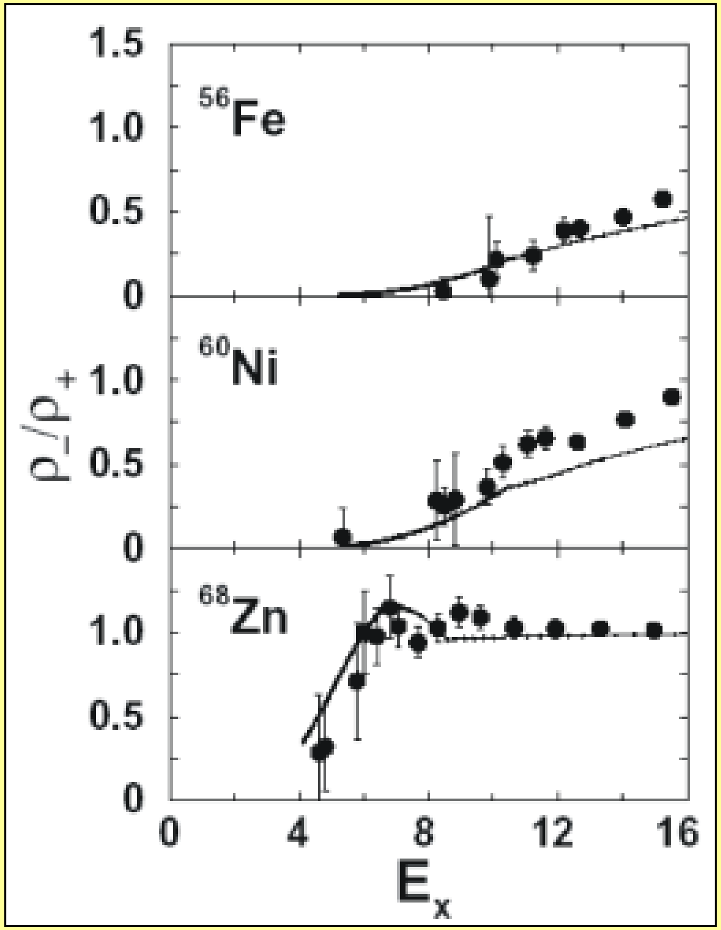
$$\rho(E_x, J) \sim e^{2\sqrt{a(E_x-\delta)}} \cdot e^{-\frac{J(J+1)}{2\sigma^2}}$$

- Many-body density of states
  - two-component Fermi gas
  - shell effects, deformations

- HF-BCS
  - microscopic statistical model (partition function, MSk7 force, local renormalization)
  - shell effects, pairing correlations, deformation effects, collective excitations

- Monte-Carlo shell model calculations
  - parity dependence?

Monte-Carlo Shell Model Predictions: pf + $g_{9/2}$ Shell



$\Delta_{pf-g_{9/2}}$ small

ρ_- important at low energies

- Total level density (not spin projected) shows strong parity dependence*
- Questioned by recent experiments (^{45}Sc)**

* Y. Alhassid, G.F. Bertsch, S. Liu, and H. Nakada, PRL 84 (2000) 4313

** S.J. Lokitz, G.E. Mitchell, and J.F. Shriner, Jr., PRC 71 (2005) 064315

Experimental Techniques

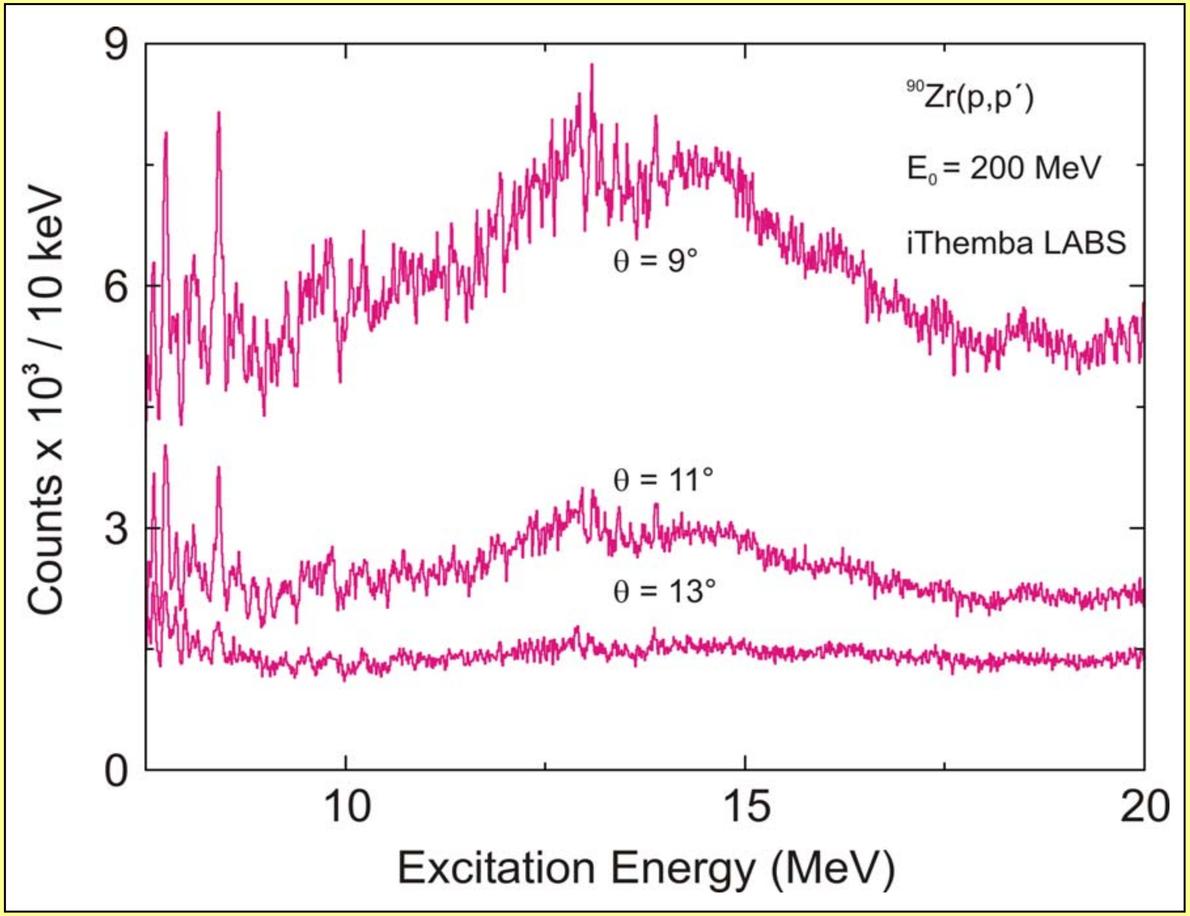
- Selectivity
  - hadron scattering at extremely forward angles and intermediate energies
  - electron scattering at 180° and low momentum transfers
- High resolution
  - lateral and angular dispersion matching
  - faint beam method*
- Level density
  - fluctuation analysis**
- Background
  - discrete wavelet transform***

* H. Fujita et al., NIM A484 (2002) 17

** P.G. Hansen, B. Jonson, and A. Richter, NPA 518 (1990) 13

*** Y. Kalmykov et al., PRL 96 (2006) 012502

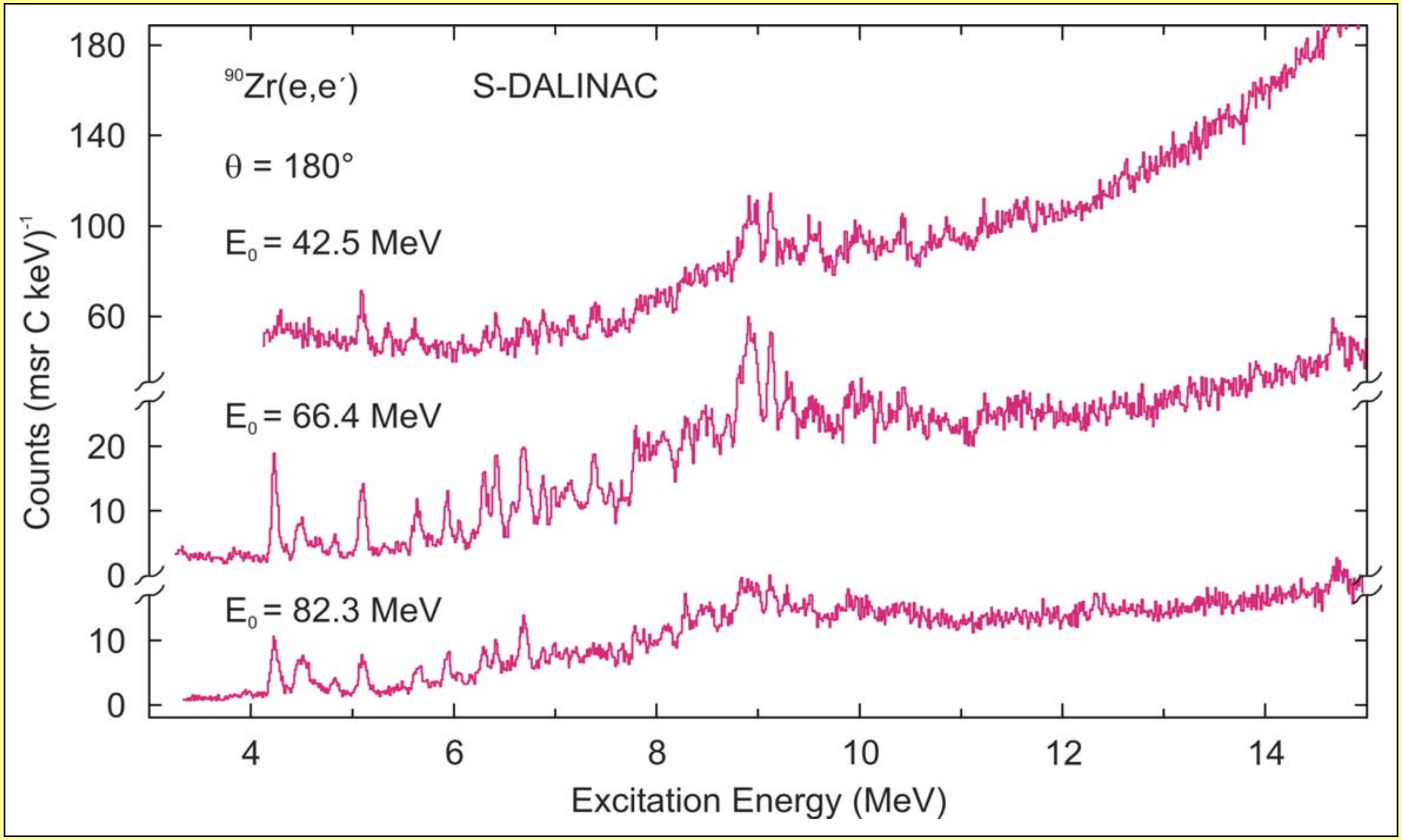
Fine Structure of the ISGQR: A = 90



- Selective excitation of 2^+ states

A. Shevchenko et al., PRL 93 (2004) 122501

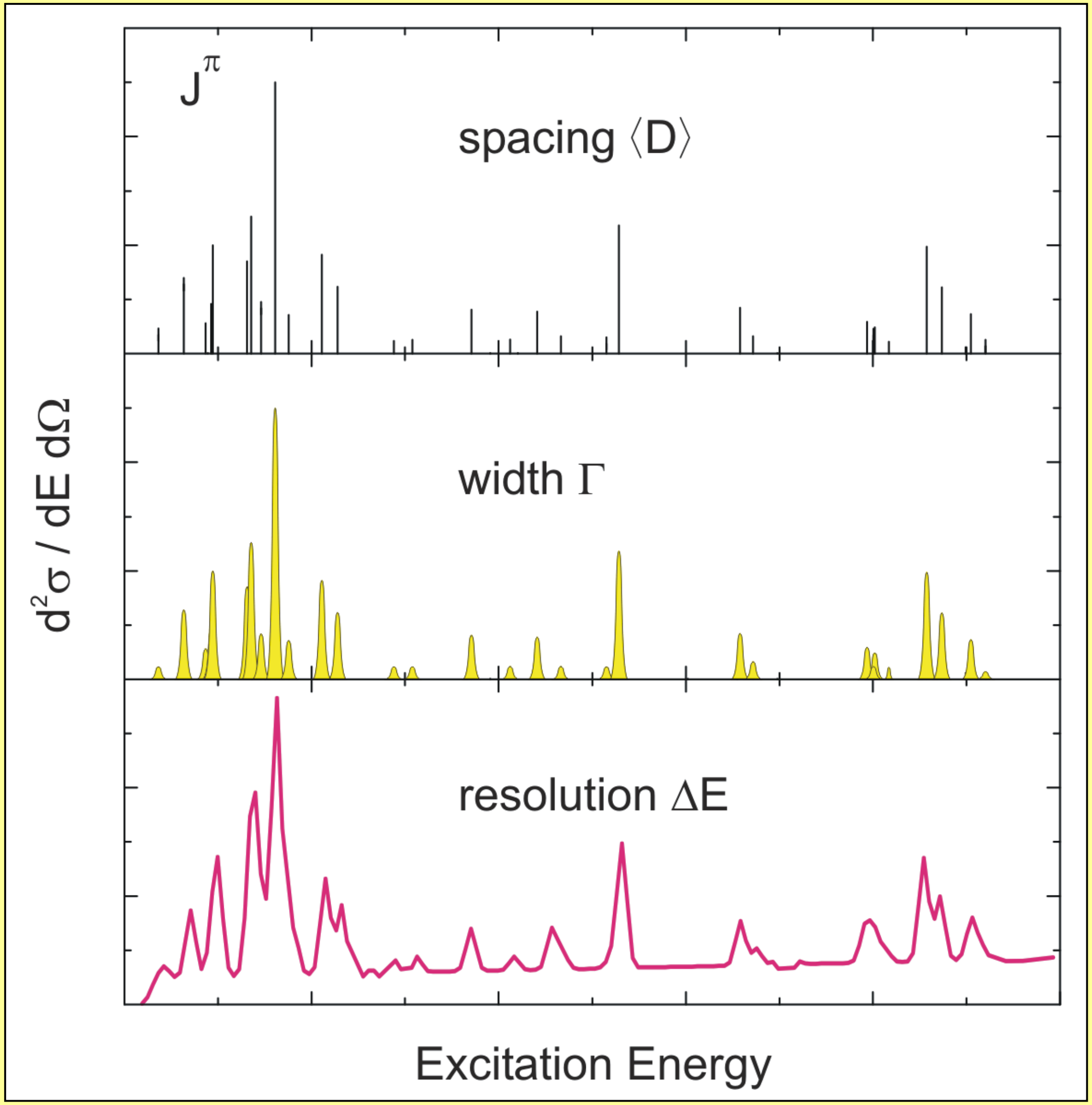
Fine Structure of the M2 Resonance: A = 90



- Selective excitation of 2^- states

P. von Neumann-Cosel et al., PRL 82 (1999) 1105

Fluctuations and Level Densities



- D / ⟨D⟩ Wigner

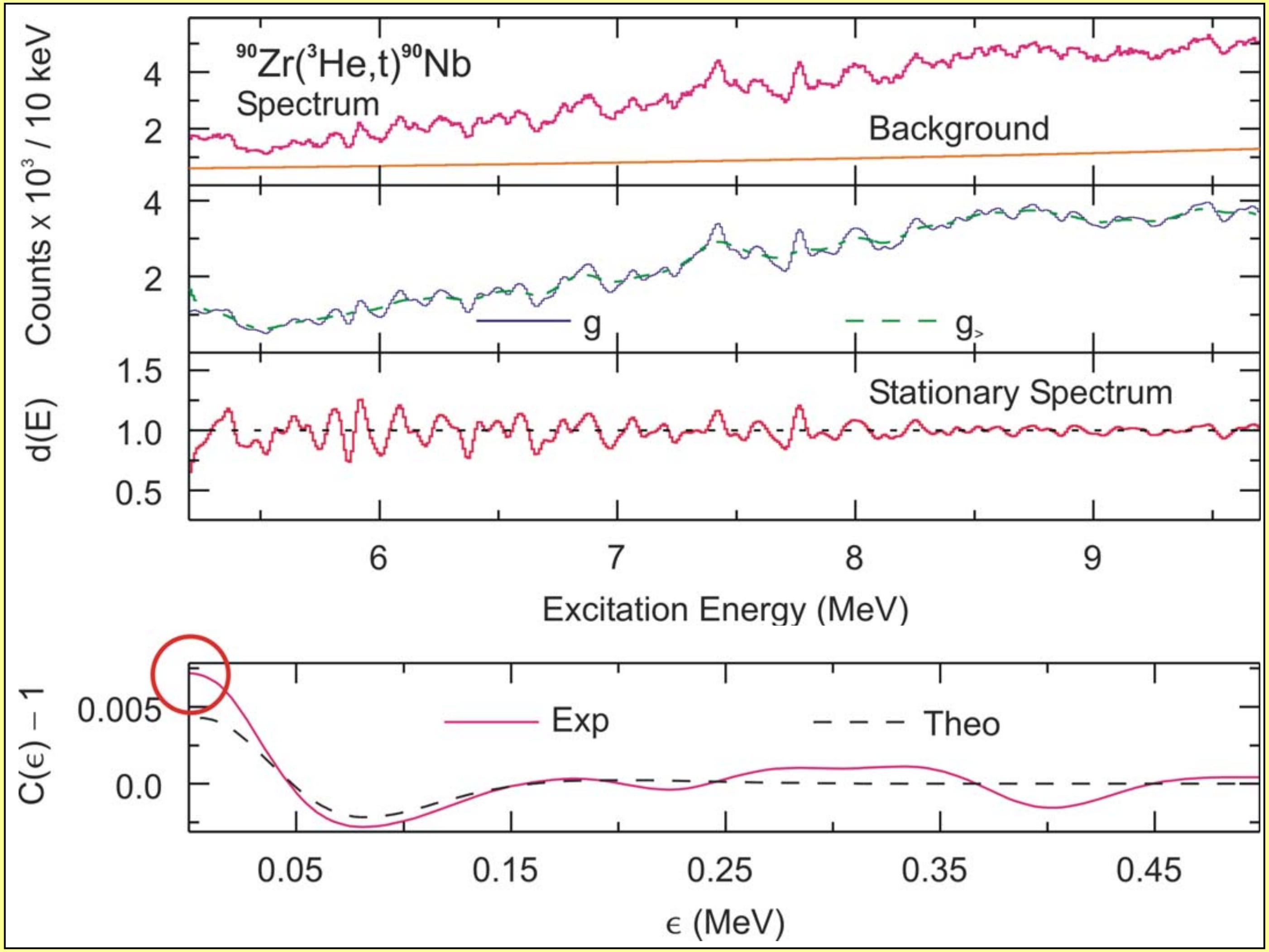
  I / ⟨I⟩ Porter-Thomas

  → **4th Lecture**

- Γ < ⟨D⟩

- Γ < ⟨D⟩ < ΔE

Fluctuation Analysis



- Background from wavelet analysis
- Statistics, local features
- Local fluctuations
- Autocorrelation function

Autocorrelation Function and Mean Level Spacing

- $$C(\varepsilon) = \frac{\langle\, d(E_x)\, d(E_x+\varepsilon)\,\rangle}{\langle\, d(E_x)\,\rangle\,\langle\, d(E_x+\varepsilon)\,\rangle}$$ autocorrelation function

- $$C(\varepsilon=0) - 1 = \frac{\langle\, d^2(E_x)\,\rangle - \langle\, d(E_x)\,\rangle^2}{\langle\, d(E_x)\,\rangle^2}$$ variance

- $$C(\varepsilon) - 1 = \frac{\alpha\,\langle\, D\,\rangle}{2\sigma\sqrt{\pi}} \times f(\sigma,\varepsilon)$$ level spacing ⟨D⟩

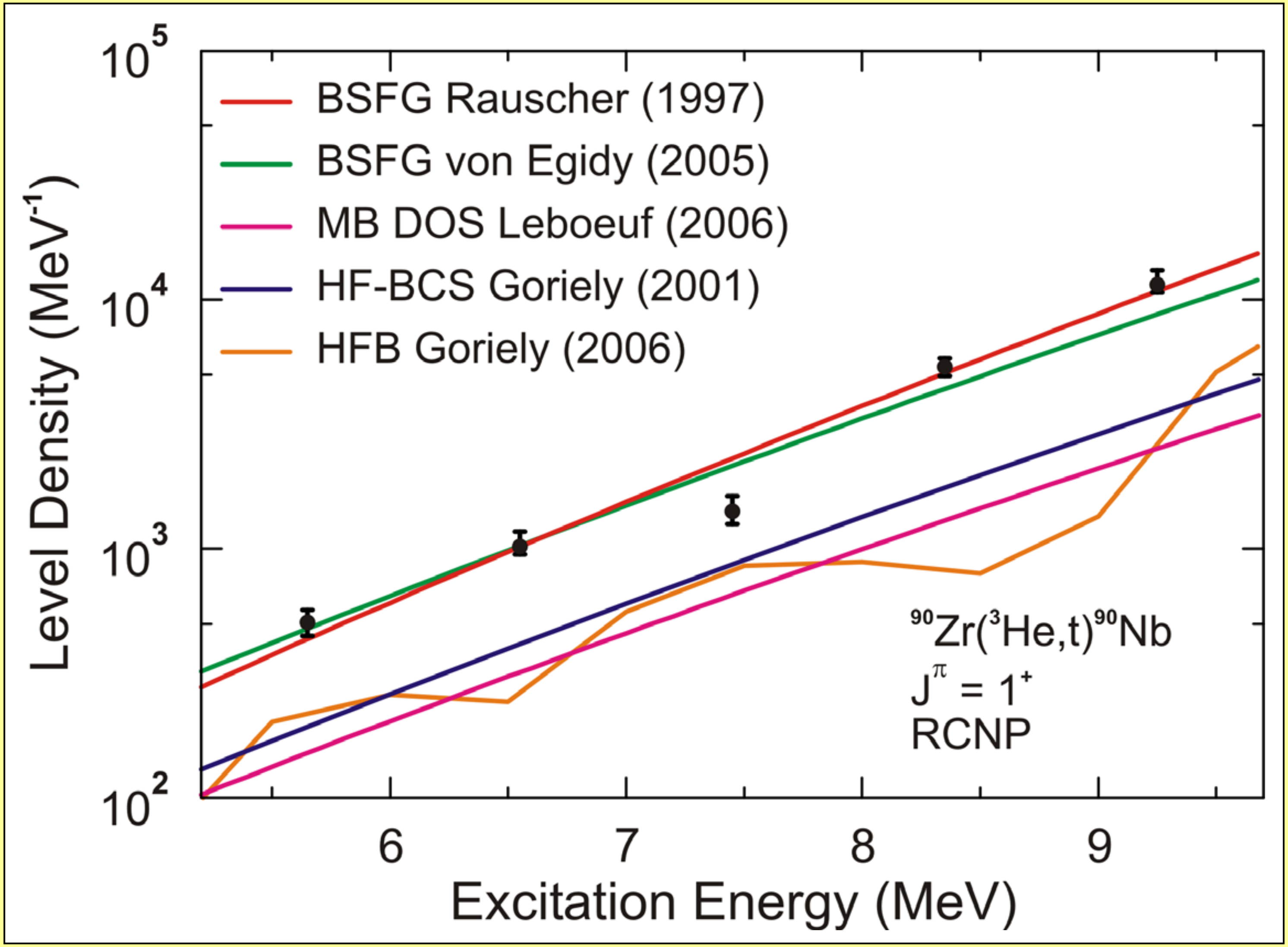
- $\alpha = \alpha_{PT} + \alpha_{W}$ selectivity

- σ resolution

S. Müller, F. Beck, D. Meuer, and A. Richter, PLB 113 (1982) 362

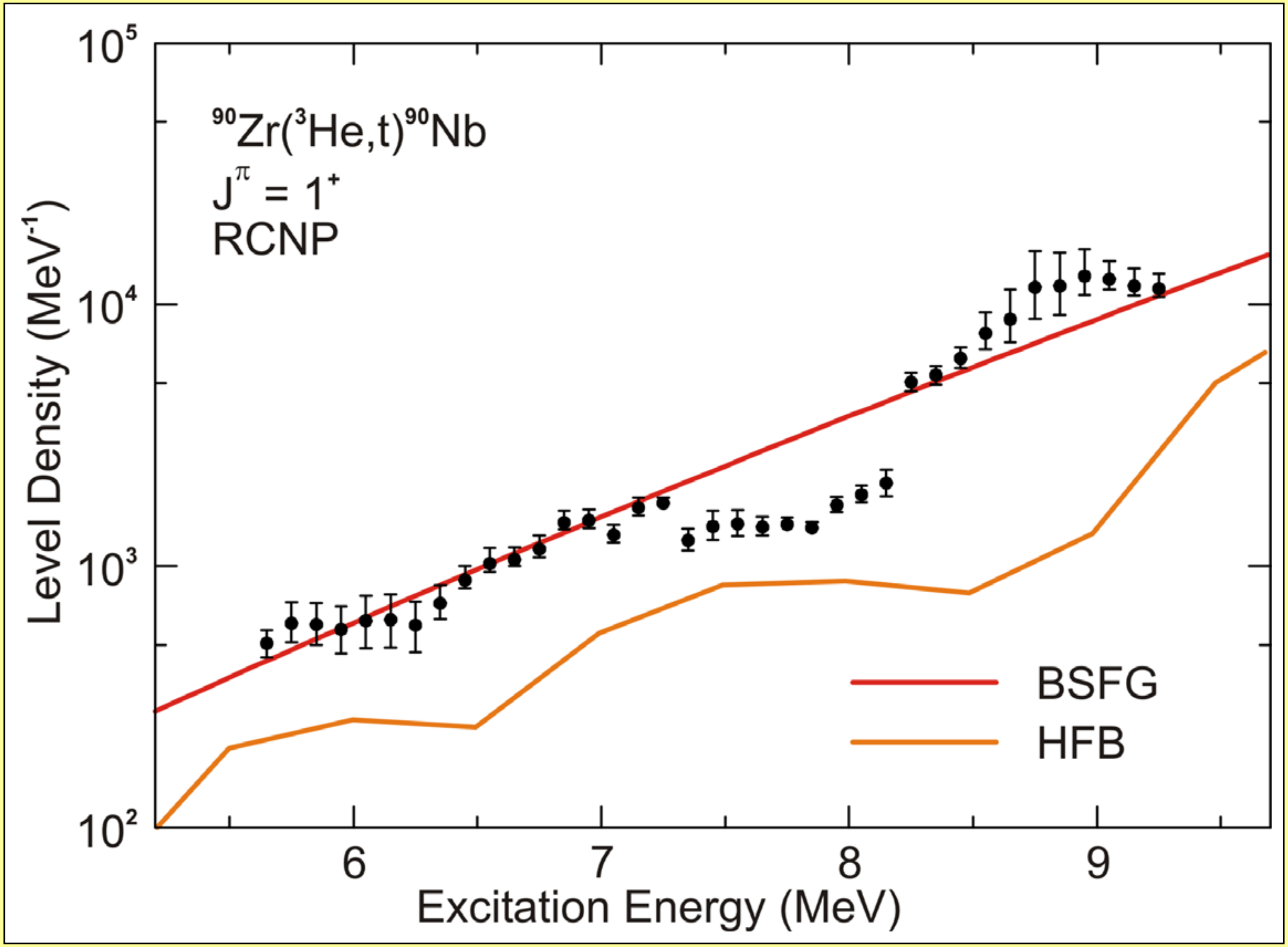
P.G. Hansen, B. Jonson, and A. Richter, NPA 518 (1990) 13

Results and Model Predictions: A = 90, $J^{\pi} = 1^{+}$



Y. Kalmykov et al., PRL 96 (2006) 012502

Fine Structure of Level Density: A = 90, $J^{\pi} = 1^{+}$



Phenomenological and Microscopic Models

- Different quality of model predictions

- BSFG, MB DOS
  parameters fitted to experimental data
  no distinction of parity

- HF-BCS
  microscopic
  no distinction of parity

- HFB, SMMC
  fully microscopic calculation of levels
  with spin and parity

- HFB
  fine structure of level densities

Ingredients of HFB

- Nuclear structure: HFB calculation with a conventional Skyrme force
  - single particle energies
  - pairing strength for each level
  - quadrupole deformation parameter
  - deformation energy

- Collective effects
  - rotational enhancement
  - vibrational enhancement
  - disappearance of deformation at high energies

Ingredients of SMMC

- Partition function of many-body states with good J^π

$$Z_J^\pi(\beta) = \mathrm{Tr}_{J,\pi}\, e^{-\beta H}$$

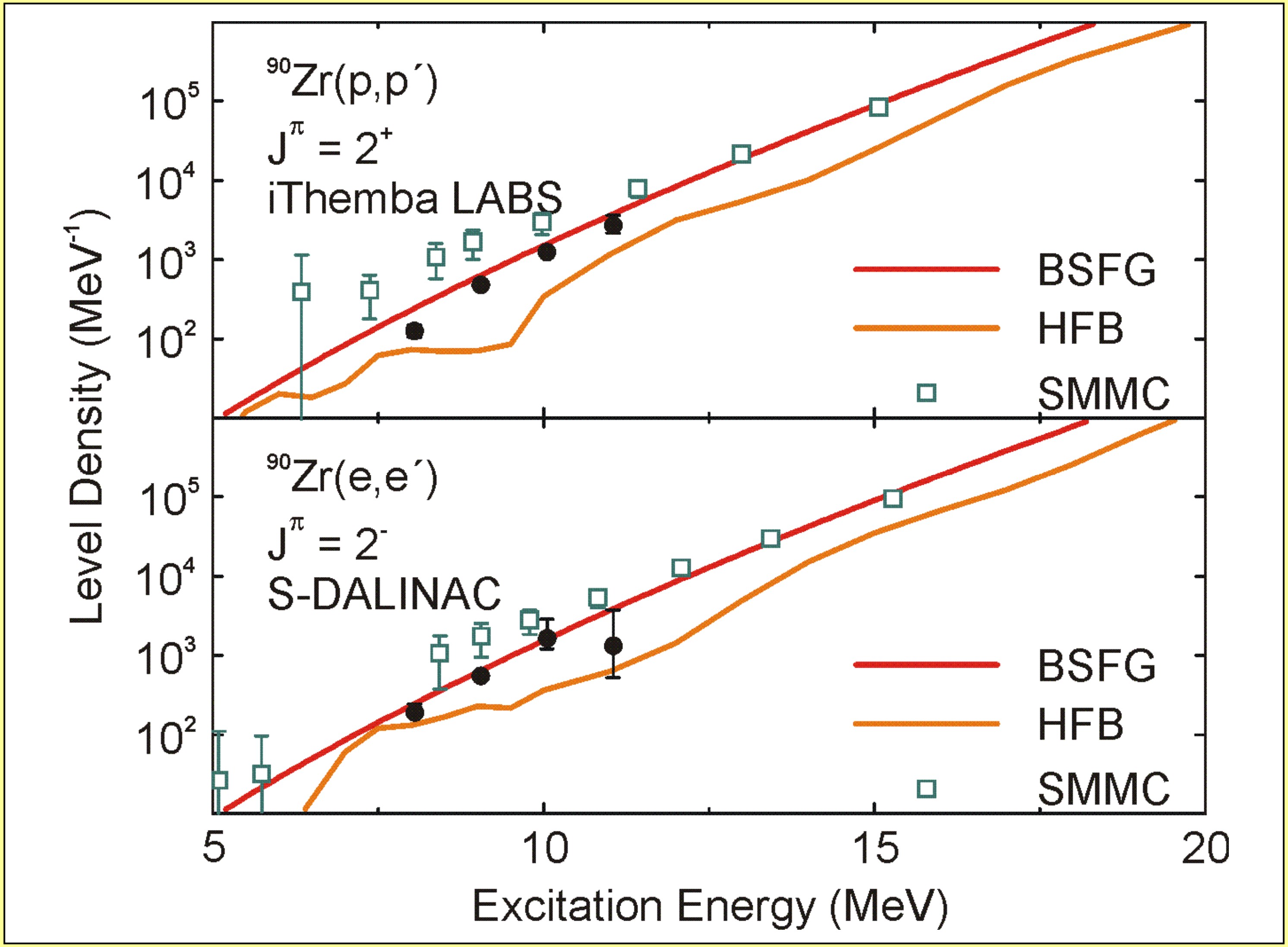
- Expectation values at inverse temperature $\beta = 1/kT$

$$E_J^\pi(\beta) = \frac{\int dE'\, e^{-\beta E'}\, E'\, \rho_J^\pi(E')}{Z_J^\pi(\beta)}$$

- Level density from inverse Laplace transform in the saddle-point approximation

$$\rho_J^\pi(\beta) = \frac{e^{\beta E_J^\pi + \ln Z_J^\pi(\beta)}}{\sqrt{-2\pi\, \frac{dE_J^\pi(\beta)}{d\beta}}}$$

Level Density of 2^+ and 2^- States: ^{90}Zr



Y. Kalmykov et al., PRL 96 (2006) 012502

Equilibration of Parity-Projected Level Densities

- ^{58}Ni

  $\rho_- \approx \rho_+$ at $E_x \approx 20$ MeV

- ^{90}Zr

  $\rho_- \approx \rho_+$ at $E_x \approx 5 - 10$ MeV

- Two energy scales which determine ρ_-/ρ_+

  pair-breaking

  ➡ 5 – 6 MeV for intermediate mass nuclei

  shell gap between opposite-parity states near the Fermi level

  ➡ depends strongly on the shell structure, e.g. ^{68}Zn $\Delta_{pf\text{-}g9/2}$ is small

- Core breaking

  e.g. near shell closure ^{58}Ni $\Delta_{sd\text{-}pf}$ transitions are important

  ➡ ρ_- would be enlarged

Summary

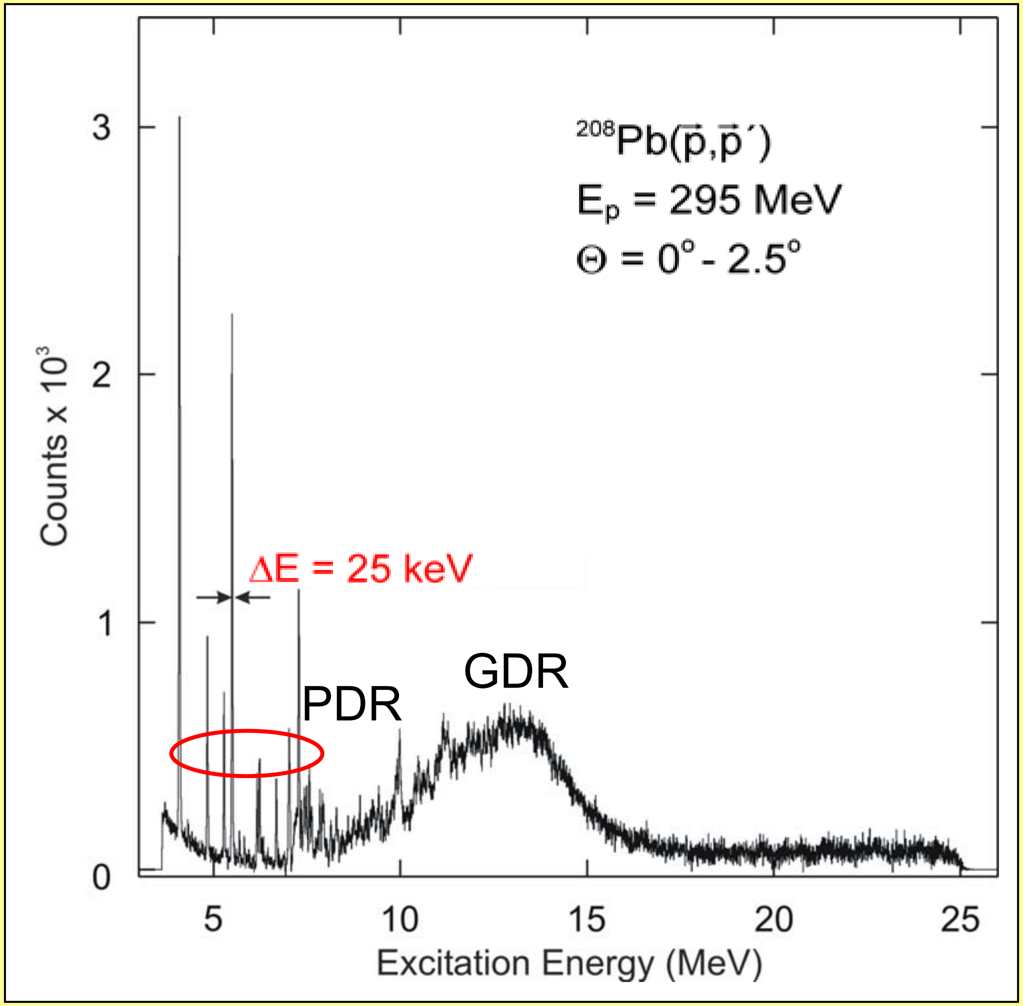
- Fine structure is a general phenomenon of many-body systems (particles, nuclei, atoms, molecules, clusters, condensates,…)

- Quantitative analysis of nuclear giant resonances with wavelets

- Origin of scales in GR's:
  - collective damping: low-lying surface vibrations
  - non-collective damping: stochastic coupling

- New method for level densities

Outlook



→ **3rd Lecture**