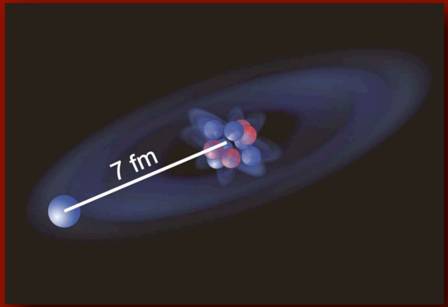
Canada's National Laboratory for Particle and Nuclear Physics
Laboratoire national canadien pour la recherche en physique nucléaire et en physique des particules

Halo Nuclei: Theory and Precision Experiments

Sonia Bacca



Nuclear Halo


Moon Halo

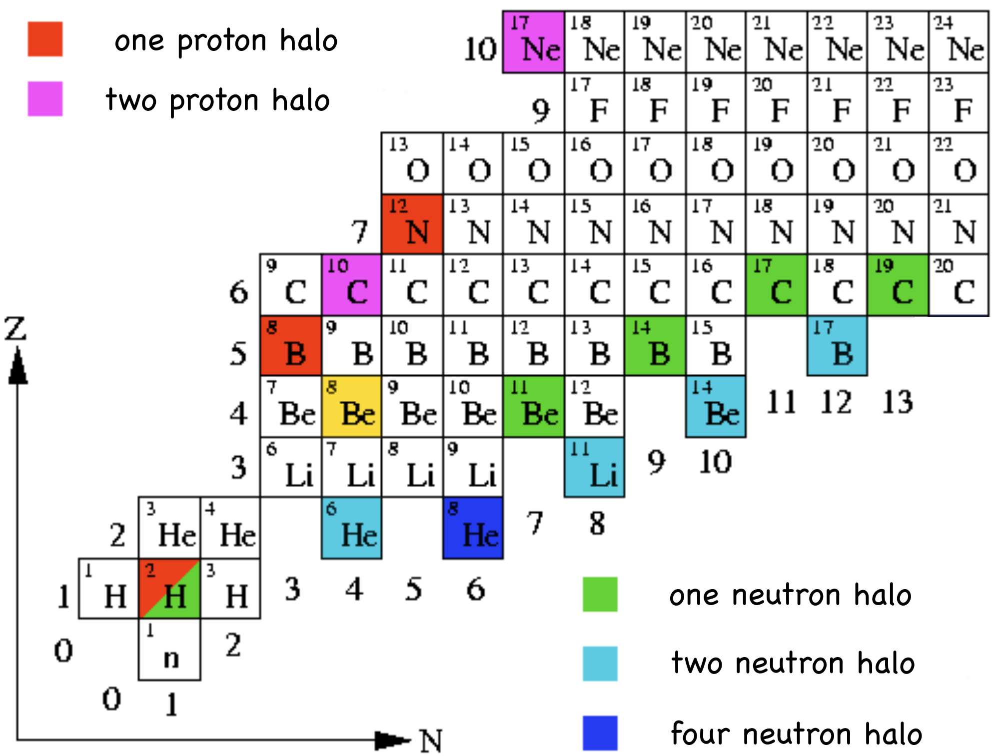
Joint 2010 National Nuclear Physics Summer School (NNPS)
and
2010 TRIUMF Summer Institute (TSI)

June 21 - July 2, 2010   Vancouver, BC, Canada

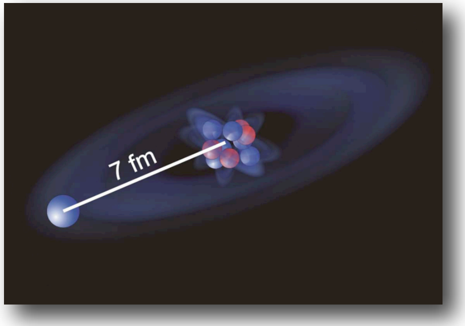
Outline

- What are halo nuclei?
- Why are halo nuclei interesting?
- Summary on experimental advances
- Theory: Different approaches to the potentials
  Different ab-initio approaches to the many-body problem
- Towards halo nuclei from EFT: ^{6}He and ^{8}He
- Summary and Outlook

Halo Nuclei



- Exotic nuclei with an interesting structure



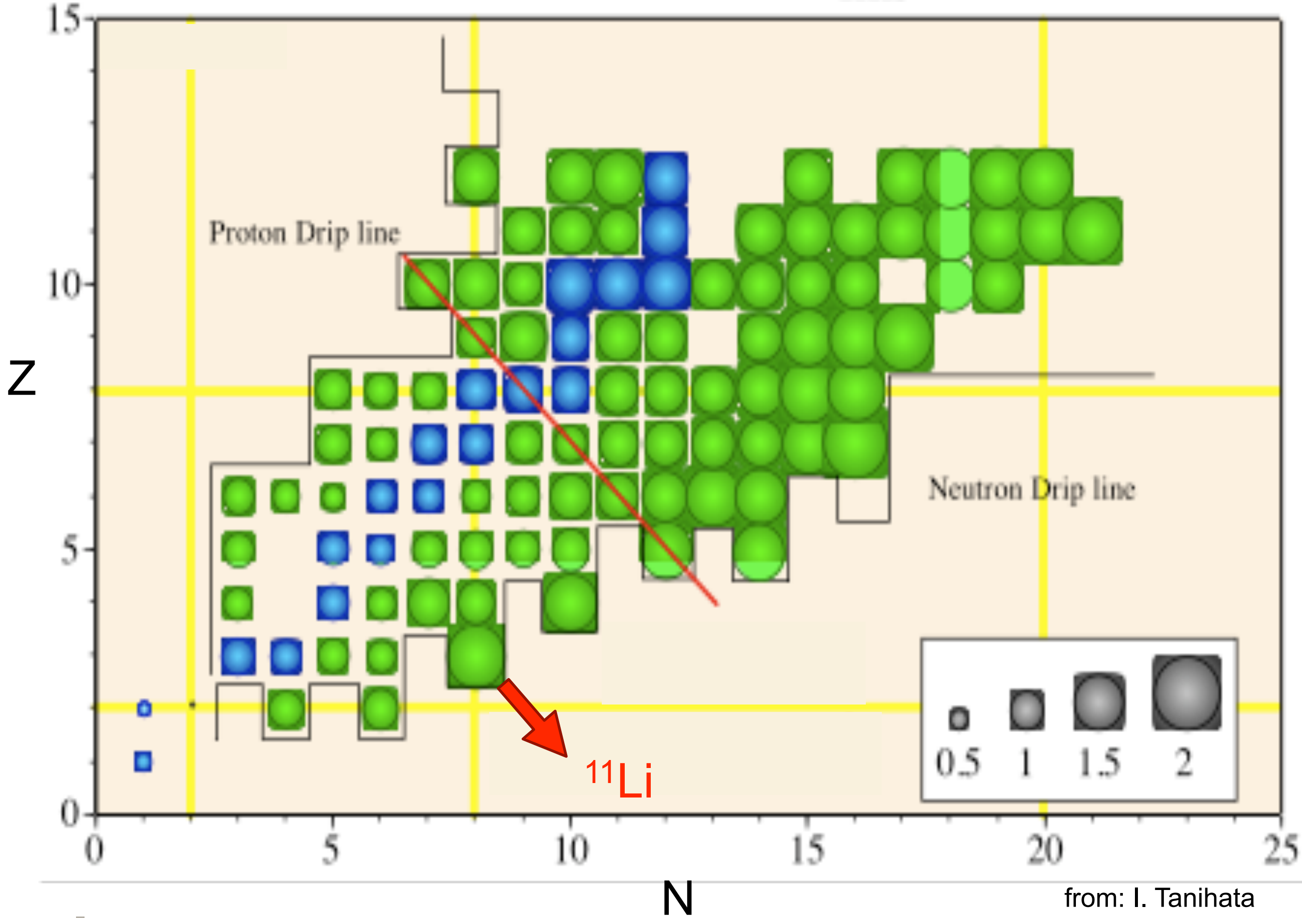
- Neutron halos: Large n/p ratio (neutron-rich)

Halo	n/p
^{6}He	2
^{8}He	3
^{11}Li	2.66
^{12}C	1

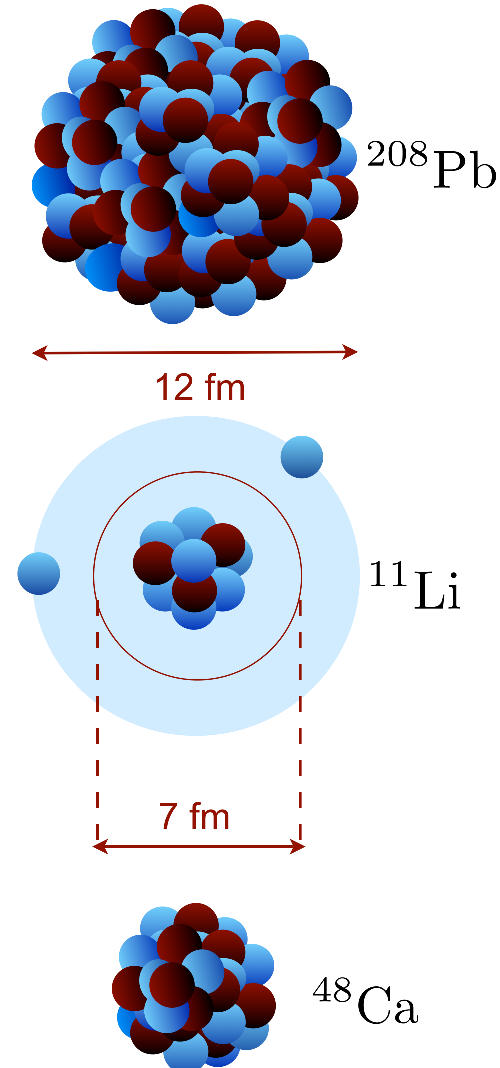
Halo Nuclei

- Large size

Nuclear radius for stable nuclei: $R_N \sim r_0 A^{1/3}$ with $r_0 \sim 1.2$ fm



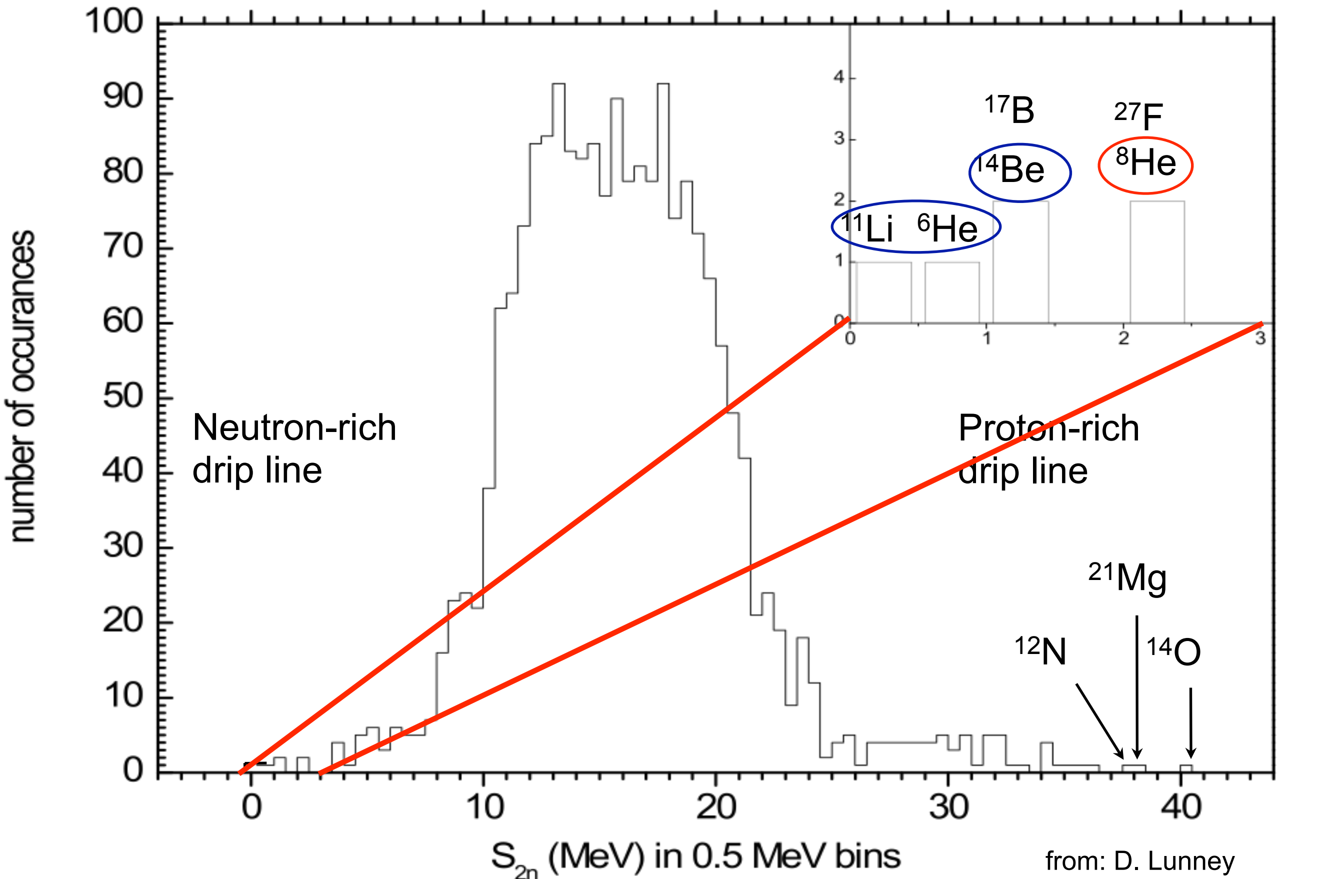
from: I. Tanihata



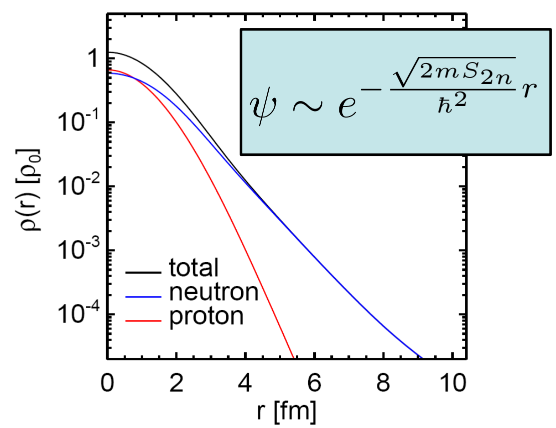
Halo Nuclei

- Small nucleon(s) separation energies

$$S_{2n} = BE(Z,N) - BE(Z,N-2)$$

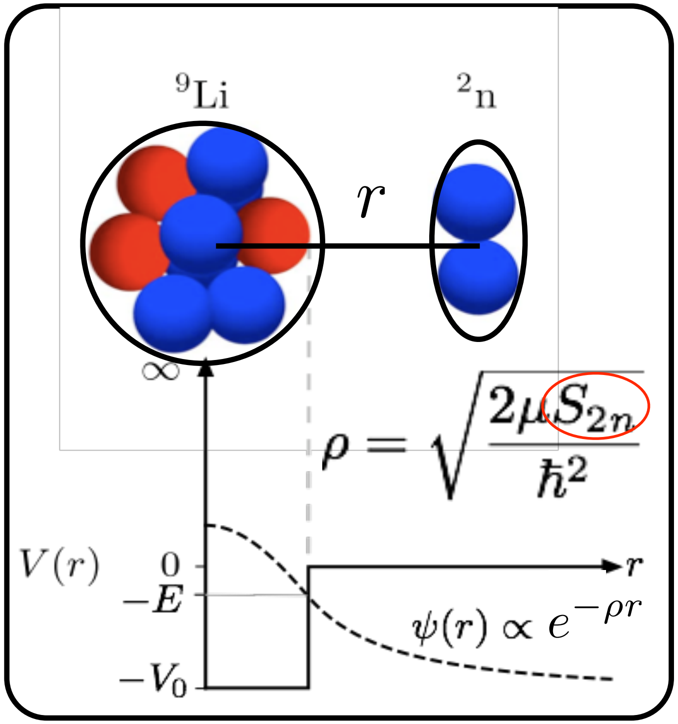


Long tail in the w.f.

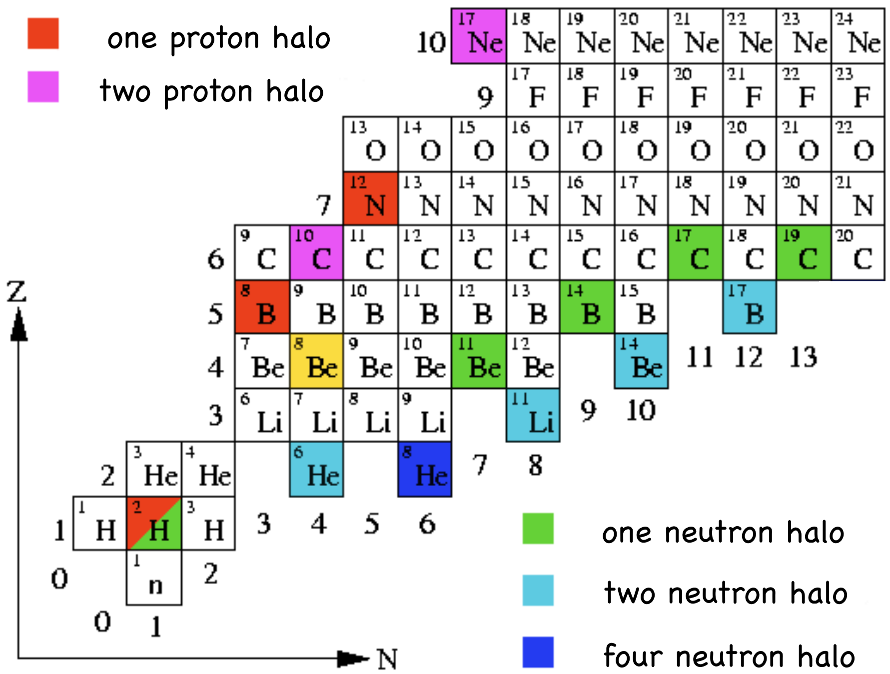


Problem #1

- Why does the w.f. have that exponential fall off?



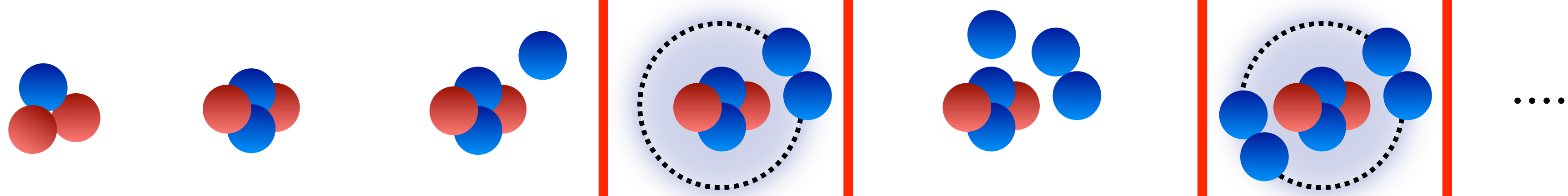
Why are they interesting?



- Their behavior deviates from nuclei in the stability line: we want to understand why?
- Enormous progress from the experimental point of view: new precision era!
- Test our understanding of their exotic structure by comparing theory-experiment
- For the very light halo we can challenge ab-initio methods: test our knowledge on nuclear forces

The helium isotope chain

$^3\mathrm{He}$	$^4\mathrm{He}$	$^5\mathrm{He}$	$^6\mathrm{He}$	$^7\mathrm{He}$	$^8\mathrm{He}$	....



bound	bound	unbound	bound halo	unbound	bound halo

Borromean system

lives 806 ms

Most exotic nucleus "on earth"

$$\frac{N}{Z} = 3$$

lives 108 ms

Even if they are exotic short lived nuclei, they can be investigated experimentally.
From a comparison of theoretical predictions with experiment we can test our knowledge on nuclear forces in the neutron rich region

Halo Nuclei - Experiment

New Era of Precision Measurements for masses and radii

- Masses (and thus binding energies) are measured with Penning traps

  TITAN TRIUMF

  Can reach a relative precision of 10^{-8}

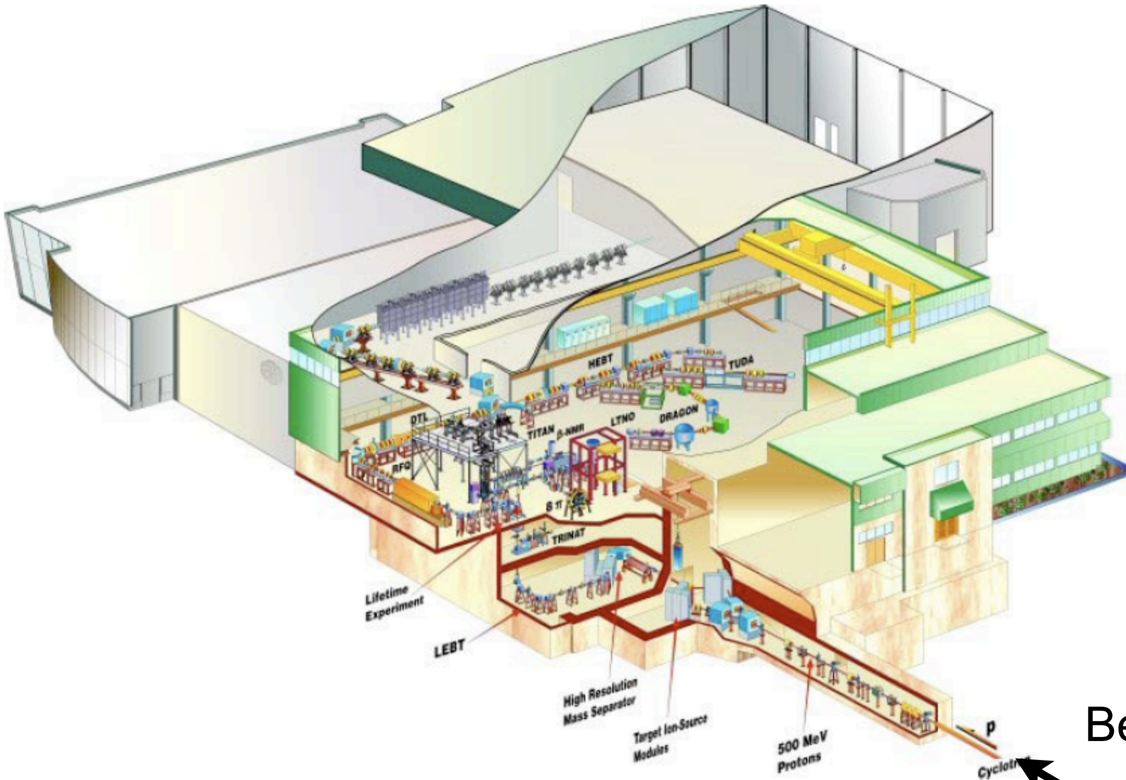
- Charge radii are measured with Laser Spectroscopy

  ARGONNE
  GANIL
  ISOLDE

Halo Nuclei - Experiment

New Era of Precision Measurements for masses and radii

Production of rare isotopes at TRIUMF



Beam

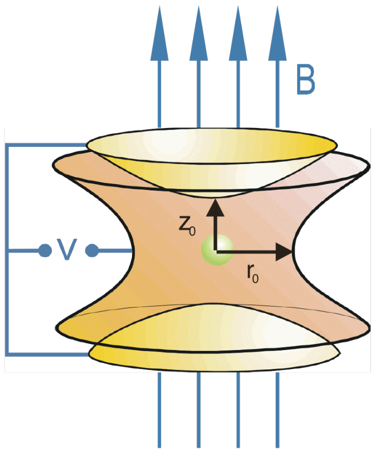
Cyclotron

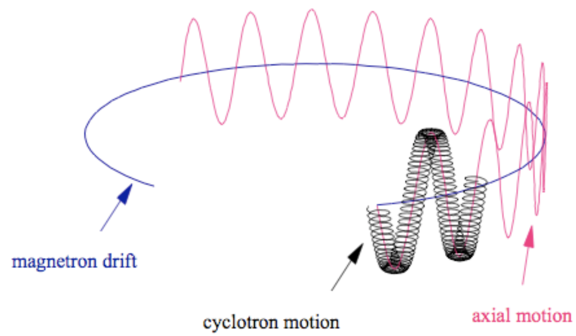
Halo Nuclei - Experiment

New Era of Precision Measurements for masses and radii

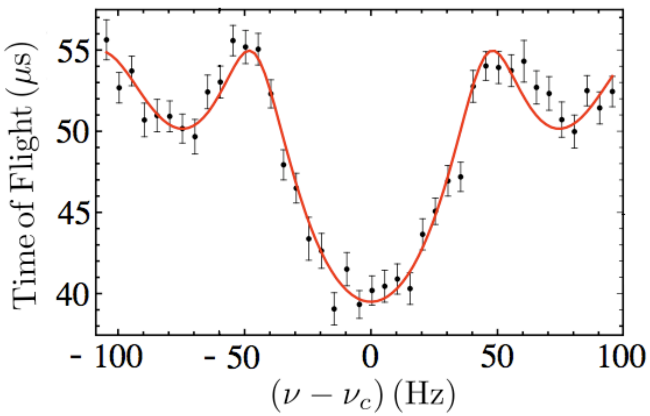
Penning Trap TITAN TRIUMF

Superposition of strong homogeneous magnetic field and a weak electrostatic quadrupole electric field





ion eigenmotions are known



$$\nu_c = \frac{1}{2\pi}\frac{qB}{m} \qquad \frac{\delta m}{m} = \frac{\delta\nu_c}{\nu_c} = \frac{\delta\nu_c 2\pi m}{qB}$$

$$\delta\nu_c \sim \frac{1}{T}\frac{1}{\sqrt{N}}$$

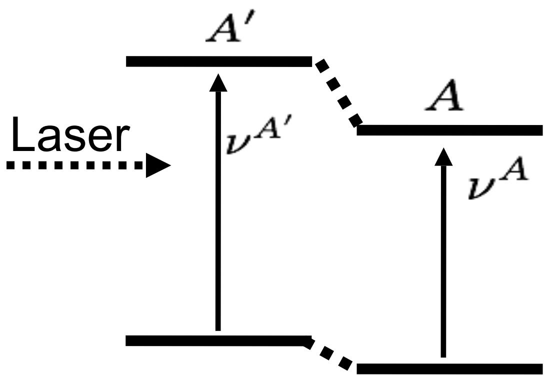
$$\frac{\delta m}{m} = \frac{m}{qTB\sqrt{N}}$$

known from the beam

Halo Nuclei - Experiment

New Era of Precision Measurements for masses and radii

Laser Spectroscopy for radii



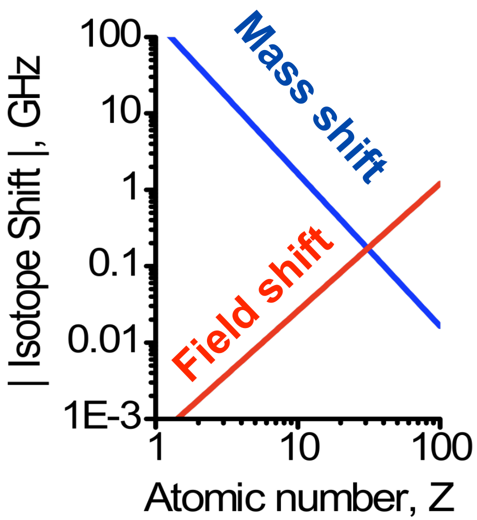
Experiment

$$\overbrace{\delta\nu^{A,A'} = \nu^{A'} - \nu^{A}}$$

Theory: from precise atomic structure calculations

$$= \underbrace{\delta\nu^{mass}_{A,A'}}_{\text{Mass shift}} + \underbrace{K\delta\langle r^2\rangle_{AA'}\text{ch}}_{\text{Field shift}}$$

Mass shift Field shift

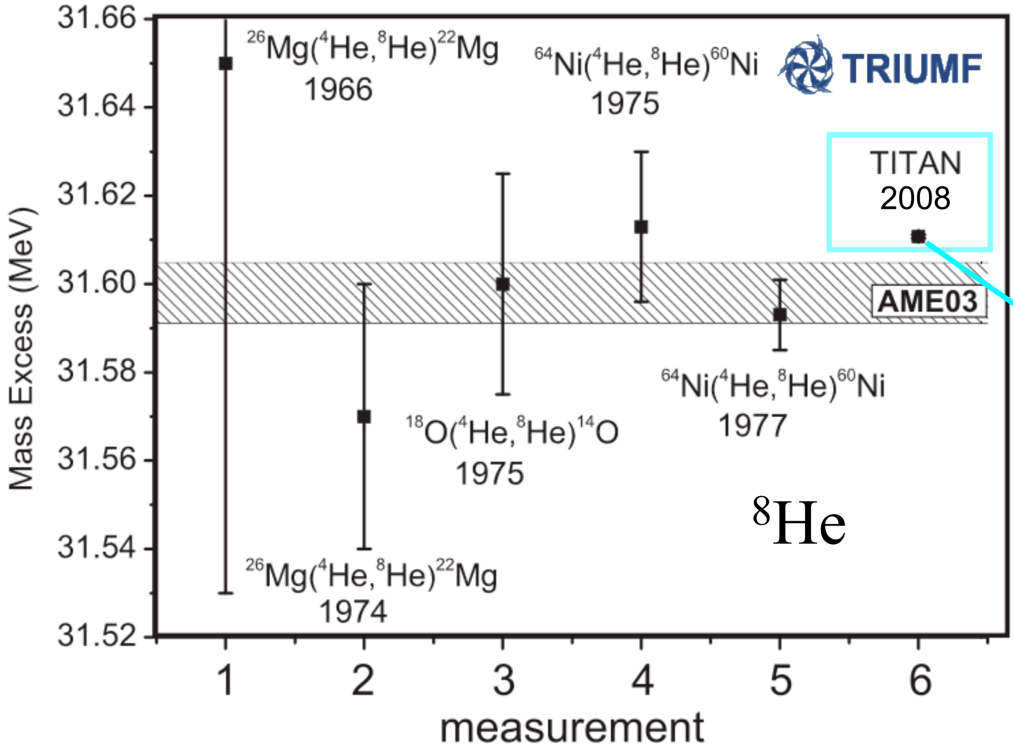


- Mass shifts dominates for light nuclei
- Nuclear masses are input for calculations of K → can be the largest source of systematic errors if not known precisely
- Precise mass measurements are key for a better determination of radii

Halo Nuclei - Experiment

New Era of Precision Measurements for masses and radii

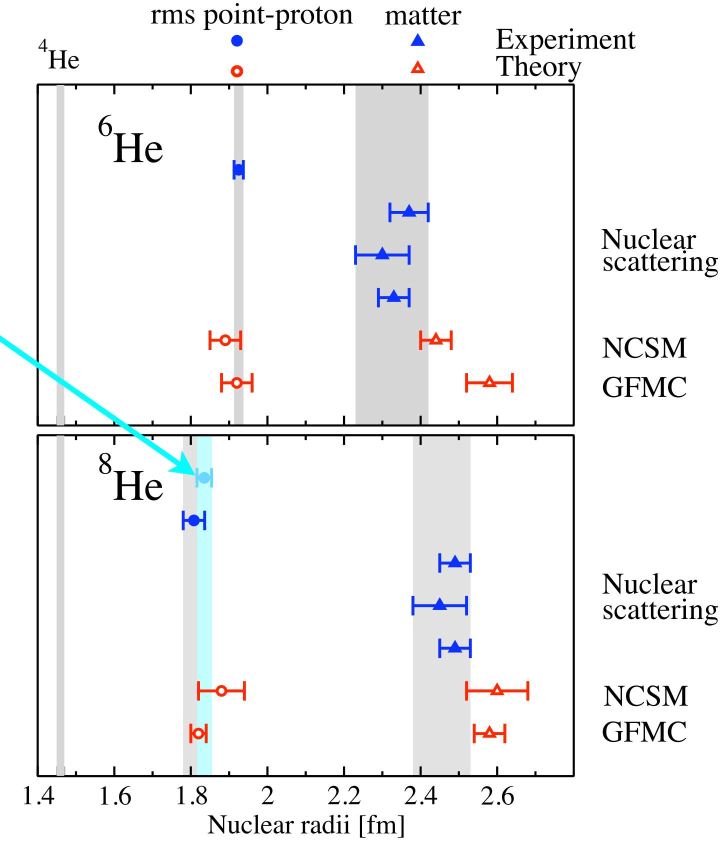
Mass measurement of ^{8}He with the Penning trap



TRIUMF, Ryjkov et al. PRL 101, 012501 (2008)

Masses and radii of helium isotopes are important challenges for theory!

Measurement of charge radii via isotope shift



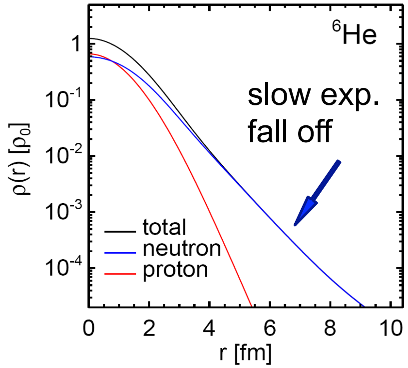
ARGONNE, Wang et al. PRL 93, 142501 (2004)
GANIL, Mueller et al. PRL 99, 252501 (2007)

$$\langle r_p^2 \rangle = \langle r_{ch}^2 \rangle - \langle R_p^2 \rangle - \frac{3}{4M_p^2} - \frac{N}{Z}\langle R_n^2 \rangle$$

Halo Nuclei -Theory

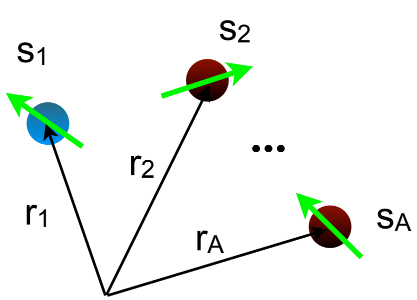
Why are halo nuclei a challenge to theory?

- It is difficult to describe the long extended wave function
- They test nuclear forces at the extremes, where less is known



Ab-initio calculations:

treat the nucleus as an A-body problem



full antisymmetrization of the w.f.

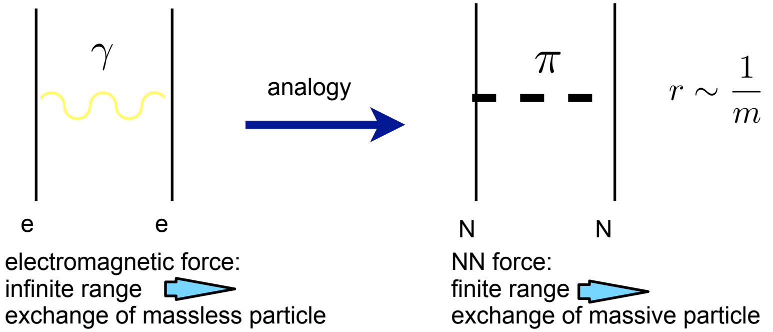
use modern Hamiltonians to predict halo properties

$$H = T + V_{NN} + V_{3N} + \ldots$$

Methods: GFMC, NCSM, CC, HH, ...

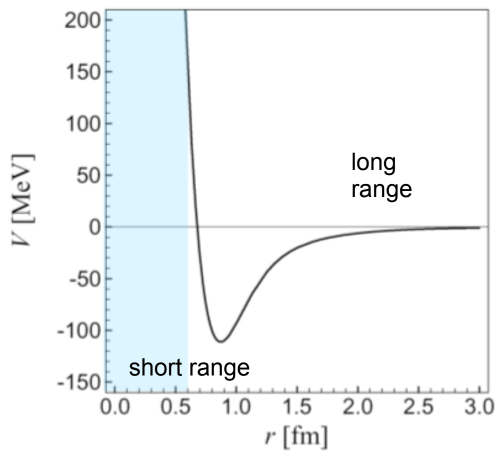
History of the nuclear interactions: NN potentials

- 1935 Yukawa: one pion exchange (OPE) model

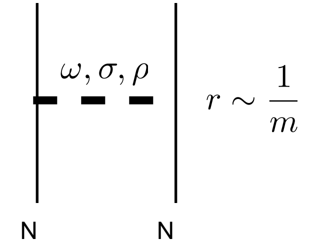


REALISTIC NN POTENTIALS: fit to NN scattering data

- 1960s one boson exchange model



OPE +



long range

short range

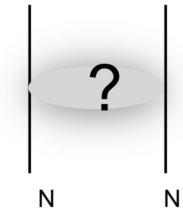
...

- 1970s: two-pion exchange added

...

- 1980s: OPE + phenomenology
  40 parameters to fit phase shifts AV18

OPE +



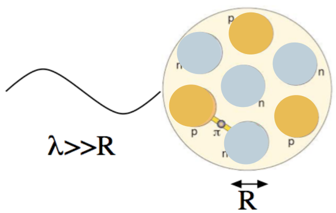
long range

short range phenomenology

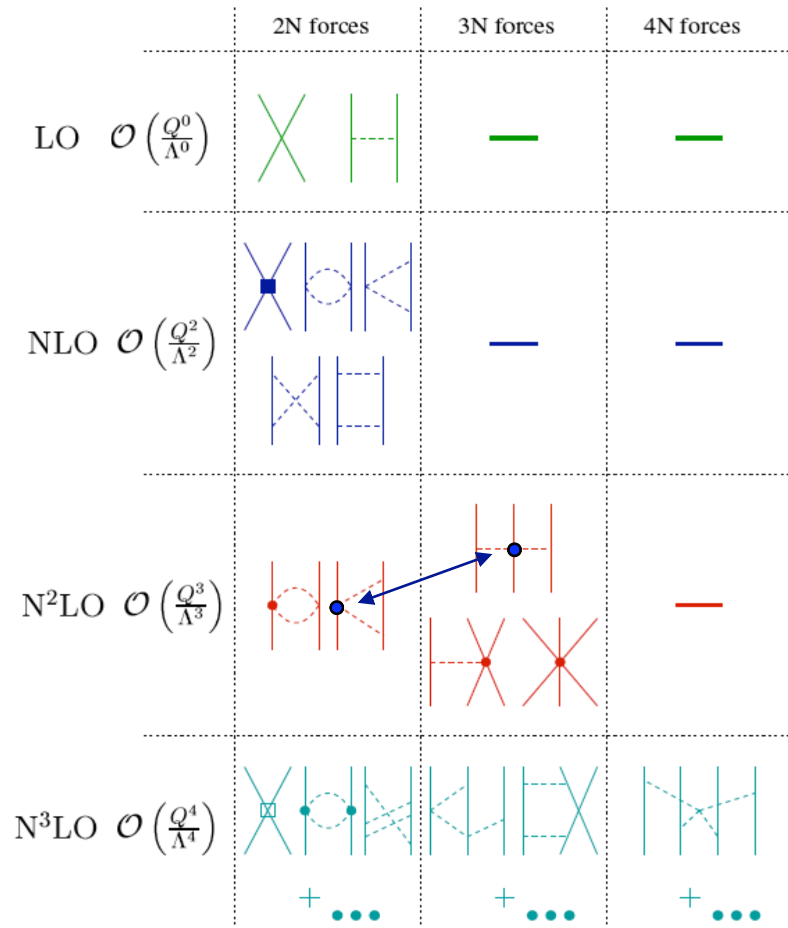
- 1990-today: new vision of effective field theory

Nuclear Forces Frontiers

Effective Field Theory: Bridges the non-perturbative low-energy regime of QCD with forces among nucleons



$$\frac{1}{\lambda} = Q \ll \Lambda_b = \frac{1}{R}$$



Use effective degrees of freedom: p,n,pions

Construct the most general Hamiltonian which is consistent with the chiral symmetry of QCD

Have a systematic expansion of the Hamiltonian in terms of diagrams

$$\mathcal{L} = \sum_k c_k \left(\frac{Q}{\Lambda_b}\right)^k$$

Power counting

$$k = -4 + 2N + 2L + \sum_i (d_i + n_i/2 - 2)$$

Fix the short range couplings on experiment

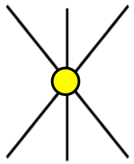
Problem #2

Given the power counting:

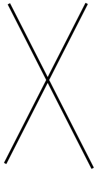
$$k = -4 + 2N + 2L + \sum_i (d_i + n_i/2 - 2)$$

- N: # of nucleons
- L: # of loops
- $\sum_i$: # of nucleon vertices
- d_i: # of derivatives, momenta or pion mass
- n_i: # of nucleon lines

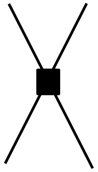
- Calculate the order k of this three-body diagram



- What is the difference between these two diagrams?



$k = 0$



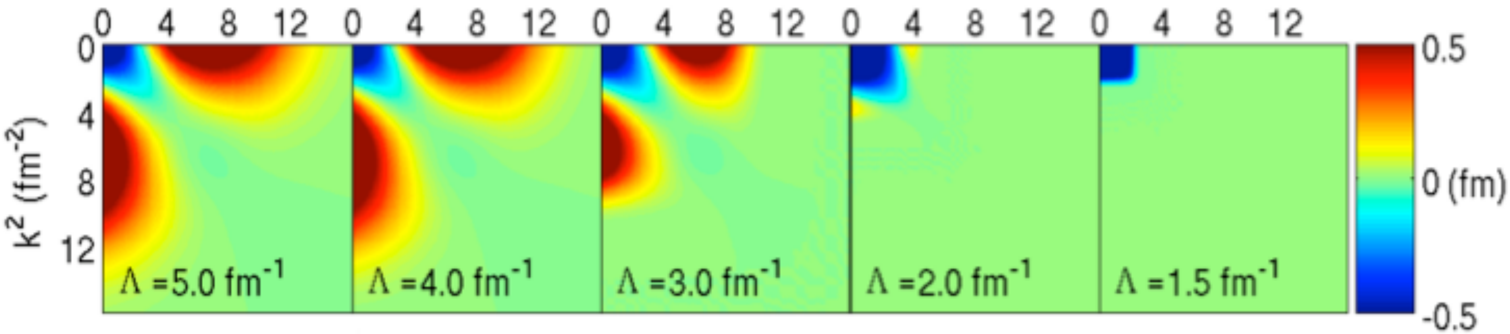
$k = 2$

Effective field theory potentials and low-momentum evolution

Evolution of 2N forces: phase-shift equivalent

Low-momentum interactions: Bogner, Kuo, Schwenk (2003) need smaller basis

Like acting with a unitary transformation $U^{-1}VU$ still preserve phase-shifts and properties of 2N systems

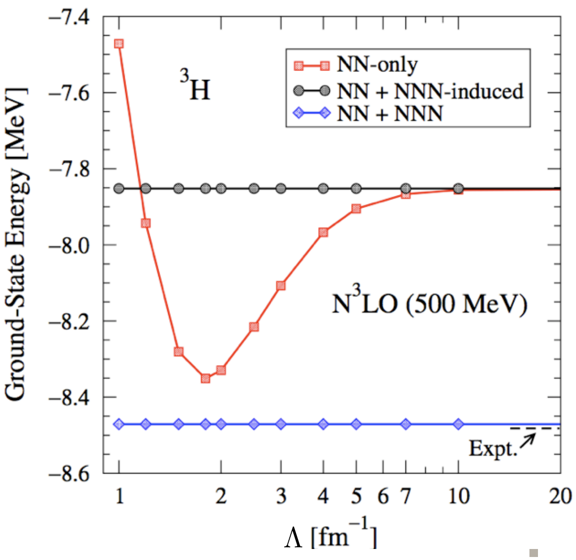


$\Lambda \sim \frac{1}{r}$ "freedom"

$$H(\Lambda) = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \ldots$$

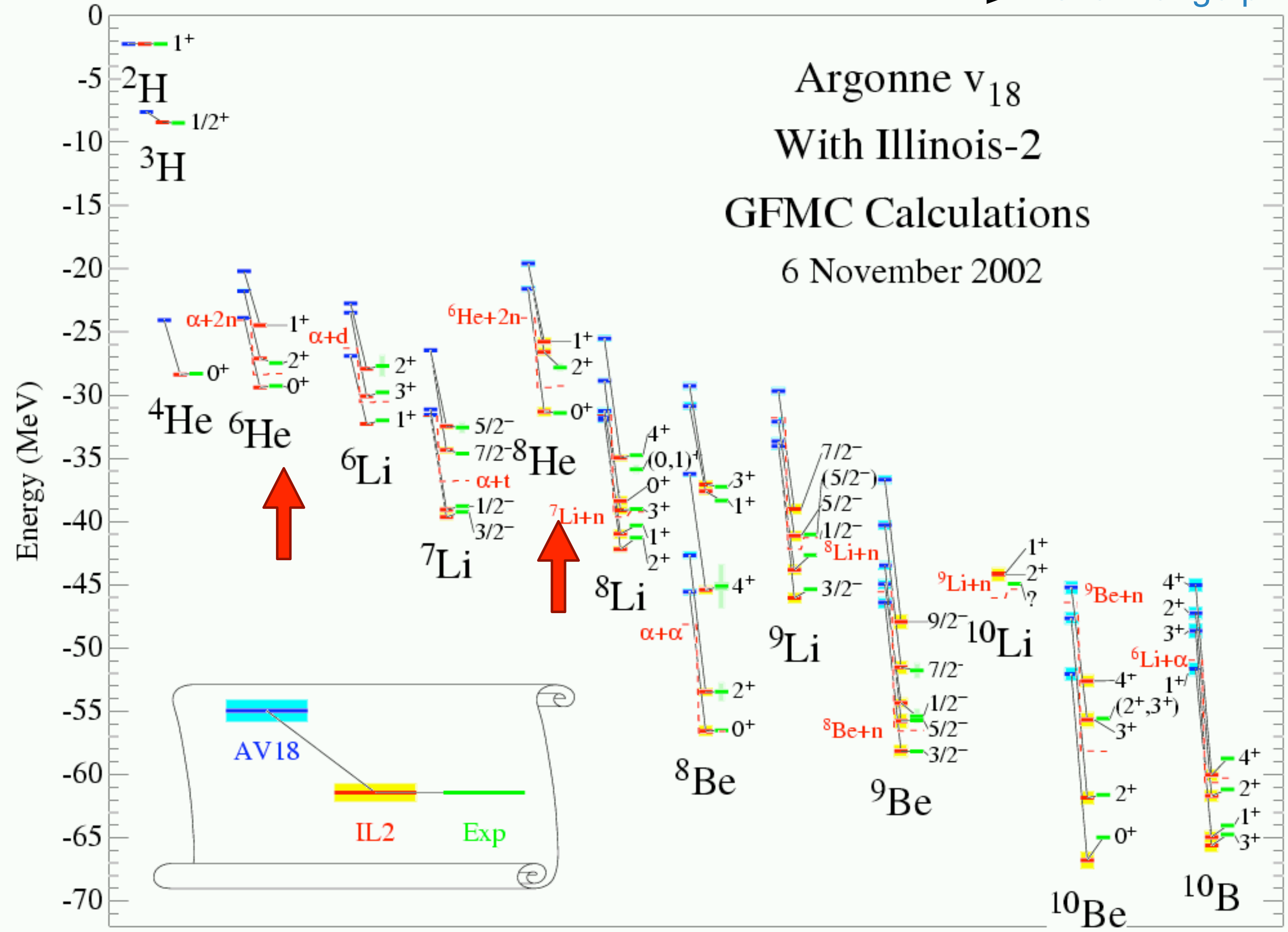
Variation of the cutoff provides a tool to estimate the effect of 3N forces

Can evolve consistently 3N forces: Jurgenson, Navratil, Furnstahl, (2009)



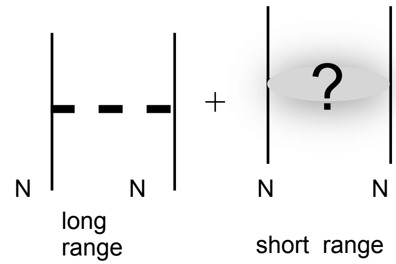
Ab-initio Calculations for Halo Nuclei

GFMC Quantum Monte Carlo Method,
Uses local two- and three-nucleon forces → short range phenomenology



Pieper et al. (2002)

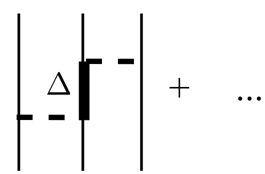
AV18 - NN force



does not bind the helium halo with respect to 2n emission

IL2 - 3N force

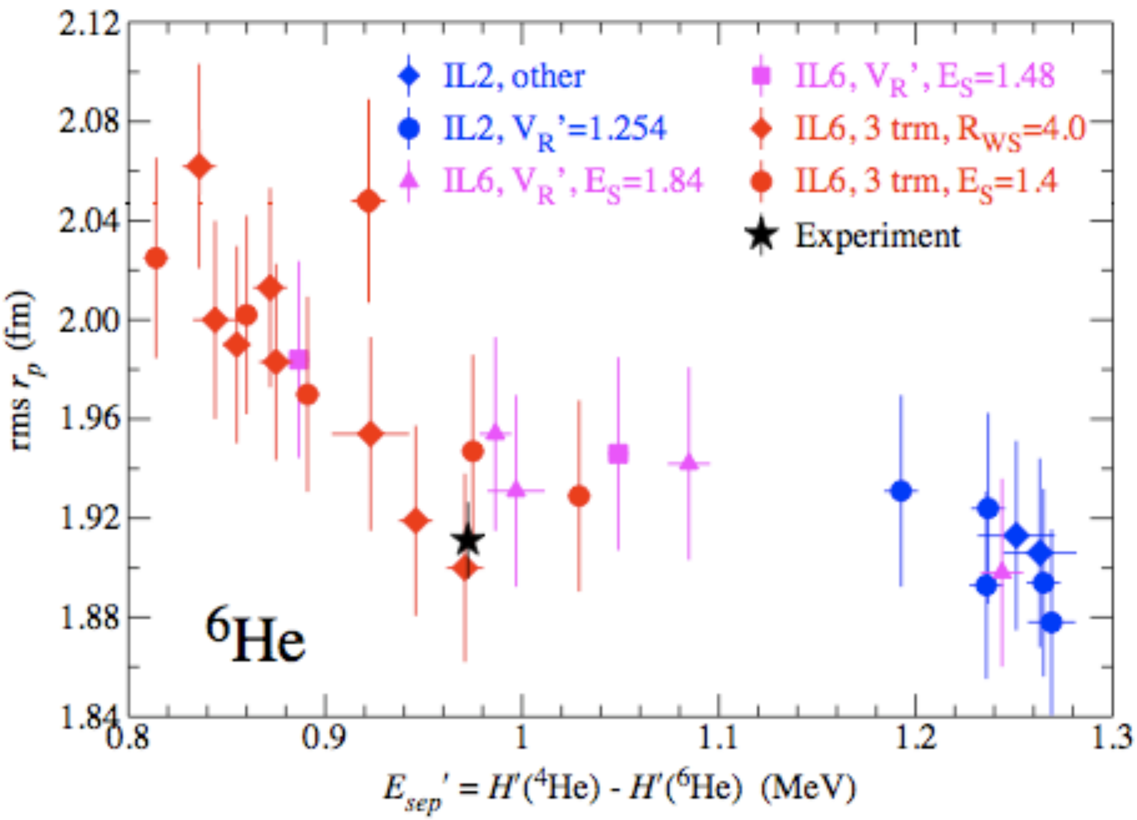
$$V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{3\pi,R} + V_{ijk}^{R}$$



N.B.: parameters of the IL2 force are obtained from a fit of 17 states of A<9 including the binding energy of ^{6}He and ^{8}He

Ab-initio Calculations for Halo Nuclei

- GFMC estimation of the proton radius -



S.C. Pieper, arXiv:0711.1500, proceedings of Enrico Fermi School

Ab-initio Calculations for Halo Nuclei

NCSM

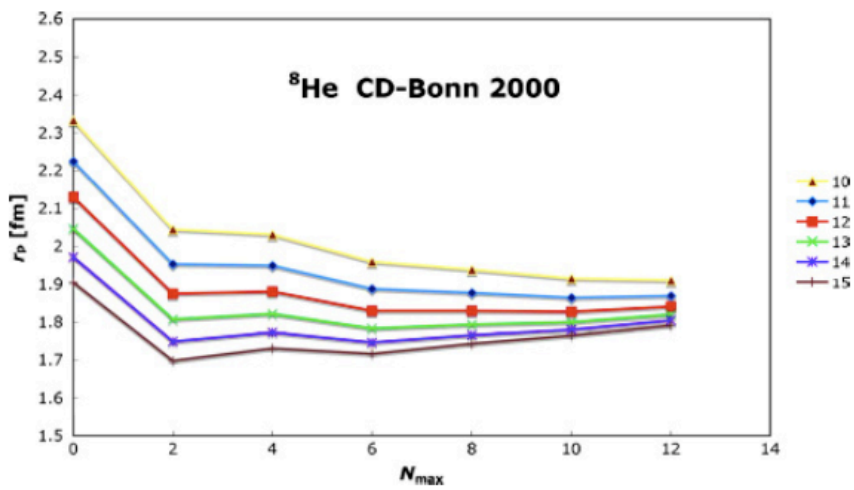
Diagonalization Method using Harmonic Oscillator Basis $\psi_{nl}(r) \sim e^{-\nu r^2} L_n^{l+1/2}(2\nu r^2) \qquad \nu = m\omega/2\hbar$

Can use non-local two- and ~~three-nucleon~~ forces

so far not for halo nuclei in large spaces

Helium Isotopes

Caurier and Navratil, PRC 73, 021302(R) (2006)

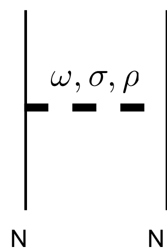


Slow convergence and HO parameter dependence in radius

CD-Bonn → meson exchange theory

short range phenomenology

OPE +



long range

short range

E_B [MeV]	Expt.	CD-Bonn 2000
^{4}He	28.296	26.16 (6)
^{6}He	29.269	26.9 (3)
^{8}He	31.408 (7)	26.0 (4)

Towards Halo Nuclei from EFT

Ideally we want:

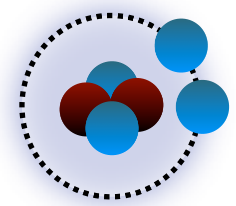
1. To use methods that enable to incorporate the correct asymptotic of the w.f. for loosely bound systems

2. To obtain convergent calculations, with no dependence on the model space parameters

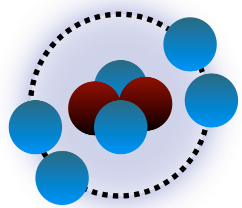
3. To systematically study the cutoff (in)dependence of predicted observables with two- and three-body forces

Two methods that enable us to achieve point 1. and 2.:

- Hyper-spherical Harmonics Expansion for ^{6}He



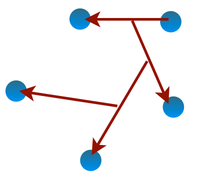
- Cluster Cluster Theory for ^{8}He



Hyper-spherical Harmonics

- Few-body method - uses relative coordinates

$$|\psi(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A)\rangle = |\varphi(\vec{R}_{CM})\Psi(\vec{\eta}_1,\vec{\eta}_2,\ldots,\vec{\eta}_{A-1})\rangle$$



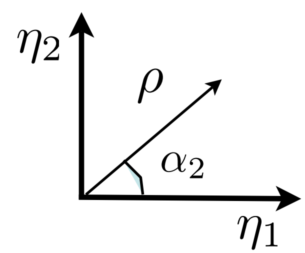
$\vec{\eta}_0 = \sqrt{A}\vec{R}_{CM} \quad \vec{\eta}_1, \ldots, \vec{\eta}_{A-1}$

Recursive definition of hyper-spherical coordinates

$$\rho, \Omega \qquad \rho^2 = \sum_{i=1}^{A} r_i^2 = \sum_{i=1}^{A-1} \eta_i^2$$

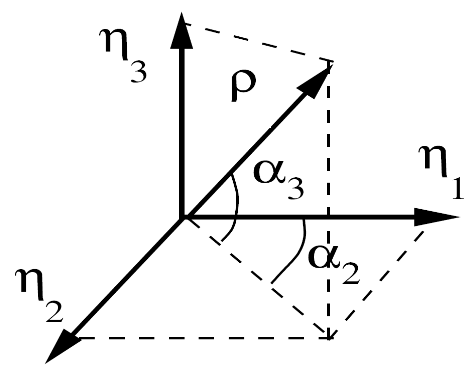
A=3

$$\begin{cases} \vec{\eta}_1 = \{\eta_1, \theta_1, \phi_1\} \\ \vec{\eta}_2 = \{\eta_2, \theta_2, \phi_2\} \end{cases} \qquad \begin{cases} \rho = \sqrt{\eta_1^2 + \eta_2^2} \\ \sin\alpha_2 = \frac{\eta_2}{\rho} \end{cases}$$



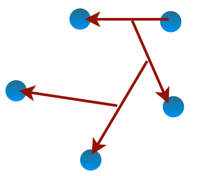
A=4

$$\begin{cases} \vec{\eta}_1 = \{\eta_1, \theta_1, \phi_1\} \\ \vec{\eta}_2 = \{\eta_2, \theta_2, \phi_2\} \\ \vec{\eta}_3 = \{\eta_3, \theta_3, \phi_3\} \end{cases} \qquad \begin{cases} \rho = \sqrt{\eta_1^2 + \eta_2^2 + \eta_3^2} \\ \sin\alpha_2 = \frac{\eta_2}{\rho} \\ \sin\alpha_3 = \frac{\eta_3}{\rho} \end{cases}$$



Hyper-spherical Harmonics

- Few-body method - uses relative coordinates $|\psi(\vec{r}_1,\vec{r}_2,\dots,\vec{r}_A)\rangle = |\varphi(\vec{R}_{CM})\Psi(\vec{\eta}_1,\vec{\eta}_2,\dots,\vec{\eta}_{A-1})\rangle$



$\vec{\eta}_0 = \sqrt{A}\vec{R}_{CM} \quad \vec{\eta}_1, \dots, \vec{\eta}_{A-1}$

Recursive definition of hyper-spherical coordinates

$$\rho, \Omega \qquad \rho^2 = \sum_{i=1}^{A} r_i^2 = \sum_{i=1}^{A-1} \eta_i^2$$

$$H(\rho,\Omega) = T_\rho + \frac{K^2(\Omega)}{\rho^2}$$

$$\Psi = \sum_{[K],\nu}^{K_{max},\nu_{max}} c_\nu^{[K]} e^{-\rho/2b} \rho^{n/2} L_\nu^n(\frac{\rho}{b}) [\mathcal{Y}_{[K]}^{\mu}(\Omega) \chi_{ST}^{\bar{\mu}}]_{JT}^a$$

Asymptotic $e^{-a\rho}$ $\rho \to \infty$

Model space truncation $K \leq K_{max}$, Matrix Diagonalization $\langle\psi|H_{(2)}|\psi\rangle = \frac{A(A-1)}{2}\langle\psi|H_{(A,A-1)}|\psi\rangle$

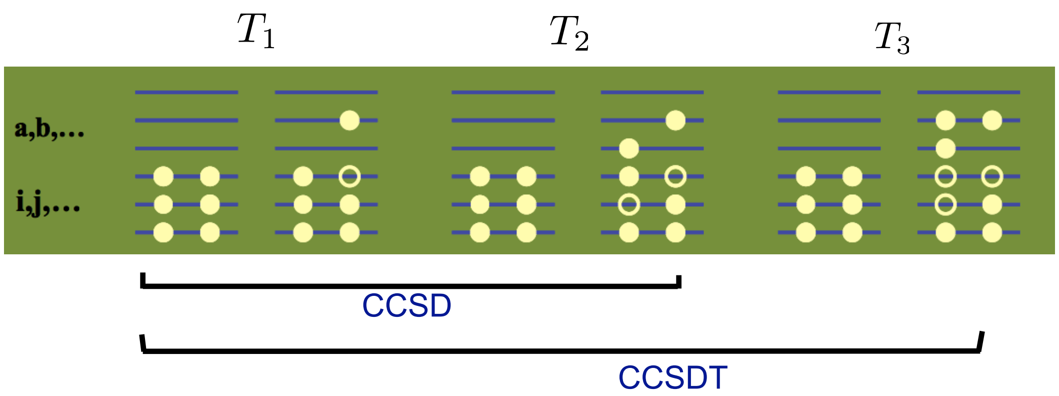
Can use non-local interactions

Most applications in few-body; challenge in A>4 Barnea and Novoselsky, Ann. Phys. 256 (1997) 192

Coupled Cluster Theory

$$|\psi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle = e^T |\phi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle \qquad T = \sum T_{(A)}$$

→ reference SD

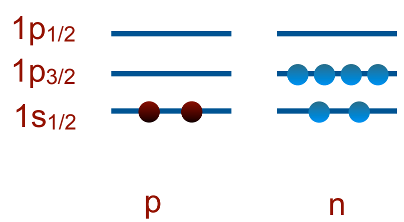


CCSD Equations

$$E = \langle \phi | e^{-T} H e^{T} | \phi \rangle$$
$$0 = \langle \phi_i^a | e^{-T} H e^{T} | \phi \rangle$$
$$0 = \langle \phi_{ij}^{ab} | e^{-T} H e^{T} | \phi \rangle$$

Asymptotic $\phi_i \sim e^{-k_i r_i} \quad r \rightarrow \infty$

Use it for ^{8}He, closed shell nucleus



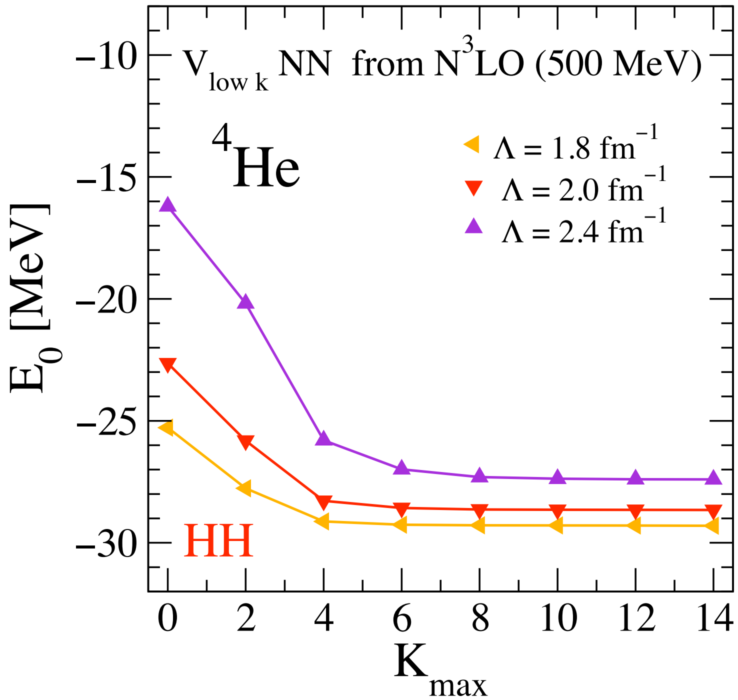
Model space truncation $N \leq N_{max}$

Can use non-local interactions

Applicable to medium-mass nuclei

Benchmark on ^{4}He

HH-CC-FY



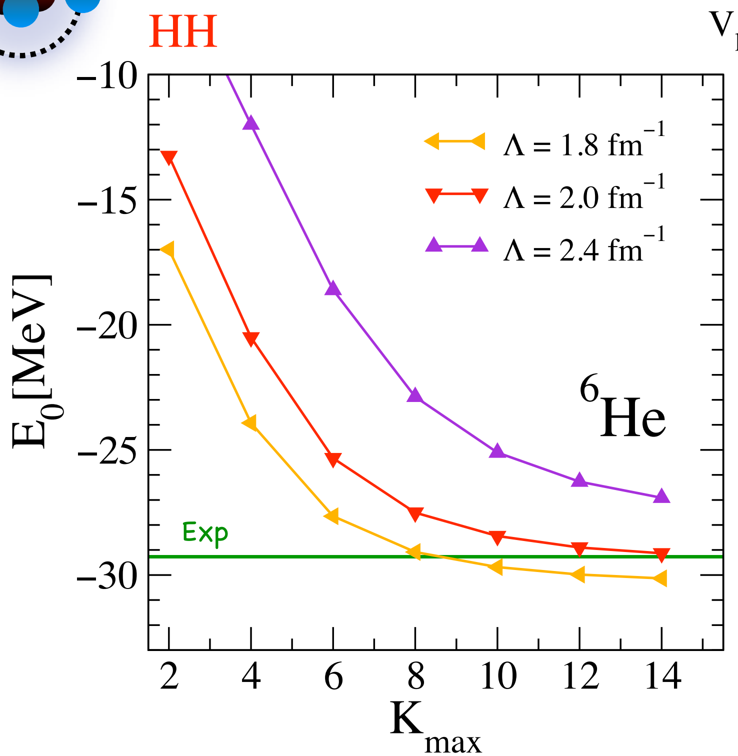
Method	$\Lambda = 2.0\ \mathrm{fm}^{-1}$	$E_0(^4\mathrm{He})$ [MeV]
Faddeev-Yakubovsky (FY)		-28.65(5)
Hyperspherical harmonics (HH)		-28.65(2)
CCSD level coupled-cluster theory (CC)		-28.44
Lambda-CCSD(T) (CC with triples corrections)		-28.63

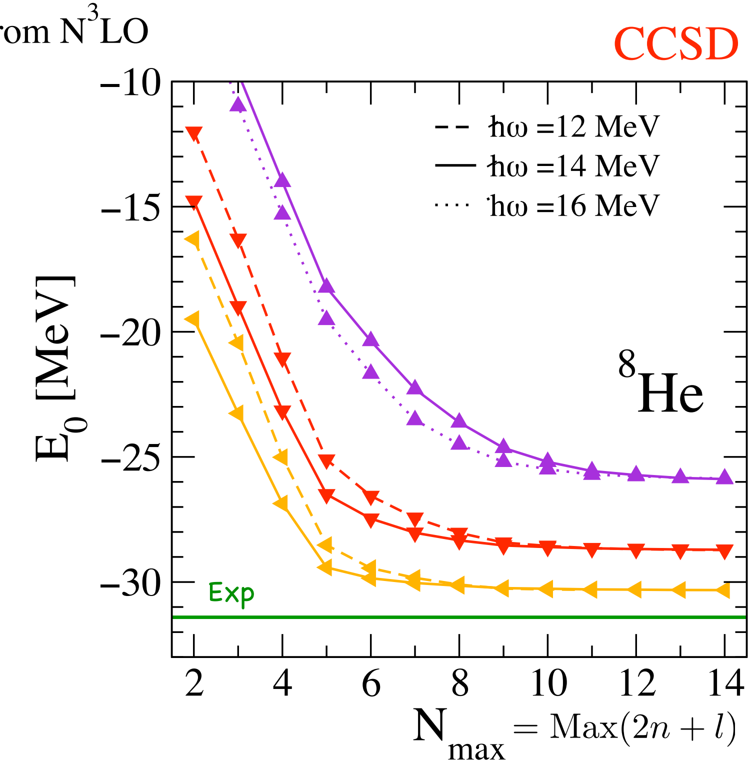
E^{exp}=-28.296 MeV

Helium Halo Nuclei

S.Bacca et al., Eur. Phys. J. A 42, 553 (2009)

$V_{\mathrm{low}\,k}$ NN from N^3LO



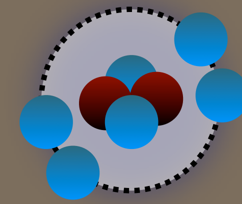


Virtually no model space dependence: can improve K_{max} convergence by exponentially extrapolate

$$E(K_{max}) = E^{\infty} + Ae^{-BK_{max}}$$

Virtually no model space dependence: can improve by adding more correlations

Binding Energy ^{8}He



- CC Theory: Add Triples Correction -

Hilbert space: 15 major shell

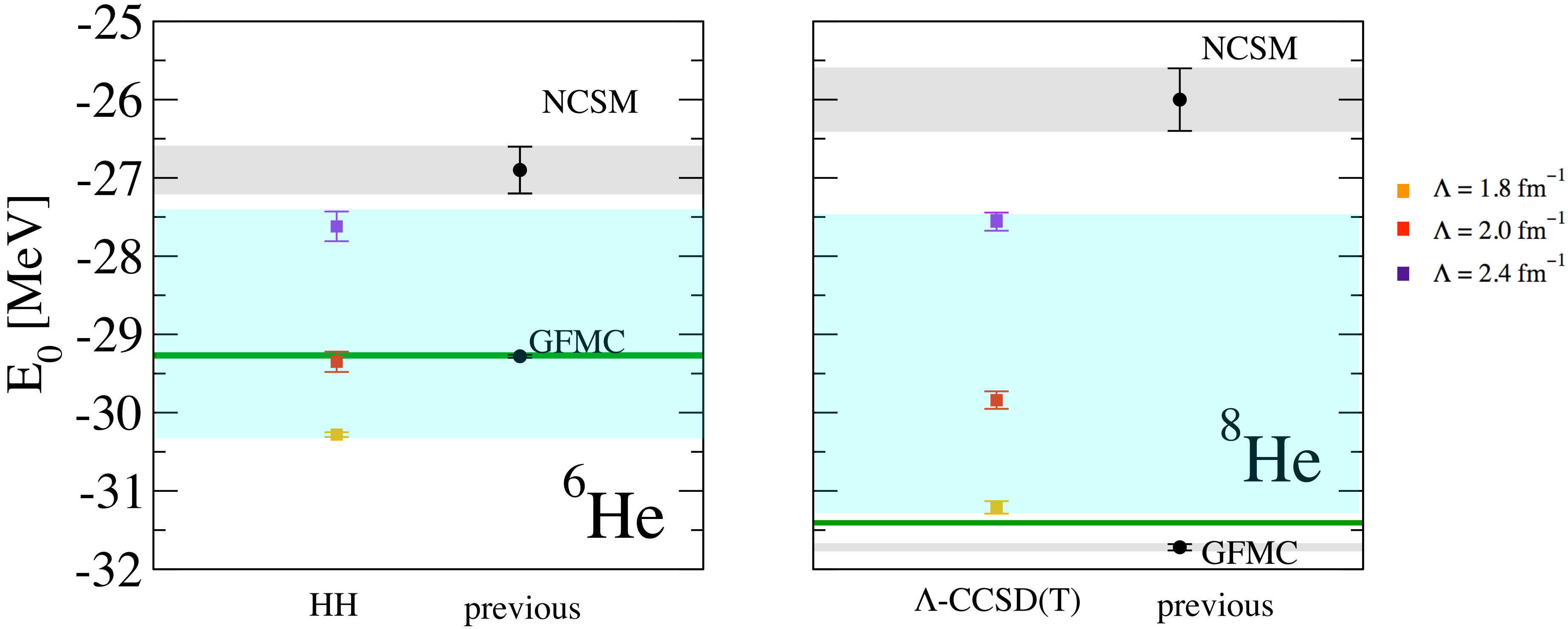
Values in MeV

Λ	E[CCSD]	E[Lambda-CCSD(T)]	Δ
1.8	-30.33	-31.21	0.88
2.0	-28.72	-29.84	1.12
2.4	-25.88	-27.54	1.66

- Triples corrections are larger for larger cutoff
- Their relative effect goes from 3 to 6%

- Q: Why do we gain more energy for larger cutoffs?

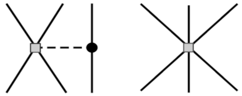
Binding Energy Summary



Experimental data

TITAN ISAC-TRIUMF

Our estimated error in neglected short range 3NFs



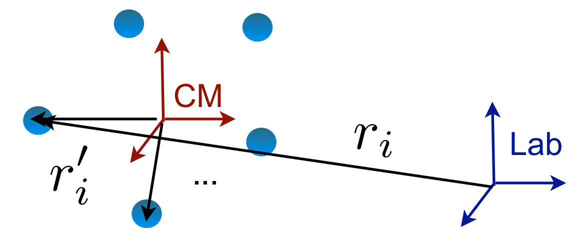
Radii of Halo Nuclei

$$\text{rms matter radius} = \langle \Psi_0 | \hat{r}^2 | \Psi_0 \rangle$$

knows about where all nucleons are

$$\hat{r}^2 = \frac{1}{A} \sum_i \hat{r'}_i^2$$

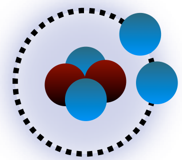
one-body operator



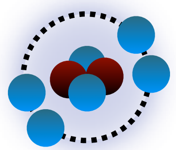
$$r'_i = r_i - R_{CM}$$

In HH for ^{6}He

$$\hat{r}^2 = \frac{1}{A} \hat{\rho}^2$$



In CC for ^{8}He: work with lab coordinates



~~$$\hat{r}^2 = \frac{1}{A} \sum_i \hat{r}_i^2$$~~

not translationally invariant

$$\hat{r}^2 = \frac{1}{A} \sum_i (\hat{r}_i - \hat{R}_{CM})^2$$

$$\text{rms point proton radius} = \langle \Psi_0 | \hat{r}_p^2 | \Psi_0 \rangle$$

knows about where protons are

$$\hat{r}_p^2 = \frac{1}{A} \sum_i \hat{r'}_i^2 \left(\frac{1 + \tau_i^z}{2} \right)$$

TRIUMF

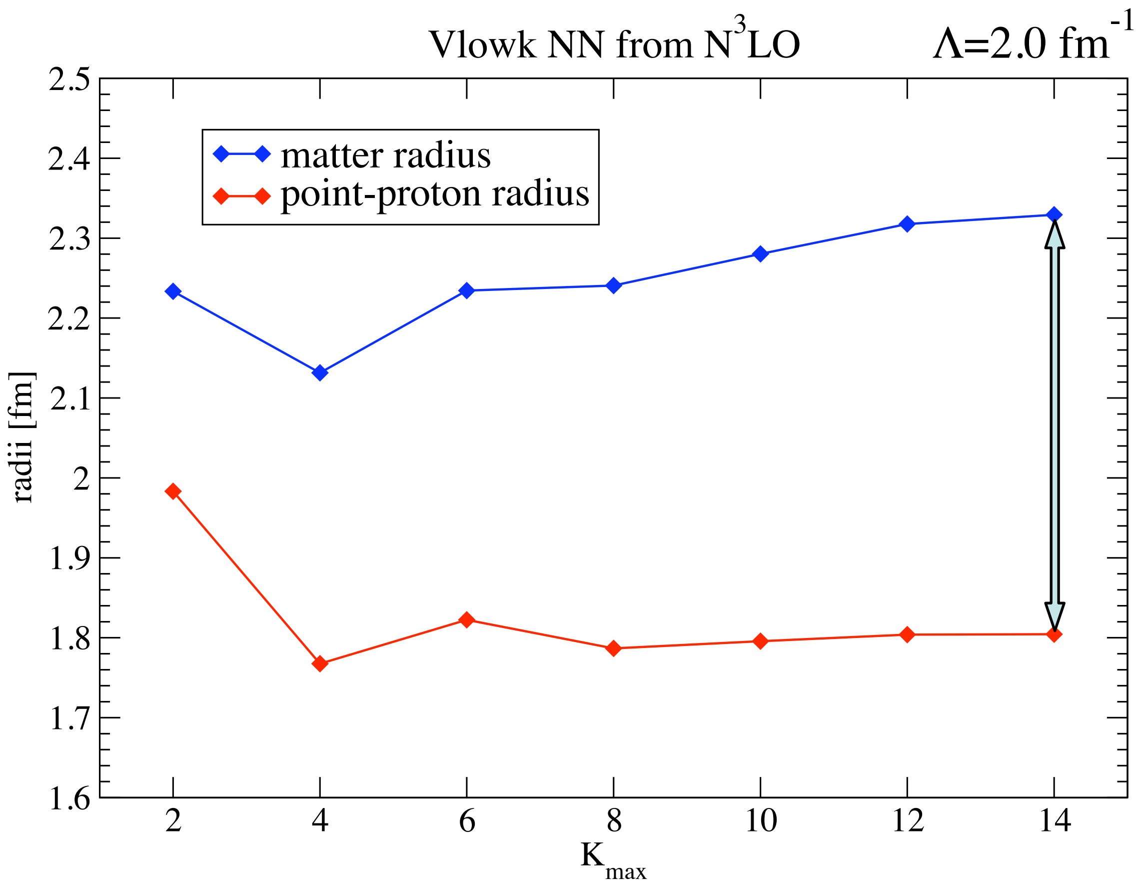
Problem #3

- Derive the expression for the translational invariant matter radius as a two-body operator

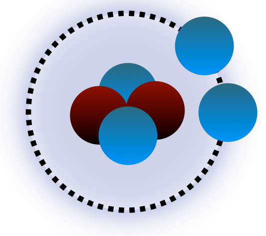
$$\hat{r}^2 = \frac{1}{A}\sum_i(\hat{r}_i - \hat{R}_{CM})^2 \qquad \hat{R}_{CM} = \frac{1}{A}\sum_j \hat{r}_j$$

- Try to do the same with the point proton radius

Radii of Halo Nuclei



^{6}He



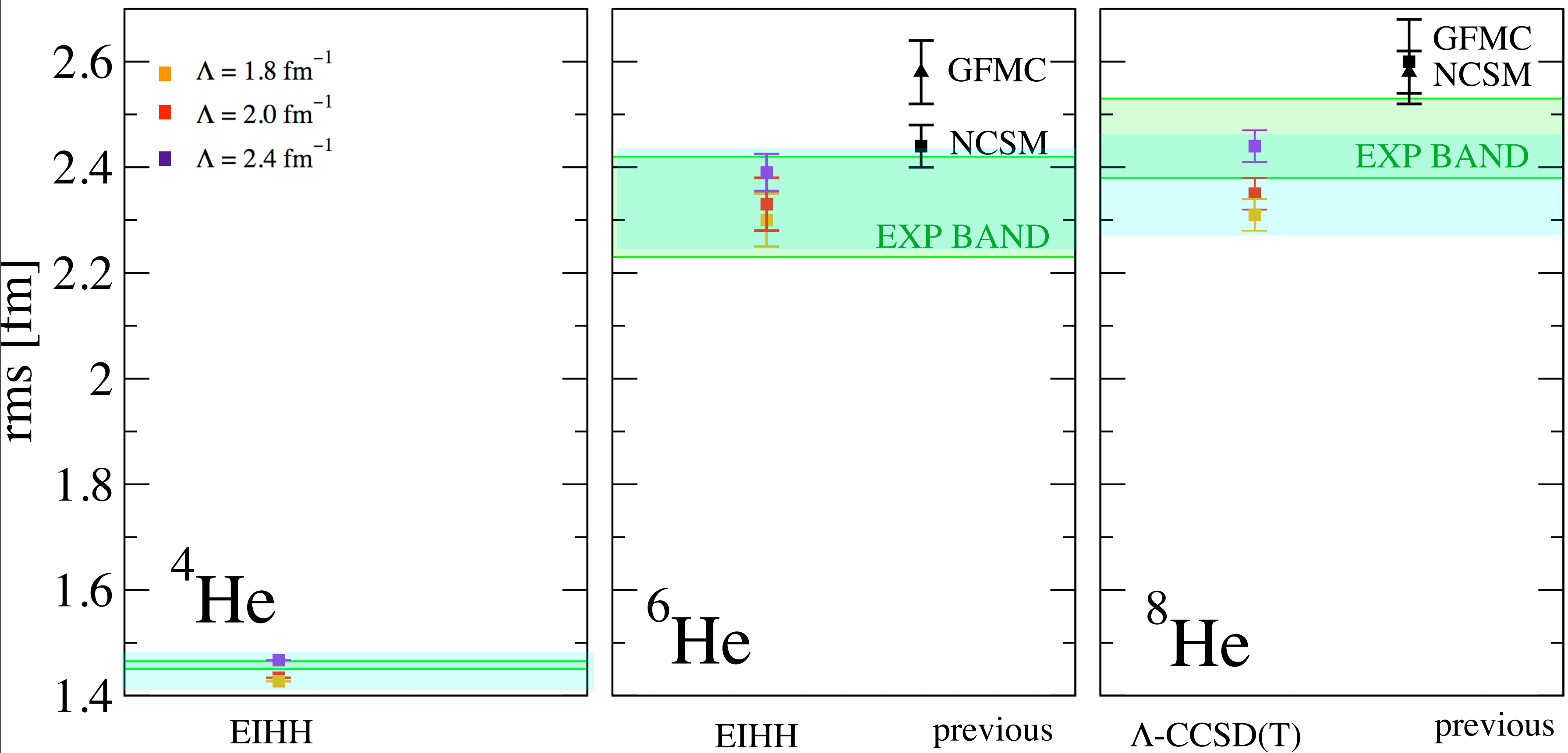
- Point proton radius converges better and are smaller than matter radii

- Q: Why is that?
  Can we understand it?

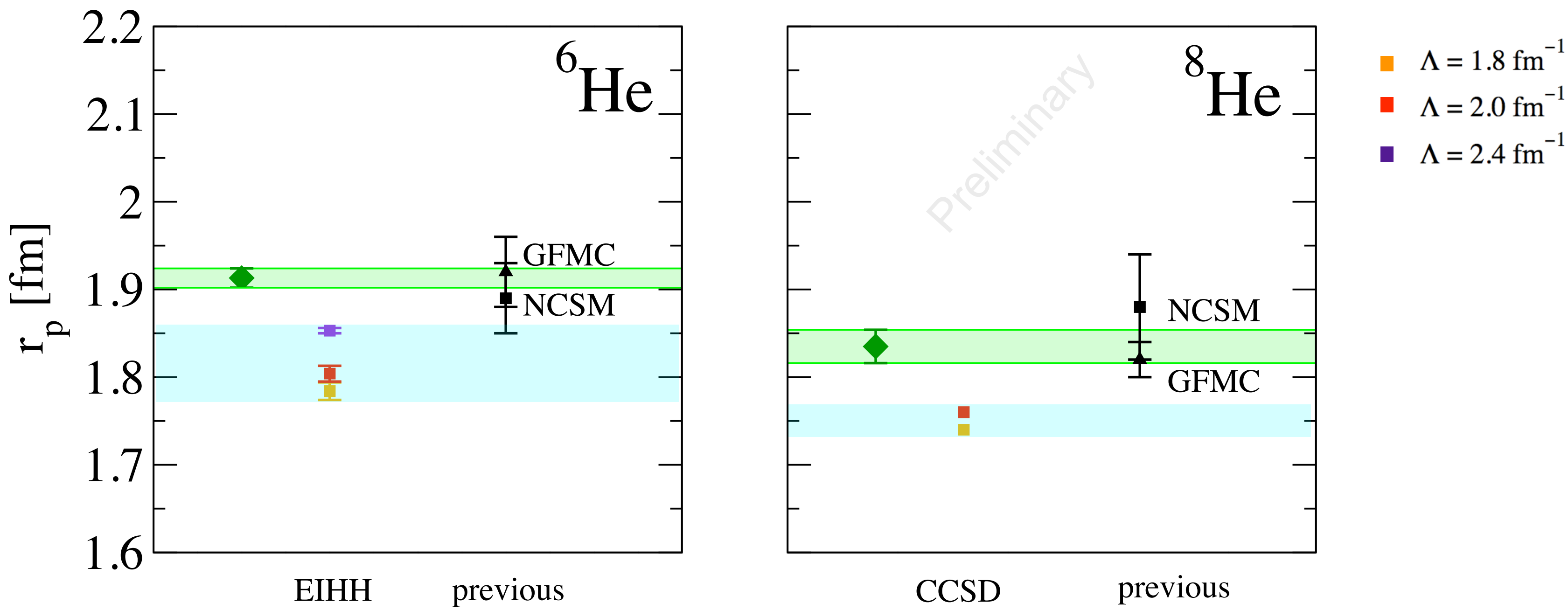
Matter radii Summary

- Benchmark on ^{4}He, CC using translational invariant operators

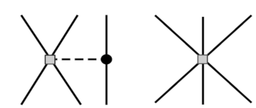
$\Lambda = 2.0 \text{fm}^{-1}$ HH 1.434 fm $\Lambda - \text{CCSD(T)}$ 1.429 fm



Proton radii Summary



◆ from laser spectroscopy using TITAN ISAC-TRIUMF binding energy as input

Our estimated error in neglected  is in percentage consistent with the results for binding energy

- The fact that for some "choice" of the NN force one gets correct radii and wrong energies (or vice-versa) shows that halo nuclei provide important tests of the different aspects of nuclear forces, which includes 3NF

Conclusions

- We provide a description of helium halo nuclei from evolved EFT interactions with the correct asymptotic in the wave function

- We estimate the effect of short range three-nucleon forces on binding energies and radii by varying the cutoff of the evolved interaction

- Our matter radii agree with experiment whereas our point-proton radii under-predict experiment

Future:

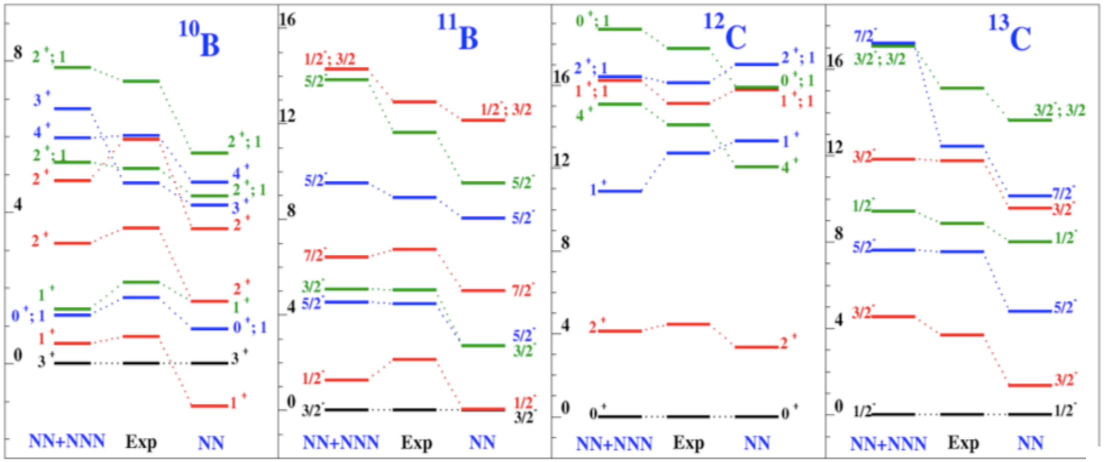
- Include three-nucleon forces

- Extend coupled cluster theory with 3NF calculations to heavier neutron rich nuclei, e.g. lithium or oxygen isotope chain

Effect of Three Nucleon Forces

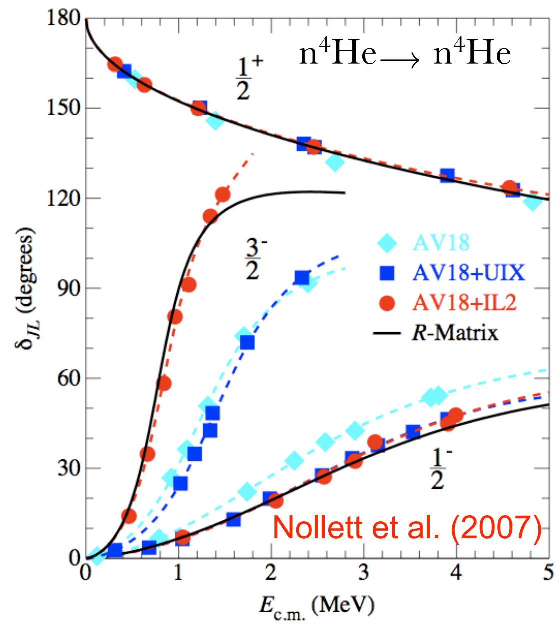
Nuclear low energy spectra

Navratil et al. (2008)



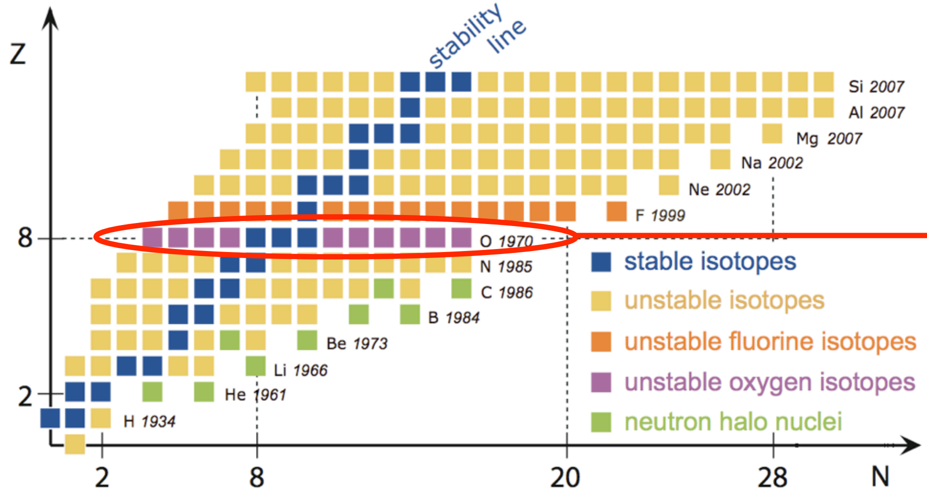
From EFT

Hadronic Reactions



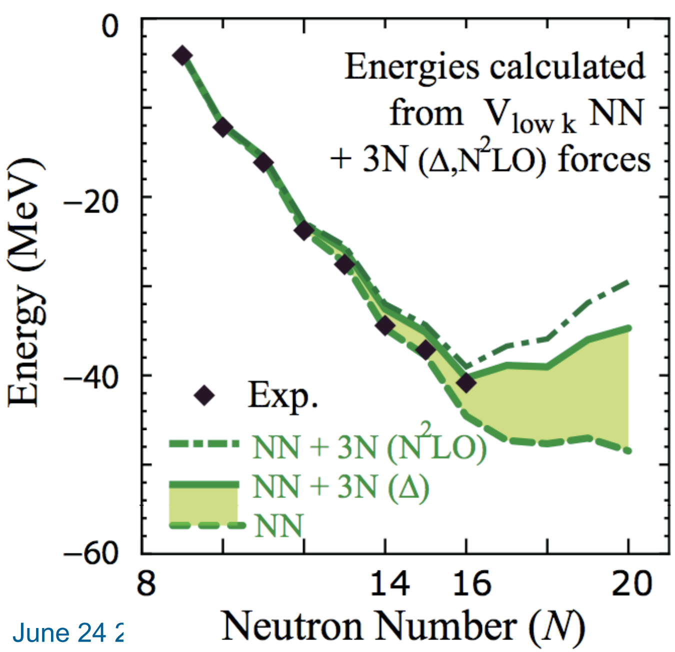
From semi-phenomenological potentials

Effect of Three Nucleon Forces



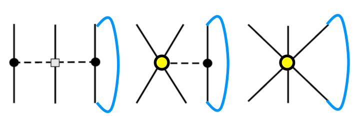
Shell Model: ^{16}O core

any SM calculation with realistic 2NF predicts bound $^{25-28}O$ in contrast with experimental observation



Otsuka, Suzuki, Holt, Schwenk, Akaishi (2009)

First results with 3NF (effective 2NF)



3NF fits to $E(^3H)$ and ^4He rms

Future:
Neutron-rich isotopes with Coupled Cluster Theory (beyond core-approximation) and 3NF