Nuclear Reactions

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Status of lecture

I. Motivation

types and characterization of Nuclear reactions

argument for transport descriptions

II. Heuristic derivation of a transport equation

test particle solution

III. Elementary derivation of Vlasov eq.

Relativistic field theory and relativistic transport eq.

IV. Quantum transport theory

V. Characterization and comparison of codes

Molecular dynamics

VI. Fluctuations and transport theories

Instabilities and phase transitions

Boltzmann-Langevin eq. and approx. treatments

VI. Overview over important results in heavy ion collisions

III.1 Elementary Derivation of Vlasov Equation (see e.g. Bertsch)

We start with the Time-dependent HF formalism:

Hamiltonian with two-body interactions

and look for solutions in Slater determinant form

$$H = \sum_i T_i + \sum_{i<j} V_{ij}$$

$$|\Psi\rangle = A \prod_i a_i^+ |0\rangle$$

The central quantity is the 1-body density

$$\rho(1,2) = \langle \Psi | \psi^+(1)\, \psi(2) | \Psi \rangle = \sum_{i \in occ} \varphi_i^*(1)\, \varphi_i(2)$$

with $\psi_i^+(1)$ a field operator and the notation $1 \equiv \{r_1, s_1, \tau_i\}$ in coordinate or $1 \equiv \{p_1, s_1, \tau_i\}$ momentum repres.

The eom for the density matix is the TDHF equation

$$\frac{\partial}{\partial t} \rho(1,2) = \tfrac{1}{i} \{h, \rho\}_{1,2}$$

Where the s.p. HF hamiltonian is

$$h(1,2) = T(1,2) + \overbrace{\sum_{3,4} (V_{13,24} - V_{14,23})\, \rho_{3,4}}^{U(1,2)}$$

Now one performs a Wigner transform

of the 1-body density matrix

$$f(r,p) = \frac{1}{(2\pi)^3} \int ds\, e^{-ips} \left\langle \psi(r + \tfrac{s}{2})\, \psi^+(r - \tfrac{s}{2}) \right\rangle$$
$$= \frac{1}{(2\pi)^3} \int dq\, e^{+iqr} \left\langle \psi(p + \tfrac{q}{2})\, \psi^+(p - \tfrac{q}{2}) \right\rangle$$

which is a Fourier transform w.r.t to the difference in coord. resp. mom. space.

It has the variables of a phase space distribution, $\int f(r,p)\,dp = \rho(r); \quad \int f(r,p)\,dr = \tilde{\rho}(p)$

and in fact, integrals over *r* and *p* give the density and momentum distribution.

But it is a quantum object, and not positive definite!

On the next page we see that the EOM for *f* obeys the Vlasov equation!

II.2 Derivation of Vlasov Equation (2)

TDHF: $\frac{\partial}{\partial t}\rho(1,2) = \frac{1}{i}\{h,\rho\}_{1,2}$ $\quad h(1,2) = T(1,2) + \sum_{3,4}(V_{13,24} - V_{14,23})\,\rho_{3,4}$

Wigner transform of s.p. density

$$f(r,p) = \frac{1}{(2\pi)^3}\int ds\, e^{-ips}\left\langle \psi(r+\tfrac{s}{2})\,\psi^+(r-\tfrac{s}{2})\right\rangle = \frac{1}{(2\pi)^3}\int ds\, e^{-ips}\;\rho(r+\tfrac{s}{2}, r-\tfrac{s}{2})$$

form time derivative of Wigner transform and use TDHF to express $\frac{\partial\rho}{\partial t}$**.**

The kinetic term gives easily $-\frac{p}{m}\nabla_r f(p,r)$

The potential term for a local potential in coord. space $U(r,r') = \delta(r-r')U(r)$ **gives**

$$\frac{1}{i}\frac{1}{(2\pi)^3}\int ds\, e^{-ips}\,\underbrace{(U(r+\tfrac{s}{2}) - U(r-\tfrac{s}{2}))}_{U(r)+\frac{1}{2}s\nabla_r U(r)+\dots}\,\rho(r+\tfrac{s}{2}, r-\tfrac{s}{2}) = \nabla_r U\,\nabla_p f + \dots = 2\sin\frac{\nabla_r^{(U)}\nabla_p^{(f)}}{2}\,U f$$

expand

Collect in first order:

$$\frac{\partial f}{\partial t} + \frac{p}{m}\nabla_r f - \nabla_r U(r)\,\nabla_p f(r,p) = 0$$

Vlasov equation

Remarks:

- **1st order gradient expansion gives a classical equation, since lhs already contained a term ~$O(\hbar)$**
- **collision term has to be added „by hand“ as before**
- **quantum statistics only contained in initial condition, but is preserved by the evolution (for Vlasov-> Liouville theorem; for coll. term explicitely via blocking terms)**

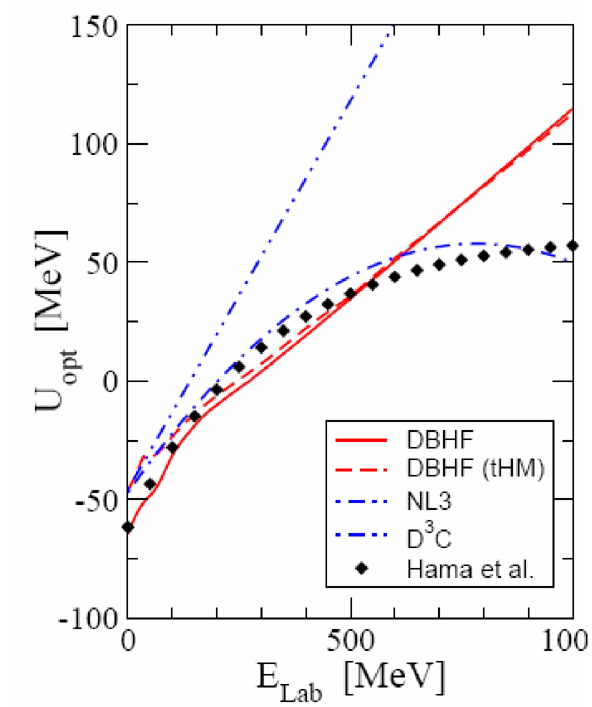
III.3 Remarks on the BUU Equation: Momentum Dependence

$$\frac{\partial f}{\partial t}+\frac{\vec{p}}{m}\vec{\nabla}^{(r)}f-\vec{\nabla}U(r)\,\vec{\nabla}^{(p)}f(\vec{r},\vec{p};t)=\int d\vec{v}_2\,d\vec{v}_{1'}\,d\vec{v}_{2'}\,v_{21}\,\sigma_{12}(\Omega)\,(2\pi)^3\left[f_{1'}f_{2'}\bar{f}_1\bar{f}_2-f_1 f_2\bar{f}_{1'}\bar{f}_{2'}\right]$$

Momentum dependence

the mean field is energy dependent for positive energies

→ known from the energy dependence of the real part of the optical potential, effect of exchange and correlations (above we assumed U local, i.e. momentum independent.



More generally introduce U(ρ,p).

In the above derivation this introduces a momentum dependent term on lhs.

$$\nabla_p U\,\nabla_r f(r,p)$$

choice of U(ρ,p) later!

III.4 Remarks on the BUU Equation: Relaxation time approximation

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\vec{\nabla}^{(r)} f - \vec{\nabla}U(r)\,\vec{\nabla}^{(p)} f(\vec{r},\vec{p};t) = \int d\vec{v}_2\, d\vec{v}_{1'}\, d\vec{v}_{2'}\, v_{21}\, \sigma_{12}(\Omega)\,(2\pi)^3 \left[f_{1'} f_{2'} \bar{f}_1 \bar{f}_2 - f_1 f_2 \bar{f}_{1'} \bar{f}_{2'} \right]$$

1. **Relaxation time approximation:**

   Write collision term in terms of in- and out-scattering transition rates $w^{\pm}$

$$:= (1-f)\tfrac{1}{2} w^{+} - f\,\tfrac{1}{2} w^{-};$$

$$:= -\tfrac{1}{2} w^{tot} (f - f_w); \quad w^{tot} = w^{+} + w^{-}; \quad f_w = \frac{w^{+}}{w^{tot}};$$

$$:= -\frac{f - f_w}{\tau}; \quad \tau = \frac{2}{w^{tot}}$$

i..e. the distribution approaches the distribution f_w with a relaxation time, which is given by the total rate, ten sum of in- and out-transition rates (which, however, is not constant!)

To take it as a constant is an approximation to the BUU eq. in the neighborhood of thermal equlilibrium. In thermal equlilibrium the solution (without potential) are the Fermion/Bose occupation probabilities

$$f_0(p) = 1/(1 \pm exp(\frac{E-\mu}{T}))$$

III.4 Relativistic transport formulation

1. Traditionally nuclear physics formulated in the Hamiltonian formalism, i.e. non-relativistically.

   simple reason ε_F~35 MeV << mc^2~1000MeV

2. About 20 years ago, starting with Walecka, a relativistic formulation has been employed. I will very briefly explain this to set the context, but it would really require another complete lecture.

3. From this, just like before, we can then derive a relativistic transport equation, which displays some new features.

4. All this has to be very brief

References:

1. Relativistic mean field model

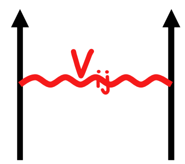
   B. Serot, J.D. Walecka, Adv. Nucl. Phys. 15, 1 (1986)

2. Relativistic transport theory

   B. Blaettel, V. Koch, U. Mosel, Rep. Prog. Phys. 56, 1 (1993) → material

III.5 „Quantenhadrodynamics“

Non-relativistic: Hamiltonian, $H = \sum_i T_i + \sum_{i<j} V_{ij}$, V_{ij} **NN-interaction**

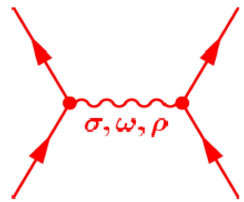


Relativistic (field theoretical): Lagrangian $L(\psi;\sigma,\omega,\pi,\eta,\delta,...)$

ψ: **Fermions: nucleon, Δ, N*,..**

ϕ: **Bosons: mesons σ,ω,ρ,...**

$$\frac{\partial L}{\partial \phi_i} - \frac{d}{dx_\mu}\frac{\partial L}{\partial_\mu \phi_i,} = 0$$



Simplest (Walecka) model:

only σ,ω mesons in linear coupling

$$L = \overline{\Psi}\left[\gamma_\mu\left(i\partial^\mu - g_\omega\omega^\mu\right) - \left(m - g_\sigma\sigma\right)\right]\Psi + L^{mes}$$

minimal coupling

*m** (Dirac) effective mass

$$L^{mes} = \frac{1}{2}\left(\partial^\mu\sigma\,\partial_\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{1}{4}F^\omega_{\mu\nu}F^{\omega\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu$$

$$F^{\omega\mu\nu} = \partial^\mu\omega^\nu - \partial^\nu\omega^\mu$$

In the static limit the minimal coupling assumption corresponds to Yukawa forces, suggested long ago by Yukawa, and successful phenomenologically

$$V_{NN} = -\frac{g_\sigma}{4\pi}\frac{e^{-m_\sigma r}}{r} + \frac{g_\omega}{4\pi}\frac{e^{-m_\omega r}}{r}$$

attractive   repulsive

III.6 Semiclassical approximation: Relativistic Mean Field (RMF)

Semiclassical approximation: meson (Boson) fields are taken as classical.

The field eqs. are then:

$$[\gamma_\mu(i\partial_\mu - g_\omega\omega_\mu) - (m - g_\sigma)]\psi = 0$$

$$(\partial_\mu\partial^\mu + m_\sigma^{\,2})\sigma = g_\sigma\langle\overline{\psi}\psi\rangle = g_\sigma\,\rho_s \quad \textbf{scalar source}$$

$$\partial_\mu F^{\mu\nu} + m_\omega^{\,2} = g_\omega\langle\overline{\psi}\gamma^\nu\psi\rangle = g_\omega\, j^\nu \quad \textbf{vector source}$$

In static, homogenious nuclear matter we obtain

- **shifted „free“ Dirac eq. with** $(\gamma_\mu p^{*\mu} + m^*)u = 0;$
- **effective momentum with vector self energy** $p^{*\mu} = p^\mu - \frac{g_\omega}{m_\omega^{\,2}}\rho_0\delta_{\mu 0} = p^\mu - \Sigma^\mu$
- **effective mass with scalar self energy** $m^* = m - \frac{g_\sigma}{m_\sigma^{\,2}}\rho_s = m - \Sigma_s$

Fitting to saturation density and energy gives

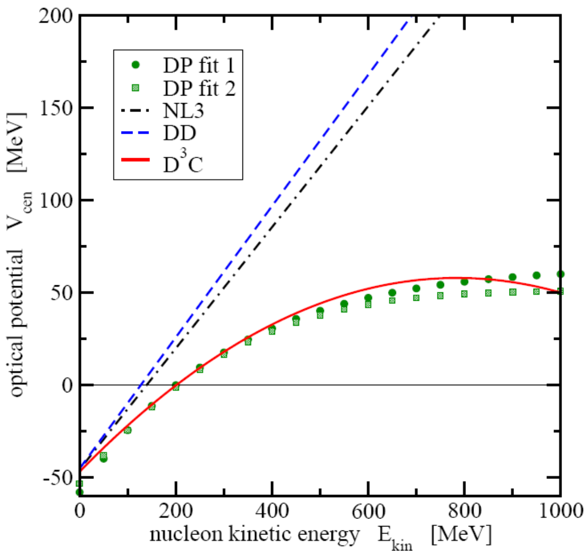
Σ_s~ -400 MeV, Σ_0~ 300 MeV, and „Schrödinger–equivalent“ potential

$$U^{SEQ} = -\Sigma_s + \frac{E}{m}\Sigma_0 + \frac{(\Sigma_s - \Sigma_0)^2}{2m}$$

and Spin-orbit potential

$$V_{so} = \frac{1}{2m^*}\frac{1}{r}\frac{d}{dr}(\Sigma_s + \Sigma_0)\,\ell\cdot s$$

explains weak and energy dependent central and strong spin-orbit interaction in nuclear systems; a major problem in the development of the nuclear shell model



Quantenhadrodynamics: Extensions

σω-model: however, not sufficient for nuclear matter properties: compressibility, effective mass, asymmetry dependence. Various extensions have been proposed

$$L = \overline{\Psi}\left[i\gamma_\mu \left(\partial^\mu + i g_\omega \omega^\mu + i g_\rho \frac{\vec{\tau}}{2}\vec{b}^\mu \right) - \left(m - g_\sigma \sigma - g_\delta \frac{\vec{\tau}}{2}\vec{\delta} \right) \right] \Psi + L^{mes}$$

isovector mesons: symmetry energy

non-linear meson self-interactions

$$L^{mes} = \frac{1}{2}\left(\partial^\mu \sigma \partial_\mu \sigma - m_\sigma^2 \sigma^2\right) - \frac{b_3}{3}\sigma^3 - \frac{b_4}{4}\sigma^4$$

density dependent coupling vertices

$\Gamma_\omega(\hat{\rho})$ $\Gamma_\rho(\hat{\rho})$ etc…

density dependent derivative coupling (D3C)

$$\Gamma_\mu = \gamma^\nu \left(g_{\nu\mu} + Y(\omega_\nu \omega_\mu) \right) + Z(\omega_\mu \sigma)$$

Full Lorentz structure:

	isoscalar	isovector
scalar	σ	δ
vector	ω	ρ

$$E_{sym} = \frac{1}{6}\frac{k_F^2}{E_F^{*2}} + \frac{1}{2}\left[f_\rho - f_\delta \left(\frac{M^*}{E^*} \right)^2 \right] \rho_B$$

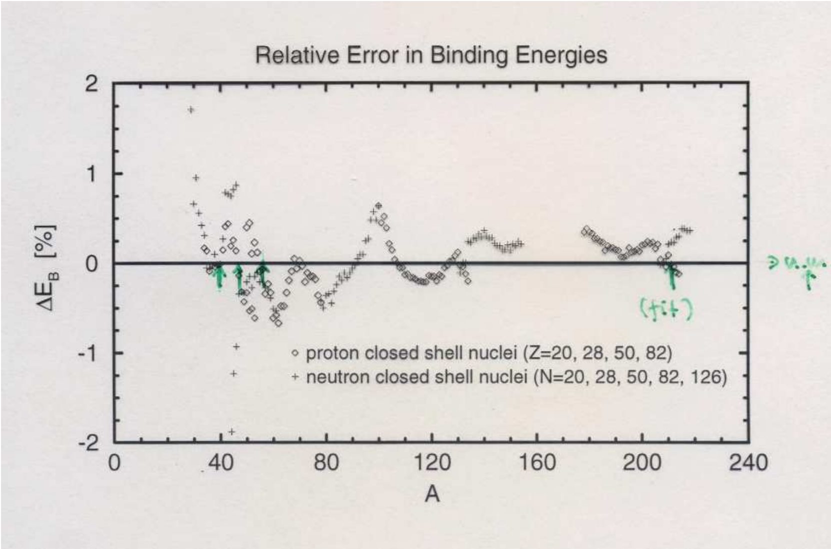
A cancellation between scalar and the isovector sector, similarly as for the isoscalar vector parts in potential

III.8 Representative Results of RMF Models

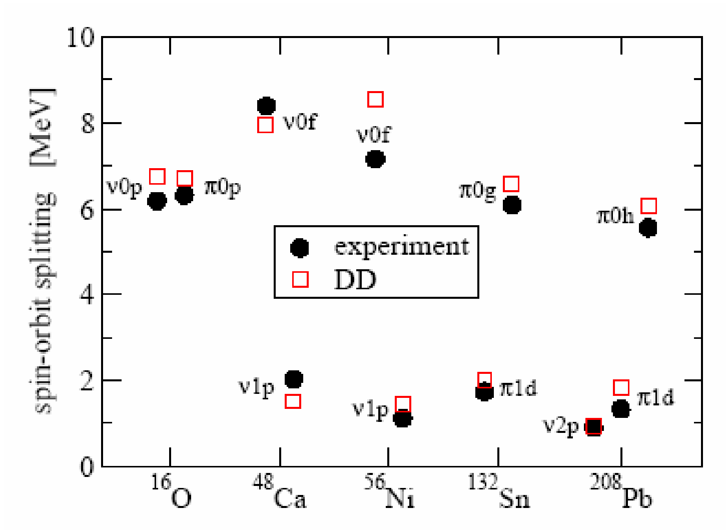
RMF model well adjusted to properties of nuclear matter, finite nuclei and neutron stars;

here some representative results for the density dependent (DD)approach: S. Typel, et al.

binding energies



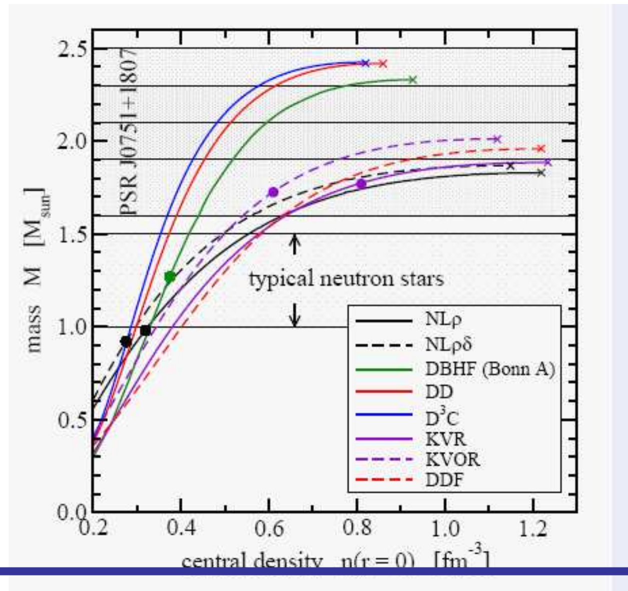
Spin-orbit splitting



Masses of neutron stars (NS):

The mass of a NS is determined by integrating the eq. for hydrostatic equil. in a gravitational field (Tolman-Oppenheimer eq.) outwards from a central censity until the pressure equals to zero. The mass of a NS for a given EOS has to be equal or heavier than the heaviest NS's observed. Most NS have masses of 1 to 1.4 solar masses.

In the figure shows results of this relationship for various nuclear EOS's, among them also RMF models (NLρδ,DD,D^{3}C,DDF)



III.9 Relativistic BUU Equation

Derived in a similar way, as for Schrödinger approach (using the (σ,ω) model for simplicity):

starting from one-body density $\rho_{\beta\alpha} = \langle \overline{\psi_\beta}(r_1)\,\psi_\alpha(r_2) \rangle$ **; α,β spinor indices**

Wigner Transform of the one-body density;
i.e. Fourier transform with respect to the relative coordinate („fast“)

$$F_{\beta\alpha}(r,p) = \frac{1}{(2\pi)^4}\int d^4s\, e^{-ip_\mu s^\mu} \left\langle \psi_\beta(r+\frac{s}{2})\,\overline{\psi_\alpha}(r-\frac{s}{2})\right\rangle$$

$$r = \tfrac{1}{2}(r_1+r_2); \quad s = \tfrac{1}{2}(r_1-r_2)$$

Contains all one-body information,
i.e. for 1-body operator O $\langle O \rangle = \int d^4x\, d^4p\, Tr(OF)$

Derive equations of motion for $F_{\beta\alpha}$: Using the Dirac eq.; one obtains expressions like

$$e^{-\frac{i}{2}\hbar\partial_\mu^{(r;\Sigma)}\partial^{(p;F)\mu}}\,\Sigma(r)F(r,p) \approx 1 - \tfrac{i}{2}\hbar\partial_\mu^{(r)}\Sigma(r)\partial^{(p)\mu}F(r,p)$$

which are evaluated in the semiclassical (gradient) approximation as before (assumption of smooth fields „slow“)

III.10 Relativistic BUU Equation (2)

Equations of motion in semiclass. approx. separated into real and imag. parts yield two equations:

1. Mass shell constraint:

$$(p^{*2} - m^{*2})F(r,p) = 0$$

$$(r;p^*) \rightarrow (\vec{r},t;\vec{p}^*); \quad p^*_0 = \sqrt{\vec{p}^{*2} + m^{*2}}$$

reduces phase space from 8- to 7-dimensional, time evolution interpretation

2. Kinetic transport) equation

a. Decomposition of F(x,p) in Lorentz invariants:

$$F(r,p) = F_s \otimes 1 + V_\mu \otimes \gamma^\mu + \underbrace{T_{\mu\nu} \otimes \sigma^{\mu\nu}}_{neglect} + \underbrace{\gamma^\mu \gamma^5 A_\mu + \gamma^5 P}_{zero\ for\ spin-saturrat.\ systems}$$

b. relation between the components

$$F_s = m^* f(r,p^*); \quad V_\mu = p^*_\mu f(r,p^*) \qquad m^* = m - \Sigma_s; \qquad F^{\mu\nu} = \partial^\mu \Sigma^\nu - \partial^\nu \Sigma^\mu$$

$$p^*_\mu = p_\mu - \Sigma_\mu$$

c. Transport (rel. Vlasov) equation:

$$\left[p^{*\mu} \partial^{(r)}_\mu + (p^*_\nu F^{\mu\nu} + m^* \partial^\mu_{(r)} m^*) \partial^{(p^*)}_\mu \right] f(r,p^*) = I_{coll}$$

d. collision term added

$$I_{coll} = \int \frac{d^3p_2}{p_{20}^*} \frac{d^3p_3}{p_{30}^*} \frac{d^3p_4}{p_{40}^*} (p^* + p_2^*)^2 \frac{d\sigma}{d\Omega} \delta(p^* + p_2^* - p_3^* - p_f^*) \left[f_3 f_4 \bar{f} \bar{f}_2 - f f_2 \bar{f}_3 \bar{f}_4 \right]$$

3. New Feature: two potentials: scalar vector → mom.dep. mean field, „Lorentz- like“ forces

IV.1 Quantum Transport Theory

We had derived the BUU transport eq.

$$\frac{\partial f}{\partial t}+\frac{\vec{p}}{m}\vec{\nabla}^{(r)}f-\vec{\nabla}U(r)\,\vec{\nabla}^{(p)}f(\vec{r},\vec{p};t)=\int d\vec{v}_2\,d\vec{v}_{1'}\,d\vec{v}_{2'}\,v_{21}\sigma_{12}(\Omega)(2\pi)^3\left[f_{1'}f_{2'}\bar{f}_1\bar{f}_2-f_1f_2\bar{f}_{1'}\bar{f}_{2'}\right]$$

or its relativistic variant.

Open questions:

→ How can the collision term be derived, instead of intuitively written down.

→ What is the deeper relationship between mf and cross section

→ how does one describe transport of particles with finite width (unstabe particles)

Use a many-body approach which takes into account the non-equlilibrium situation (Kadanoff-Baym eqs.)

Here only a sketch of the essential ingredients. More in the following refs.

L.P.Kadanoff, G. Baym, Quantum statistical mechanics, 1965
P. Danielewicz, Ann. Phys. 152, 239 (1984)
W. Botermans, R. Malfliet, Phys. Rep. 198 (1990) 115
P. Danielewicz, development of lattice methods (Trento, 2009)

or in a lecture by S. Leupold (Giessen)(in the materials on the website).

IV.2 Non-equilibrium Transport

Hierarchy of n-body Green functions (Martin-Schwinger hierarchy)

$$\underbrace{(i\gamma^{\mu}\partial_{\mu} - m)}_{D(1)} G^{(1)}(1,2) = \delta(1-1') + (12\,|V|\,1'2')\,G^{(2)}(12,1'2')$$
$$=: \delta(1-1') + \Sigma(1,1'')\,G^{(1)}(1'',2)$$

decouple formally via the self energy Σ, or practically an approximation to it, in particular, in Brueckner theory (BHF)

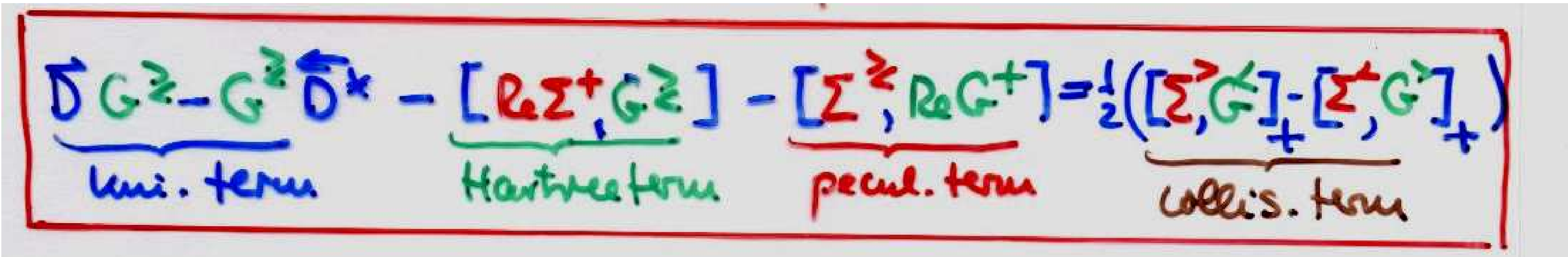
In non-equilibrium there are two independent 1-body Green functions (GF), since the propagation forward and backward in time is different. Often one uses the correlation GFs $G^{>}$ and $G^{<}$, and a variety of derived GF and self energies.

$$G^{<}(1,2) = i\langle \overline{\psi}(1)\,\psi(2)\rangle \xrightarrow[\text{Wigner transf}]{} i\,A(r,p)F(x,p)$$ **F generalized occupation**

$$G^{>}(1,2) = i\langle \psi(1)\,\overline{\psi}(2)\rangle \xrightarrow[\text{Wigner transf}]{} i\,A(r,p)(1-F(x,p))$$ **A spectral function**

$$G^{+}(1,2) = G^{cron} - G^{<}$$
$$G^{-}(1,2) = G^{cron} - G^{>}$$
retarded/advanced GF

For these one obtains with a Wigner transform and a gradient approximation the Kadanoff-Baym equations



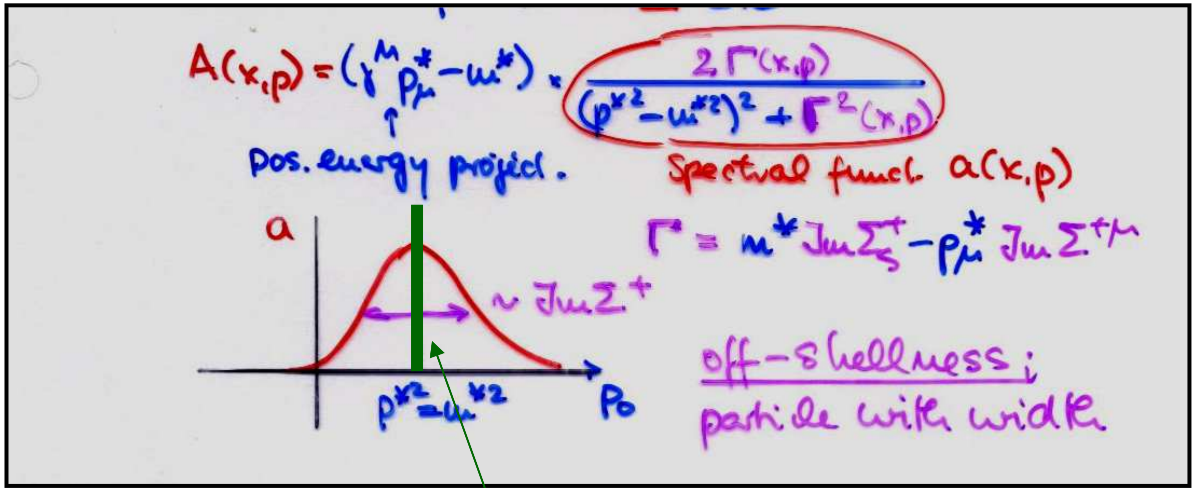
The Spectral Function

The spectral function contains the information about the decay width of a particle in medium.

Even particle which are stable in vacuum obtain a width in-medium due to collisions (imaginary part of self energy)

„Off-shell“ transport has only been invest. in a few cases.

To obtain the usual BUU transport eq. one makes the „Quasipart. Approximation (QPA)“, replacing the spectral fct. by a delta function on the mass shell.



$$a(x,k) = 2\pi\delta(k^{*2} - m^{*2})2\Theta(k^{*0})$$

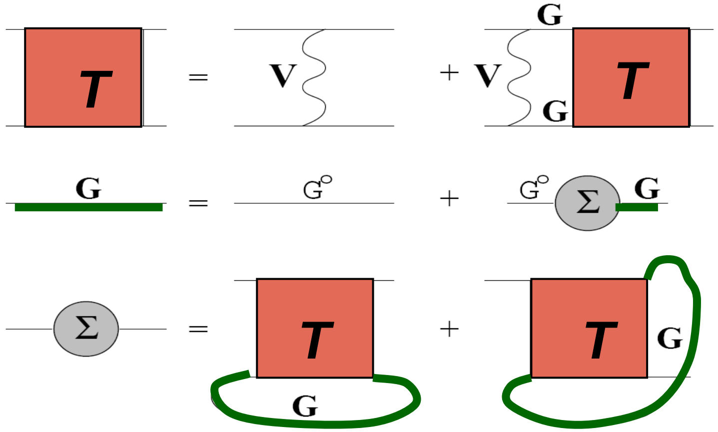
The Self Energy

The self energy is taken in the T_Matrix approximation, including exchange and two-body correlations: the Brueckner HF theory.

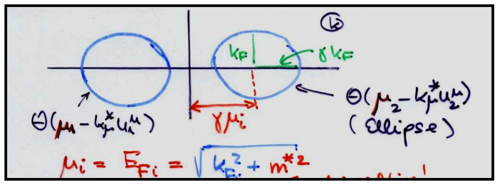
With this and the QPA one obtains BUU-like eqs.

Now the collision term appears consistently and is obtained on the same footing from the Brueckner T-Matrix.

The T-Matrix would have to be calculated consitently with the non-equil. phase space disribution, i.e. in non-eq., which is hardly possible. But there have been approx., like a two-Fermi sphere approx.



$$\left[\left(m^*\partial_x^\mu m^* - k^{*\nu}\partial_x^\mu k_\nu^*\right)\partial_\mu^k - \left(m^*\partial_k^\mu m^* - k^{*\nu}\partial_k^\mu k_\nu^*\right)\partial_\mu^x\right] f(x,\mathbf{k})$$
$$= \frac{1}{2}\int \frac{d^4k_2}{E_{k_2}^*(2\pi)^3}\frac{d^4k_3}{E_{k_3}^*(2\pi)^3}\frac{d^4k_4}{E_{k_4}^*(2\pi)^3} W(kk_2|k_3k_4)(2\pi)^4\delta^4\left(k+k_2-k_3 -\right.$$
$$\times\Big[f(x,\mathbf{k}_3)f(x,\mathbf{k}_4)\left(1-f(x,\mathbf{k})\right)\left(1-f(x,\mathbf{k}_2)\right) -$$
$$f(x,\mathbf{k})f(x,\mathbf{k}_2)\left(1-f(x,\mathbf{k}_3)\right)\left(1-f(x,\mathbf{k}_4)\right)\Big] \quad .$$



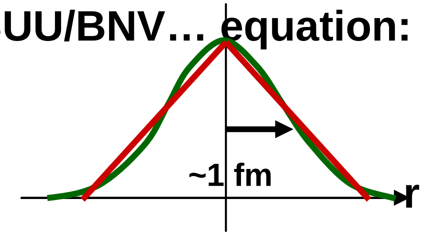
$$W(kk_2|k_3k_4) = m^*(x,k)m^*(x,k_2)m^*(x,k_3)m^*(x,k_4)$$
$$\times \; < kk_2|\mathcal{T}^+|k_3k_4 >< k_3k_4|\mathcal{T}^-|kk_2 >$$

To explain all this in detail needs much more space!
Main message: Transport theory can be placed on a solid many-body footing (which, however, has not often been employed in real calculations.)

V.1 Characterization of Codes for Transport Calculations

First family: Vlasov-type for 1-body phase space density

1. **Implementation, attempting to simulate the solution of the BUU/BNV… equation:**
   - → **test particle (TP) method**
   - → **point or finite size test partticles (Gaussians or triangles)**
   - → **MF often parametrized as Skyrme type with momentum dep. (next page)**



Energy density: $\varepsilon = \varepsilon_{kin} + \varepsilon_{pot}^{A}(rep) + \varepsilon_{pot}^{B}(attr) + \varepsilon_{pot}^{C,z}(mom.dep)$

$$\beta = \frac{N-Z}{A}$$

Generalized Bombaci-Gale-Bertsch-Das Gupta (BGBD) interaction

	isoscalar	isovector
$\varepsilon_A =$	$\frac{A}{2}\frac{\rho^2}{\rho_0}$	$-\frac{A}{3}\left(\frac{1}{2}+x_0\right)\frac{\rho^2}{\rho_0}\beta^2$
$\varepsilon_B =$	$\frac{B}{\sigma+1}\frac{\rho^{\sigma+1}}{\rho_0^{\sigma}}$	$-\frac{2}{3}\frac{B}{\sigma+1}\left(\frac{1}{2}+x_3\right)\frac{\rho^{\sigma+1}}{\rho_0^{\sigma}}\beta^2$

$$\frac{E}{A}(\rho_0 = 0.16\ fm^{-3}) = -16\,MeV$$

$$K_{NM}(\rho_0) = 215\ \ MeV, \qquad \frac{m^*}{m} = 0.67$$

Momentum Dependence (MD): isoscalar + isovector

$$\varepsilon_{C,z} = \frac{8}{5\rho_0}(C+2z)I_{np} + \frac{2}{5\rho_0}(3C-4z)(I_{nn}+I_{pp})$$

$$I_{tt'} = \left[\frac{2}{(2\pi)^3}\right]^2 \int d^3k\,d^3k'\, f_t(\vec{r},\vec{k})\, f_{t'}(\vec{r},\vec{k}')\, g(\vec{k},\vec{k}')$$

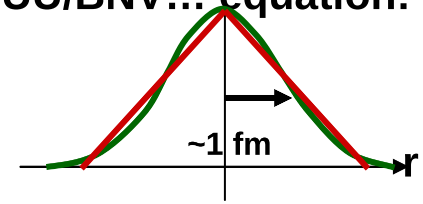
V.1 Characterization of Codes for Transport Calculations

First family: Vlasov-type for 1-body phase space density

1. **Implementation, attempting to simulate the solution of the BUU/BNV… equation:**

   → test particle (TP) method

   → point or finite size test partticles (Gaussians or triangles)



   → MF often parametrized as Skyrme type with momentum dep. (next page)

   → cross section empirical (usually free cross section, isospin dependent)

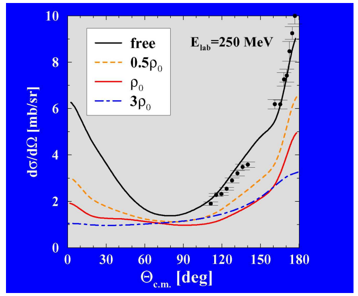
   → parallel ensemble method (collisons in sep. ensemble, MF averaged)

2. **Relativistic BUU (RBUU)**

   → relativistic variant of BUU, often also with gaussian TP

   → MF either from empirical density functional,

   i.e. RMF (non-linear or density dependent)



   → or use of Brueckner HF (Dirac-BHF) G-matrix in MF and collision term consistently

   →also including non-equlibrium effects in the two-Fermi sphere approximation

Second family: Molecular Dynamics to solve the many-body problem

V.2 Molecular Dynamics

Attempt to solve the many-body problem with assumptions: use of 2-body interaction instead of MF depending on density

1. **Classical Molecular dynamics CMD**

   point particles, deterministic,

   but possibly chaotic behaviour because

   of short range repulsion

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{r}_i = \{\boldsymbol{r}_i, \mathcal{H}\}, \qquad \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{p}_i = \{\boldsymbol{p}_i, \mathcal{H}\},$$

where the many-body Hamiltonian is of the form

$$\mathcal{H}\{\boldsymbol{r}_n, \boldsymbol{p}_n\} = \sum_{i=1}^{A} \frac{\boldsymbol{p}_i^2}{2m_i} + \sum_{i<j} V(|\boldsymbol{r}_i - \boldsymbol{r}_j|).$$

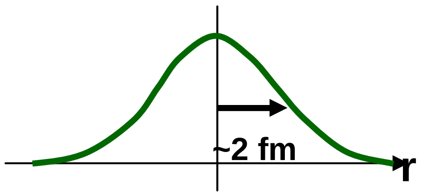
2. **Quantum molecular dynamics QMD**

   gausssian particles with large width to smooth fluctuations,

   not a wave packet, no antisymmetrization

   (thus similar to BUU with N_{TP}=1) but event generator.

   variant IQMD, isospin dependence in interactions

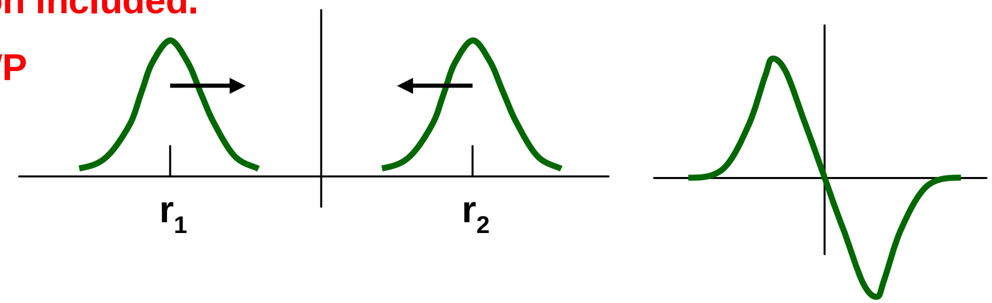


3. **Fermionic MD (FMD), Antisymmetrized MD (AMD)**

   Gaussian paricles, but antisymmetrization included.

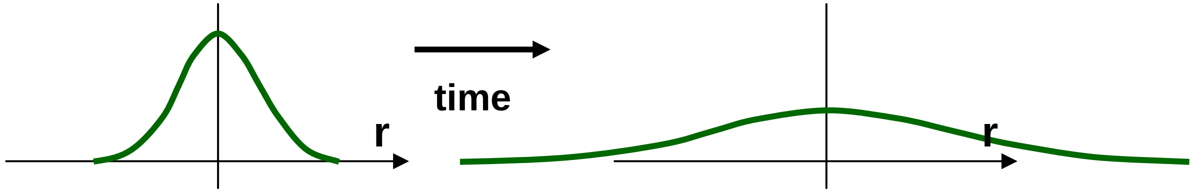
   Particle coordinates loose meaning as WP

   approach each other

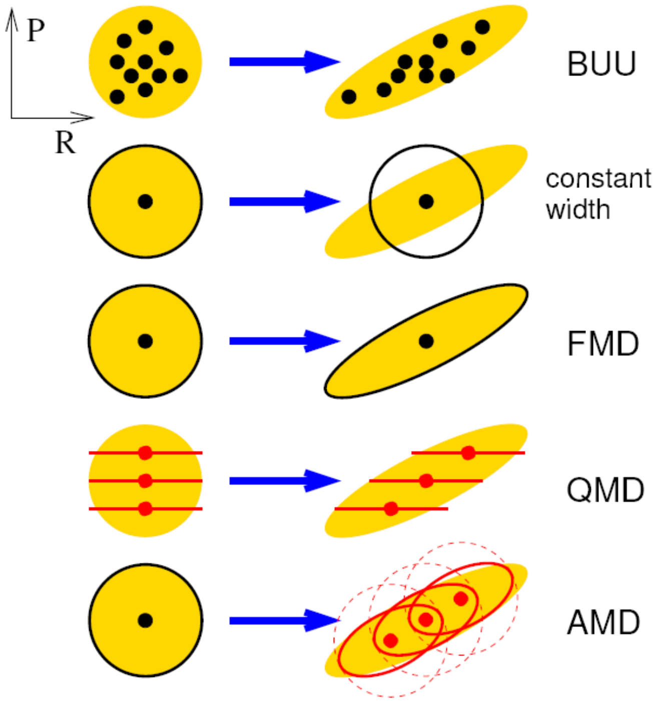


V.3 Molecular Dymnamics (2)

Spreading of wave packet:



Treated differently in the different approaches:



- BUU: **represented by test particles**
- constant width: **not included**
- FMD: **taken fulls into account, difficulties in final state**
- QMD: **represented by different events**
- AMD: **stochastic splitting of wave packet into WPs of original size**

Important in treatment of fluctuations

V.4 Code Comparison

Workshop on Simulations of Heavy Ion Collisions at Low and Intermediate Energies, ECT*, Trento, May 11-15, 2009

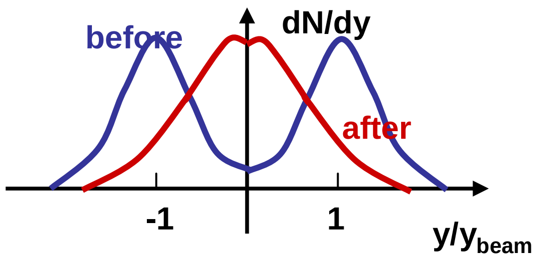
Obviously, transport codes are essential to gain information from HIC.

On the other hand they are complicated simulation programs, which contain many different strategies. It is important, to know the uncertainties associated with these implementations.

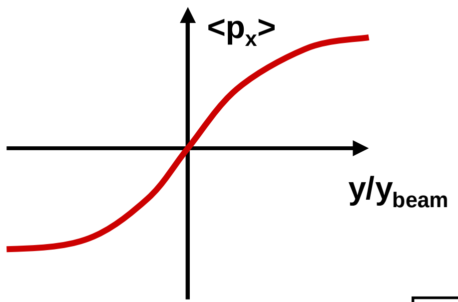
Thus we organized a workshop (working group) to attempt to compare the results from different codes, taking as far as possible the same physical input (mean field, cross section, etc.) → included codes, next page

Show some representative results for observables, which are discussed more later:

Rapidity distribution (stopping)



Transverse flow (compression)



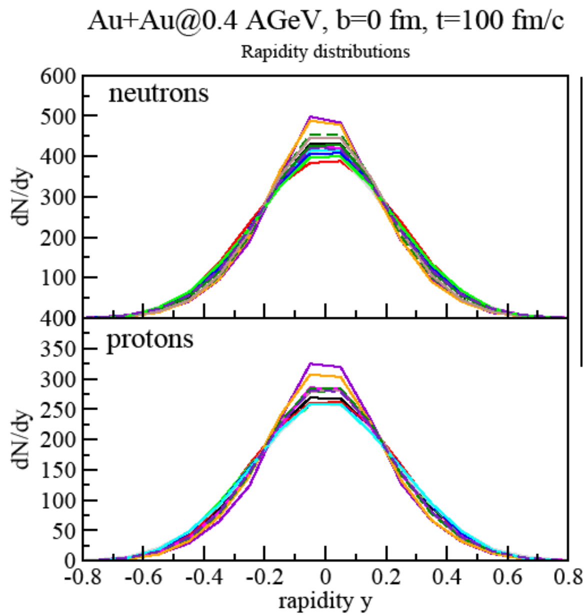
$$rapidity \quad y = \frac{1}{2}\ln\frac{1+\beta_z}{1-\beta_z}$$

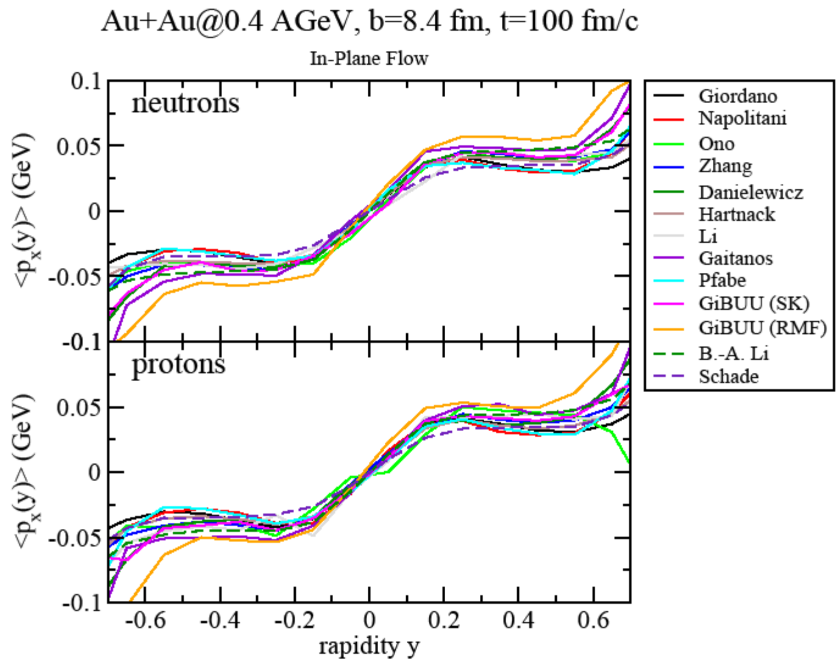
V.5 Codes, included in the comparison

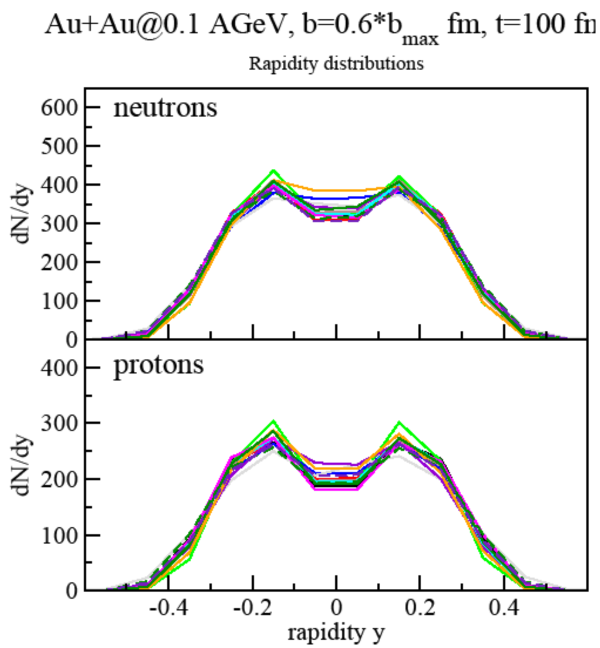
The major codes in use today are included in the comparison

Name	Code	References	EoS (hw1)	EoS (hw2)
Ono	AMD	PR**C66**(02)014603	own EoS	own EoS
Hartnack	IQMD	EPJ**A1**(98)151	as required	as required
Napolitani	BQMD	PR**202**(91)233	as required	own EoS
Zhang	ImQMD	PL**B664**(08)145; PR**C71** (05)024604; PR**C74**(06)014602	as required	as required
Danielewicz	BEM	-	as required	as required
Q.-F. Li	UrQMD	PR**C73**(06)051601; JP**G32**(06)151	as required	as required
Giordano	BNV (CT)	NP**A732**(04)202; PR**C72**(05)064609	as required	as required
Pfabe	BNV	??	as required	as required
Gaitanos	RBUU(Munich)	NP**A714**(03)643;NP**A741** (04) 209	as required	as required
GiBUU (SK)	BUU-Giessen	gibuu.physik.uni-giessen.de	as required	as required
GiBUU (RMF)	RBUU-Giessen	PL**B663**(08)197; arXiv:0904.2106v1; PR**C76**(07)044909	as required	as required
B.-A. Li	BUU	PR**160**(88)189; PR**C44**(91)450 & 2095.	as required	as required
H. Schade	BUU	??	as required	as required

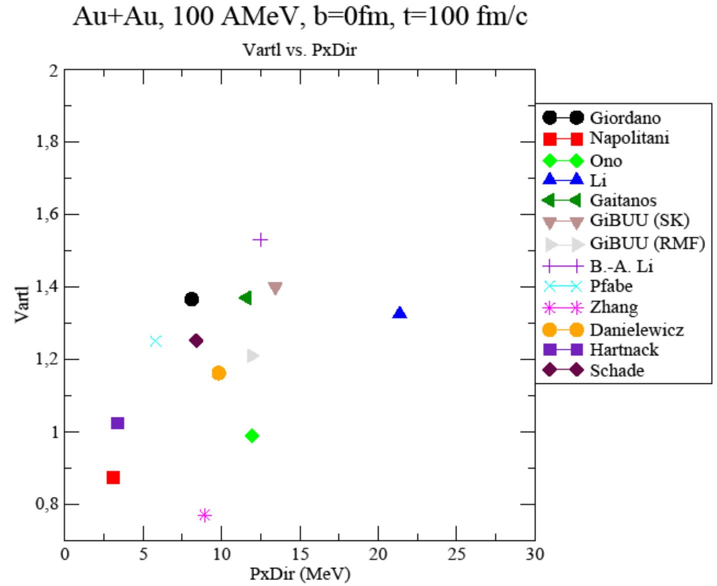
V.6 Code Comparison: Flow







Correlation between transverse flow (abcissa) and stopping (ordinate, vartl,) in ndifferent codes (100 AMeV)

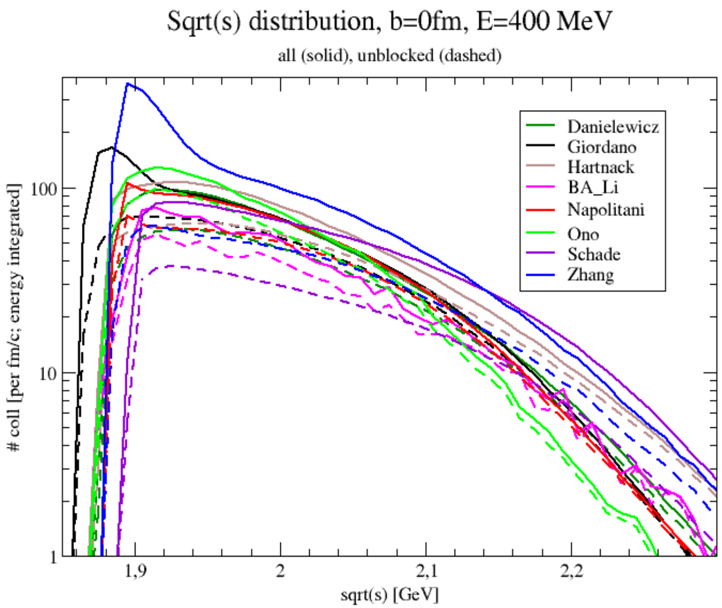


Code Comparison: Collisions

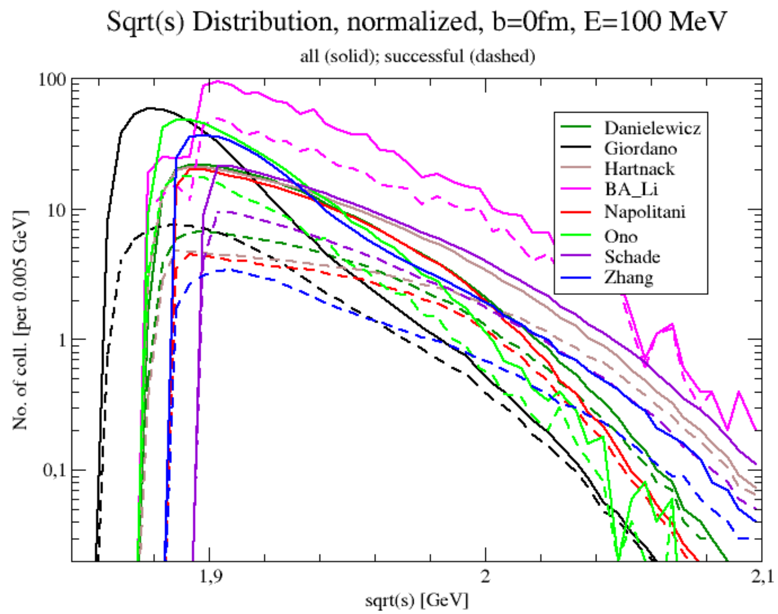
Energy distributions of collisions

solid: all attempted collisions, dashed: unblocked collisions

400 AMeV



100 AMeV



These are preliminary results!

The differences for flow observables are not drastic (even though they are sometimes of the order of physical effects of different EOS‘s). The differences in the collision histories are large. Here may lie the reason for the difference in the behaviours of the different codes.

Further studies are forthcoming.