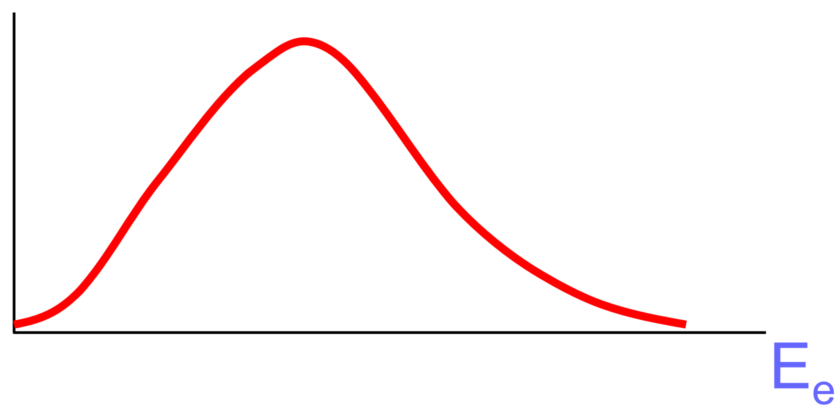
Nuclear Beta Decay

- Since 1920's physicists have observed beta decay: e.g. $^{14}C \rightarrow {}^{14}N + e^-$

- But the electron energy distribution is continuous:



- Where did the energy go??

Pauli's letter of the 4th of December 1930

Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and Li^6 *nuclei and the continuous beta spectrum, I have hit upon a deseperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses> The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...*

I agree that my remedy could seem incredible because one should have seen those neutrons very earlier if they really exist. But only the one who dare can win and the difficult situation, due to the continuous structure of the beta spectrum, is lighted by a remark of my honoured predecessor, Mr Debye, who told me recently in Bruxelles: "Oh, It's well better not to think to this at all, like new taxes". From now on, every solution to the issue must be discussed. Thus, dear radioactive people, look and judge. Unfortunately, I cannot appear in Tubingen personally since I am indispensable here in Zurich because of a ball on the night of 6/7 December. With my best regards to you, and also to Mr Back.

Your humble servant
. W. Pauli

"I have done something very bad today by proposing a particle that cannot be detected; it is something that no theorist should ever do."

- *Wolfgang Pauli*

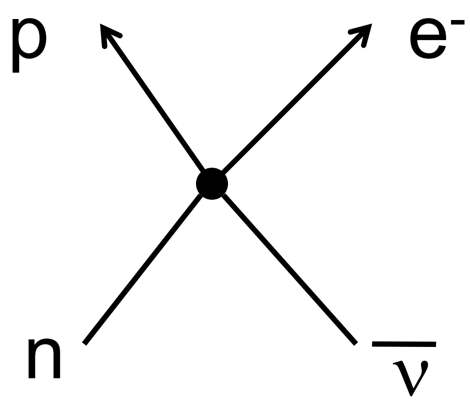
Fermi Theory of β decay (1934)

$$n \rightarrow p + e^- + \bar{\nu}$$

Golden Rule:

$$W = \frac{2\pi}{\hbar} G^2 |M|^2 \frac{dN}{dE_e}$$

density of final states



$$M = \langle p|J^\mu|n\rangle \langle e|J_\mu|\nu\rangle$$

(actually • is $G = g^2/M_W^2 \sim 1x10^{-5}$ /GeV^2)

$$dN = \frac{4\pi p_e^2 dp_e}{h^3} \frac{4\pi p_\nu^2 dp_\nu}{h^3} \delta(E_o - E_\nu - E_e)$$

(Integrate over $E_\nu = p_\nu$, $m_\nu=0$)

$$= 16\pi^2 \, p_e \, E_e \, (E_0 - E_e)^2 \, dE_e$$

So we can explain the observed spectrum:

$$W \sim p_e \, E_e \, (E_0 - E_e)^2$$

(e.g., Kurie plot)

Inverse Beta Decay

The related process also should occur:

$$\overline{\nu} + p \rightarrow n + e^+$$

Estimate cross section (neglect e^+ mass):

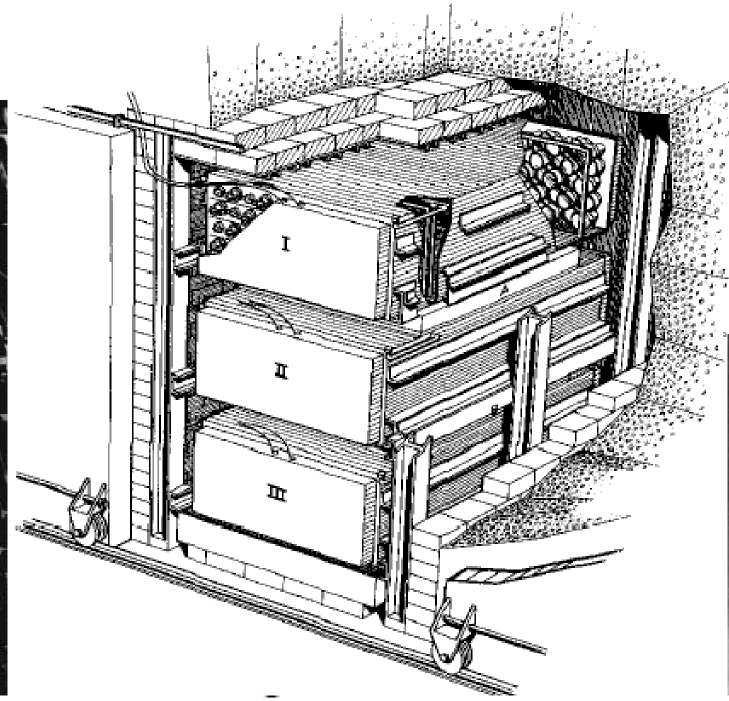
$$\sigma \sim G^2 E_\nu^2 \bullet (\hbar c)^2$$

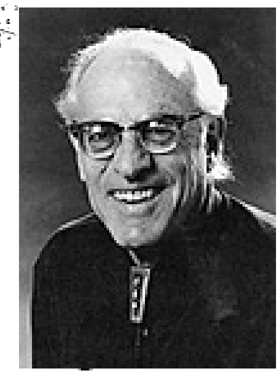
$$\sim 10^{-10}\ (10^{-6})\ (0.2 \times 10^{-13})^2\ \text{cm}^2 \quad (\text{@ } E = 1\text{MeV})$$

$$\sim 10^{-44}\ \text{cm}^2$$

$$\lambda = (\sigma\rho)^{-1} \sim (10^{-44} \times 10^{23})^{-1}\ \text{cm} = 10^{19}\ \text{cm} \sim 10\ \text{l.y.!!}$$

Discovery of the Neutrino – 1956

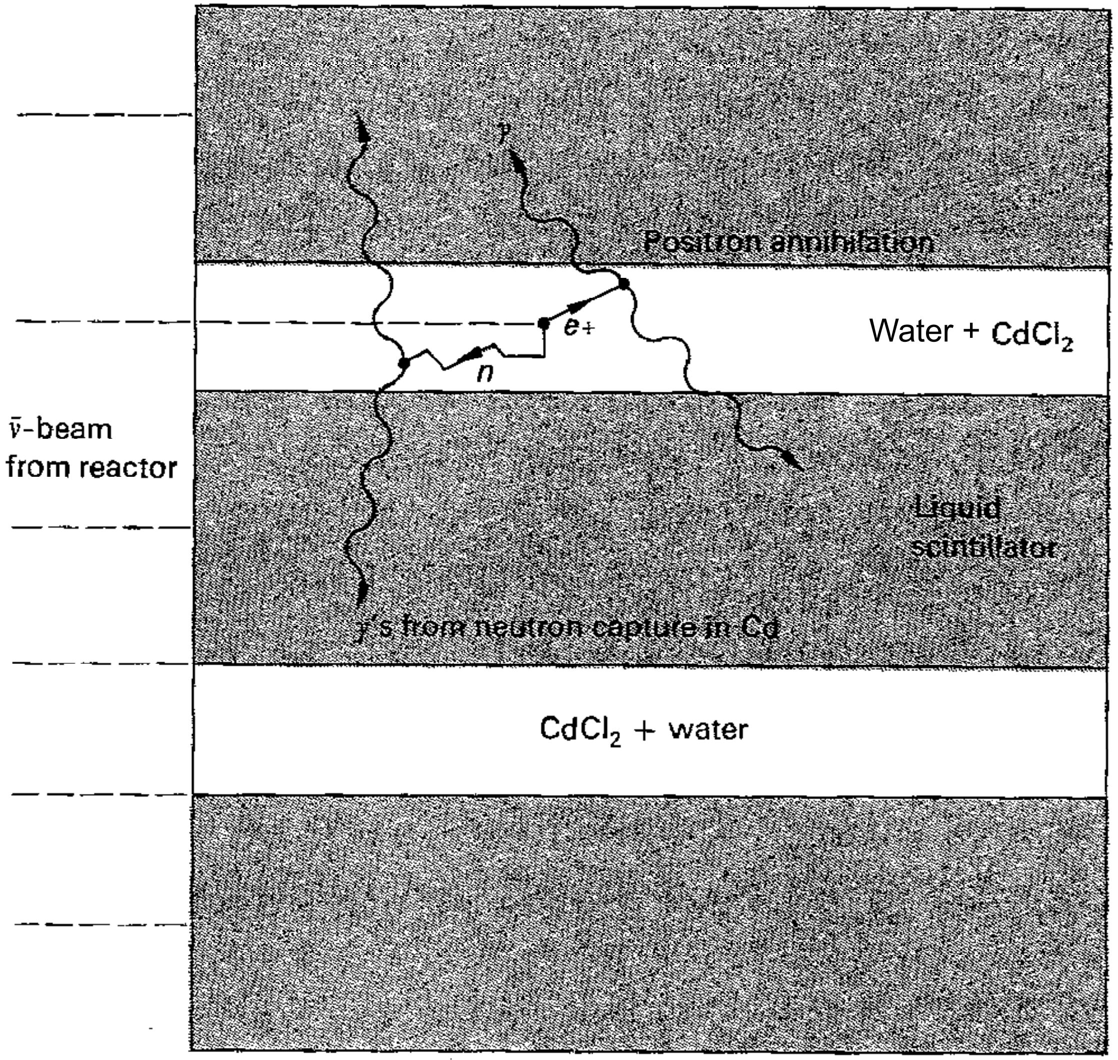




Finally, we chose to look for the reaction $\overline{\nu}_e + p \rightarrow n + e^+$. If the free neutrino exists, this inverse beta decay reaction has to be there, as Hans Bethe and Rudolf Peierls recognized, and as I'm sure did Fermi, but they had no occasion to write it down in the early days.

F. Reines, Nobel Lecture, 1995

Reines-Cowan Experiment



PHYSICAL REVIEW VOLUME 104, NUMBER 1 OCTOBER 1, 1956

Question of Parity Conservation in Weak Interactions*

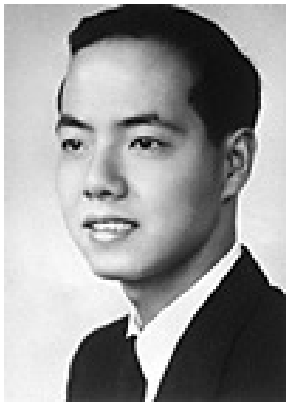
T. D. LEE, *Columbia University, New York, New York*

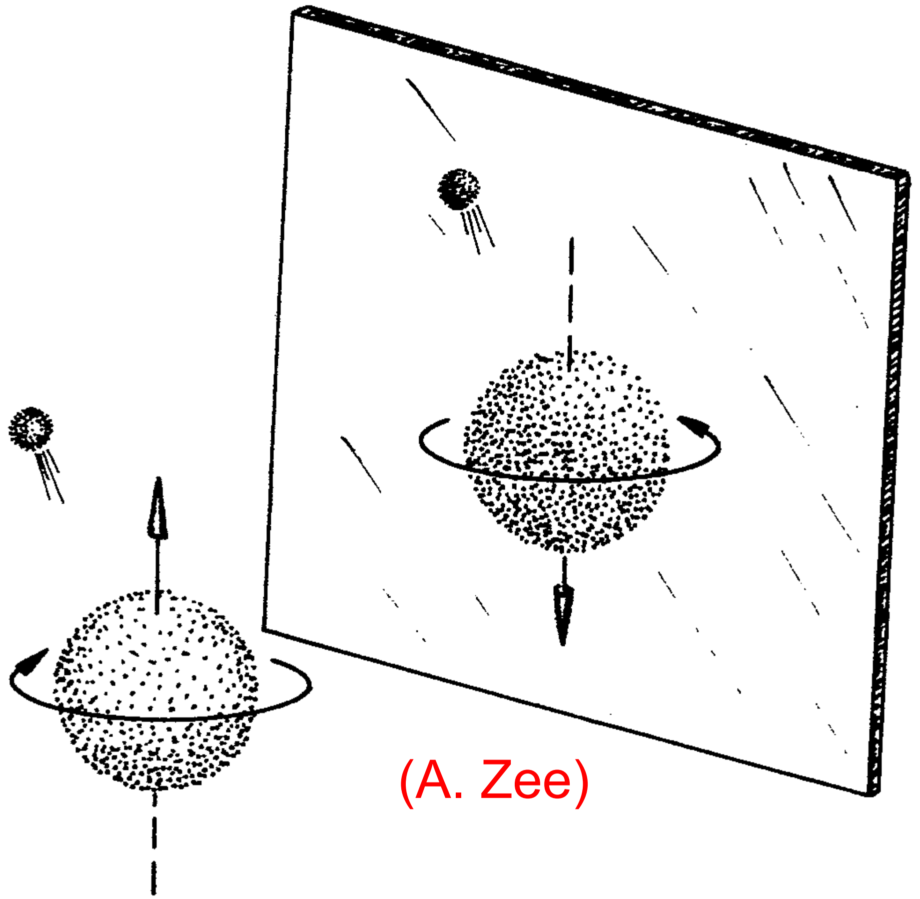
AND

C. N. YANG,† *Brookhaven National Laboratory, Upton, New York*

(Received June 22, 1956)

1956 – A Year of Revolution





(A. Zee)

FOR THE FIRST TIME –

A FORCE OF NATURE WAS <u>ASYMMETRIC</u>

Implication for the Neutrino!

PHYSICAL REVIEW VOLUME 105, NUMBER 5 MARCH 1, 1957

Parity Nonconservation and a Two-Component Theory of the Neutrino

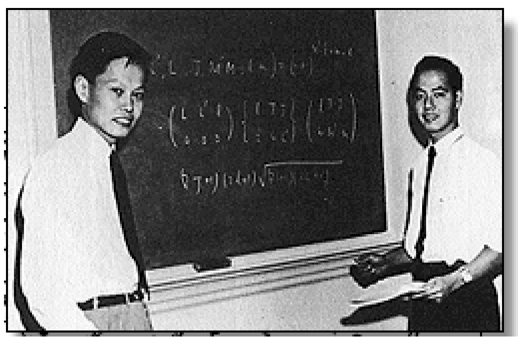
T. D. LEE, *Columbia University, New York, New York*

AND

C. N. YANG, *Institute for Advanced Study, Princeton, New Jersey*

(Received January 10, 1957; revised manuscript received January 17, 1957)

A two-component theory of the neutrino is discussed. The theory is possible only if parity is not conserved in interactions involving the neutrino. Various experimental implications are analyzed. Some general remarks concerning nonconservation are made.



In this theory the mass of the neutrino must be zero, and its wave function need only have two components instead of the usual four. That such a relativistic theory is possible is well known.[4] It was, however, always rejected because of its intrinsic violation of space inversion invariance, a reason which is now no longer valid.

Pauli (1933)

Chirality States

Dirac Eq.: $(\not{p} - m)\psi = 0$

X γ_5: $(\not{p} + m)\gamma_5\psi = 0$

Define $\psi_R \equiv ½(1+\gamma_5)$; $\psi_L \equiv ½(1-\gamma_5)$

$$\not{p}\,\psi_R - m\,\psi_L = 0$$
$$\not{p}\,\psi_L - m\,\psi_R = 0$$

$m \rightarrow 0$: $\not{p}\,\psi_R = 0$; $\not{p}\,\psi_L = 0$.

Note: $m \neq 0 \rightarrow$ must have <u>both</u> ψ_R , ψ_L

Free fermions obey the Dirac equation $(\not{p} - m)\Psi = 0$ where $\not{p} = \gamma^\mu p_\mu$.

use the representation: $\gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\vec{\gamma} = \begin{pmatrix} 0 & -\vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$ $\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

Now rewrite the four component Dirac equation as two coupled two component equations $\Psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$:

$$-m\psi_- + (E - \vec{\sigma}\cdot\vec{p})\psi_+ = 0$$
$$(E + \vec{\sigma}\cdot\vec{p})\psi_- - m\psi_+ = 0$$

In the limit $m \to 0$ these equations decouple and we obtain the Weyl equations describing states with a definite helicity $\vec{\sigma}\cdot\hat{p}\psi_\pm = \pm\psi_\pm$.

ψ_- is the left-handed neutrino (E>0)
or a right-handed antineutrino (E<0).

ψ_+ describes ν_R or $\overline{\nu}_L$.

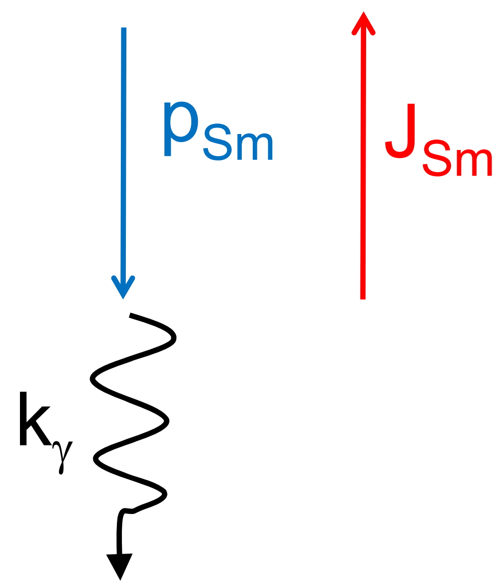
Neutrino Helicity Measurement

$$^{152}\mathrm{Eu}\,(0^+) + e^- \rightarrow {}^{152}\mathrm{Sm}^*(1^+) + \nu \rightarrow {}^{152}\mathrm{Sm}(0^+) + \gamma$$

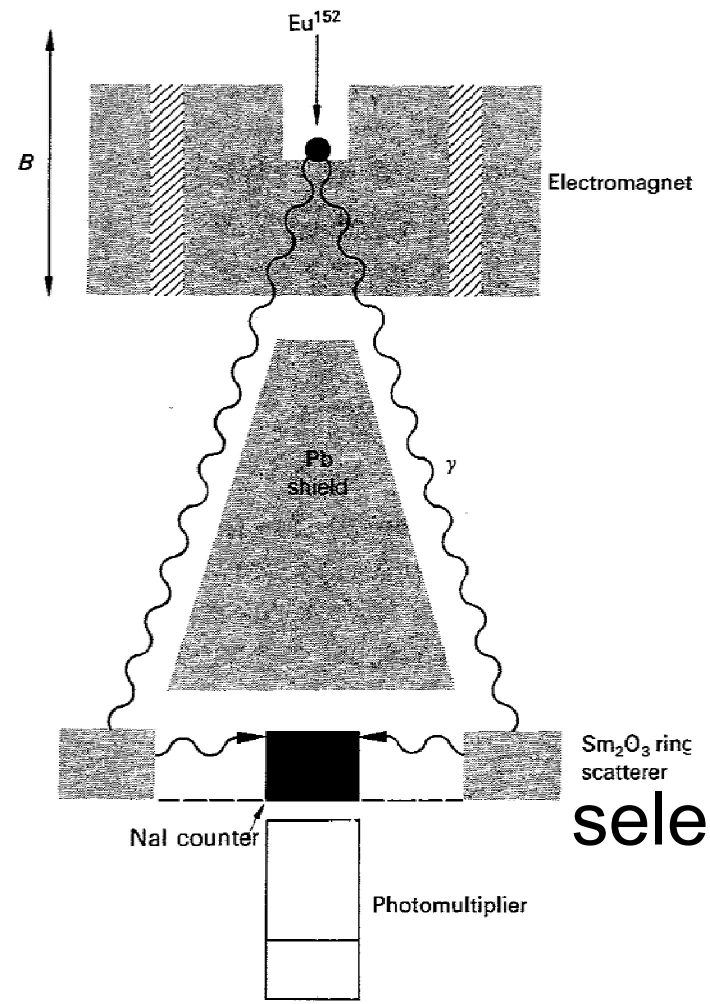
Let $\vec{p}_\nu = p_\nu\,\hat{z}$ so that $\vec{p}_{Sm} = -\,p_{Sm}\,\hat{z}$

LH $\nu \rightarrow \vec{J}_{Sm} \sim +\,\hat{z}$

$\rightarrow \vec{J}_\gamma \sim +\,\hat{z}$ (LH)



Neutrino Helicity Measurement (1958)



γ spin analyzer

select $E_\gamma \rightarrow\ p_{Sm} \downarrow$

The STANDARD MODEL (1967)

$$\psi_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \qquad \psi_R = \ell_R$$

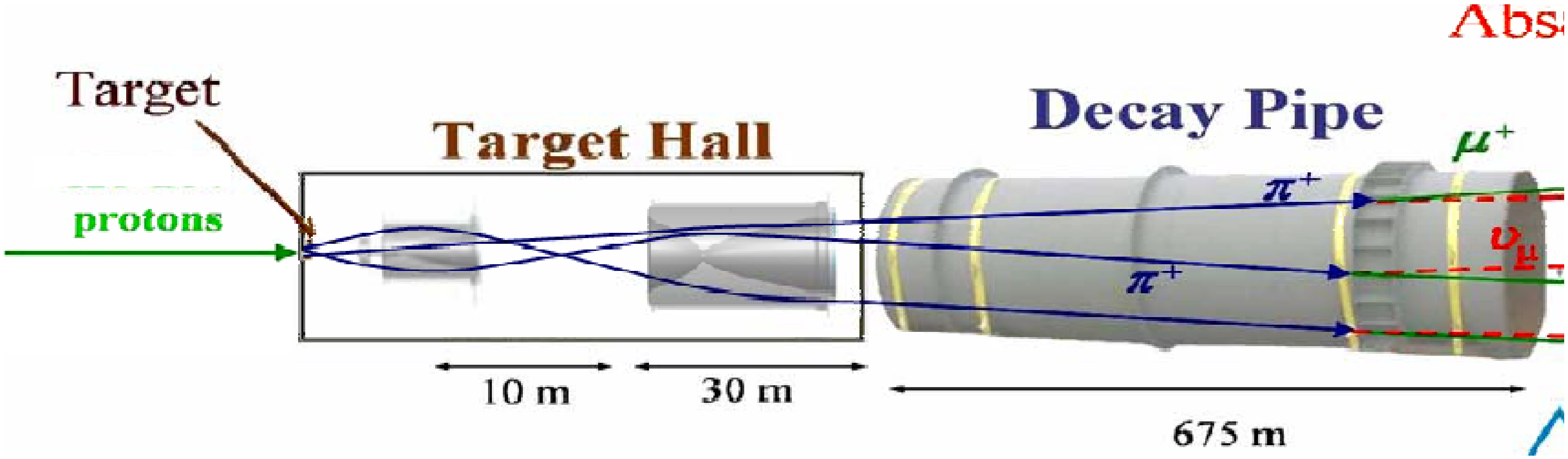
ν_R does not exist!

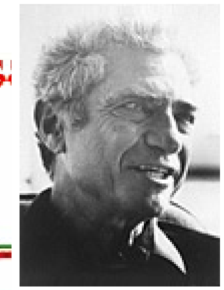
$$\mathscr{L}_F = \sum_i \overline{\psi}_i \left(i \, \partial\!\!\!/ - m_i - \frac{g m_i H}{2 M_W} \right) \psi_i$$
$$- \frac{g}{2\sqrt{2}} \sum_i \overline{\psi}_i \, \gamma^\mu \, (1 - \gamma^5)(T^+ \, W^+_\mu + T^- \, W^-_\mu) \, \psi_i$$
$$- e \sum_i q_i \, \overline{\psi}_i \, \gamma^\mu \, \psi_i \, A_\mu$$
$$- \frac{g}{2 \cos\theta_W} \sum_i \overline{\psi}_i \, \gamma^\mu (g^i_V - g^i_A \gamma^5) \, \psi_i \, Z_\mu \; .$$

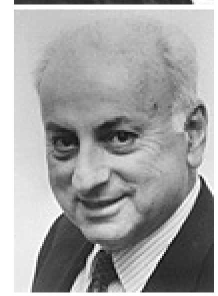
Neutral currents!

$$g^i_V \equiv t_{3L}(i) - 2 q_i \sin^2\theta_W \; ,$$
$$g^i_A \equiv t_{3L}(i) \; ,$$

- 1960's – ν were studied with accelerator experiments: $\nu_e \neq \nu_\mu$

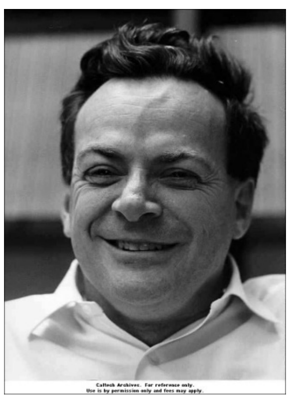






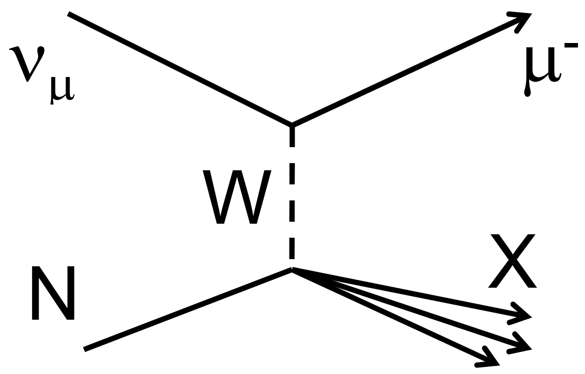
$$\pi^+ \rightarrow \mu^+ \nu_\mu$$

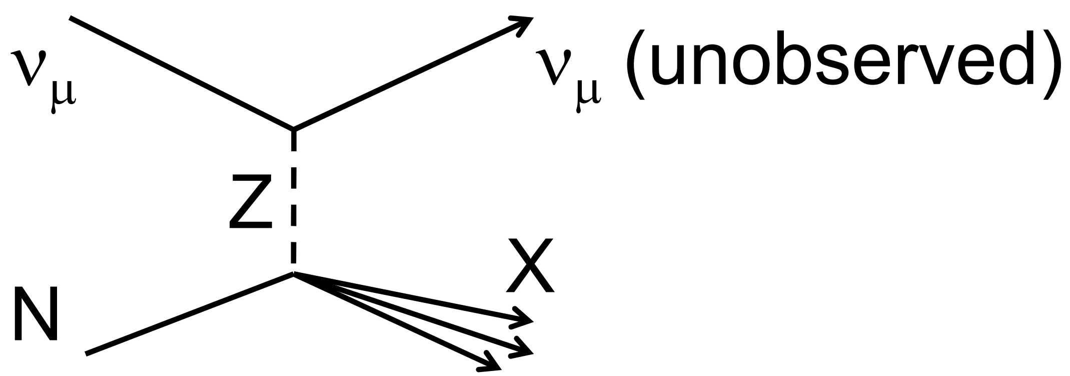
$$\nu_\mu + A \rightarrow X + \mu^+ \text{, but never } e^+$$



"All you have to do is imagine something that does practically nothing.
You can use your son-in-law as a prototype."

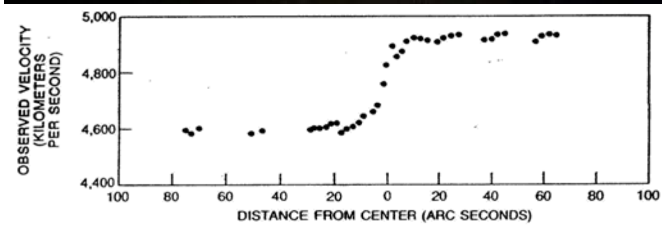
Neutral Current Discovery (1973)

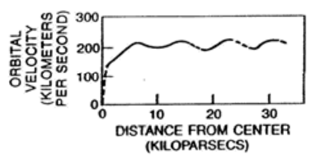




Major Triumph for the Standard Model!!

A Puzzle from Astrophysics





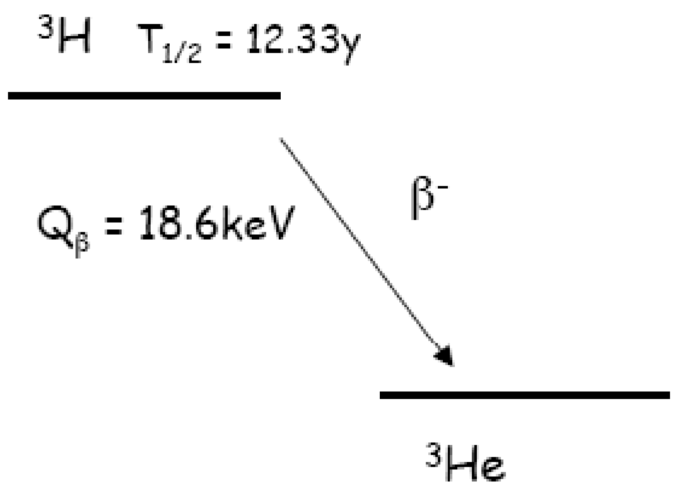
Could neutrinos have mass and explain the **Dark Matter?**

Galactic Rotation Curves → gravitational clustering of neutral matter

Neutrino mass and three-body decays

(in particular β decay where two light particles are emitted):

$Z^A \rightarrow (Z+1)^A + e^- + \bar{\nu}_e$



$E_\nu + T_e = Q_\beta$

(electron kinetic energy, since Q_β represents atomic mass diff.)

Momentum is automatically conserved (nuclear recoil).

For $T_e \sim Q_\beta$ neutrino can become nonrelativistic

Electron spectrum in nuclear β decay

The transition probability per unit time is given by Fermi's golden rule

$$dW = (2\pi)^{-5} d^3\mathbf{p_e} d^3\mathbf{p}_{\bar{\nu}} \delta(E_e + E_\nu - \Delta)|A_{fi}|^2$$

Since the lepton energies are of order m_e their deBroglie wavelength ($\lambda \sim \hbar/m_e c = 4 \cdot 10^{-11}$cm) is much larger than the nuclear radius, and we can neglect the variation of their wave function over the nuclear volume. In addition, we neglect all recoil terms $(E_e + E_\nu)/m_N$ and terms involving nucleon velocity. *This is so called allowed approximation,* $p_e \cdot R \ll 1$.

$$A_{fi} = \frac{G_F}{\sqrt{2}} \cos\theta_C \left[C_V \langle 1 \rangle j_0(0) - C_A \langle \vec{\sigma} \rangle \vec{j}(0) \right]$$

Here $C_V = 1$ and $C_A = 1.26$ are coupling constants and $\langle 1 \rangle$ and $\langle \vec{\sigma} \rangle$ are the Fermi and Gammow-Teller nuclear matrix elements. The lepton currents $j(0)$ and $\vec{j}(0)$ depend on lepton spins and directions. After squaring, summing over spins, and integrating over angles we obtain the transition probability

$$dW = \frac{G_F^2 \cos^2\theta_C}{2\pi^3} \xi F(Z+1, E_e) E_e p_e E_\nu p_\nu dE_e$$

Here $\xi = C_V^2 \langle 1 \rangle^2 + C_A^2 \langle \sigma \rangle^2$ and $F(Z, E)$ is the easily calculable correction for the Coulomb effect on the emitted electron (ratio of the square of the electron wave function at $r = R$ with and without Coulomb field.)

Electron spectrum in nuclear β decay, continued

In the last formula one must substitute

$$E_\nu = \Delta - E_e = Q_\beta - T_e \quad \text{and} \quad p_\nu = \sqrt{E_\nu^2 - m_\nu^2}$$

The last expression gives the clue to the sensitivity to the neutrino mass. Obviously, when $p_\nu \rightarrow 0$ or $T_e \rightarrow Q_\beta$, i.e., near the endpoint of β spectrum, that sensitivity is more pronounced

Kurie plot

Lets call $dW/dE \equiv N(E)$ and consider

$$K(E) \equiv \left[\frac{N(E)}{F(Z,E)p_e E_e}\right]^{1/2} \sim const \times [p_\nu E_\nu]^{1/2}$$

For massles neutrinos this quantity, called Kurie plot, is a straight line $K(E_e) \sim \Delta - E_e$, with the intersect at the endpoint Δ (or Q_β if the electron kinetic energy is used).

Rewriting $[p_\nu E_\nu]^{1/2} = (\Delta - E_e)\left[1 - \frac{m_\nu^2}{(\Delta - E_e)^2}\right]$

One can see that

- The spectrum ends at $\Delta - m_\nu$ and not at Δ as for massless neutrinos
- More importantly, the slope of Kurie plot becomes infinite near the endpoint for massive neutrinos.
- Effects like finite resolution, background, etc. will make the slope less steep, unlike the neutrino mass, that makes it steeper.
- Thus, if such effects are incorrectly included we might get $m_\nu^2 < 0$ from a fit to the data

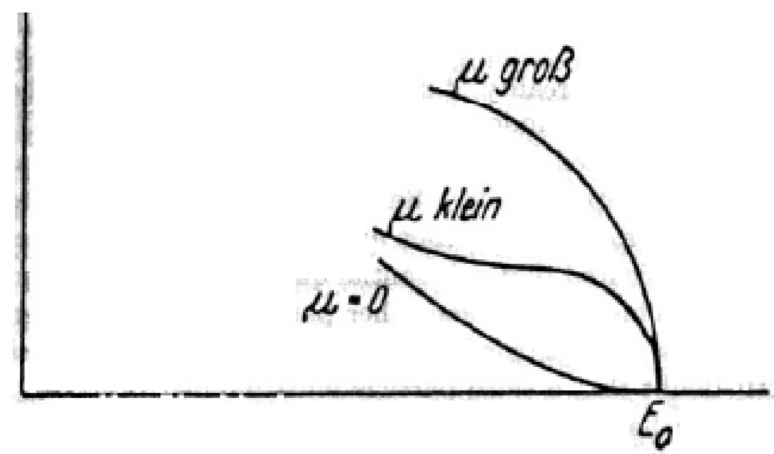
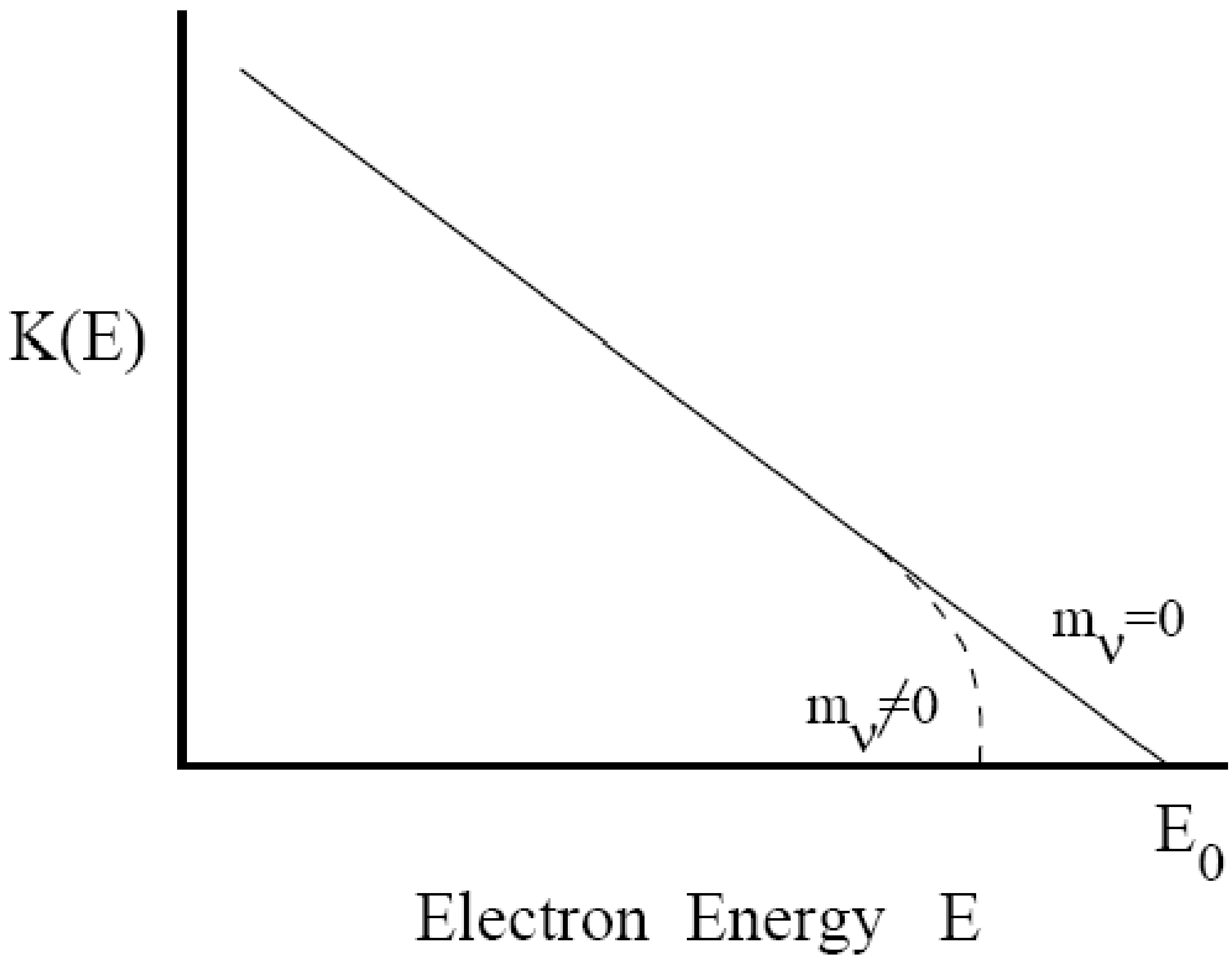


Fig. 1.2. Graph from Fermi's famous paper on the theory of beta decay, showing how the shape of the emitted electron's energy spectrum varies with neutrino mass



K(E)
m_ν=0
m_ν≠0
E_0
Electron Energy E

Tritium Kurie Plot

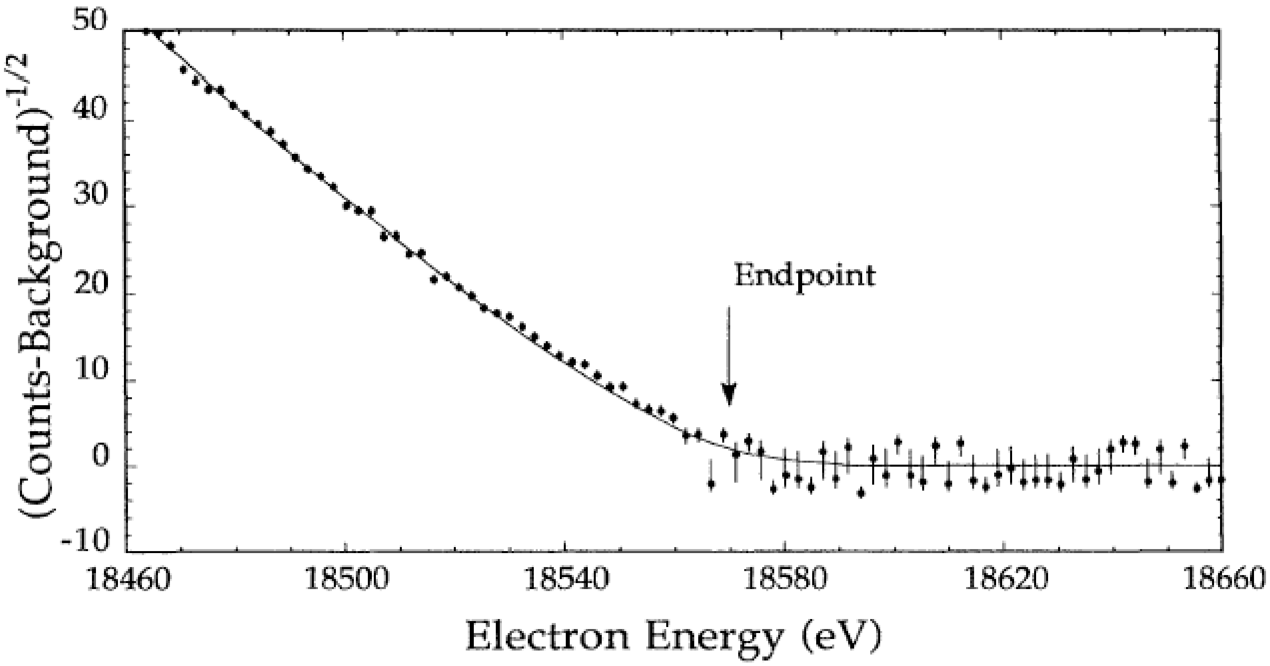
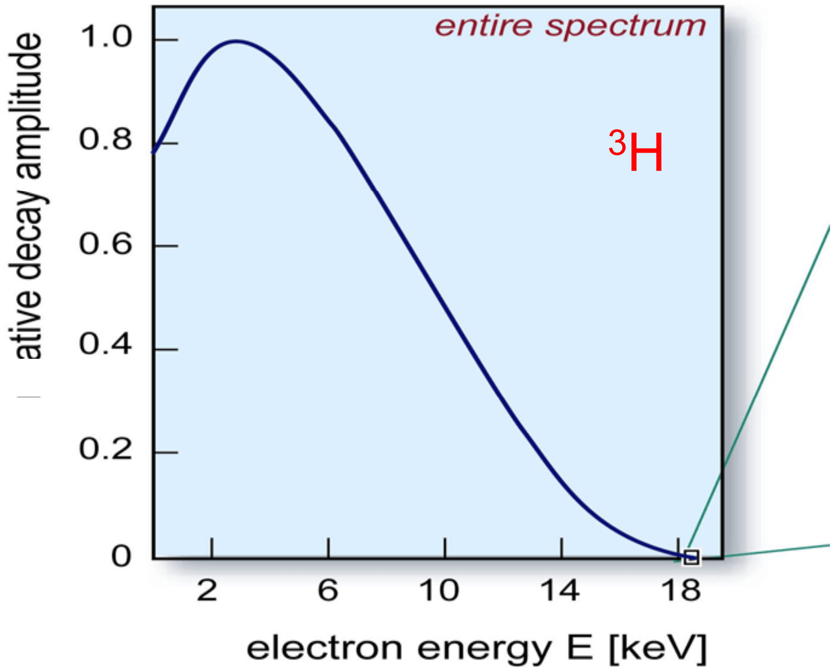
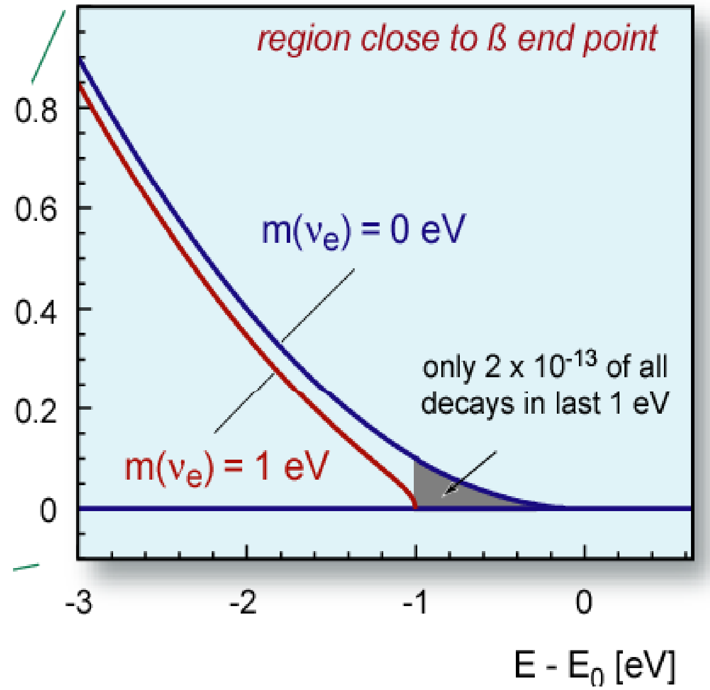


FIG. 1. The Kurie plot of the end point region for molecular tritium. The solid curve is a fit to the data of run (b) with zero neutrino mass. The subtracted background is 26 counts/channel.

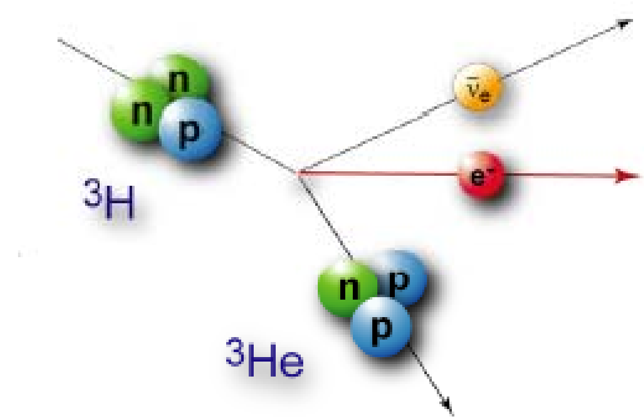
Tritium Beta decay and neutrino mass



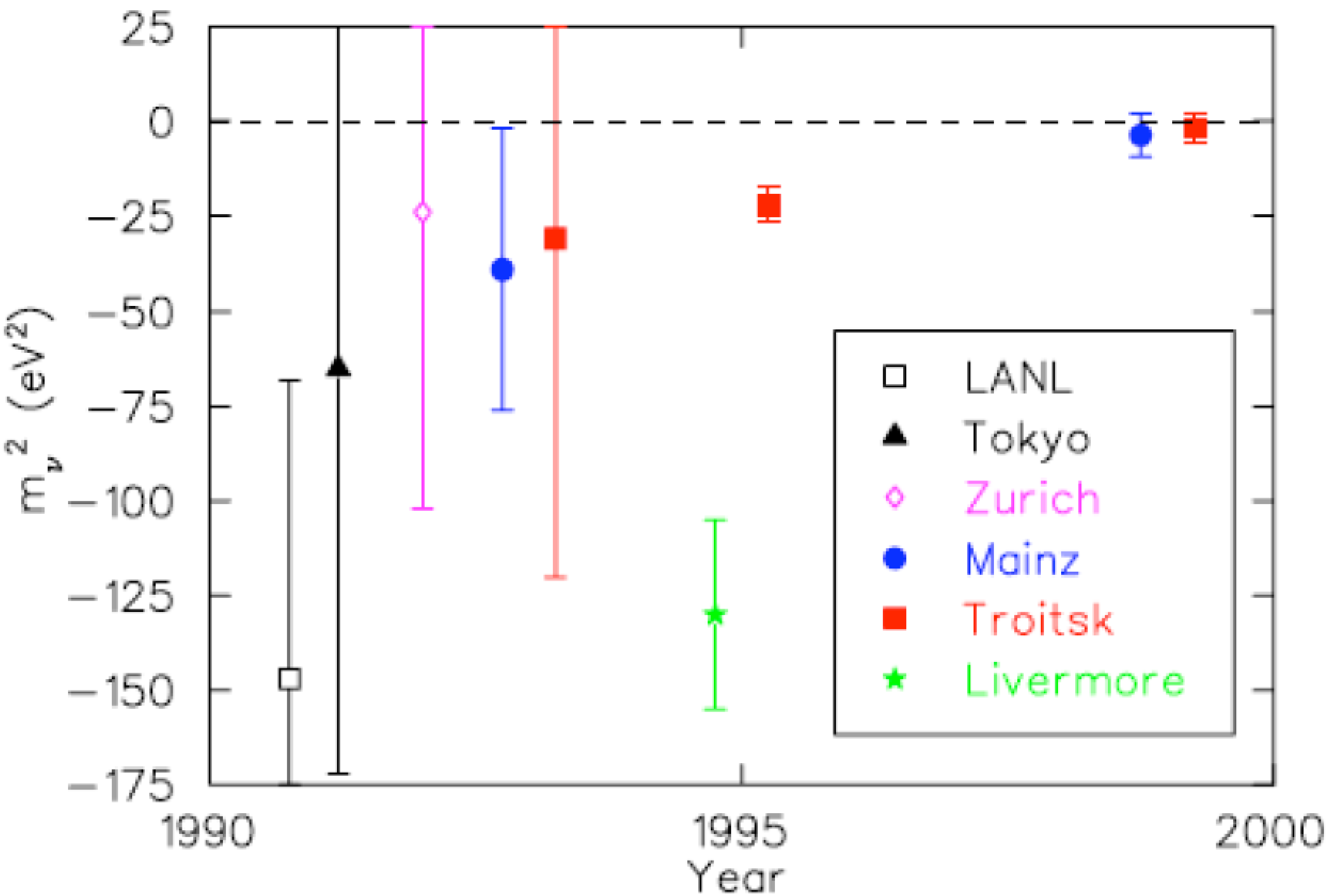


Requirements:

- Strong source
- Excellent energy resolution
- Small endpoint energy E_0
- Long term stability
- Low background rate



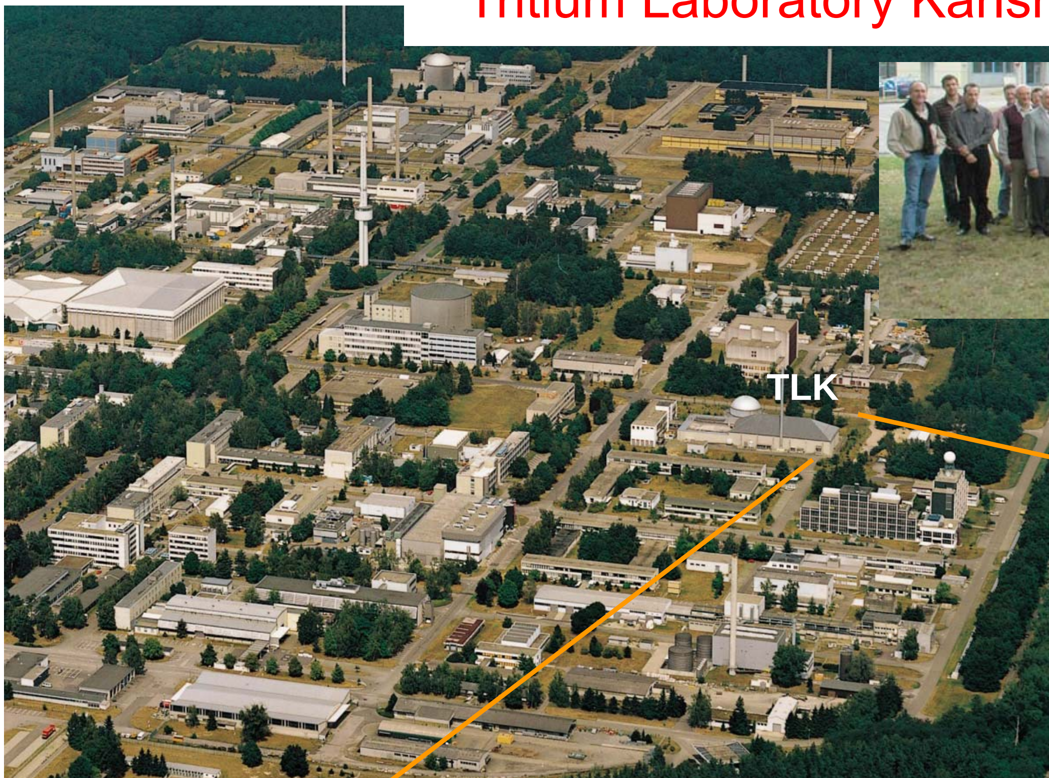
Previous Tritium Measurements

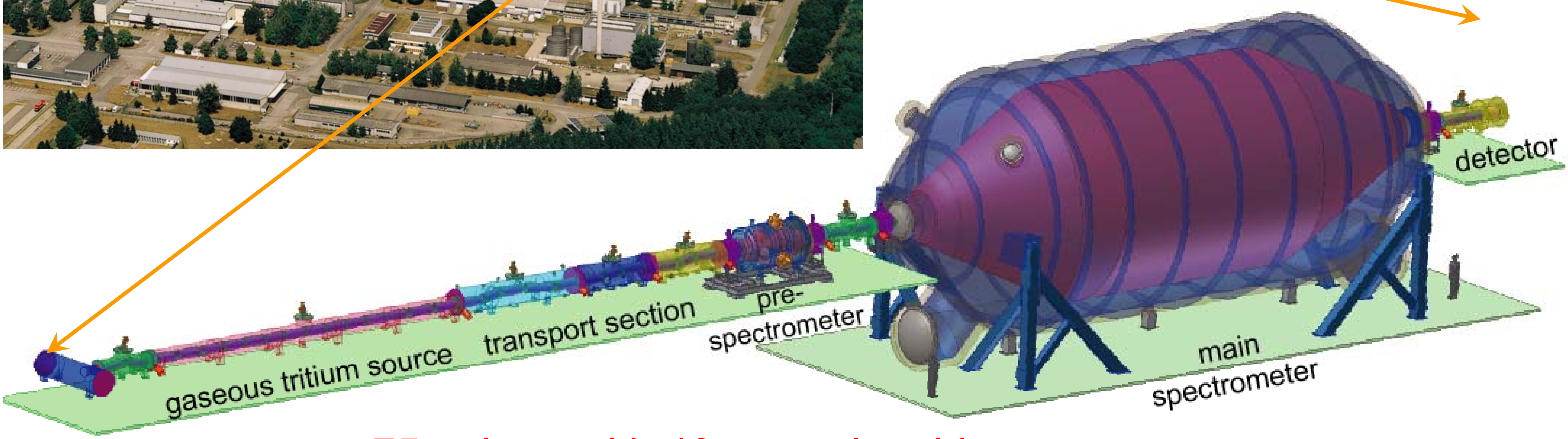


KATRIN

at Forschungszentrum Karlsruhe
unique facility for closed T_2 cycle:
Tritium Laboratory Karlsruhe

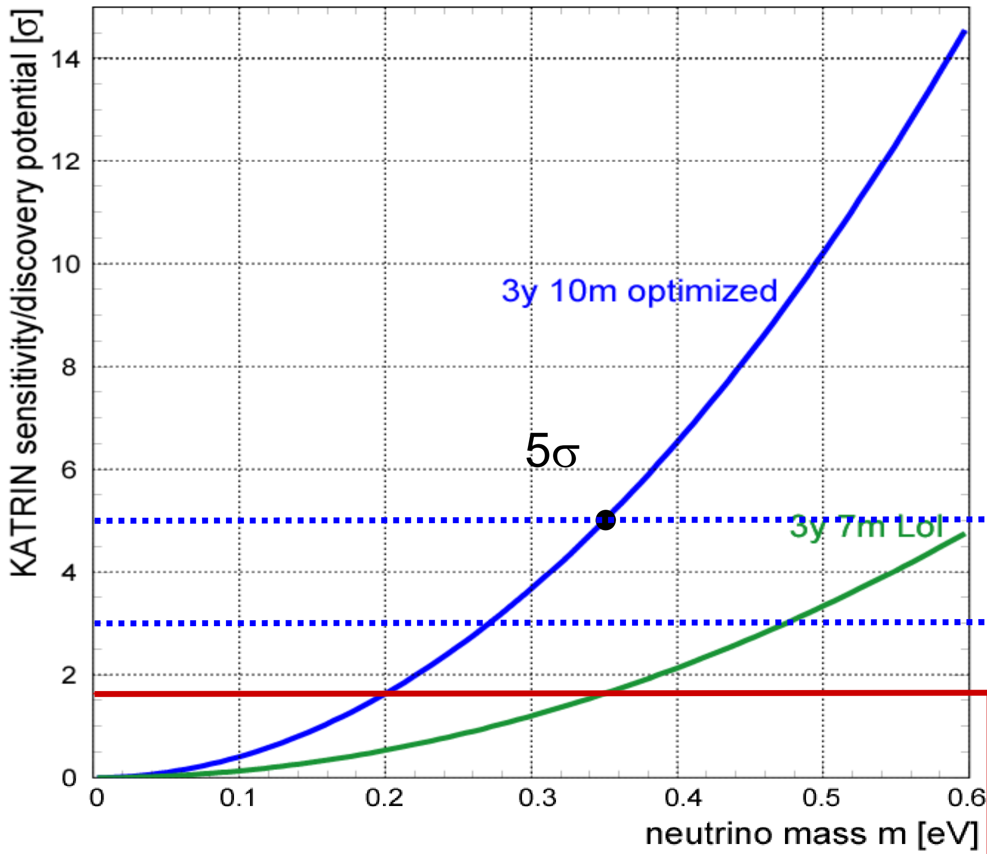
5 countries
13 institutions
100 scientists





~ 75 m long with 40 s.c. solenoids

KATRIN:
sensitivity and discovery potential



Expectation for 3 full beam years:

$\sigma_{syst} \sim \sigma_{stat}$

discovery potential:

m_ν = 0.35eV (5σ)

m_ν = 0.3eV (3σ)

Sensitivity: $m_\nu < 0.2$eV (90%CL)

⇒ KATRIN will improve the sensitivity by 1 order of magnitude

will check the whole cosmological relevant mass range

will detect background neutrinos (if they are degen.)

Perhaps neutrinos are not so useless after all

10.3 Neutrino Oscillations

We begin with a brief introduction to the physics of neutrino oscillations in free space. We discuss the case of two flavors of neutrino, ν_μ and ν_e. The generalization to three flavors is straightforward. These neutrinos are those created (and absorbed) via weak interaction processes. However, we postulate that they are not the mass eigenstates. Rather, they are a mixture of two mass eigenstates designated ν_1 and ν_2 with masses m_1 and m_2 ($m_1 \neq m_2$). The weak interaction states are obtained by a unitary transformation of these mass eigenstates:

$$|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle \quad (10.28)$$

$$|\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle \quad (10.29)$$

where θ is a mixing angle and a parameter of the theory. A weak interaction process like nuclear beta decay generates a ν_e, which then propagates as a function of time as

$$|\nu(t)\rangle = e^{-iE_1 t}\cos\theta|\nu_1\rangle + e^{-iE_2 t}\sin\theta|\nu_2\rangle\ . \quad (10.30)$$

At $t = 0$ we have a pure ν_e but because of the phase slippage as a function of time, the relative degree of ν_μ and ν_e in the state vector varies. If the mass difference is small, $\Delta m^2 \equiv m_2^2 - m_1^2 \ll p^2$ ($p \cong E_1 \cong E_2$ is the momentum) then the energies are related by

$$E_1 - E_2 \cong \frac{m_2^2 - m_1^2}{2p} \, . \tag{10.31}$$

Then the probabilities for detecting a ν_e or ν_μ at a distance $x(= t)$ are given by

$$P_e(x) = |\langle \nu_e | \nu \rangle_t|^2 = 1 - \sin^2 2\theta \sin^2 \left(\frac{\pi x}{L} \right) \tag{10.32}$$

$$P_\mu(x) = |\langle \nu_\mu | \nu \rangle_t|^2 = \sin^2 2\theta \sin^2 \left(\frac{\pi x}{L} \right) \tag{10.33}$$

where the characteristic oscillation length (in vacuum) is defined by

$$L \equiv \frac{4\pi p}{\Delta m^2} \, . \tag{10.34}$$

Length & Energy Scales

$E_\nu = 1$ MeV, $\Delta m^2 = 1$ eV^2, → L = 1.24 meters

(P_e →minimum)

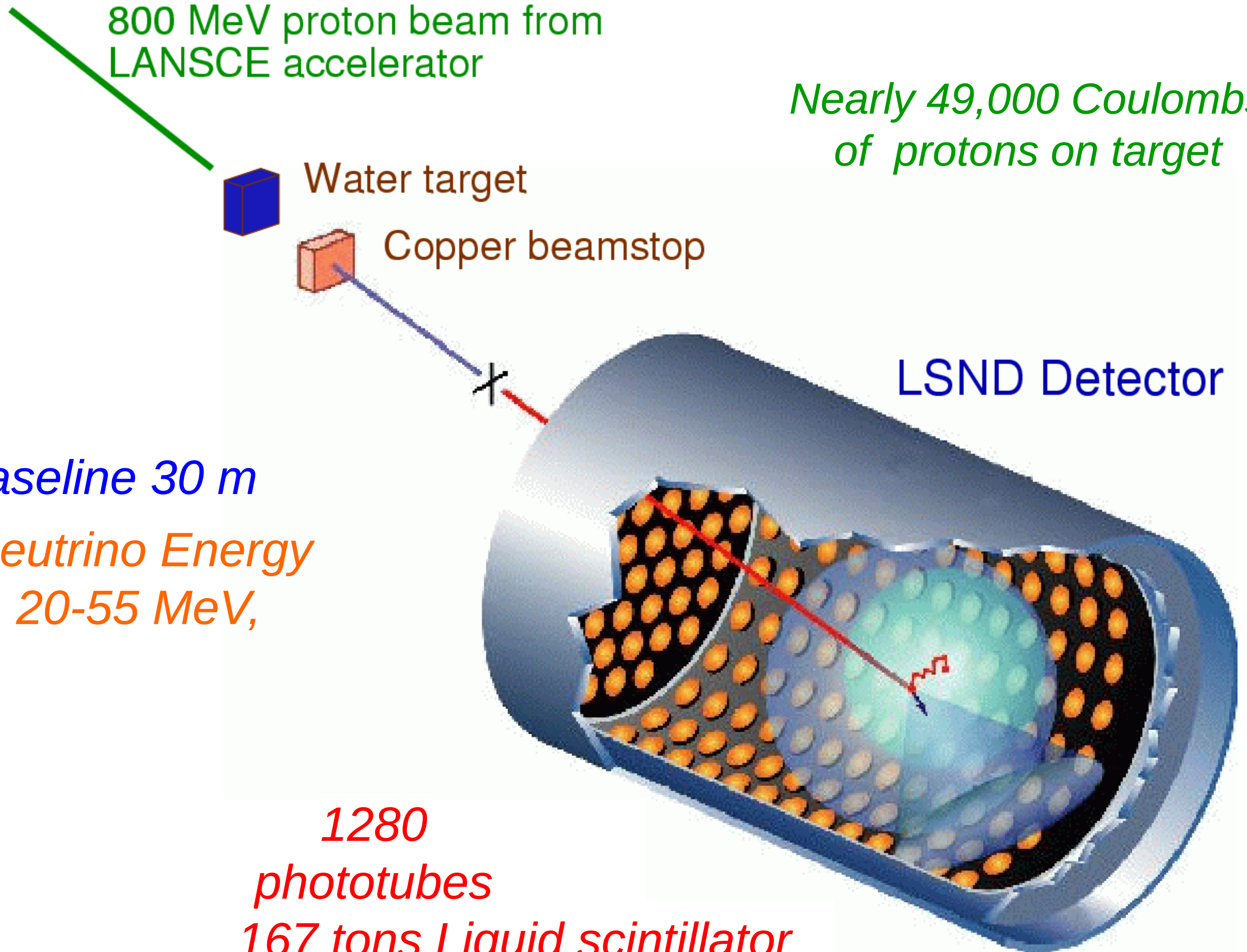
E_ν= 1 GeV, $\Delta m^2 = 10^{-3}$ eV^2 , L = 1240 km Super-K

E_ν= 1 MeV, $\Delta m^2 = 10^{-3}$ eV^2 , L = 1.2 km Chooz, Palo Verde

E_ν= 1 MeV, $\Delta m^2 = 10^{-5}$ eV^2 , L = 125 km KamLAND

Kamioka Liquid scintillator Anti-Neutrino Detector

The LSND Experiment (1993-98)



Nearly 49,000 Coulombs of protons on target

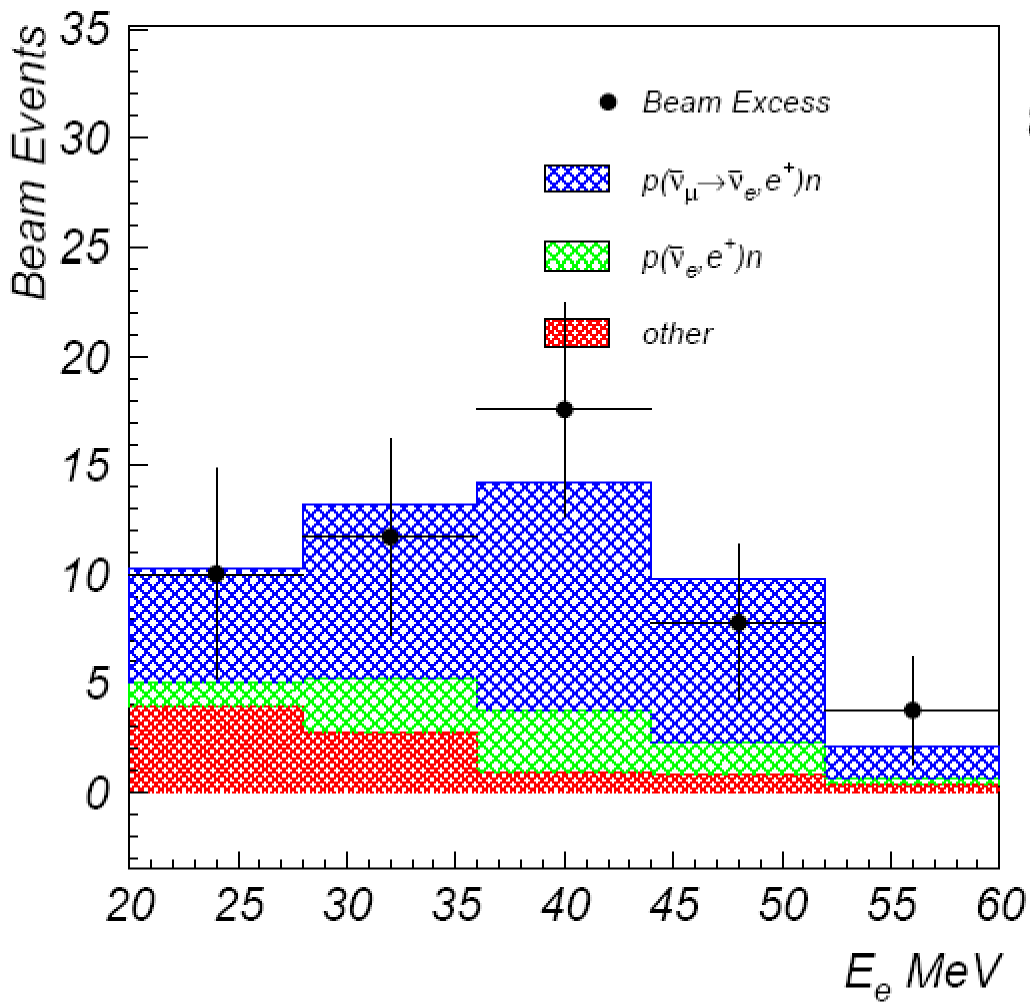
Baseline 30 m

Neutrino Energy 20-55 MeV,

1280 phototubes

167 tons Liquid scintillator

LSND Result

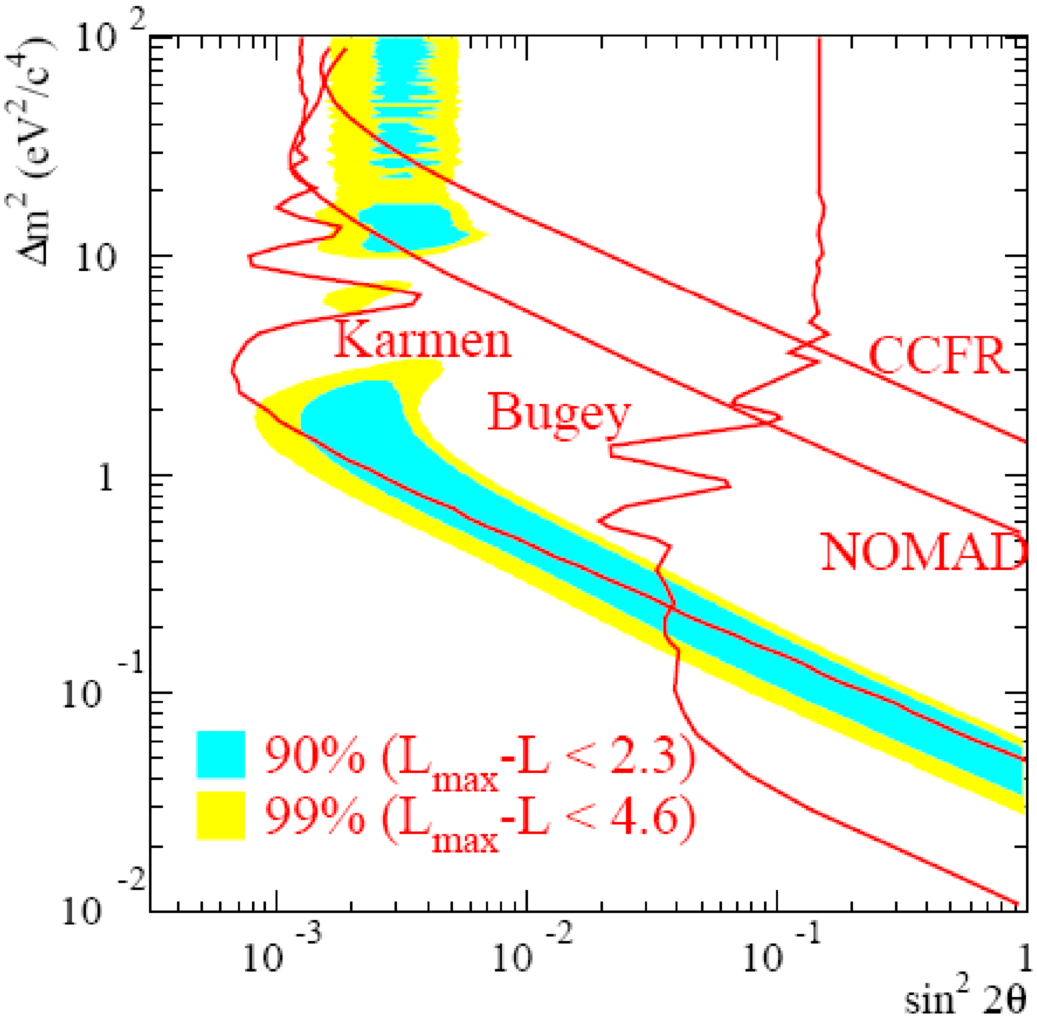


Excess:

$87.9 \pm 22.4 \pm 6.0 \; \bar{\nu}_e p \rightarrow e^+ n$ events

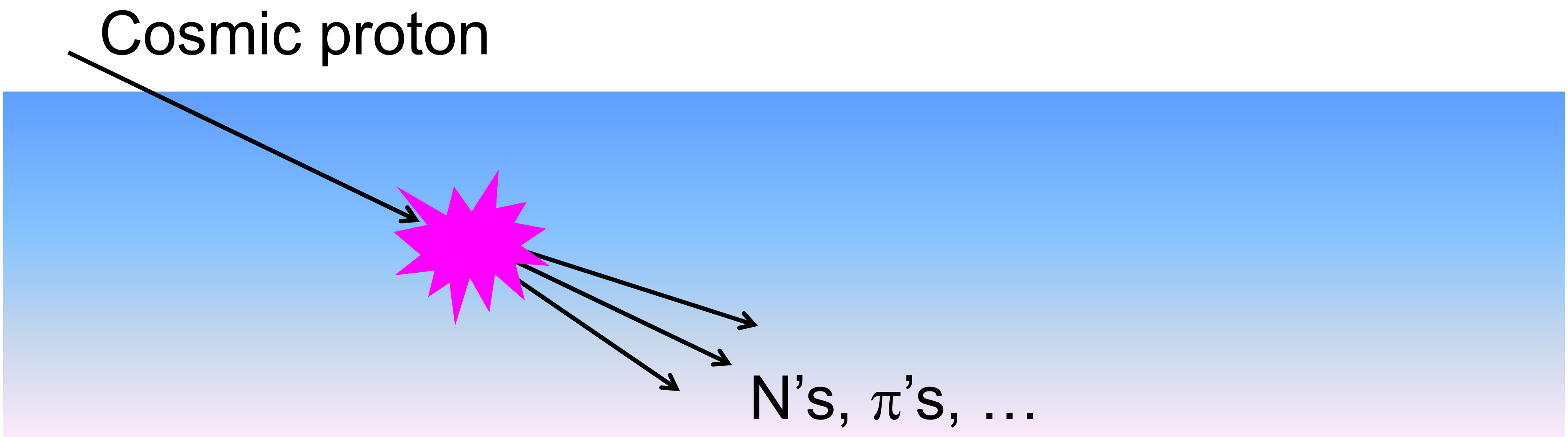
Osc. Prob.:

$(0.264 \pm 0.067 \pm 0.045)\%$



LSND was not unconfirmed –
(New MiniBOONE result)

Atmospheric neutrinos



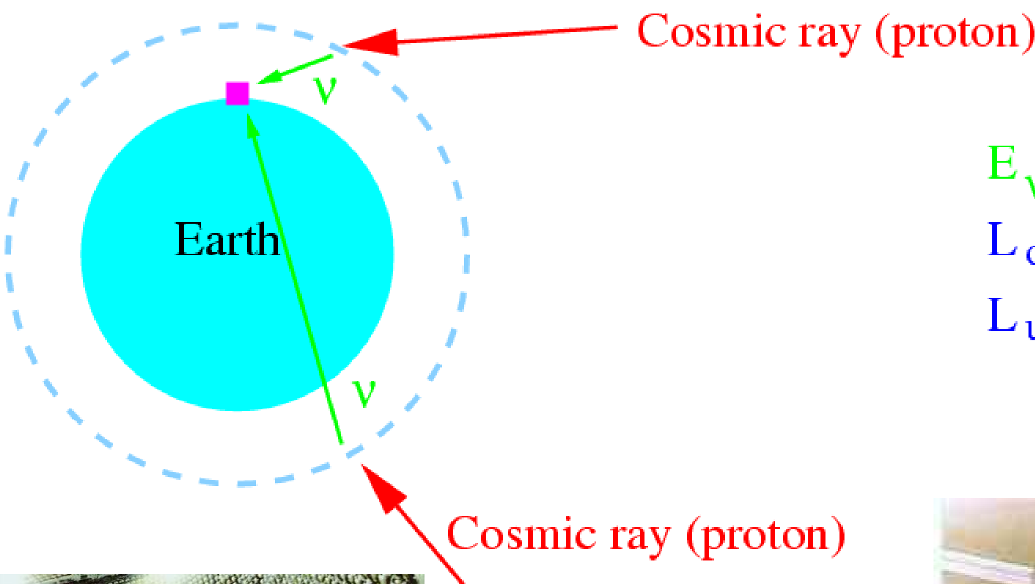
$$\pi \rightarrow \mu + \nu_\mu$$

$$\mu \rightarrow e + \bar{\nu}_\mu + \nu_e$$

$$\frac{\nu_\mu + \bar{\nu}_\mu}{\nu_e + \bar{\nu}_e} = 2$$

Super – Kamiokande

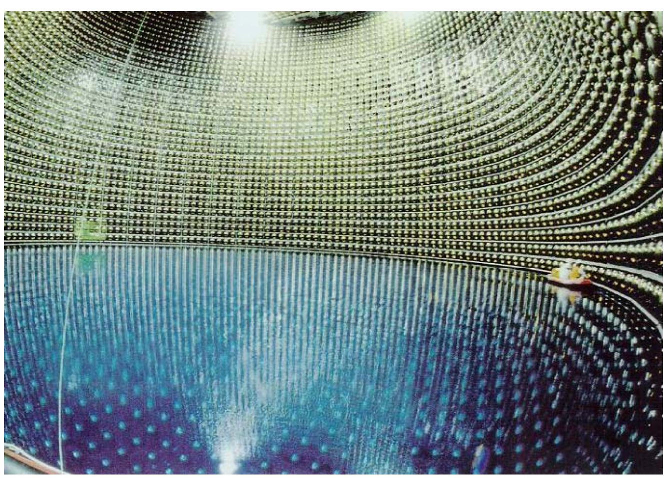
Atmospheric Neutrino Oscillations



E_{ν} ~ 0.5 – 5 GeV

L_{down} ~ 100 km

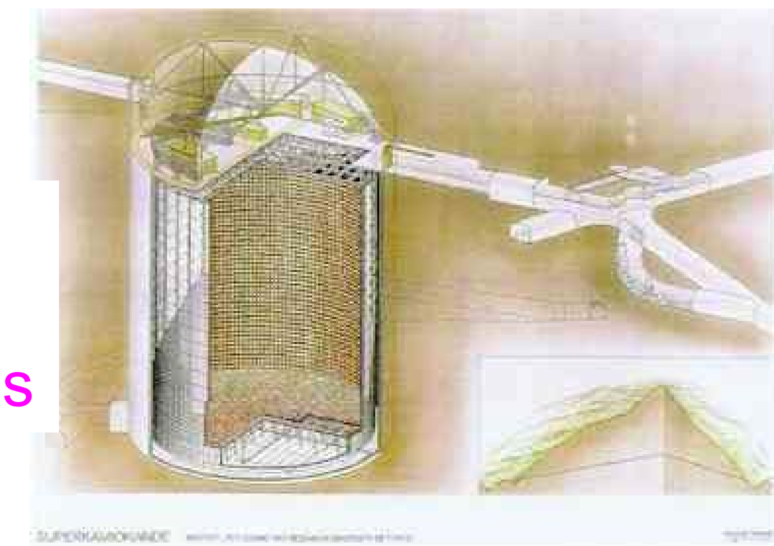
L_{up} ~ 10,000 km

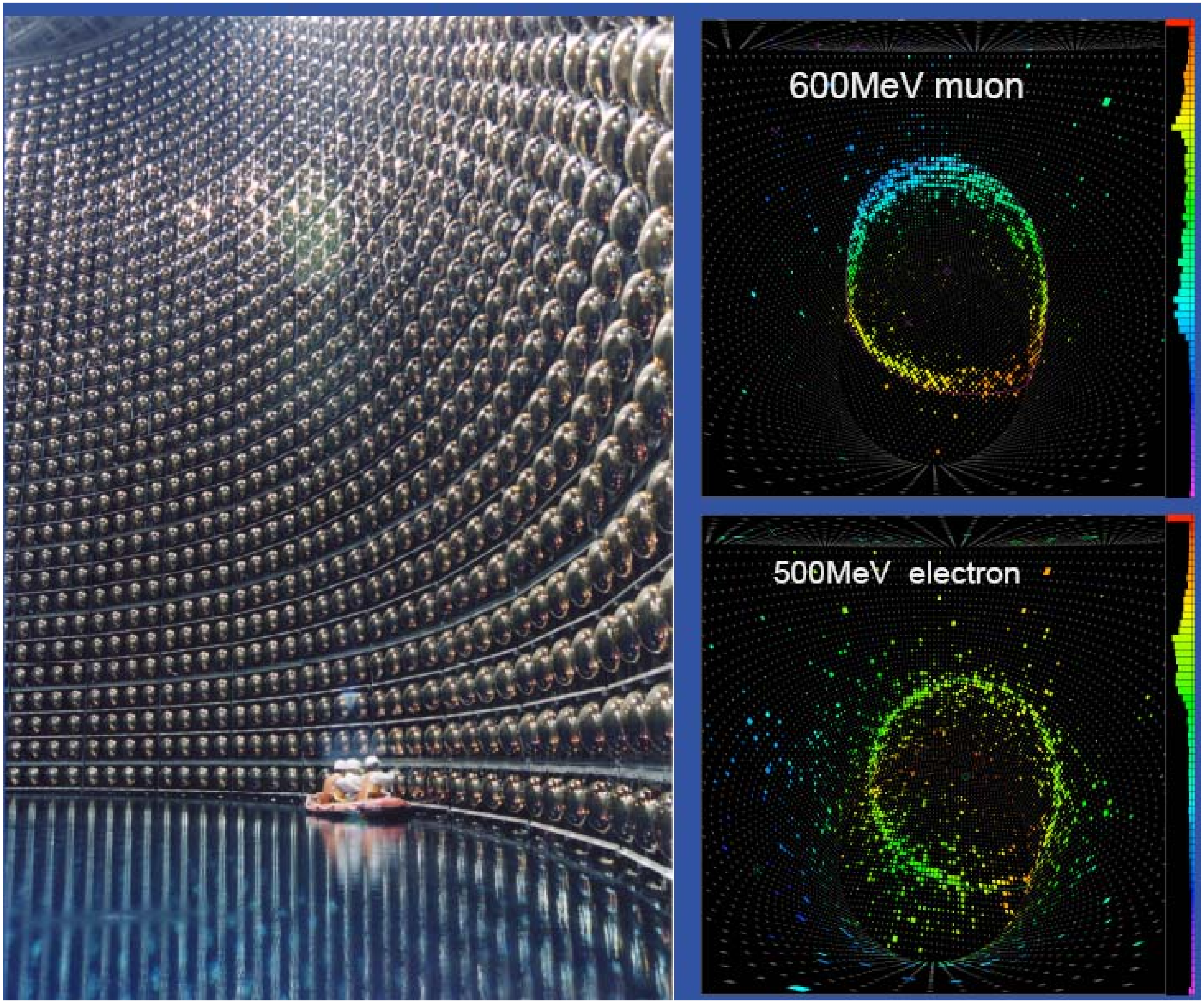


30 kton

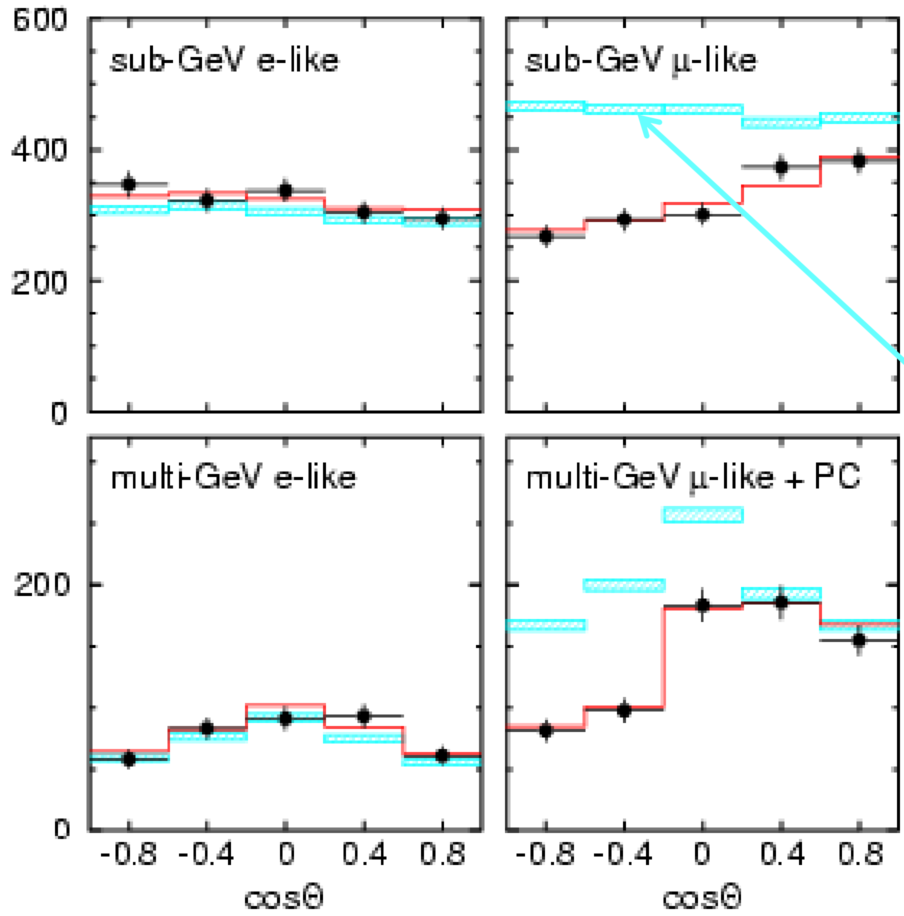
$H_2$0 Cherenkov

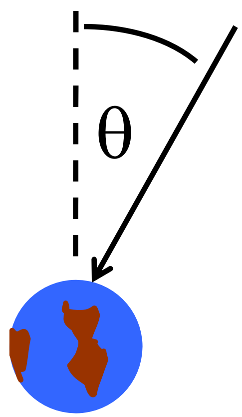
11000 20” PMT’s




600MeV muon
500MeV electron

SuperKamiokande Result (1998)





Monte Carlo

Where are the upward muon neutrinos??

Implications of SuperK Observation

- $\Delta m^2 = 2.5 \times 10^{-3}\ eV^2 \neq 0 \;\; \rightarrow$ at least 1 $m_\nu \neq 0$
- Mixing angle is quite large ($\theta \sim 45°$)

Note: SuperK seems to be observing

$$\nu_\mu \rightarrow \nu_\tau$$

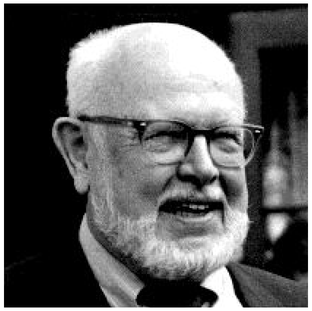
My deepest personal interest is in experimental data, in the analysis of the data and in the proper use of the data in theoretical stellar models. I continue to be encouraged in this regard by this one-hundred and nine year old quotation from Mark Twain:

> *There is something fascinating about science. One gets such wholesale returns of conjecture out of such a trifling investment of fact.*
>
> \- Life on the Mississippi 1874

For me Twain's remark is a challenge to the experimentalist. The experimentalist must try to eliminate the word "trifling" through his endeavors in uncovering the facts of nature.

W.A. Fowler
Nobel Lecture,
1983

Maybe there is something wrong with these astrophysical neutrinos???



We need a “laboratory” experiment!!


Stay Tuned !!