QCD at High Temperature

(Theory)

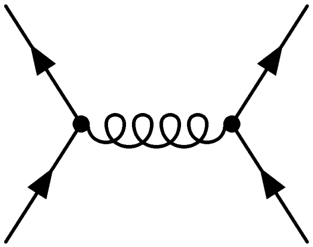
The High T Phase: Qualitative Argument

High T phase: Weakly interacting gas of quarks and gluons?

typical momenta $p \sim 3T$

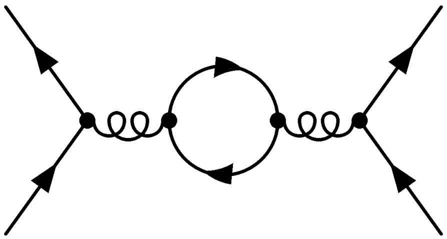
Large angle scattering involves large momentum transfer

effective coupling is small



Small angle scattering is screened (not anti-screened!)

coupling does not become large



Quark Gluon Plasma

Basic Thermodynamics

Massless particles, zero baryon density $(\zeta(3) = 1.2)$

$$n = g\frac{\zeta(3)}{\pi^2}T^3 \begin{cases} 1 \\ 3/4 \end{cases} \qquad \epsilon = g\frac{\pi^2}{30}T^4 \begin{cases} 1 & \text{bosons} \\ 7/8 & \text{fermions} \end{cases}$$

$$s/n = 2\pi^4/(45\zeta(3)) \simeq 3.6 \qquad P = \epsilon/3$$

massless quarks and gluons

$$g_{eff} = 2 \times 8 \times 1 + 4 \times 3 \times 2 \times 7/8 = 37$$

spin $\times$ color $\times$ boson $+$ spin $\times$ color $\times$ flavors $\times$ fermion

massless pions

$$g = (N_f^2 - 1) \times 1 = 3$$

First Approach: Bag Model

Low temperature: Pions

$$\epsilon = \frac{3\pi^2}{30} T^4 \qquad\qquad P = \frac{3\pi^2}{90} T^4$$

High temperature: Quarks and gluons

$$\epsilon = \frac{37\pi^2}{30} T^4 \qquad\qquad P = \frac{37\pi^2}{90} T^4$$

Include vacuum energy $T_{\mu\nu} = B g_{\mu\nu}$ (QCD cosmological constant)

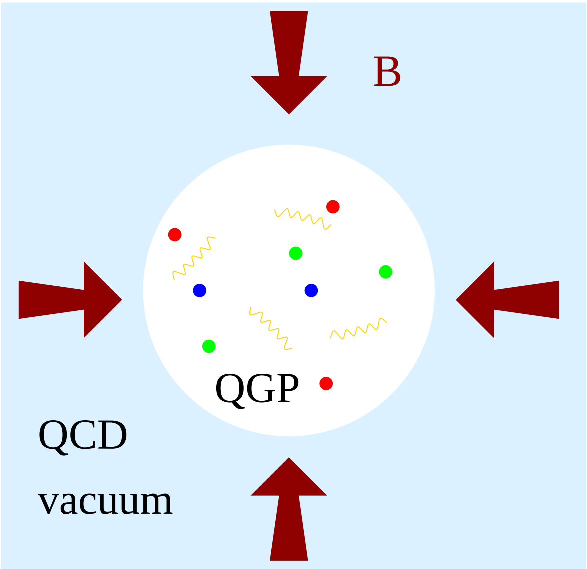
$$\epsilon_{vac} = -P_{vac} = -B \qquad\qquad \epsilon_{vac} = -\frac{b}{32}\langle \frac{\alpha}{\pi} G^2 \rangle \simeq -0.5 \text{ GeV}/\text{fm}^3$$

trace anomaly relation

Critical temperature: equate pressures

$$\frac{3\pi^2}{90}T^4 + B = \frac{37\pi^2}{90}T^4$$

$$T_c = \left(\frac{45B}{17\pi^2}\right)^{1/4} \simeq 180\ \text{MeV}$$



Pressure is continuous, but energy density jumps

$$\begin{aligned}\epsilon(T_c^-) &= \frac{3\pi^2}{30}T_c^4 \simeq 100\ \text{MeV}/\text{fm}^3 \\ \epsilon(T_c^+) &= \frac{37\pi^2}{30}T_c^4 + B \simeq 2000\ \text{MeV}/\text{fm}^3\end{aligned}$$

Second Approach: Sigma Model

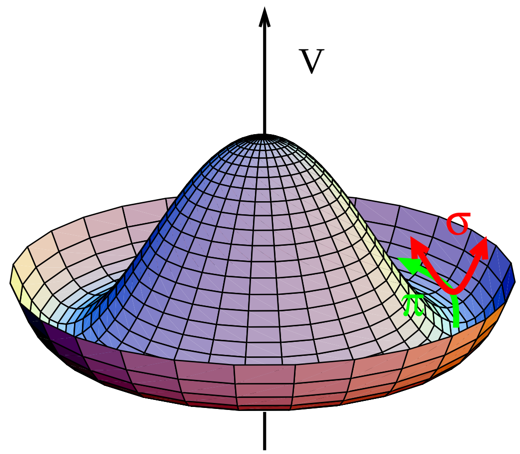
Simple model based on linear representation of $SU(2)_L \times SU(2)_R$

$$\phi^a = (\sigma, \vec{\pi}) \qquad\qquad O(4) = SU(2)_L \times SU(2)_R$$

Chirally symmetric lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^a)^2 + V(\phi^a \phi^a)$$

$$V(\phi^a \phi^a) = -\frac{\mu^2}{2}(\phi^a \phi^a) + \frac{\lambda}{4}(\phi^a \phi^a)^2$$



Minimum of potential

$$\frac{\partial V}{\partial \phi^a} = \phi^a(-\mu^2 + \lambda \phi^a \phi^a) = 0 \qquad \phi_0^a = (\sigma_0, \vec{0}) \;\; \sigma_0^2 = \mu^2/\lambda \equiv f_\pi^2$$

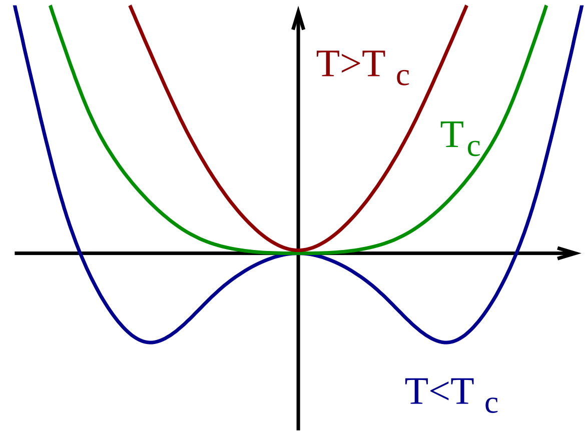
Direction fixed by explicit breaking $\mathcal{L}_{SB} = -c\sigma$

Thermal Fluctuations

Thermal averages

$$\vec{\pi}_T = 0$$

$$\sigma_T^2 = f_\pi^2 \left(1 - \frac{1}{f_\pi^2}\langle\tilde{\phi}^a\tilde{\phi}^a\rangle\right)$$



Gaussian fluctuations ($m = 0$)

$$\langle\tilde{\phi}^a\tilde{\phi}^a\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\omega_k}\frac{1}{e^{\beta\omega_k} - 1} = \frac{T^2}{12}$$

Critical temperature (3 light d.o.f.)

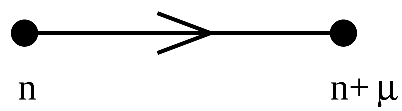
$$\sigma_T^2 = f_\pi^2\left(1 - \frac{T^2}{3f_\pi^2}\right) \qquad T_c = \sqrt{3}f_\pi \simeq 160 \text{ MeV}$$

Lattice QCD

Euclidean partition function

$$Z = \int dA_\mu d\psi \exp(-S) = \int dA_\mu \det(i\not{D}\ ) \exp(-S_G)$$

Lattice discretization:



$$U_\mu(n) = \exp(igaA_\mu(n))$$

$$D_\mu \phi \rightarrow \frac{1}{a}[U_\mu(n)\phi(n+\mu) - \phi(n)]$$

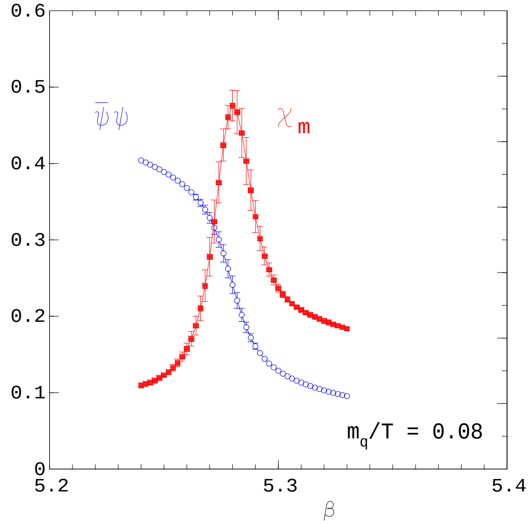
$$(G^a_{\mu\nu})^2 \rightarrow \frac{1}{a^4}\mathrm{Tr}[U_\mu(n)U_\nu(n+\mu)U_{-\mu}(n+\mu+\nu)U_{-\nu}(n+\nu) - 1]$$

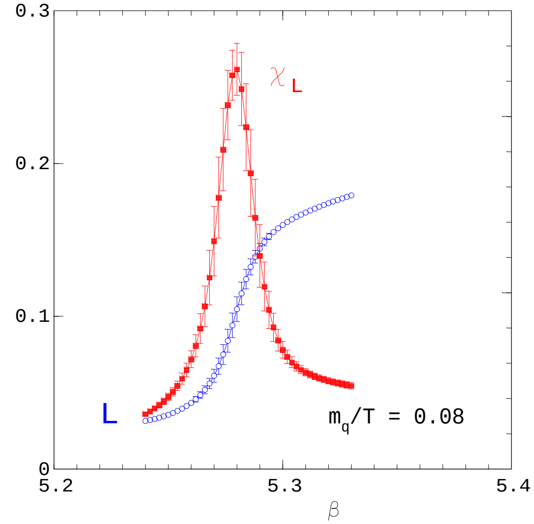
Monte Carlo:

$$\int dA_\mu\ e^{-S} \rightarrow \{U_\mu^{(1)}(n), U_\mu^{(2)}(n), \ldots\}$$

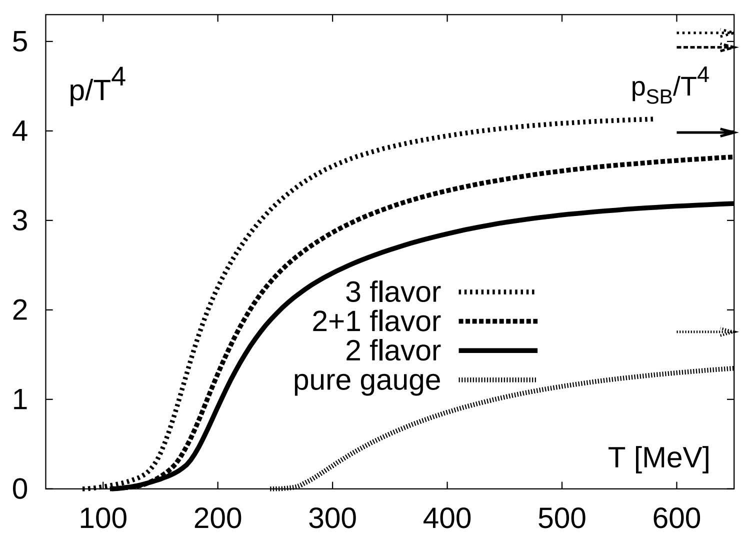
Lattice Results

order parameters

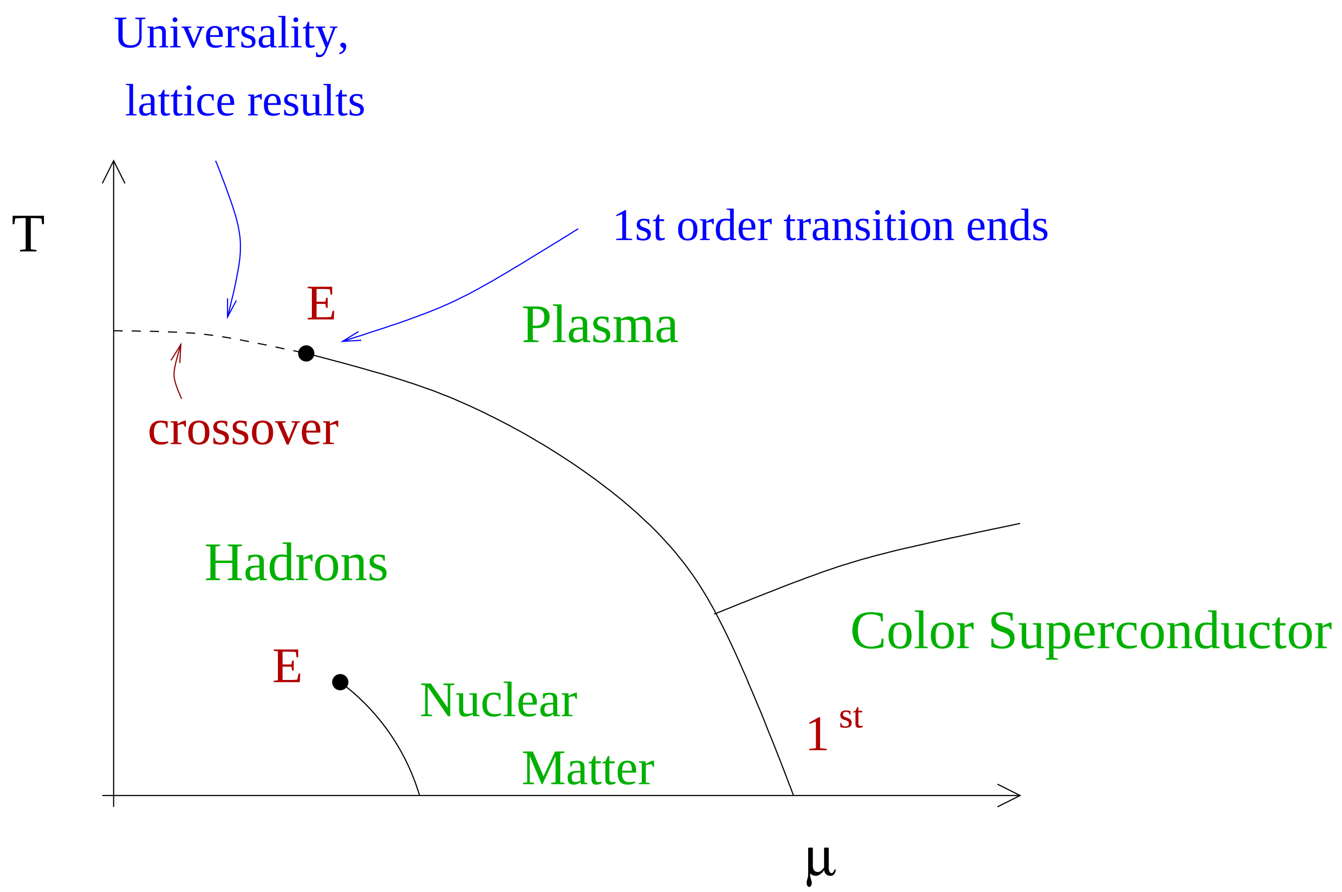




equation of state



Phase Diagram: First Version



critical endpoint (E) persists even if $m \neq 0$

Weakly coupled QGP

Basic object: Partition function

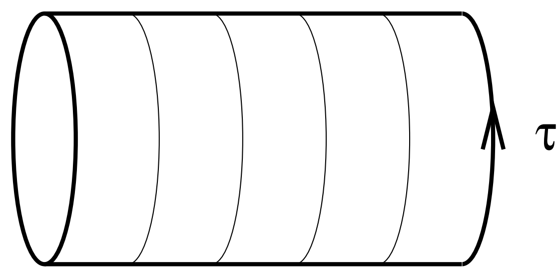
$$Z = \mathrm{Tr}[e^{-\beta H}], \quad \beta = 1/T \qquad\qquad F = T\log(Z)$$

Basic trick

$$Z = \mathrm{Tr}[e^{-i(-i\beta)H}] \qquad\qquad \text{imaginary time evolution}$$

Path integral representation ($\tau = it$)

$$Z = \int dA_\mu d\psi \exp\left(-\int_0^\beta d\tau \int d^3x \; \mathcal{L}_E\right)$$



$$A_\mu(\vec{x},\beta) = A_\mu(\vec{x},0); \; \psi(\vec{x},\beta) = -\psi(\vec{x},0)$$

Fourier representation

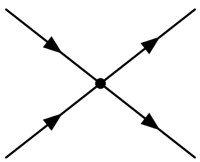
$$A_\mu(\vec{x},\tau) = \sum_n \int d^3k A^n_\mu(\vec{k}) e^{i(\vec{k}\vec{x}+\omega_n\tau)}$$

Matsubara frequencies

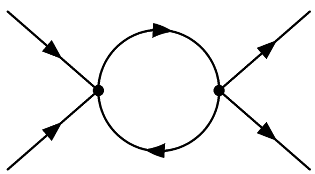
$$\omega_n = 2\pi n T \qquad \text{bosons}$$

$$\omega_n = (2n+1)\pi T \qquad \text{fermions}$$

Feynman rules: Euclidean QCD with discrete energies



$$T \sum_n \int \frac{d^3 p}{(2\pi)^3}$$



$$(2\pi)^3 \delta^3 (\sum \vec{p}_i) \delta_{\sum n_i}$$

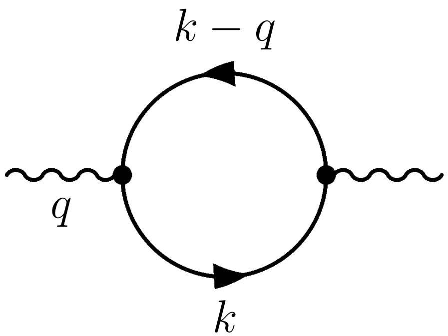
Typical Matsubara Sums

$$\sum_k \frac{1}{x^2 + k^2} = \frac{2\pi}{x} \left(\frac{1}{2} + \frac{1}{e^{2\pi x} - 1} \right) \qquad \text{bosons}$$

$$\sum_k \frac{1}{x^2 + (2k+1)^2} = \frac{\pi}{x} \left(\frac{1}{2} - \frac{1}{e^{\pi x} + 1} \right) \qquad \text{fermions}$$

Gluon Polarization Tensor

Warmup: Photon polarization function $\Pi_{\mu\nu}$



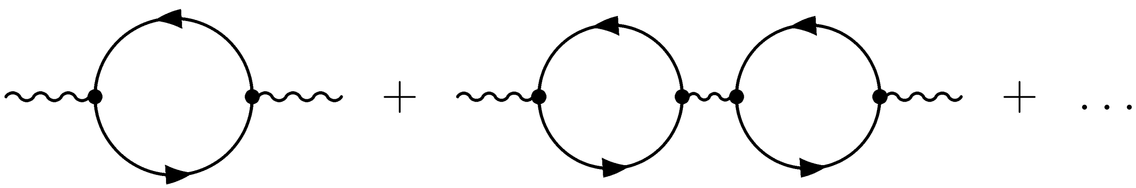
$$= e^2 T \sum_n \int \frac{d^3k}{(2\pi)^3} \mathrm{tr}[\gamma_\mu \not{k} \gamma_\nu (\not{k} - \not{q})] \Delta(k)\Delta(k-q)$$

Hard Thermal Loop (HTL) limit ($q \ll k \sim T$)

$$\Pi_{\mu\nu} = 2m^2 \int \frac{d\Omega}{4\pi} \Big(\frac{i\omega \hat{K}_\mu \hat{K}_\nu}{q \cdot \hat{K}} + \delta_{\mu 4}\delta_{\nu 4} \Big) \qquad \hat{K} = (-i, \hat{k})$$

$2m^2 = \frac{1}{3} e^2 T^2$ Debye mass

Significance of $\Pi_{\mu\nu}$

$D_{\mu\nu} =$  $= \dfrac{1}{(D^0_{\mu\nu})^{-1} + \Pi_{\mu\nu}}$

$D_{00}(\omega = 0, \vec{q})$ determines static potential

$$V(r) = e \int \frac{d^3q}{(2\pi)^3} \frac{e^{iqr}}{\vec{q}^2 + \Pi_{00}} \simeq -\frac{e}{r} \exp(-m_D r)$$

screened Coulomb potential

D_{ij} determines magnetic interaction

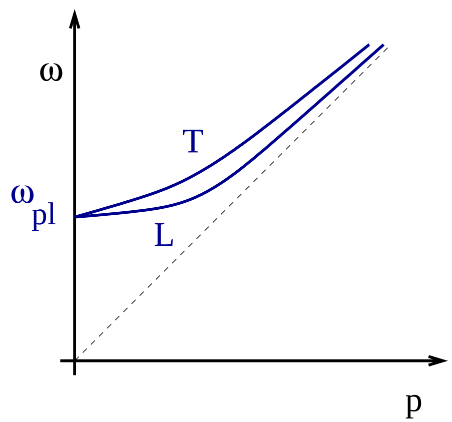
$$\Pi_{ii}(\omega \to 0, 0) = 0$$

no magnetic screening

$$\mathrm{Im}\Pi_{ii}(\omega, q) \sim \frac{\omega}{q} m_D^2 \Theta(q - \omega)$$

Landau damping

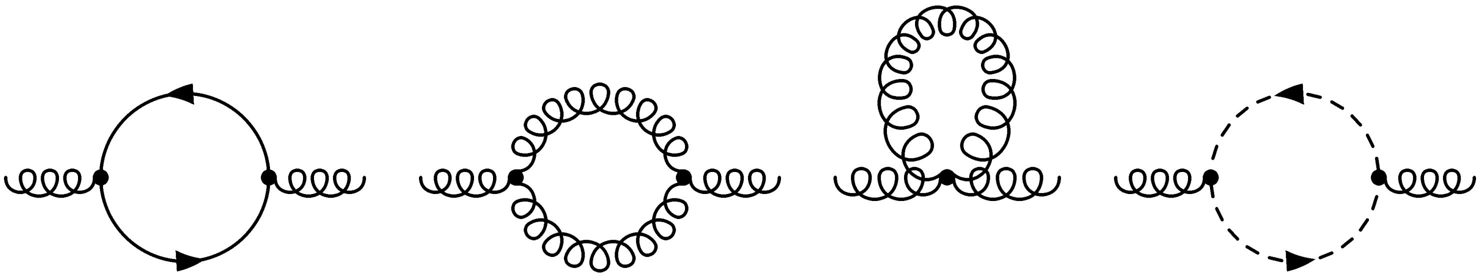
Poles of propagator: Plasmon dispersion relation



$$\text{pole}: \; D^{-1}_{L,T}(\omega, q) = 0$$

$$q \to 0: \quad \omega_L^2 = \omega_T^2 = \frac{1}{3} m_D^2$$

QCD looks more complicated



same result as QED with $m_D^2 = g^2T^2(1 + N_f/6)$

Conclusion: Perturbative Quark Gluon Plasma

quasi-quarks and quasi-gluons

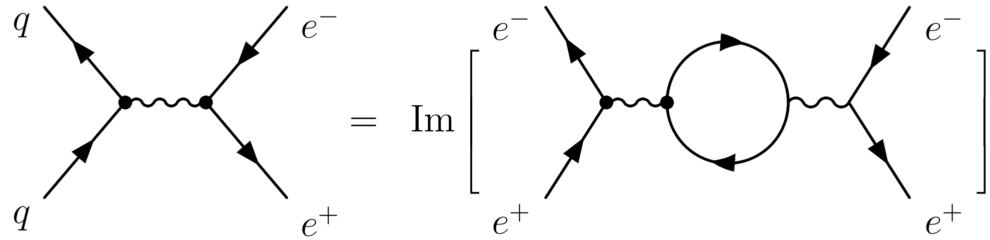
typical energies, momenta $\omega, p \sim T$

effective masses $m \sim gT$, width $\gamma \sim g^2T$

Note that $\gamma \ll \omega$ (long lived quasi-particles)

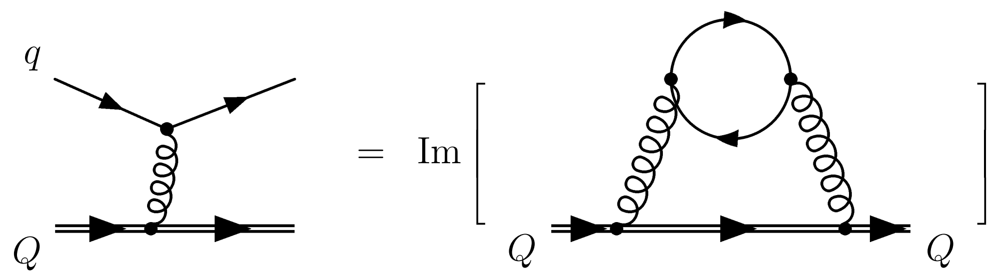
Physical Applications

Dilepton production



$$\frac{dR}{d^4q} = \frac{\alpha^2}{48\pi^2}\Big(12\sum_q e_q^2\Big)e^{-E/T}$$

Collisional energy loss



$$\frac{dE}{dx} = \frac{8\pi}{3}\alpha_s^2T^2\Big(1+\frac{N_f}{6}\Big)\log\Big(c\frac{\sqrt{ET}}{m_D}\Big) \qquad E \gg M^2/T$$

$E = 20$ GeV: $dE/dx \simeq 0.3\ \mathrm{GeV/fm}$ for c, b quarks

note: for light quarks radiative energy loss dominates

Kinetic Theory

Quasi-Particles ($\gamma \ll \omega$): introduce distribution function $f_p(x,t)$

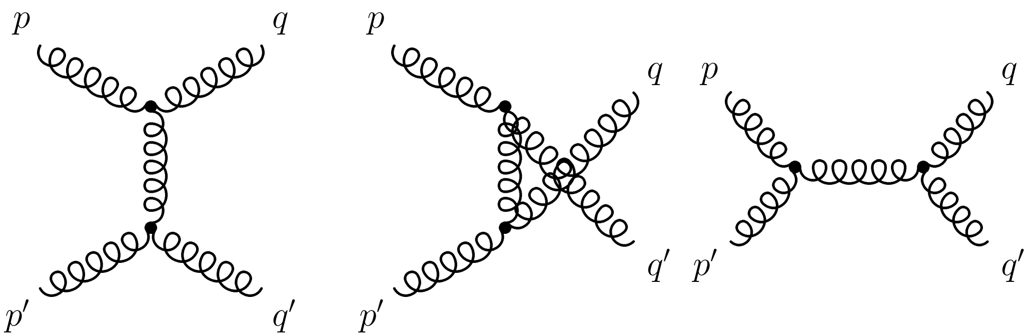
$$N = \int \frac{d^3p}{E_p}\, f_p \qquad\qquad T_{ij} = \int d^3p\, \frac{p_i p_j}{E_p} f_p,$$

Boltzmann equation

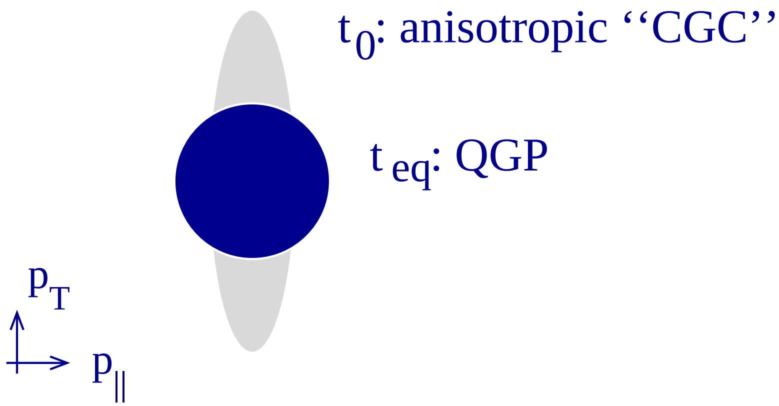
$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p + \vec{F}\cdot\vec{\nabla}_p f_p = C[f_p]$$

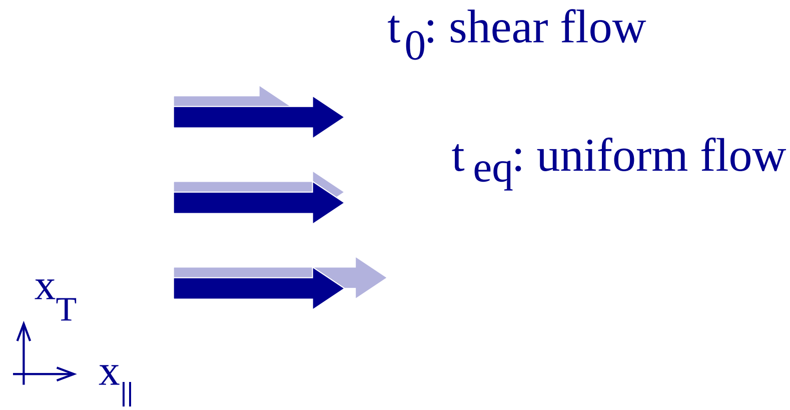
Collision term $C[f_p] = C_{gain} - C_{loss}$

$$C_{loss} = \int dp'dqdq'\; f_p f_{p'} w(p,p';q,q') \qquad\qquad C_{gain} = \ldots$$



Applications: Equilibration, transport coefficients, . . .





Linearized theory (Chapman-Enskog): $f_p = f_p^0(1 + \chi_p/T)$

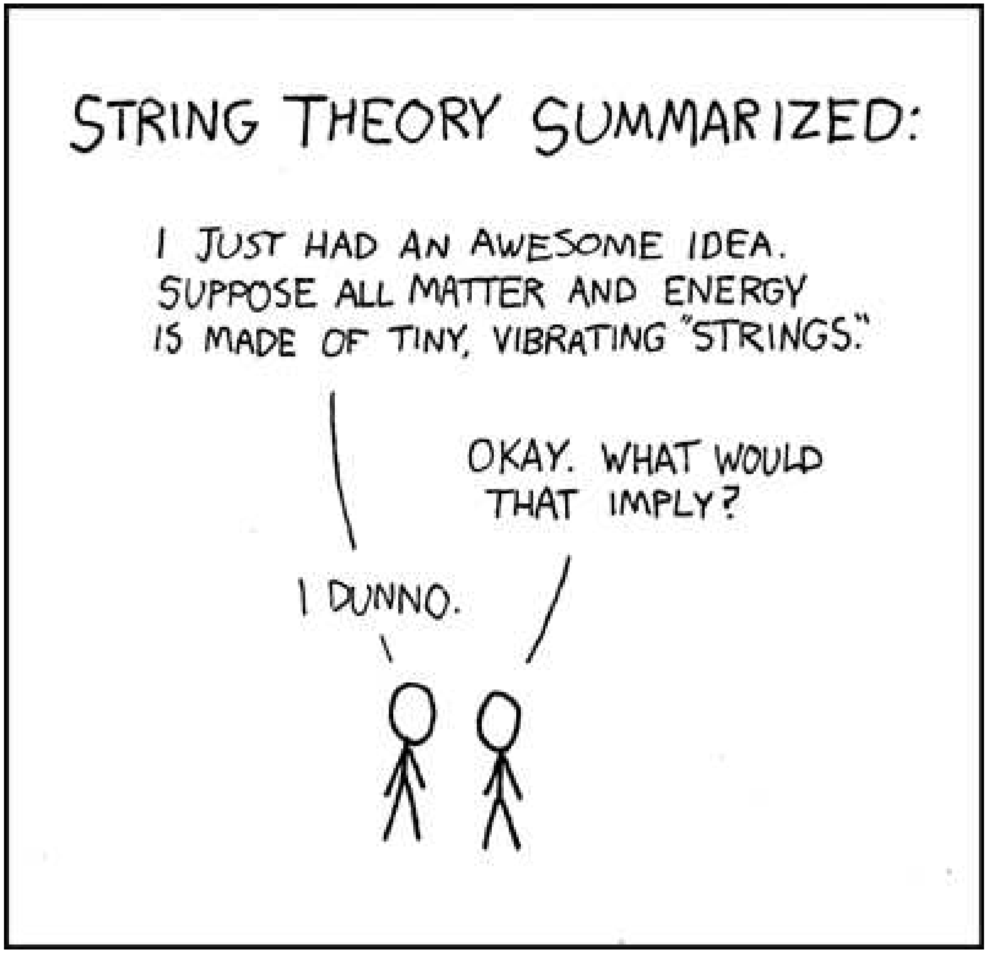
suitable for transport coefficients

Example: shear viscosity $\chi_p = g_p p_i p_j v_{ij}$ $(v_{ij} = \partial_i v_j + \partial_j v_i - trace)$

$$\eta \geq \frac{\langle\chi|X\rangle^2}{\langle\chi|C|\chi\rangle} \qquad \langle\chi|X\rangle = \int d^3p\, f_p^0\, (\chi_p \cdot p_i p_j v_{ij})$$

$$QCD \qquad \eta = \frac{0.34T^3}{\alpha_s^2 \log(1/\alpha_s)}$$

And now for something completely different . . .



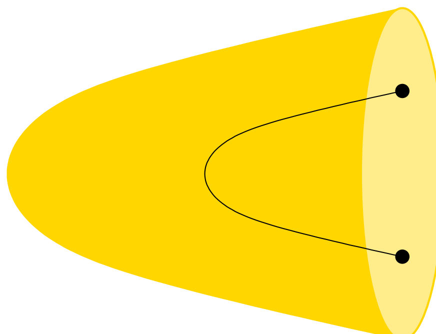
Gauge Theory at Strong Coupling: Holographic Duals

The AdS/CFT duality relates

large N_c (Conformal) gauge theory in 4 dimensions	$\Leftrightarrow$	string theory on 5 dimensional Anti-de Sitter space $\times S_5$
correlation fcts of gauge invariant operators	$\Leftrightarrow$	boundary correlation fcts of AdS fields

$$\langle \exp \int dx \; \phi_0 \mathcal{O} \rangle =$$

$$Z_{string}[\phi(\partial AdS) = \phi_0]$$



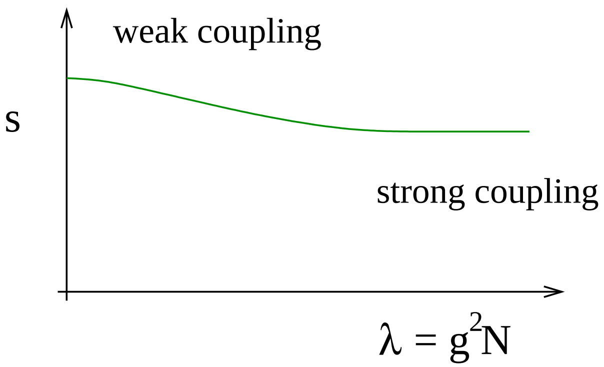
The correspondence is simplest at strong coupling $g^2 N_c$

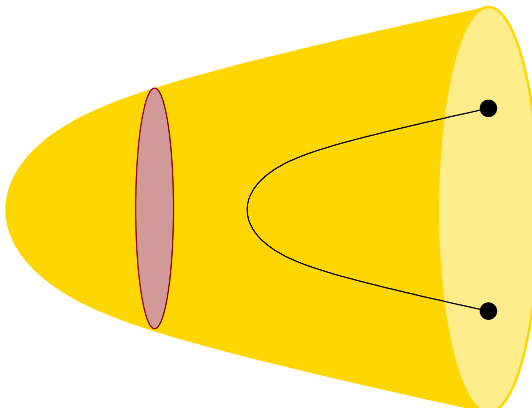
strongly coupled gauge theory $\Leftrightarrow$ classical string theory

Holographic Duals at Finite Temperature

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT temperature	$\Leftrightarrow$	Hawking temperature of black hole
CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon





$$s(\lambda \to \infty) = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} s(\lambda = 0)$$

Gubser and Klebanov

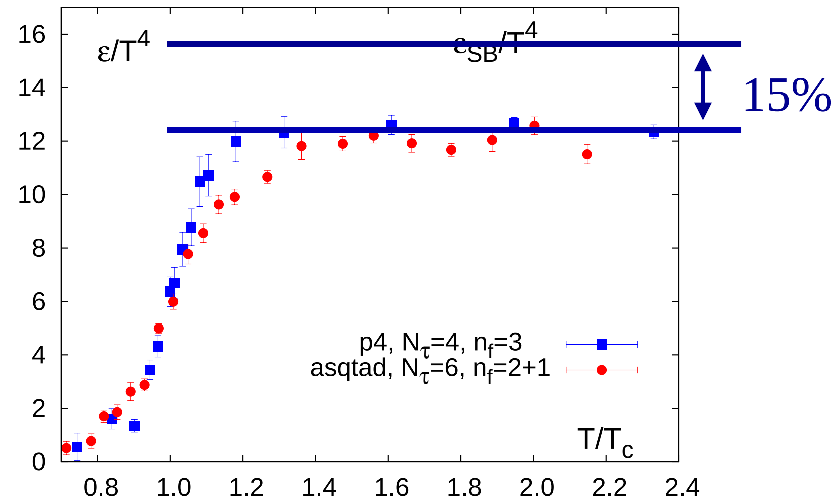
Relevance to QCD?

$\mathcal{N}=4$ QCD

gluons, gluinos [4], Higgses [6]
(all in adjoint representation)
exact conformal symmetry
no chiral symmetry breaking
no confinement
no phase transition

QCD

Matter content not relevant in QGP?
approximately conformal for $T > T_c$?



Ultimate goal: Find holographic dual of QCD

Holographic Duals: Transport Properties

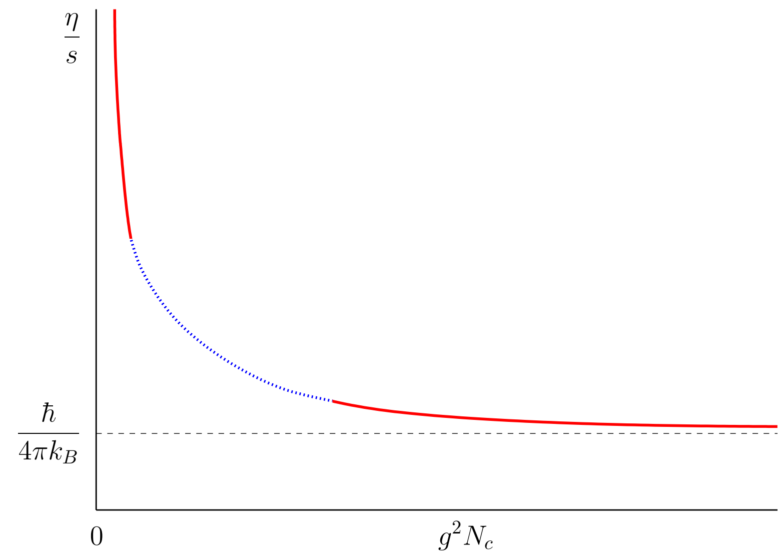
Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon
shear viscosity	$\Leftrightarrow$	Graviton absorption cross section $\sim$ area of event horizon

Strong coupling limit

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Son and Starinets



Strong coupling limit universal? Provides lower bound for all theories?

Summary (Theory)

Lattice QCD: single chiral and deconfinement crossover transition

$T_c \sim 185$ MeV, $\epsilon_{cr} \sim 1.5\,\mathrm{GeV}/\mathrm{fm}^3$

Weakly coupled Quark Gluon Plasma

Quark and gluon quasi-particles, $\gamma \ll \omega$

Thermodynamics: Stefan-Boltzmann gas

Transport: long equilibration times, $\eta/s \simeq 1/\alpha_s^2 \gg 1$

Strongly coupled plasma

No quasi-particles, no kinetics, only hydrodynamics

Thermodynamics: Stefan-Boltzmann law

Transport: fast equilibration, $\eta/s \simeq 1/(4\pi) < 1$