Selected Topics in Lattice Quantum ChromoDynamics

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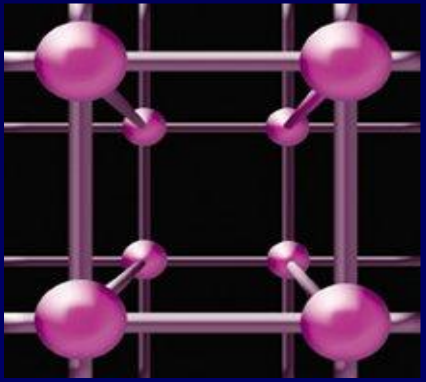
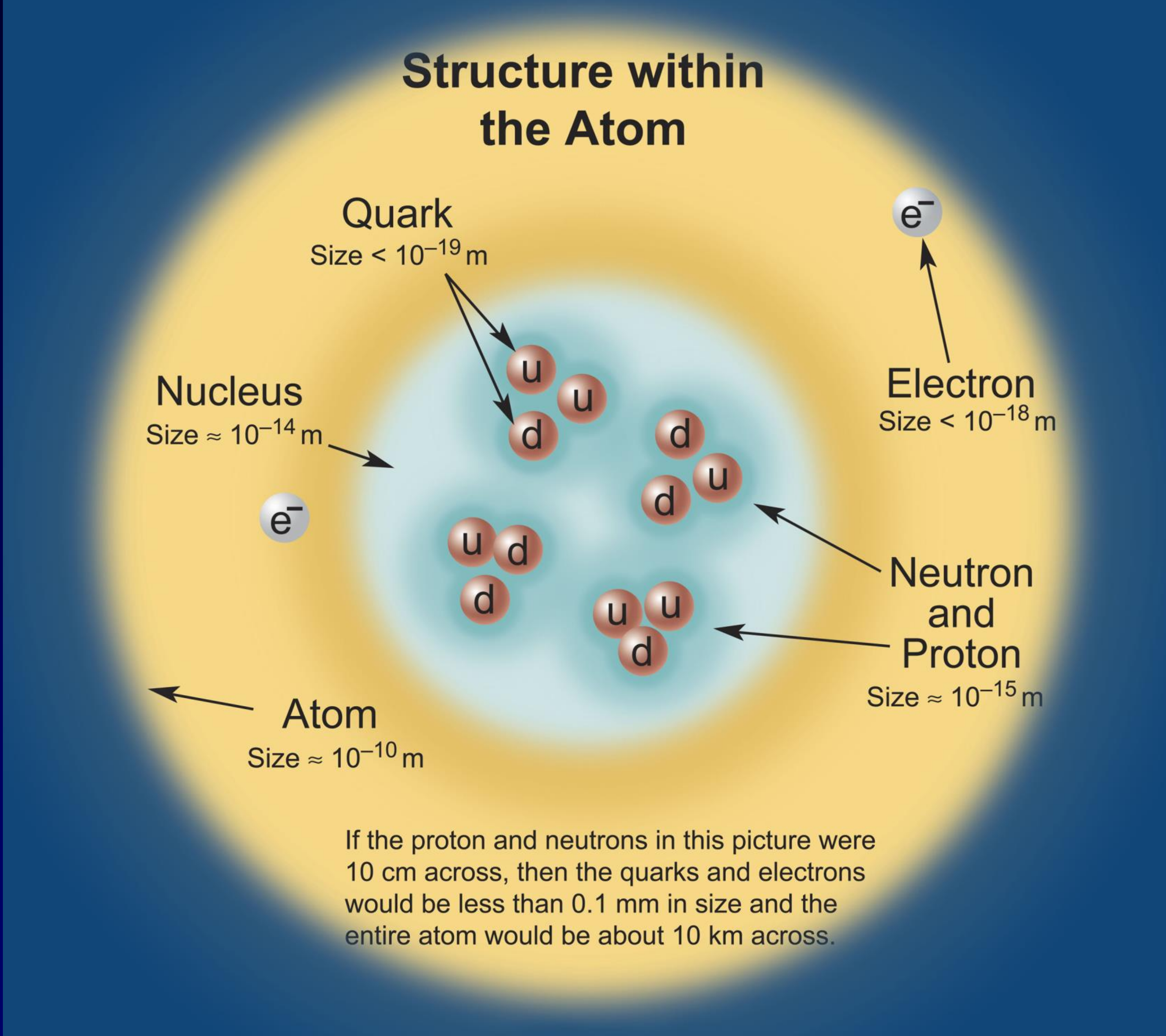
- Key ideas and techniques of lattice QCD
- A few examples relevant to nuclear physics
  - hadron spectrum
  - nuclear force
- Background-field calculations
  - magnetic moments
  - polarizabilities
- Outlook



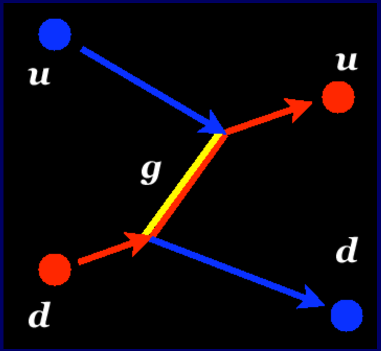

Structure within the Atom
Quark
Size < 10^{-19} m
e⁻
Nucleus
Size ≈ 10^{-14} m
Electron
Size < 10^{-18} m
u
u
d
d
u
d
e⁻
u
d
d
u
u
d
Neutron and Proton
Size ≈ 10^{-15} m
Atom
Size ≈ 10^{-10} m
If the proton and neutrons in this picture were 10 cm across, then the quarks and electrons would be less than 0.1 mm in size and the entire atom would be about 10 km across.

The Four Fundamental Interactions

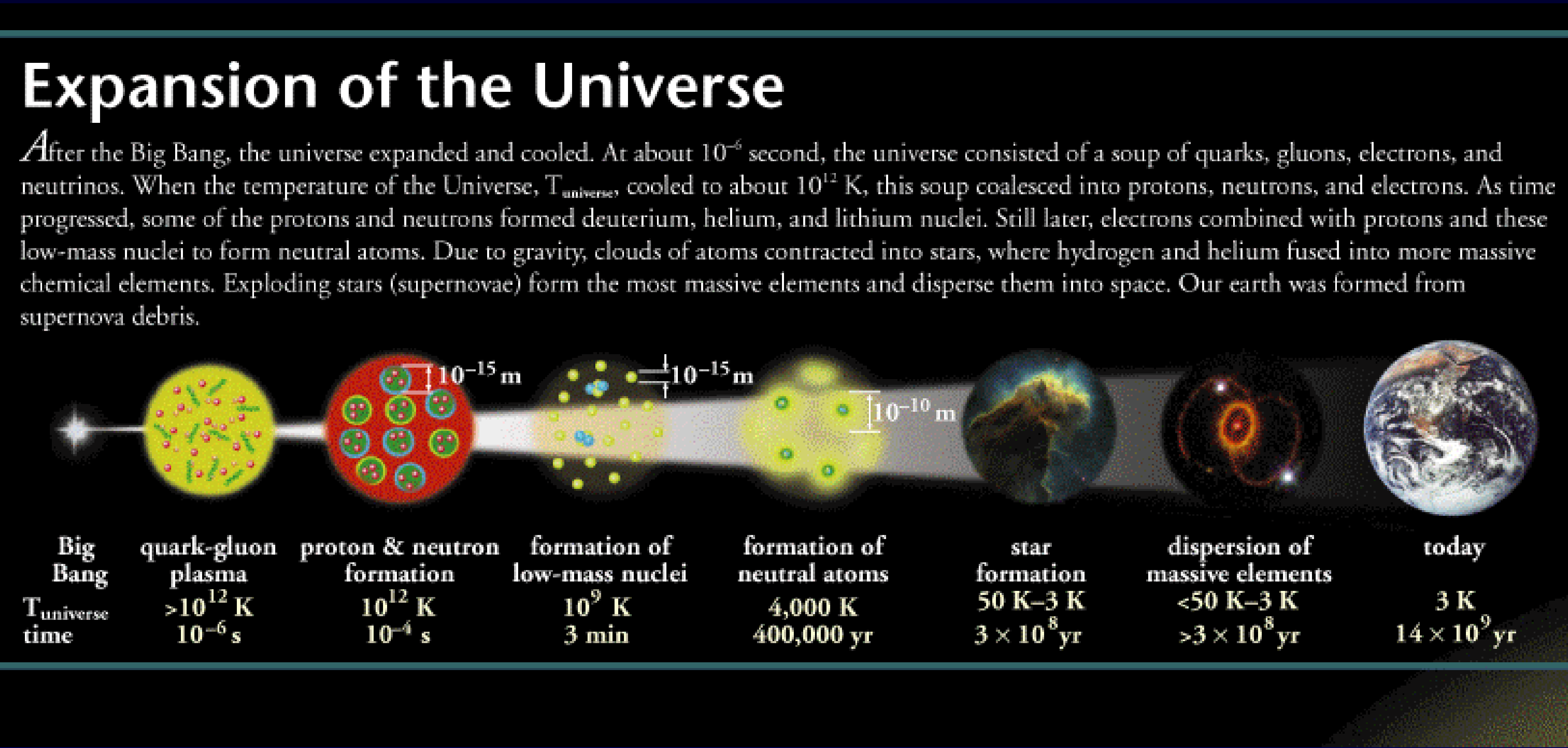
Property	Gravitational Interaction	Weak Interaction (Electroweak)	Electromagnetic Interaction	Strong Interaction
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge
Particles experiencing:	All	Quarks, Leptons	Electrically Charged	Quarks, Gluons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons
Strength at 10^{-18} m	10^{-41}	0.8	1	25
Strength at 3×10^{-17} m	10^{-41}	10^{-4}	1	60

Quarks and Strong Interactions

	name	mass (GeV/c^2)	EM charge (e)
u	up	0.003	+2/3
d	down	0.006	–1/3
s	strange	0.08	–1/3
c	charm	1.3	+2/3
b	bottom	4.5	–1/3
t	top	174	+2/3



But, it is a long, hard struggle from quarks and gluons to …



- Study of the very small is closely related to the study of the very big.

The Particle Zoo (excitation spectrum of QCD)

Baryons (3 quarks)

p	P_{11}	****
n	P_{11}	****
$N(1440)$	P_{11}	****
$N(1520)$	D_{13}	****
$N(1535)$	S_{11}	****
$N(1650)$	S_{11}	****
$N(1675)$	D_{15}	****
$N(1680)$	F_{15}	****
$N(1700)$	D_{13}	***
$N(1710)$	P_{11}	***
$N(1720)$	P_{13}	****
$N(1900)$	P_{13}	**
$N(1990)$	F_{17}	**
$N(2000)$	F_{15}	**
$N(2080)$	D_{13}	**
$N(2090)$	S_{11}	*
$N(2100)$	P_{11}	*
$N(2190)$	G_{17}	****
$N(2200)$	D_{15}	**
$N(2220)$	H_{19}	****
$N(2250)$	G_{19}	****
$N(2600)$	$I_{1,11}$	***
$N(2700)$	$K_{1,13}$	**

$\Delta(1232)$	P_{33}	****
$\Delta(1600)$	P_{33}	***
$\Delta(1620)$	S_{31}	****
$\Delta(1700)$	D_{33}	****
$\Delta(1750)$	P_{31}	*
$\Delta(1900)$	S_{31}	**
$\Delta(1905)$	F_{35}	****
$\Delta(1910)$	P_{31}	****
$\Delta(1920)$	P_{33}	***
$\Delta(1930)$	D_{35}	***
$\Delta(1940)$	D_{33}	*
$\Delta(1950)$	F_{37}	****
$\Delta(2000)$	F_{35}	**
$\Delta(2150)$	S_{31}	*
$\Delta(2200)$	G_{37}	*
$\Delta(2300)$	H_{39}	**
$\Delta(2350)$	D_{35}	*
$\Delta(2390)$	F_{37}	*
$\Delta(2400)$	G_{39}	**
$\Delta(2420)$	$H_{3,11}$	****
$\Delta(2750)$	$I_{3,13}$	**
$\Delta(2950)$	$K_{3,15}$	**
$\Theta(1540)^+$		***
$\Phi(1860)$		*

Λ	P_{01}	****
$\Lambda(1405)$	S_{01}	****
$\Lambda(1520)$	D_{03}	****
$\Lambda(1600)$	P_{01}	***
$\Lambda(1670)$	S_{01}	****
$\Lambda(1690)$	D_{03}	****
$\Lambda(1800)$	S_{01}	***
$\Lambda(1810)$	P_{01}	***
$\Lambda(1820)$	F_{05}	****
$\Lambda(1830)$	D_{05}	****
$\Lambda(1890)$	P_{03}	****
$\Lambda(2000)$		*
$\Lambda(2020)$	F_{07}	*
$\Lambda(2100)$	G_{07}	****
$\Lambda(2110)$	F_{05}	***
$\Lambda(2325)$	D_{03}	*
$\Lambda(2350)$	H_{09}	***
$\Lambda(2585)$		**

Σ^+	P_{11}	****
Σ^0	P_{11}	****
Σ^-	P_{11}	****
$\Sigma(1385)$	P_{13}	****
$\Sigma(1480)$		*
$\Sigma(1560)$		**
$\Sigma(1580)$	D_{13}	**
$\Sigma(1620)$	S_{11}	**
$\Sigma(1660)$	P_{11}	***
$\Sigma(1670)$	D_{13}	****
$\Sigma(1690)$		**
$\Sigma(1750)$	S_{11}	***
$\Sigma(1770)$	P_{11}	*
$\Sigma(1775)$	D_{15}	****
$\Sigma(1840)$	P_{13}	*
$\Sigma(1880)$	P_{11}	**
$\Sigma(1915)$	F_{15}	****
$\Sigma(1940)$	D_{13}	***
$\Sigma(2000)$	S_{11}	*
$\Sigma(2030)$	F_{17}	****
$\Sigma(2070)$	F_{15}	*
$\Sigma(2080)$	P_{13}	**
$\Sigma(2100)$	G_{17}	*
$\Sigma(2250)$		***
$\Sigma(2455)$		**
$\Sigma(2620)$		**
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Ξ^0	P_{11}	****
Ξ^-	P_{11}	****
$\Xi(1530)$	P_{13}	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	D_{13}	***
$\Xi(1950)$		***
$\Xi(2030)$		***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-		****
$\Omega(2250)^-$		***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**
Λ_c^+		****
$\Lambda_c(2593)^+$		***
$\Lambda_c(2625)^+$		***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2880)^+$		**
$\Sigma_c(2455)$		****
$\Sigma_c(2520)$		***
Ξ_c^+		***
Ξ_c^0		***
$\Xi_c'^+$		***
$\Xi_c'^0$		***
$\Xi_c(2645)$		***
$\Xi_c(2790)$		***
$\Xi_c(2815)$		***
Ω_c^0		***
Ξ_{cc}^+		*
Λ_b^0		***
Ξ_b^0, Ξ_b^-		*

Mesons (quark-antiquark)

LIGHT UNFLAVORED
($S = C = B = 0$)

	$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$
• π^0	$1^-(0^{-+})$
• η	$0^+(0^{-+})$
• $f_0(600)$	$0^+(0^{++})$
• $\rho(770)$	$1^+(1^{--})$
• $\omega(782)$	$0^-(1^{--})$
• $\eta'(958)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$
• $a_0(980)$	$1^-(0^{++})$
• $\phi(1020)$	$0^-(1^{--})$
• $h_1(1170)$	$0^-(1^{+-})$
• $b_1(1235)$	$1^+(1^{+-})$
• $a_1(1260)$	$1^-(1^{++})$
• $f_2(1270)$	$0^+(2^{++})$
• $f_1(1285)$	$0^+(1^{++})$
• $\eta(1295)$	$0^+(0^{-+})$
• $\pi(1300)$	$1^-(0^{-+})$
• $a_2(1320)$	$1^-(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$
$h_1(1380)$	$?^-(1^{+-})$
• $\pi_1(1400)$	$1^-(1^{-+})$
• $\eta(1405)$	$0^+(0^{-+})$
• $f_1(1420)$	$0^+(1^{++})$
• $\omega(1420)$	$0^-(1^{--})$
$f_2(1430)$	$0^+(2^{++})$
• $a_0(1450)$	$1^-(0^{++})$
• $\rho(1450)$	$1^+(1^{--})$
• $\eta(1475)$	$0^+(0^{-+})$
• $f_0(1500)$	$0^+(0^{++})$
$f_1(1510)$	$0^+(1^{++})$
• $f_2'(1525)$	$0^+(2^{++})$
$f_2(1565)$	$0^+(2^{++})$
$h_1(1595)$	$0^-(1^{+-})$
• $\pi_1(1600)$	$1^-(1^{-+})$
$a_1(1640)$	$1^-(1^{++})$
$f_2(1640)$	$0^+(2^{++})$
• $\eta_2(1645)$	$0^+(2^{-+})$
• $\omega(1650)$	$0^-(1^{--})$
• $\omega_3(1670)$	$0^-(3^{--})$

	$I^G(J^{PC})$
• $\pi_2(1670)$	$1^-(2^{-+})$
• $\phi(1680)$	$0^-(1^{--})$
• $\rho_3(1690)$	$1^+(3^{--})$
• $\rho(1700)$	$1^+(1^{--})$
$a_2(1700)$	$1^-(2^{++})$
• $f_0(1710)$	$0^+(0^{++})$
$\eta(1760)$	$0^+(0^{-+})$
• $\pi(1800)$	$1^-(0^{-+})$
$f_2(1810)$	$0^+(2^{++})$
• $\phi_3(1850)$	$0^-(3^{--})$
$\eta_2(1870)$	$0^+(2^{-+})$
$\rho(1900)$	$1^+(1^{--})$
$f_2(1910)$	$0^+(2^{++})$
• $f_2(1950)$	$0^+(2^{++})$
$\rho_3(1990)$	$1^+(3^{--})$
• $f_2(2010)$	$0^+(2^{++})$
$f_0(2020)$	$0^+(0^{++})$
• $a_4(2040)$	$1^-(4^{++})$
• $f_4(2050)$	$0^+(4^{++})$
$\pi_2(2100)$	$1^-(2^{-+})$
$f_0(2100)$	$0^+(0^{++})$
$f_2(2150)$	$0^+(2^{++})$
$\rho(2150)$	$1^+(1^{--})$
$f_0(2200)$	$0^+(0^{++})$
$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
$\eta(2225)$	$0^+(0^{-+})$
$\rho_3(2250)$	$1^+(3^{--})$
• $f_2(2300)$	$0^+(2^{++})$
$f_4(2300)$	$0^+(4^{++})$
• $f_2(2340)$	$0^+(2^{++})$
$\rho_5(2350)$	$1^+(5^{--})$
$a_6(2450)$	$1^-(6^{++})$
$f_6(2510)$	$0^+(6^{++})$

OTHER LIGHT

Further States

STRANGE
($S = \pm1$, $C = B = 0$)

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K^0_S	$1/2(0^-)$
• K^0_L	$1/2(0^-)$
$K_0^*(800)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED
($C = \pm1$)

• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^\pm$	$1/2(2^+)$
$D^*(2640)^\pm$	$1/2(?^?)$

CHARMED, STRANGE
($C = S = \pm1$)

• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{sJ}^*(2317)^\pm$	$0(0^+)$
• $D_{sJ}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}(2573)^\pm$	$0(?^?)$

BOTTOM
($B = \pm1$)

	$I^G(J^{PC})$
• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B^0_s/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
$B_J^*(5732)$	$?(?^?)$

BOTTOM, STRANGE
($B = \pm1$, $S = \mp1$)

• B^0_s	$0(0^-)$
B^*_s	$0(1^-)$
$B^*_{sJ}(5850)$	$?(?^?)$

BOTTOM, CHARMED
($B = C = \pm1$)

• $B_c^\pm$	$0(0^-)$

$c\bar{c}$

• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
$h_c(1P)$	$?^?(?^{??})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
$\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
$\psi(3836)$	$0^-(2^{--})$
$X(3872)$	$?^?(?^?)$
• $\psi(4040)$	$0^-(1^{--})$
• $\psi(4160)$	$0^-(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$

$b\bar{b}$

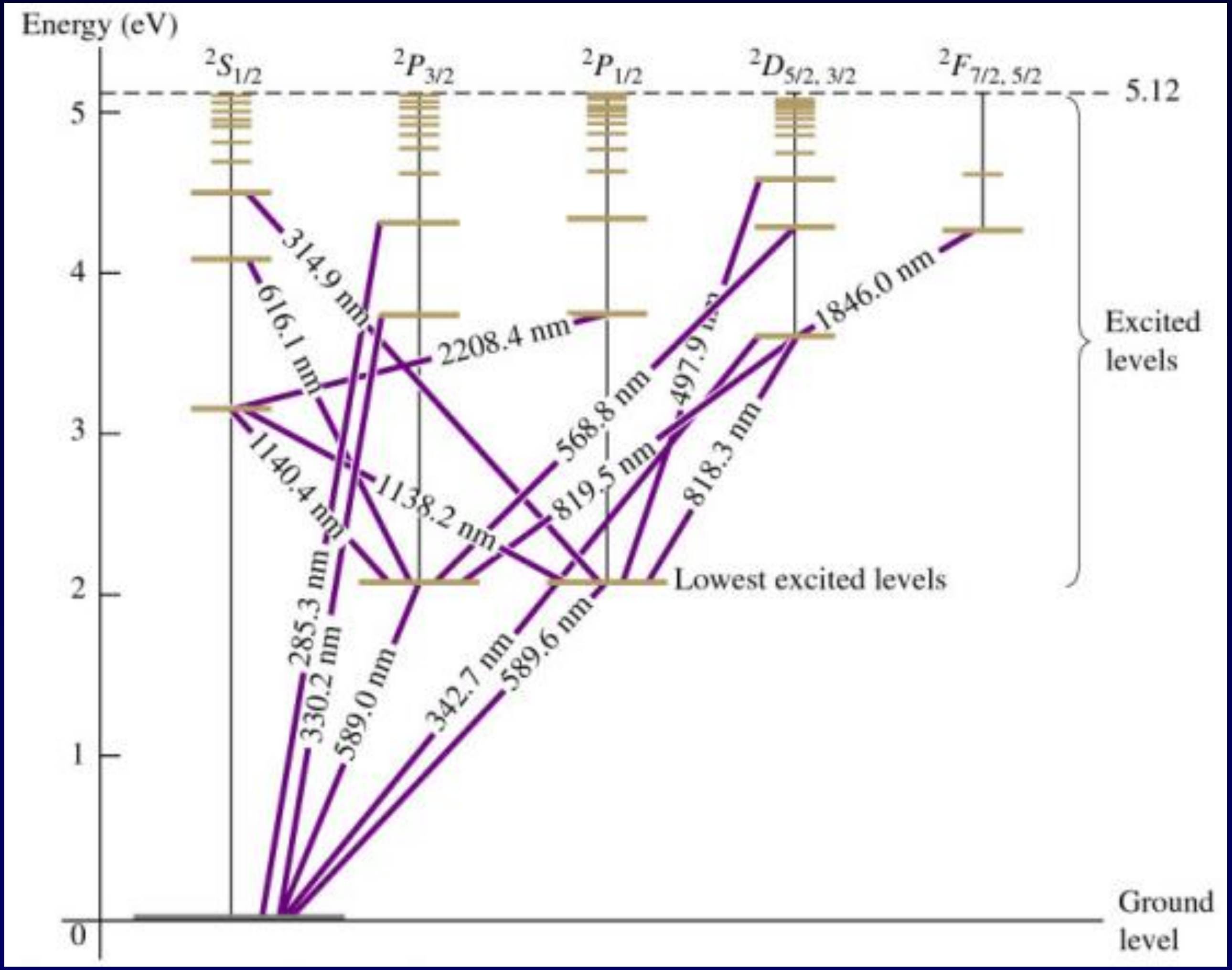
$\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

NON-$q\bar{q}$ CANDIDATES

NON-$q\bar{q}$ CANDIDATES

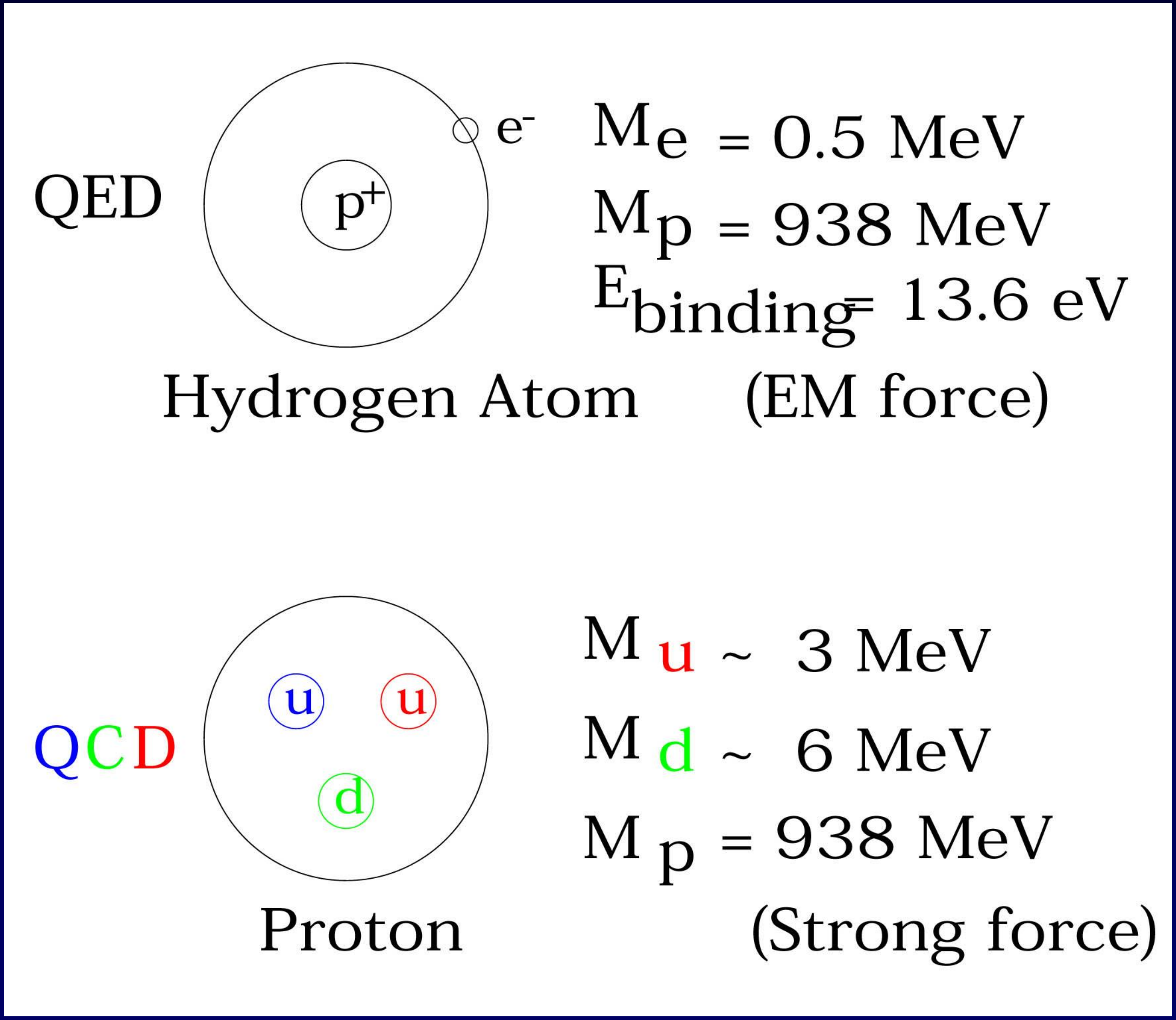
$$L_{QCD} = \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \bar{q}(\gamma^\mu D_\mu + m_q)q$$

A quantum mechanical analogy (the hydrogen atom)



$$H = -\frac{\hbar^2}{2m}\nabla^2 + V(r), \quad H\Psi = E\Psi$$

EM force vs. Strong force



- How does the proton get its mass?

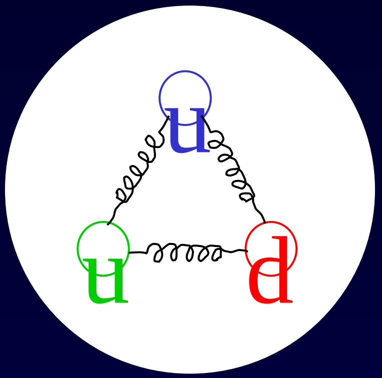
Two kinds of quarks

- Constituent quarks
  - $m_u = m_d \sim 340$ MeV, $m_s \sim 500$ MeV
- Most of the proton mass comes from the quark masses.
- The mass splittings are provided by residual pair-wise interactions.

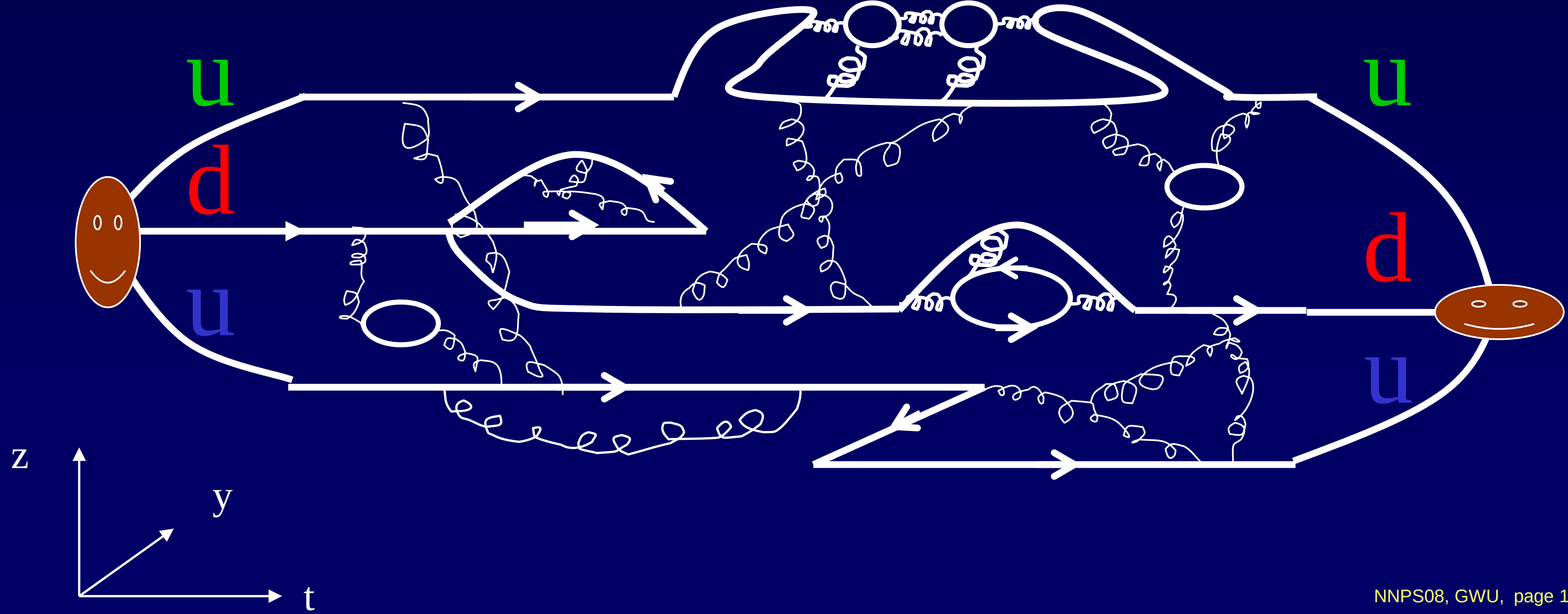
- QCD quarks
  - $m_u \sim 3$ MeV, $m_d \sim 6$ MeV, $m_s \sim 100$ MeV
- Most of the mass and splittings come from interactions, to all orders of interaction.
- The quark masses only contribute to about 1% of the proton mass.

quark model → **QCD**

The proton in quark model:



The proton in QCD:



Quantum Chromodynamics (QCD)

--- underlying theory of the strong interaction (one of the four fundamental interactions in nature besides gravity, electromagnetism and the weak interaction)

$$L_{QCD} = \frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \bar{q}(\gamma^{\mu}D_{\mu} + m_q)q$$

Field strength tensor : $F_{\mu\nu} = \partial G_{\mu} - \partial G_{\nu} + g[G_{\mu}, G_{\nu}]$

Covariant derivative : $D_{\mu} = \partial_{\mu} + gG_{\mu}$

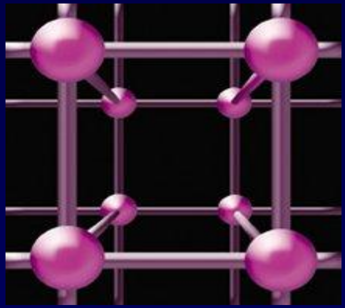
- Chiral symmetry and its spontaneous breaking
- Asymptotic freedom: perturbative at high energies
- Confinement: non-perturbative at low energies

Quantum Chromodynamics (continued)

Basic computational methodology: From Action to Answers

$$S_{QCD} = \int d^4x \left[\tfrac{1}{2} \mathrm{Tr} F_{\mu\nu} F^{\mu\nu} + \bar{q}(\gamma^{\mu} D_{\mu} + m_q) q \right]$$

All physics is computed from path integrals



$$\left\langle \hat{O} \right\rangle \equiv \frac{\int DG_{\mu} Dq D\bar{q} e^{-S_{QCD}} \hat{O}(G_{\mu}, q, \bar{q})}{\int DG_{\mu} Dq D\bar{q} e^{-S_{QCD}}}$$

which can be evaluated on a space-time lattice that involves millions of degrees of freedom. (hence the field is called *lattice QCD* or *lattice gauge theory*)

Path Integral Method

(Richard Feynmann)

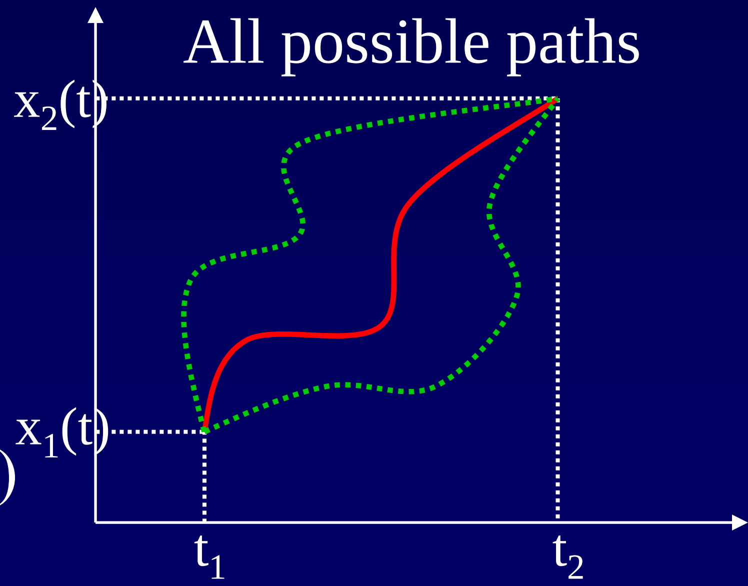
- Applicable to any problem that can be cast into the form

$$\langle O \rangle \equiv \frac{\int Dx\, e^{-S} O(x)}{\int Dx\, e^{-S}}$$

with an action, for example:

$$S = \int_{t_1}^{t_2} dt \left\{ \frac{1}{2} m \dot{x}^2(t) + V[x(t)] \right\}$$

- Successfully used in
  - statistical physics
  - quantum chemistry
  - condensed matter physics
  - biological physics
  - quantum field theories (QED,QCD, …)
  - and more

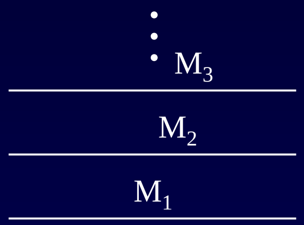


How to compute proton's mass in QCD?

$$\boldsymbol{G(\vec{\mathrm{p}},\mathrm{t}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle 0 \,|\, T[\eta(x)\bar{\eta}(0)] \,|\, 0 \rangle}$$

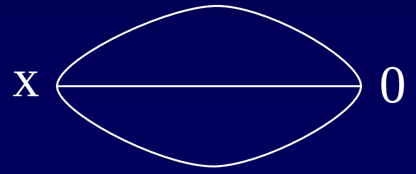
Interpolating field $\boldsymbol{\eta(x) = \varepsilon^{abc}[u^{aT}(x)C\gamma_5 d^b(x)]u^c(x)}$

with quantum numbers of the proton $\boldsymbol{I(J^P) = \frac{1}{2}\left(\frac{1}{2}^+\right)}$



- On the hadronic level, saturated by intermediate states

$$G(\vec{\mathrm{p}},\mathrm{t}) = \sum_n \lambda_n^2 e^{-E_n(\vec{p})t} \quad \text{with } E_n(\vec{p}) = \sqrt{\vec{p}^{\,2} + M_n^2}$$



- On the quark level, contracting out quark pairs

$$G(\vec{\mathrm{p}},\mathrm{t}) = -\varepsilon^{abc}\varepsilon^{a'b'c'} \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \left[S_u^{aa'} \gamma_5 C S_d^{bb'T} C\gamma_5 S_u^{cc'} + S_u^{aa'} Tr\left(\gamma_5 C S_d^{bb'T} C\gamma_5 S_u^{cc'} \right) \right]$$

where S_q^{ab} is the fully-interacting quark propagator.

Quark propagator in QCD: building block of hadrons

$$M^{-1}(x,0) = \langle 0 | T[q(x)\bar{q}(0) | 0 \rangle \equiv \frac{\int DGDqD\bar{q}\left[q(x)\bar{q}(0)\right]e^{-S_{QCD}}}{\int DGDqD\bar{q}e^{-S_{QCD}}}$$

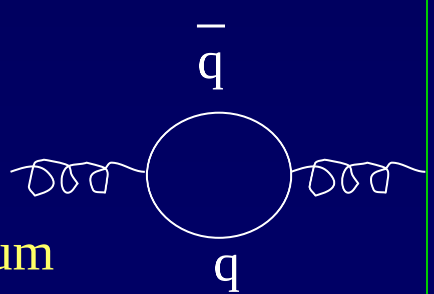
$$S_{QCD} = \int d^4x \left[\tfrac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \bar{q}Mq\right] \qquad M = \gamma^\mu D_\mu + m_q$$

Quark fields can be integrated out by Grassmann algebra:

$$\int DqD\bar{q}e^{-\bar{q}Mq} = \det(\mathrm{M}), \; \int DqD\bar{q}(q\bar{q})e^{-\bar{q}Mq} = \det(\mathrm{M})\mathrm{M}^{-1}$$

$$M^{-1}(x,0) = \frac{\int DG \,\det M \; e^{-S_G} M^{-1}}{\int DG \,\det M \; e^{-S_G}}$$

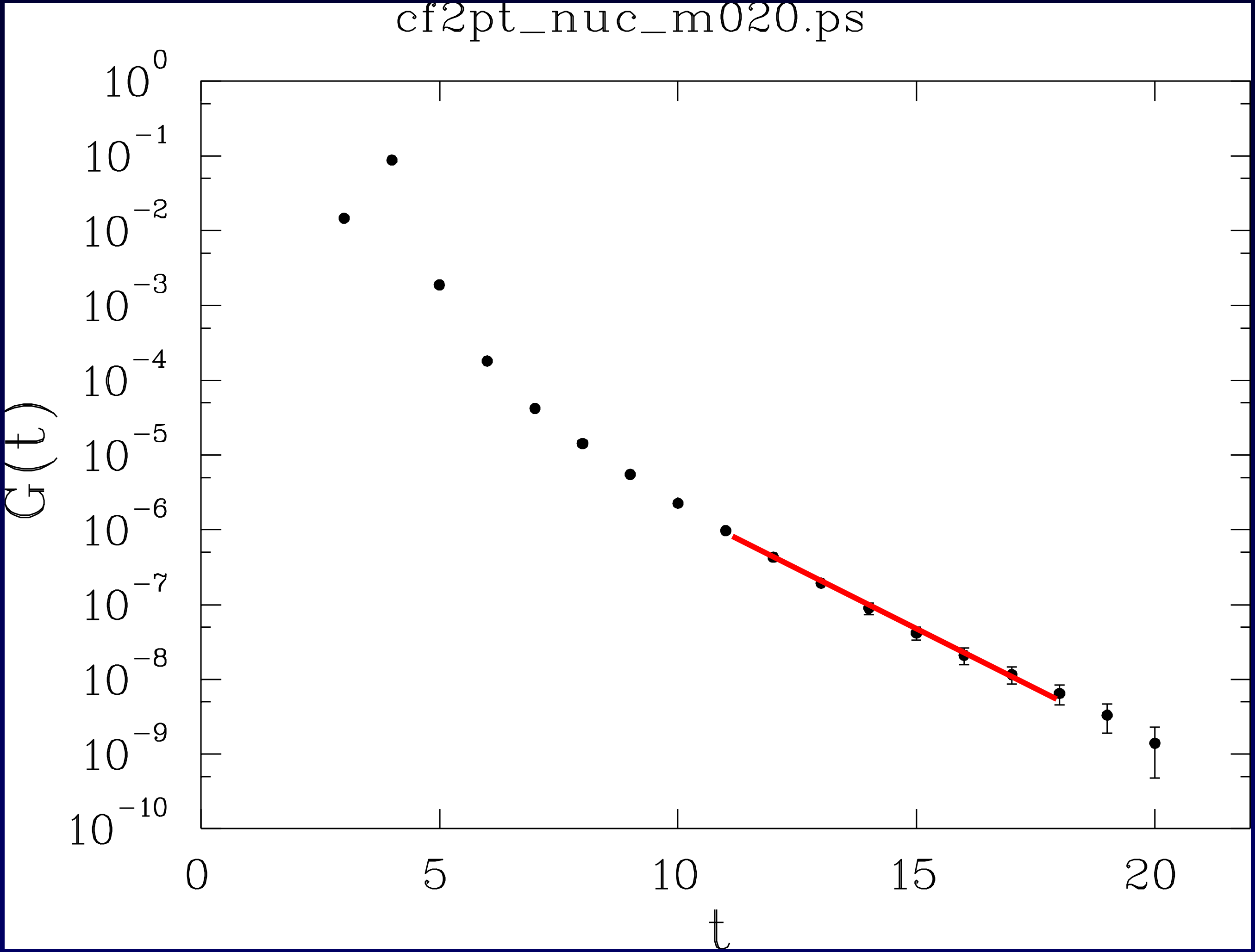
- Path integrals over gauge fields
  - Monte-Carlo with weighting factor $\det M \; e^{-S_G}$
- Quenched approximation (set detM=1)
  - Physically, suppress quark-antiquark bubbles in the vacuum



Sample correlation function

$$G(t) = \sum_{n=1}^{\infty} A_n e^{-M_n t}$$

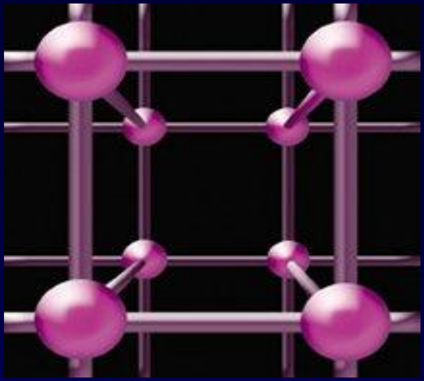
$$\xrightarrow{t \text{ large}} A_1 e^{-M_1 t}$$



- 20^4 lattice, a=0.15 fm, overlap fermions, m_0a=0.02, 80 configs

Key Ideas of Lattice QCD

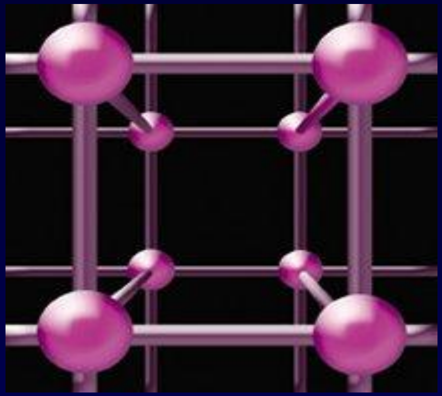
- Discretization of 4-dimensional space-time by a lattice of spacing a, size $V=N_x \times N_y \times N_z \times N_t$
  - For example, $16^3 x28$ lattice at a=0.2 fm.
    So the physical volume is $(3.2\ \text{fm})^3 x5.6$ fm
  - Quarks live on the sites
  - Gluons live on the links



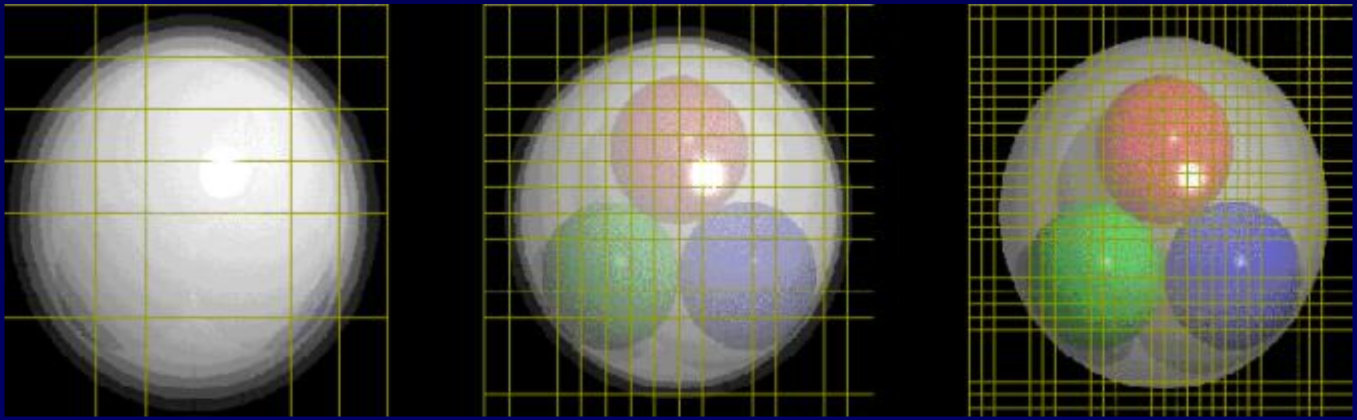
- Real time → imaginary time (Minkowski space → Euclidean space)
  - Space and time on equal footing
  - Probability interpretation (e^{-S}), instead of oscillating phase (e^{iS})
  - amenable to Monte-Carlo methods
- Infinite degrees of freedom → finite (albeit still millions)
  - Can be manipulated by a computer
- Many-body, interacting problem → treated exactly by path integrals
  - Evaluated numerically

The real world is approached by

- Continuum limit
  - finer grid ($a \to 0$)
  - larger lattice ($L \to \infty$)



- Chiral limit ($m_q \to 0$)

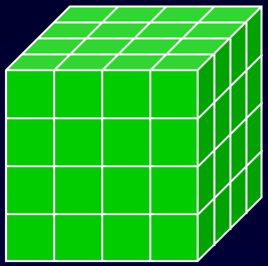


Computational methods in lattice QCD

- Finite-element analysis
  - space-time as a lattice
  - path integrals
- Monte-Carlo Simulations
  - Metropolis, heat-bath, molecular dynamics, hybrid, …
- Linear Algebra
  - inversion of large matrices (typically on the order of a few million by a few million); large memory and disk demands
  - Conjugate Gradient, Krylov muti-mass shifted solvers
- Statistical data analysis
  - χ^2 minimization
  - Bayesian methods (priors, maximum entropy)
  - Variational methods

Supercomputers

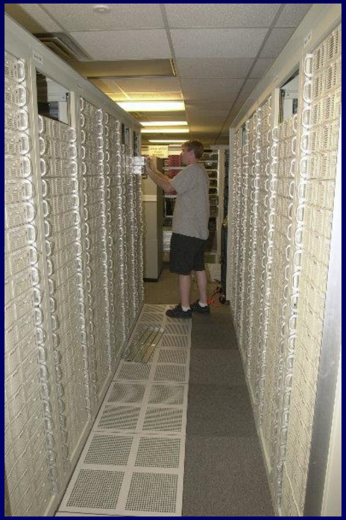
$$\text{Cost} \sim \frac{L^4}{a^{6.5} m_q^{2.5}}$$



- As we approach reality ($a \to 0$, $L \to \infty$, $m_q \to 0$), the cost of lattice simulations goes up dramatically
  - need supercomputers, parallel processing, faster networks, faster algorithms, …
  - Lattice QCD is one of the fields driving the technology of HPC.

The 6080-processor IBM-SP at NERSC

The 256-processor Intel cluster at JLab



The 160-core GWU QCD Cluster

Technical challenges of lattice QCD

- Construction and isolation of states
  - operators
  - matrix elements
  - renormalization factors
- Approaching the real world
  - continuum limit ($a \longrightarrow 0$, $V \longrightarrow \infty$)
  - chiral limit ($m_q \longrightarrow 0$)
  - dynamical quarks
- Doing things faster
  - faster algorithms
  - faster computers

Lattice QCD is an exact solution of QCD in the following sense: All the errors (systematic and statistical) are controlled and can be removed with increasing computing power.

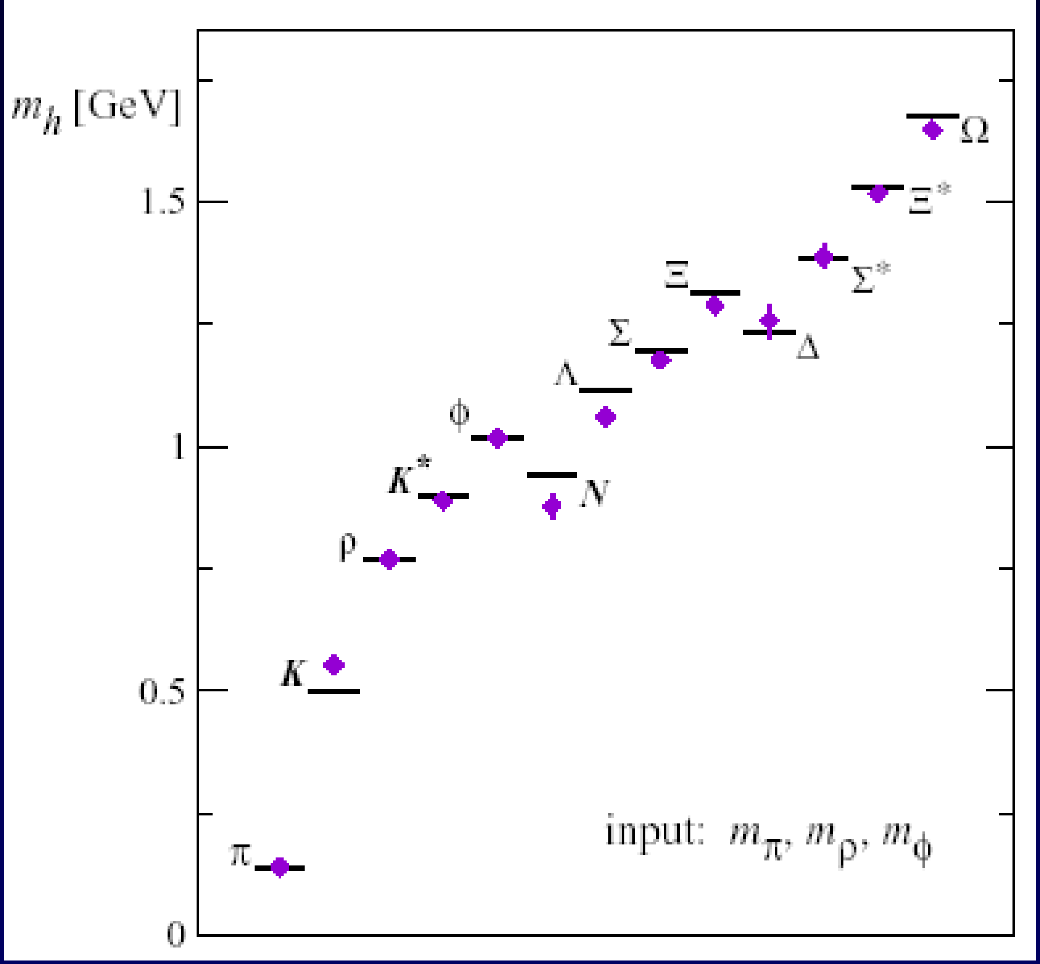
Some Lattice QCD Collaborations

- USQCD
  - MILC
  - LHPC
  - NPLQCD
  - χQCD
  - polarQCD
  - HPQCD
  - …
- UKQCD
- CP-PACS
- JLQCD
- ALPHA
- …

A Few Examples

New Topic

Example 1: particle mass spectrum



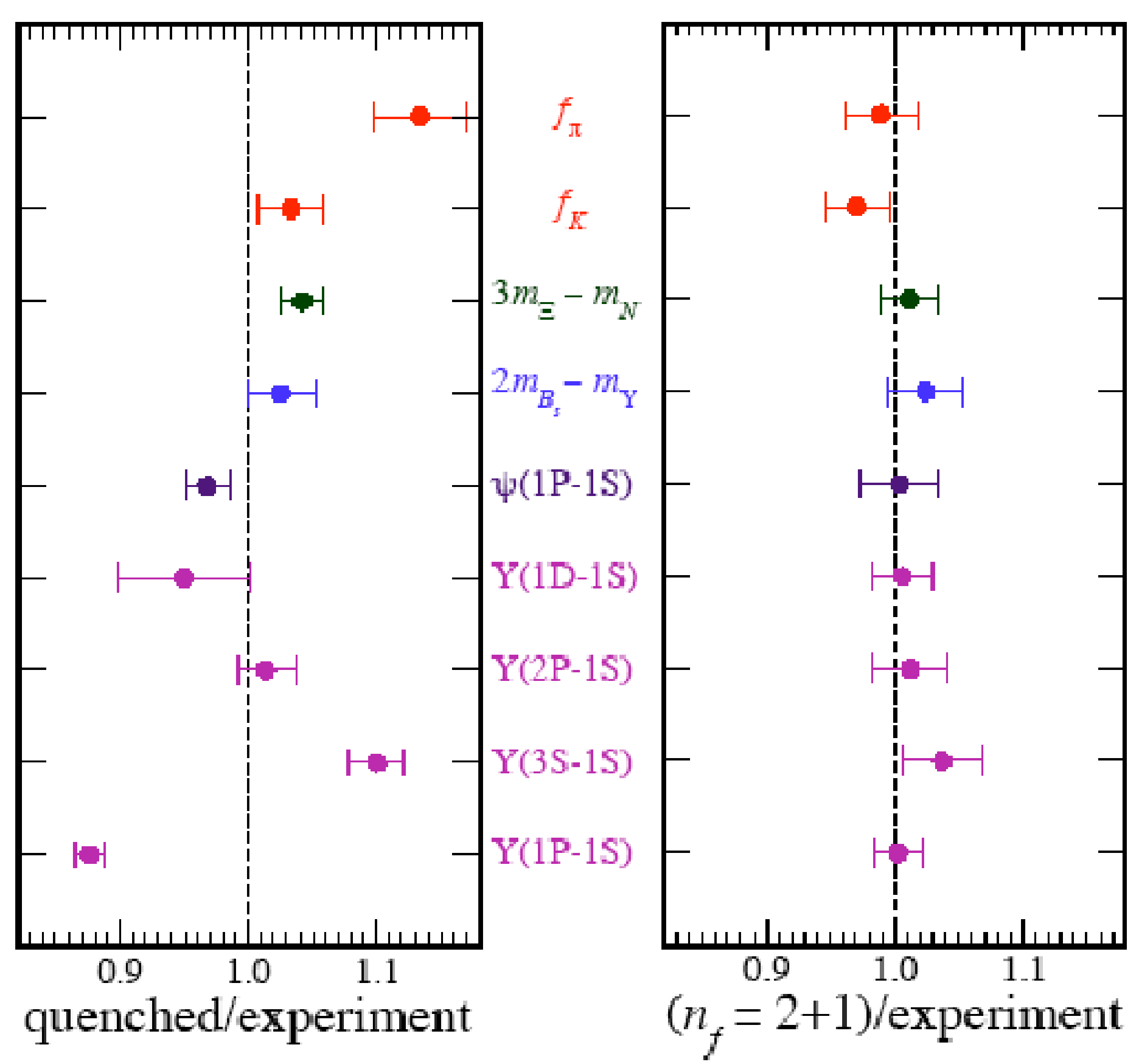
- CP-PACS, heplat/0206090.
- Lattices: 32^3x56 to 64^3x128
- Spacing 0.1 fm to 0.05 fm
- M_π/ M_ρ is 0.75 to 0.4
- 1 to 3 % statistical error
- 2% systematic error
- Took more than a year of running on a dedicated computer sustaining 300 Gflops.
- 1 Gflop = 10^9 floating-point operations per second

The computed quenched light hadron spectrum is within 7% of the experiment. The remaining discrepancy is attributed to the quenched approximation.

Outlook on hadron spectrum on the lattice

- Smaller quark masses
  - quark actions that preserve chiral symmetry (overlap fermions, domain-wall, …)
- Chiral extrapolations
  - bridge the gap to the physical point (pion 140 MeV)
- Tailor-made operators, higher excited states, and higher spin states, hybrid hadrons, multi-quark states
  - non-local operators, anisotropic lattices, variational analysis
- Dynamical configurations
  - Tera-flop computers

Example 2: effects of the quenched approximation

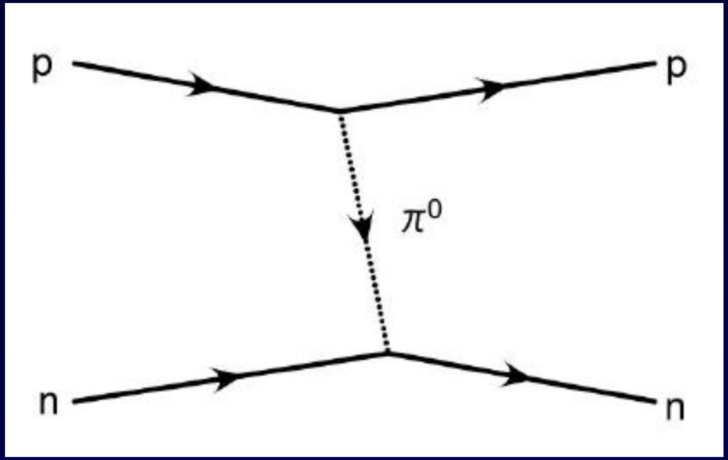


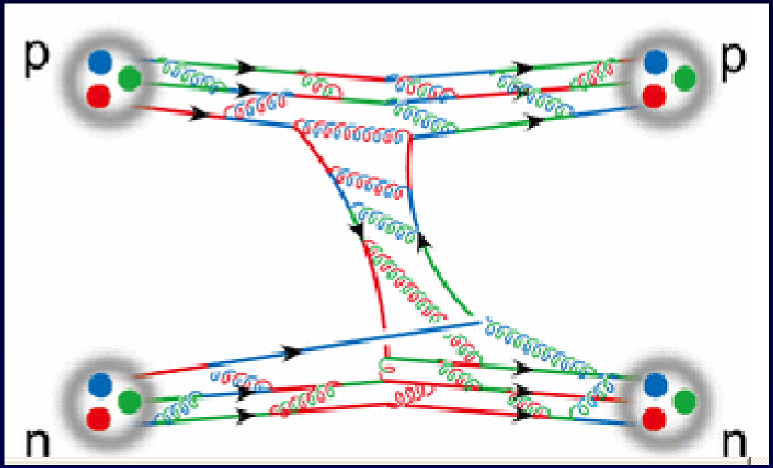
Selected physics quantities are reproduced to 3% level.

But there's a lot more to do …

U.S. lattice community, PRL92 (2004)

Example 3: the nuclear force from lattice QCD





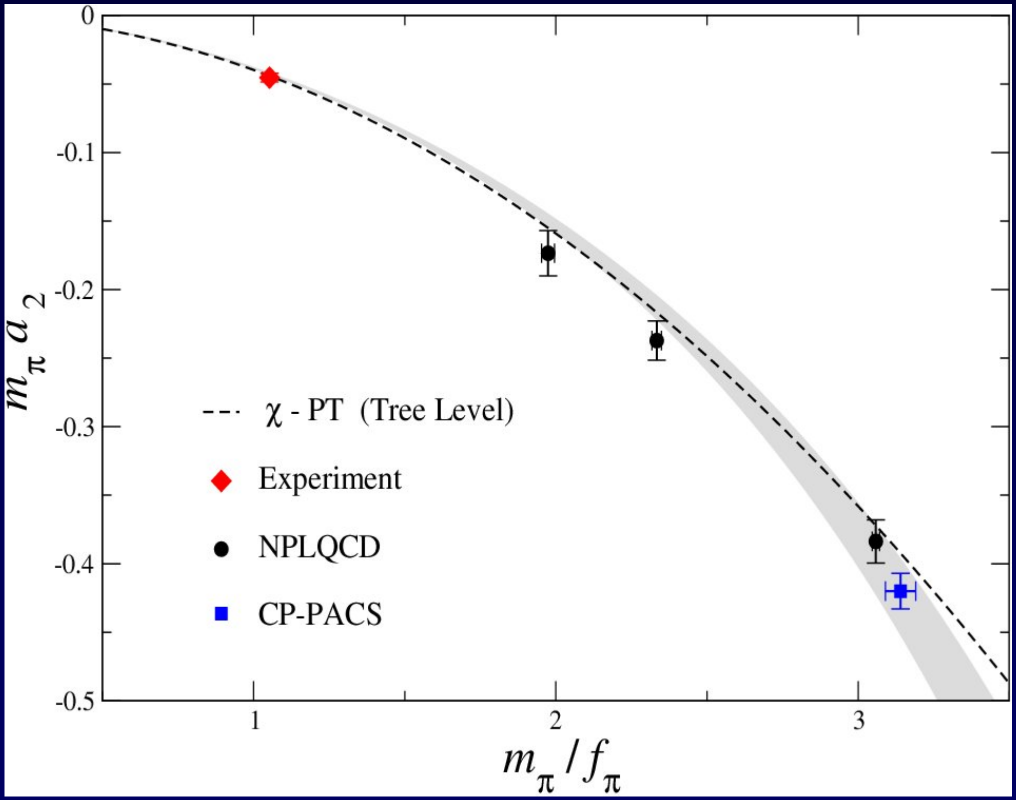
- Consistent and rigorous pathway from QCD

Lattice QCD → EFT → Many-Body

- Can we understand the nuclear force in terms of quarks and gluons?
- What are the effective degrees of freedom?
- Why meson exchange models of the nuclear force work so well?
- What is the nature of the repulsive core?

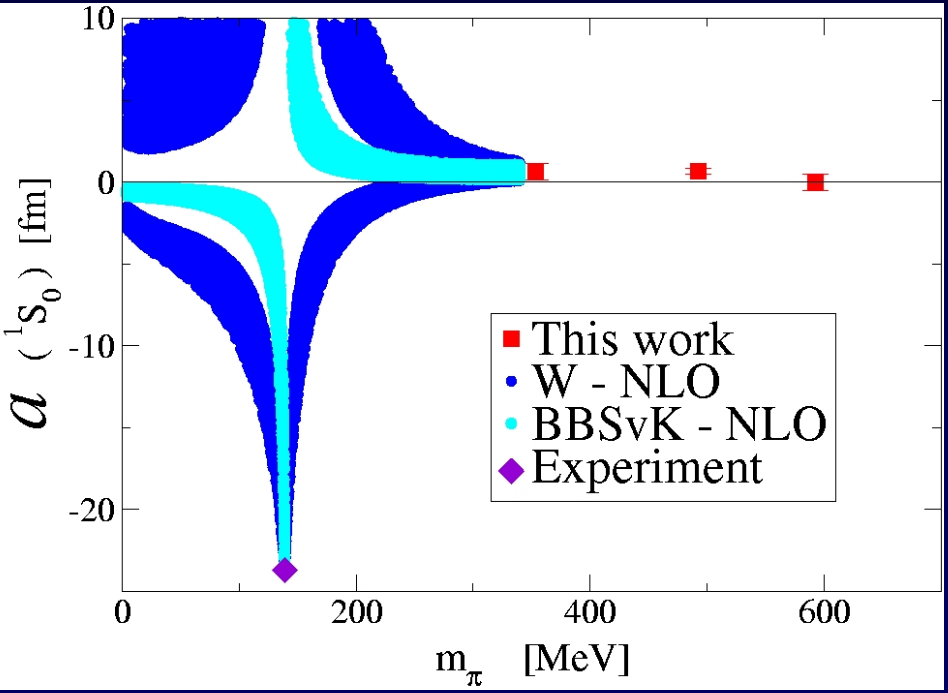
Lattice QCD is starting to address these questions.

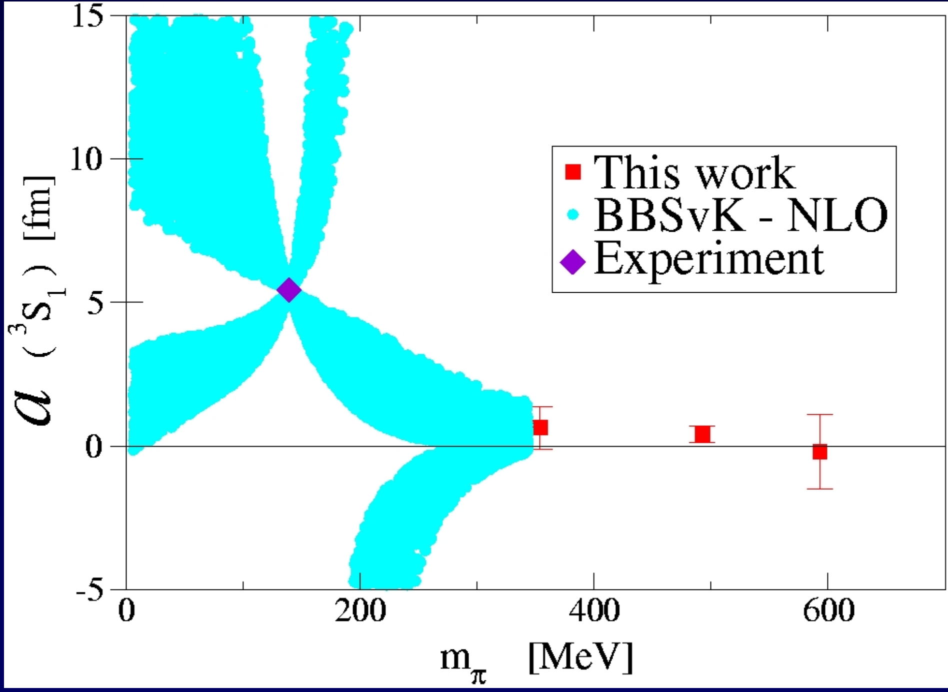
"I=2 pi-pi Scattering from Fully-Dynamical Mixed-Action Lattice QCD", NPLQCD, Phys. Rev. D73 (2006) 054503



- $M_\pi a_2$ = -0.0426 +- 0.0006 +- 0.0003 +- 0.0018

"Nucleon-Nucleon Scattering From Fully-Dynamical Lattice QCD", NPLQCD, Phys.Rev.Lett. 97 (2006) 012001





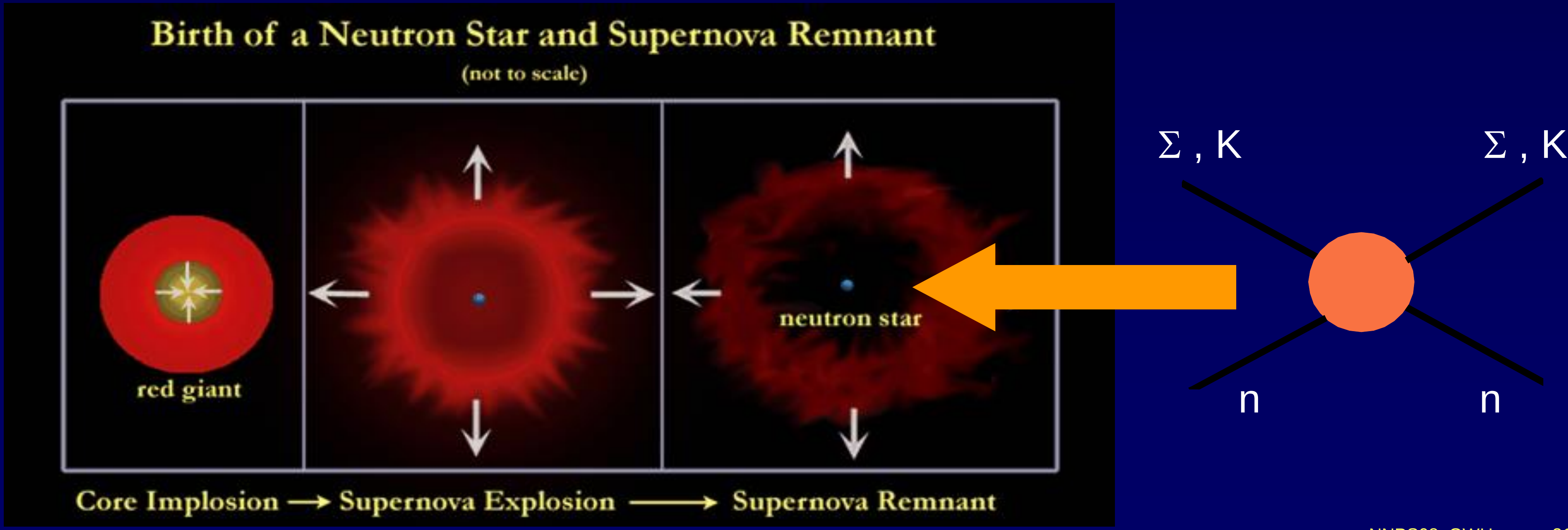
"Hyperon-Nucleon Scattering from Fully-Dynamical Lattice QCD", NPLQCD, hep-lat/0612026

Channel	m_π (MeV)	Range	ΔE (MeV)	$\|\mathbf{k}\|$ (MeV)	δ (degrees)	$-(k\cot\delta)^{-1}$ (fm)
$n\Lambda$ $^1S_0$	$592 \pm 1 \pm 10$	8-12	$-9 \pm 8 \pm 20$	–	–	$0.8 \pm 1.4 \pm 0.4$
	$493 \pm 1 \pm 8$	6-9	$29.8 \pm 5.4 \pm 2.5$	$197 \pm 24 \pm 4$	$-32.3 \pm 8.1 \pm 2.8$	$0.63 \pm 0.12 \pm 0.014$
	$354 \pm 1 \pm 6$	5-9	$56.8 \pm 6.0 \pm 5.5$	$255 \pm 22 \pm 13$	$-53.4 \pm 8.5 \pm 10.1$	$1.04 \pm 0.24 \pm 0.15$
$n\Lambda$ $^3S_1$	$592 \pm 1 \pm 10$	8-13	$-13 \pm 13 \pm 8$	–	–	$3 \pm 14 \pm 2$
	$493 \pm 1 \pm 8$	7-11	$-4 \pm 13 \pm 14$	–	–	$(-\infty, \infty)$
	$354 \pm 1 \pm 6$	5-10	$23 \pm 17 \pm 4$	$168 \pm 62 \pm 14$	$-23 \pm 18 \pm 4$	$0.50 \pm 0.26 \pm 0.06$
$n\Sigma^-$ $^1S_0$	$592 \pm 1 \pm 10$	9-13	$-17 \pm 11 \pm 27$	–	–	$(-\infty, \infty)$
	$493 \pm 1 \pm 8$	5-9	$24.9 \pm 7.8 \pm 3.0$	$179 \pm 28 \pm 11$	$-27.2 \pm 9.0 \pm 3.8$	$0.57 \pm 0.13 \pm 0.05$
$n\Sigma^-$ $^3S_1$	$592 \pm 1 \pm 10$	6-10	$38.5 \pm 8.8 \pm 5.0$	$226 \pm 26 \pm 15$	$-44.3 \pm 9.8 \pm 5.4$	$0.85 \pm 0.20 \pm 0.10$
	$493 \pm 1 \pm 8$	6-10	$53 \pm 14 \pm 5$	$261 \pm 35 \pm 13$	$-58 \pm 15 \pm 5$	$1.19 \pm 0.51 \pm 0.15$

More channels are forthcoming.

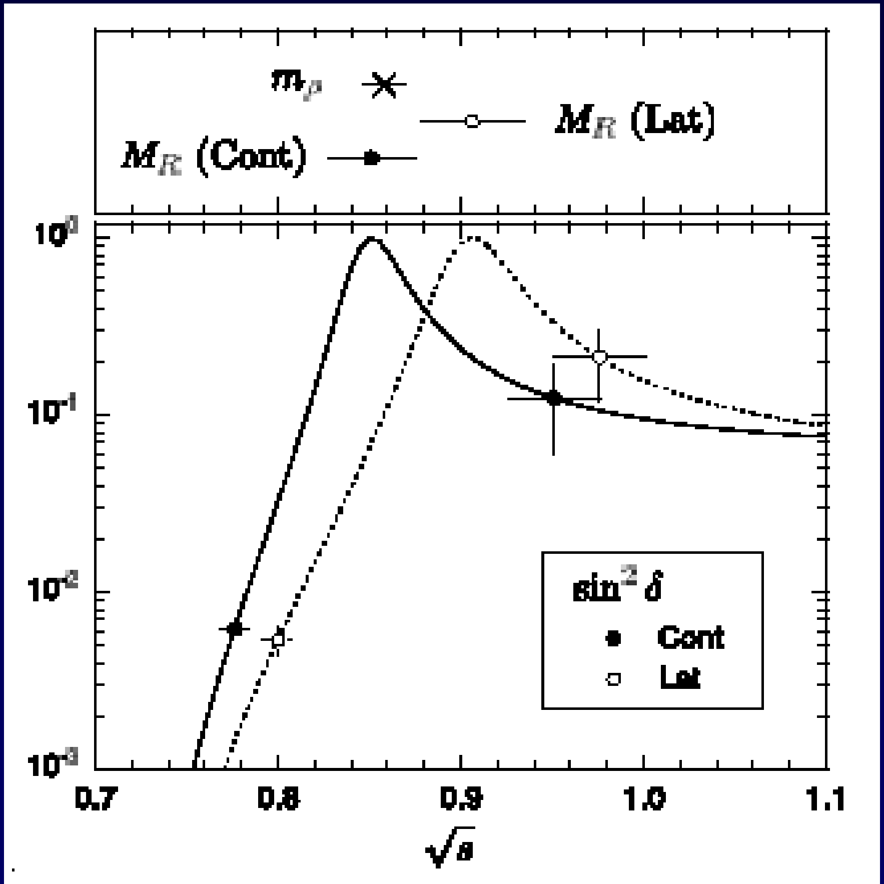
Lattice QCD and Astrophysics: can provide crucial input where experiments are not possible

- Supernova Remnant ?
  - neutron stars (equation of state)
  - blackholes
  - kaon condensation?



"ππ to ρ decay width", CPPACS, PRD76(2007)094506

$\sin^2\delta$



fit to Breit-Wigner form

signature of resonance :

$\delta > 0$ for $\sqrt{s} < m_\rho$ (attractive)

$\delta < 0$ for $\sqrt{s} > m_\rho$ (repulsive)

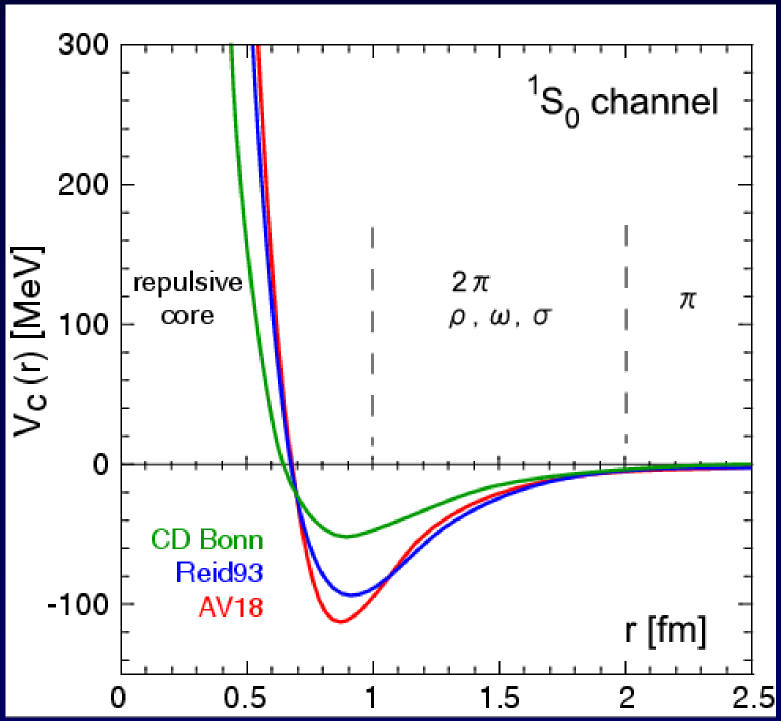
$g_{\rho\pi\pi} = 5.82 \pm 0.55$

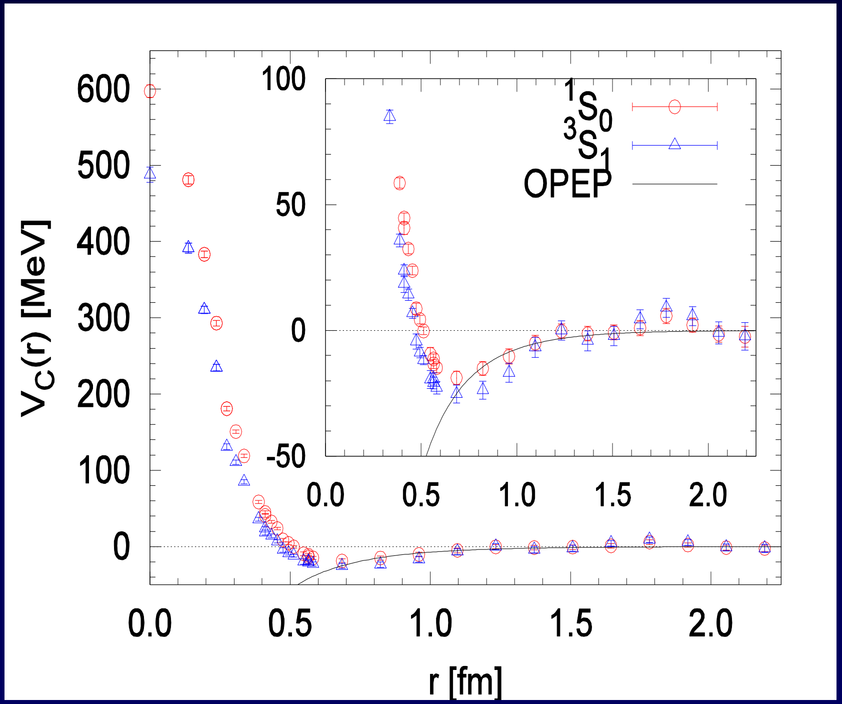
$M_R = 0.906 \pm 0.028$

$\Gamma = 162 \pm 35$ MeV

($\Gamma^{\mathrm{exp}} = 150$ MeV)

“NN Potential from Wave Functions”, S. Aoki, Lattice 2007





$$[H_0 + V(\mathbf{r})]\varphi(\mathbf{r},E) = E\varphi(\mathbf{r},E)$$

Yukawa $\sim e^{-m_\pi r}/r$

$$\varphi(\mathbf{r},E) = \langle 0|N(\mathbf{x},0)N(\mathbf{y},0)|2N;E\rangle$$

Background-Field Calculations

New Topic

Hadron Structure via Background Fields

Interaction energy of a hadron in the presence of external electromagnetic fields:

$$H = -\vec{\mu}\cdot\vec{B} - \frac{1}{2}\alpha E^2 - \frac{1}{2}\beta B^2$$
$$-\frac{1}{2}\gamma_{E1}\sigma\cdot\vec{E}\times\dot{\vec{E}} - \frac{1}{2}\gamma_{M1}\sigma\cdot\vec{B}\times\dot{\vec{B}}$$
$$+\gamma_{E2}\sigma_i E_{ij} B_j - \gamma_{M2}\sigma_i B_{ij} E_j$$
$$-\frac{1}{12}\alpha_{E2} E_{ij}^2 - \frac{1}{12}\beta_{M2} B_{ij}^2 + \cdots$$

Time and spatial derivatives: $\dot{E} = \frac{\partial E}{\partial t},\; E_{ij} = \frac{1}{2}(\nabla_i E_j + \nabla_j E_i)$, etc

Probe of internal structure of the system in increasingly finer detail.

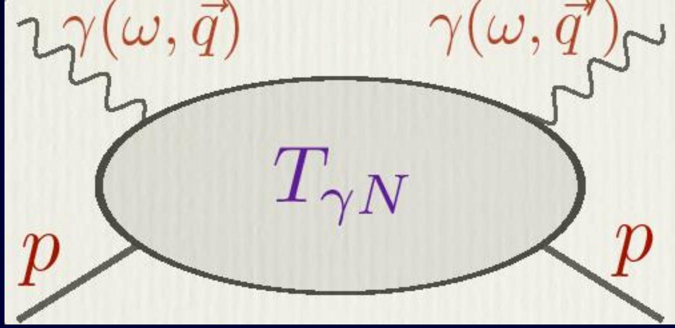
μ, α, β:

static bulk response

others :

spatial and time resolution

Compton Scattering



Low-energy expansion of real Compton scattering amplitude on the nucleon

$$
\begin{aligned}
T_{\gamma N} = {} & A_1(\omega,\theta)\,\vec{\epsilon}\,' \cdot \vec{\epsilon} + A_2(\omega,\theta)\,\vec{\epsilon}\,' \cdot \hat{k}\,\vec{\epsilon} \cdot \hat{k}' + i\,A_3(\omega,\theta)\,\vec{\sigma} \cdot (\vec{\epsilon}\,' \times \vec{\epsilon}) + i\,A_4(\omega,\theta)\,\vec{\sigma} \cdot (\hat{k}' \times \hat{k})\,\vec{\epsilon}\,' \cdot \vec{\epsilon} \\
& + i\,A_5(\omega,\theta)\,\vec{\sigma} \cdot \left[(\vec{\epsilon}\,' \times \hat{k})\,\vec{\epsilon} \cdot \hat{k}' - (\vec{\epsilon} \times \hat{k}')\,\vec{\epsilon}\,' \cdot \hat{k} \right] + i\,A_6(\omega,\theta)\,\vec{\sigma} \cdot \left[(\vec{\epsilon}\,' \times \hat{k}')\,\vec{\epsilon} \cdot \hat{k}' - (\vec{\epsilon} \times \hat{k})\,\vec{\epsilon}\,' \cdot \hat{k} \right]
\end{aligned}
$$

$$
\begin{aligned}
A_1(\omega,\theta) &= -Z^2 \frac{e^2}{M_N} + \frac{e^2}{4M_N^3}\left(\mu^2(1+\cos\theta) - Z^2\right)(1-\cos\theta)\,\omega^2 + 4\pi(\alpha + \beta\cos\theta)\omega^2 + \mathcal{O}(\omega^4)\,, \\
A_2(\omega,\theta) &= \frac{e^2}{4M_N^3}(\mu^2 - Z^2)\omega^2\cos\theta - 4\pi\beta\omega^2 + \mathcal{O}(\omega^4)\,, \\
A_3(\omega,\theta) &= \frac{e^2\omega}{2M_N^2}\left(Z(2\mu - Z) - \mu^2\cos\theta\right) + 4\pi\omega^3\left(\gamma_1 - (\gamma_2 + 2\gamma_4)\cos\theta\right) + \mathcal{O}(\omega^5)\,, \\
A_4(\omega,\theta) &= -\frac{e^2\omega}{2M_N^2}\mu^2 + 4\pi\omega^3\gamma_2 + \mathcal{O}(\omega^5)\,, \\
A_5(\omega,\theta) &= \frac{e^2\omega}{2M_N^2}\mu^2 + 4\pi\omega^3\gamma_4 + \mathcal{O}(\omega^5)\,, \\
A_6(\omega,\theta) &= -\frac{e^2\omega}{2M_N^2}Z\mu + 4\pi\omega^3\gamma_3 + \mathcal{O}(\omega^5)\,,
\end{aligned}
$$

polarizabilities: α, β, γ_1, γ_2, γ_3, γ_4

Experimental information on nucleon polarizabilities

- Proton electric polarizability (α_p) is around 12 in units of 10^{-4} fm^3.
- Proton magnetic polarizability (β_p) is around 2 in units of 10^{-4} fm^3.
- α_n is about the same as α_p
- β_n is about the same as β_p
- Experiments are under way or planned for other polarizabilities at electron accelerators around the world (HiγS, MAXlab, JLab, …)

Polarizabilities on the Lattice

Measure mass shifts in progressively-small external electric and magnetic fields, specially designed to isolate them:

$$H = -\vec{\mu}\cdot\vec{B} - \frac{1}{2}\alpha E^2 - \frac{1}{2}\beta B^2$$
$$-\frac{1}{2}\gamma_{E1}\sigma\cdot\vec{E}\times\dot{\vec{E}} - \frac{1}{2}\gamma_{M1}\sigma\cdot\vec{B}\times\dot{\vec{B}}$$
$$+\gamma_{E2}\sigma_i E_{ij} B_j - \gamma_{M2}\sigma_i B_{ij} E_j$$
$$-\frac{1}{12}\alpha_{E2}E_{ij}^2 - \frac{1}{12}\beta_{M2}E_{ij}^2 + \cdots$$

Small field expansion:

$$\delta m(B) = m(B) - m(0) = c_1 B + c_2 B^2 + c_3 B^3 + c_4 B^4 + \cdots$$

A computational trick

- We generate two sets of quark propagators, one with the original set of fields, the other with the fields reversed.
- The mass shift in the presence of small fields is

$$\delta m(B) = m(B) - m(0) = c_1 B + c_2 B^2 + c_3 B^3 + c_4 B^4 + \cdots$$

- At the cost of a factor of two,
  – by taking the average, $[\delta m(B) + \delta m(-B)]/2$, we get the leading quadratic response with the odd-powered terms eliminated. (magnetic polarizability)
  – by taking the difference, $[\delta m(B) - \delta m(-B)]/2$, we get the leading linear response with the even-powered terms eliminated. (magnetic moment)
- Our calculation is equivalent to 11 mass spectrum calculations.
  – 5 original fields, 5 reversed, plus the zero-field to set the baseline

Introduction of an external electromagnetic field on the lattice

- Minimal coupling in the QCD covariant derivative in Euclidean space

$$D_\mu \to \partial_\mu + gG_\mu + qA_\mu$$

- Recall that SU(3) gauge field is introduced by the link variables

$$U_\mu(x) = \exp(iagG_\mu)$$

- It suggests multiplying a U(1) phase factor to the links

$$U'_\mu(x) = \exp(iaqA_\mu)U_\mu$$

- This should be done in two places where the Dirac operator appears: both in the dynamical gauge generation and quark propagator generation

For Example

$$\vec{B} = \nabla \times \vec{A}$$

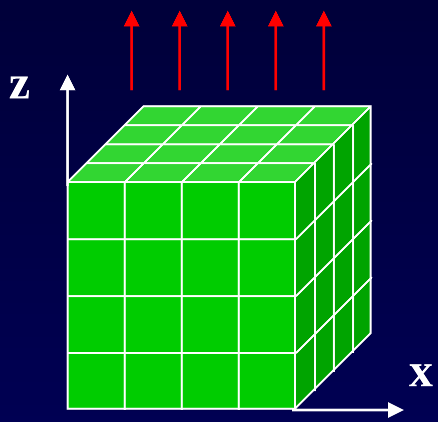
$$\vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}$$

- To apply magnetic field **B** in the z-direction, one can choose the 4-vector potential

$$A_\mu \equiv (\phi, \vec{A}) = (0,0,Bx,0)$$

then the y-link is modified by a x-dependent phase factor

$$U_y \to \exp(iqaBx)U_y$$



- To apply electric field **E** in the x-direction, one can choose the 4-vector potential

$$A_\mu = (0, Et, 0, 0)$$

then the x-link is modified by a t-dependent phase factor

$$U_x \to \exp(iqaEt)U_x$$

Relevant Literature on External Field Method

- "Lattice quantum-chromodynamics calculation of some baryon magnetic moments", Bernard, Draper, Olynyk, PRL49 (1982) 1076; NPB220 (1983) 508
- "A study of hadron electric polarizability in quenched lattice QCD", Fiebig, Wilcox, Woloshyn, NPB324, 47 (1989)
- "Electric Polarizability of Neutral Hadrons from Lattice QCD", Christensen, Wilcox, Lee, Zhou, PRD72, 034503 (2005)
- "Baryon magnetic moments in the background field method", Lee, Kelly, Zhou, Wilcox, PLB627, 71 (2005)
- "Magnetic polarizability of hadrons from lattice QCD in the background field method", Lee, Zhou, Wilcox, Christensen, PRD73, 034503 (2006)
- "Electricmagnetic and spin polarizabilitites in lattice QCD", Detmold, Tiburzi, Walker-Loud, PRD73 (2006) 114505
- "Neutron electric dipole moment with external electric field method in lattice QCD", Shintani et al, CP-PACS collaboration, PRD75, 034507 (2007)
- "Neutron electric polarizability from unquenched lattice QCD using the background field approach", M. Engelhardt, LHPC collaboration, PRD76, 114502 (2007)

Computational Demands

$$U_y \to \exp(iqaBx)U_y$$

- Consider quark propagator generation

$$\frac{\int DG_\mu \, \Pi\det(\not{D}+m_q)\, e^{-S_G} (\not{D}+m_q)^{-1}}{\int DG_\mu \, \Pi\det(\not{D}+m_q)\ e^{-S_G}}$$

$$D_\mu \to \partial_\mu + gG_\mu + qA_\mu$$

- Fully dynamical: For each value of external field, a new dynamical ensemble is needed that couples to the sea u-quark (q=1/3), d- and s-quark (q=-2/3). Valence quark propagator is then computed on the ensembles with matching values
- Re-weighting: Perturbative expansion of action in terms of external field
- U(1) quenched: no field in the sea, only in the valence
  - any gauge ensemble can be used to compute valence quark propagators.

Magnetic Moments

New Topic

Magnetic Moment: two methods

- Form factor method: $G_M(Q^2=0)$
  - Since the minimum momentum on the lattice is non-zero ($p=2\pi/L$), extrapolation to zero momentum transfer is required.
  - Three-point function calculations

- Background field method
  - direct access
  - Two-point function calculations
  - but no Q^2 dependence

Magnetic moment in background field

- For a particle of spin s and mass m in small fields,

$$E_{\pm} = m \pm \mu B$$

where upper sign means spin-up and lower sign spin-down, and

$$\mu = g \frac{e}{2m} s$$

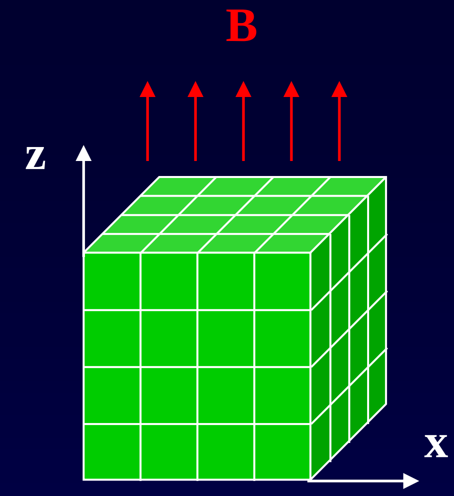
- g factor (magnetic moment in natural magnetons) is extracted from

$$g = m \frac{(E_{+} - m) - (E_{-} - m)}{eBs}$$

- Look for the slope (g-factor) in the mass shift as a function of the field

$$\delta m = g(eB)$$

Lattice details

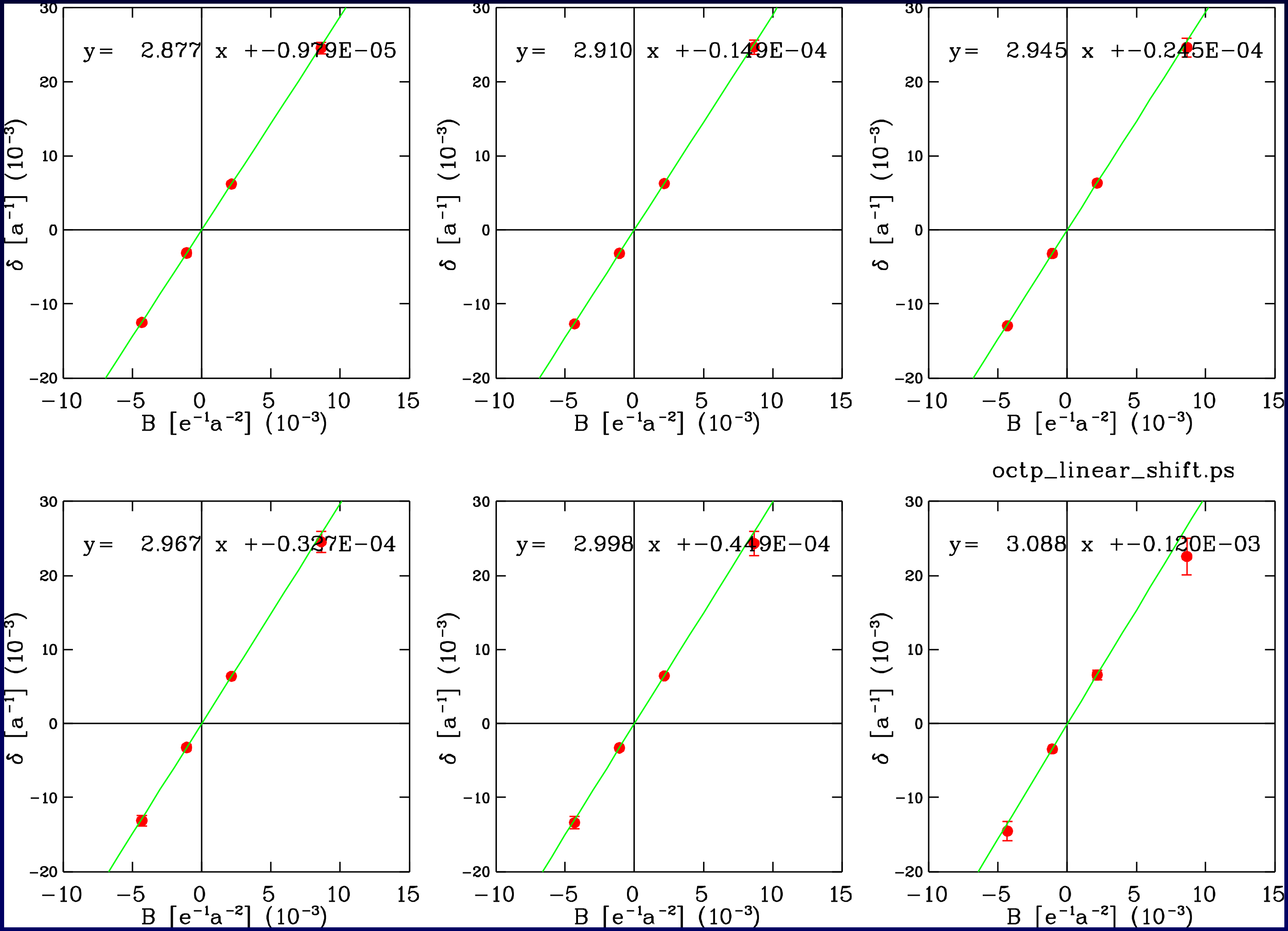


- Standard Wilson gauge action
  - 24^4 lattice, β=6.0 (or a $\approx$ 0.1 fm)
  - 150 configurations
- Standard Wilson fermion action
  - κ=0.1515, 0.1525, 0.1535, 0.1540, 0.1545, 0.1555
  - Pion mass about 1015, 908, 794, 732, 667, 522 MeV
  - Strange quark mass corresponds to κ=0.1535 (or m_π~794 MeV)
  - Source location (x,y,z,t)=(12,1,1,2)
  - Boundary conditions: periodic in y and z, fixed in x and t
- The following 5 dimensionless numbers $\eta \equiv qBa^2$ =+0.00036, -0.00072, +0.00144, -0.00288, +0.00576 correspond to 4 small B fields

  eBa^2 = -0.00108, 0.00216, -0.00432, 0.00864 for both u and d (or s) quarks.
  - Small in the sense that the mass shift is only a fraction of the proton mass: $\mu B/m$ ~ 1 to 5% at the smallest pion mass. In physical units, B ~ 10^{13} Tesla.

Proton mass shifts

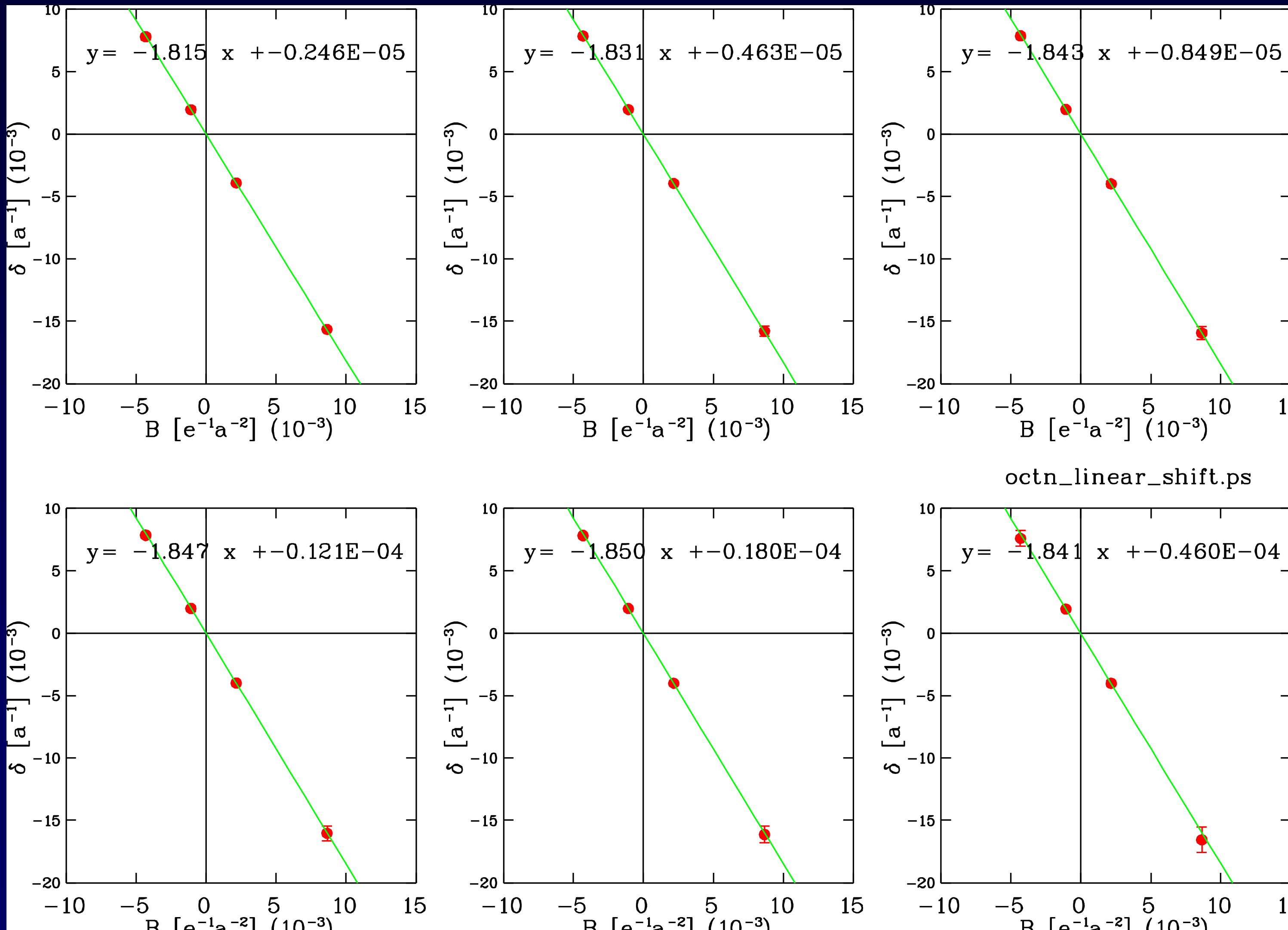
$\delta m = g(eB$



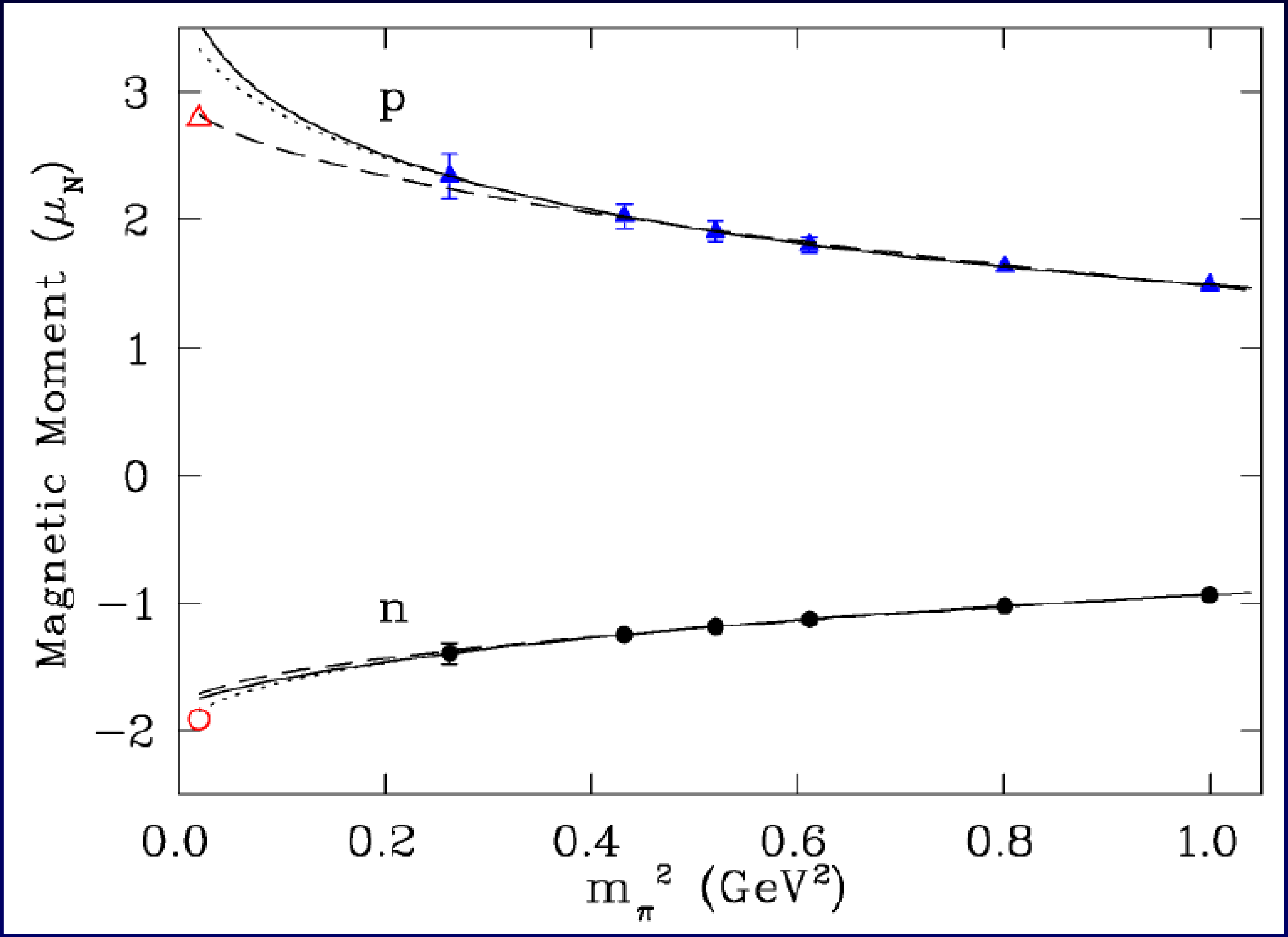
- We use the 2 smallest fields to fit the line.

$\delta m = g(eB)$

Neutron mass shifts



Proton and neutron magnetic moments



Chiral Extrapolation

- To one meson loop, χPT predicts

$$\mu = \mu_0 + c_1 m_\pi + c_2 m_\pi^2 \log m_\pi^2 + c_3 m_\pi^2 + \cdots$$

but only applicable in small mass region.

- Encapsulating form (Leinweber, Lu and Thomas, PRD60 (1999) 034014)

$$\mu = \frac{\mu_0}{1 + \alpha m_\pi + \beta m_\pi^2}$$

Pade ansatz

- For small mass,

$$\mu = \mu_0 [1 - \alpha m_\pi + (\alpha^2 - \beta) m_\pi^2 + \cdots]$$

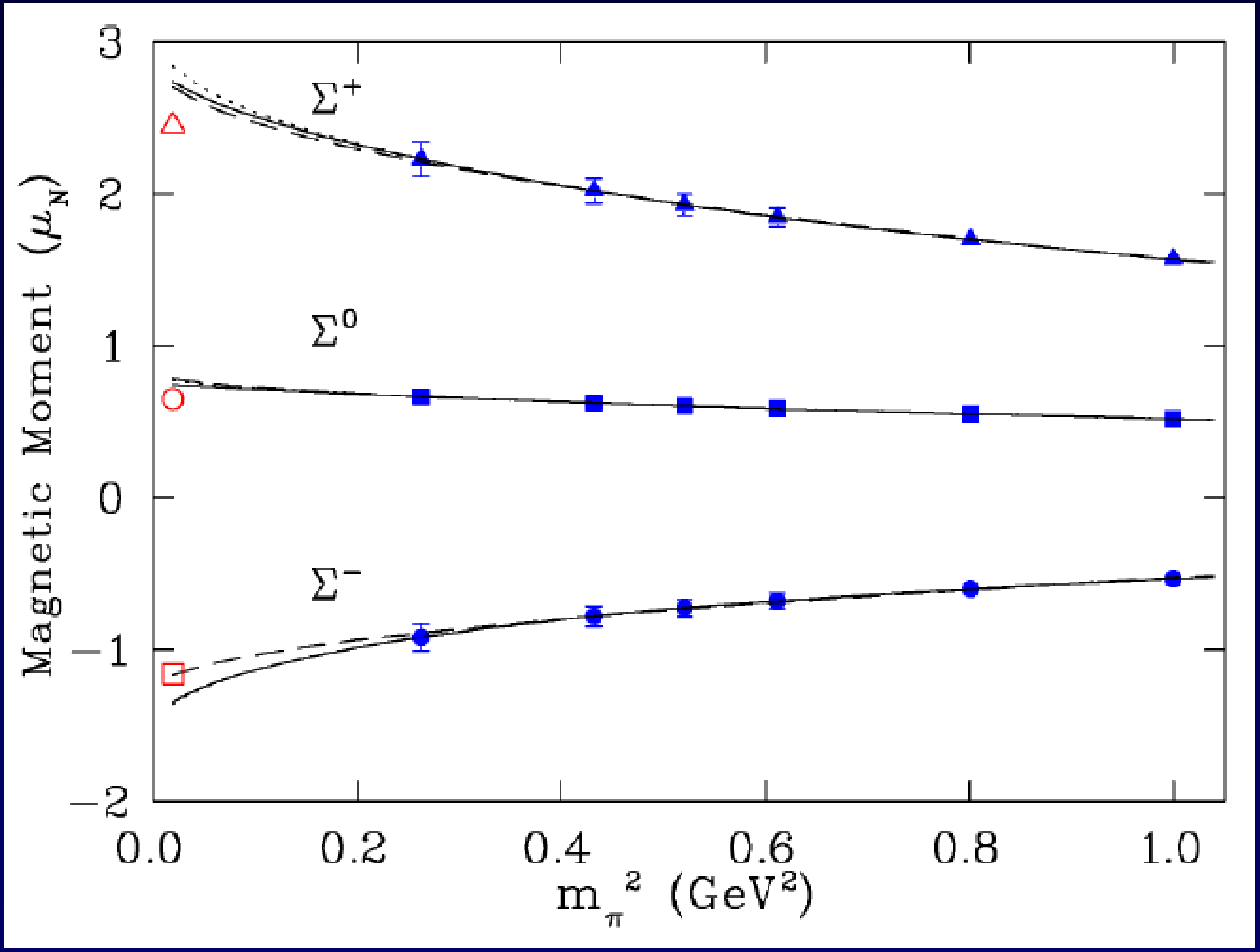
- For large mass,

$$\mu = \frac{\mu_0}{\beta m_\pi^2} \left(1 - \frac{\alpha}{\beta m_\pi} + \cdots \right) \propto \frac{1}{m_q}$$

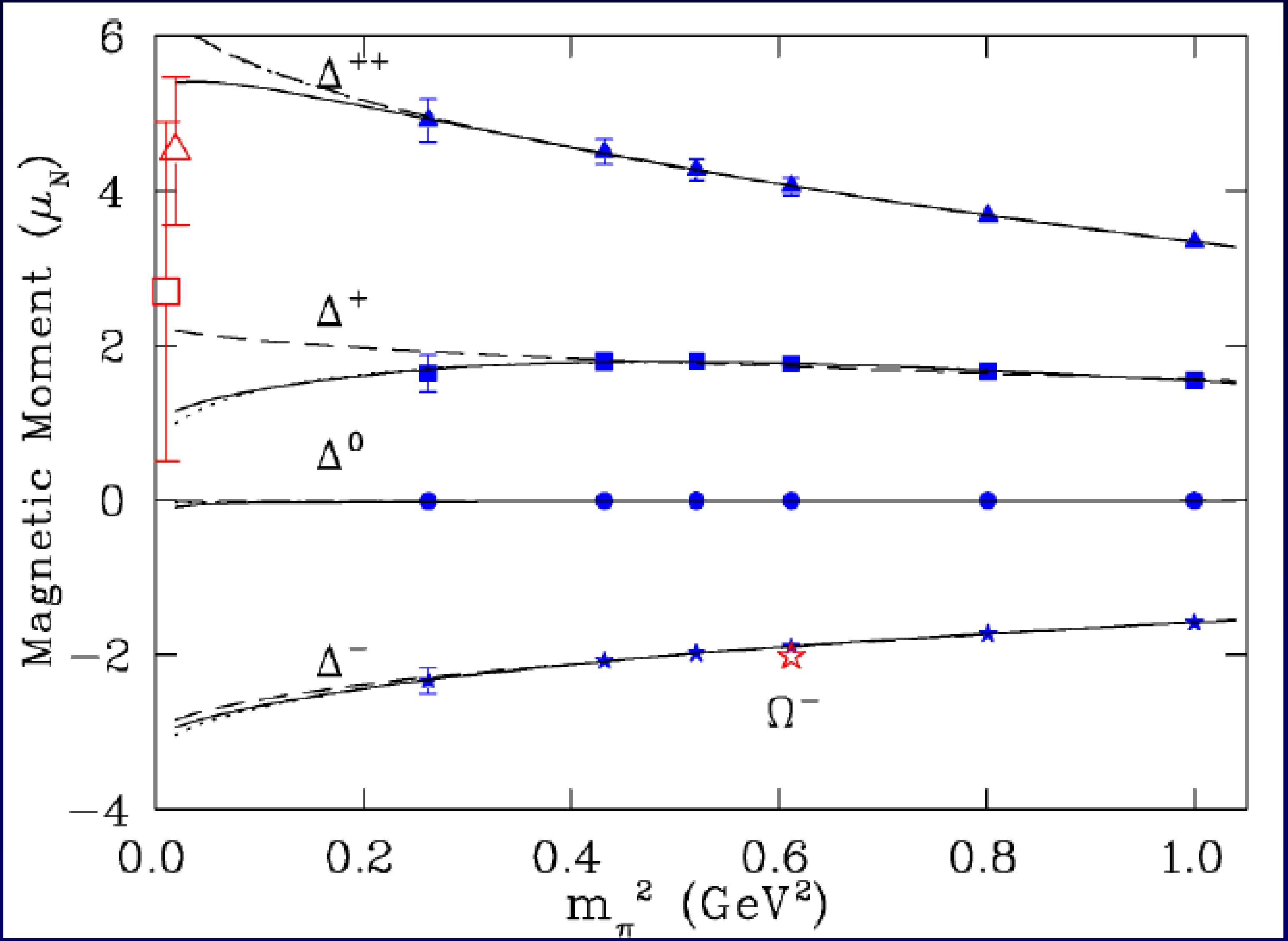
Magnetic Moments in ChPT

- Leinweber, Lu, Thomas, PRD60 (1999) 034014
- Hackett-Jones, Leinweber, Thomas, PLB489 (2000) 143
- Leinweber, Thomas, Young, PRL86 (2001) 5011
- Borasoy, Lewis, Ouimet, PRD65 (2002) 114023
- Arndt, Tiburzi, PRD68 (2003) 114503
- Beane, Savage, PRD68 (2003) 114502
- Hemmert, Procura, Weise, NPA721 (2003) 938
- Young, Leinweber, Thomas, hep-lat/0311038
- Young, Leinweber, Thomas, hep-lat/0406001
- Leinweber, PRD69 (2004) 014005

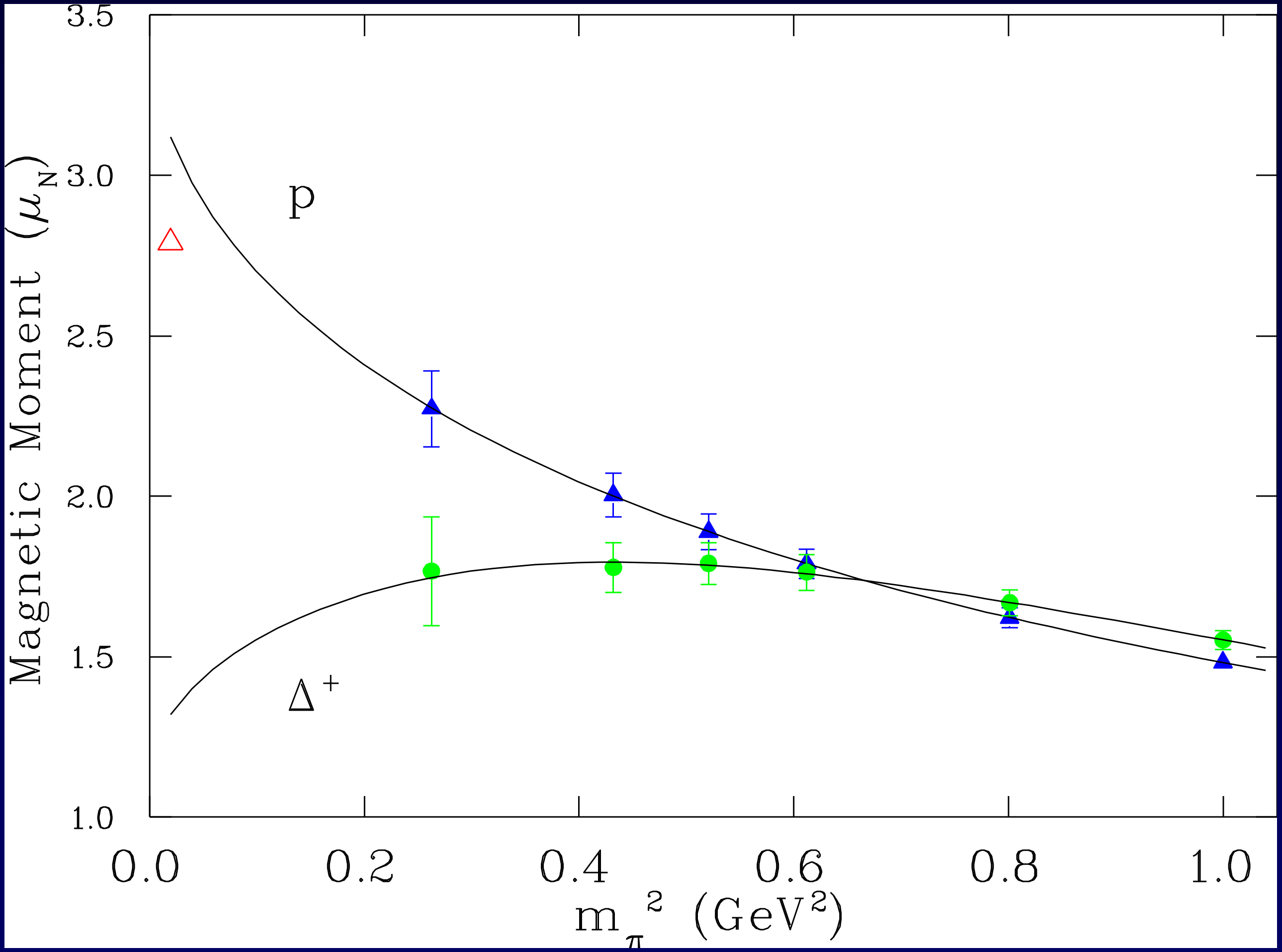
Octet Sigma magnetic moments



Delta magnetic moments



Proton and Δ^+ magnetic moments



Curvatures expected from ChPT.

Magnetic moments for other hadrons

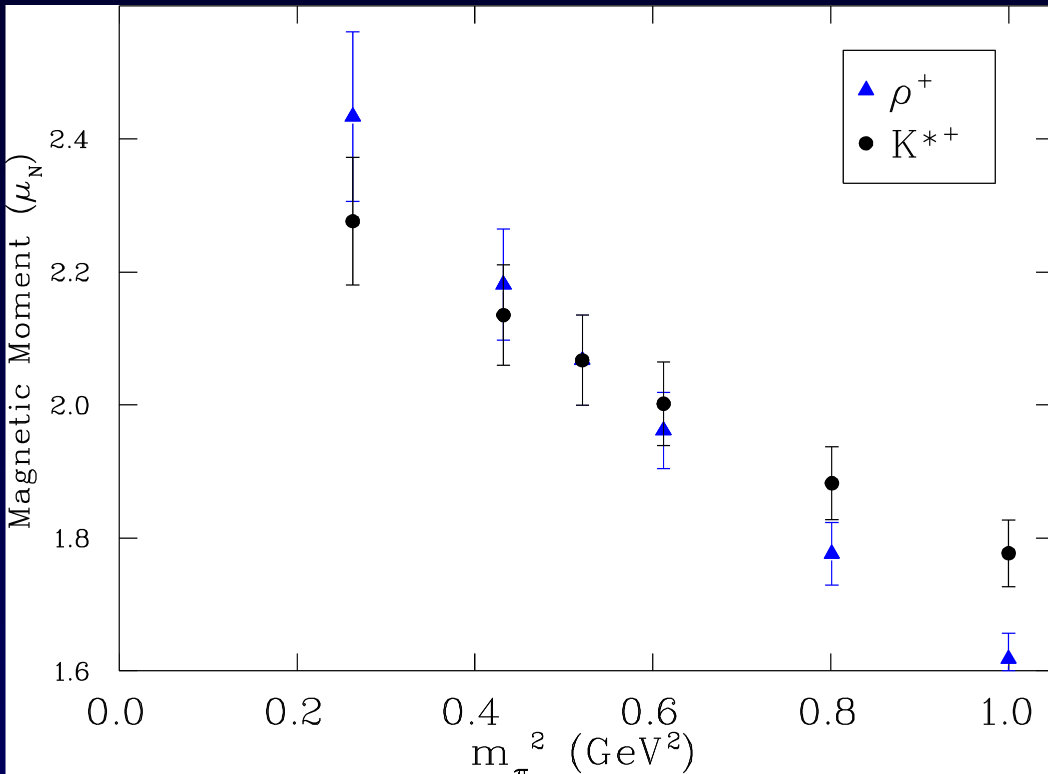
Table 1

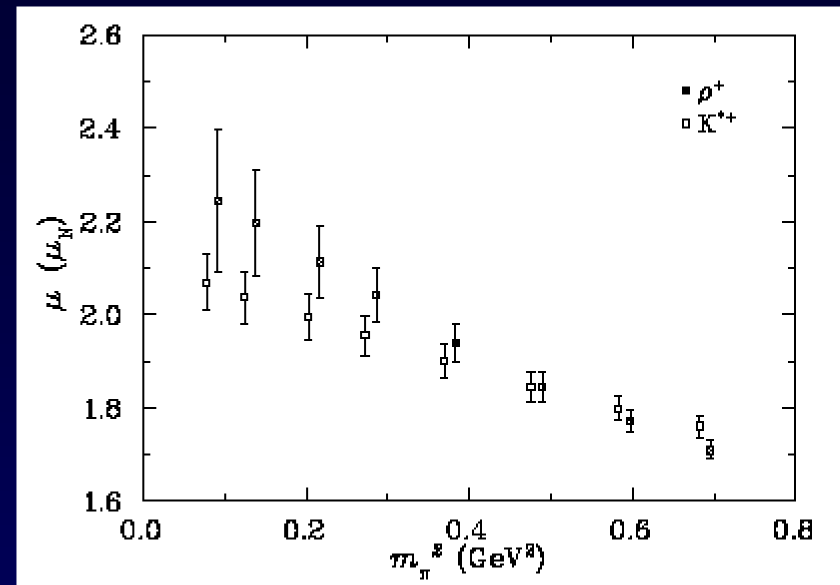
The computed magnetic moments for the baryon octet and decuplet in nuclear magnetons (μ_N) as a function of the pion mass. The extrapolated values are based on Eq (10). The experimental values are taken from the PDG [16]

κ	0.1515	0.1525	0.1535	0.1540	0.1545	0.1555	Extrap.	Expt.
m_π (GeV)	1.015	0.908	0.794	0.732	0.667	0.522		
p	1.48(3)	1.63(4)	1.80(6)	1.90(8)	2.02(10)	2.34(17)	3.04(6)	2.79
n	−0.94(2)	−1.02(2)	−1.13(3)	−1.18(4)	−1.25(5)	−1.39(8)	−1.84(3)	−1.91
Σ^+	1.57(4)	1.70(5)	1.85(6)	1.93(7)	2.02(8)	2.23(11)	2.87(3)	2.45
Σ^0	0.52(1)	0.55(1)	0.59(2)	0.61(2)	0.63(2)	0.67(3)	0.76(1)	0.65
Σ^-	−0.54(4)	−0.60(4)	−0.68(6)	−0.73(6)	−0.78(7)	−0.92(9)	−1.48(5)	−1.16
Ξ^0	−1.07(4)	−1.10(4)	−1.13(4)	−1.15(4)	−1.17(5)	−1.21(7)	−1.37(1)	−1.25
Ξ^-	−0.76(6)	−0.77(7)	−0.77(8)	−0.77(8)	−0.77(9)	−0.78(11)	−0.82(1)	−0.65
Λ	−0.59(2)	−0.60(2)	−0.60(2)	−0.61(2)	−0.61(2)	−0.62(3)	−0.70(1)	−0.61
Δ^{++}	3.35(7)	3.68(9)	4.06(12)	4.28(14)	4.50(16)	4.92(28)	5.24(18)	4.52(1.00)
Δ^+	1.55(4)	1.67(6)	1.77(8)	1.80(10)	1.80(11)	1.64(24)	0.97(8)	
Δ^0	−0.002(0)	−0.003(0)	−0.004(1)	−0.005(1)	−0.007(1)	−0.011(6)	−0.035(2)	
Δ^-	−1.58(4)	−1.73(5)	−1.89(7)	−1.98(8)	−2.07(10)	−2.34(17)	−2.98(19)	
$\Sigma*^+$	1.40(5)	1.56(6)	1.72(8)	1.80(10)	1.86(11)	1.88(17)	1.27(6)	
$\Sigma*^0$	−0.13(1)	−0.09(1)	−0.03(0)	−0.01(0)	0.03(0)	0.08(2)	0.33(5)	
$\Sigma*^-$	−1.68(5)	−1.76(7)	−1.83(9)	−1.87(10)	−1.89(11)	−1.93(16)	−1.88(4)	
$\Xi*^0$	−0.09(2)	−0.06(1)	−0.022(5)	−0.002(1)	0.018(6)	0.05(3)	0.16(4)	
$\Xi*^-$	−0.59(7)	−0.61(8)	−0.62(9)	−0.62(10)	−0.63(10)	−0.63(11)	−0.62(1)	

F.X. Lee, R. Kelly, L. Zhou, W. Wilcox, Phys. Lett. B 627, 71 (2005)

Vector Meson Magnetic Moment





Background field method

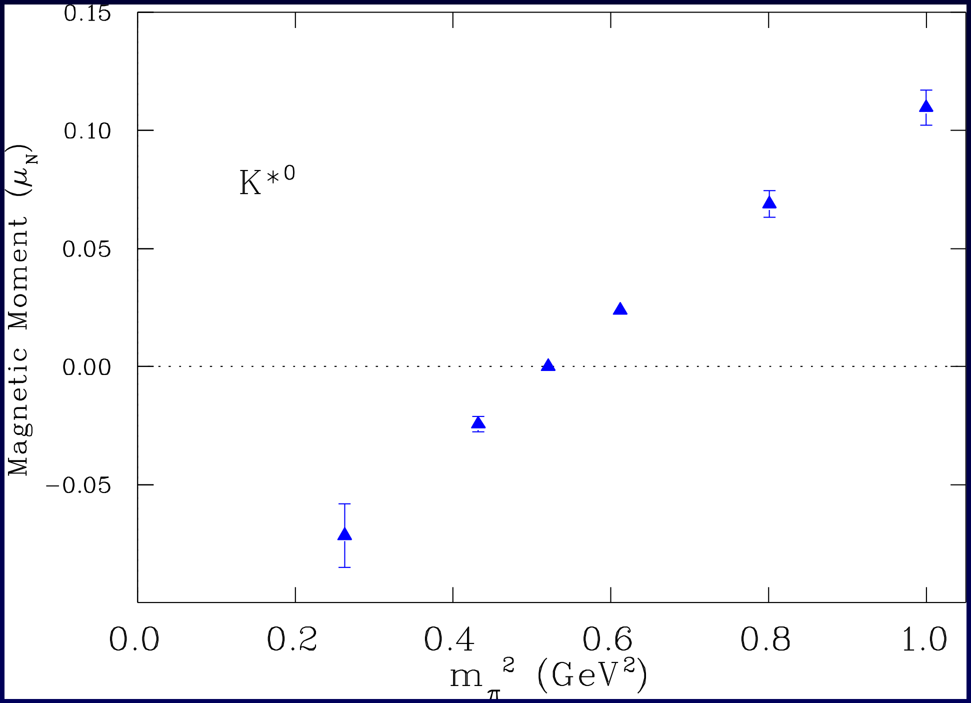
hep-lat/0710.2329, polarQCD

Form factor method

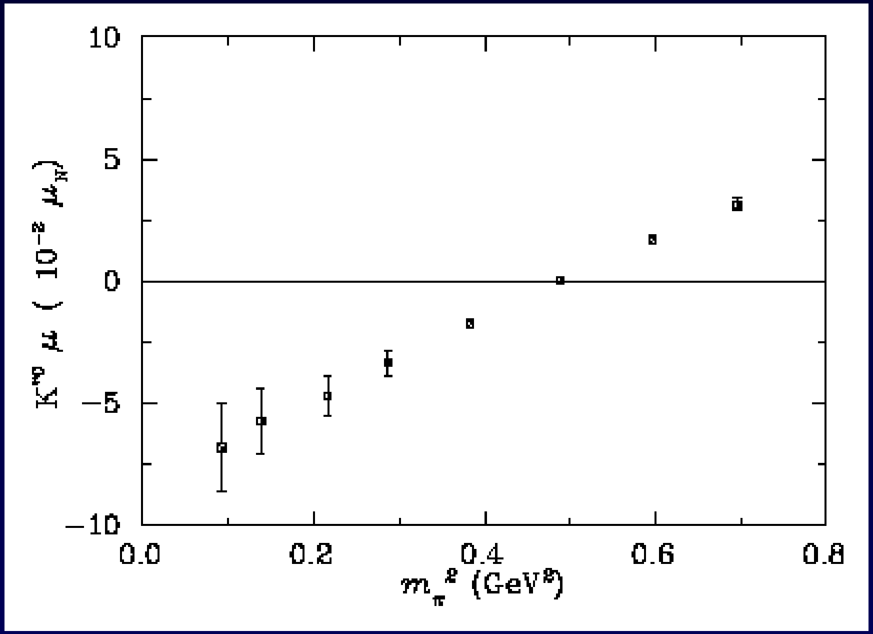
hep-lat/0703014, Adelaide group

Also agrees with that from the Charge Overlap Method by W. Andersen and W. Wilcox, Annals Phys. 255, 34 (1997)

K^{*0} Meson Magnetic Moment



Background field method



Form factor method

We also computed axial and tensor mesons. The results are in preparation for a publication.

Polarizabilities

New Topic

Polarizability: a quantum mechanical example

- Hydrogen atom placed in static electric and magnetic fields
  - 2nd order Stark effect: electric polarizability

$$\Delta E^{(2)} = -\frac{9}{4}a_0^3 E^2 = -\frac{1}{2}\alpha E^2,$$

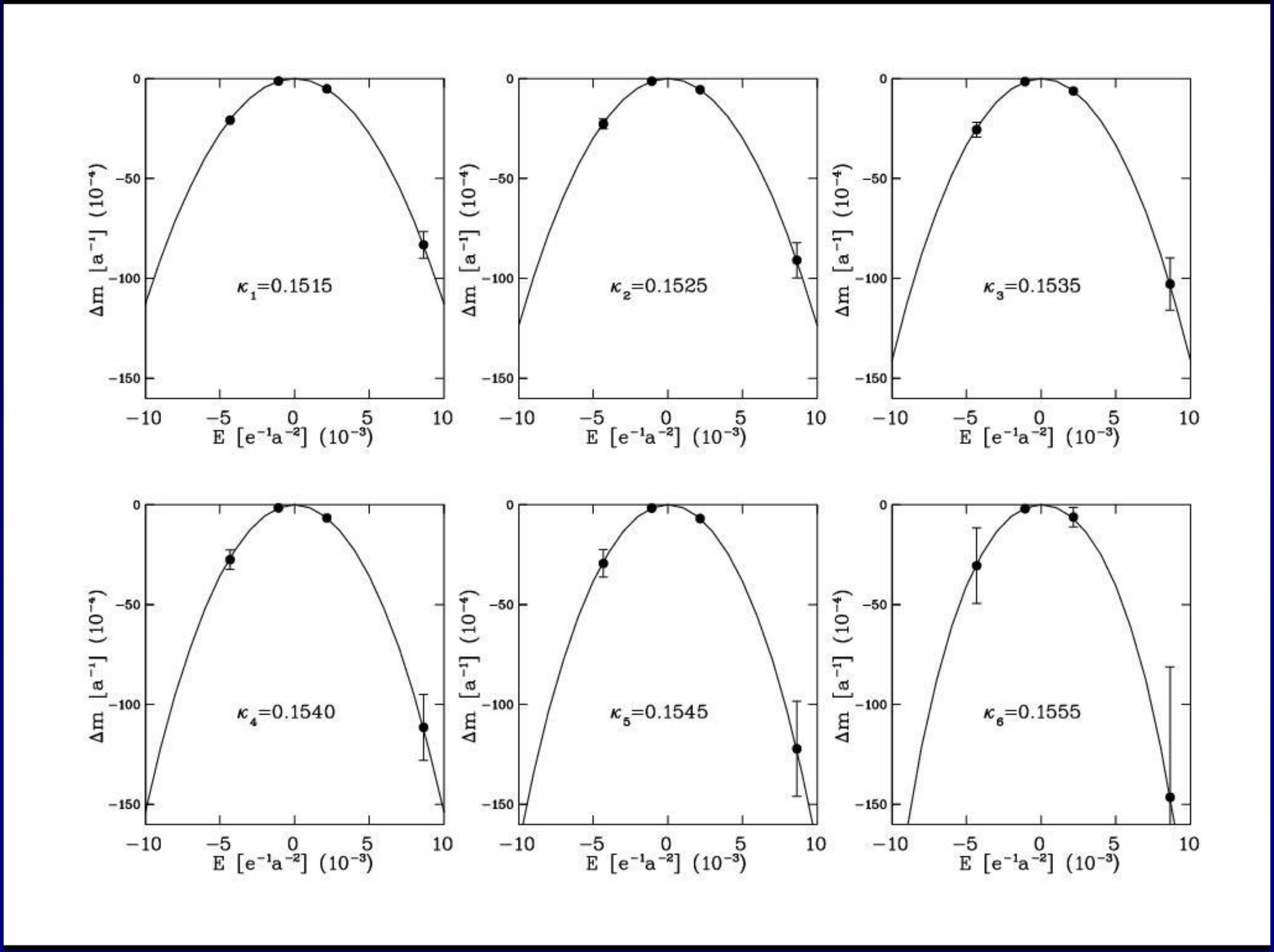
$$\text{so } \alpha = 4.5 a_0^3 \approx 0.67\ \text{fm}^3$$

  - 2nd order Zeeman effect: magnetic polarizability

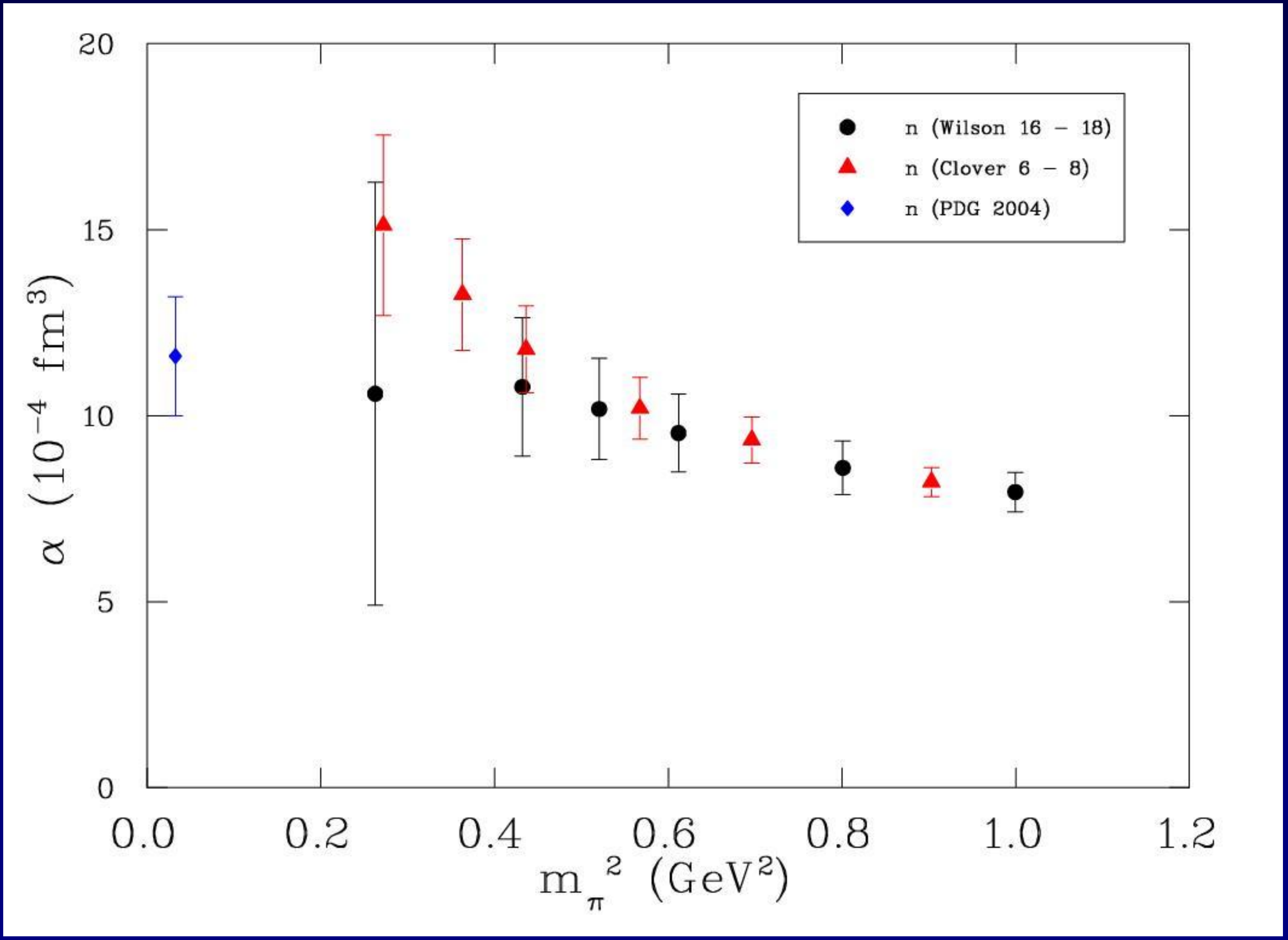
$$\Delta E^{(2)} = -\frac{1}{2}\beta B^2$$

Neutron Mass Shift in Electric Field

$$\Delta m = -\frac{1}{2}\alpha E^2$$



Electric Polarizability of neutron



Electric Polarizabilities of Neutral Particles

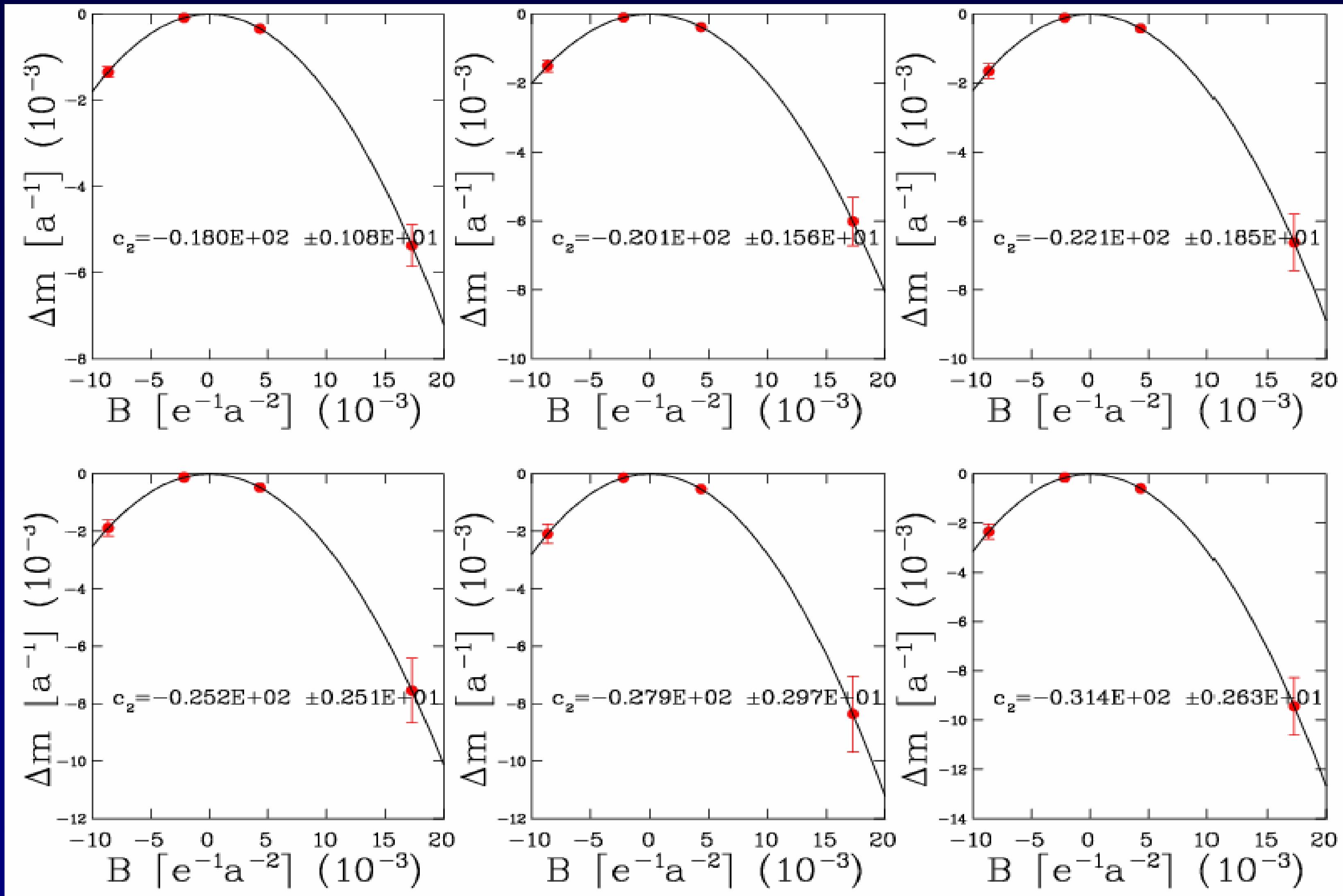
TABLE II: The electric polarizabilities from the calculation with Wilson action using six κ values. The units of the electric polarizability are 10^{-4} fm^3. The pion masses were fit on time steps $11-13$ from the propagator origin and are given in GeV.

Christensen, Wilcox, Lee, Zhou, Phys.Rev. D72 (2005) 034503

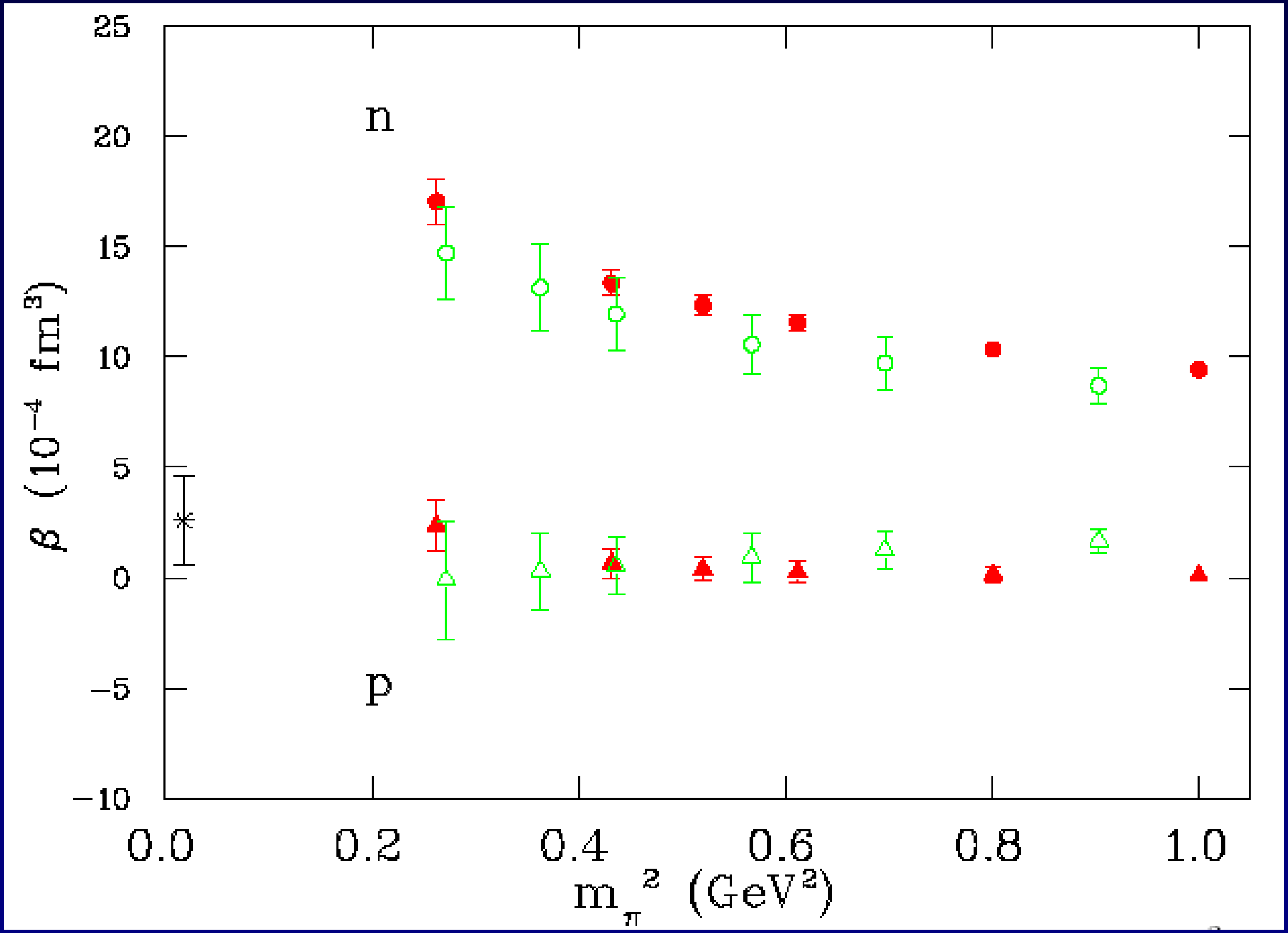
κ	0.1515	0.1525	0.1535	0.1540	0.1545	0.1555	
m_π	$1.000 \pm .005$	$0.895 \pm .006$	$0.782 \pm .006$	$0.721 \pm .006$	$0.657 \pm .007$	$0.512 \pm .007$	
Meson							fit range
ρ^0	5.0 ± 0.5	4.8 ± 0.7	4.4 ± 0.9	4.0 ± 1.2	3.5 ± 1.6	2.8 ± 4.8	13-15
K^{*0}	1.2 ± 0.2	1.2 ± 0.2	1.3 ± 0.3	1.3 ± 0.3	1.3 ± 0.4	1.2 ± 0.6	10-12
Baryon octet							
n	7.9 ± 0.5	8.6 ± 0.7	9.5 ± 1.0	10.2 ± 1.4	10.8 ± 1.9	10.6 ± 5.7	14-16
Σ^0	7.7 ± 0.7	8.1 ± 0.9	8.8 ± 1.3	9.3 ± 1.5	10.0 ± 1.9	11.5 ± 4.0	14-16
Λ^0_o	8.2 ± 0.7	8.8 ± 0.8	9.7 ± 1.1	10.2 ± 1.2	10.8 ± 1.6	13.2 ± 3.2	14-16
Ξ^0	9.3 ± 0.9	9.7 ± 1.0	10.0 ± 1.2	10.2 ± 1.4	10.3 ± 1.6	10.1 ± 2.3	14-16
Baryon decuplet							
Δ^0	1.7 ± 0.1	1.6 ± 0.2	1.5 ± 0.3	1.5 ± 0.4	1.6 ± 0.6	2.3 ± 1.2	9-11
Σ^{*0}	1.7 ± 0.2	1.6 ± 0.2	1.5 ± 0.4	1.5 ± 0.4	1.6 ± 0.5	2.0 ± 0.9	9-11
Ξ^{*0}	1.7 ± 0.2	1.6 ± 0.3	1.5 ± 0.4	1.5 ± 0.4	1.5 ± 0.5	1.6 ± 0.8	9-11

Neutron Mass Shifts in Magnetic Field

$$\Delta m = -\frac{1}{2}\beta B^2$$



Magnetic Polarizability of the Nucleon



Magnetic Polarizabilities: baryon octet

TABLE III. The calculated magnetic polarizabilities for the octet baryons as a function of the pion mass from the Wilson action. The pion mass is in GeV and the magnetic polarizability is in 10^{-4} fm^3. The time window from which each polarizability is extracted is given in the last column. The errors are statistical.

κ	0.1515	0.1525	0.1535	0.1540	0.1545	0.1555	fit range
m_π	1.000	0.895	0.782	0.721	0.657	0.512	
p	0.09 ± 0.29	0.14 ± 0.37	0.26 ± 0.48	0.40 ± 0.56	0.64 ± 0.67	2.36 ± 1.20	12-14
n	9.4 ± 0.2	10.4 ± 0.3	11.6 ± 0.4	12.3 ± 0.5	13.4 ± 0.6	17.0 ± 1.1	12-14
Σ^+	-0.15 ± 0.36	0.09 ± 0.42	0.24 ± 0.50	0.40 ± 0.56	0.61 ± 0.64	1.60 ± 1.00	12-14
Σ^0	8.0 ± 0.3	8.5 ± 0.3	9.1 ± 0.4	9.6 ± 0.5	10.1 ± 0.6	11.9 ± 0.9	12-14
Σ^-	-11.8 ± 0.3	-12.6 ± 0.3	-13.6 ± 0.4	-14.2 ± 0.4	-14.7 ± 0.4	-16.1 ± 0.5	12-14
Ξ^0	10.7 ± 0.3	11.3 ± 0.4	11.9 ± 0.4	12.3 ± 0.4	12.8 ± 0.5	13.9 ± 0.7	12-14
Ξ^-	-12.6 ± 0.3	-13.1 ± 0.3	-13.8 ± 0.4	-14.1 ± 0.4	-14.6 ± 0.4	-15.6 ± 0.5	12-14
Λ^8	9.1 ± 0.4	9.9 ± 0.5	10.8 ± 0.6	11.4 ± 0.7	12.1 ± 0.8	14.0 ± 1.2	12-14
Λ^C	9.3 ± 0.4	10.2 ± 0.5	11.2 ± 0.6	11.9 ± 0.7	12.6 ± 0.9	15.0 ± 1.4	12-14
Λ^S	3.6 ± 0.1	3.4 ± 0.2	3.3 ± 0.2	3.3 ± 0.3	3.2 ± 0.4	3.2 ± 1.0	5-7

F.X. Lee, L. Zhou, W. Wilcox, J. Christensen, Phys. Rev. D73 (2006) 034503

Magnetic Polarizabilities: baryon decuplet

TABLE IV. The calculated magnetic polarizabilities for the decuplet baryons as a function of the pion mass from the Wilson action. The pion mass is in GeV and the magnetic polarizability is in 10^{-4} fm^3. The time window from which each polarizability is extracted is given in the last column. The errors are statistical.

κ	0.1515	0.1525	0.1535	0.1540	0.1545	0.1555	fit range
m_π	1.000	0.895	0.782	0.721	0.657	0.512	
Δ^{++}	-39.9 $\pm$ 0.7	-43.9 $\pm$ 0.8	-48.7 $\pm$ 1.0	-51.5 $\pm$ 1.1	-54.7 $\pm$ 1.3	-63.1 $\pm$ 1.9	10-12
Δ^{+}	-2.5 $\pm$ 0.2	-3.0 $\pm$ 0.3	-3.5 $\pm$ 0.5	-3.9 $\pm$ 0.6	-4.3 $\pm$ 0.7	-5.1 $\pm$ 1.1	10-12
Δ^{0}	7.6 $\pm$ 0.2	8.2 $\pm$ 0.3	8.8 $\pm$ 0.4	9.2 $\pm$ 0.5	9.6 $\pm$ 0.6	10.9$\pm$ 1.0	10-12
Δ^{-}	-10.1 $\pm$ 0.2	-11.1 $\pm$ 0.2	-12.4 $\pm$ 0.2	-13.1 $\pm$ 0.3	-14.0 $\pm$ 0.3	-16.2 $\pm$ 0.5	10-12
Σ^{*+}	-2.9 $\pm$ 0.3	-3.2 $\pm$ 0.4	-3.6 $\pm$ 0.5	-3.9 $\pm$ 0.6	-4.2 $\pm$ 0.6	-5.1 $\pm$ 0.9	10-12
Σ^{*0}	7.9 $\pm$ 0.2	8.4 $\pm$ 0.3	8.9 $\pm$ 0.4	9.2 $\pm$ 0.5	9.5 $\pm$ 0.6	10.3 $\pm$ 0.8	10-12
Σ^{*-}	-10.9 $\pm$ 0.2	-11.7 $\pm$ 0.2	-12.6 $\pm$ 0.3	-13.1 $\pm$ 0.3	-13.7 $\pm$ 0.3	-15.0 $\pm$ 0.4	10-12
Ξ^{*0}	8.1 $\pm$ 0.3	8.5 $\pm$ 0.3	9.0 $\pm$ 0.4	9.2 $\pm$ 0.5	9.4 $\pm$ 0.5	9.8 $\pm$ 0.7	10-12
Ξ^{*-}	-11.9 $\pm$ 0.2	-12.4 $\pm$ 0.2	-12.8 $\pm$ 0.3	-13.1 $\pm$ 0.3	-13.4 $\pm$ 0.3	-14.0 $\pm$ 0.3	10-12
Ω^{-}			-12.4 $\pm$ 0.2				10-12

PRD73 (2006) 034503

What's Next ?

$$H = -\vec{\mu}\cdot\vec{B} - \frac{1}{2}\alpha E^2 - \frac{1}{2}\beta B^2$$
$$-\frac{1}{2}\gamma_{E1}\sigma\cdot\vec{E}\times\dot{\vec{E}} - \frac{1}{2}\gamma_{M1}\sigma\cdot\vec{B}\times\dot{\vec{B}}$$
$$+\gamma_{E2}\sigma_i E_{ij} B_j - \gamma_{M2}\sigma_i B_{ij} E_j$$
$$-\frac{1}{12}\alpha_{E2} E_{ij}^2 - \frac{1}{12}\beta_{M2} E_{ij}^2 + \cdots$$

- Compute higher-order polarizabilities
  - Need non-uniform, sourceless fields
  - For example, to extract α_{E2} and γ_{E2}, choose

$$A_\mu = (0, -cxt, 0, -czt), \quad \vec{E} = (cz, 0, cx)$$

  - For example, to extract β_{M2} and γ_{M2}, choose

$$A_\mu = (0, -bxy, 0, bzy), \quad \vec{B} = (bz, 0, bx)$$

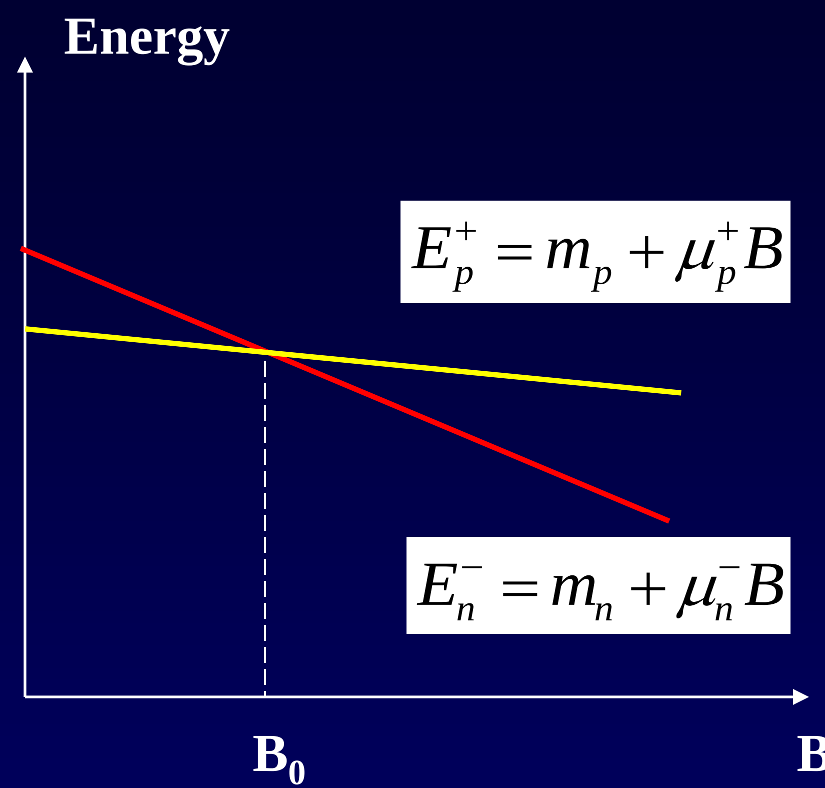
  - α and β must be re-measured and subtracted

- The path to unquenched calculations
  - Use CP-PACS 2+1 flavor dynamical gauge ensembles (Iwasaki glue + clover). But still U(1) quenched
  - Introduce U(1) fields in the dynamical gauge generation

Beta-decay of proton in magnetic field

- At sufficiently large B fields (**10^{16} Tesla**), proton can become heavier than neutron, allowing the 'β-decay' of the proton:

$$p \rightarrow n + e^{+} + \nu_e$$



- As compared to the natural neutron β-decay:

$$n \rightarrow p + e^{-} + \bar{\nu}_e$$

Such process can take place in stars where extremely strong magnetic field exists.

Long-term Goal of Lattice QCD

- To build all strong-interaction physics on the foundation of QCD.
  - mass spectrum
  - decay rates
  - form factors and transitions
  - electromagnetic properties
  - strangeness content of the nucleon
  - the nuclear force
  - matter at finite temperature and density
  - …
- Then tell us how the physical world works from this point of view
  - "the universe from scratch"