Neutrinos and Big Bang Nucleosynthesis

Expansion of the Universe

Edwin Hubble

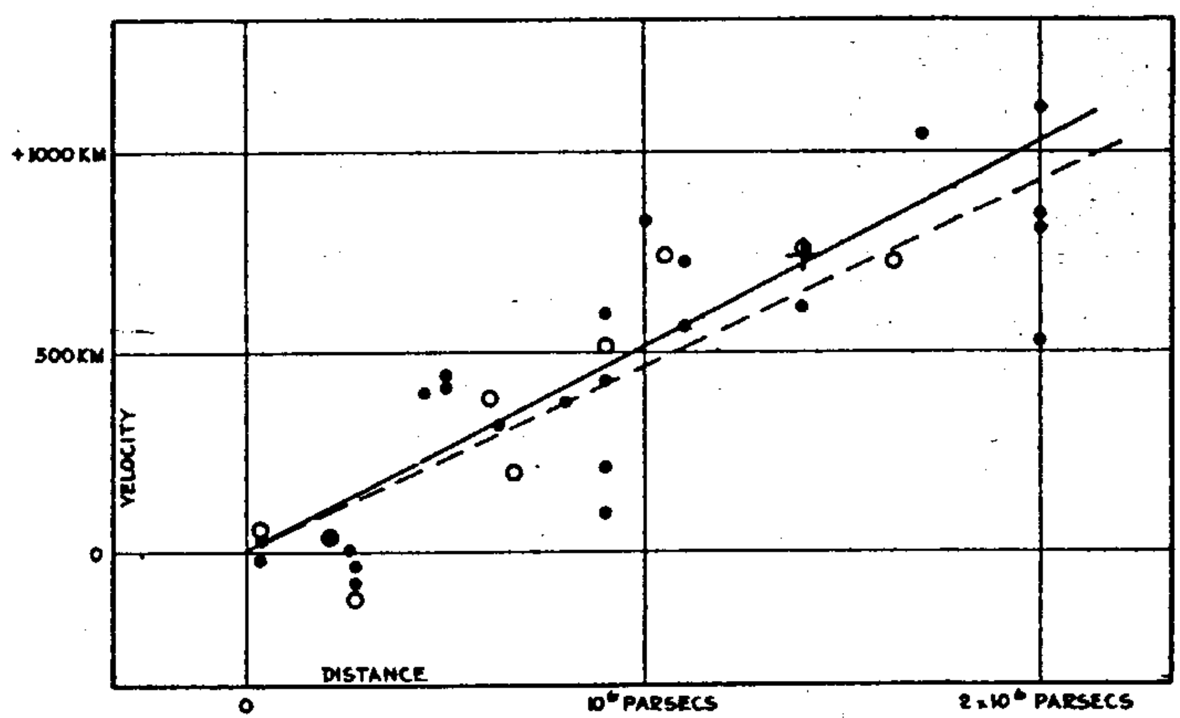
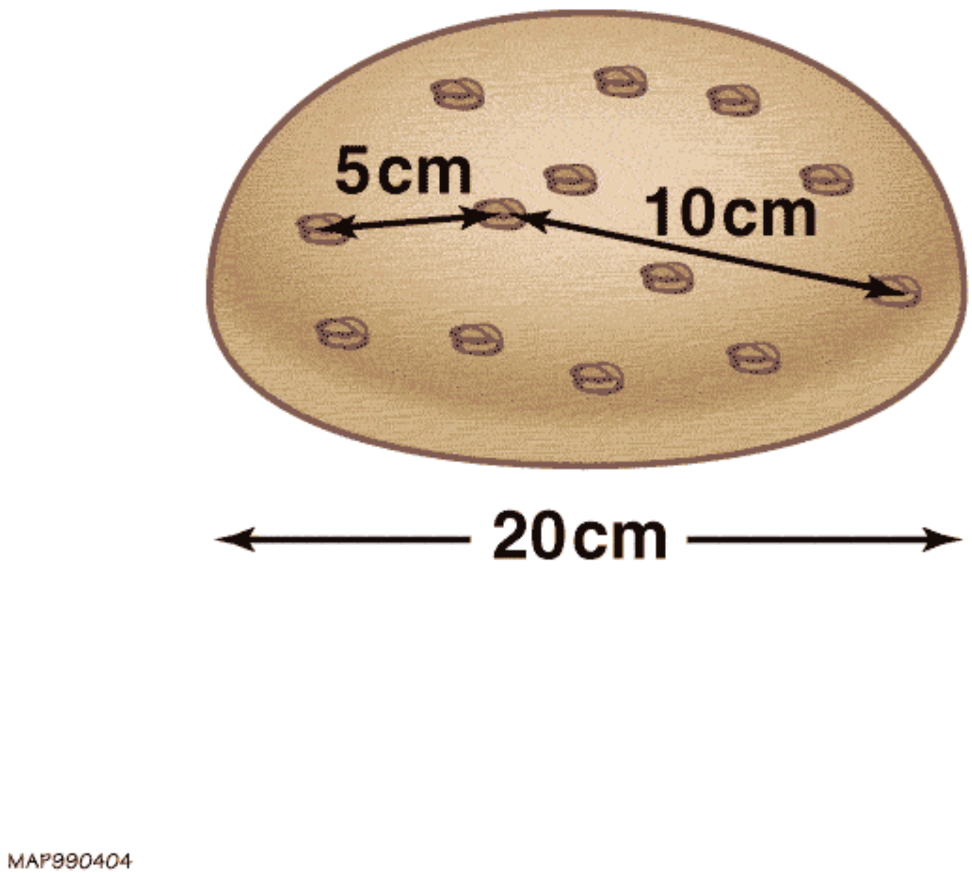
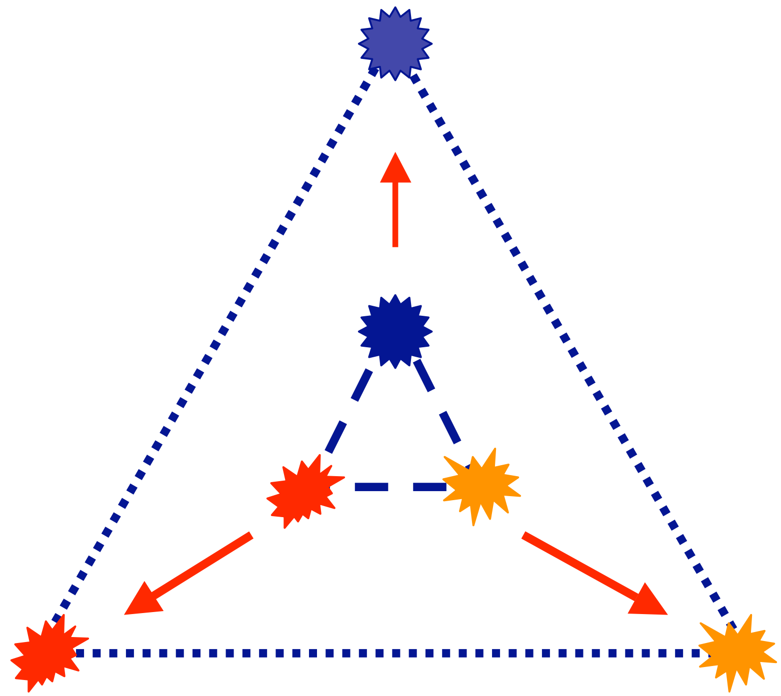


FIGURE 1



From WMAP

Expansion of the Universe



Universe remains homogeneous and isotropic during expansion

Any length scale $\ell(t) = R(t)\ell_0$

$$\dot{\ell} = \dot{R}\ell_0 = \frac{\dot{R}}{R}\ell(t)$$

Hubble's constant $H(t) = \frac{\dot{R}}{R}$

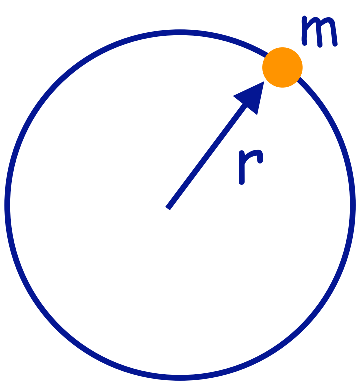
units $\frac{\mathrm{km/s}}{\mathrm{Megaparsec(Mpc)}}$

1 pc = 3.09×10^{13} km ≈ 3.26 light years

Observations $\frac{50\mathrm{km/s}}{\mathrm{Mpc}} \leq H \leq \frac{100\mathrm{km/s}}{\mathrm{Mpc}}$

$H^{-1} = 9.78\times10^{9}$ years

h = H/100 km s^{-1} Mpc^{-1}



Enclosed mass: $M = \frac{4\pi}{3}\rho_m r^3$

$$m\ddot{r} = -\frac{GmM}{r^2}$$

$$E = \frac{1}{2}m\dot{r}^2 - \frac{GmM}{r}$$

$$= \frac{1}{2}mr^2\left[\left(\frac{\dot{r}}{r}\right)^2 - \frac{8\pi G}{3}\rho_m\right]$$

H^2 → $\left(\frac{\dot{r}}{r}\right)^2$

E < 0 ⇒ the mass m is bound and cannot escape to infinity

E > 0 ⇒ expansion can continue

E = 0 ⇒ critical density, ρ_c

$$\rho_c = \frac{H^2}{G}\frac{3}{8\pi} = \left(\frac{H}{\frac{75\text{ km./s}}{\text{Mpc.}}}\right)^2 \times 1.05\times10^{-29}\text{g}/\text{cm}^3$$

Planck Mass: $$\frac{Gm_p^2}{\hbar c} = 1$$

$m_p = 2.17\times10^{-5}$ g $\qquad m_pc^2 = 1.2\times10^{19}$ GeV

Planck time $$\tau_p = \frac{\hbar}{m_pc^2} \sim 10^{-43} s$$

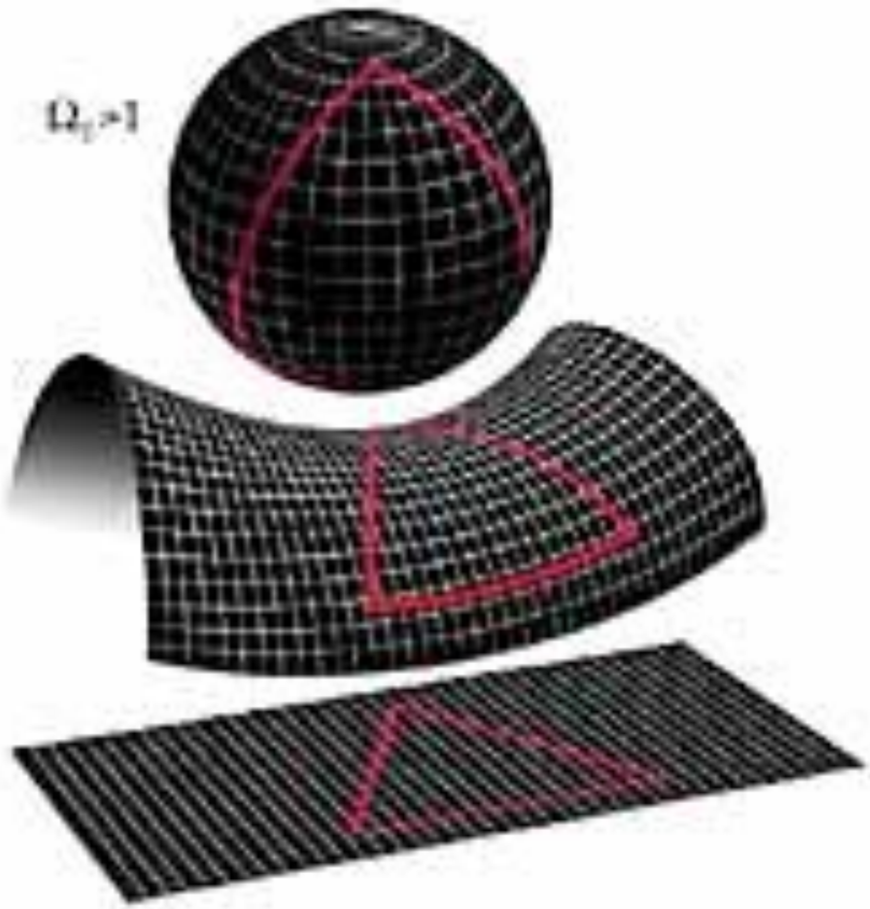
$$\rho_c = \left(\frac{3}{8\pi}\right) m_p^2 H^2$$

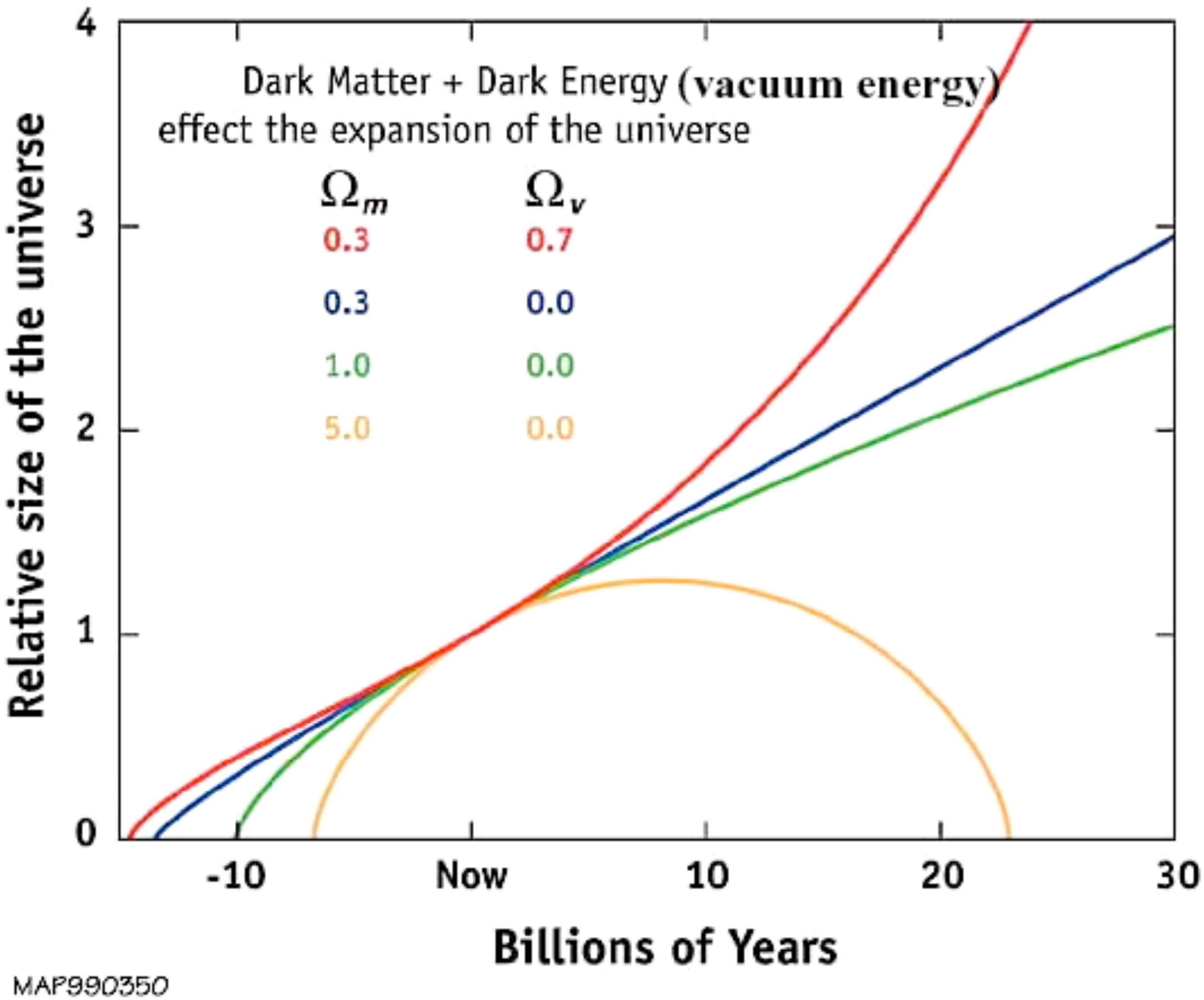
$\rho_c c^2 = 9.45\times10^{-9}$ ergs/cm^3

Hence if $\rho > \rho_c$ the universe will begin to contract

For a flat universe, k=0

$$\Omega = \rho/\rho_c = \Omega_\nu + \Omega_\gamma + \underbrace{\Omega_{baryon} + \Omega_{dark\text{-}matter}}_{\approx 0.3} + \underbrace{\Omega_{vacuum}}_{\approx 0.65} \approx 1$$




Dark Matter + Dark Energy (vacuum energy)
effect the expansion of the universe
Ω_m
Ω_v
0.3
0.7
0.3
0.0
1.0
0.0
5.0
0.0
Relative size of the universe
0
1
2
3
4
-10
Now
10
20
30
Billions of Years
MAP990350

From WMAP

Elementary Statistical Mechanics

Particle Number

$$N = g\frac{V}{(2\pi)^3}\int d^3\mathbf{p}\left[\frac{1}{\exp\left(E_{\mathbf{p}}-\mu\right)\pm 1}\right]$$

Energy

$$E = g\frac{V}{(2\pi)^3}\int d^3\mathbf{p}\left[\frac{E_{\mathbf{p}}}{\exp\left(E_{\mathbf{p}}-\mu\right)\pm 1}\right]$$

$E_p = (p^2 + m^2)^{1/2}$

μ: Chemical potential

g: Spin degrees of freedom

\+ fermions

\- bosons

Relativistic Limit (T>>μ, T>>m)

Energy density:

$$\rho = \frac{E}{V} = \begin{cases} g\left(\frac{\pi^2}{30}\right) T^4 \\ g\ \frac{7}{8} \times \left(\frac{\pi^2}{30}\right) T^4 \end{cases}$$

Number density:

$$n = \frac{N}{V} = \begin{cases} g\left(\frac{\zeta(3)}{\pi^2}\right) T^3 \\ g\ \frac{3}{4} \times \left(\frac{\zeta(3)}{\pi^2}\right) T^4 \end{cases}$$

Effective energy density:

$$g_s\left(\frac{\pi^2}{30}\right) T^4$$

$$g_s = \sum_{\text{bosons}} g_b + \frac{7}{8} \sum_{\text{fermions}} g_f$$

Cosmic microwave background photons (CMBR)

Temperature: $T=2.7^{o}K$, $g=2$, two polarization components

Number density: $N/V = (2/\pi^2)\ 1.202\ T^3 \sim 398$ photons/cm^3

Energy density: $E/V = 2\ (\pi^2/30)\ T^4 \sim 5 \times 10^{-13}$ ergs/cm^3

$\Omega_\gamma = (\rho_\gamma / \rho_c) \sim 10^{-4}$, does not close the universe

Recall that $H^2 \sim \rho$

Small R, Radiation-dominated universe (very early Universe) $H^2 \sim R^{-4}$ ⇒ $R \sim t^{1/2}$

Large R, Matter-dominated universe (later Universe) $H^2 \sim R^{-3}$ ⇒ $R \sim t^{2/3}$

Species will remain in thermal equilibrium as long as their interaction rate exceeds the expansion rate

$$n\langle \sigma E \rangle \gg H = \frac{\dot{R}}{R}$$

$$\rho_c = \frac{3m_p^2}{8\pi} H^2 \propto g_s \left(\frac{\pi^2}{30}\right) T^4$$

$H = 1.66\,(g_s)^{1/2}\, T^2/m_p$

$\sigma_\nu \sim G_F^2\, E^2 \sim G_F^2\, T^2$

$n_\nu \sim T^3$

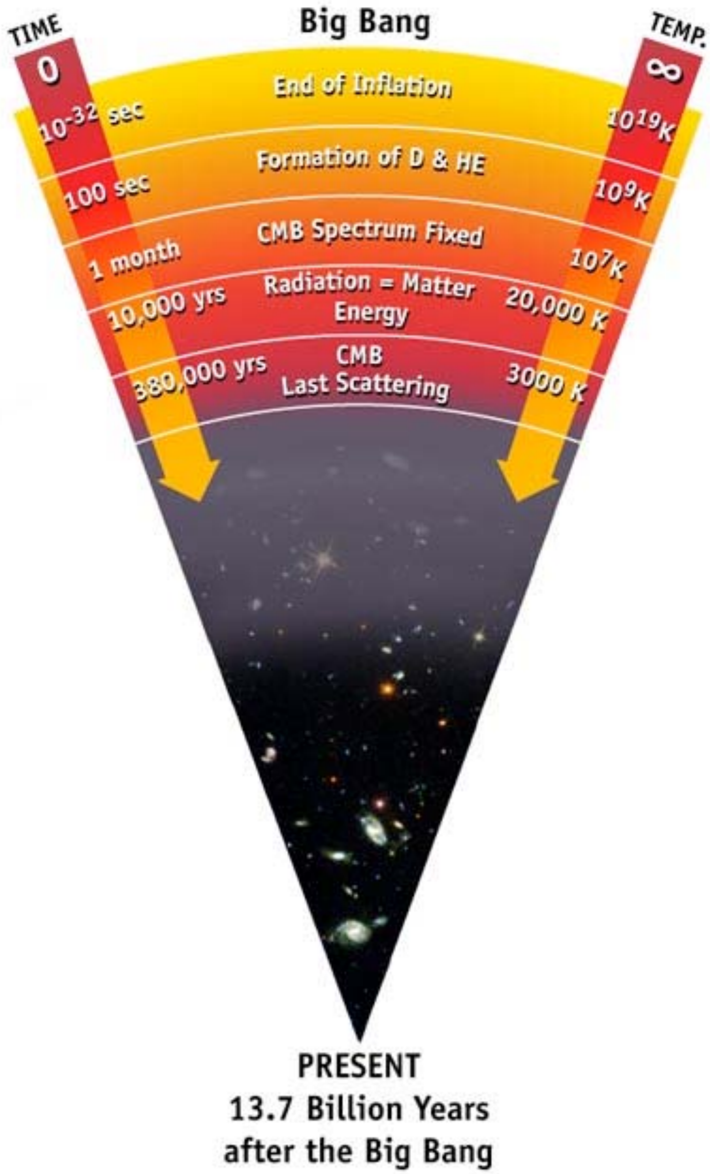
$\Gamma \sim G_F^2\, T^5$

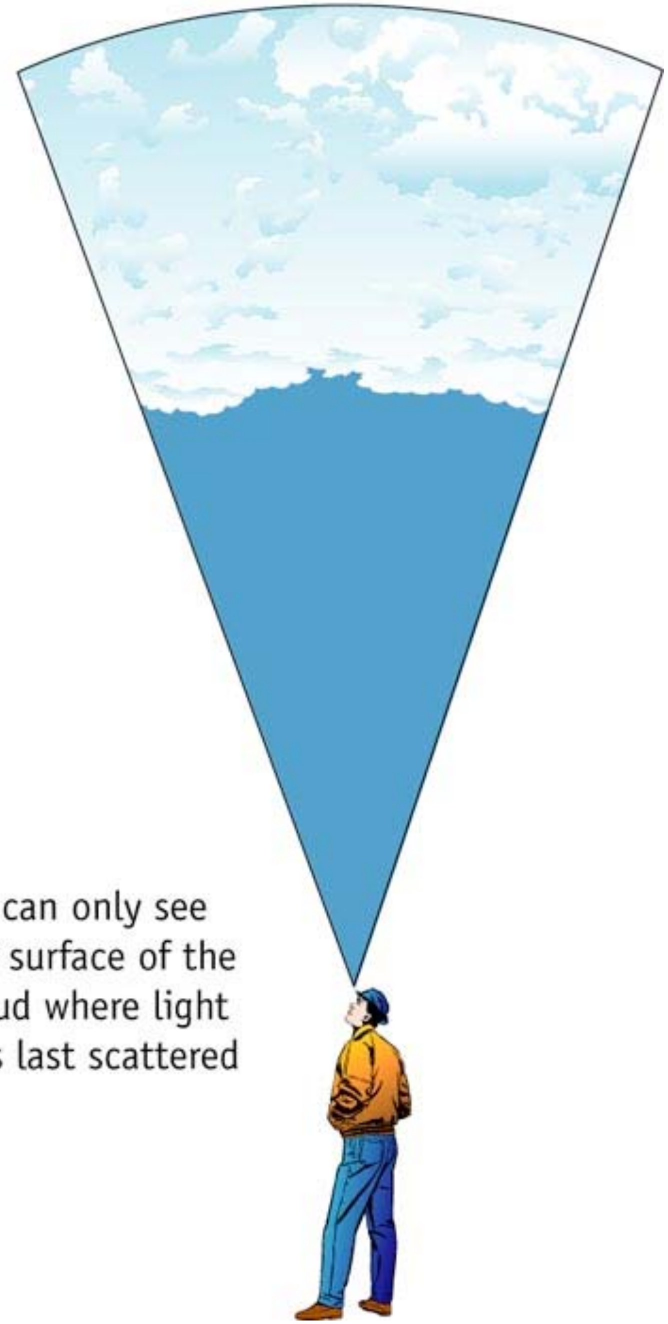
Freeze-out when $\Gamma = H$

$$T^3_{\text{freeze-out}} = \frac{1.66\,(g_s)^{1/2}}{m_p\, G_F^2}$$

Thermal history of the Universe

kT less than	Particles in equilibrium	g_s
1 eV	γ	2
$m_e c^2$	γ, $e^\pm$	11/2
$m_\mu c^2$	$\gamma, \nu_e, \nu_\mu, \nu_\tau$, $e^\pm$	43/4
$m_\pi c^2$	$\gamma, \nu_e, \nu_\mu, \nu_\tau$, $e^\pm$, $\mu^\pm$	57/4
Λ_{QCD}	$\gamma, \nu_e, \nu_\mu, \nu_\tau$, $e^\pm$, $\mu^\pm$, $\pi^\pm$, π^0	69/4
$m_s c^2$	$\gamma, \nu_e, \nu_\mu, \nu_\tau$, $e^\pm$, $\mu^\pm$, $u, \bar{u}, d, \bar{d}, g$	205/4





We can only see the surface of the cloud where light was last scattered

The cosmic microwave background Radiation's "surface of last scatter" is analogous to the light coming through the clouds to our eye on a cloudy day.

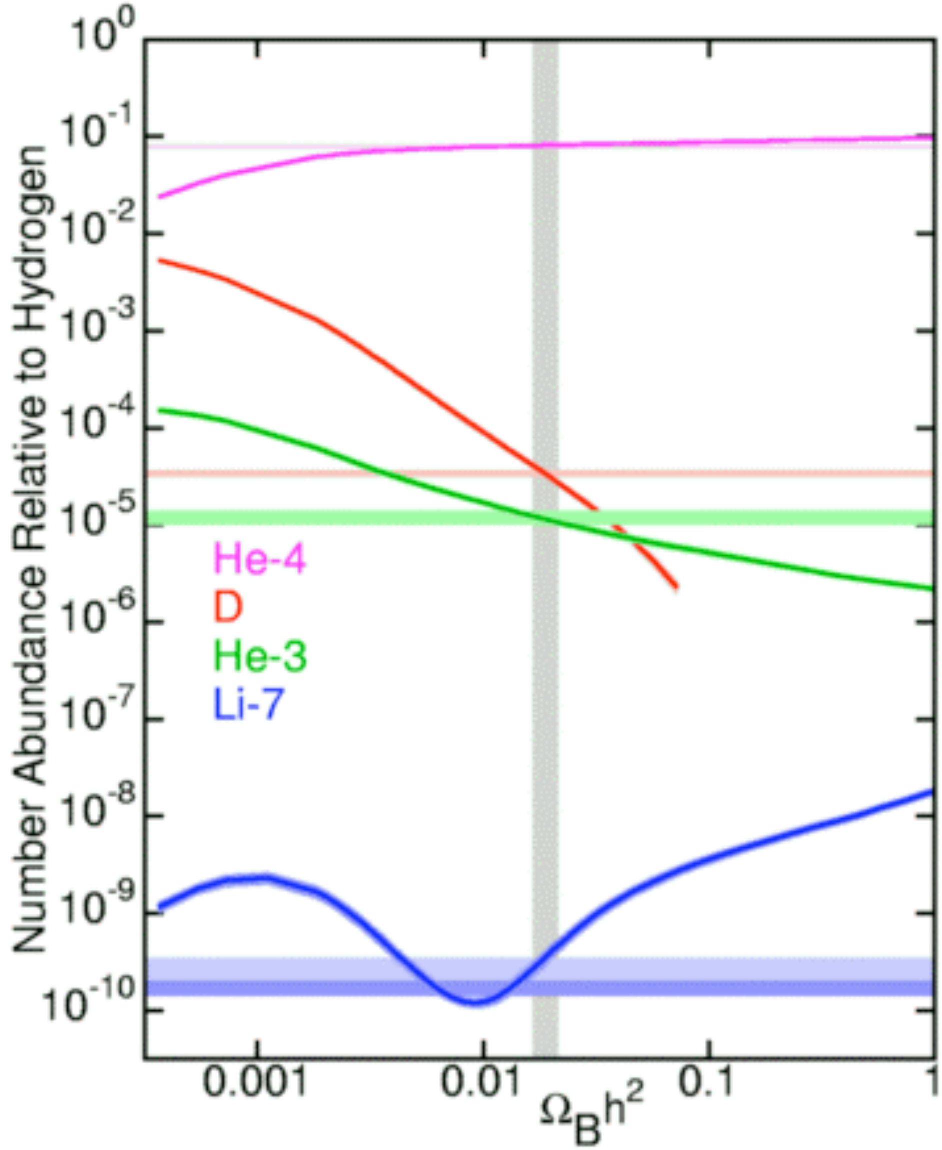
^{4}He equilibrium abundance

$$Y_\alpha = \frac{2\,(N_n/N_p)}{N_n + N_p}$$

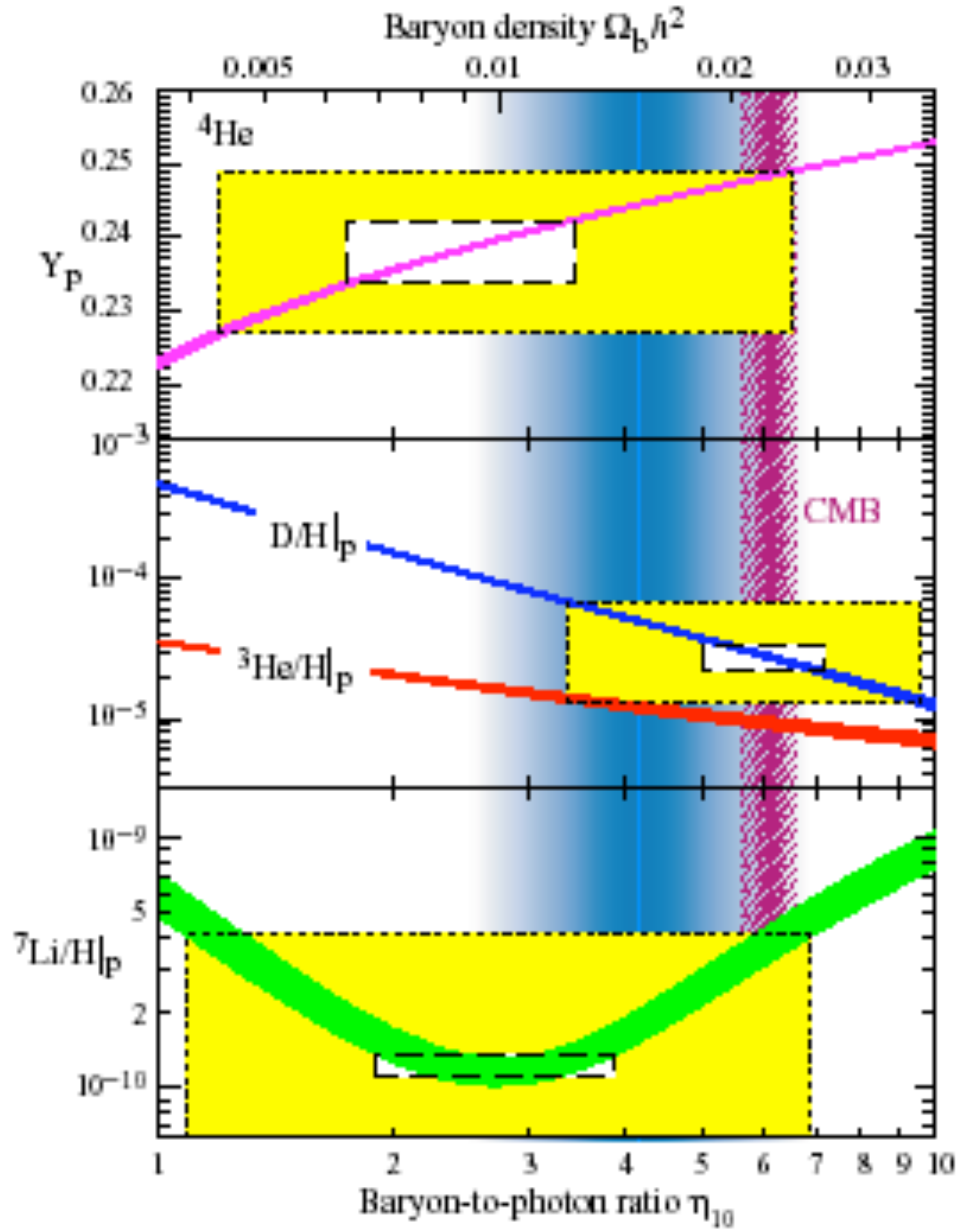
$$(N_n/N_p) = \exp(-\Delta m/T)$$

neutron-proton mass difference

freeze-out temperature $T \sim 0.8$ MeV



Number Abundance Relative to Hydrogen
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
He-4
D
He-3
Li-7
0.001
0.01
0.1
1
$\Omega_B h^2$



Baryon density $\Omega_b h^2$
0.005
0.01
0.02
0.03
0.26
0.25
0.24
0.23
0.22
^{4}He
Y_p
10^{-3}
10^{-4}
10^{-5}
CMB
$D/H|_p$
$^3He/H|_p$
10^{-9}
5
2
10^{-10}
$^7Li/H|_p$
1
2
3
4
5
6
7
8
9
10
Baryon-to-photon ratio η_{10}

From PDG

Recall that

$$T_{freeze\text{-}out} = \frac{1.66\,(g_s)^{1/2}}{m_p\, G_F^2}$$

Y_α depends on $T_{freeze\text{-}out}$ which in turn depends on g_s

Neutrino counting !

$$\delta Y_\alpha \propto \delta g_s \propto \delta N_\nu$$

Neutrino decoupling

$dS = dE/T \Rightarrow$ entropy per unit volume $\sim g_s\, T^3$

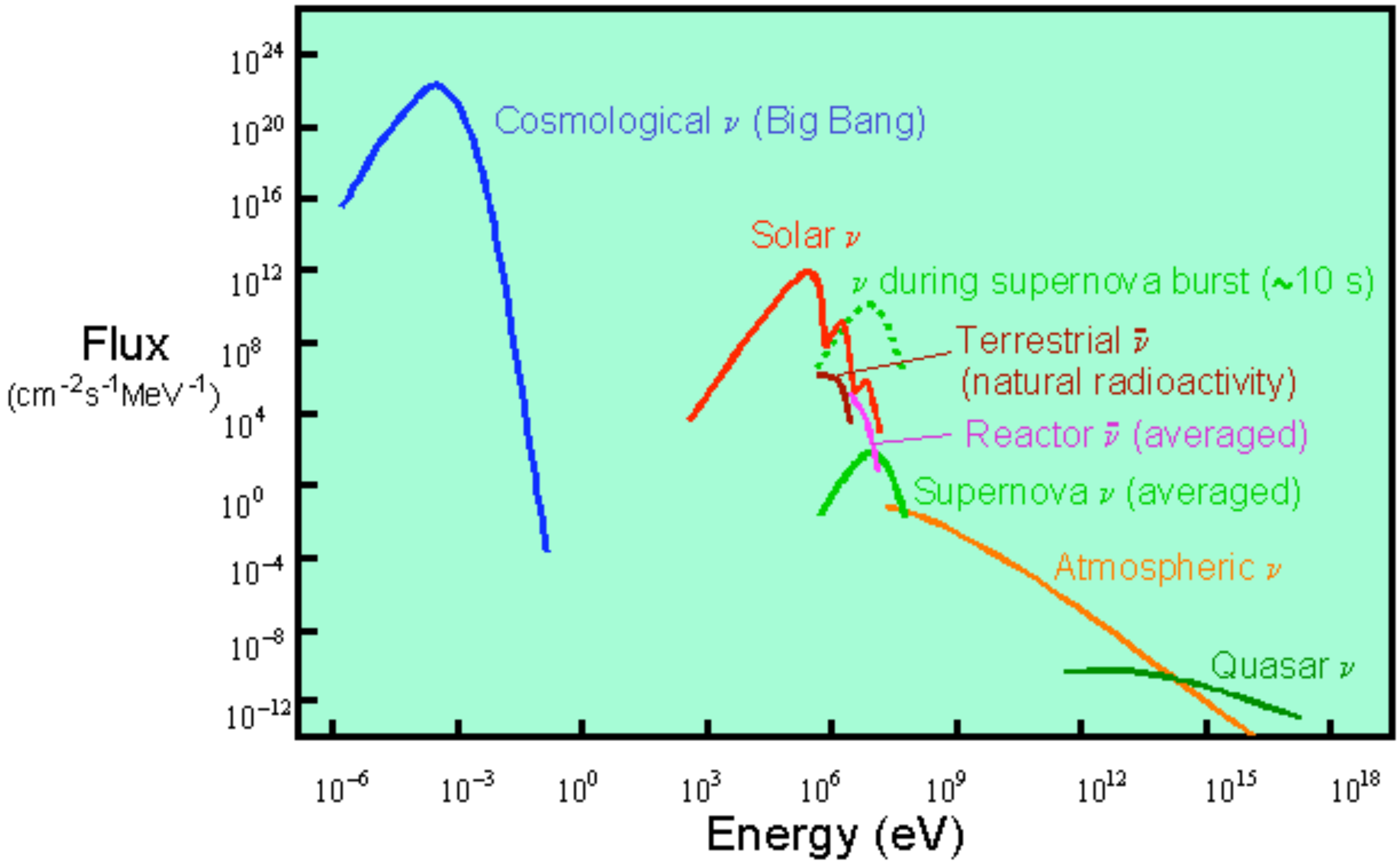
Before e^+e^- annihilation: $g_s^{(b)} = 2 + (7/8) \times 4 = 11/2$

After e^+e^- annihilation: $g_s^{(a)} = 2$

Entropy conservation: $g_{before}\, T^3_{before} = g_{after}\, T^3_{after}$

$$T_{after} = (11/4)^{1/3}\, T_{before}$$

$T_\gamma = 2.7\ ^oK$ → $T_\nu = 1.9\ ^oK$

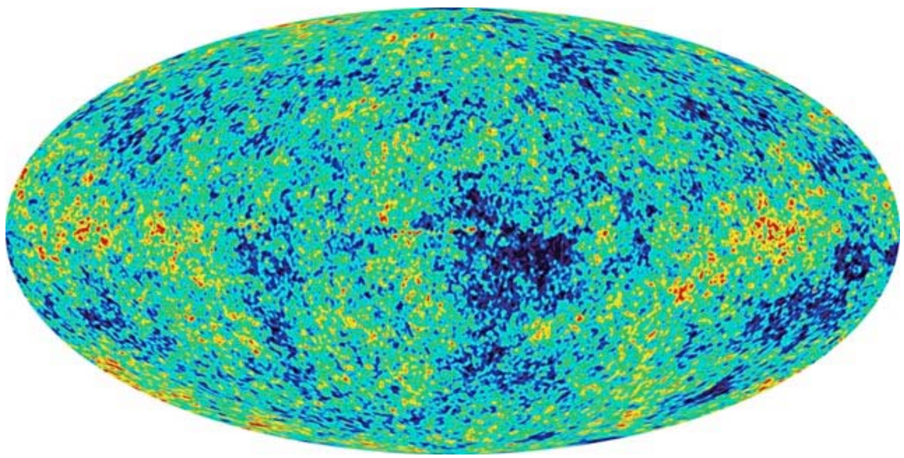


Flux on earth of neutrinos from various sources, in function of energy

Probing the Cosmic Microwave Background Radiation (CMBR)

CMBR is anisotropic, its temperature varies slightly at different locations

$$\Delta T(\Omega) = \sum_{\ell>0} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\Omega)$$

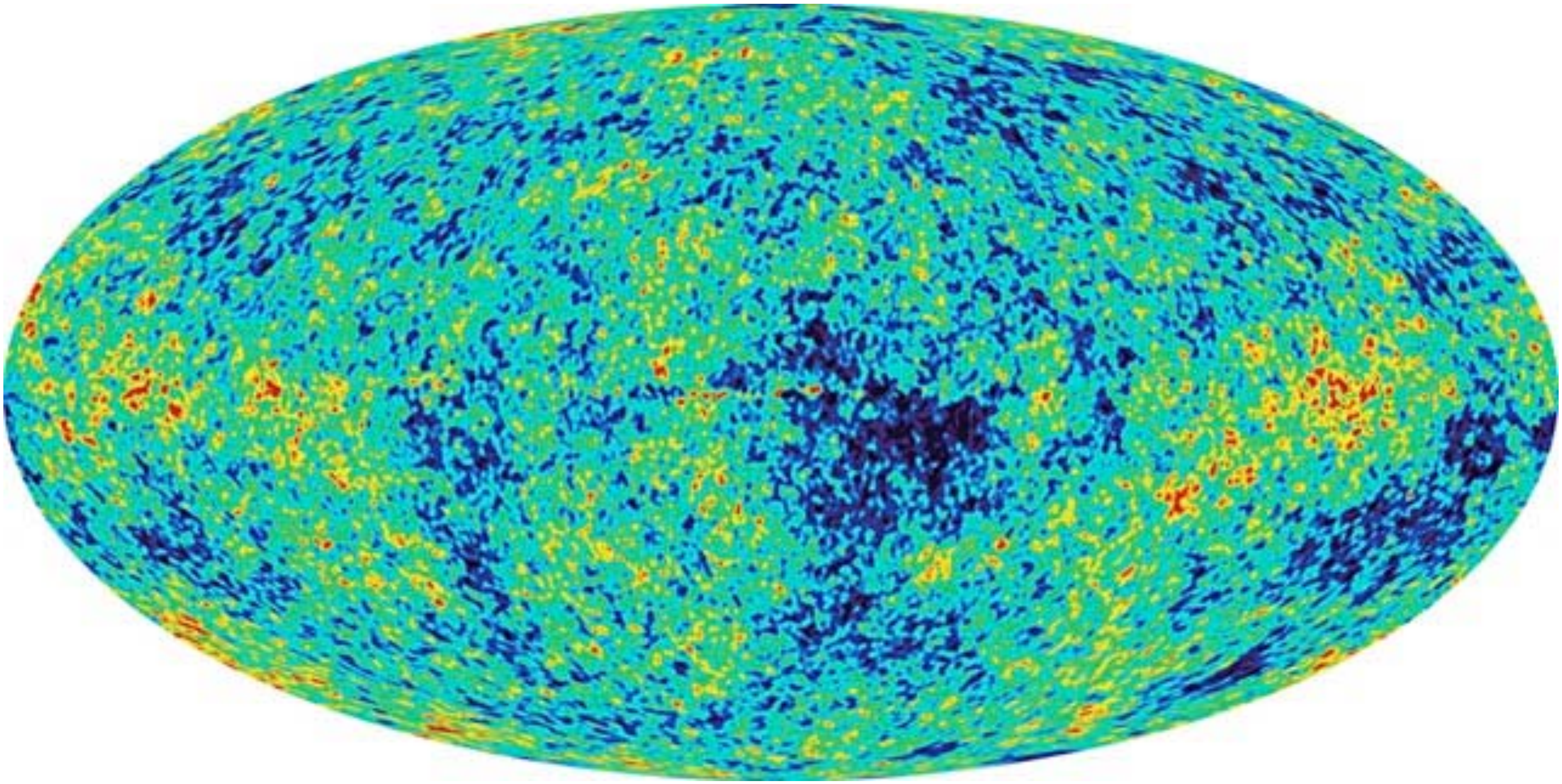


$$\langle a_{\ell m} \rangle = 0$$

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell$$

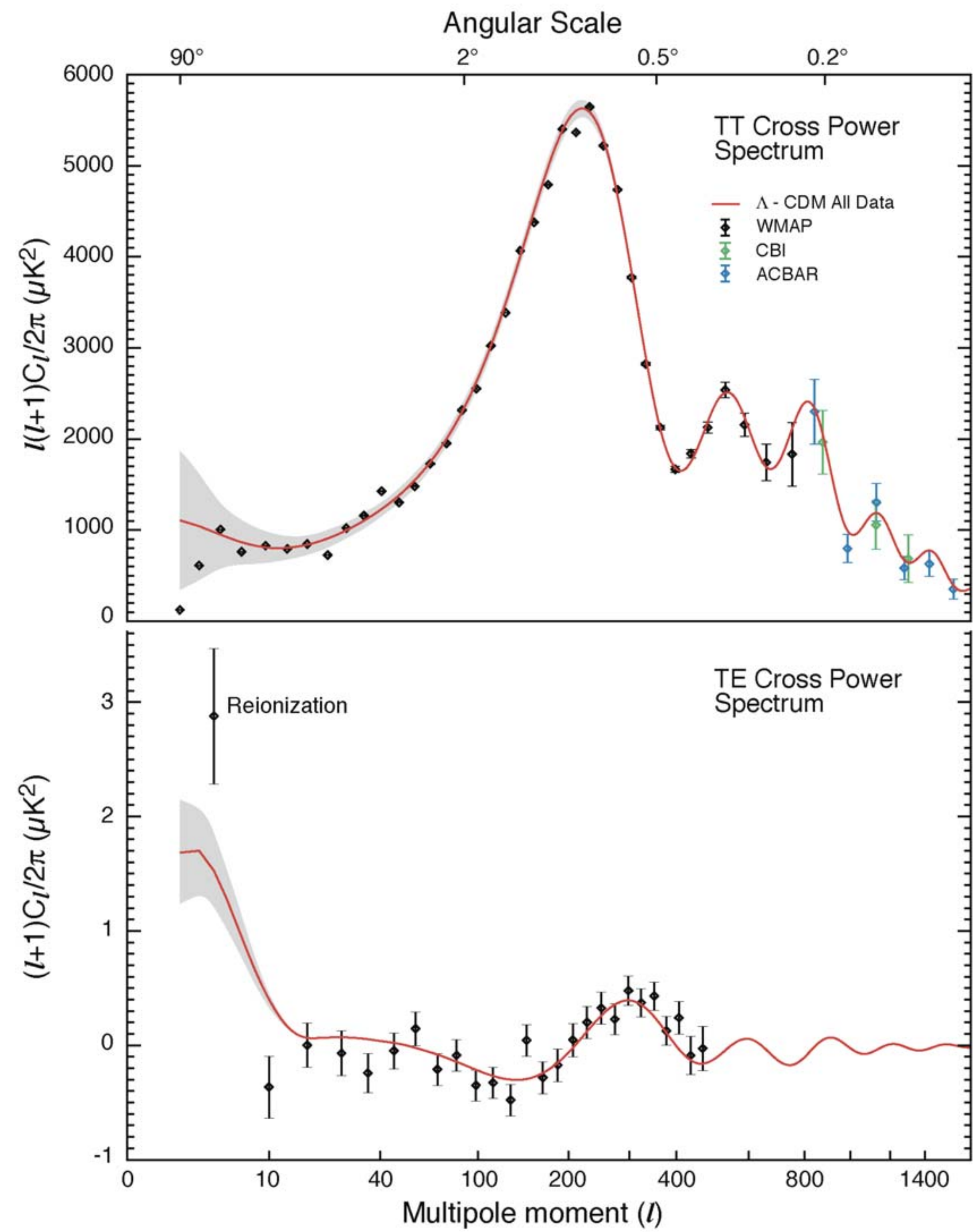
$$C_\ell^{\rm sky} = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2$$

WMAP results:



Dark Blue = -200 μK   Red = +200 μK

WMAP results



In the matter-dominated epoch massive neutrinos cluster on very large scales, but free-stream out of smaller scale fluctuations. This suppresses the amplitude of the fluctuations.

$$\Omega_\nu \sim \left\{\frac{m_\nu}{92\ \text{eV}}\right\}\left\{\frac{T\gamma}{2.75\ \text{K}}\right\}^3$$

Combination of several experiments:

$\Omega_\nu h^2 < 0.0072$ (95% C.L.)

Cosmological Implications

Atmospheric neutrinos: $\Delta m_{23}^2 \approx 2.0 \times 10^{-3}$ eV^2

$\therefore$ One neutrino mass > 0.04 eV

SNO + KamLAND: $\Delta m_{12}^2 \approx 7.1 \times 10^{-5}$ eV^2

$\therefore$ One neutrino mass > 0.008 eV

Limits on "ν_e mass" give: $m(\nu_{1,2,3}) <$ 2.2 eV

Σ neutrino masses: $0.048 < m_1+m_2+m_3 < 6.6$ eV

$$0.001 < \Omega_\nu < 0.13$$

Stay tuned…

Summary of Methods to Obtain Neutrino Masses

Single beta decay	$\Sigma_\iota\, m_i^2\, \lvert U_{ei}\rvert^2$	Sensitivity 0.2 eV
Double beta decay	$m_{\beta\beta} = \lvert\Sigma_\iota\, m_i\, \lvert U_{ei}\rvert^2\, \varepsilon_i\rvert$ ε_i = Majorana phases	Sensitivity 0.01 eV
Neutrino oscillations	$\delta m^2 = m_1^2 - m_2^2$	Observed $\sim 10^{-5}$ eV^2
Cosmology	$\Omega_\nu \rightarrow \Sigma_\iota\, m_i$	Observed ~ 0.1 eV

Only double beta decay is sensitive to Majorana nature.

Where do we stand?

A perspective

Fundamental discoveries are recently made

- SNO, 2002: Discovery of the non-electron neutrino component of the solar flux ($\Rightarrow$ neutrino oscillations); measurement of the total solar neutrino flux.
- SuperK, 1998:Discovery of atmospheric neutrino flux variations ($\Rightarrow$ neutrino oscillations).
- Baksan, Kamioka, IMB, 1987: Detection of neutrinos from Supernova 1987A (neutrino flux consistent with neutron star binding energy, cooling time is near that expected).
- Irvine, 1987: Detection of two-neutrino double-beta decay.
- MSW, 1986: Recognition that matter enhances neutrino oscillations.

..that broadly impact physics, astronomy, and cosmology

- Massive neutrinos: Beyond the Standard Model of elementary particles.
- Neutrino mixing angles are close to maximal: Impacts on leptogenesis; explosion mechanism and nucleosynthesis in core-collapse supernovae.
- Total solar neutrino flux is measured: The theory of main sequence stellar evolution is verified.
- Direct neutrino mass measurements:

  Neutrino component of dark matter.

  WMAP put significant limits on Ω_ν

Backup slides

Applications of ν-nucleus interactions

Theory and applications of Detector Response: Detectors for solar, atmospheric, accelerator, and reactor neutrinos.

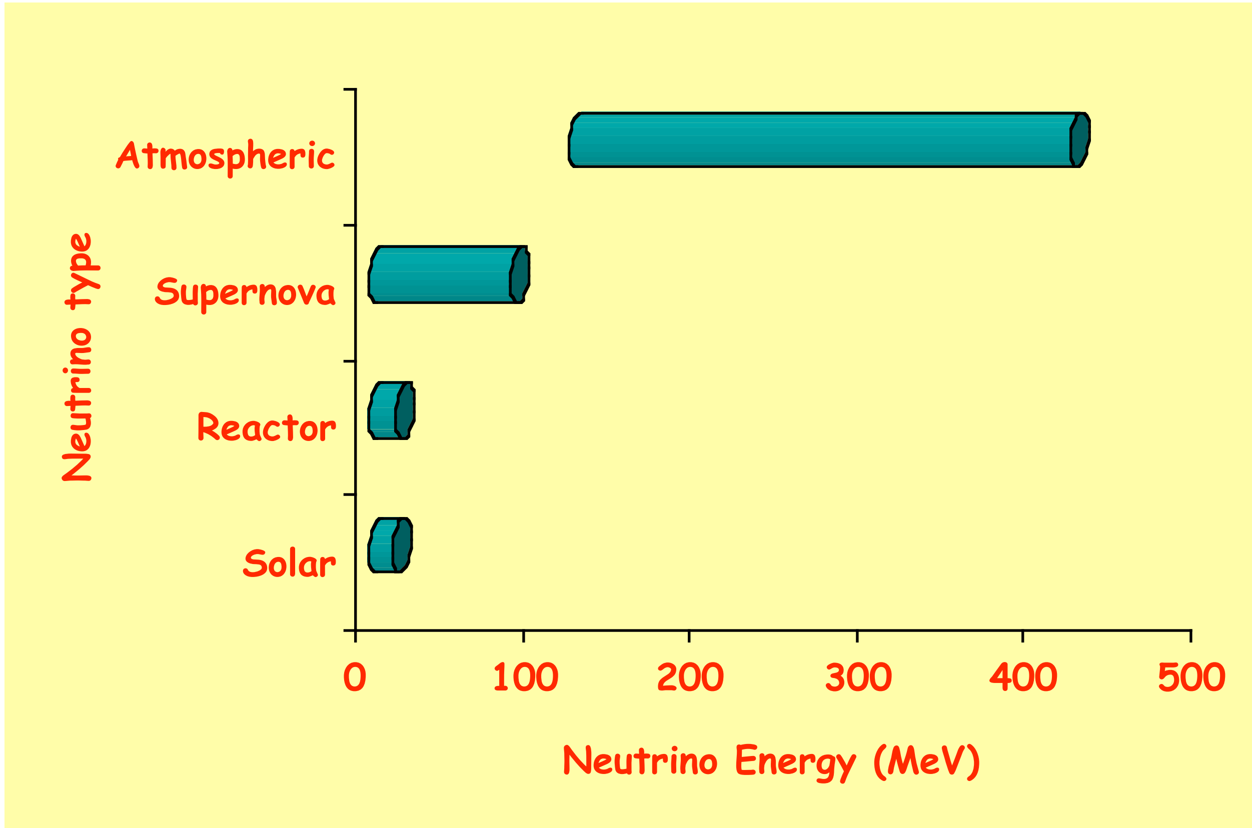
Input into astrophysics: Neutrino reactions in core-collapse supernovae, supernova nucleosynthesis, gamma-ray burst nucleosynthesis.

Tests of nuclear structure calculations: Shell Model, effective field theories.

Fundamental physics at low energies: Determining proton strange form factors.

Neutrino-nucleus cross-sections

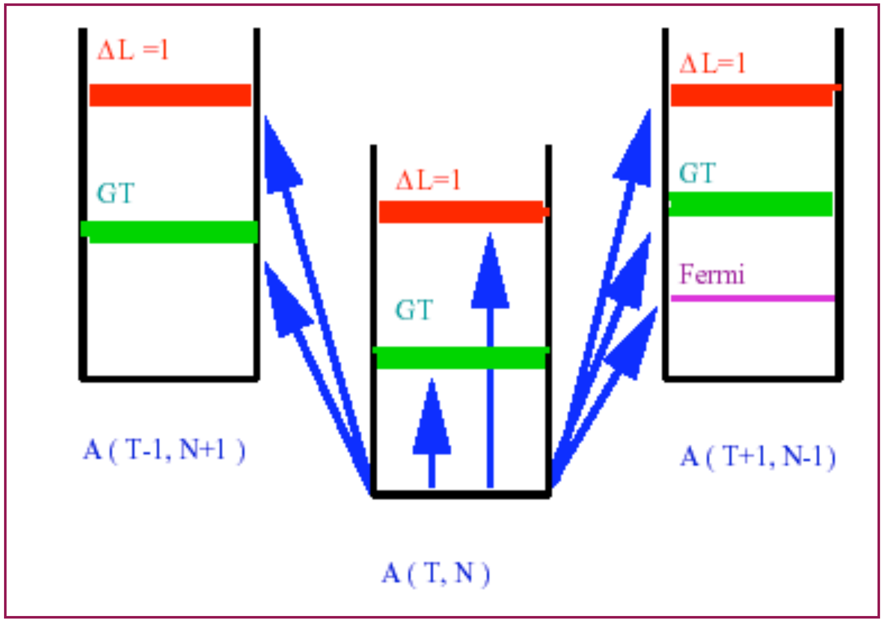
Low Energy (0 < E <~200 MeV)	**Non-relativistic many-body theories (Shell Model, RPA); effective field theory**	Solar, reactor, and supernova neutrinos; beam-stops
Intermediate E (~200 MeV < E < ~200 GeV)	**Relativistic; hadronic ⇒ partonic d.o.f, superscaling ideas; quasielastic and resonance regime**	Atmospheric neutrinos MINERVA, SciBooNE, etc.
High Energy (~200 GeV < E < EeV)	**Deep-inelastic scattering, partonic d.o.f., x-scaling**	Atmospheric ν's, neutrinos from point sources Icecube, Antares



We need to know the response of the nucleus to neutrinos with a wide range of energies.

What happens when a 50 MeV neutrino hits a nucleus? Where is the strength? What is g_A/g_V?

At the lowest energies Shell Model is the best approach. Gamow-Teller strength is quenched in the Shell Model:



Nucleus	^{128}Sn	^{130}Sn	^{132}Sb	^{132}Te	^{133}Te
Transition	$0^+ \to 1^+$	$0^+ \to 1^+$	$4^+ \to 3,4,5^+$	$0^+ \to 1^+$	$\frac{3}{2}^+ \to \frac{1}{2},\frac{3}{2},\frac{5}{2}^+$
$T_{1/2}$exp.	59.07m	3.72m	2.79m	3.2d	12.5m
$T_{1/2}$calc. (0.74)	32.21m	2.47m	1.56m	1.73d	6.42m
Renorm.	0.54	0.6	0.55	0.54	0.53

Beta-decay rates from Nowacki

As the incoming neutrino energy increases, the contribution of the states which are not well-known increase, including first- and even second-forbidden transitions.

At higher energies where the rate is sensitive to total strength and the energy of giant resonances there is a tendency to use RPA.

However…

Using Shell Model to calculate neutrino-nucleus cross-sections one needs to use effective operators when one employs effective Hamiltonians. However in most calculations the systematics of BE(2) transitions are characterized by effective charges and Gamow-Teller distributions by quenching. Different calculations disagree by as much as 30% from from one another. To understand neutrino-nucleus collisions at energies up to 60 MeV we need 1) A more consistent treatment of the Shell Model in a bigger basis spaces; and 2) More data. We should also be careful with the QRPA calculations.

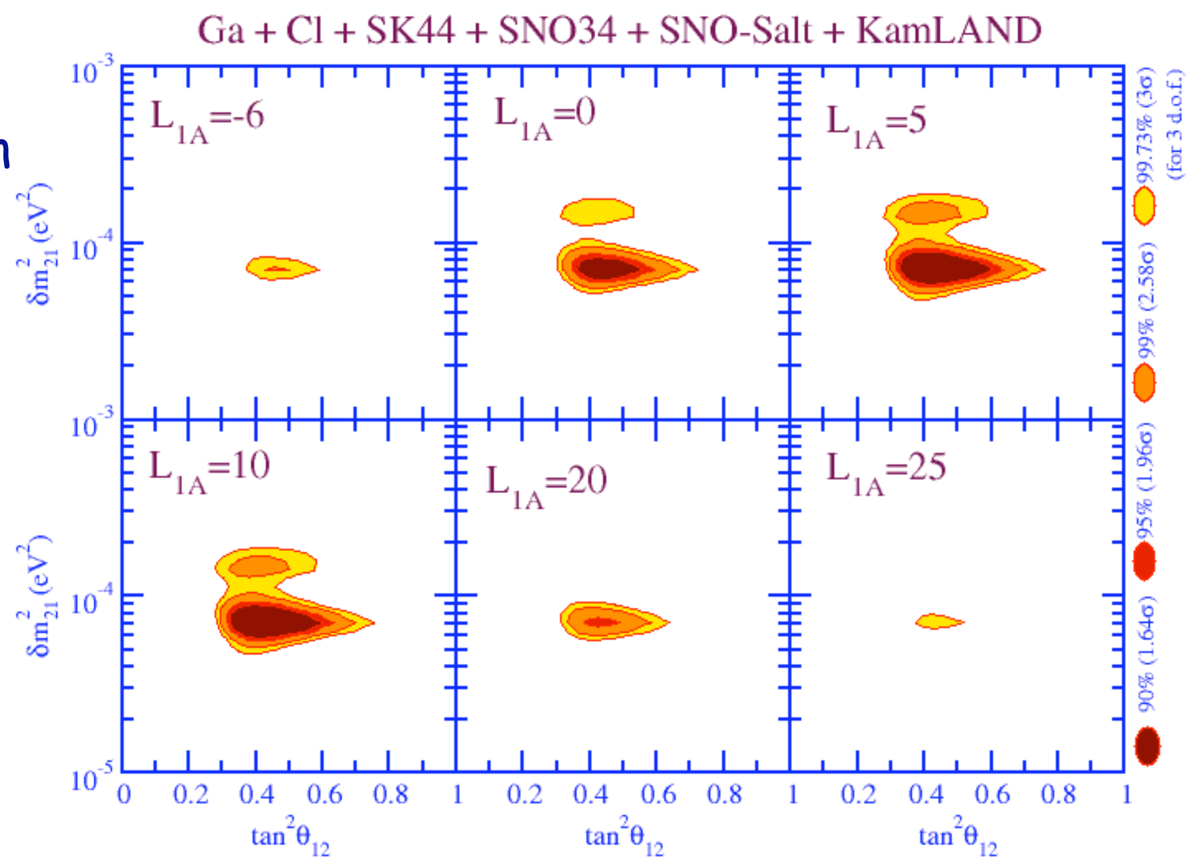
We should also question the effective interactions utilized. For example, Otsuka, et al. find that inclusion of a tensor force in the Shell Model may increase the Gamow-Teller strength distribution at higher energies.

This is bread and butter nuclear structure physics!

Effective Field theory Approach to low-energy neutrino-deuteron scattering Butler, Chen

Below the pion threshold ${}^3S_1 \rightarrow {}^3S_0$ transition dominates and one only needs the coefficient of the two-body counter term, L_{1A} (isovector two-body axial current)

L_{1A} can be obtained by comparing the cross section $\sigma(E) = \sigma_0(E) + L_{1A}\, \sigma_1(E)$ with cross-section calculated using other approaches or measured experimentally



A.B. Balantekin and H. Yuksel

Not easy to study three-body systems

Note that an observation of 0νββ double beta decay does not necessarily imply the existence of a light Majorana neutrino!

$A_L \sim m_{\beta\beta}$

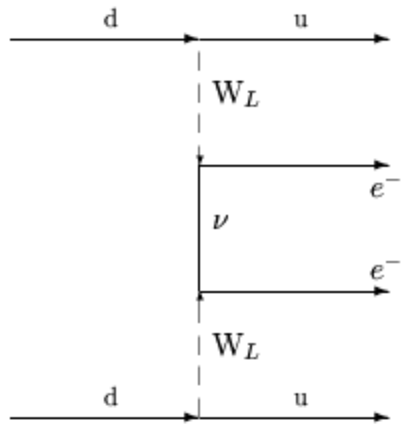
$A_H \sim M_W^4 / \Lambda^5$

for $\Lambda \sim 1$ TeV both mechanisms may contribute equally.

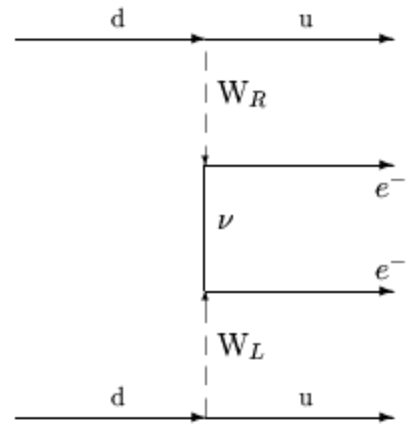
Lepton Flavor violation involving charged leptons may provide a "diagnostic tool" for establishing the mechanism of 0νββ decay

Ramsey-Musolf & Vogel

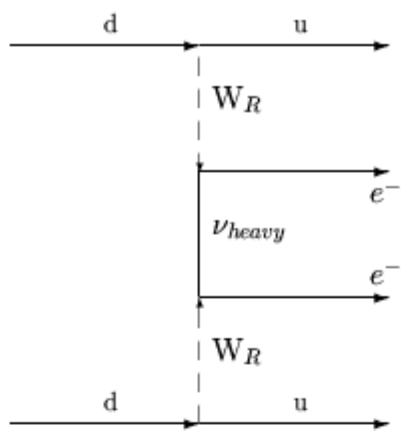
Exchange of a **light** neutrino, only left-handed currents



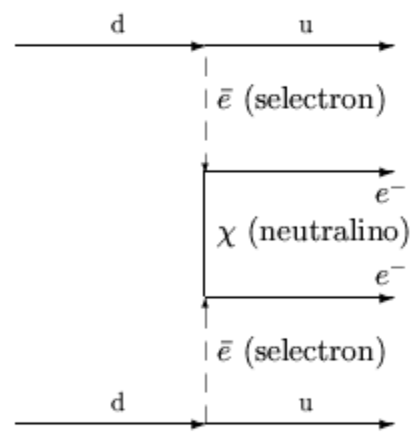
Exchange of a light or heavy neutrino and one **right-handed W_R**



Exchange of a **heavy** neutrino, short range hadron physics at play

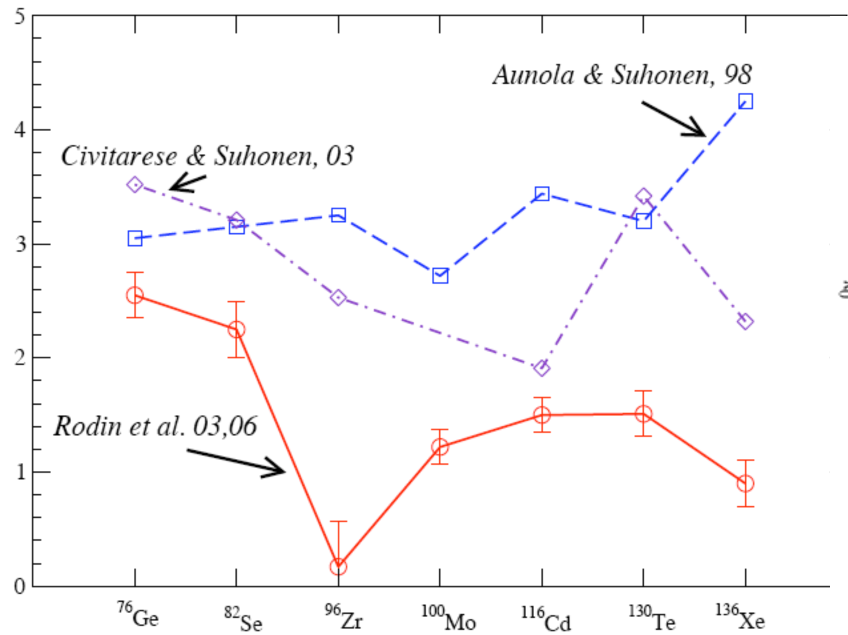


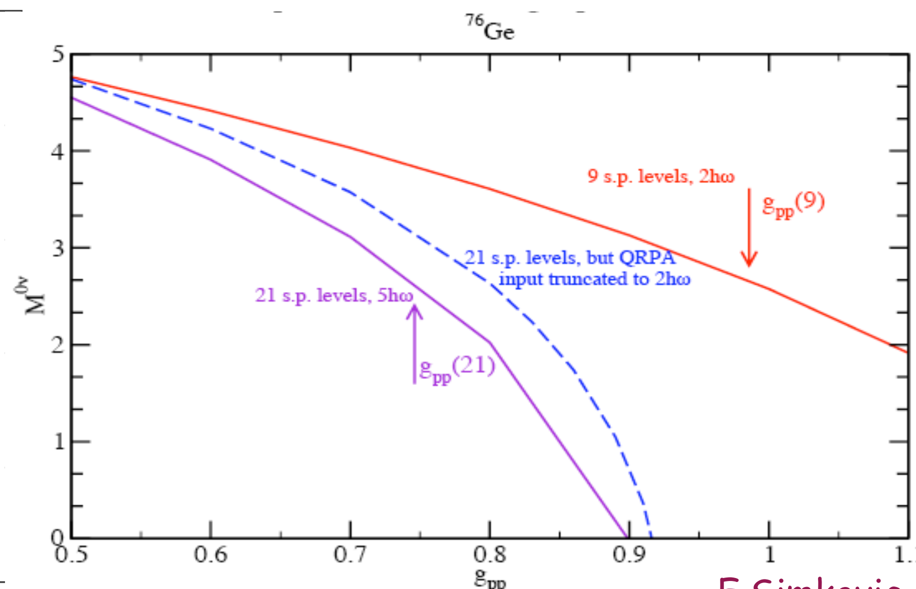
Exchange of **supersymmetric** particles, R-symmetry violated



There is a big spread in the matrix elements calculated using QRPA

In QRPA the dependence of the 0νβ β matrix element on s.p. states is reduced if the coupling strength g_{pp} is adjusted to the 2νββ rate:





F.Simkovic

Most of this spread can be traced to different choices of the initial and final states, choice of the value of the parameter g_{pp}, and corrections for the short-range nuclear repulsion.