



Neutrino physics and astrophysics:  
Nuclear Physics 2008 Summer School  
Lectures

A.B. Balantekin, University of Wisconsin



Right now we are in the middle of twin revolutions in neutrino physics and astrophysics/cosmology ...



Wolfgang Pauli,  
father of neutrino  
and Pauli exclusion  
principle

# Physicist goes to a ball

or

## Mystery of Missing Energy

*Original: Photograph of 1930 0393*  
Abschrift/15.12.96 PW

Offener Brief an die Gruppe der Radioaktiven bei der  
Gauvereins-Tagung zu Tübingen.

Abschrift

Physikalisches Institut  
der Eidg. Technischen Hochschule  
Zürich

Zürich, 4. Dez. 1930  
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Überbringer dieser Zeilen, den ich halbvollst  
anzuhören bitte, Ihnen das Näherem auseinandersetzen wird, bin ich  
angesichts der "falschen" Statistik der  $N$ - und  $Li-6$  Kerne, sowie  
des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg  
verfallen um den "Wechselatz" (1) der Statistik und den Energiesatz  
zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale  
Teilchen, die ich Neutronen nennen will, in den Kernen existieren,  
welche den Spin  $1/2$  haben und das Ausschliessungsprinzip befolgen und  
sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie  
sich mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen  
könnte von derselben Grössenordnung wie die Elektronenmasse sein und  
jedemfalls nicht grösser als  $0,01$  Protonenmasse. Das kontinuierliche  
beta-Spektrum wäre dann verständlich unter der Annahme, dass beim  
beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert  
wird, derart, dass die Summe der Energien von Neutron und Elektron  
konstant ist.

Cosmic Gall  
John Updike

Neutrinos they are very small.  
They have no charge and have no mass  
And do not interact at all.  
The earth is just a silly ball  
To them, through which they simply pass,  
Like dustmaids down a drafty hall  
Or photons through a sheet of glass.  
They snub the most exquisite gas,  
Ignore the most substantial wall,  
Cold-shoulder steel and sounding brass,  
Insult the stallion in his stall,  
And, scorning barriers of class,  
Infiltrate you and me! Like tall  
And painless guillotines, they fall  
Down through our heads into the grass.  
At night, they enter at Nepal  
And pierce the lover and his lass  
From underneath the bed – you call  
It wonderful; I call it crass.

Neutrino mass eigenstates are a combination of weak-interaction eigenstates: neutrinos mix!

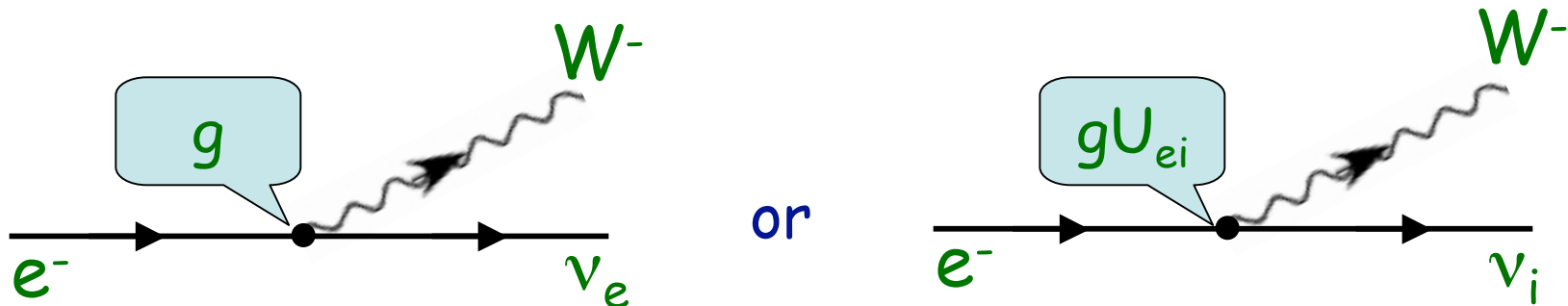
$$\nu_{\alpha} = \sum_i U_{\alpha i} \nu_i$$

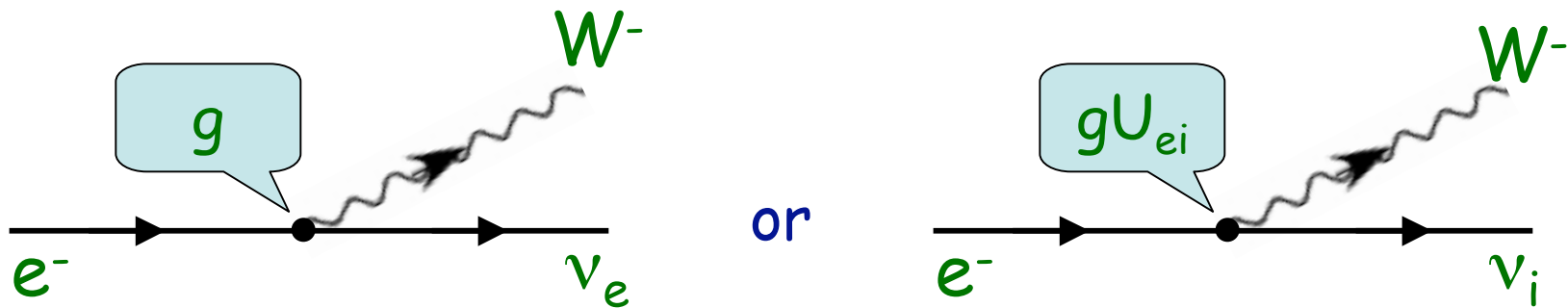
$$\alpha = e, \mu, \tau$$

$$i = 1, 2, 3, \dots$$

$$U \text{ is unitary: } U^{\dagger}U = UU^{\dagger} = 1$$

If the neutrino mass were zero this would be nothing more than a change of basis in the Standard Model:





$$L_{SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left( \bar{\ell}_{L\alpha} \gamma^\lambda \nu_{L\alpha} W_\lambda^- + \bar{\nu}_{L\alpha} \gamma^\lambda \ell_{L\alpha} W_\lambda^+ \right)$$

$$= -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left( \bar{\ell}_{L\alpha} \gamma^\lambda U_{\alpha i} \nu_{Li} W_\lambda^- + \bar{\nu}_{Li} \gamma^\lambda U_{\alpha i}^* \ell_{L\alpha} W_\lambda^+ \right)$$

Left-handed

$$\begin{array}{ccc}
 \text{Atmospheric} & \text{Reactor, long-baseline} & \text{solar} \\
 U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 & & \times \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 \begin{array}{l} c_{ij} \equiv \cos \theta_{ij} \\ s_{ij} \equiv \sin \theta_{ij} \end{array} & & \text{Majorana phases}
 \end{array}$$

When  $\nu_i \rightarrow e^{i\phi} \nu_i$  then  $U_{\alpha i} \rightarrow e^{i\phi} U_{\alpha i}$

When  $\ell_\alpha \rightarrow e^{i\phi} \ell_\alpha$  then  $U_{\alpha i} \rightarrow e^{-i\phi} U_{\alpha i}$

One can multiply any one row or any one column by the same phase by redefining the phases of the fields!

Except when the neutrino mass eigenstates are their own antiparticles ("Majorana neutrinos"):

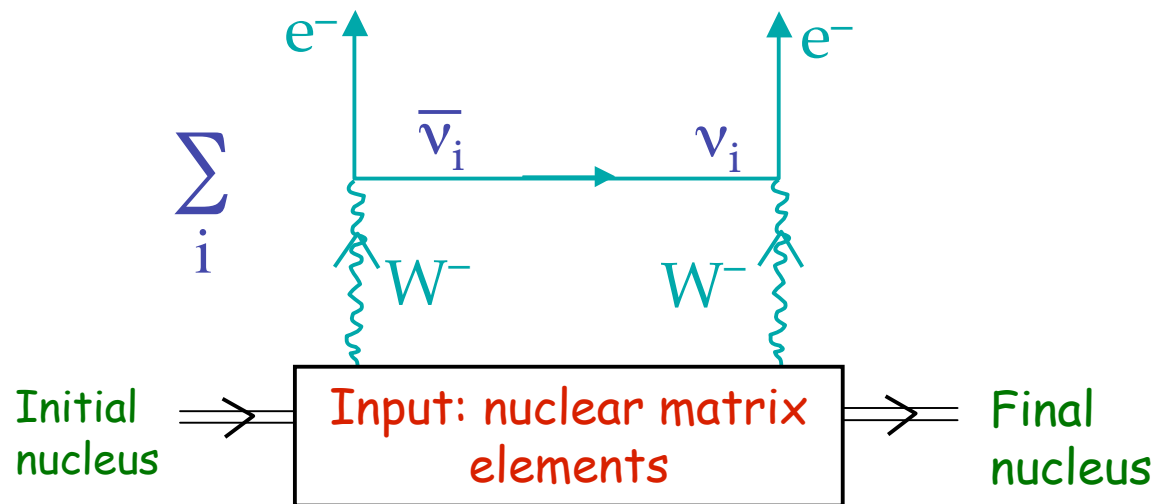
$$\nu_i = \nu_i^c = C \bar{\nu}_i^T$$

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Only for Majorana neutrinos



Majorana nature of the neutrinos permit  
neutrinoless double beta decay:



- Symmetries, in particular weak isospin invariance, define the Standard Model.
- In the Standard Model, the left-handed and the right-handed components of the neutrino are treated differently:  $\nu_L$  sits in an weak-isospin doublet ( $I_W=1/2$ ) together with the left-handed component of the associated charged lepton, whereas  $\nu_R$  is an weak-isospin singlet ( $I_W=0$ ).
- A mass term connects left- and right-handed components. The usual Dirac mass term is  $L = m\bar{\psi}\psi = m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$ . But such a neutrino mass term breaks the weak-isospin symmetry, hence it is NOT permitted in the Standard Model.
- The right-handed component of the neutrino carries no weak isospin quantum numbers. This permits *Majorana neutrino mass* in the Standard Model if one only uses right-handed neutrinos.

Majorana mass term:

$$m_R \overline{\nu_R^c} \nu_R$$

- Such a mass term violates lepton number conservation since it implies that neutrinos are their antiparticles.
- It is permitted by the weak-isospin invariance of the Standard Model.
- Neutrino mass terms are not included in the fundamental Lagrangian of the Standard Model. They arise from new physics. Of course it is possible to write down an *effective* Lagrangian for the neutrino mass in terms of only the Standard Model fields if you give up renormalizability.

## A note on dimensional counting

- Lagrangian,  $L$ , has dimensions of energy (or mass).
- $L = \int d^3x \mathcal{L} \Rightarrow$  Lagrangian density,  $\mathcal{L}$ , has dimensions of energy/volume or  $M^4$ .
- Define the scaling dimension of  $x$ ,  $[x]$  to be  $-1 \Rightarrow$  scaling dimension of momentum (or mass) is  $[m] = +1$  (recall that  $(p \cdot x / \hbar)$  is dimensionless and we take  $[\hbar] = 0$ ).
- Clearly  $[L] = 4$ . This should be true for any Lagrangian density of any theory.
- Consider the mass term for fermions,  $L_m = m \bar{\Psi}\Psi$ . Then  $[\bar{\Psi}\Psi] = 3$  or  $[\Psi] = 3/2$ .
- In the Standard Model the Higgs field vacuum expectation value gives the particle mass:  $L = H \bar{\Psi}\Psi$ . Hence  $[H] = 1$ .

Using the Standard Model degrees of freedom one can parameterize the neutrino mass by a dimension 5 operator.  
(Recall that  $I_3^W = 1/2$  for the  $\nu_L$  and  $-1/2$  for  $H_{SM}$ ).

$$L = X_{\alpha\beta} H_{SM} H_{SM} \overline{\nu_{L\alpha}^c} \nu_{L\beta} / \Lambda$$

$$v^2 X_{\alpha\beta} / \Lambda = U m_\nu^{\text{diagonal}} U^T$$

This term is not renormalizable!

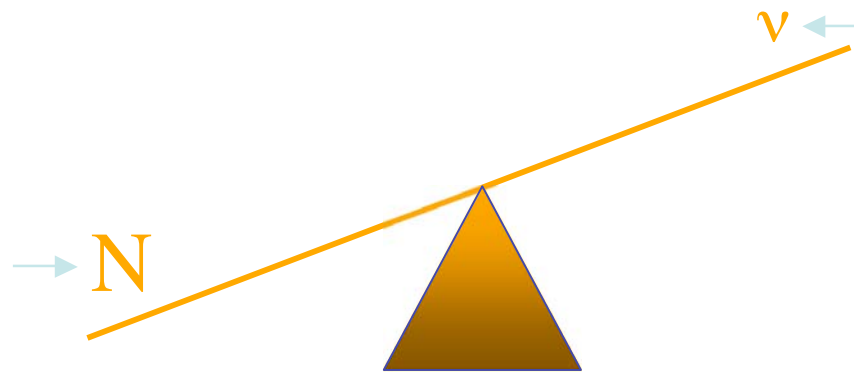
There are other ways to obtain neutrino mass:

$$L = H_{I=1} \overline{\nu_{L\alpha}}^c \nu_{L\beta}$$

Note: This Higgs is not in the Standard Model!

## See-Saw Mechanism

$$\begin{pmatrix} 0 & h_v v \\ h_v v & M_R \end{pmatrix}$$



One of the most exciting discoveries in the physics during the last two decades was that neutrinos have mass!

This was achieved through the observation of neutrino oscillations!



## Neutrino Oscillations

$$\nu_e(t=0) = \cos \theta \nu_1 + \sin \theta \nu_2$$



$$\nu_e(t) = \cos \theta \exp(iE_1 t) \nu_1 + \sin \theta \exp(iE_2 t) \nu_2$$

$$P(\nu_e \rightarrow \nu_e) = |\nu_e(L)|^2$$

Phase Difference:  $E_2 - E_1 \propto m_2^2 - m_1^2 = \delta m^2$

$$\begin{bmatrix} \Psi_e \\ \Psi_\mu \end{bmatrix} = \begin{bmatrix} \cos \theta_v & \sin \theta_v \\ -\sin \theta_v & \cos \theta_v \end{bmatrix} \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix},$$

$$\begin{aligned} & i\hbar \frac{\partial}{\partial t} \begin{bmatrix} \Psi_e(t) \\ \Psi_\mu(t) \end{bmatrix} \\ &= \frac{1}{4E} \begin{bmatrix} -\delta m^2 \cos 2\theta_v & \delta m^2 \sin 2\theta_v \\ \delta m^2 \sin 2\theta_v & \delta m^2 \cos 2\theta_v \end{bmatrix} \begin{bmatrix} \Psi_e(t) \\ \Psi_\mu(t) \end{bmatrix} \\ & \quad \delta m^2 = m_2^2 - m_1^2 \quad (m_2 > m_1) \end{aligned}$$

Appearance probability of the other flavor

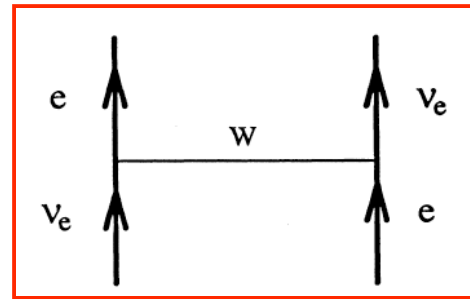
$$P = \sin^2 2\theta_v \sin^2 [1.27 \delta m^2 (\text{eV}^2) L / E (\text{m/MeV})].$$

## MSW effect

In vacuum:  $E^2 = p^2 + m^2$

In a potential:  $(E - \Phi)^2 = p^2 + m^2 \Rightarrow m_{\text{eff}}^2 = m^2 + 2E\Phi$

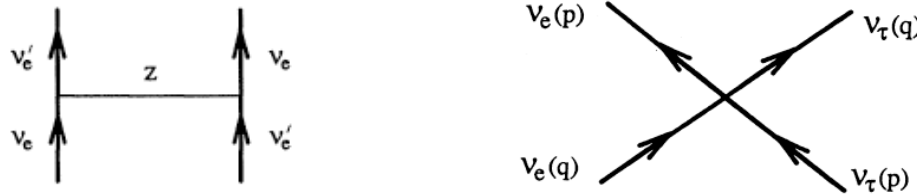
The potential is provided by the coherent forward scattering of  $\nu_e$ 's off the electrons in dense matter.



There is a similar term with Z-exchange. But since it is the same for all neutrino flavors, it does not contribute to phase differences *unless* we invoke a sterile neutrino.

$$i \frac{\partial}{\partial t} \begin{pmatrix} \psi_e \\ \psi_x \end{pmatrix} = \begin{pmatrix} -\frac{\delta m^2}{4p} \cos 2\theta + \frac{1}{\sqrt{2}} G_F N_e & \frac{\delta m^2}{4p} \sin 2\theta \\ \frac{\delta m^2}{4p} \sin 2\theta & \frac{\delta m^2}{4p} \cos 2\theta - \frac{1}{\sqrt{2}} G_F N_e \end{pmatrix} \begin{pmatrix} \psi_e \\ \psi_x \end{pmatrix}$$

If the neutrino density itself is also very high then one has to consider the effects of neutrinos scattering off other neutrinos. This is the case for a core-collapse supernova.



$$i \frac{\partial}{\partial t} \begin{pmatrix} \psi_e \\ \psi_x \end{pmatrix} = \frac{1}{2} \begin{pmatrix} A + B - \Delta \cos 2\theta & B_{e\mu} + \Delta \sin 2\theta \\ B_{\mu e} + \Delta \sin 2\theta & -A - B + \Delta \cos 2\theta \end{pmatrix} \begin{pmatrix} \psi_e \\ \psi_x \end{pmatrix}$$

$$\Delta = \frac{\delta m^2}{2p}, \quad A = \sqrt{2} G_F N_e$$

$$B = \sqrt{2} G_F \int dq (1 - \cos \theta_{pq}) \left[ \left( |\psi_e(q, t)|^2 - |\psi_x(q, t)|^2 \right) \right]$$

$$B_{ex} = 2\sqrt{2} G_F \int d^3q (1 - \cos \theta_{pq}) (\psi_e(q, t) \psi_\mu^*(q, t))$$

The most copious neutrino source for Earth is the Sun unless you are really close to a nuclear reactor!

Let us take a brief detour into this modest main sequence star.

# Stellar Equilibrium

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p - \rho \nabla \phi$$

Equation of motion

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} = 0$$

Equation of continuity

$$\nabla^2 \phi = 4\pi G \rho$$

Poisson's Equation

$$\frac{1}{p} \frac{Dp}{Dt} - \Gamma_1 \frac{1}{\rho} \frac{D\rho}{Dt} = \frac{\Gamma_3 - 1}{p} (\rho \epsilon - \nabla \cdot \mathbf{F})$$

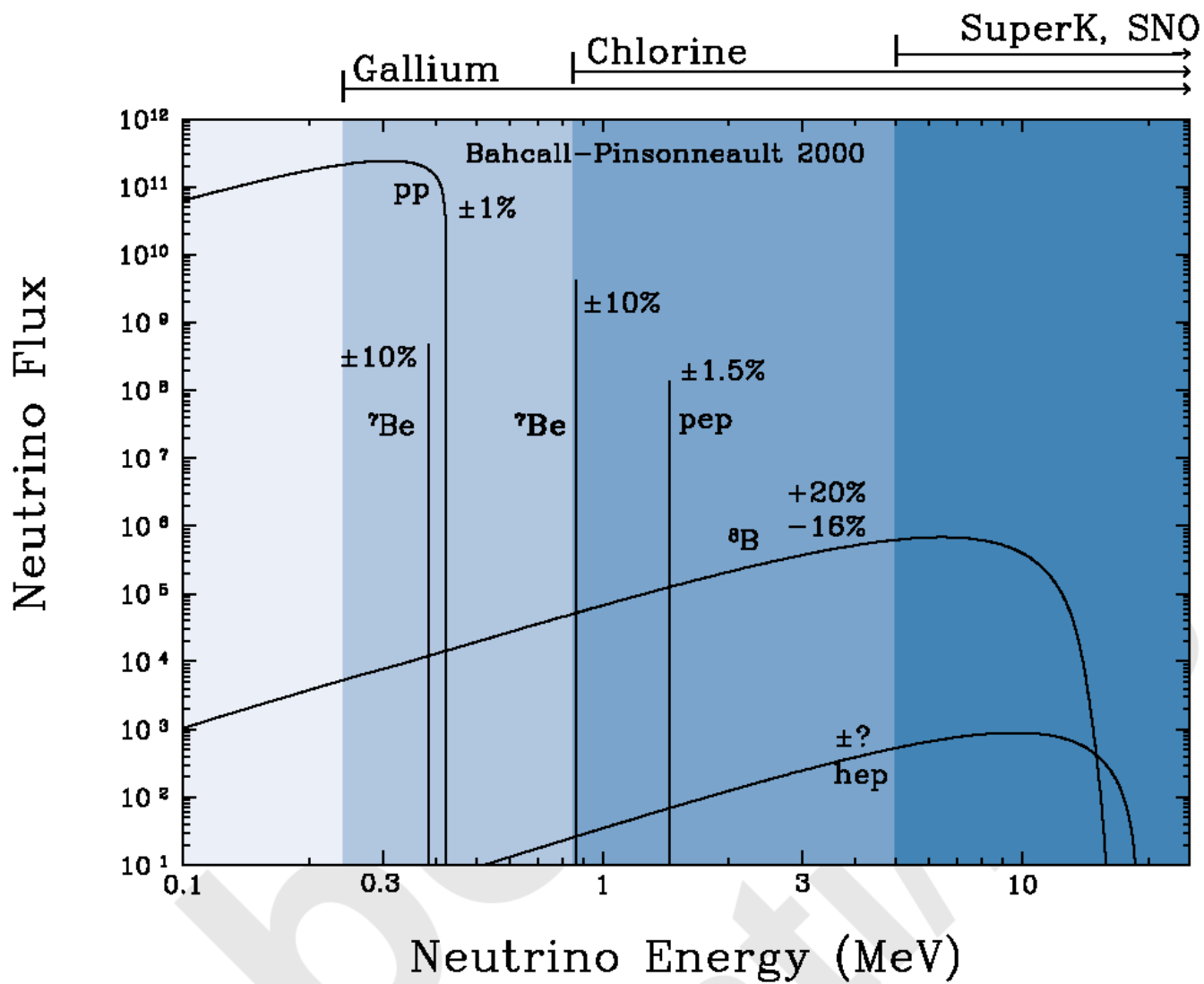
Energy Equation

$$\Gamma_1 = \left( \frac{\partial \log p}{\partial \log \rho} \right)_s, \Gamma_3 - 1 = \left( \frac{\partial \log T}{\partial \log \rho} \right)_s$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$$

$\epsilon$  = Rate of nuclear energy generation

$\mathbf{F}$  = Energy flux



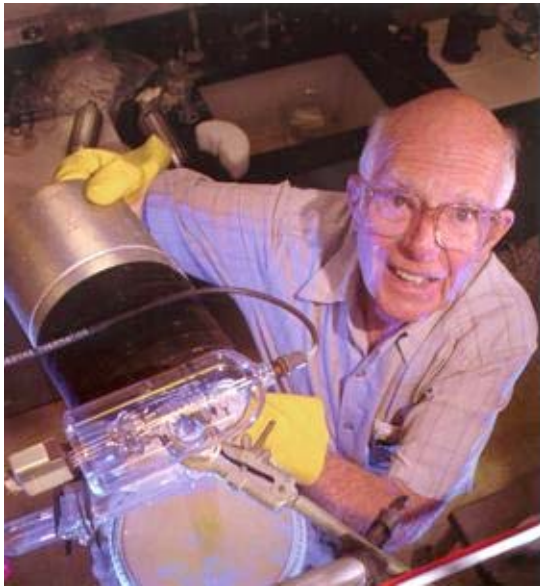
Continuous fluxes in  $/\text{cm}^2/\text{s}/\text{MeV}$   
 Discrete fluxes in  $/\text{cm}^2/\text{s}$

# Solar neutrino experiments

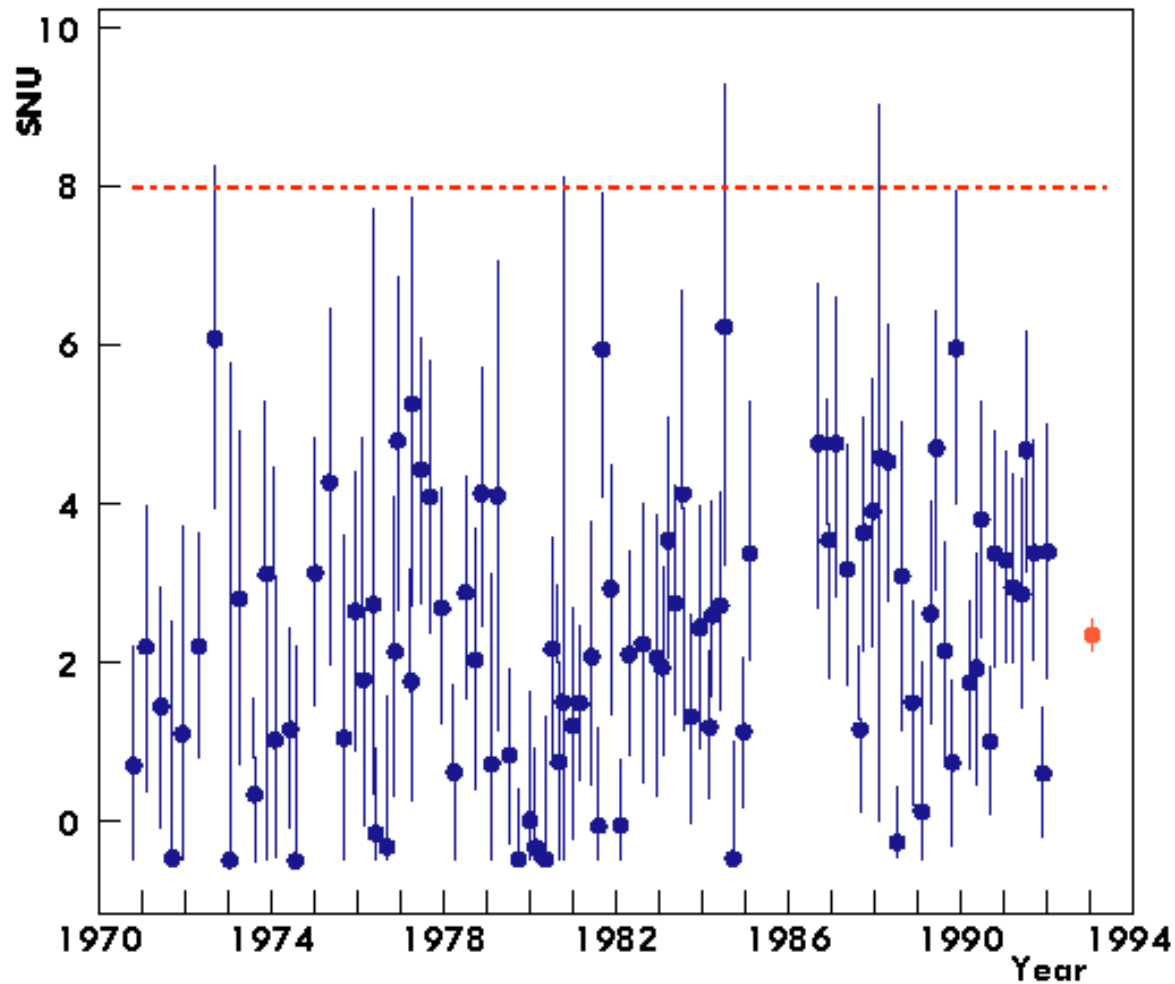
- **Radiochemical experiments:** Measure integrated (over time and energy) count rate.  
1 SNU (solar neutrino unit) =  $10^{-36}$  neutrinos captured per atom per second.  
Homestake (Cl), SAGE (Ga), Gallex (Ga), GNO (Ga)
- **Real-time counting experiments:** Measure the energy spectra in real time.  
Kamiokande, SuperKamiokande, SNO



# Homestake Experiment



Then: Homestake Count Rate

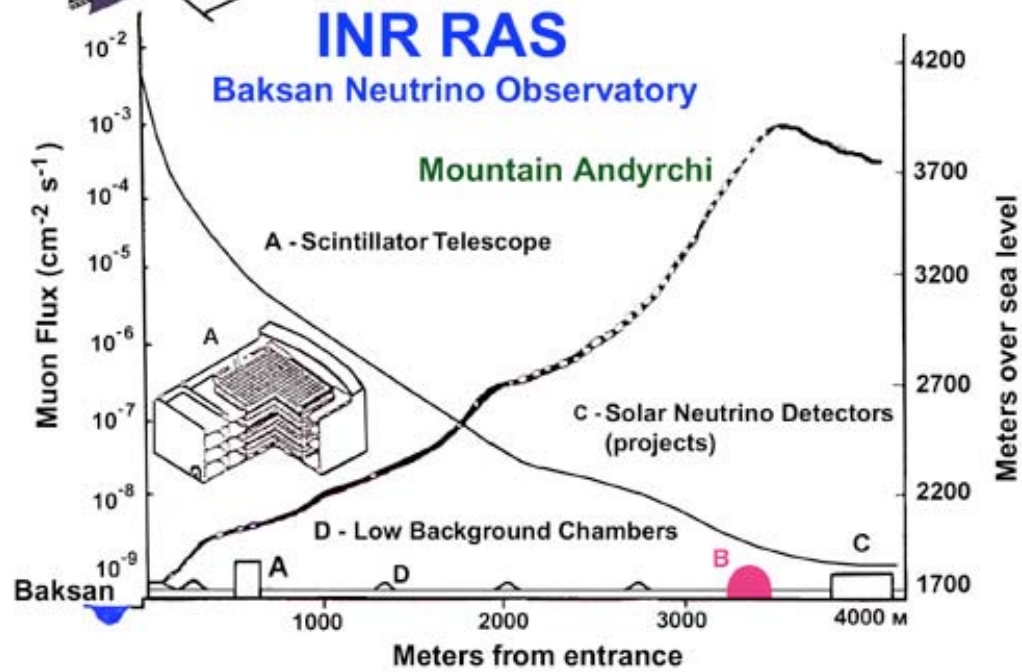
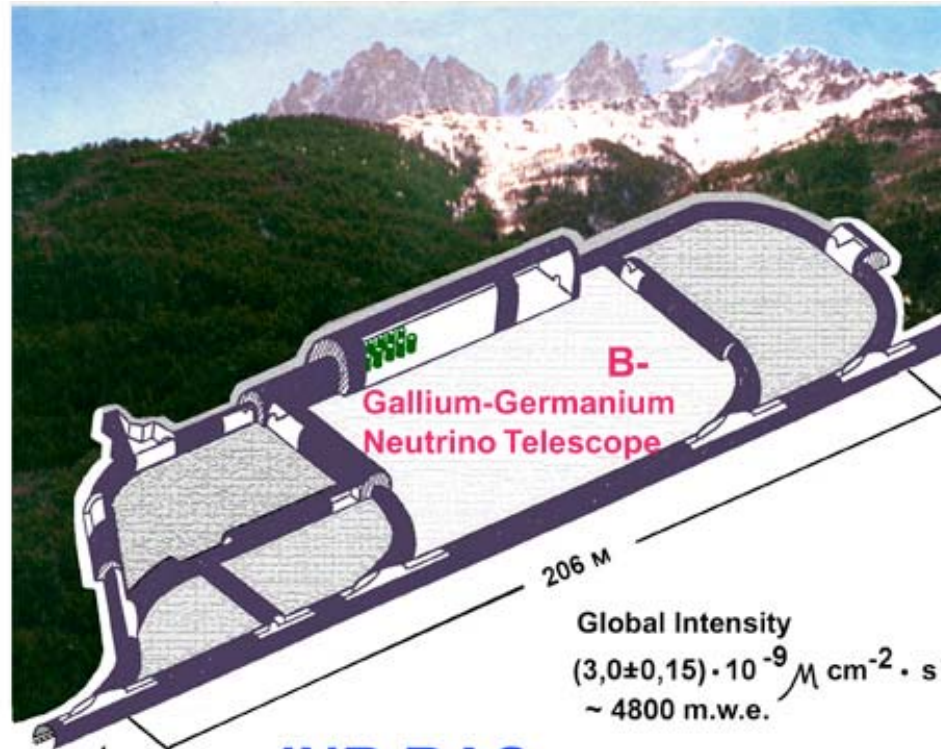


Those of us who believed the data thought that neutrino mixing angles are, like quark mixing angles, small for vacuum mixing, but, thanks to MSW, matter-enhanced oscillations do the reduction. Most people did not even believe the data.

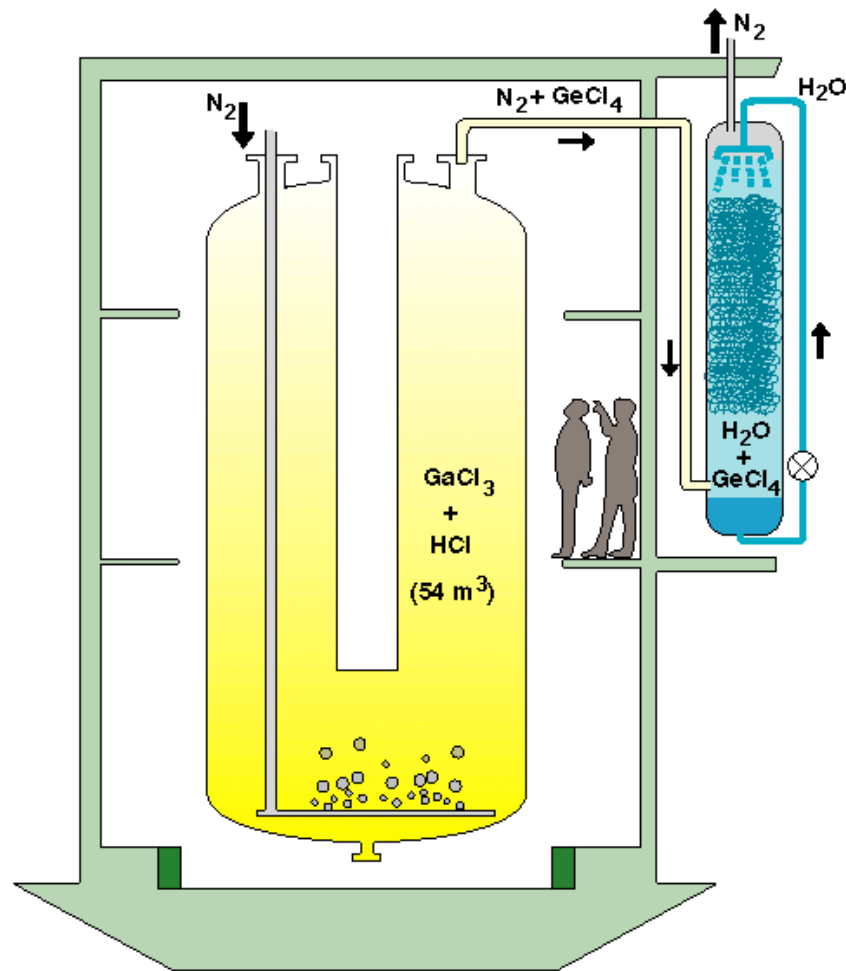
# SAGE Experiment

Mt. Andyrchi, Caucasus  
Near Mt. Elbrus  
Kabardino-Balkaria  
Russia

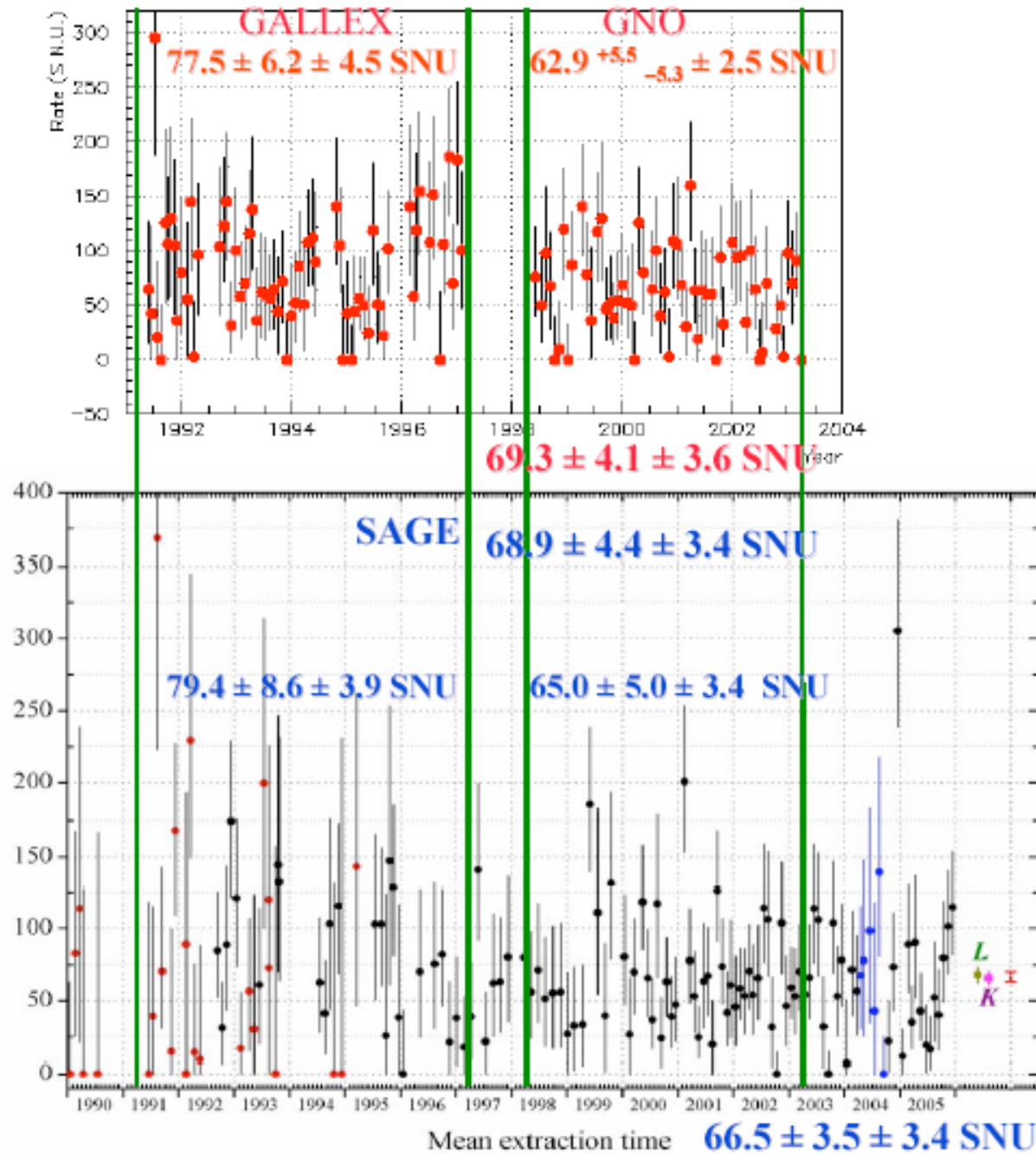




# Gallex and GNO

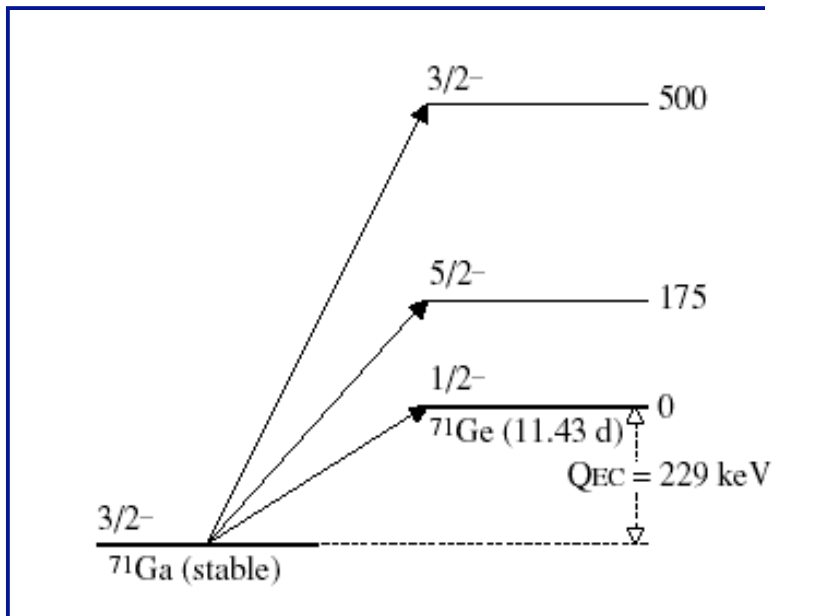
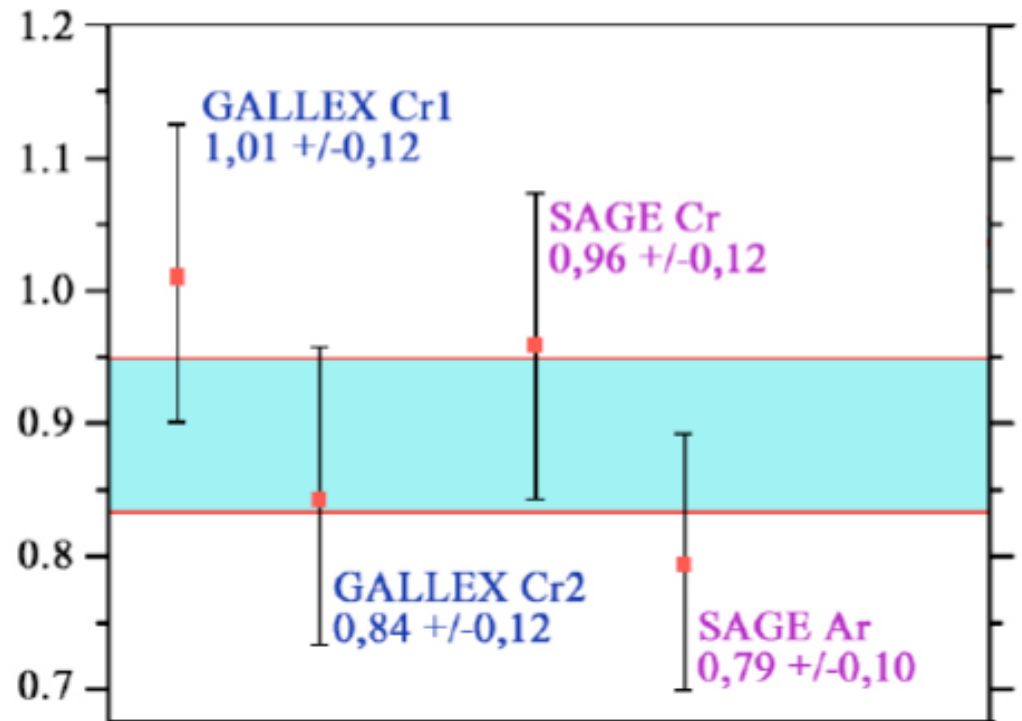


Now:



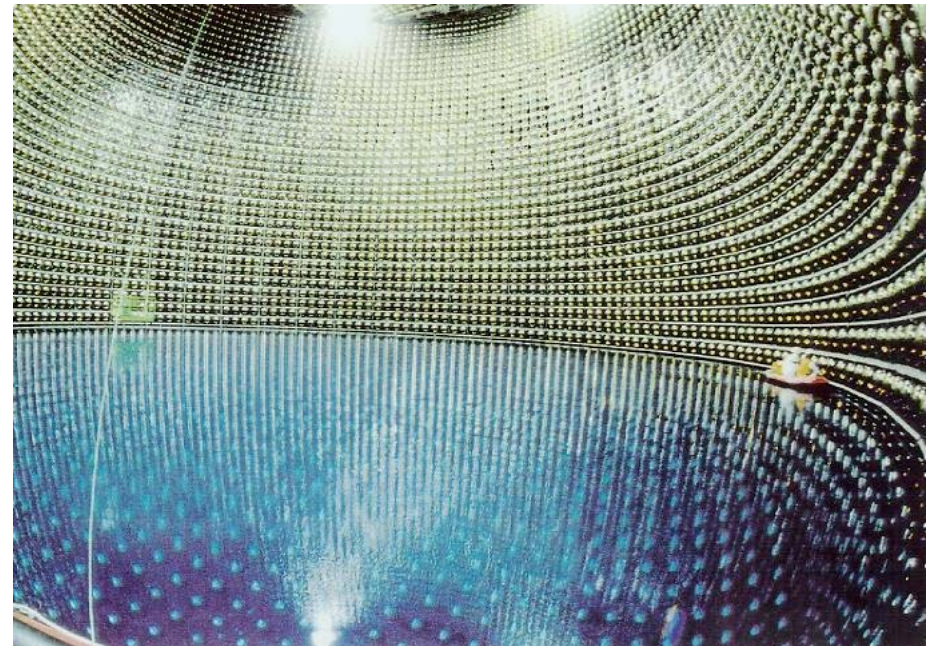
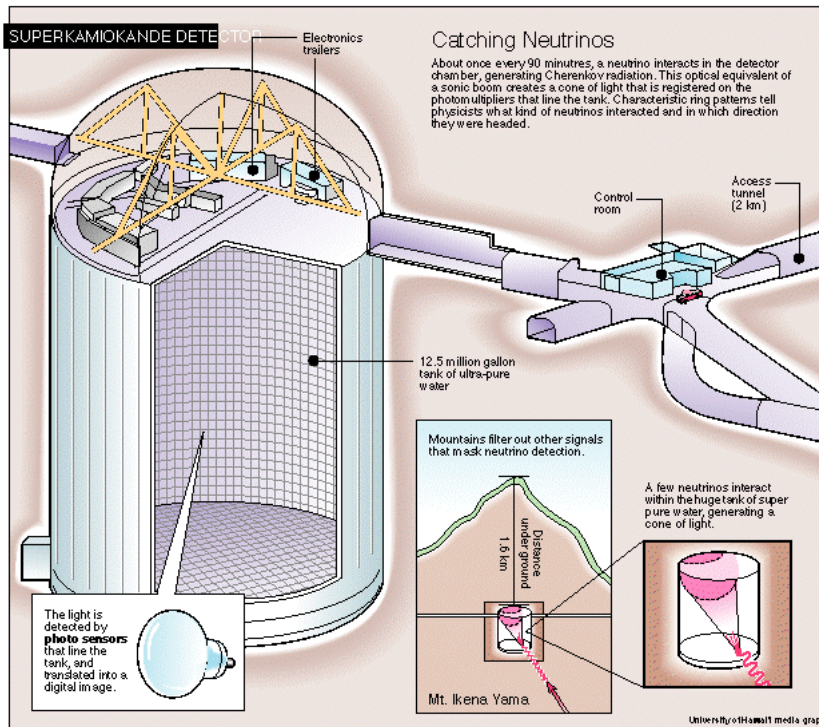
Adopted  
from  
Gavrin

What are the Ga source experiments telling us?



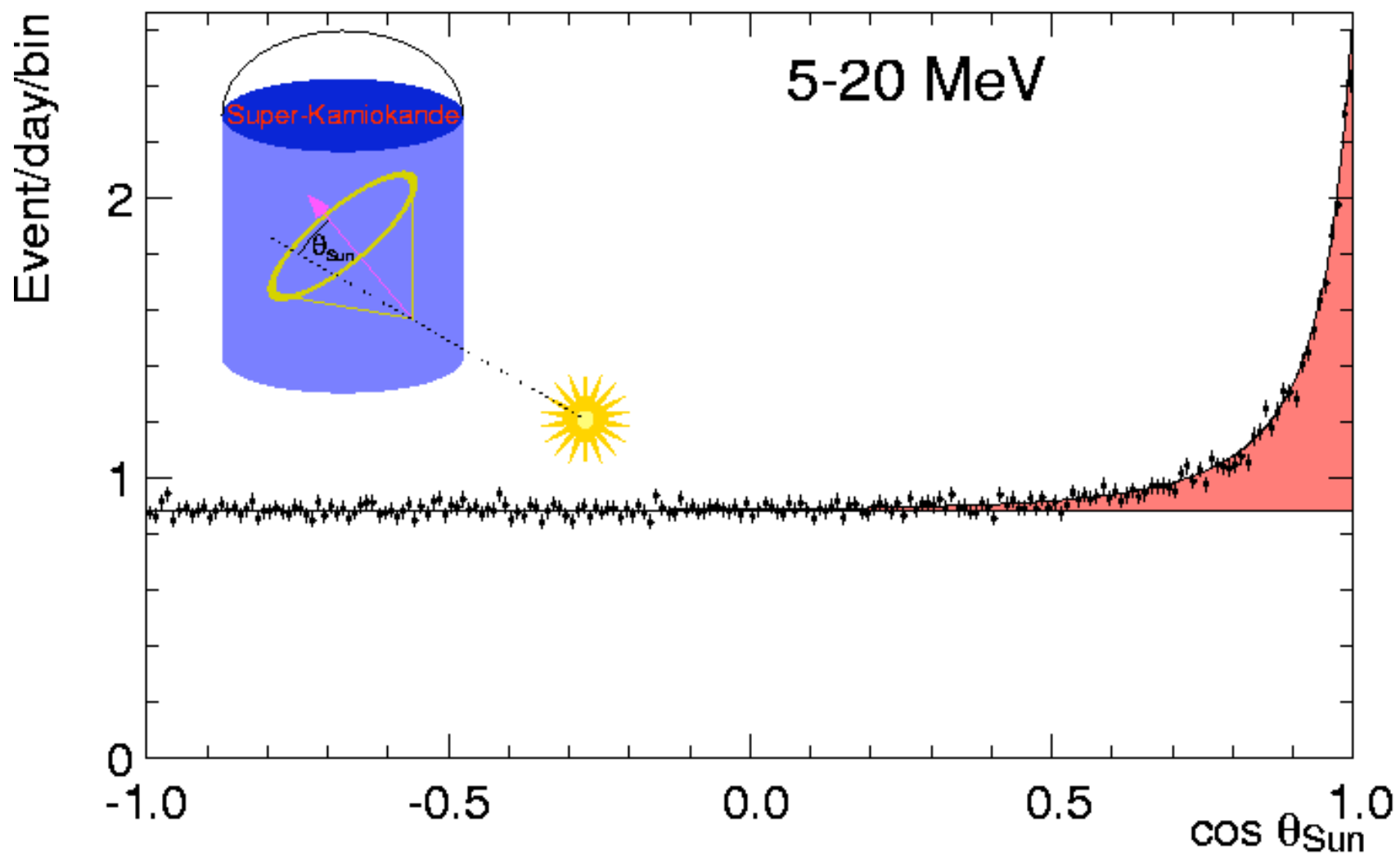
Ground state cross-section is fixed from the beta-decay (via detailed balance). Correction is due to the excited states, which contribute little to the solar neutrino capture rate.

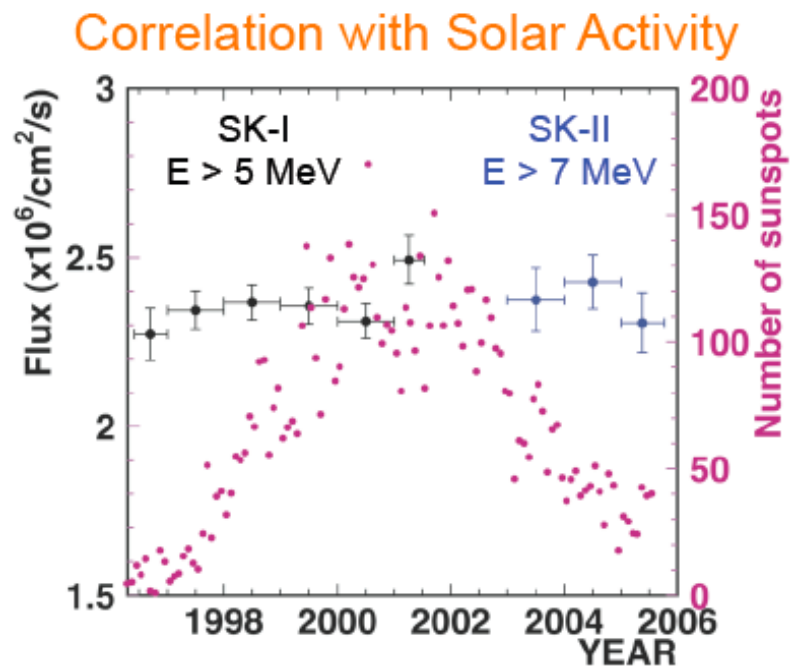
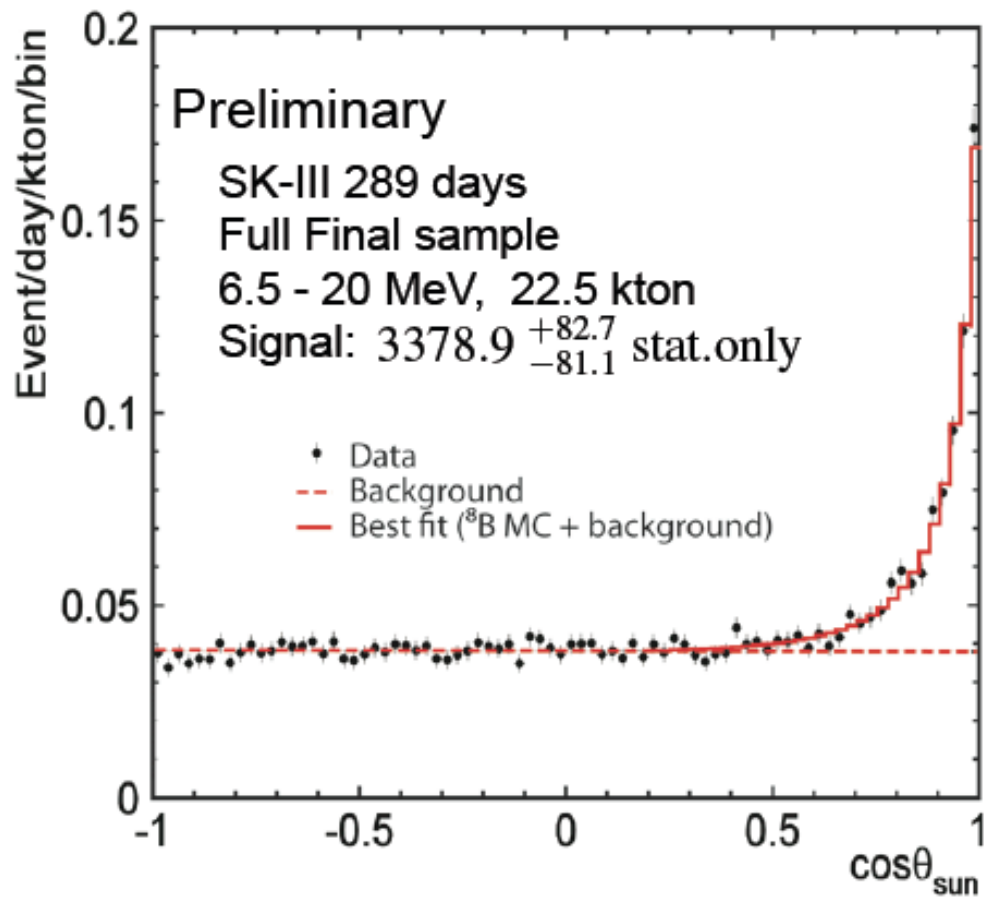
# SuperKamiokande Detector





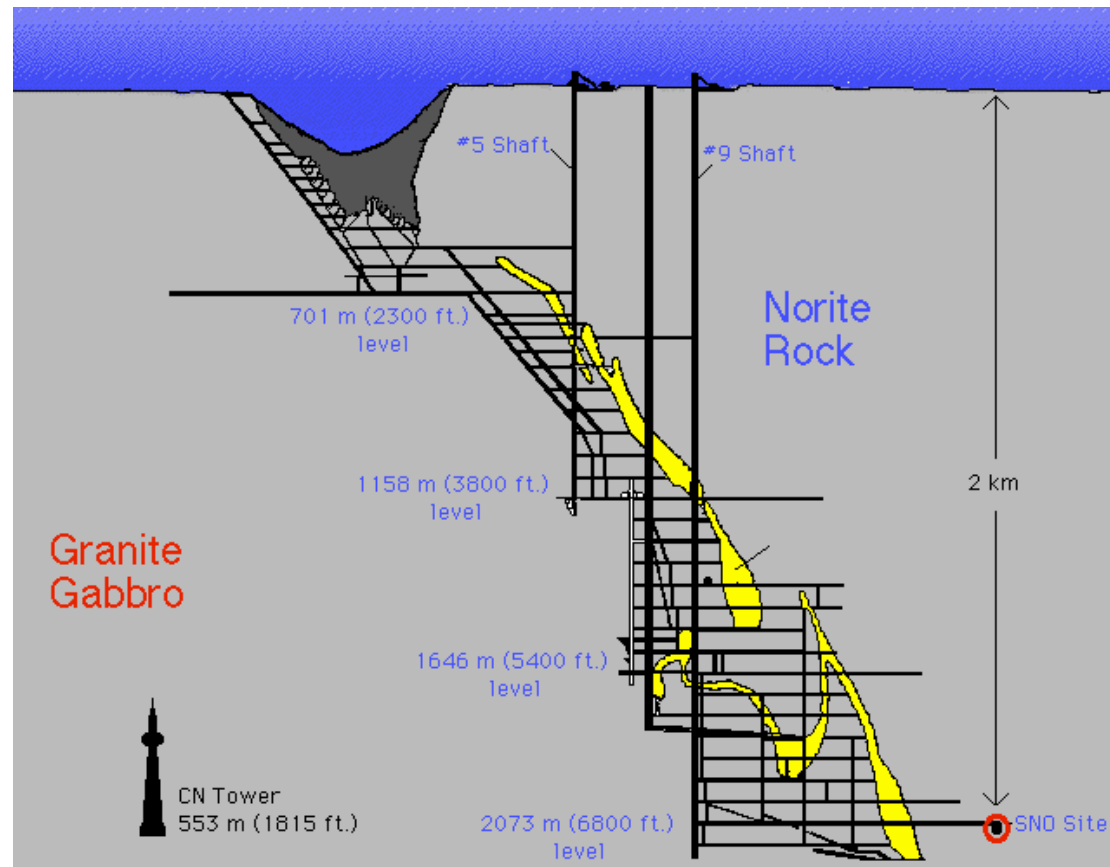
SuperKamiokande-I  $^8\text{B}$  solar  $\nu$ 's

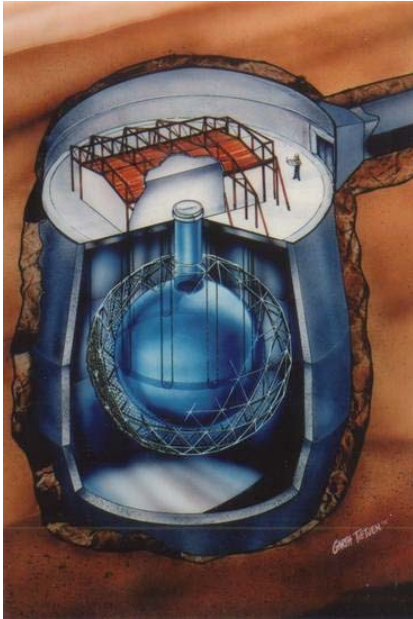




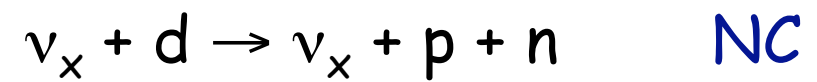
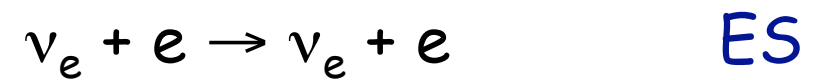
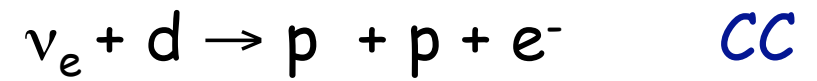
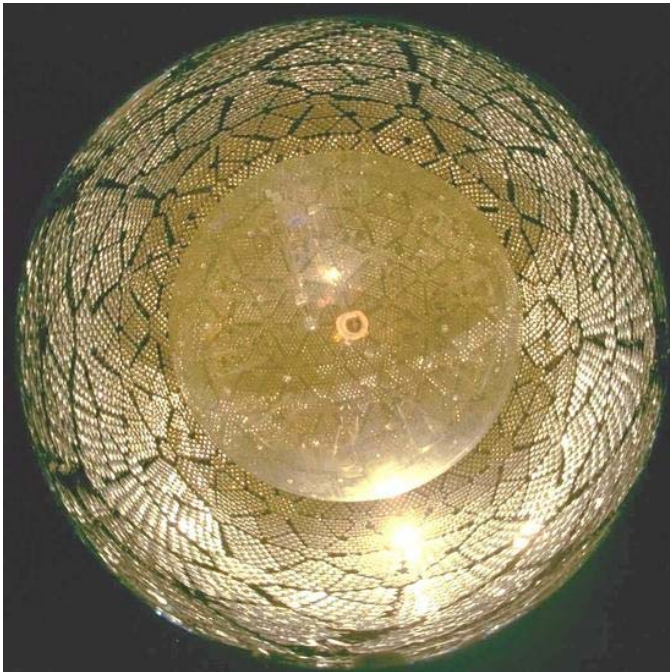
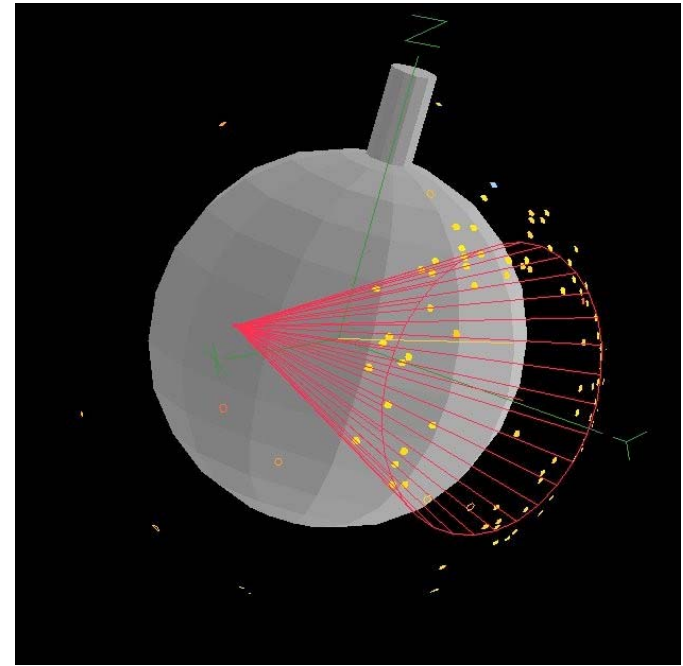
*No correlation with solar cycle  
minima or maximum seen*

# Sudbury Neutrino Observatory

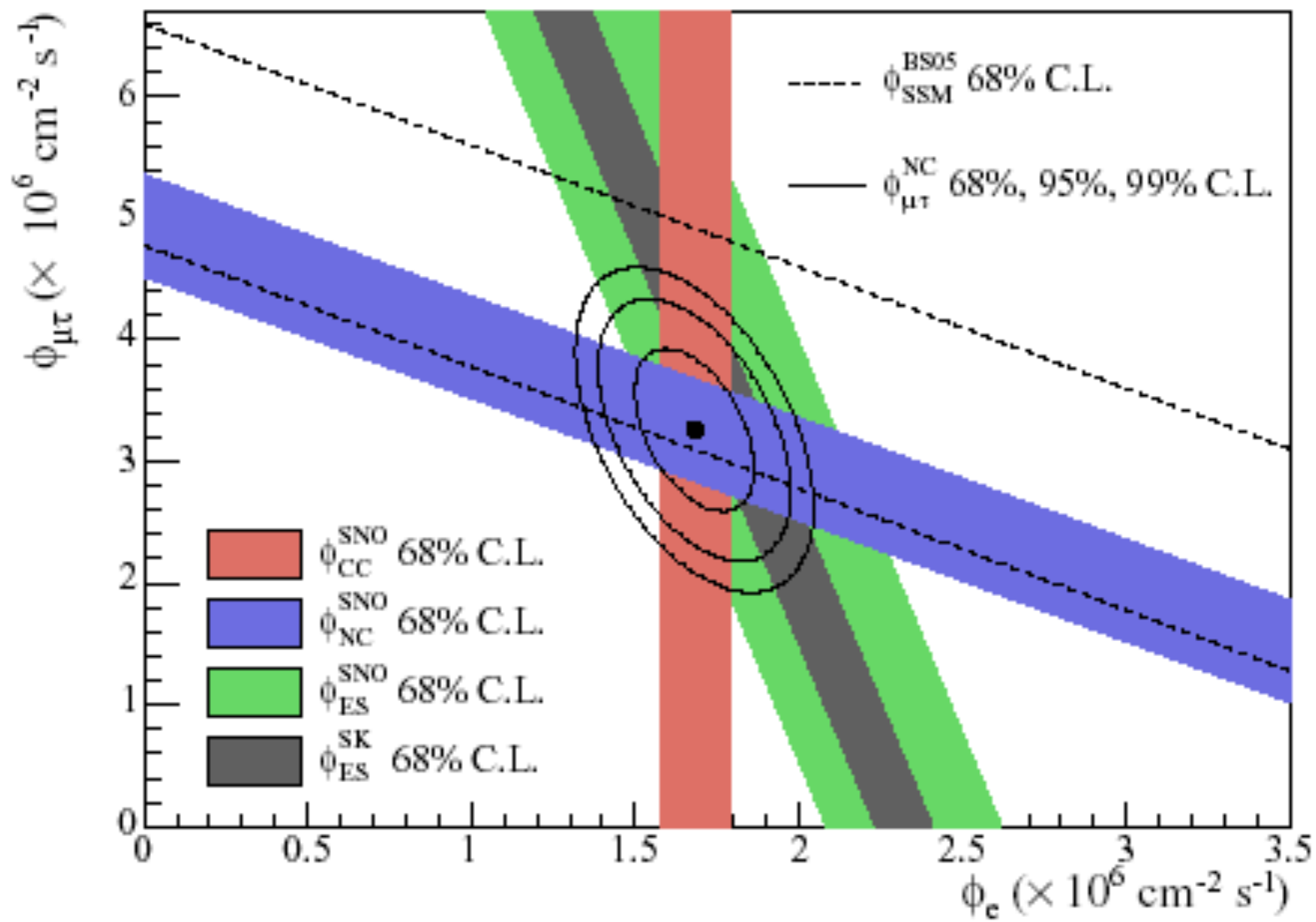




## Sudbury Neutrino Observatory



# SNO



Preliminary

## Fluxes

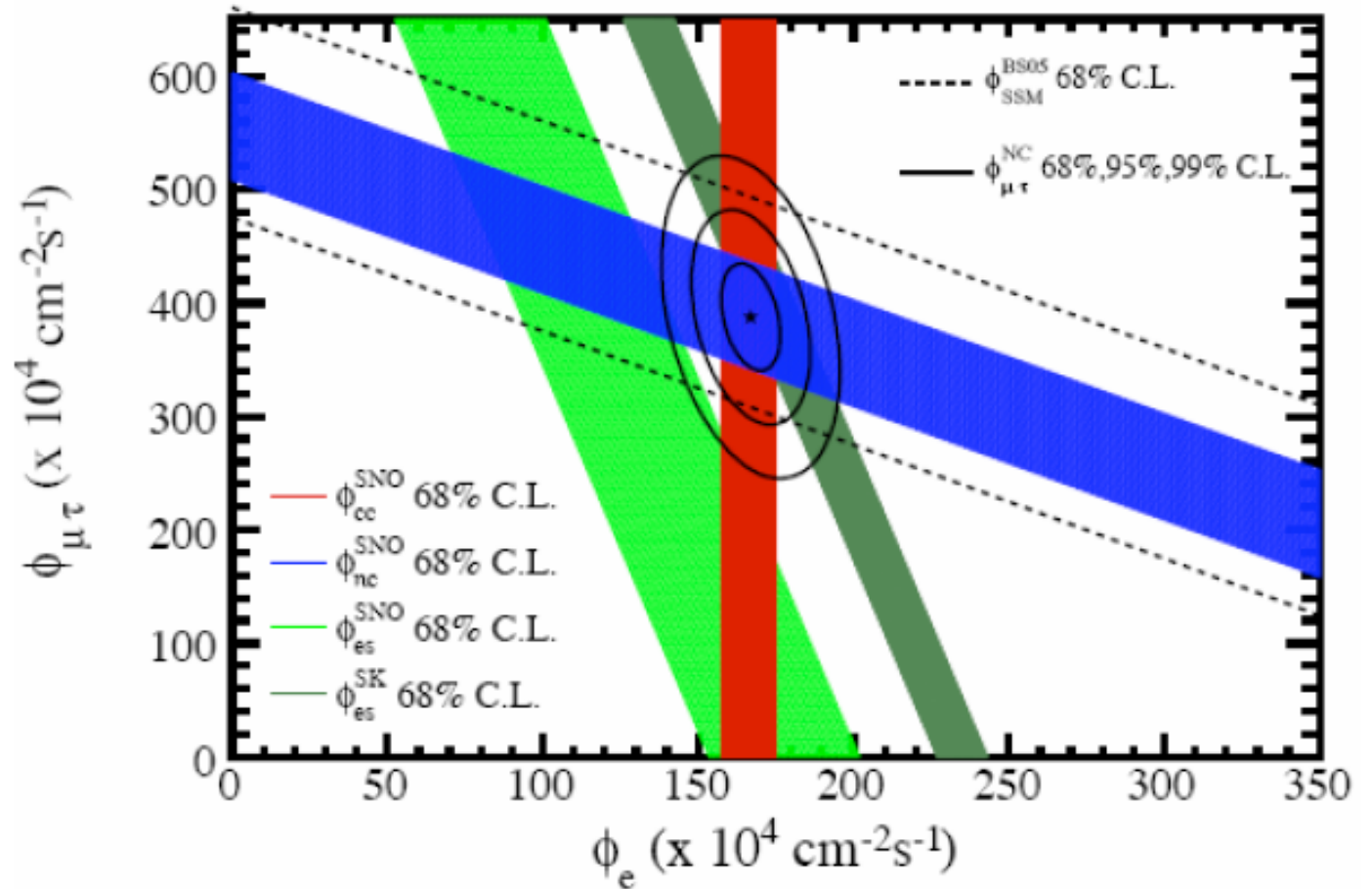
( $10^4 \text{ cm}^{-2} \text{ s}^{-1}$ )

$\nu_e$ : 167(9)

$\nu_{ES}$ : 177(26)

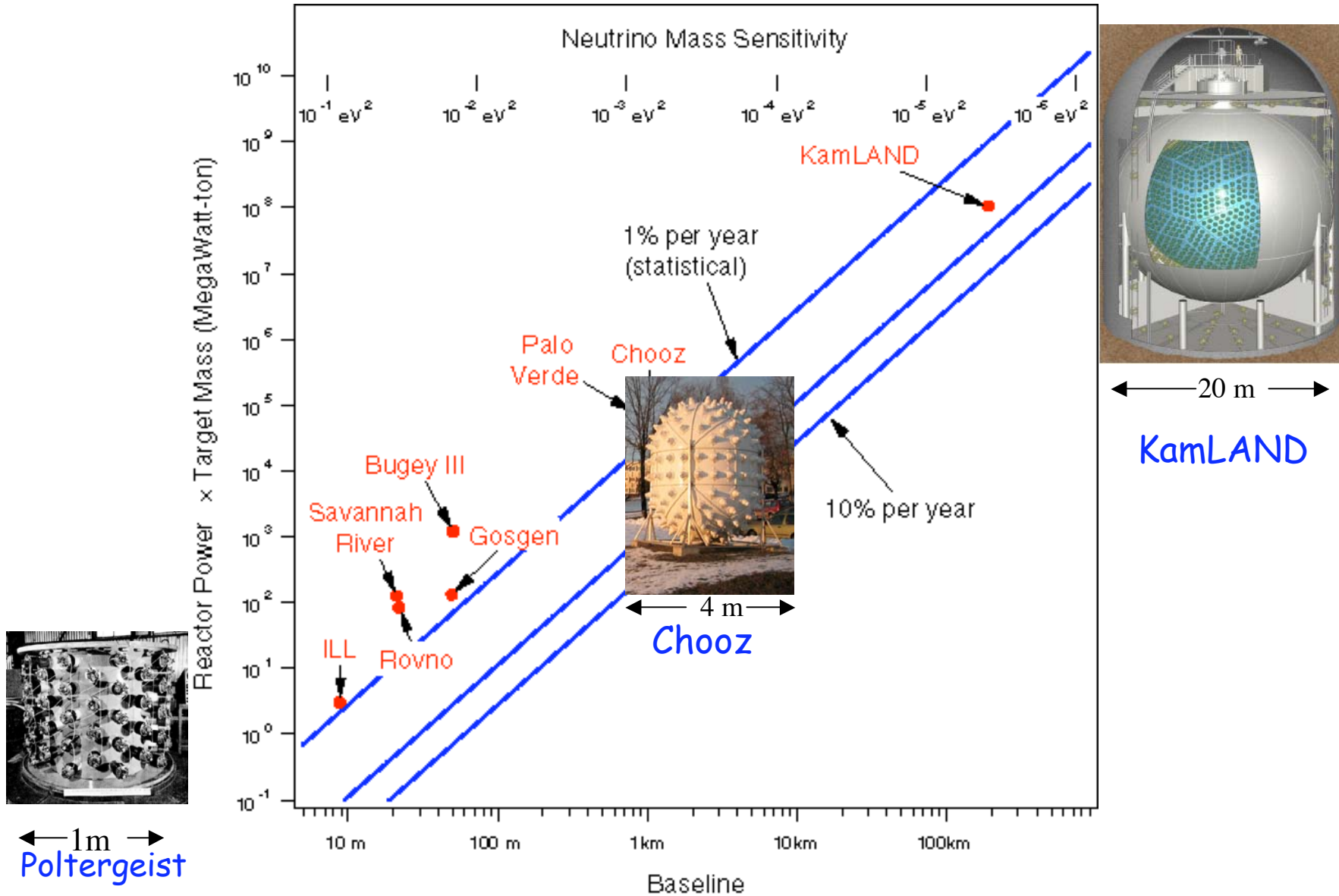
$\nu_{\text{total}}$ : 554(48)

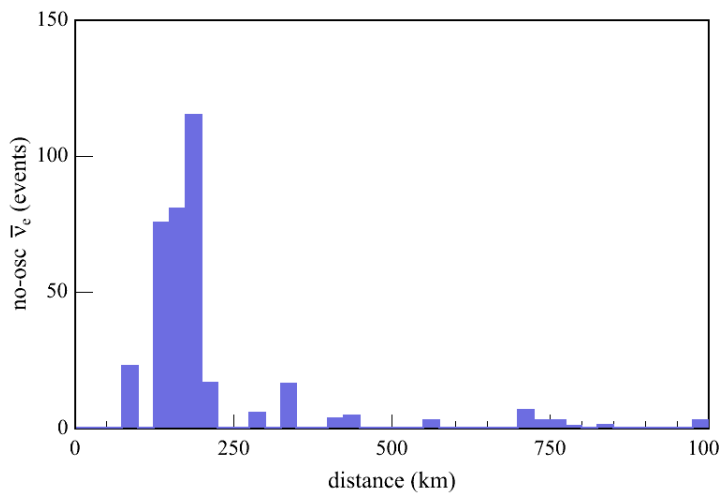
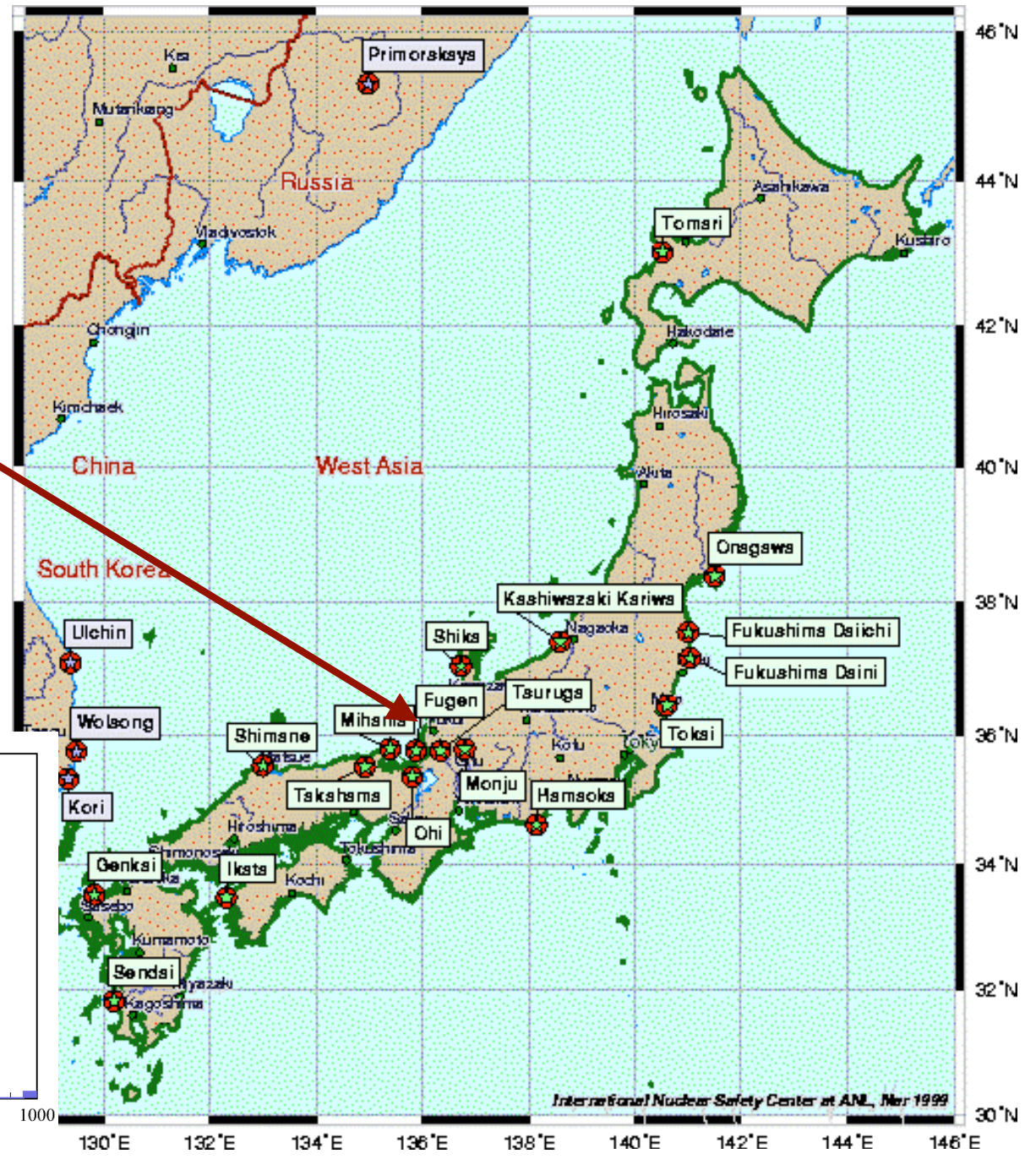
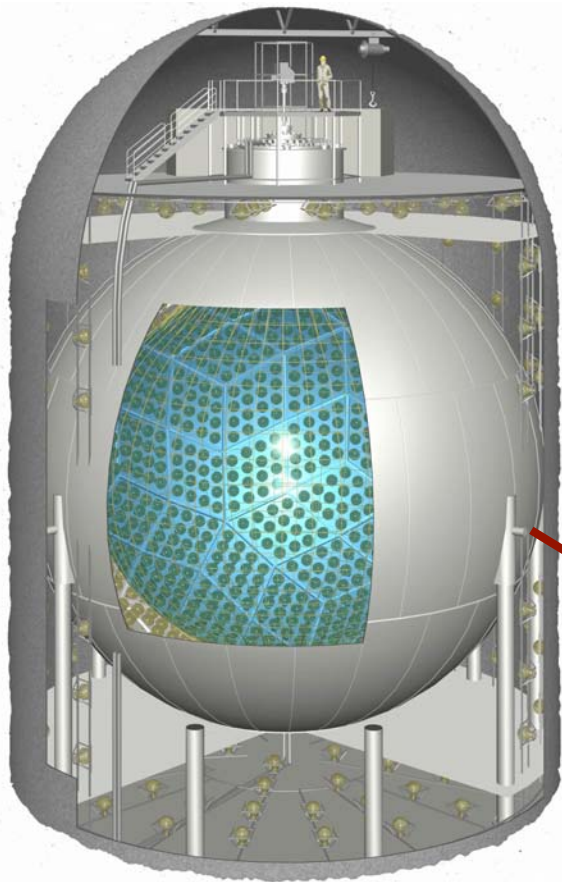
$\nu_{SSM}$ : **569(91)**



From NEUTRINO 2008 presentation

# Long Baseline Reactor Neutrino Experiments

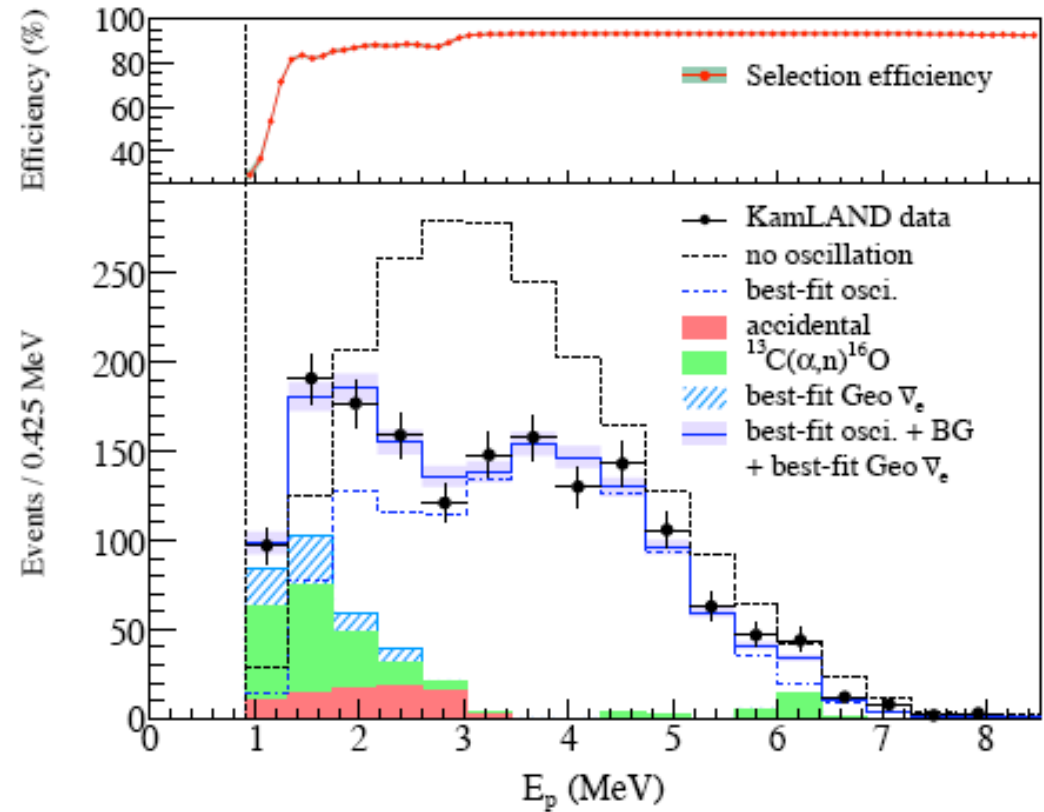
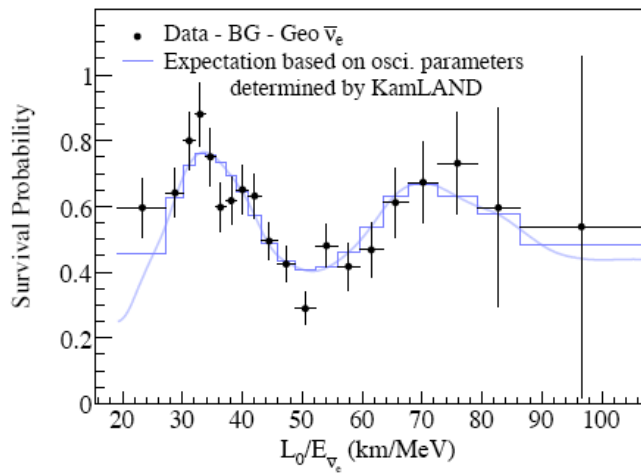
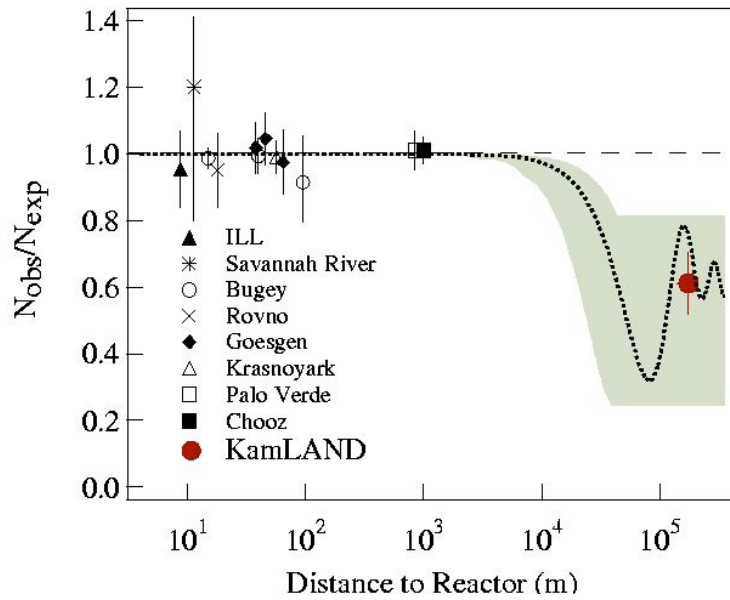
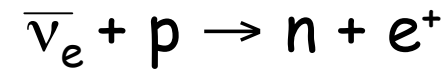




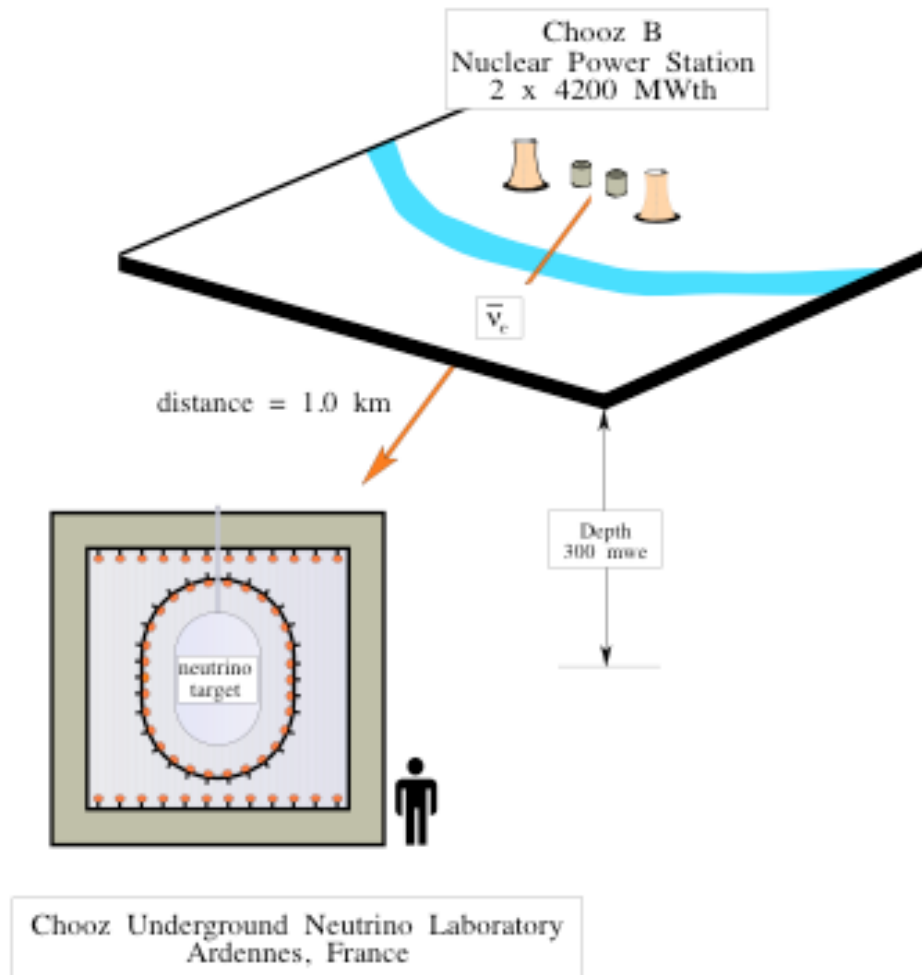
International Nuclear Safety Center at ANL, Mar 1999



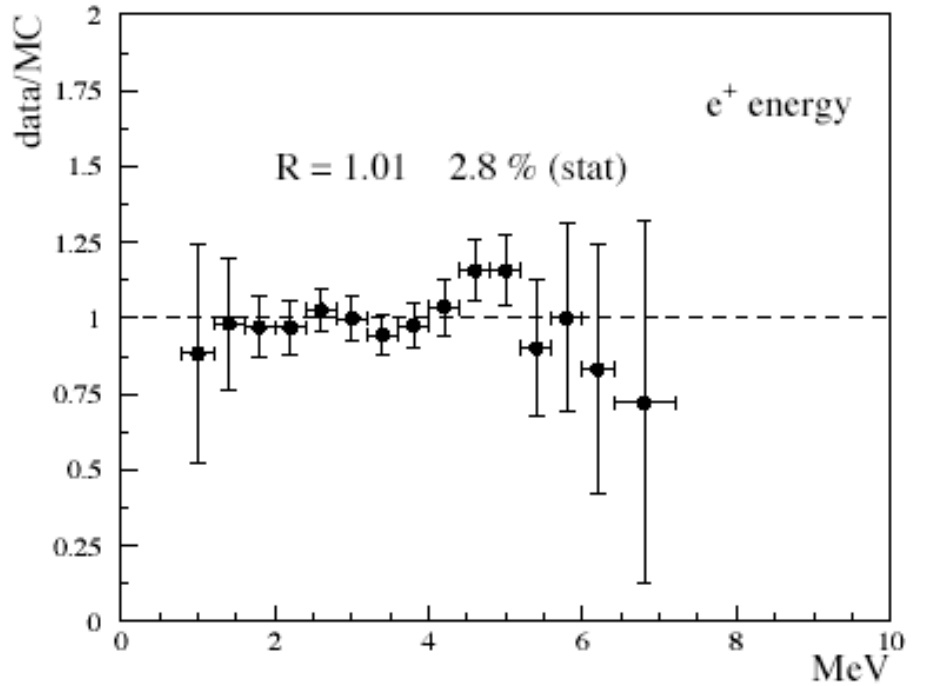
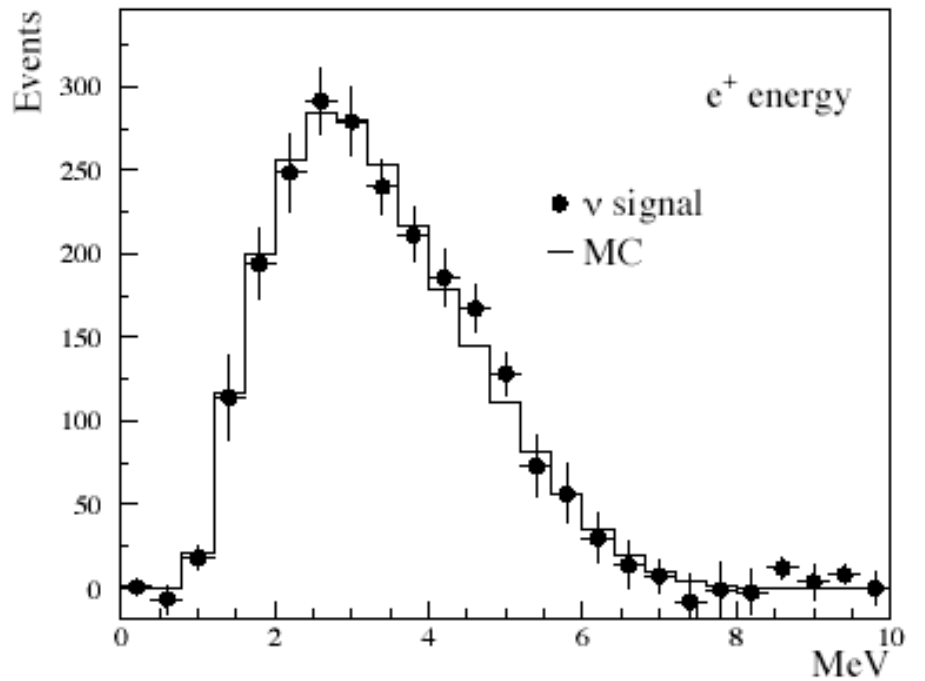
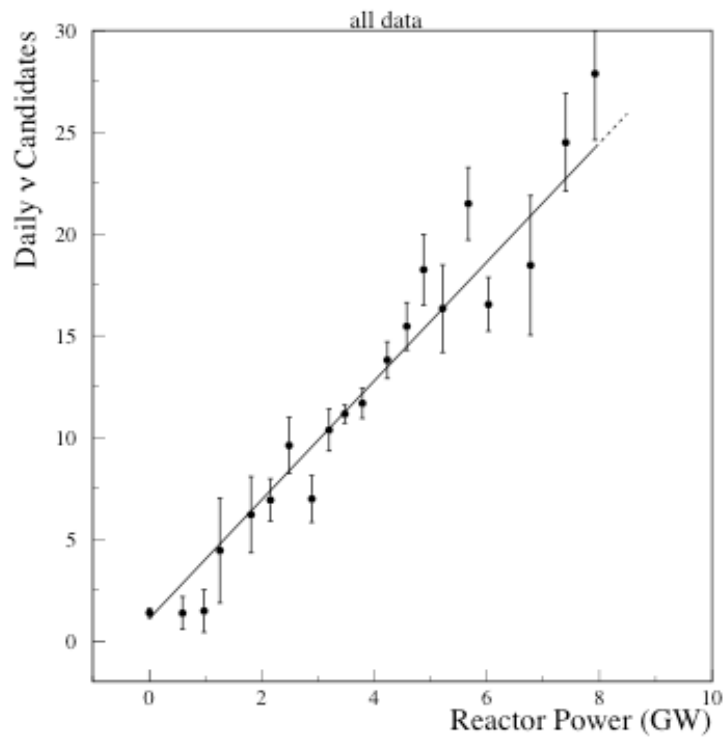
# KamLAND



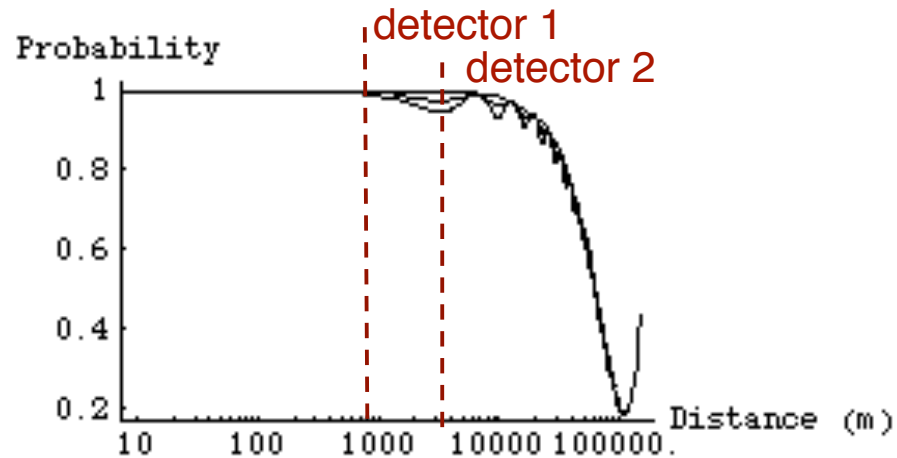
# CHOOZ



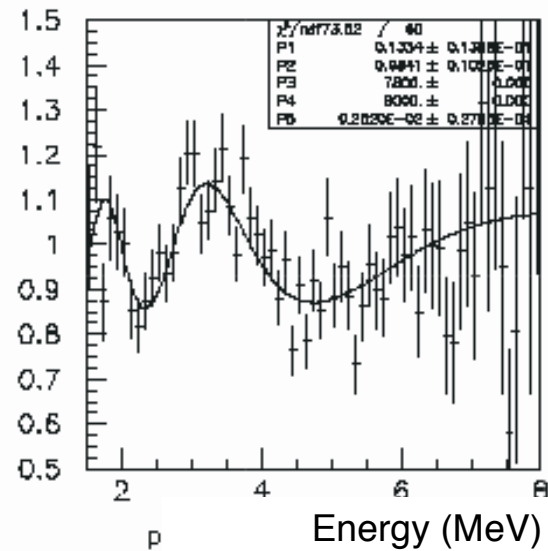
# CHOOZ



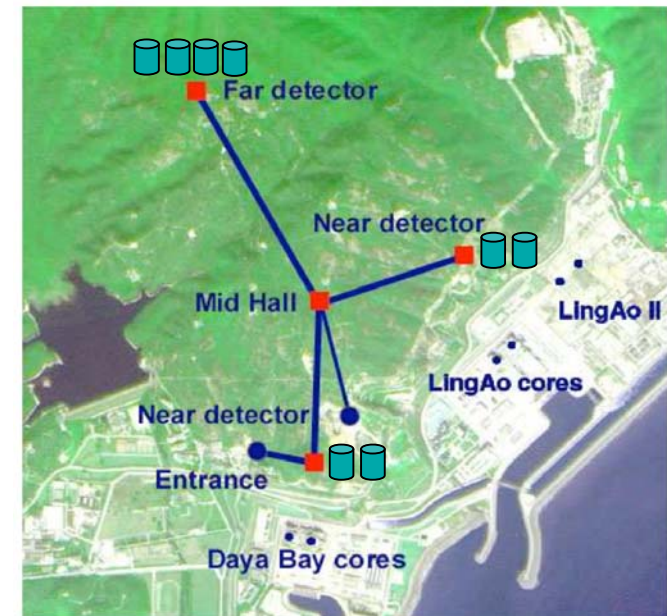
# Two-Detector Reactor Experiment to measure $\theta_{13}$



Spectral Ratio



# Daya Bay Reactor Experiment, China



# Atmospheric Neutrinos

## ATMOSPHERIC NEUTRINOS

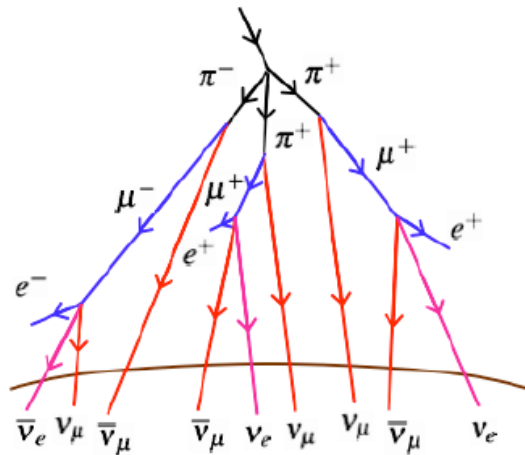
Cosmic Rays + O, N  $\rightarrow$   $\pi^\pm(K^\pm)$

$$\begin{aligned} \pi^\pm(K^\pm) &\rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu), \\ \mu^\pm &\rightarrow e^\pm + \nu_e(\bar{\nu}_e) + \bar{\nu}_\mu(\nu_\mu). \end{aligned}$$

$$\tau = (\nu_e + \bar{\nu}_e) / (\nu_\mu + \bar{\nu}_\mu) \sim 0.5$$

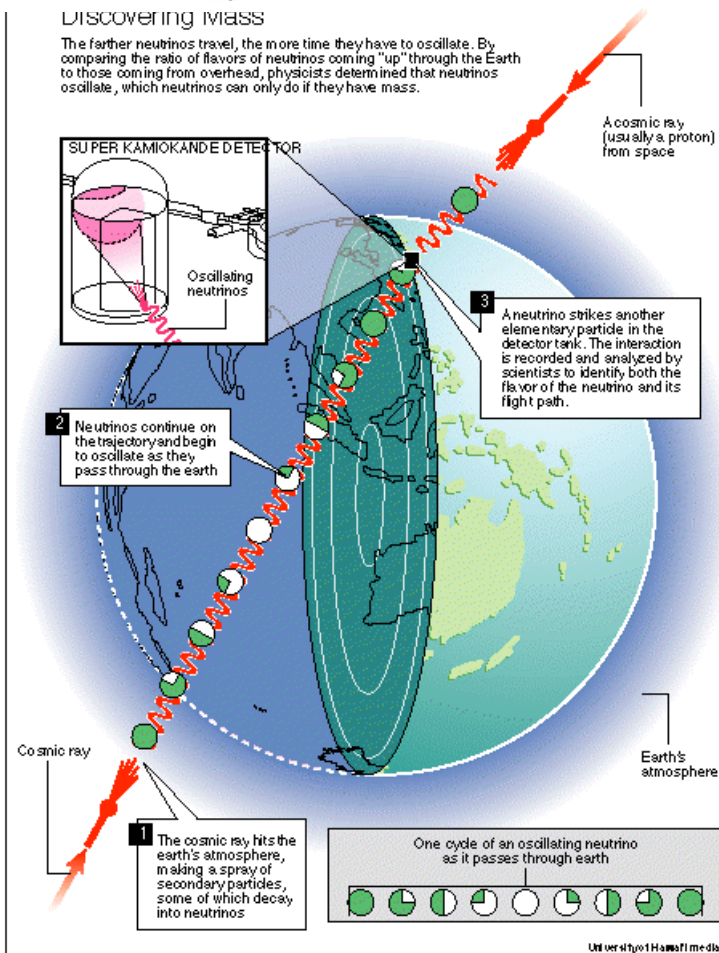
Including the effects of muon polarization  $\tau \sim 0.45$ .

$$R = \frac{(\nu_\mu/\nu_e)_{\text{data}}}{(\nu_\mu/\nu_e)_{\text{Monte Carlo}}}$$

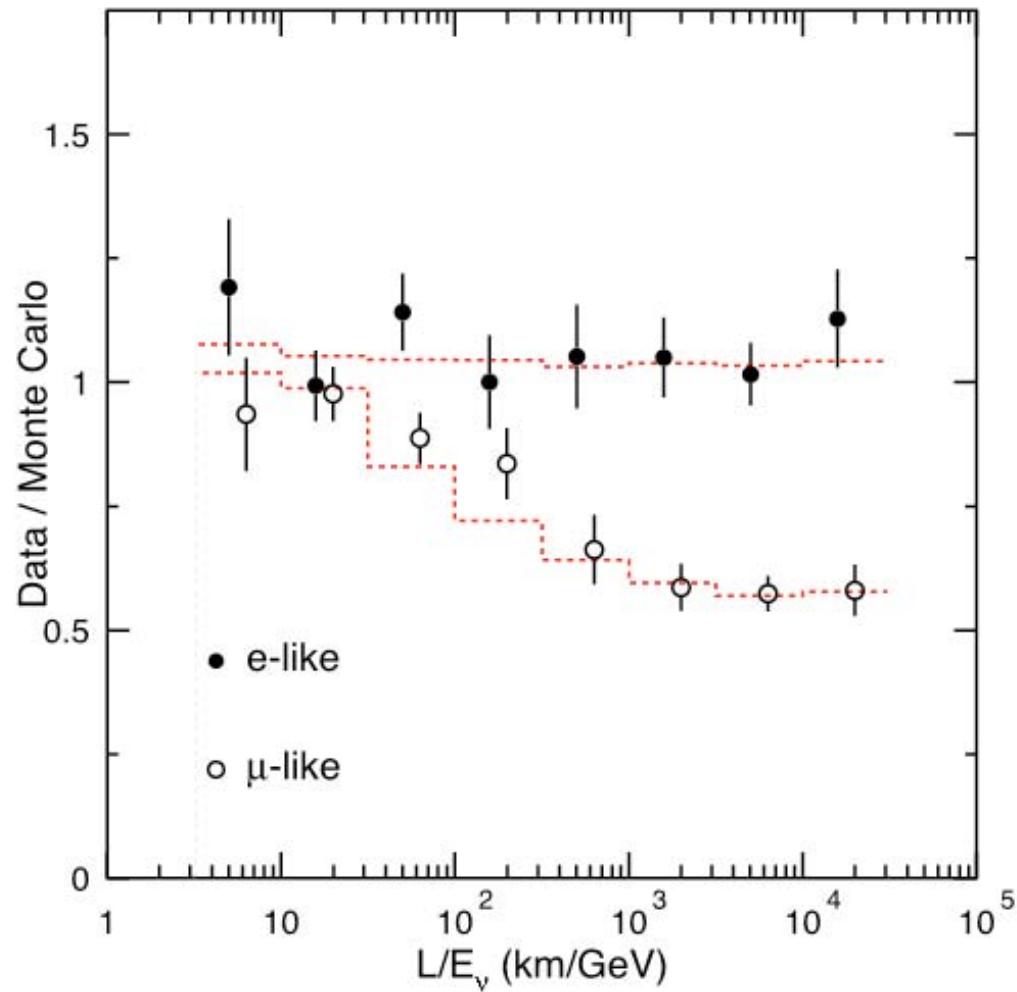


### Discovering mass

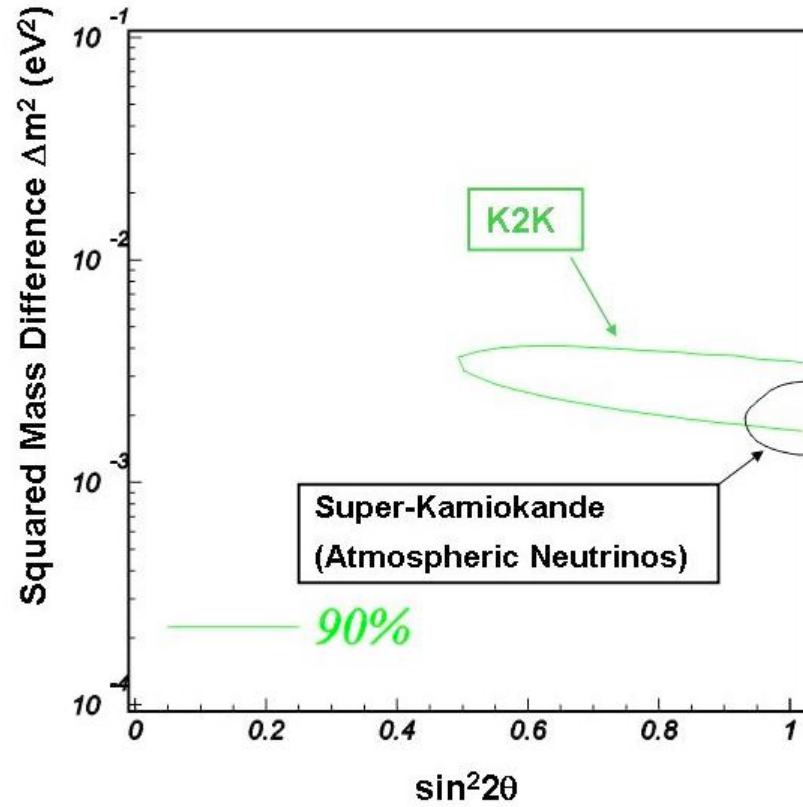
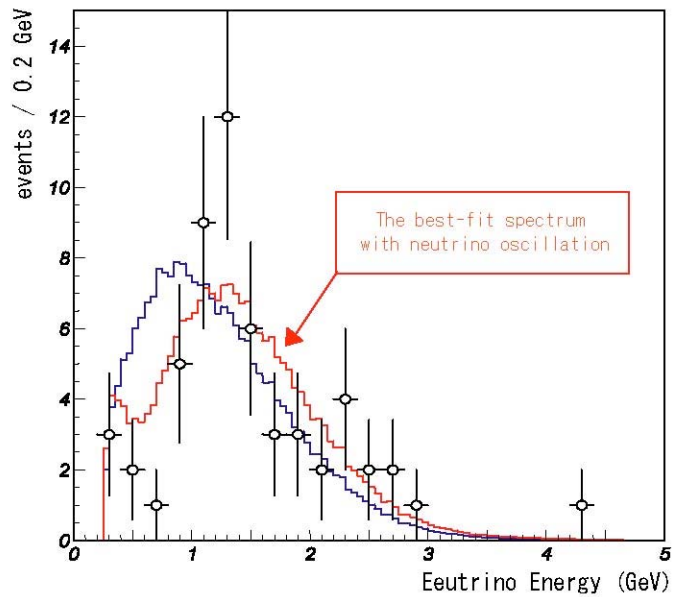
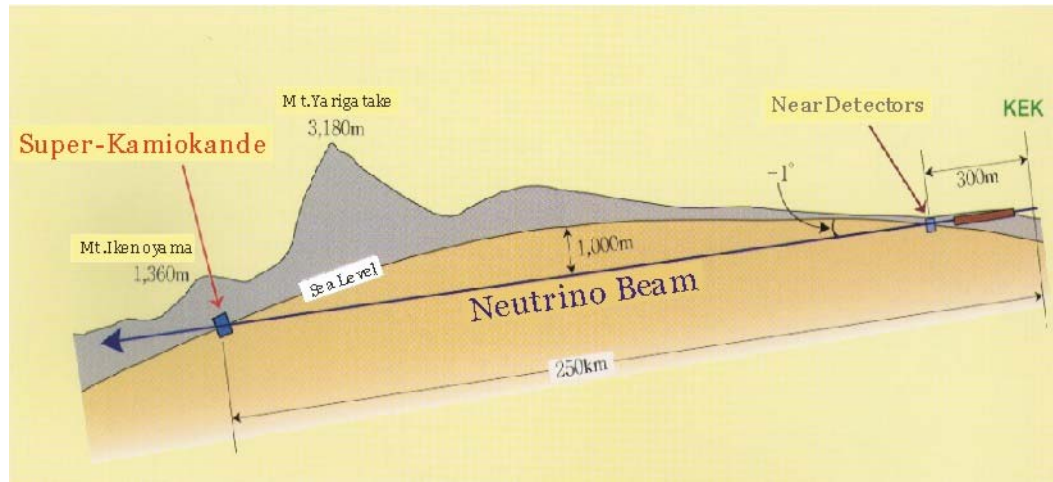
The farther neutrinos travel, the more time they have to oscillate. By comparing the ratio of flavors of neutrinos coming "up" through the Earth to those coming from overhead, physicists determined that neutrinos oscillate, which neutrinos can only do if they have mass.



# Atmospheric Neutrino Oscillations

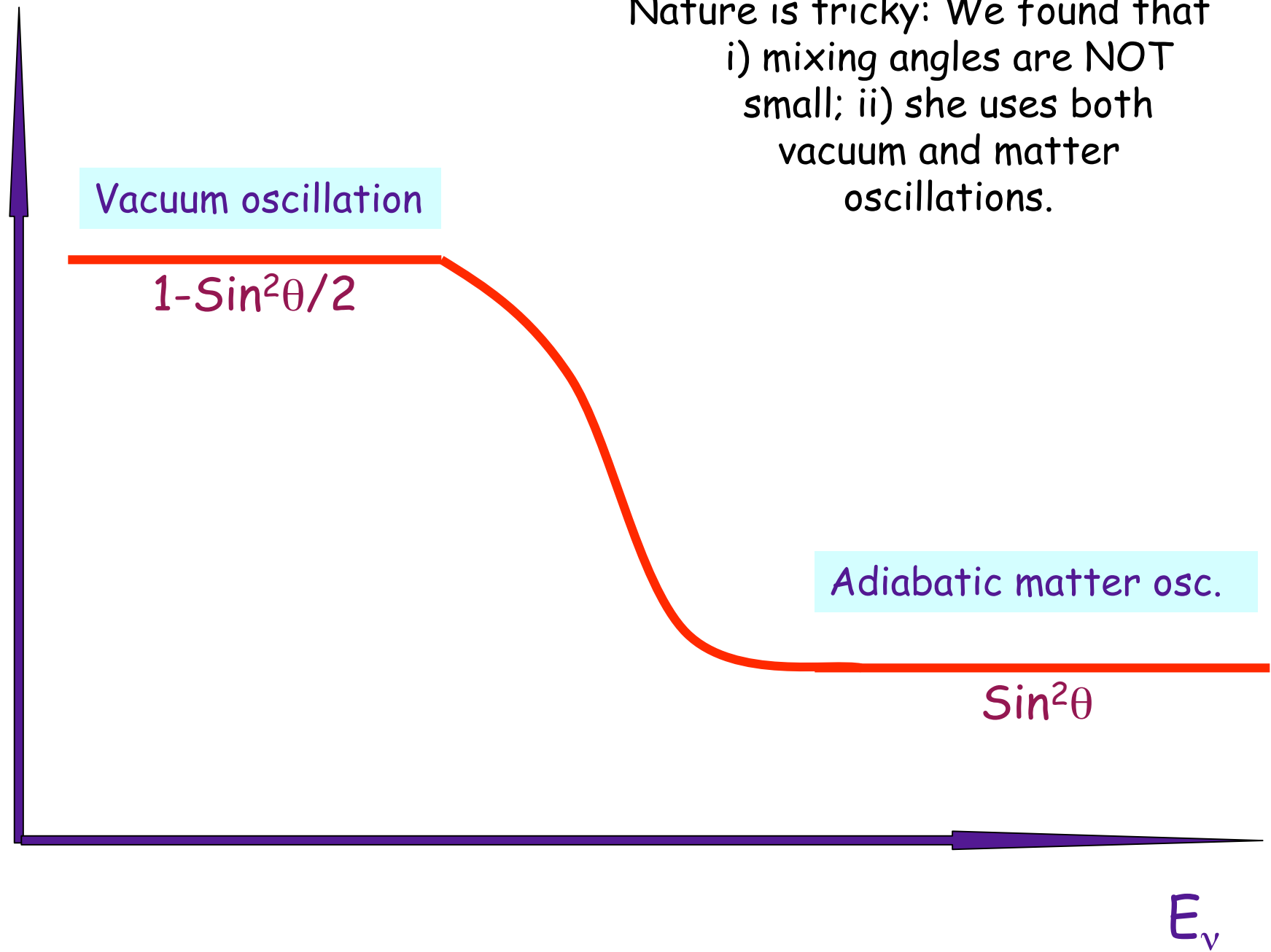


# K2K to SuperKamioka





$P(\nu_e \rightarrow \nu_e)$



Vacuum oscillation

$1 - \sin^2\theta/2$

Adiabatic matter osc.

$\sin^2\theta$

$E_\nu$

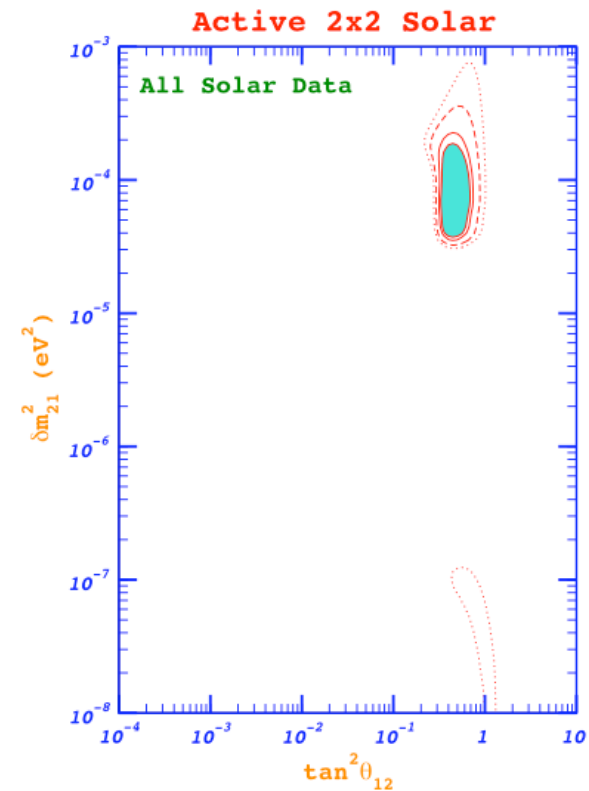
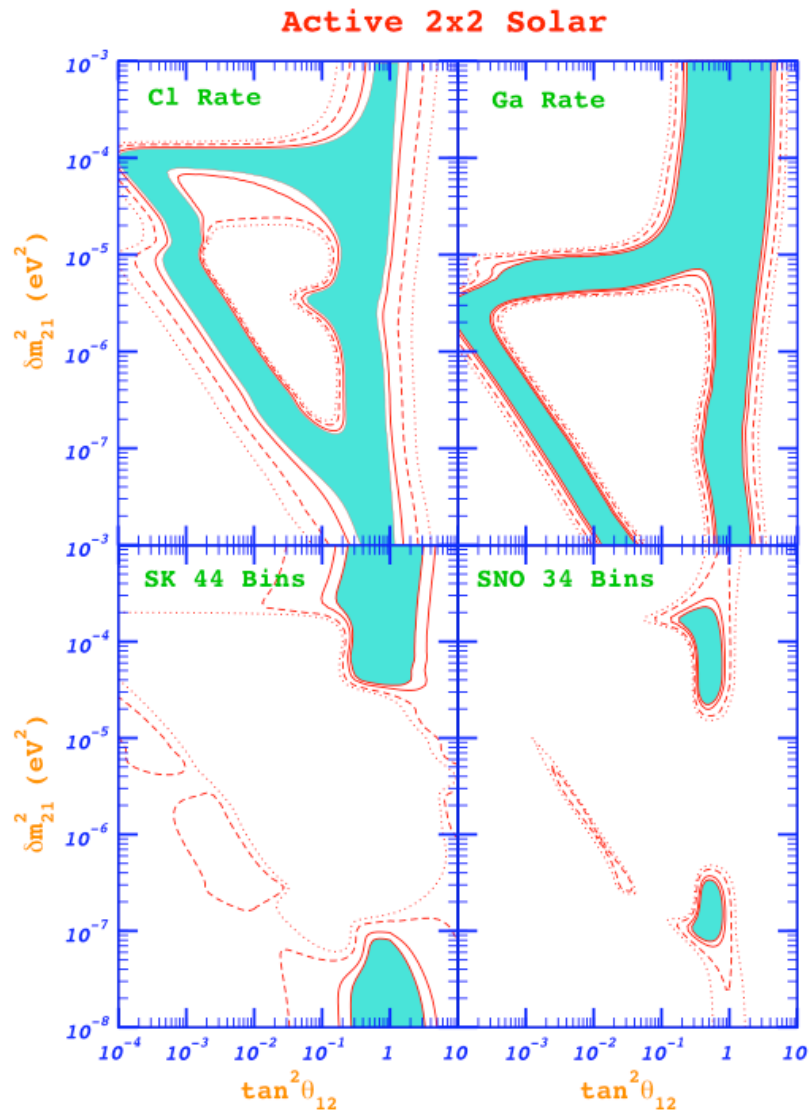
Nature is tricky: We found that  
i) mixing angles are NOT small;  
ii) she uses both vacuum and matter oscillations.

Typically solar neutrino analyses assume that  $\nu_e$  mixes with a combination of  $\nu_\mu$  and  $\nu_\tau$ . This is exact only when  $\theta_{13}$  is zero. When  $\theta_{13}$  is non-zero, but small we can use

$$P_{3 \times 3}(\nu_e \rightarrow \nu_e) = \cos^4 \theta_{13} P_{2 \times 2}(\nu_e \rightarrow \nu_e \text{ calc. with } \cos^2 \theta_{13} N_e) + \sin^4 \theta_{13}$$

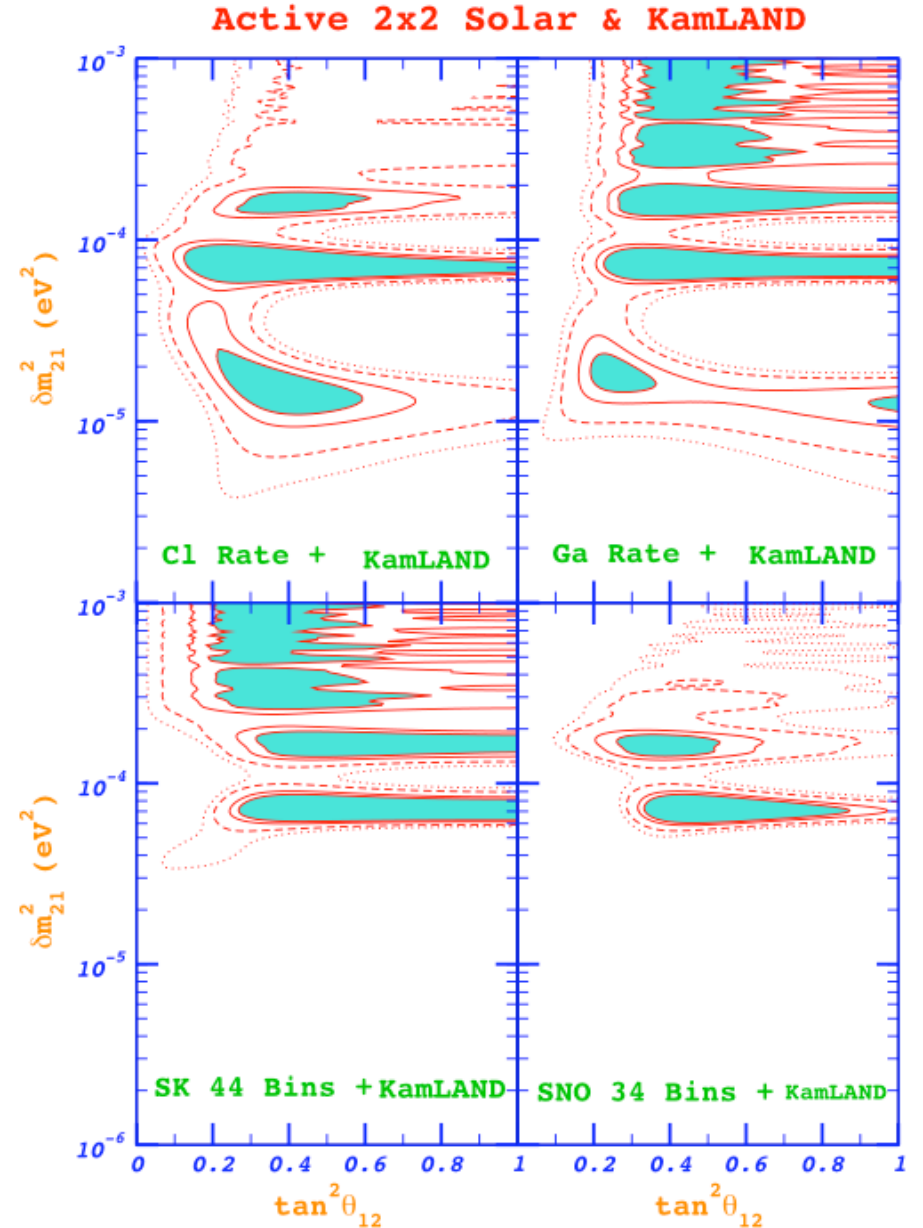
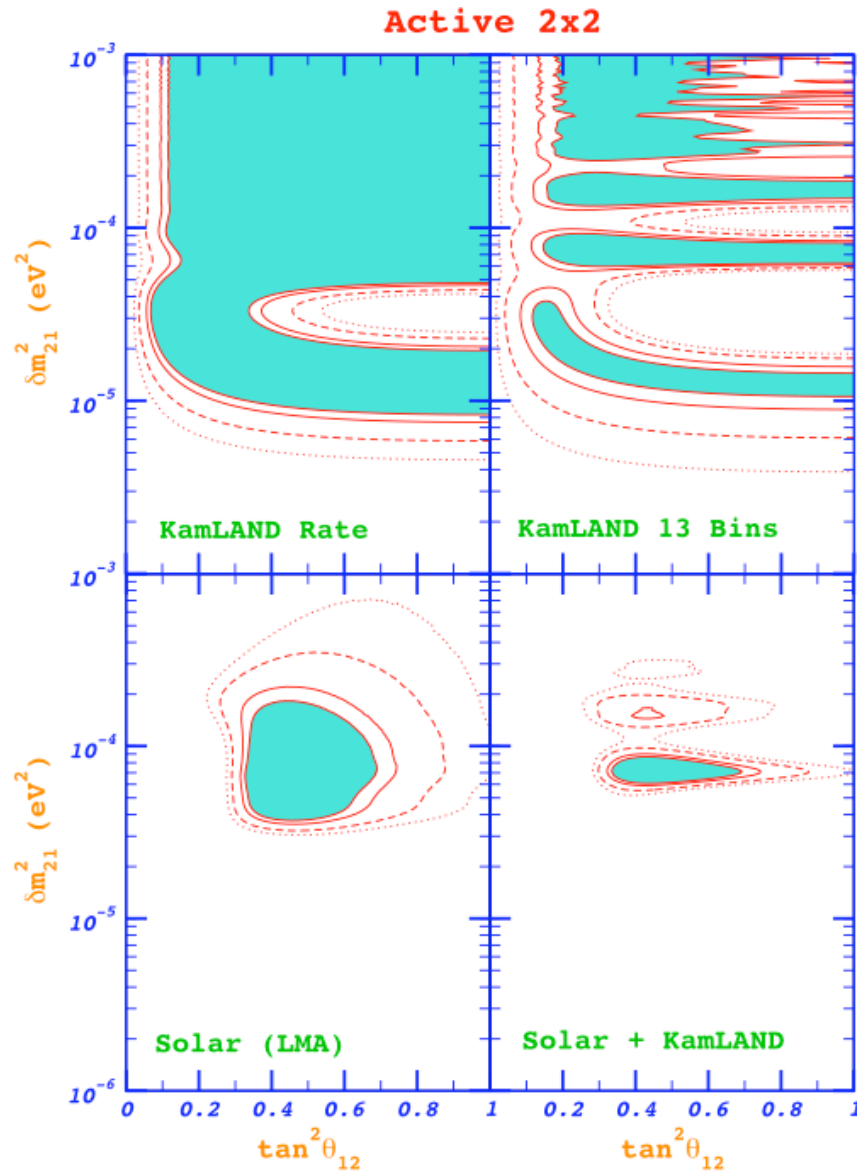
This works both for vacuum and matter oscillations...

# A global analysis of the solar neutrino data

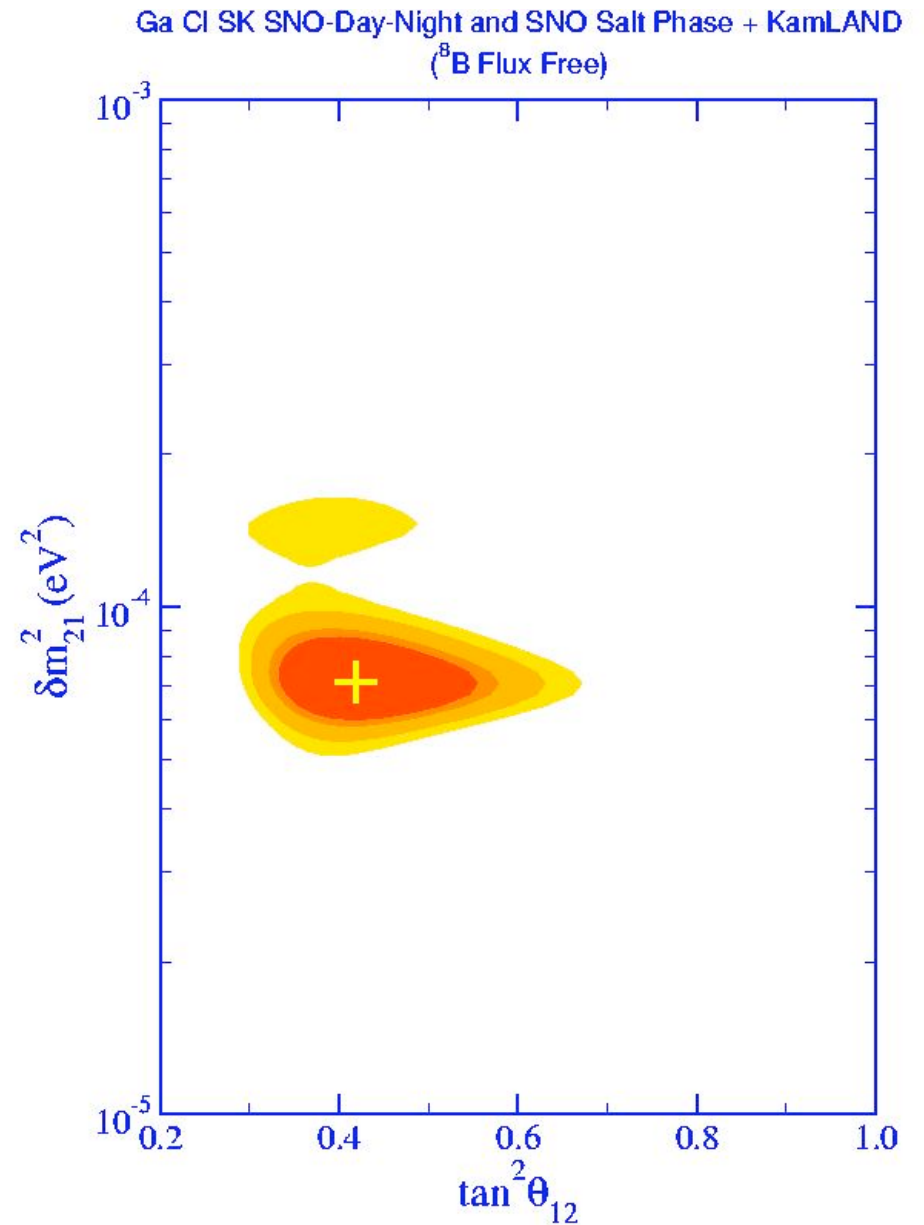
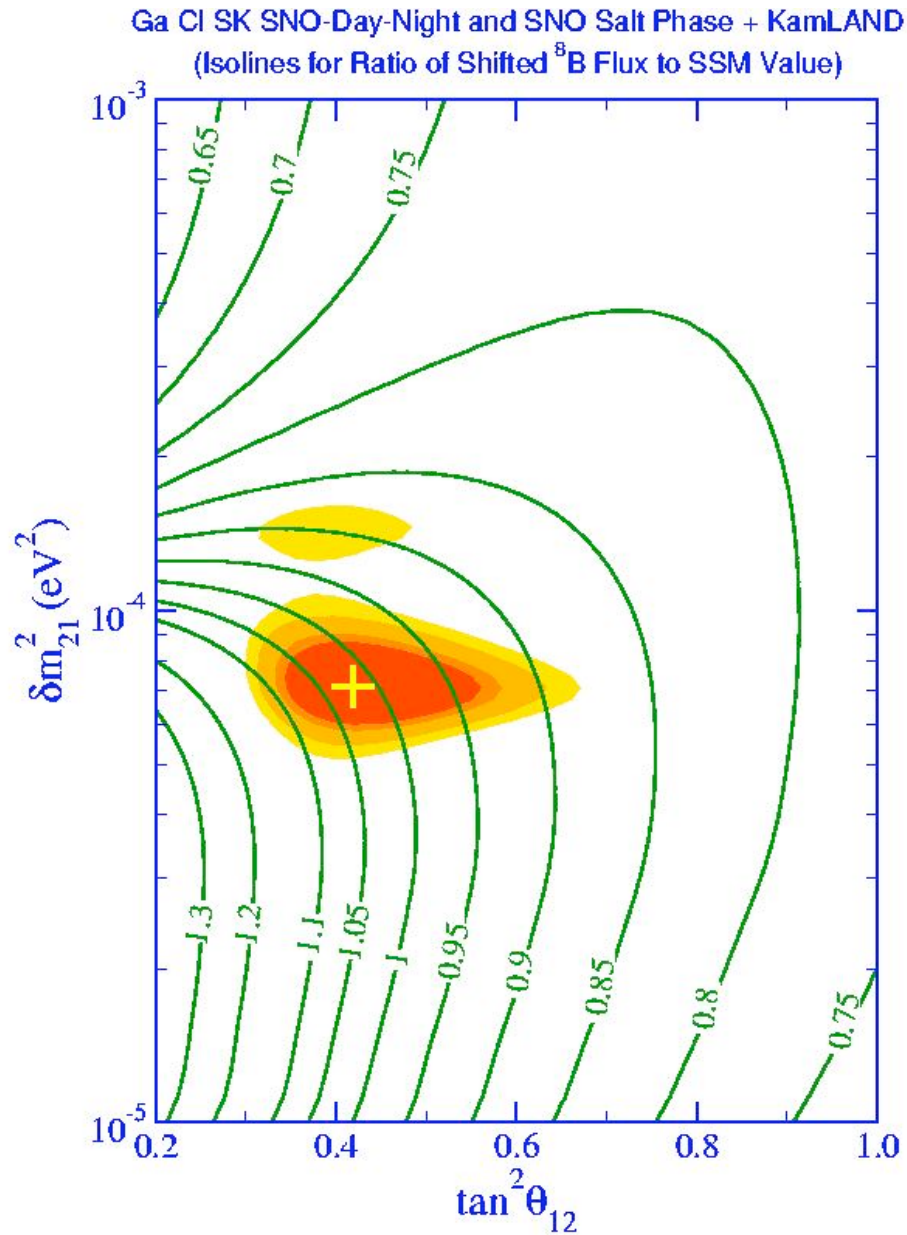


Balantekin & Yuksel, J.  
Phys. G 29, 665 (2003).

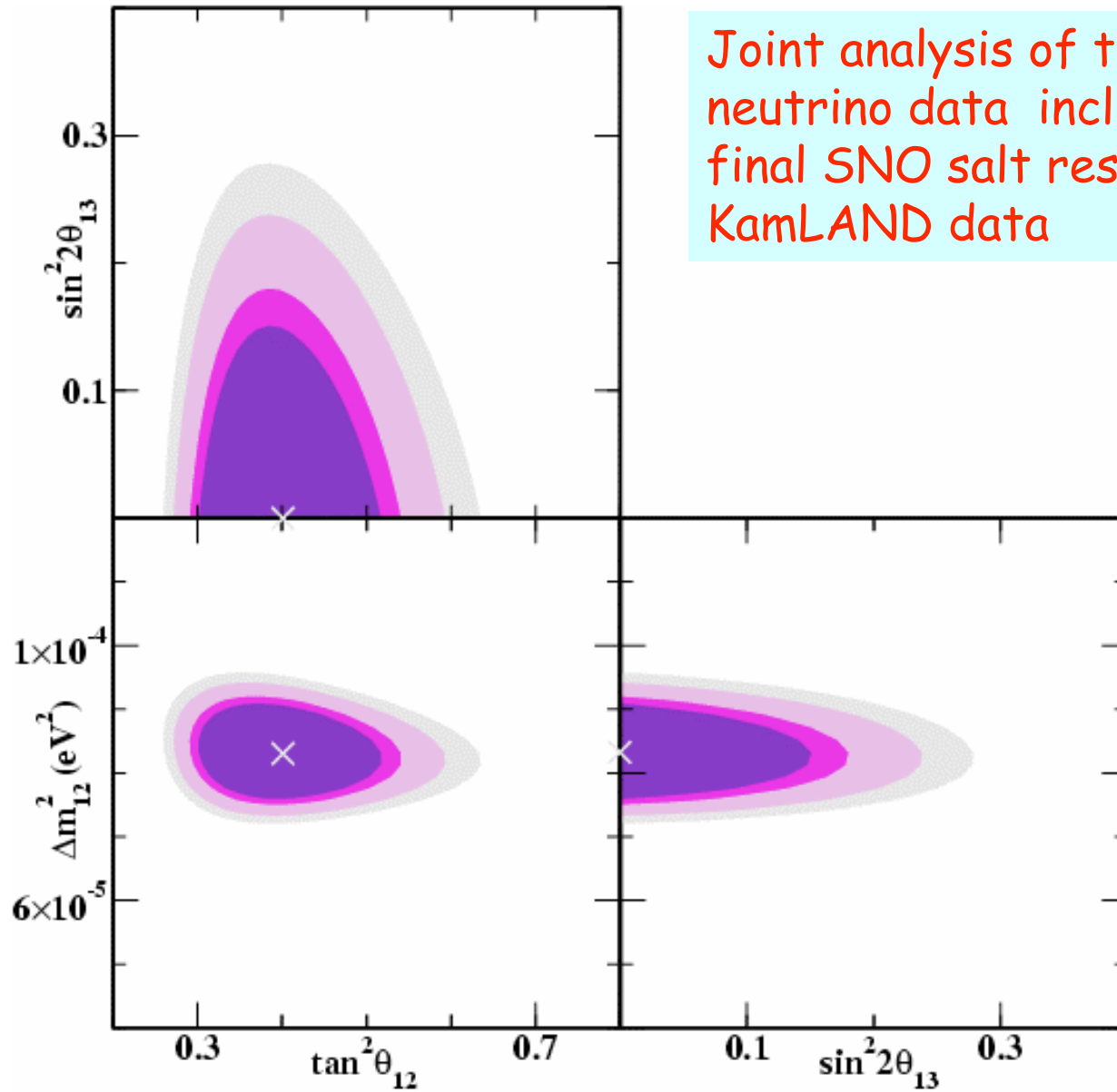
# Solar + KamLAND Global Analysis



SNO first Salt Results , Balantekin and Yuksel, PRD 68, 113002 (2003)

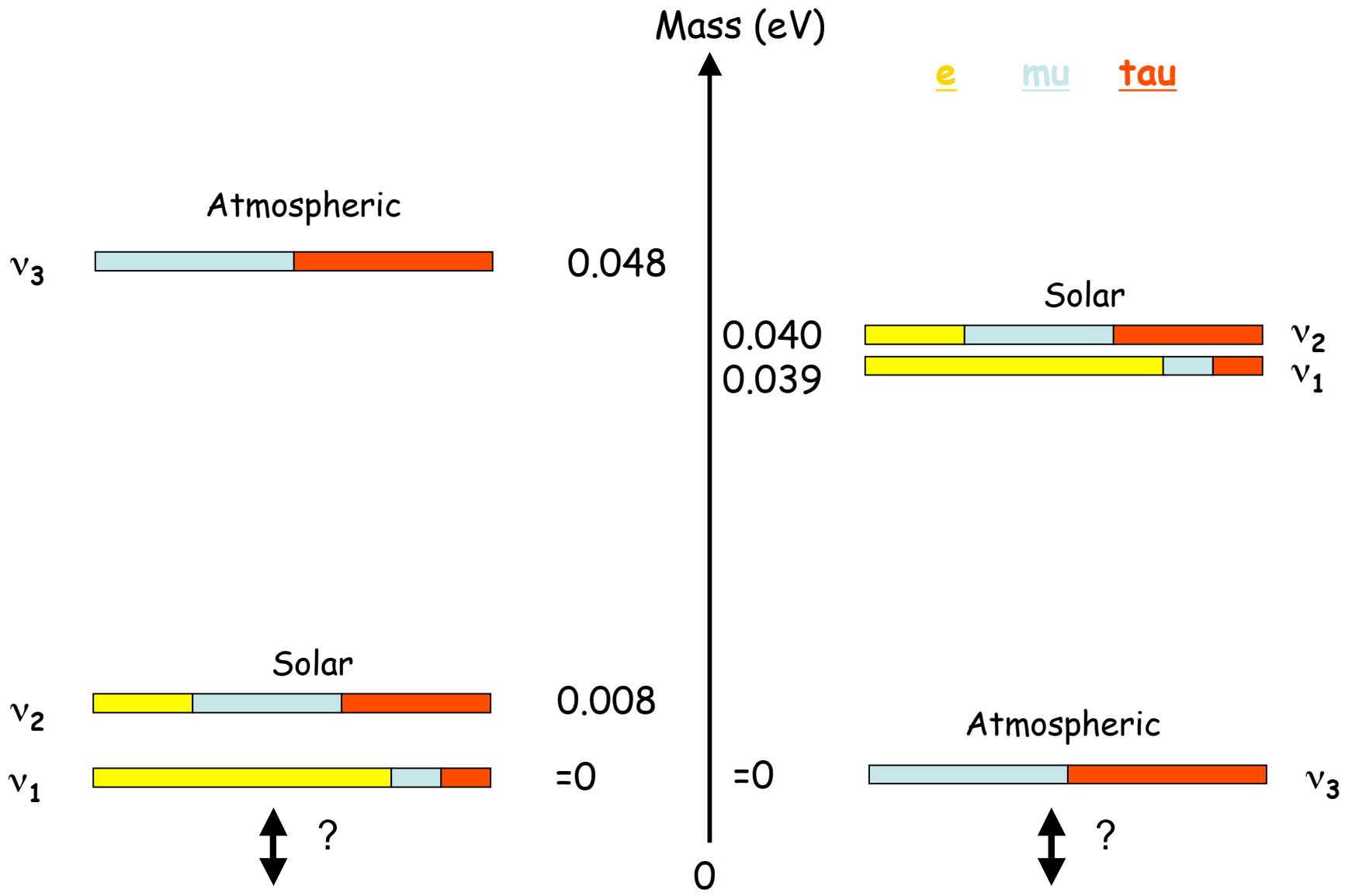


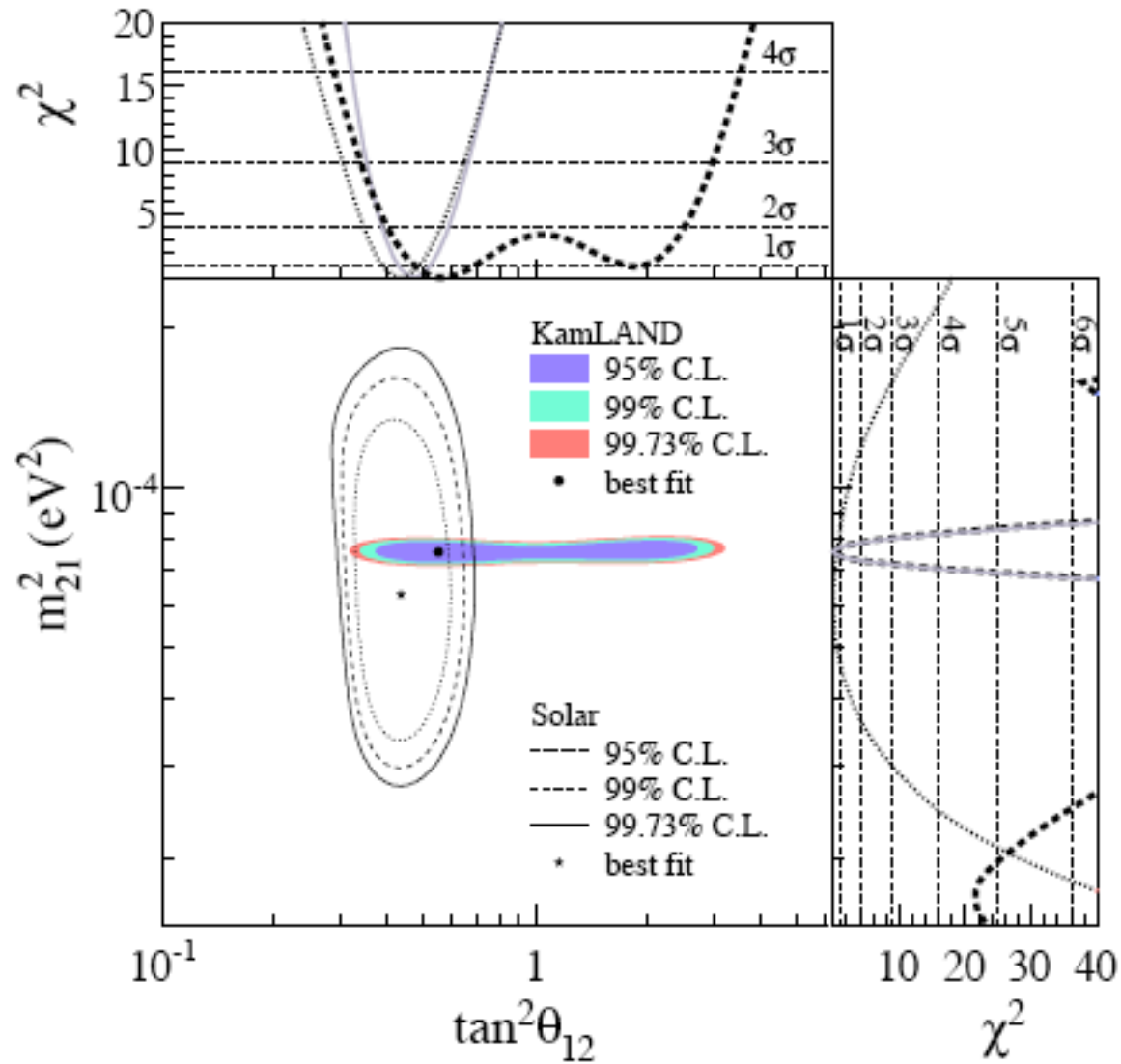
Joint analysis of the solar neutrino data including final SNO salt results and KamLAND data



Balantekin, et al., PLB 613, 61 (2005)

# Neutrino Masses and Flavor Content

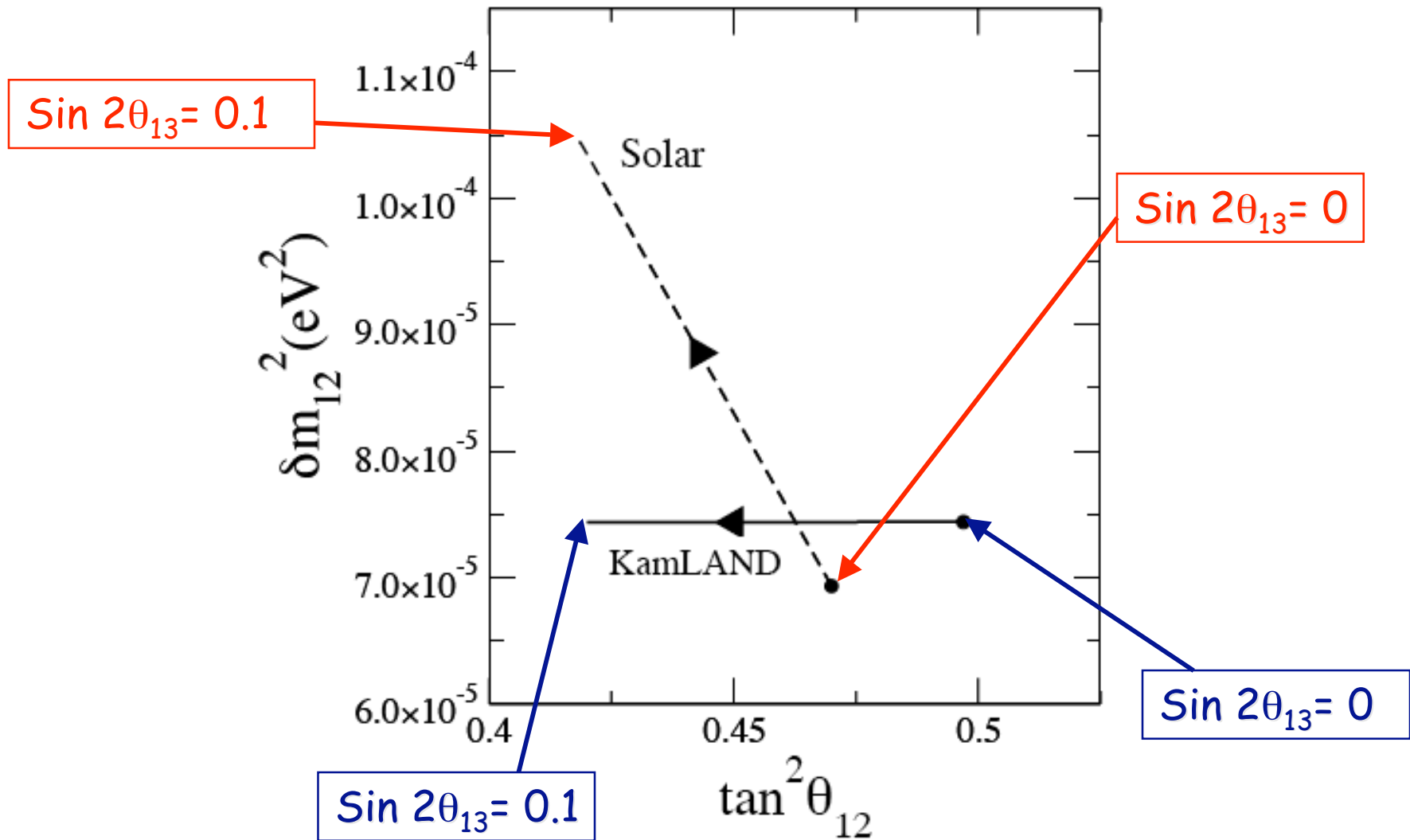


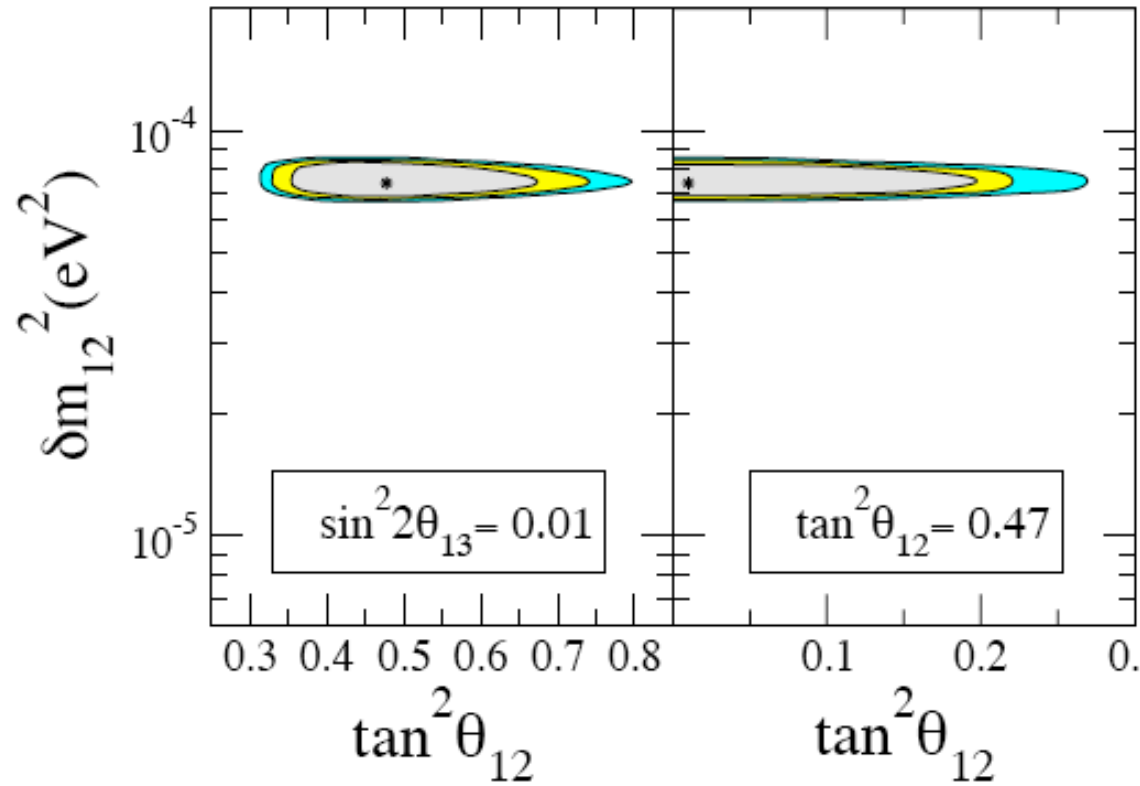


KamLAND and solar best fit values are not the same!  
 CPT-violation? Other new physics?

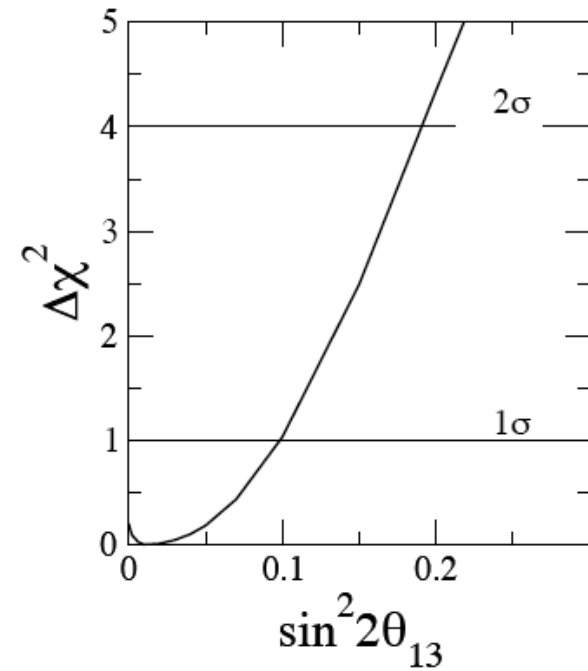


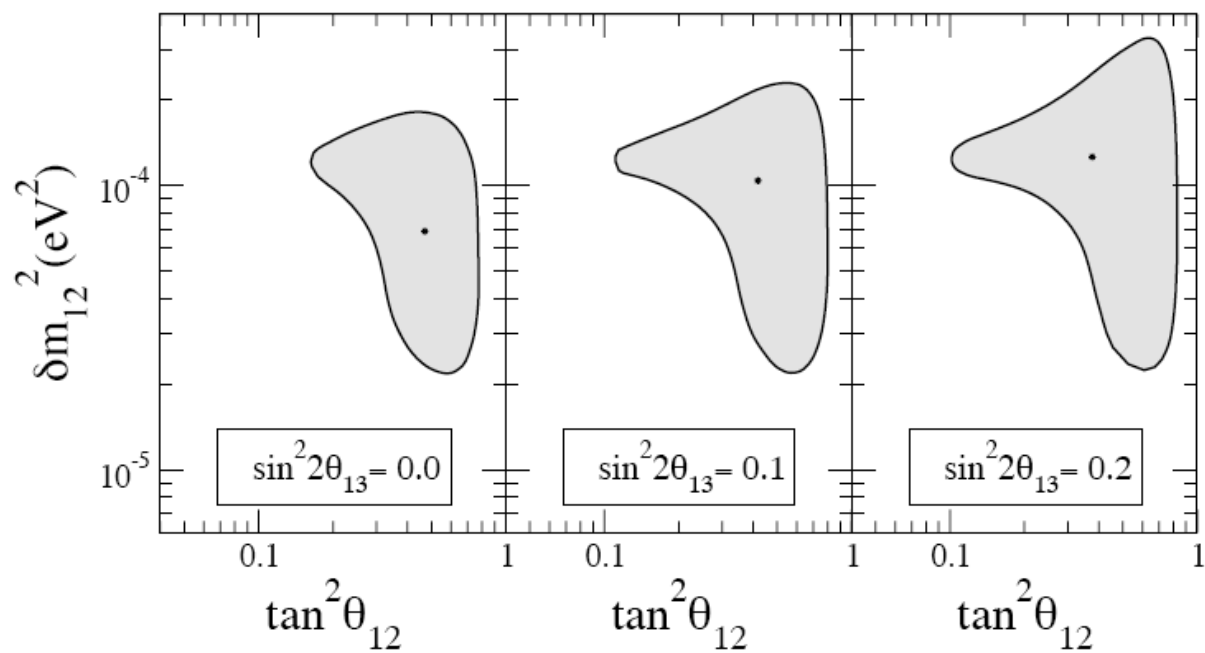
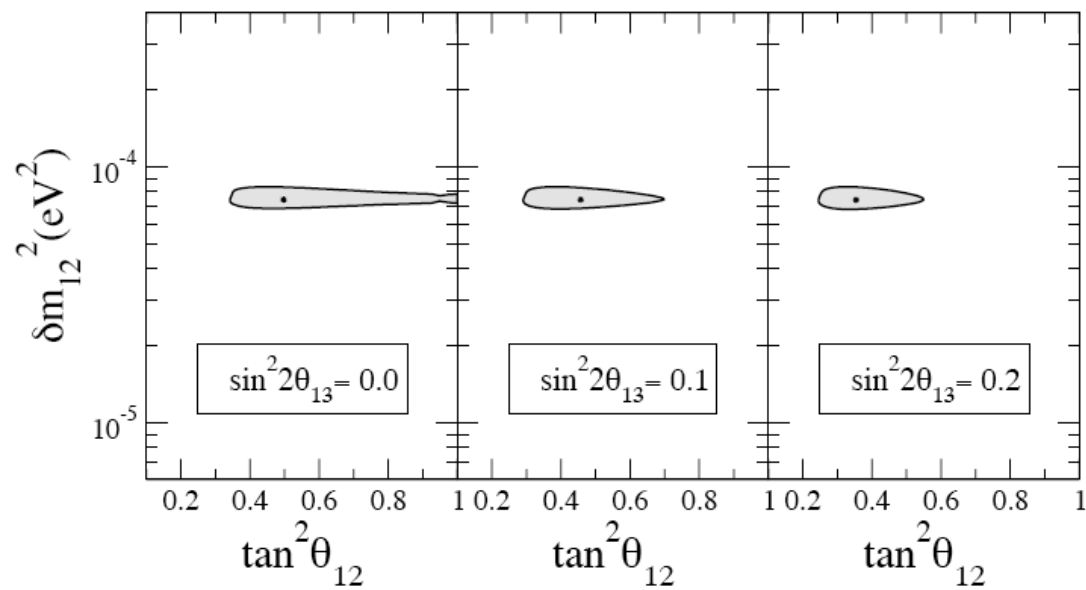
....or is it simply ignoring  $\theta_{13}$  ?





A.B. Balantekin, D. Yilmaz,  
 arXiv:0804.3345

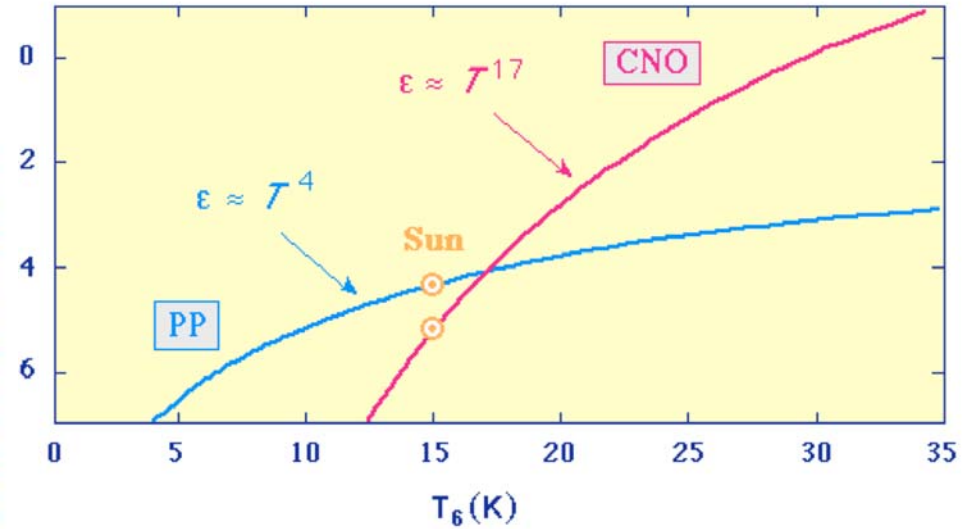
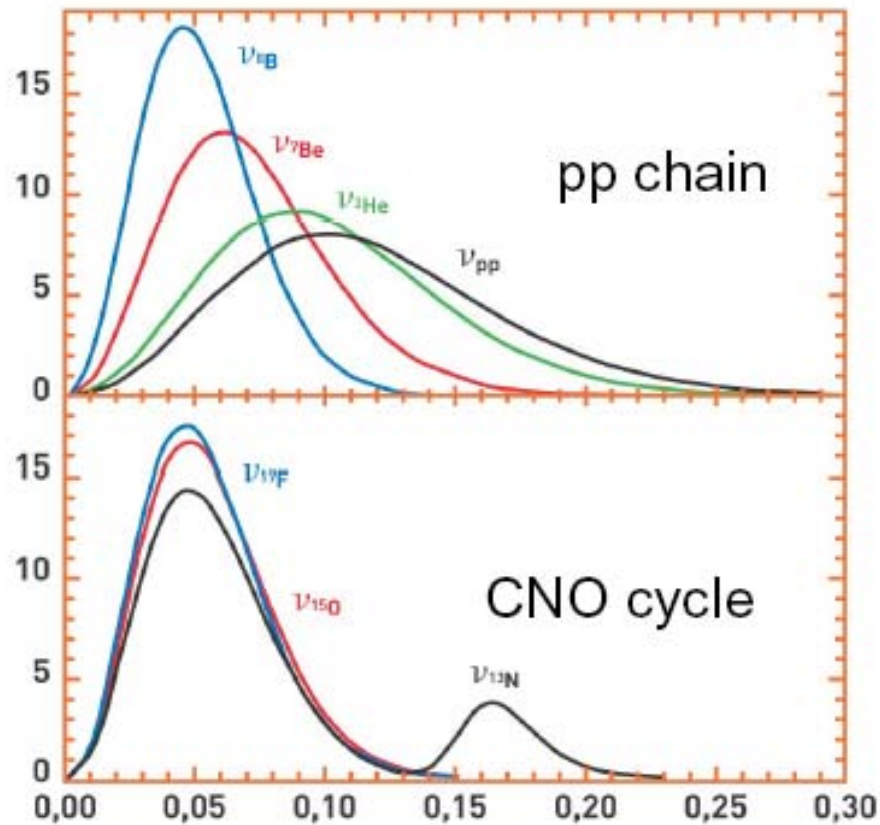




Are nuclear fusion reactions the only source of solar energy? To answer this question we need to accurately measure solar neutrino luminosity!

- This is a crucial test of energy generation during the main stage of stellar evolution.
- It is a test independent of the detailed dynamics of the solar models.
- It requires pp, pep,  ${}^7\text{Be}$ , and CNO neutrino fluxes.
- Present uncertainty is very big, but a few percent of accuracy is within reach.

# How much does the CNO cycle contribute in the Sun?



In SSM CNO cycle contribute about 0.8% of the neutrino flux. Data are consistent with this. A more precise measurement of the CNO contribution will provide a test of SSM.