Electromagnetic structure of light nuclei and the current operator

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Outline

- Nuclear forces: a brief overview
- Nucleon-nucleon interaction: the $v8$ potential
- Meson exchange potentials and construction of the effective propagator
- The nuclear electromagnetic currents
- Preliminary results and conclusions

Nuclear Forces

- quantum chromodynamics (QCD) is the fundamental theory of strong interaction; on this basis, the nucleon-nucleon (NN) interaction is completely determined by the underlying quark-gluon dynamics
- due to the difficulties of solving QCD in the low-energy regime, we are ”far” from a quantitative understanding of the NN force from this point of view $\rightarrow$ introduction nuclear models

Basic Model

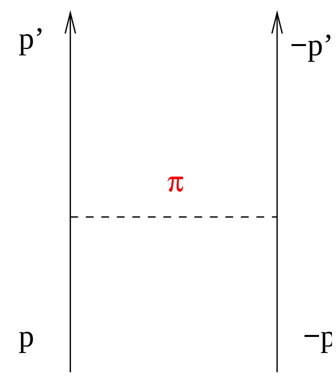
- nuclear degrees of freedom: nucleons (p, n)
  $\rightarrow$ energies below the π threshold ($K^{CM} \simeq 140\ MeV$)

- nucleon typical velocity $v^2 \sim 0.05$
  $\rightarrow$ non relativistic treatment; hence, assuming that the nucleus is a composite system of A interacting nucleons, the Hamiltonian is given by:

$$H = \sum_{i=1}^{A} \left(m_i + \frac{\mathbf{p}_i^2}{2m_i} \right) + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + ...$$

v_{ij} and V_{ijk} are the 2- and 3-nucleon interaction operators;
realistic models of v_{ij}, V_{ijk} are based on experimental data fitting, therefore the quark-gluon dynamics is included in the parametrization

One-pion-exchange potential (OPEP)



- fortunately, there is a strong experimental support to the theoretical expectation, that the long range part of the potential is due to pion exchange processes, therefore only their short range parts need to be modeled; all realistic model contain the OPEP

- meson-exchange idea goes back to Yukawa who, in 1935, stated that the nuclear force is mediated by a massive-particle exchange, which suggests itself as a tool for describing the finite range of the nuclear force (range$\sim \frac{1}{m_{meson}}$); the π meson was then observed in 1947

- the interaction of the pion field with a Pauli nucleon is described by the Hamiltonian:

$$
\begin{aligned}
H_{\pi NN} &= -\frac{f_{\pi NN}}{m_\pi}\int d^3r\, \psi^\dagger(\mathbf{r})\boldsymbol{\sigma}\cdot[\nabla\phi(\mathbf{r})\cdot\boldsymbol{\tau}]\psi(\mathbf{r}) \\
\psi(\mathbf{r}) &= \sum_{\mathbf{p},s,t}\frac{1}{\sqrt{2E_{\mathbf{p}}}} b_{\mathbf{p},s,t}e^{i\mathbf{p}\cdot\mathbf{r}}\chi^s\eta^t \\
\phi_\pm(\mathbf{r}) &= \sum_{\mathbf{k}}\frac{1}{\sqrt{2\omega_{\mathbf{k}}}}\left[a_{\mathbf{k}\mp}e^{i\mathbf{k}\cdot\mathbf{r}} + a^\dagger_{\mathbf{k}\pm}e^{-i\mathbf{k}\cdot\mathbf{r}}\right] \\
\phi_0(\mathbf{r}) &= \sum_{\mathbf{k}}\frac{1}{\sqrt{2\omega_{\mathbf{k}}}}\left[a_{\mathbf{k}0}e^{i\mathbf{k}\cdot\mathbf{r}} + a^\dagger_{\mathbf{k}0}e^{-i\mathbf{k}\cdot\mathbf{r}}\right]
\end{aligned}
$$

the $f_{\pi NN}$ is the dimensionless pion-nucleon coupling constant

- the evaluation of the one-pion-exchange amplitude, which by definition equals the OPEP in the momentum space, gives:

$$
v^{\pi}{}_{ij} = -\frac{f^2_{\pi NN}}{m^2_\pi}\frac{\boldsymbol{\sigma}_i\cdot\mathbf{q}\boldsymbol{\sigma}_j\cdot\mathbf{q}}{q^2+m^2_\pi}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j
$$

- the configuration space $v_\pi(\mathbf{r})$ is obtained from its Fourier transform:

$$
\begin{aligned}
{v^{\pi}}_{ij} &= -\frac{f^2_{\pi NN}}{4\pi}\frac{m_\pi}{3}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\left[T_\pi S_{ij} + [Y_\pi - \frac{4\pi}{m^3}_\pi\delta(\mathbf{r})]\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right] \\
S_{ij} &= 3\boldsymbol{\sigma}_i\cdot\hat{\mathbf{r}}\boldsymbol{\sigma}_j\cdot\hat{\mathbf{r}} - \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j
\end{aligned}
$$

S_{ij} is the tensor operator, Y_π and T_π are functions of r

- the description of the long-range part of the NN interaction depends on only one parameter i.e. the pion-nucleon coupling constant $f_{\pi NN}$; the value obtained in 1993 from NN scattering data is:

$$\frac{f^2_{\pi NN}}{4\pi} = 0.075$$

- the Nijmegen group fitted the NN scattering data assuming that the long range part is given by the exchange of a pseudoscalar-isovector particle of mass m, the best fit was obtained with $m = m_\pi$

Realistic model

- we can separate the NN potential v_{ij} as follows:

$$\begin{aligned} v_{ij} &= v_{ij}^{\gamma} + v_{ij}^{\pi} + v_{ij}^{R} \\ &= v_{ij}^{\gamma} + \underbrace{v_{ij}^{\pi} + v_{ij}^{IR} + v_{ij}^{SR}}_{strong\, terms} \end{aligned}$$

  v^R contains all interactions other than the electromagnetic and the OPEP; primarily determined from experimental data; many possible description of v^R are available

- the v_{ij} is dominated by v^{π} for $r > 1.4$ fm
- v^{IR} (intermediate range attraction) significant for $r < 1.4$ fm
- v^{SR} (repulsive core) important for $r < 0.7$ fm

The $v8$ potential

- the strong interaction part of v_{ij} can be expressed as a sum of operators:

$$v_{ij} = \sum_p v_p(r_{ij}) O^p_{ij};$$
$$v_p(r) = v^\pi_p(r) + v^{IR}_p(r) + v^{SR}_p(r)$$

where the static operator $O^{p=1,6}$ are:

$$O^{p=1,6} = 1(c), \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j(\tau), \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j(\sigma), \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j(\sigma\tau),$$
$$S_{ij}(t), S_{ij}\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j(t\tau)$$

and the two isoscalar spin-orbit operators are:

$$O^{p=7,8} = \mathbf{L} \cdot \mathbf{S}(b), \mathbf{L} \cdot \mathbf{S}\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j(b\tau)$$

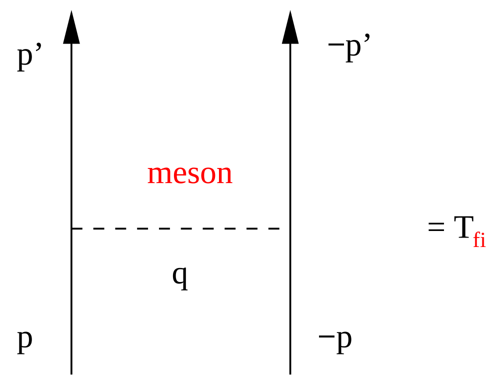
The $v8$ potential in the momentum space

$$
\begin{aligned}
v^8 &= v^c(q) + q^2 v^\sigma(q)\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + v^t(q)\boldsymbol{\sigma}_i\cdot\mathbf{q}\boldsymbol{\sigma}_j\cdot\mathbf{q} \\
&\quad + v^b(q)\frac{i}{2}(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot(\mathbf{p\prime}\times\mathbf{p}) + [...]\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j
\end{aligned}
$$

$$
\left\{
\begin{aligned}
v^c(q) &= \mathcal{F}[v^c(r)] \\
v^\sigma(q) &= \mathcal{F}[v^\sigma(r)] + \mathcal{F}[v^t(r)] \\
v^t(q) &= -3\mathcal{F}[v^t(r)] \qquad = -3\tfrac{4\pi}{q^2}\int dr\, r^2 j_2(qr)v^t(r) \\
v^b(q) &= \mathcal{F}[v^b(r)]
\end{aligned}
\right.
$$

same for $v^{\alpha,\tau}$, $\alpha = c, \sigma, t, b$

The meson exchange potential



- the idea is to evaluate the invariant amplitude T_{fi} induced by the one-boson-exchange (OBE) Lagrangians; the invariant amplitude is related to the non-relativistic (NR) potential trough the following relation:

$$v_{fi} = \frac{m}{E} T_{fi}$$

where $m = \frac{m_p + m_n}{2}$ and $E = \sqrt{\mathbf{p}^2 + m^2}$

- and subsequently make a link between the so obtained NN potentials and the $v8$ one

The One Boson Exchange (OBE) Lagrangians

scalar

$$-g^{S0}\bar{\psi}\psi\phi^{S0} \qquad -g^{S1}\bar{\psi}\boldsymbol{\tau}\psi\cdot\vec{\phi}^{S1}$$

pseudo-scalar

$$-ig^{PS0}\bar{\psi}\gamma_5\psi\phi^{PS0} \qquad -ig^{PS1}\bar{\psi}\gamma_5\boldsymbol{\tau}\psi\cdot\vec{\phi}^{PS1}$$

vector

$$-g^{V0}\bar{\psi}\gamma^{\mu}\psi\phi^{V0}{}_{\mu} \qquad -g^{V1}\bar{\psi}\gamma^{\mu}\boldsymbol{\tau}\psi\cdot\vec{\phi}^{V1}{}_{\mu}$$

tensor

$$\frac{-g^{T0}}{2m^{T0}}\bar{\psi}\sigma^{\mu\nu}\psi\partial_{\nu}\phi^{T0}_{\mu} \qquad \frac{-g^{T1}}{2m^{T1}}\bar{\psi}\sigma^{\mu\nu}\boldsymbol{\tau}\psi\cdot\partial_{\nu}\vec{\phi}^{T1}_{\mu}$$

$$
\begin{aligned}
v^{S0} &= -\frac{g_{S0}^2}{q^2+m_{S0}^2}\left[1+\frac{q^2}{8m^2}-\frac{i}{4m^2}(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot\mathbf{p\prime}\times\mathbf{p}\right] \\
v^{PS0} &= -\frac{g_{PS0}^2}{q^2+m_{PS0}^2}\frac{1}{4m^2}\boldsymbol{\sigma}_i\cdot\mathbf{q}\boldsymbol{\sigma}_j\cdot\mathbf{q} \\
v^{V0} &= \frac{g_{V0}^2}{q^2+m_{V0}^2}\left[1-\frac{q^2}{8m^2}-\frac{q^2}{4m^2}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j+\frac{1}{4m^2}\boldsymbol{\sigma}_i\cdot\mathbf{q}\boldsymbol{\sigma}_j\cdot\mathbf{q}+\right. \\
&\qquad \left.\frac{i3}{4m^2}(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot\mathbf{p\prime}\times\mathbf{p}\right] \\
v^{T0} &= \underbrace{\frac{g_{T0}^2}{q^2+m_{T0}^2}}_{bare\;propagator}\left[-\frac{q^2}{4m^2}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j+\frac{1}{4m^2}\boldsymbol{\sigma}_i\cdot\mathbf{q}\boldsymbol{\sigma}_j\cdot\mathbf{q}\right]
\end{aligned}
$$

the $v^{\alpha,1}$ α=S, PS, V, T are obtained by multiplying by $\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$ and $(\alpha,0)\leftrightharpoons(\alpha,1)$

The $v8$ potential in the momentum space (bis)

$$
\begin{aligned}
v^8 &= v^c(q) + q^2 v^\sigma(q)\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + v^t(q)\boldsymbol{\sigma}_i \cdot \mathbf{q}\boldsymbol{\sigma}_j \cdot \mathbf{q} \\
&+ v^b(q)\frac{i}{2}(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \cdot (\mathbf{p'} \times \mathbf{p}) + [...]\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j
\end{aligned}
$$

$$
\left\{
\begin{array}{lcc}
v^c(q) & = & \mathcal{F}[v^c(r)] \\
v^\sigma(q) & = & \mathcal{F}[v^\sigma(r)] + \mathcal{F}[v^t(r)] \\
v^t(q) & = & -3\mathcal{F}[v^t(r)] \\
v^b(q) & = & \mathcal{F}[v^b(r)]
\end{array}
\right.
$$

same for $v^{\alpha,\tau}$, $\alpha = c, \sigma, t, b$

The effective propagators D

- now we are able to construct the effective propagators by comparing the OBE potential with the $v8$
- define

$$D^{\alpha,\beta} = \frac{g^2_{\alpha,\beta}}{q^2 + m^2_{\alpha,\beta}}$$

$\alpha = S, PS, V, T$, $\beta = 0, 1$

- hence we are left with the following relations:

$$D^{S0}(q) = \frac{m^2}{2}v^b - \frac{3}{4}v^c \qquad D^{PS0}(q) = -4m^2v^t - 4m^2v^\sigma$$

$$D^{V0}(q) = \frac{m^2}{2}v^b + \frac{1}{4}v^c \qquad D^{T0}(q) = -\frac{1}{4}v^c - 4m^2v^\sigma - \frac{m^2}{2}v^b$$

Electromagnetic current

- in order to study and predict the behavior of our system in presence of an external probe, the model must contain currents operator describing the interaction of the nuclei with the external fields
- those operators are expanded as a sum of $1-$, $2-$, ... nucleon operators
- the charge density $\rho(\mathbf{q})$ and the current density $\mathbf{j}(\mathbf{q})$ of the nucleus have the form:

$$
\begin{aligned}
\rho(\mathbf{q}) &= \sum_{i=1}^{A} \rho_i(\mathbf{q}) + \sum_{i<j} \rho_{ij}(\mathbf{q}) + \dots \\
\mathbf{j}(\mathbf{q}) &= \sum_{i=1}^{A} \mathbf{j}_i(\mathbf{q}) + \sum_{i<j} \mathbf{j}_{ij}(\mathbf{q}) + \dots
\end{aligned}
$$

One body current operators

- the one body operators describe the current of a single nucleon; nucleons in nuclei absorb photons via their charges and magnetic moments
- the one body charge operator is given by:

$$\rho_i(\mathbf{q}) = \frac{1}{2}[G_E^S(q) + G_E^V(q)\tau_{i,z}]e^{i\mathbf{q}\cdot\mathbf{r}_i}$$

- the one body convention current operator is given by:

$$\mathbf{j}_{c,i}(\mathbf{q}) = \frac{1}{4m_i}[G_E^S(q) + G_E^V(q)\tau_{i,z}][\nabla_i\prime e^{i\mathbf{q}\cdot\mathbf{r}_i} - e^{i\mathbf{q}\cdot\mathbf{r}_i}\nabla_i]$$

- the one body current operator due to the nucleon magnetic moment interaction with the field is:

$$\mathbf{j}_{m,i}(\mathbf{q}) = e^{i\mathbf{q}\cdot\mathbf{r}_i}\frac{1}{2m_i}[G_M^S(q) + G_M^V(q)\tau_{i,z}](-i)(\boldsymbol{\sigma}_i \times \mathbf{q})$$

Two body current operators

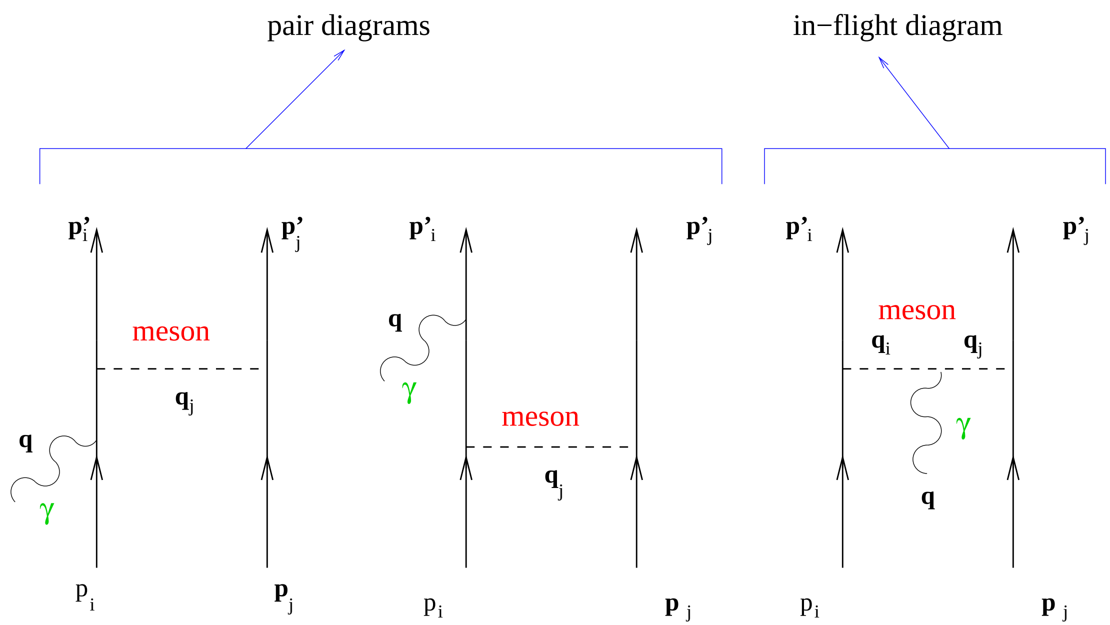


photo-meson currents:the photon produces a meson by hitting the nucleon, the meson is virtual and has to be reabsorbed by an other nucleon

meson currents: the photon is absorbed by the current of a charged meson being exchanged by the nucleons

- the photo-meson and meson-exchange currents are derived by evaluating the invariant amplitudes associated to the Feynman diagrams induced by the OBE Lagrangians
- the bare propagators are then replaced by the effective propagators fixed by the chosen NN potential
- the present method generalizes the procedure adopted in L. E. Marcucci *et all* Phys.Rev. **C**72, 014001,2005 to obtain the effective π and ρ meson propagators

The $PS1$ current operator in the momentum space

$$\mathbf{j}^{PS1} \quad (\mathbf{q}_i, \mathbf{q}_j) = \frac{-i}{4m^2} F_1^V (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_z D^{PS1}(q_j) \boldsymbol{\sigma}_i (\boldsymbol{\sigma}_j \cdot \mathbf{q}_j) + i \leftrightharpoons j$$

$$\frac{-i}{4m^2} (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_z \frac{\mathbf{q}_i - \mathbf{q}_j}{q_i^2 - q_j^2} \boldsymbol{\sigma}_i \cdot \mathbf{q}_i \boldsymbol{\sigma}_j \cdot \mathbf{q}_j \left[D^{PS1}(q_i) - D^{PS1}(q_j) \right]$$

$\mathbf{q}_i$, $\mathbf{q}_j$ are the fractional momenta delivered to nucleons i and j;

$\mathbf{q}_i = \mathbf{p}_{i'} - \mathbf{p}_i$ same for j

The current operators in the confi guration space

- the configuration-space expressions of the currents are obtained from:

$$\begin{aligned} \mathbf{j}^{\alpha,\beta}(\mathbf{q};\mathbf{r}_i,\mathbf{r}_j) &= \int \frac{d\mathbf{q}_i}{(2\pi)^3}\frac{d\mathbf{q}_j}{(2\pi)^3} e^{i\mathbf{q}_i\cdot\mathbf{r}_i} e^{i\mathbf{q}_j\cdot\mathbf{r}_j} \\ &\times (2\pi)^3\delta(\mathbf{q}_i+\mathbf{q}_j-\mathbf{q})\,\mathbf{j}^{\alpha,\beta}(\mathbf{q}_i,\mathbf{q}_j) \end{aligned}$$

with $\alpha = S\,,PS\,,V\,,T$ and $\beta = 0\,,1$

The magnetic form factor

- we fix the coordinate system so that the momentum transfer $\mathbf{q}$ lies in the x direction, and the z axis is taken as the quantization axis for the magnetic quantum number
- the elastic form factor is then:

$$F(q) = \sqrt{2}\langle 0_R | j_y(q\hat{\mathbf{x}}) | 0 \rangle$$

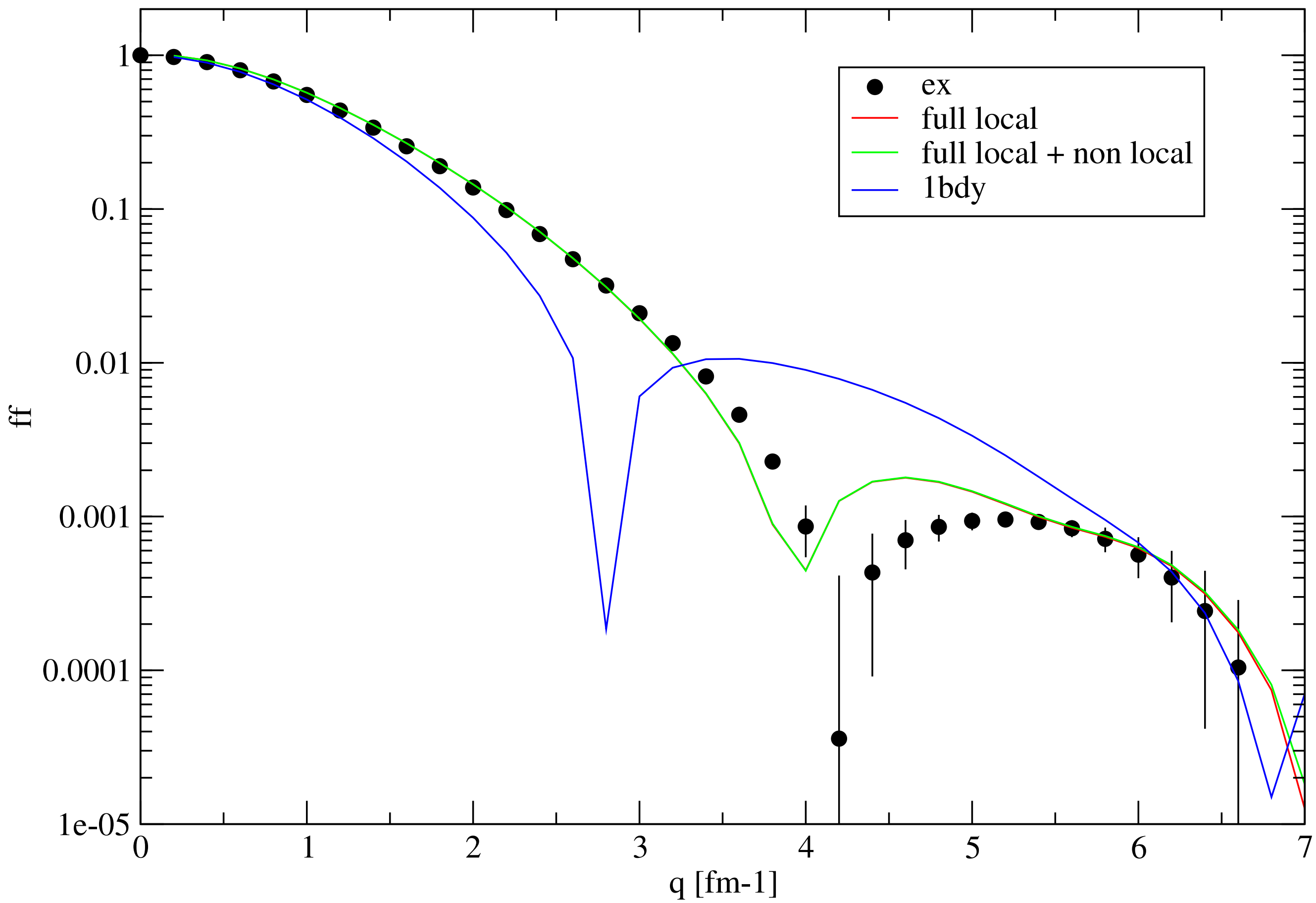
where $|0_R\rangle$ denotes the ground state recoiling with momentum $\mathbf{q} = q\hat{\mathbf{x}}$; in the limit $q \to 0$ the elastic form factor behaves like:

$$F(q) \simeq \frac{1}{\sqrt{2}} \frac{q}{m} \mu$$

where μ is the magnetic moment of the nucleus in nuclear magnetons the normalized magnetic from factor is then defined as:

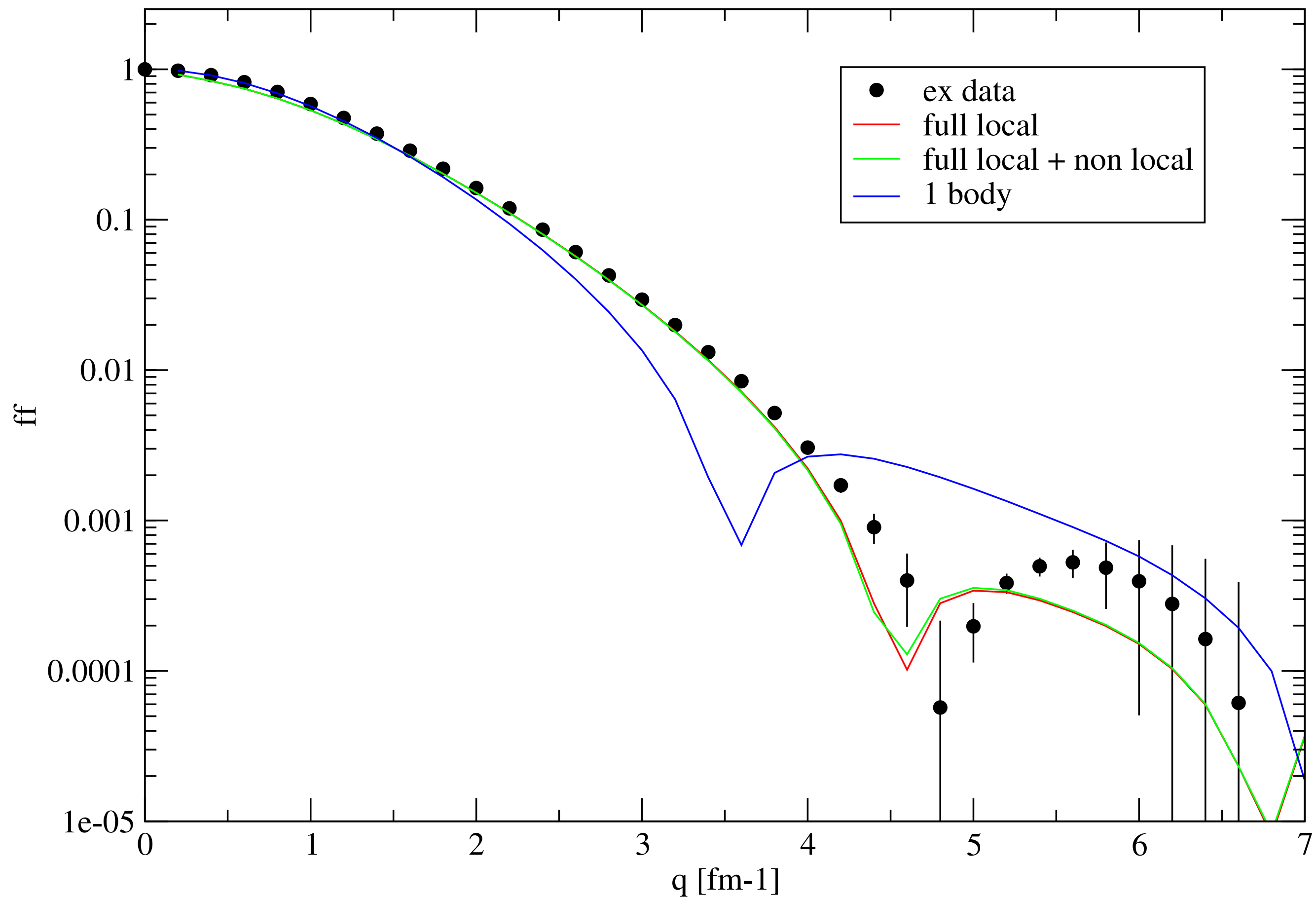
$$F^M(q) = \sqrt{2} \frac{m}{\mu} \frac{F(q)}{q}$$

he3 - v8



ex
full local
full local + non local
1bdy
ff
q [fm-1]
1
0.1
0.01
0.001
0.0001
1e-05
0
1
2
3
4
5
6
7

h3 form factor - v8



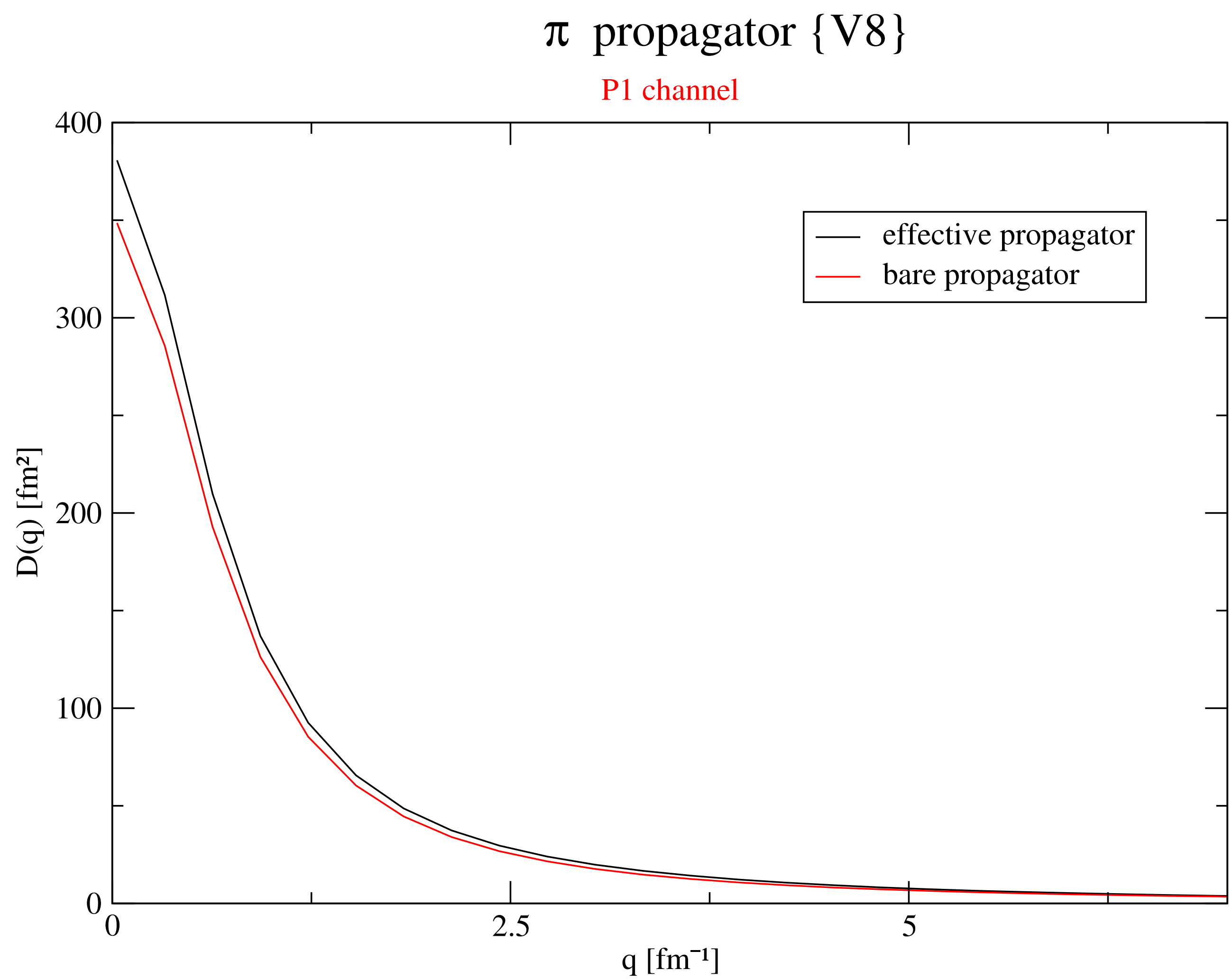
Summary

- we constructed the two body electromagnetic currents starting from the meson-exchange mechanism theoretical insight
- we fixed the effective propagators such to reproduce the $v8$ structure
- non local terms have been included in the evaluation of the currents operators (with the exception of the $V1$ non local contribution)
- a good agreements with the experimental data will lead to a systematic use of the effective propagator in the evaluation of the currents operators

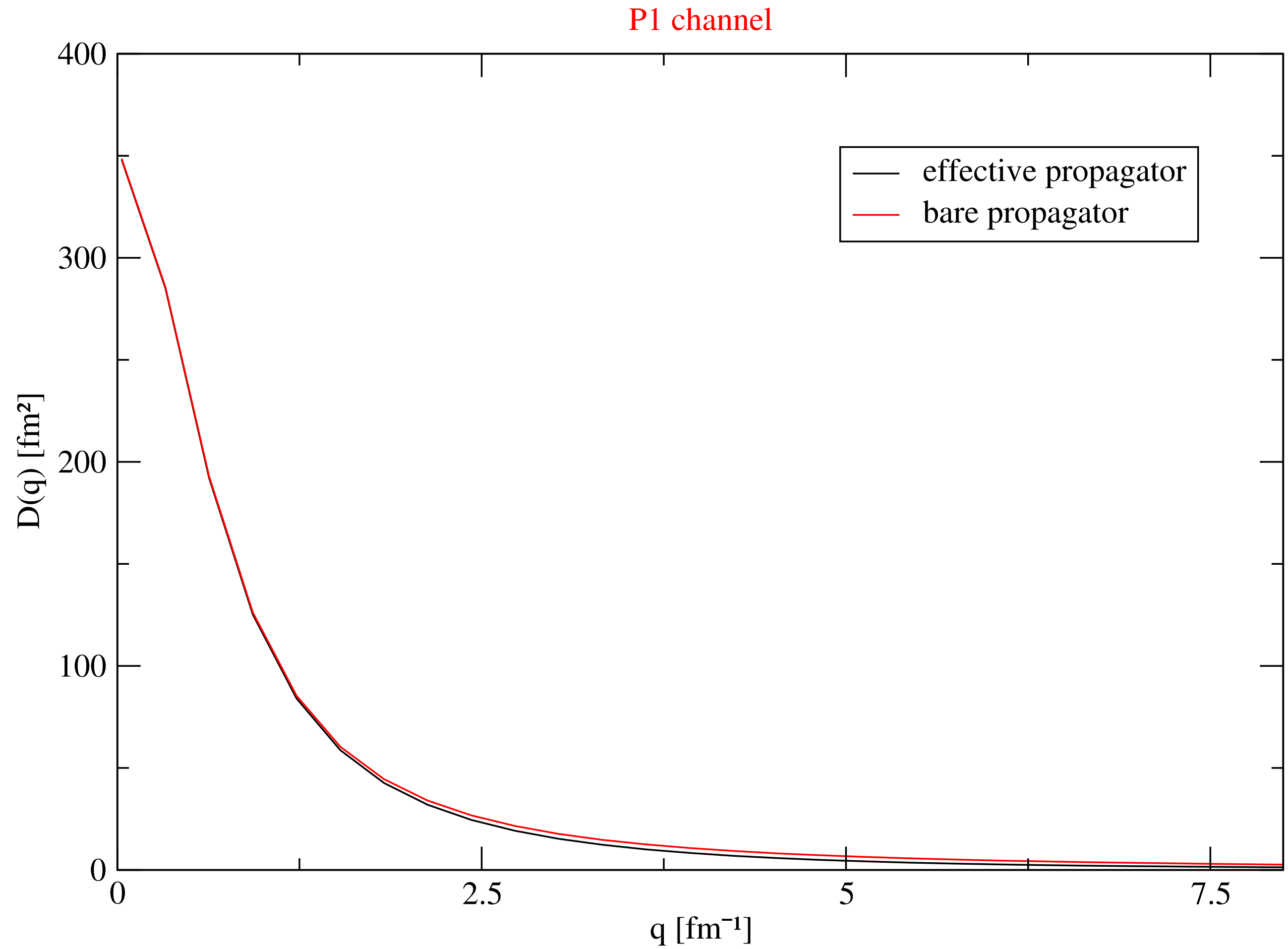
CD Bonn Potential

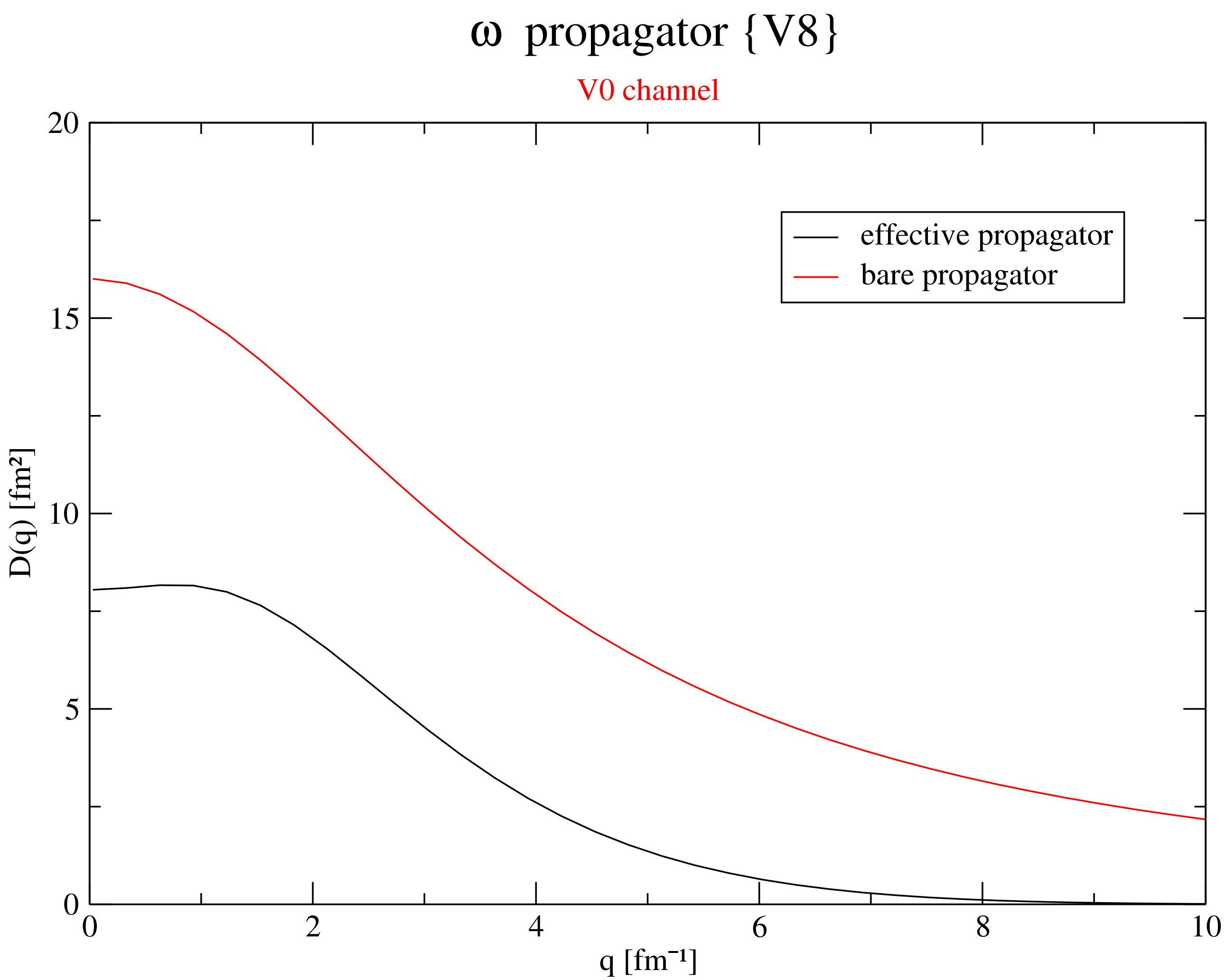
	Mass (MeV)	I	J^π	$\frac{g^2}{4\pi}$	$\frac{g^T}{g_V}$	
$\pi^\pm$	139.56995	1	0^-	13.6		$PS1$
π^0	134.9764	1	0^-	13.6		$PS1$
η	547.3	0	0^-	0.4		$PS0$
$\rho^\pm, \rho^0$	769.9	1	1^-	0.84	6.1	$V1$; $T1$
ω^0	781.94	0	1^-	20.0	0.0	$V0$; $T0$
σ	400-1200	0	0^+			$S0$

R.Machleidt, Phys.Rev. C**63**, 014001 (2001)

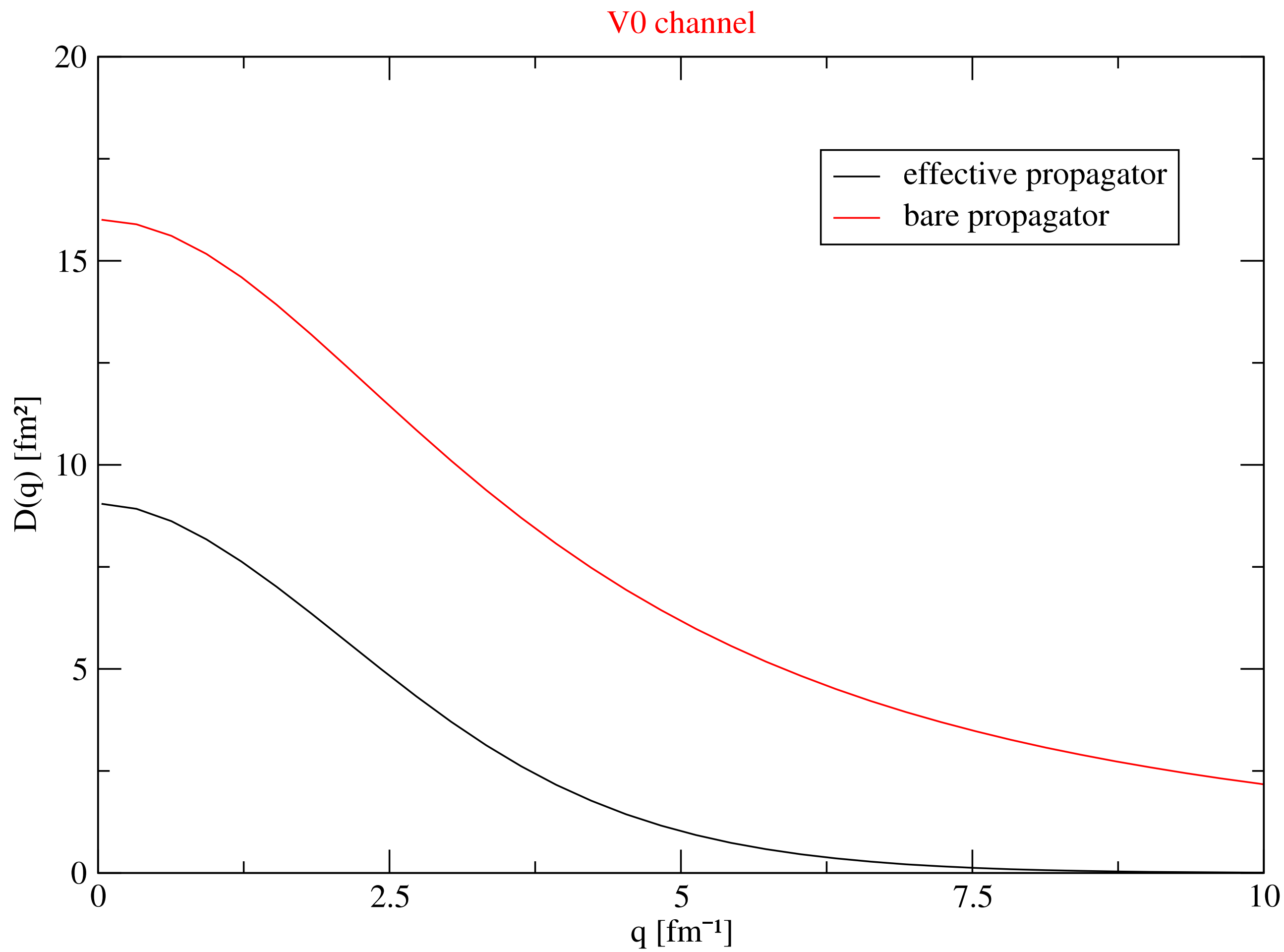

π propagator {V8}
P1 channel
400
300
200
100
0
D(q) [fm²]
0
2.5
5
q [fm⁻¹]
effective propagator
bare propagator

π propagator {V18}

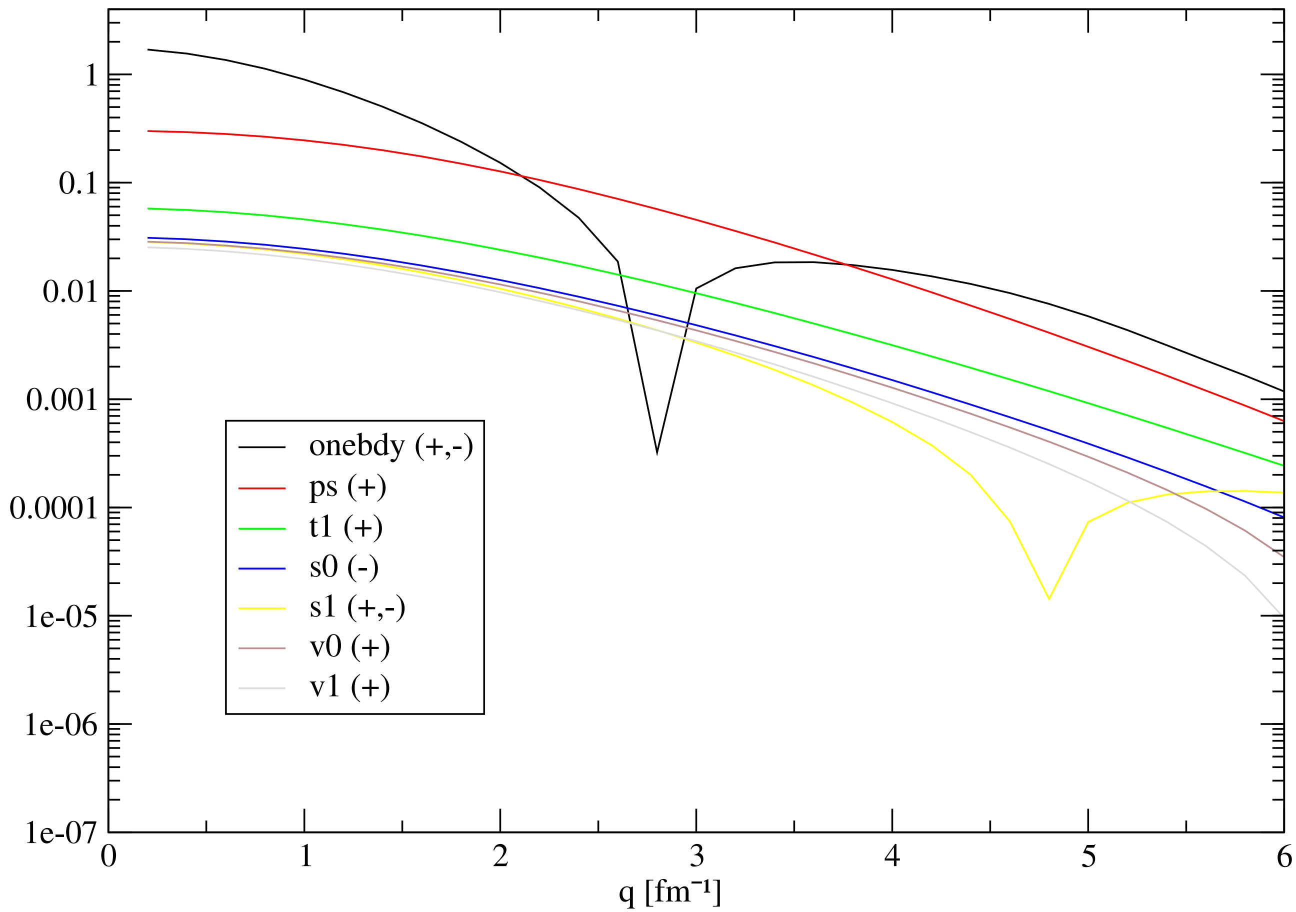



ω propagator {V8}
V0 channel
20
15
10
5
0
0
2
4
6
8
10
D(q) [fm²]
q [fm⁻¹]
effective propagator
bare propagator

ω propagator {V18}



^{3}He currents [V8]



The $V1$ current operator in the momentum space

$$\mathbf{j}^{V1}(\mathbf{q}_i,\mathbf{q}_j) = \frac{i}{4m^2}(\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_z D^{V1}(q_j)\,\cdot$$
$$\Big\{F_1^V\left[-\mathbf{q}_j + i\boldsymbol{\sigma}_i \times (\mathbf{p}_{j'} + \mathbf{p}_j) + i\boldsymbol{\sigma}_i \times (\boldsymbol{\sigma}_j \times \mathbf{q}_j)\right] +$$
$$(F_1^S\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + F_1^V\tau_{j,z})\left[-i(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\times\mathbf{q}_j - (\mathbf{p}_{j'}+\mathbf{p}_j)\right]\Big\}\; i \leftrightharpoons j$$
$$+ \quad \mathbf{j}^{V1}_{NC}$$

$\mathbf{j}^{V1}_{NC}$ is the non local piece of the vector current which has not been evaluated in the configuration-space yet