Pairing phase transition in mesoscopic systems

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Outline

- Exact solution of pairing
- Distribution of zeros of the grand canonical and canonical partition function and the classification of phase transition
- The effect of magnetic field to the type of phase transition
- Summary

Exact solution of pairing*

- Two body Hamiltonian

$$H = 2\sum_{1} \varepsilon_1 n_1 - \sum_{12} G_{12} p_1^{\dagger} p_2$$

- Introducing quasi spin operators

$$[p_1^{\dagger}, p_2] = 2\delta_{12} p_1^z \qquad p_1^z = (n_1 - \frac{1}{2})$$

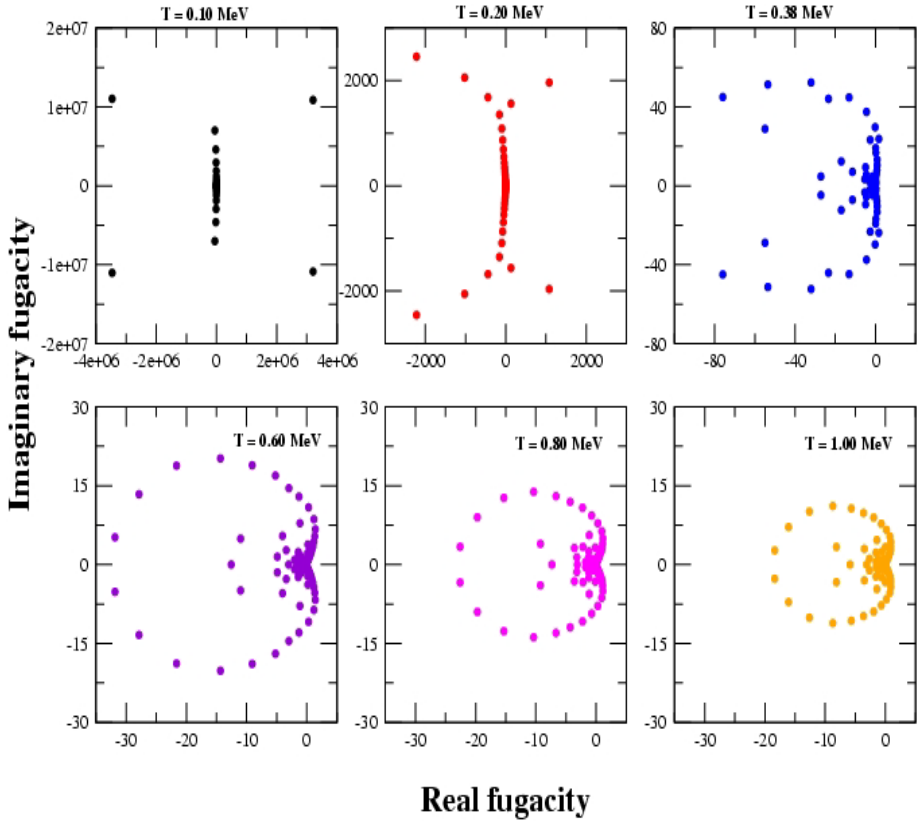
- Now Hamiltonian can be written as

$$H = \sum_{1} \varepsilon_1 + \sum_{1} 2(\varepsilon_1 - \frac{G_{11}}{2}) p_1^z - \sum_{12} G_{12} (\vec{p}_1 . \vec{p}_2 - p_1^z p_2^z)$$

*A.Volya, B.A.Brown, V.Zelevinsky Phys. Lett. B 509 (2001) 37-42

Distribution of zeros (DOZ) of the grand canonical partition function (GCPF) and canonical partition function (CPF)

Yang-Lee's Theory *: Phase transition can be characterized by the analytic behavior of DOZ near a critical point z_0



Two levels < N > = 50 and G=1.00

$$\mathbf{Z}(\beta,\mu)=\sum_N z^N Z(\beta,N)=0$$

We can express DOZ in the complex chemical potential plane

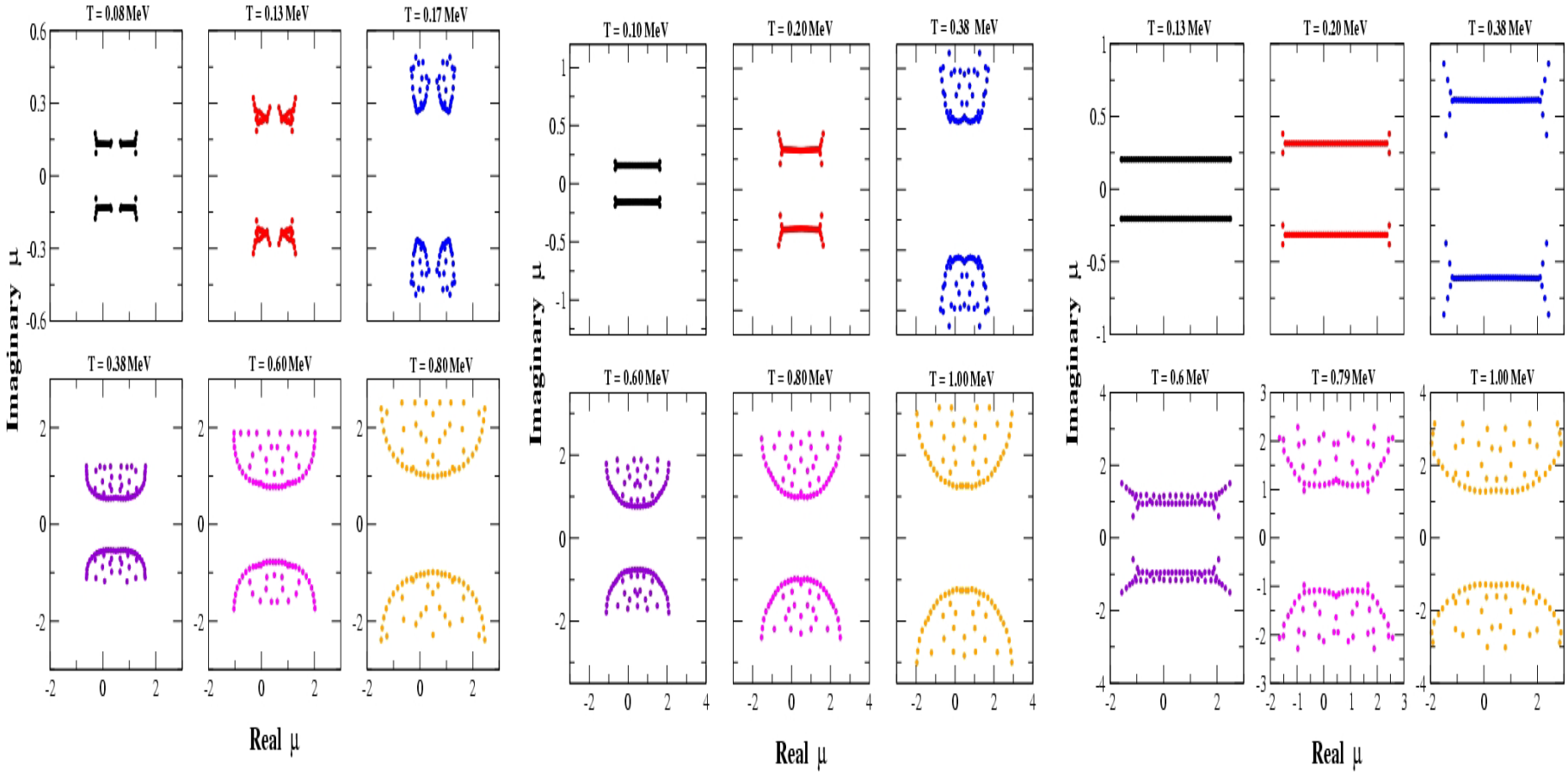
$$z=e^{\beta\mu}$$

*C.N. Yang and T.D. Lee, Phys.Rev.87,(1952) 404

G=0.50

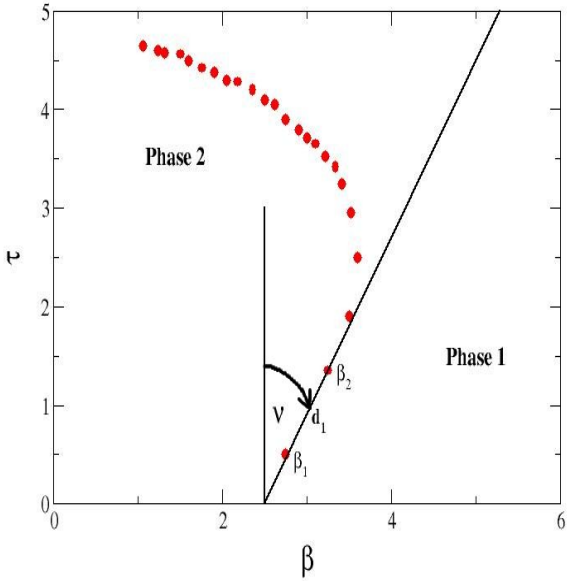
G=1.00

G=2.00



DOZ of GCPF in the complex chemical potential plane for two levels system and $< N > = 50$ particles

DOZ of CPF in the complex temperature plane*



$$\mathrm{B} = \beta + i\tau \qquad Z(\mathrm{B}, N) = \sum_{\alpha,s} g_{\alpha,s} \exp(-E_{\alpha,s}(\beta + i\tau)) = 0$$

$$\upsilon = \arctan\left(\frac{\beta_2 - \beta_1}{\tau_2 - \tau_1}\right) \qquad \alpha = \frac{\ln \Phi(\tilde{\tau}_2) - \ln \Phi(\tilde{\tau}_1)}{\ln \tilde{\tau}_2 - \ln \tilde{\tau}_1}$$

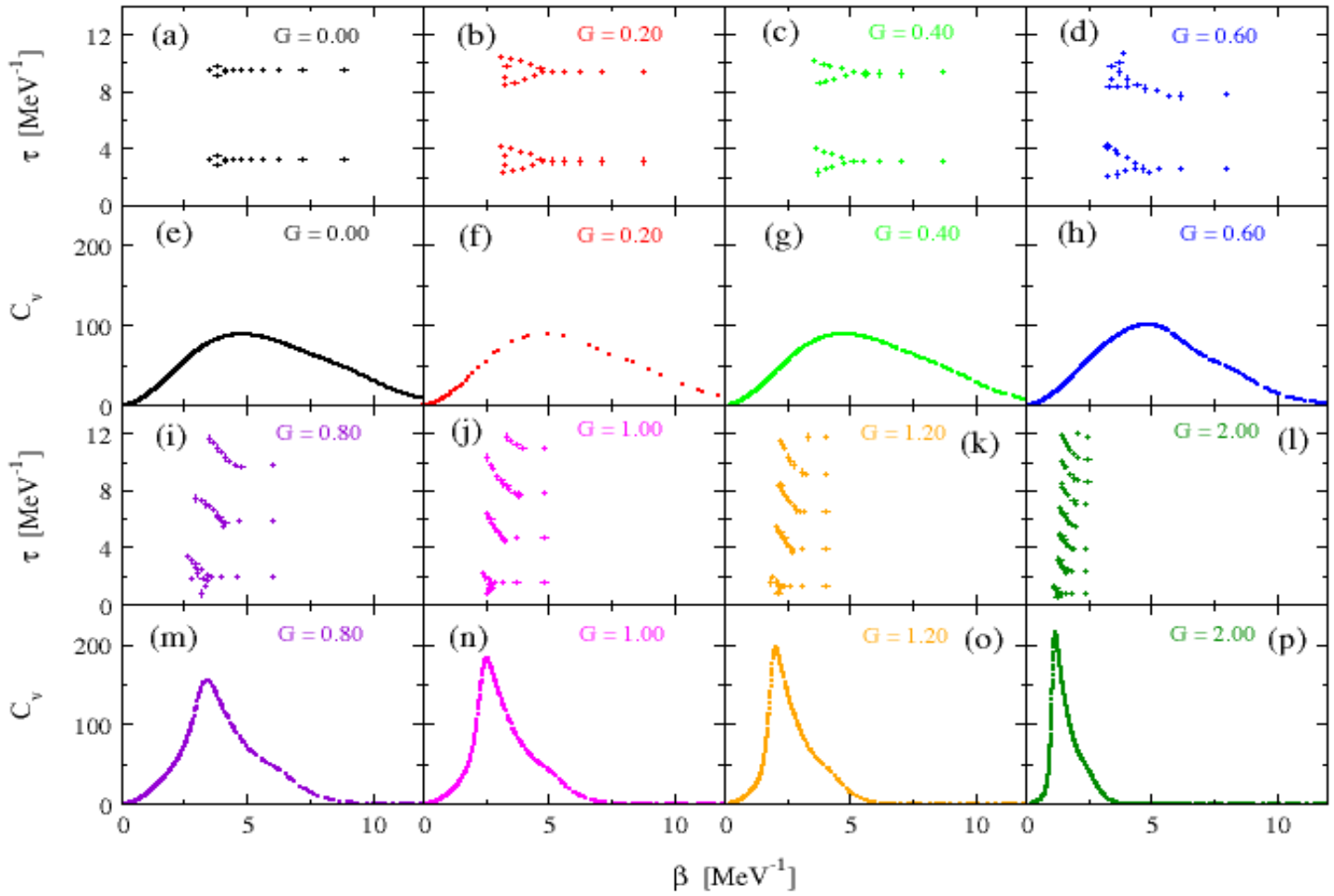
$$\Phi(\tilde{\tau}_j) = \frac{1}{d_j} \qquad \tilde{\tau}_j = \frac{(\tau_j + \tau_{j+1})}{2}$$

Classification of Phase transitions

- **1st order phase transition : □ < 0 or □ = □ = 0**
- **2nd order phase transition : 0 < □ < 1 and □ = 0 or □ □ 0**
- **Higher order phase transition : □ > 1**

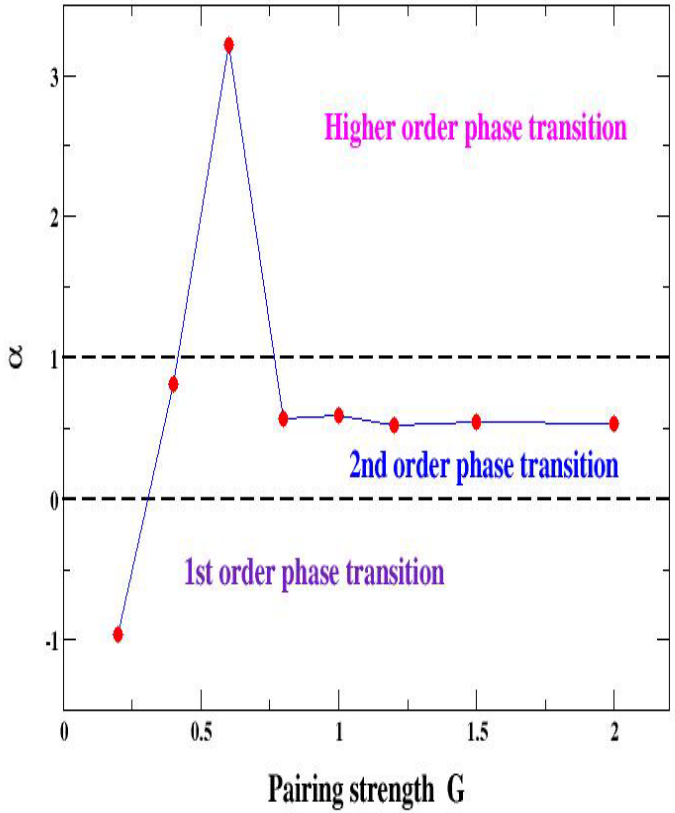
*P.Borrmann, O.Mulken, J.Harting, Phys.Rev.Lett 84 (2000) 3511-3514

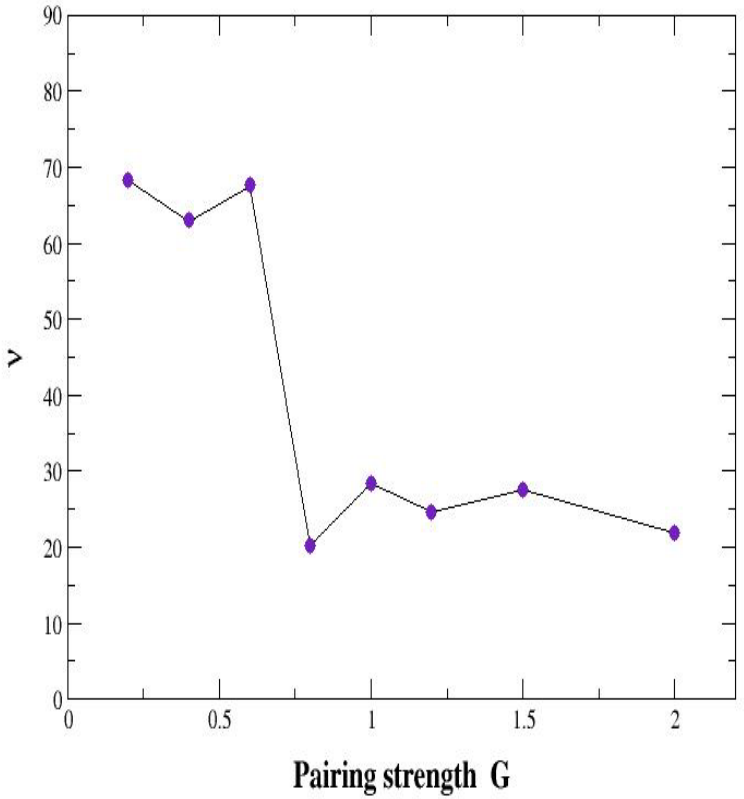
A. Schiller et al Phys.Rev.C 66 (2002) 024322



Evolution of DOZ in the complex temperature plane for two levels system and N=100 particles

Classification of the Phase Transition for Two levels system and N=100 particles





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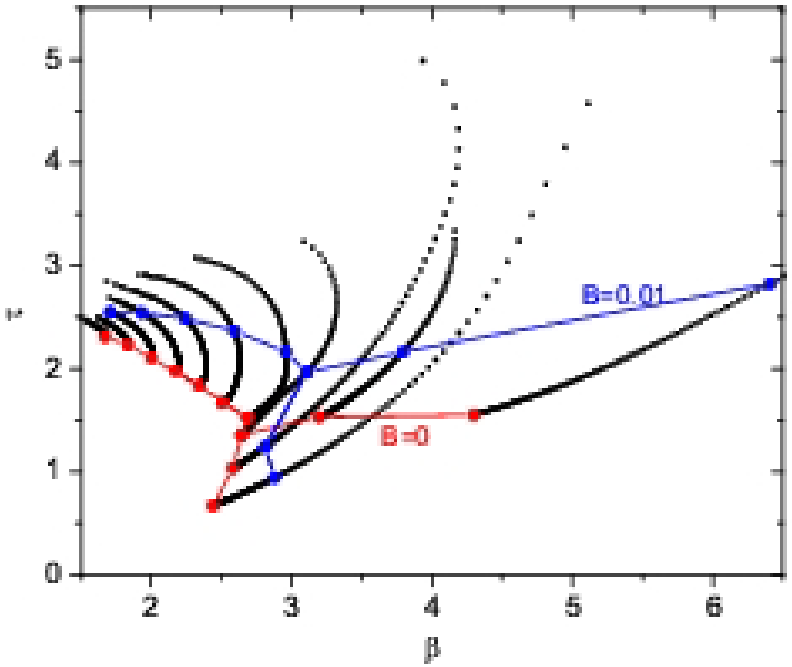
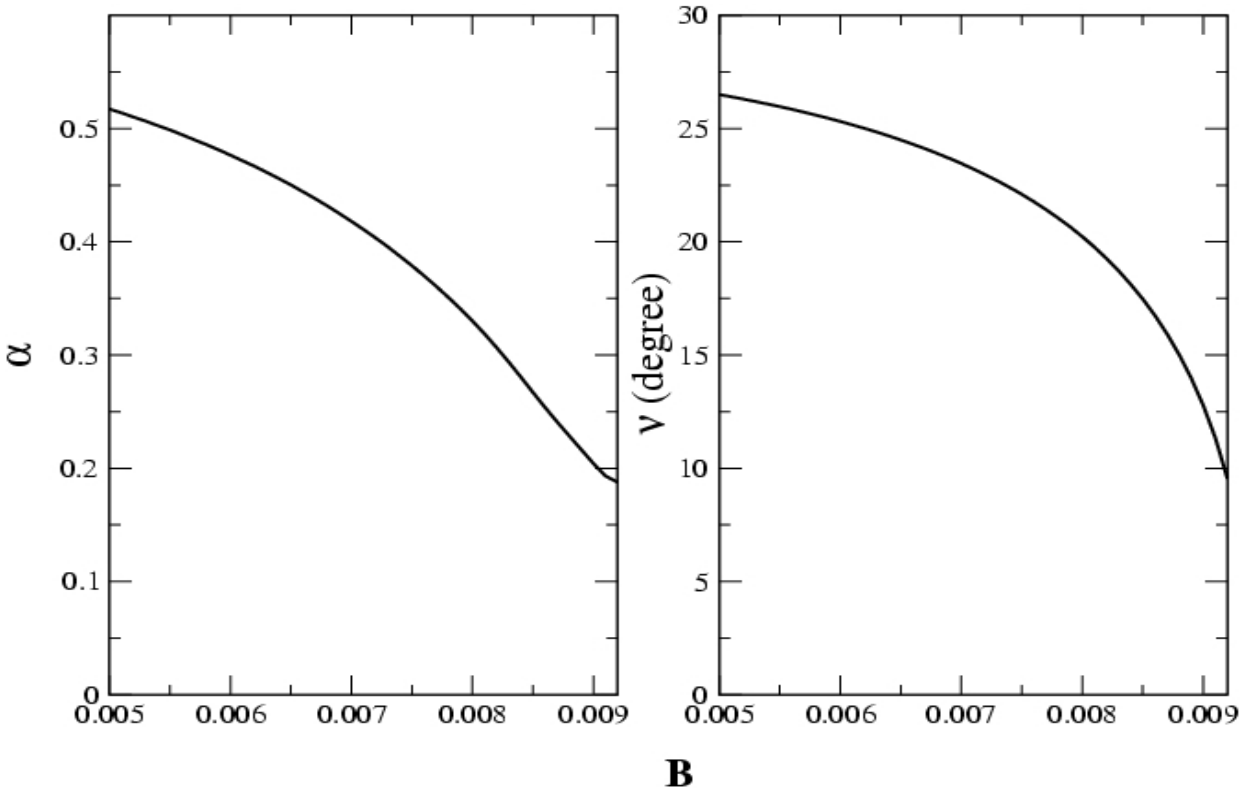
The effect of magnetic field to the type of phase transition

$$H = H_B - gJ.B$$



Evolution of DOZ for 2 level system, N=60 and G=1.00



The evolution of critical parameters as a function of B in system with N=100 and V=1.00

Summary

- DOZ of GCPF and CPF provide rich informations about pairing phase transition in the mesoscopic system, Including the classification of phase transitions

- Magnetic field can change the type of phase transition