Lattice QCD:

Using Lattice Field Theory to Understand the Structure of Hadrons

J.W. Negele

18th National Nuclear Physics Summer School

Indiana University July 29, 2006

Collaborators

MIT

B. Bistrovic
J. Bratt
D. Dolgov
O. Jahn
A. Pochinsky
D. Sigaev

JLab

R. Edwards
D. Richards

William & Mary, JLab

K. Orginos

Arizona

D. Renner

Yale

G. Fleming

T. U. Munchen

Ph. Haegler

DESY Zeuthen

W. Schroers

U Cyprus

C. Alexandrou
G. Koutsou
Ph. Leontiou

Athens

A. Tsapalis

ETH, CERN

Ph. de Forcrand

Julich

Th. Lippert

Wuppertal

K. Schilling

Outline

- Introduction
  - QCD
  - Lattice Field Theory
- Highlights from hadron structure
  - Deep inelastic scattering
  - Quark distributions
  - Form factors and generalized form factors
  - Transverse structure
  - Origin of nucleon spin
  - Baryon shapes
  - Exotics
- Insight into how QCD works
- Summary and future challenges

Introduction

- How do hadrons arise from QCD?
- Lagrangian constrained by Lorentz invariance, gauge invariance and renormalizability:

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu} D_{\mu} - m)\psi - \frac{1}{4} F^{2}_{\mu\nu}$$

$$D_{\mu} = \partial_{\mu} - igA_{\mu} \qquad F_{\mu\nu} = \frac{i}{g}[D_{\mu}, D_{\nu}]$$

- Deceptively simple Lagrangian produces amazingly rich and complex structure of strongly interacting matter in our universe

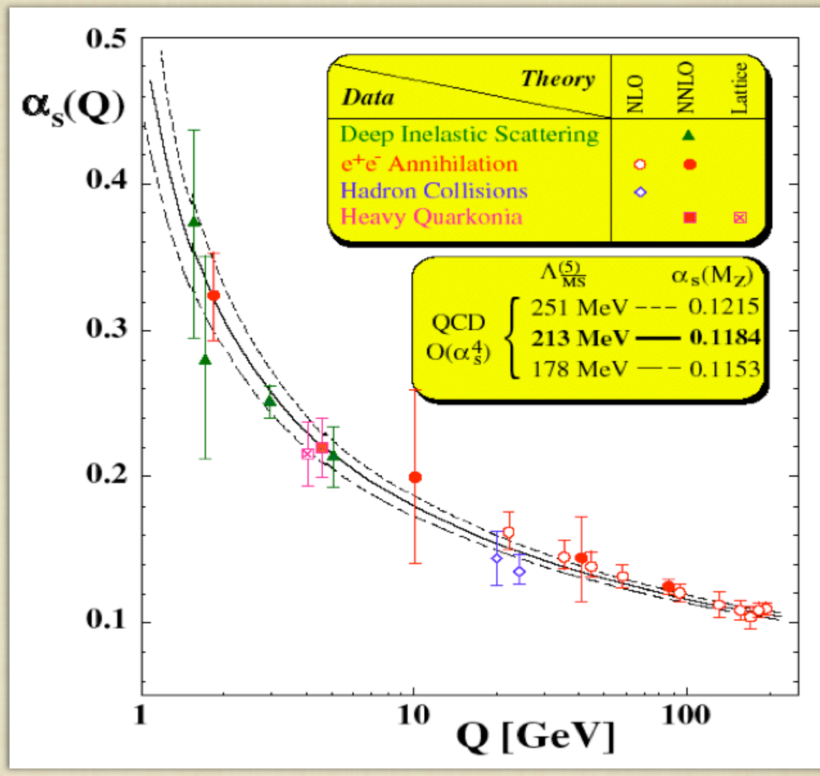
QCD and Asymptotic Freedom

David J. Gross
Kavli Institute for Theoretical Physics
University of California, Santa Barbara, USA

H. David Politzer
California Institute of Technology (Caltech), Pasadena, USA

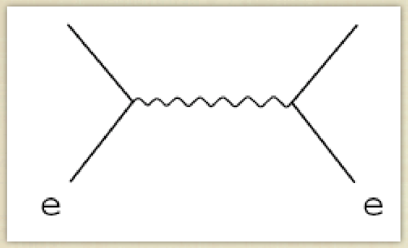
Frank Wilczek
Massachusetts Institute of Technology (MIT), Cambridge, USA

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics for 2004 "for the discovery of asymptotic freedom in the theory of the strong interaction" jointly to David J. Gross, H. David Politzer and Frank Wilczek

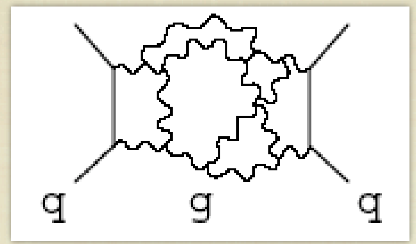


Nonperturbative QCD

QED



QCD



- Fundamental differences relative to QED
  - Self-interacting: highly nonlinear
  - Interaction increases at large distance: Confinement
  - Interaction decreases at small distance: Asymptotic Freedom
  - Strong coupling: $\alpha_s \gg \alpha_{em}$
  - Topological excitations
- Solution of QCD
  - Present analytical techniques inadequate
  - Numerical evaluation of path integral on space-time lattice

Profound differences between hadrons and other many-body systems

- Atoms, molecules, nuclei,...
  - Constituents can be removed
  - Exchanged boson generating interaction may be subsumed into static potential
    - photons → Coulomb potential
    - Mesons → N-N potential
  - Most of mass from fermion constituents
- Nucleons
  - Quarks are confined
  - Gluons are essential degrees of freedom
    - Carry half of momentum
    - Nonperturbative topological excitations
  - Most of mass generated by interactions

Goals

- Quantitative calculation of hadron observables from first principles
  - Agreement with experiment
  - Credibility for predictions and guiding experiment
- Insight into how QCD works
  - Mechanisms
    - Paths that dominate action - instantons
    - Variational wave functions
  - Dependence on parameters
    - N_c, N_f, gauge group
    - m_q

Basic Ideas in Lattice QCD

Evolution in Euclidean time

$$|\psi\rangle \equiv \sum_n e^{-\beta E_n} C_n |\psi_n\rangle \rightarrow C_0 e^{-\beta E_0} |\psi_0\rangle$$

Lattice Regularization

$$\phi(x) \rightarrow \phi(x_n), \quad x_n = na$$

Path Integral

$$e^{-\beta \hat{H}} \rightarrow \int D[x(\tau)] e^{-\int_0^\beta d\tau S[x(\tau)]}$$

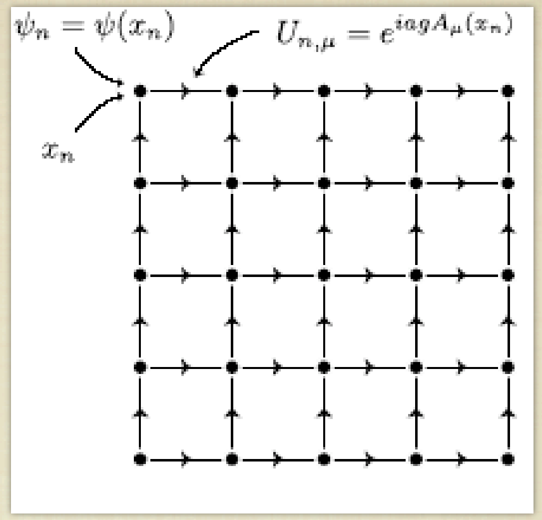
Stochastic Solution

$$\int dx f(x) P(x) = \frac{1}{N} \sum_{x_i \in P} f(x_i) + \mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$$

Lattice QCD

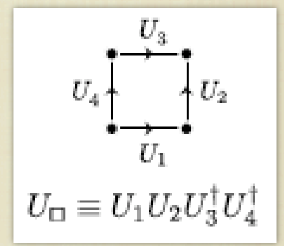
Euclidean time

$$e^{i\int dt d^3x \mathcal{L}} \rightarrow e^{-\int d\tau d^3x \mathcal{L}}$$



Discrete space-time lattice

Discrete Action



$$S(U) = \sum_{\Box} \frac{2N}{g^2}(1 - N^{-1}\mathrm{ReTr}U_\Box) \rightarrow \frac{1}{4}F_{\mu\nu}^2$$

$$\bar{\psi}M(U)\psi = \sum_n [\bar{\psi}_n\psi_n + \kappa(\bar{\psi}_n(1-\gamma_\mu)U_{n,\mu}\psi_{n+\mu} + \bar{\psi}_{n+\mu}(1+\gamma_\mu)U_{n,\mu}^\dagger\psi_n]$$

$$\rightarrow \bar{\psi}(\partial\!\!\!/ + m + ig A\!\!\!/)\psi$$

Lattice QCD

$$\langle T\, e^{-\beta H} \psi\psi\psi \cdots \bar\psi\bar\psi\bar\psi \rangle$$

$$= \frac{1}{Z} \int \mathcal{D}[\psi]\mathcal{D}[\bar\psi]\mathcal{D}[A] e^{-\int d^4x \left[\bar\psi(\partial\!\!\!/ + m + igA\!\!\!/)\psi + \frac{1}{4}F^2_{\mu\nu}\right]} \psi\psi\psi \cdots \bar\psi\bar\psi\bar\psi$$

$$\to \prod_n \frac{1}{Z} \int d\psi_n\, d\bar\psi_n\, dU_n e^{-\sum\limits_n \left[\bar\psi M(U)\psi + S(U)\right]} \psi\psi\psi \cdots \bar\psi\bar\psi\bar\psi$$

$$= \prod_n \int dU_n \underbrace{\frac{1}{Z} \det M(U) e^{-S(U)}}_{\text{Sample with M.C.}} \sum M^{-1}(U) M^{-1}(U) \cdots M^{-1}(U)$$

$$\to \frac{1}{N} \sum_{\substack{i=1 \\ U_i \in \frac{\det M(U)}{Z} e^{-S(U)}}}^{N} M^{-1}(U_i) M^{-1}(U_i) M^{-1}(U_i)$$

Lattice QCD - summing over paths

$$\langle T\, e^{-\beta H} \psi\psi\psi \cdots \bar{\psi}\bar{\psi}\bar{\psi} \rangle = \prod_n \int dU_n \frac{1}{Z} \det M(U) e^{-S(U)} \sum M^{-1}(U) M^{-1}(U) \cdots M^{-1}(U)$$

- $M^{-1} = (1+ \kappa U)^{-1}$ connects Ψ's with line of U's → Sum over valence quark paths
- det M generates closed loops of U's Sum over sea quark excitations
- S(U) tiles with plaquettes → Sum over all gluons
- $32^3 \times 64$ lattices → 10^8 gluon variables

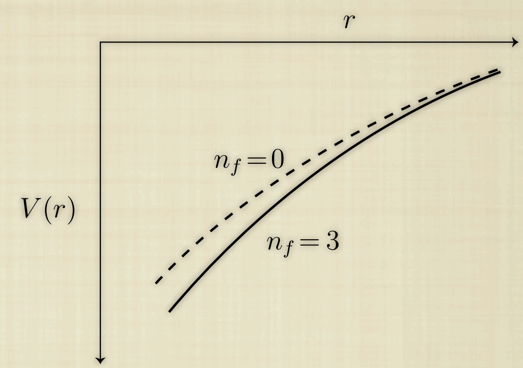
Computational Issues

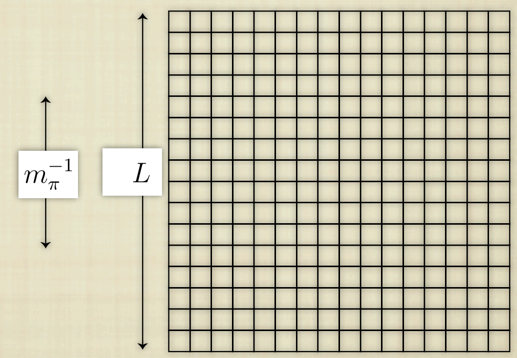
- Fermion determinant - Full QCD
- Small lattice spacing
- Small quark mass
- Large lattice volume

$$\frac{1}{m_\pi} \leq \frac{L}{4}$$

L(fm)	m_π (Mev)
1.6	500
4.0	200
5.7	140

- Cost ~ $(m_q)^{-2.5}(m_\pi)^{-4}$ ~ $(m_\pi)^{-9}$





Current Status

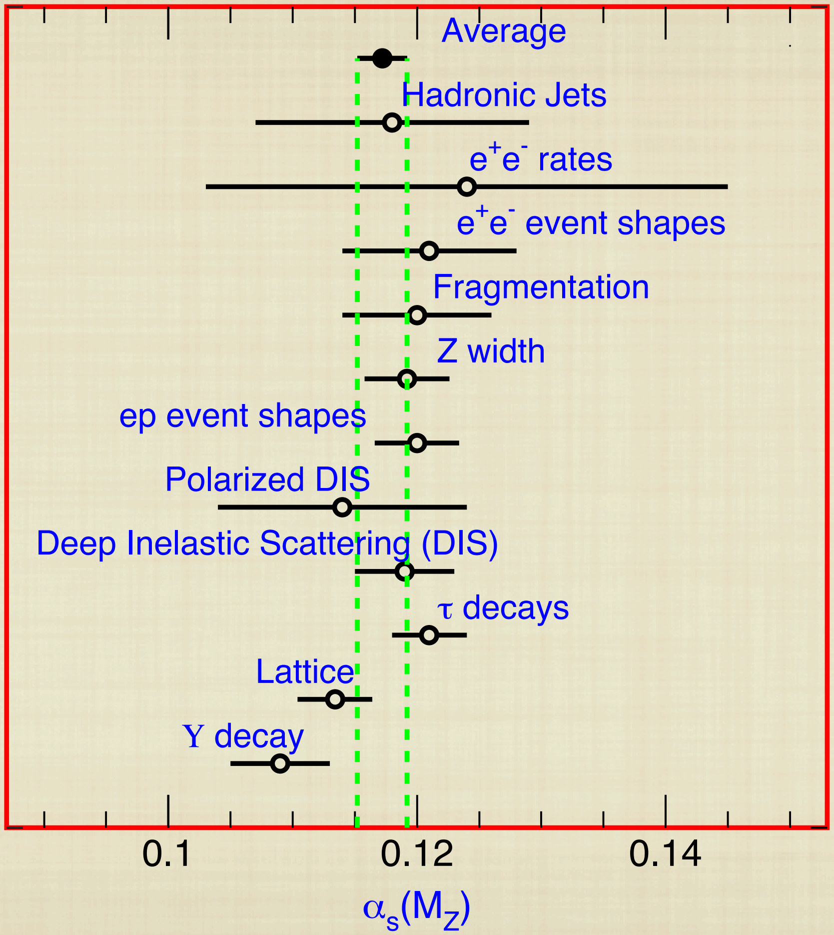
- Include fermion determinant - Full QCD
- Precision results in heavy quark systems
- $(m_\pi)^{-9}$ limited past nucleon structure to "heavy pion world" - $m_\pi \geq 500$ MeV
- Beginning to explore physical "light pion world" role of chiral symmetry
- Terascale resources required for physical regime
- DOE: ~8 sustained Teraflops

International Resource Estimates by S. Gottlieb

Final estimates consider multiple users for Julich, KEK, Munich, Edinburgh

Location	type	size	peak	est. perf.	total
Paris-Sud	apeNEXT	1 racks	0.8 TF	0.4 TF	0.4
Bielefeld	apeNEXT	6 (3) racks	4.9 TF	2.5 TF	
DESY (Zeuthen)	apeNEXT	3 racks	2.5 TF	1.2 TF	
Julich	BlueGene/L	8 racks	45.8 TF	11.5 TF $\times 1/2$?	10–15
Munich	SGI Tollhouse	3328 nodes	70 TF	14 TF?? $\times$?	
Rome	apeNEXT	12 (8) racks	9.8 TF	4.9 TF	5
KEK	BlueGene/L	10 racks	57.3 TF	14.3 TF	
Tsukuba	PACS-CS	2560 nodes	14.3 TF	3.3 TF	14–18
KEK	Hitachi		2.1 TF	1 TF ?	
Edinburgh	QCDOC	12 racks	9.8 TF	4.2 TF	4–5
Edinburgh	BlueGene/L	1 racks	5.7 TF	1.4 TF $\times$?	

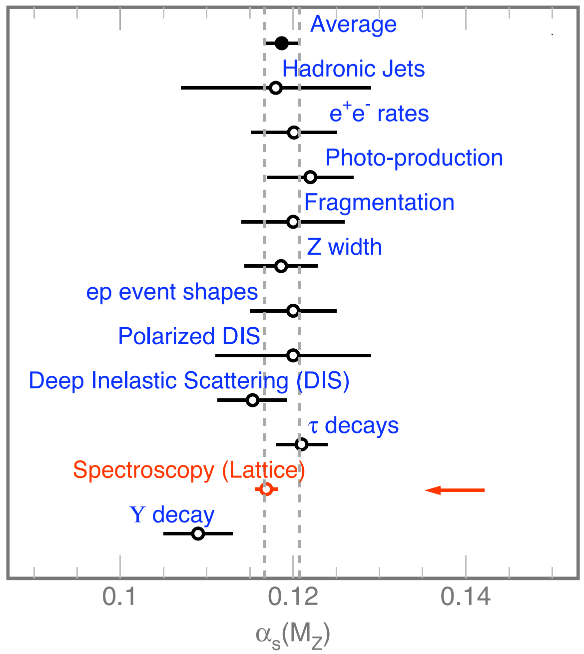
Precision agreement in heavy quark systems



$\alpha_s(M_Z)$ from Particle Data Group

Precision agreement in heavy quark systems

Context:

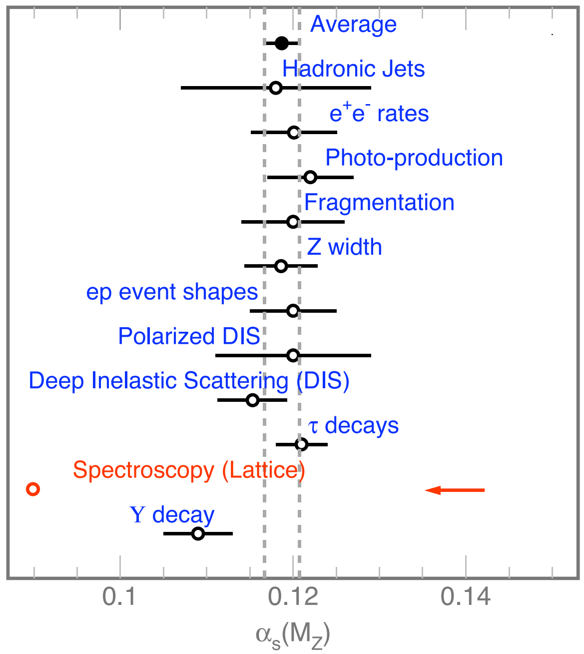


Mason et al, hep-lat/0503005v1 (2005); Particle Data Group (2004)

$\alpha_S(M_Z)$ from Particle Data Group

Precision agreement in heavy quark systems

And without light-quark vacuum polarization:



Mason et al, hep-lat/0503005v1 (2005); Particle Data Group (2004)

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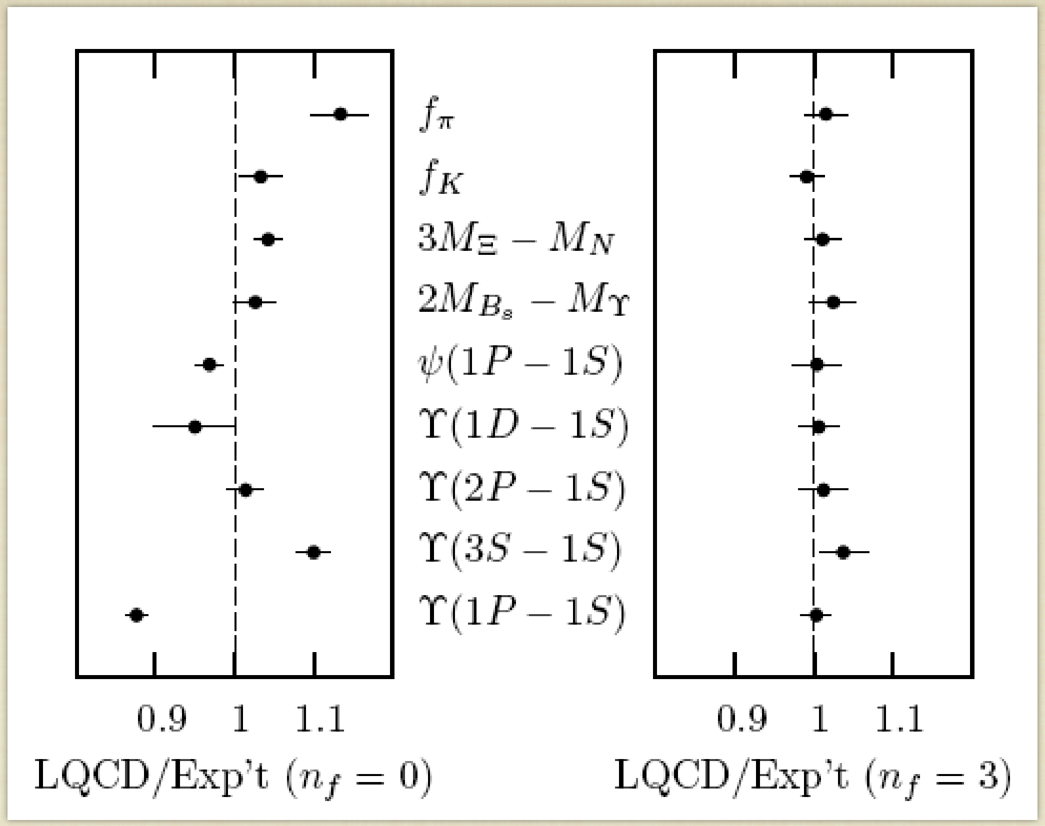
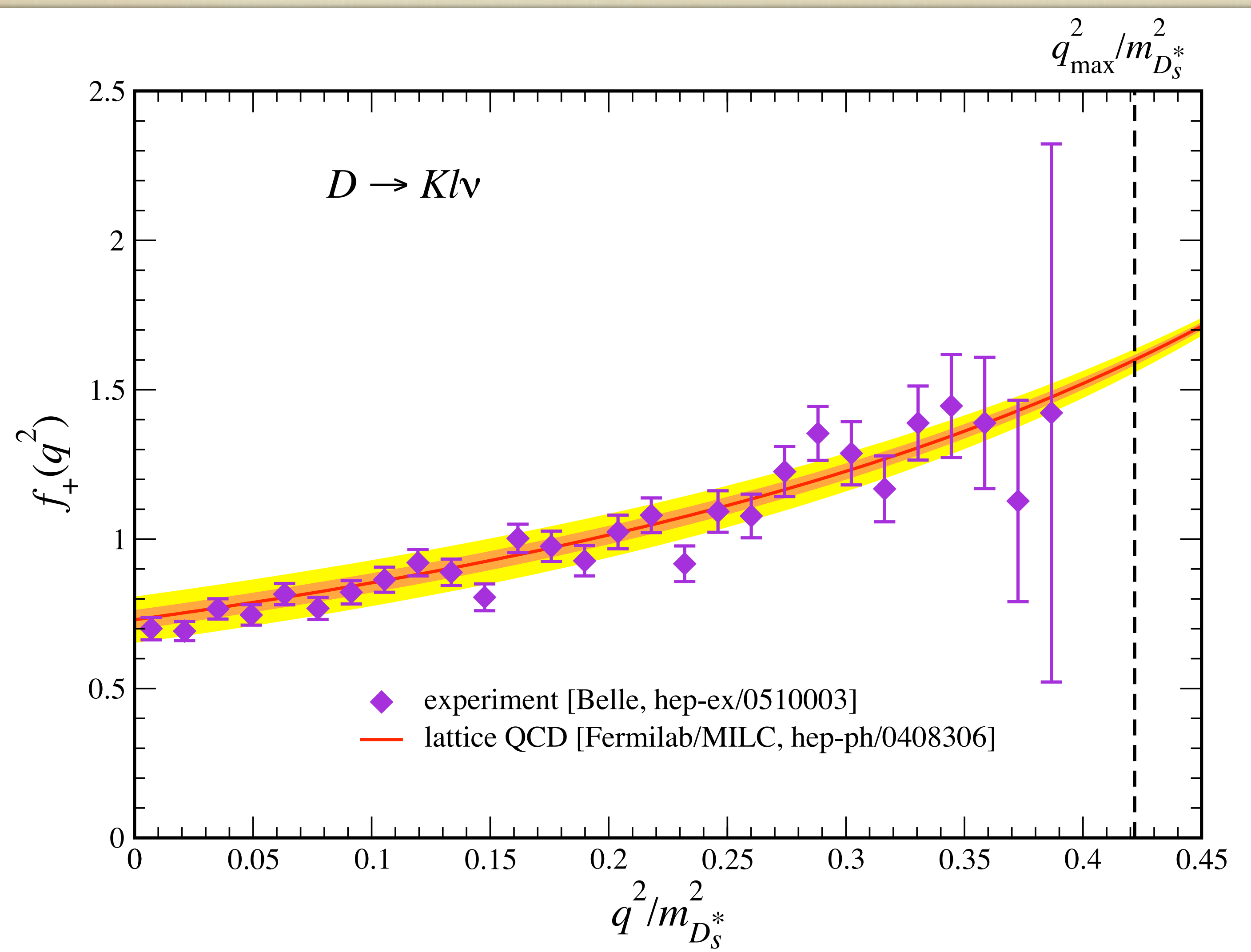
Precision agreement in heavy quark systems



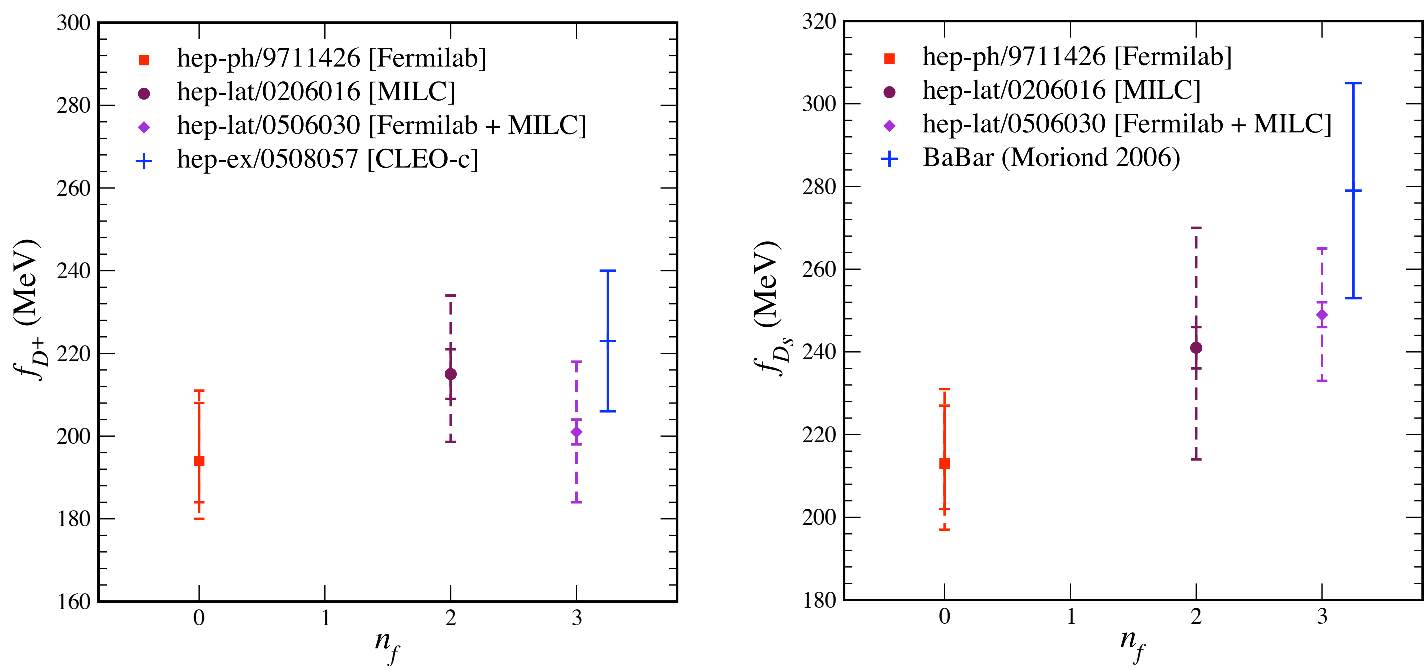
- "Gold Plated Observables" (Davies et. al. hep-lat/0304004)
  - Staggered quarks
  - Asqtad improved action
  - a = 0.13, 0.09 fm

Lattice QCD Predictions



Lattice QCD Predictions

D meson decay constants



$$\frac{\sqrt{m_D} f_D}{\sqrt{m_{D_s}} f_{D_s}} = 0.786 \pm 0.042 \quad \text{lattice}$$

$$= 0.779 \pm 0.093 \quad \text{CLEO/BaBar}$$

Mass of B_c meson

Hadron structure revealed by high energy scattering

- High energy scattering measures correlation functions along light cone
  - Asymptotic freedom: reaction theory perturbative
  - Unambiguous measurement of operators in light cone frame
  - Must think about physics on light cone
- Parton distribution q(x) gives longitudinal momentum distribution of light-cone wave function
- Generalized parton distribution $q(x, r_\perp)$ gives transverse spatial structure of light-cone wave function

Parton and generalized parton distributions

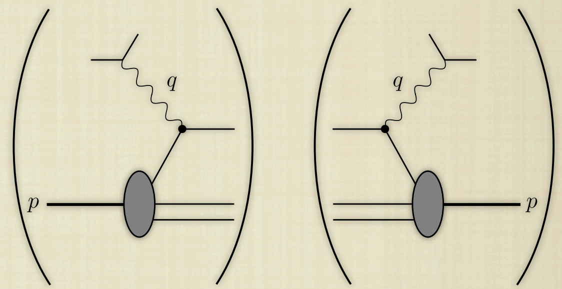
High energy scattering: light-cone correlation function $(\lambda = p^+ x^-)$

$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}(-\frac{\lambda}{2}n) \not{n} \mathcal{P} e^{-ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(\alpha n)} \psi(\frac{\lambda}{2}n)$$

Deep inelastic scattering: diagonal matrix element

$$\langle P|\mathcal{O}(x)|P\rangle = q(x)$$

$$[\not{n} \to \not{n}\gamma_5 : \quad \Delta q(x)]$$

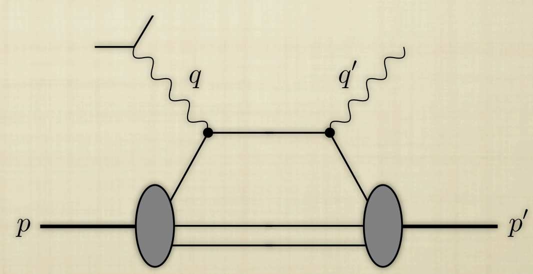


Deeply virtual Compton scattering: off-diagonal matrix element

$$\langle P'|\mathcal{O}(x)|P\rangle = \langle\gamma\rangle H(x,\xi,t) + \tfrac{i\Delta}{2m}\langle\sigma\rangle E(x,\xi,t)$$

$$\Delta = P' - P, \quad t = \Delta^2, \quad \xi = -n\cdot\Delta/2$$

$$[\not{n} \to \not{n}\gamma_5 : \quad \tilde{E}(x,\xi,t), \tilde{H}(x,\xi,t)]$$



Moments of parton distributions

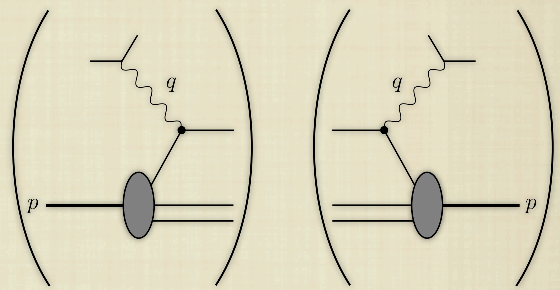
Expansion of $\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}(-\frac{\lambda}{2}n) \not{n} \mathcal{P} e^{-ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(\alpha n)} \psi(\frac{\lambda}{2}n)$

Generates tower of twist-2 operators

$$\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}} = \overline{\psi}_q \gamma^{\{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n\}} \psi_q$$

Diagonal matrix element

$$\langle P|\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}}|P\rangle \sim \int dx\, x^{n-1} q(x)$$



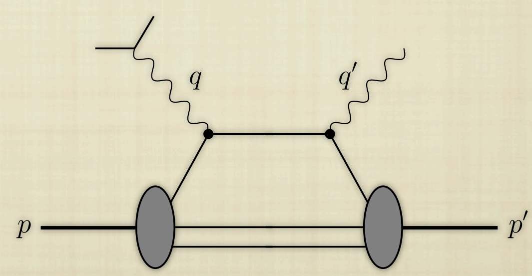
Off-diagonal matrix element

$$\langle P'|\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}}|P\rangle \to A_{ni}(t), B_{ni}(t), C_{n0}(t)$$

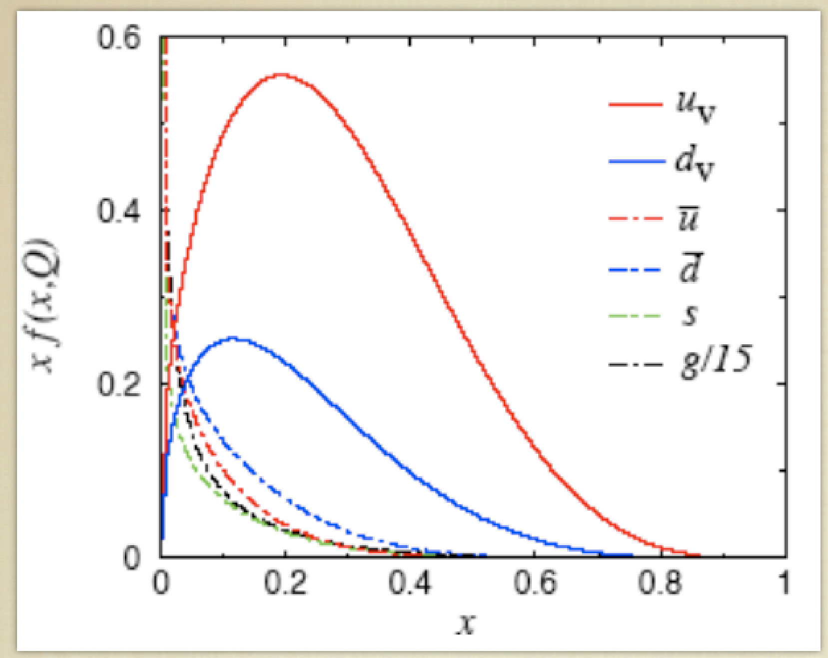
$$\int dx\, x^{n-1} H(x,\xi,t) \sim \sum \xi^i A_{ni}(t) + \xi^n C_{n0}(t)$$

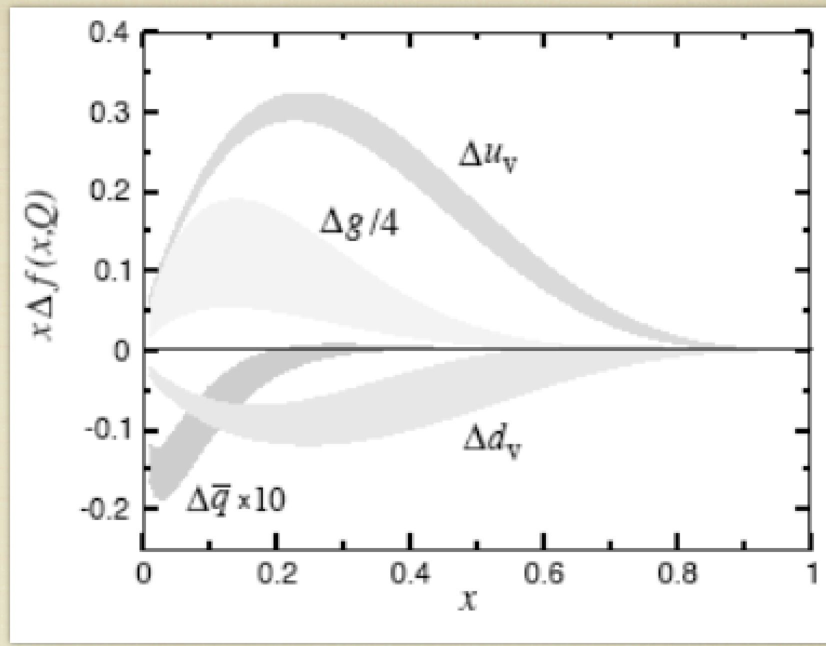
$$\int dx\, x^{n-1} E(x,\xi,t) \sim \sum \xi^i B_{ni}(t) - \xi^n C_{n0}(t)$$

$[\not{n} \to \not{n}\gamma_5 : \quad \tilde{A}_{ni}(t), \tilde{B}_{ni}(t)]$



Moments of parton distributions





$$\langle p|\bar\psi\gamma_\mu D_{\mu_1}\cdots D_{\mu_n}\psi|p\rangle \to \quad \langle x^n\rangle_q = \int_0^1 dx x^n[q(x)+(-1)^{(n+1)}\bar q(x)]$$

$$\langle p|\bar\psi\gamma_5\gamma_\mu D_{\mu_1}\cdots D_{\mu_n}\psi|p\rangle \to \quad \langle x^n\rangle_{\Delta q} = \int_0^1 dx x^n[\Delta q(x)+(-1)^{(n)}\Delta\bar q(x)]$$

$$\langle p|\bar\psi\gamma_5\sigma_{\mu\nu} D_{\mu_1}\cdots D_{\mu_n}\psi|p\rangle \to \quad \langle x^n\rangle_{\delta q} = \int_0^1 dx x^n[\delta q(x)+(-1)^{(n+1)}\delta\bar q(x)]$$

where $\quad q = q_\uparrow + q_\downarrow, \quad \Delta q = q_\uparrow - q_\downarrow, \quad \delta q = q_\top + q_\perp,$

Lattice operators: irreducible representations of hypercubic group with minimal operator mixing and minimal non-zero momentum components

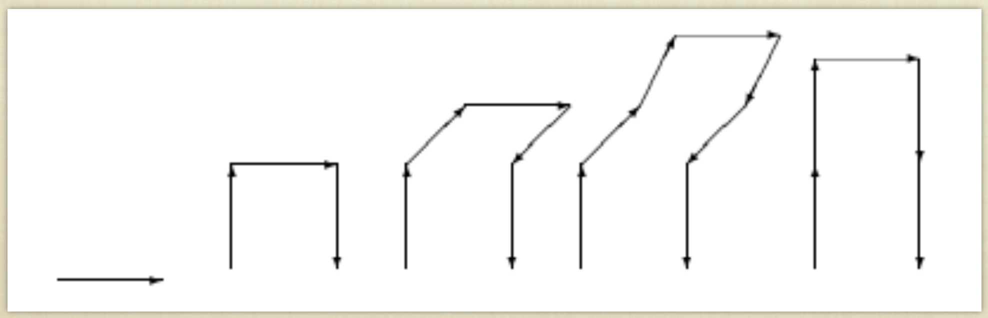
$\langle x\rangle_q^{(a)}$	6_3^+	$\bar{\psi}\gamma_{\{1}\overleftrightarrow{D}_{4\}}\psi$
$\langle x\rangle_q^{(b)}$	3_1^+	$\bar{\psi}\gamma_4\overleftrightarrow{D}_4\psi - \frac{1}{3}\sum_{i=1}^{3}\bar{\psi}\gamma_i\overleftrightarrow{D}_i\psi$
$\langle x^2\rangle_q$	8_1^-	$\bar{\psi}\gamma_{\{1}\overleftrightarrow{D}_1\overleftrightarrow{D}_{4\}}\psi - \frac{1}{2}\sum_{i=2}^{3}\gamma_{\{i}\overleftrightarrow{D}_i\overleftrightarrow{D}_{4\}}\psi$
$\langle x^3\rangle_q$	2_1^+	$\bar{\psi}\gamma_{\{1}\overleftrightarrow{D}_1\overleftrightarrow{D}_4\overleftrightarrow{D}_{4\}}\psi + \bar{\psi}\gamma_{\{2}\overleftrightarrow{D}_2\overleftrightarrow{D}_3\overleftrightarrow{D}_{3\}}\psi - \{3 \leftrightarrow 4\}$
$\langle 1\rangle_{\Delta q}$	4_4^+	$\bar{\psi}\gamma^5\gamma_3\psi$
$\langle x\rangle_{\Delta q}^{(a)}$	6_3^-	$\bar{\psi}\gamma^5\gamma_{\{1}\overleftrightarrow{D}_{3\}}\psi$
$\langle x\rangle_{\Delta q}^{(b)}$	6_3^-	$\bar{\psi}\gamma^5\gamma_{\{3}\overleftrightarrow{D}_{4\}}\psi$
$\langle x^2\rangle_{\Delta q}$	4_2^+	$\bar{\psi}\gamma^5\gamma_{\{1}\overleftrightarrow{D}_3\overleftrightarrow{D}_{4\}}\psi$
$\langle 1\rangle_{\delta q}$	6_1^+	$\bar{\psi}\gamma^5\sigma_{34}\psi$
$\langle x\rangle_{\delta q}$	8_1^-	$\bar{\psi}\gamma^5\sigma_{3\{4}\overleftrightarrow{D}_{1\}}\psi$
d_1	6_1^+	$\bar{\psi}\gamma^5\gamma_{[3}\overleftrightarrow{D}_{4]}\psi$
d_2	8_1^-	$\bar{\psi}\gamma^5\gamma_{[1}\overleftrightarrow{D}_{\{3]}\overleftrightarrow{D}_{4\}}\psi$

Full QCD Calculations

Collaboration	m_π (MeV)	Gluon action	Quark action
LHPC / SESAM	> 650	Wilson	Wilson
QCDSF / UKQCD	> 550	Wilson	Clover improved Wilson
RBCK	> 500	DBW2	Domain wall
LHPC / MILC	> 350	Asqtad	Staggered sea HYP Domain wall valence

Asqtad Action: $O(a^2)$ perturbatively improved

- Symansik improved glue
  - $S_g(U) = C_0 W^{1\times 1} + C_1 W^{1\times 2} + C_2 W^{cube}$
- Smeared staggered fermions $S_f(V,U)$
  - Fat links remove taste changing gluons
  - Tadpole improved



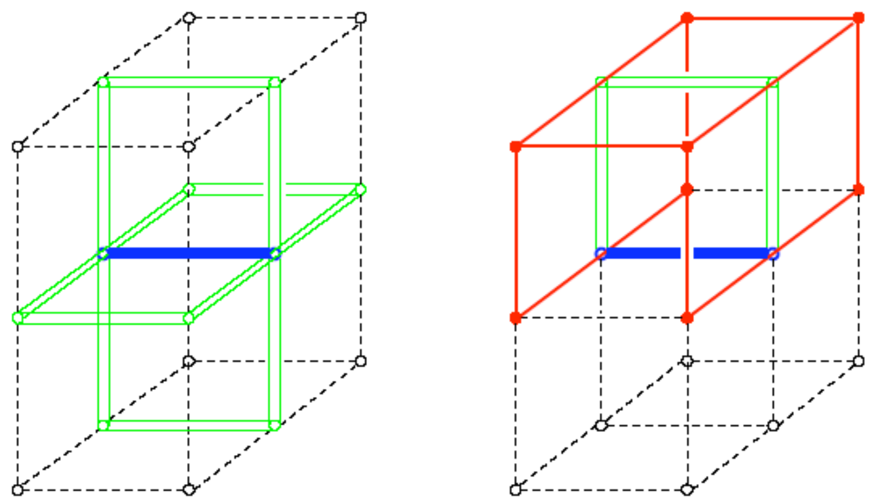
HYP Smearing

- Three levels of SU(3) projected blocking within hypercube
- Minimize dislocations - important for DW fermions

$$V_{i,\mu} = Proj_{SU(3)}[(1-\alpha_1)U_{i,\mu} + \frac{\alpha_1}{6}\sum_{\pm\nu\neq\mu} \tilde{V}_{i,\nu;\mu}\tilde{V}_{i+\hat{\nu},\mu;\nu}\tilde{V}^{\dagger}_{i+\hat{\mu},\nu;\mu}],$$

$$\tilde{V}_{i,\mu;\nu} = Proj_{SU(3)}[(1-\alpha_2)U_{i,\mu} + \frac{\alpha_2}{4}\sum_{\pm\rho\neq\nu,\mu} \bar{V}_{i,\rho;\nu\mu}\bar{V}_{i+\hat{\rho},\mu;\rho\nu}\bar{V}^{\dagger}_{i+\hat{\mu},\rho;\nu\mu}],$$

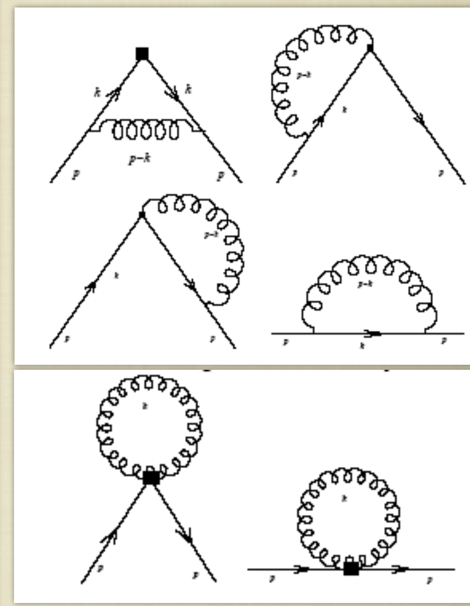
$$\bar{V}_{i,\mu;\nu\rho} = Proj_{SU(3)}[(1-\alpha_3)U_{i,\mu} + \frac{\alpha_3}{2}\sum_{\pm\eta\neq\rho,\nu,\mu} U_{i,\eta}U_{i+\hat{\eta},\mu}U^{\dagger}_{i+\hat{\mu},\eta}].$$



Perturbative renormalization

HYP smeared domain wall fermions - B. Bistrovic

operator	$H(4)$	NOS	HYP	APE
$\bar{q}[\gamma_5]q$	$1_1^{\pm}$	0.792	0.981	1.046
$\bar{q}[\gamma_5]\gamma_\mu q$	$4_4^{\mp}$	0.847	0.976	0.994
$\bar{q}[\gamma_5]\sigma_{\mu\nu} q$	$6_1^{\mp}$	0.883	0.992	0.993
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu\}}q$	$6_3^{\pm}$	0.991	0.979	0.954
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu\}}q$	$3_1^{\pm}$	0.982	0.975	0.951
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	$8_1^{\mp}$	1.134	0.988	0.934
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	mixing	5.71×10^{-3}	1.88×10^{-3}	8.21×10^{-4}
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	$4_2^{\mp}$	1.124	0.987	0.934
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_\alpha D_{\beta\}}q$	$2_1^{\pm}$	1.244	0.993	0.919
$\bar{q}[\gamma_5]\sigma_{\mu\{\nu}D_{\alpha\}}q$	$8_1^{\pm}$	1.011	0.994	0.964
$\bar{q}[\gamma_5]\gamma_{[\mu}D_{\nu]}q$	$6_1^{\mp}$	0.979	0.982	0.989
$\bar{q}[\gamma_5]\gamma_{[\mu}D_{\{\nu]}D_{\alpha\}}q$	$8_1^{\pm}$	0.955	0.959	0.965



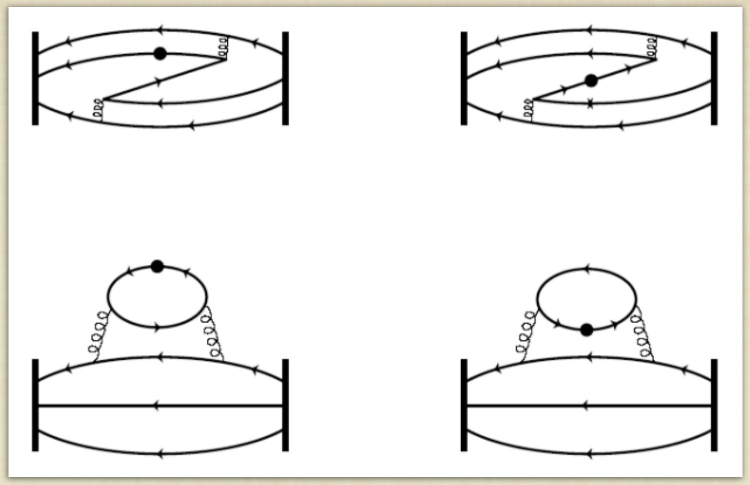
$$O_i^{\overline{MS}}(Q^2) = \sum_j \left(\delta_{ij} + \frac{g_0^2}{16\pi^2}\frac{N_c^2-1}{2N_c}\left(\gamma_{ij}^{\overline{MS}}\log(Q^2a^2) - (B_{ij}^{LATT} - B_{ij}^{\overline{MS}})\right)\right)\cdot O_j^{LATT}(a^2)$$

Numerical calculations

- Improved staggered sea quarks (MILC configurations)
  - $N_F = 3$, $a = 0.125$ fm
- Domain wall valence quarks
  - $L_S = 16$, $M = 1.7$
  - Masses and volumes:

m_π	configs	Vol	L (fm)
761	425	20^3	2.5
693	350	20^3	2.5
544	564	20^3	2.5
486	498	20^3	2.5
354	655	20^3	2.5
354	270	28^3	3.5

Hadron matrix elements on the lattice



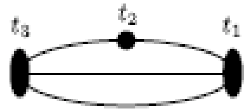
- Measure $\langle \mathcal{O} \rangle$ for m_q, a, L
- Connected diagrams
- Disconnected diagrams (cancel for $\langle \mathcal{O} \rangle_u - \langle \mathcal{O} \rangle_d$)
- Extrapolate $m_q : m_\pi \rightarrow 140 \text{ MeV}$
  $a \rightarrow \sim 0.05 \text{ fm}$
  $L \rightarrow \sim 5 \text{ fm}$

Matrix elements on the lattice

$J^\dagger$: Current with quantum numbers of proton

$|\psi_J\rangle = J^\dagger|\Omega\rangle$ Trial function

$$\langle T J(t_3)\,\mathcal{O}(t_2)\,J^\dagger(t_1)\rangle = \sum_{m,n}\langle\psi_J|n\rangle\langle n|\mathcal{O}|m\rangle\langle m|\psi_J\rangle e^{-E_n(t_3-t_2)-E_m(t_2-t_1)}$$



$$\xrightarrow[\substack{t_3-t_2\gg 1\\ t_2-t_1\gg 1}]{} \left|\langle\psi_J|0\rangle\right|^2\langle 0|\mathcal{O}|0\rangle e^{-E_0(t_3-t_1)}$$

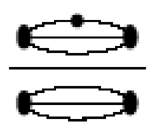
Normalize:

$$\langle T J(t_3)\,J^\dagger(t_1)\rangle = \sum_n\left|\langle\psi_J|n\rangle\right|^2 e^{-E_n(t_3-t_1)}$$

$$\xrightarrow[t_3-t_1\gg 1]{} \left|\langle\psi_J|0\rangle\right|^2 e^{-E_0(t_3-t_1)}$$

$\Longrightarrow$

$$\langle 0|\mathcal{O}|0\rangle = \frac{\langle J\mathcal{O}J^\dagger\rangle}{\langle JJ^\dagger\rangle} =$$



Overdetermined system for form factors

Calculate ratio

$$R_{\mathcal{O}}(\tau,P',P)=\frac{C^{3\mathrm{pt}}_{\mathcal{O}}(\tau,P',P)}{C^{2\mathrm{pt}}(\tau_{\mathrm{snk}},P')}\left[\frac{C^{2\mathrm{pt}}(\tau_{\mathrm{snk}}-\tau+\tau_{\mathrm{src}},P)\;C^{2\mathrm{pt}}(\tau,P')\;C^{2\mathrm{pt}}(\tau_{\mathrm{snk}},P')}{C^{2\mathrm{pt}}(\tau_{\mathrm{snk}}-\tau+\tau_{\mathrm{src}},P')\;C^{2\mathrm{pt}}(\tau,P)\;C^{2\mathrm{pt}}(\tau_{\mathrm{snk}},P)}\right]^{1/2}$$

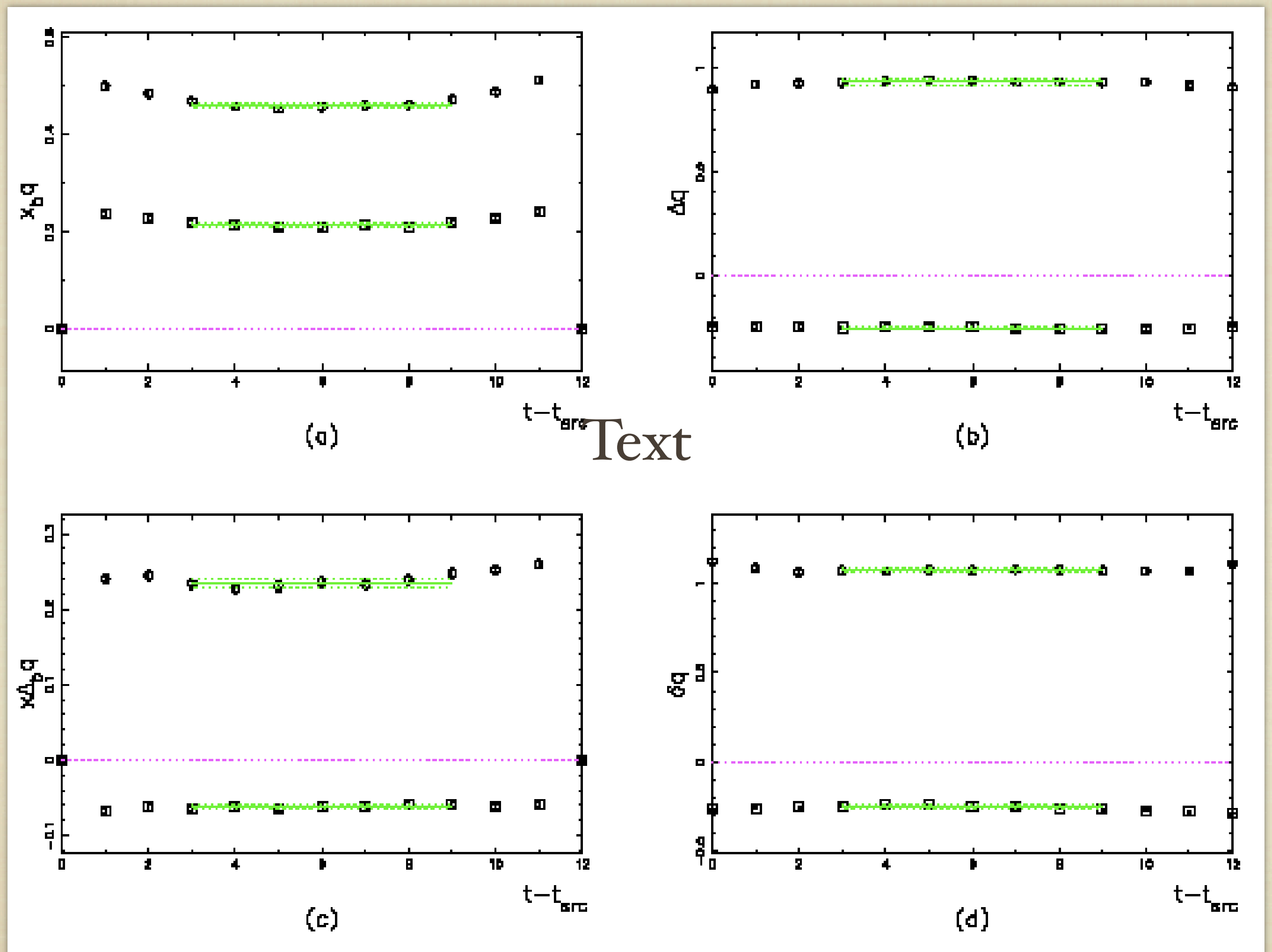
Schematic form

$$\begin{aligned}
\langle \mathcal{O}_i^{cont}\rangle &= \sum_j a_{ij}\mathcal{F}_j \\
\langle \mathcal{O}_i^{cont}\rangle &= \sqrt{E'E}\sum_j Z_{ij}\overline{R}_j \\
\overline{R}_i &= \frac{1}{\sqrt{E'E}}\sum_{jk} Z^{-1}_{ij}a_{jk}\mathcal{F}_k \\
&\equiv \sum_j a'_{ij}\mathcal{F}_j\,.
\end{aligned}$$

Plateaus for operators

$\langle x \rangle_q$ $\langle 1 \rangle_{\Delta q}$

$\langle x \rangle_{\Delta q}$ $\langle 1 \rangle_{\delta q}$



Nucleon axial charge in full lattice QCD

- Why g_A?

- Matrix element of axial current $A_\mu = \bar{q}\gamma_\mu\gamma_5 \frac{\vec{\tau}}{2} q$

$$\langle N(p+q)|A_\mu|N(p)\rangle = \bar{u}(p+q)\frac{\vec{\tau}}{2}\left[g_A(q^2)\gamma_\mu\gamma_5 + g_P(q^2)q_\mu\gamma_5\right]u(p)$$

$$g_A(0) = 1.2695 \pm 0.0029$$

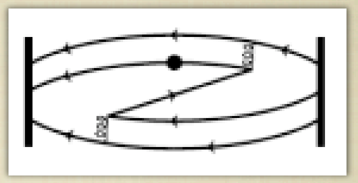
- Adler Weisberger $g_A^2 - 1 \sim \int(\sigma_{\pi^+p} - \sigma_{\pi^-p})$

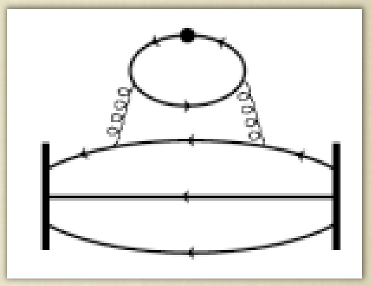
- Goldberger Treiman $g_A \to f_\pi g_{\pi NN}/M_N$

- Spin content $\langle 1\rangle_{\Delta q} = \int_0^1 dx[\Delta q(x) + \Delta\bar{q}(x)]$

$$g_A = \langle 1\rangle_{\Delta u} - \langle 1\rangle_{\Delta d} \qquad \Sigma = \langle 1\rangle_{\Delta u} + \langle 1\rangle_{\Delta d} + \langle 1\rangle_{\Delta s}$$

Nucleon axial charge

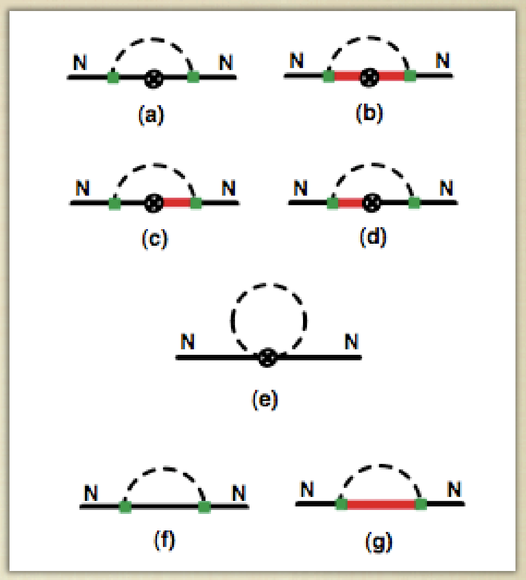




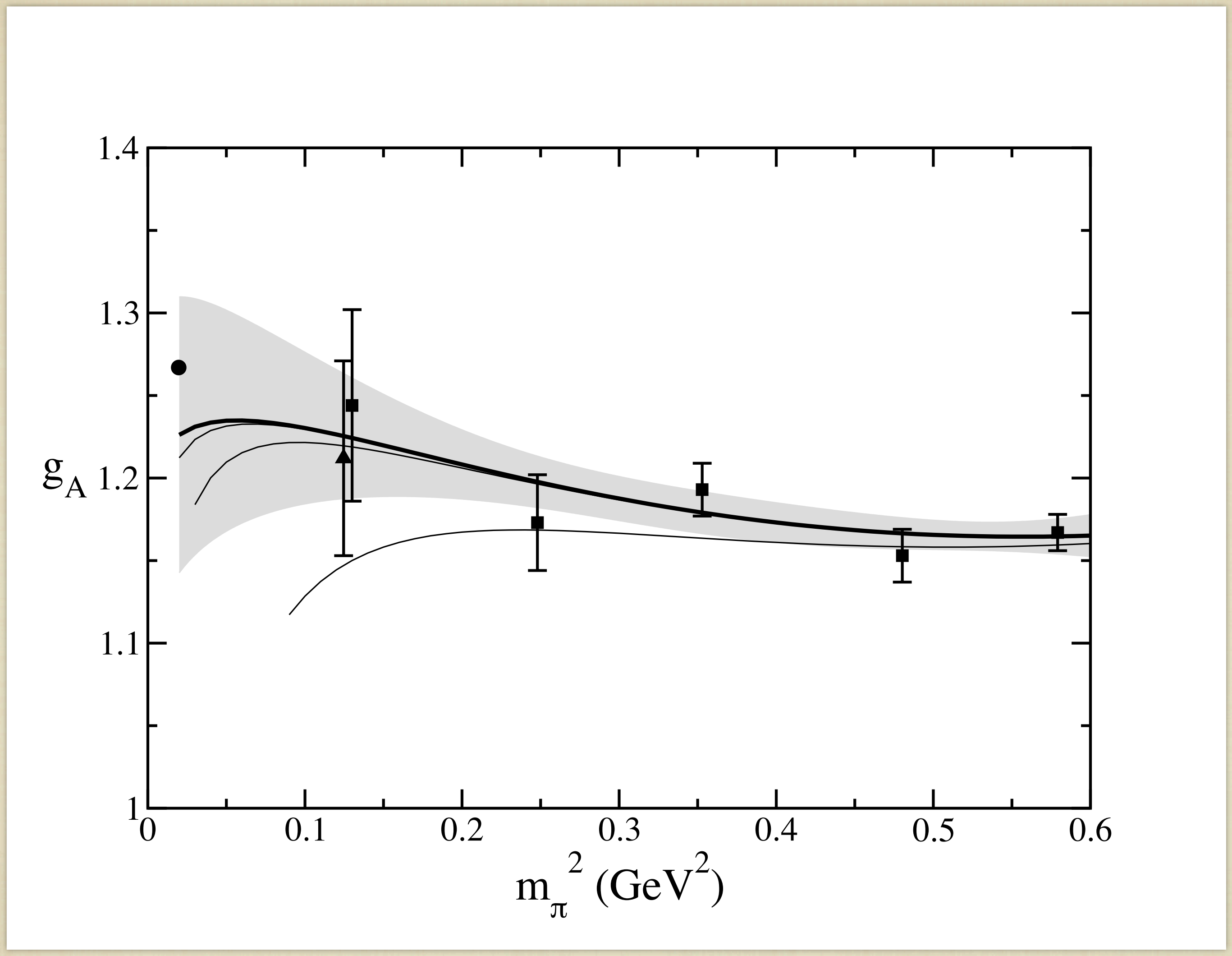
- Gold-Plated observable
  - Accurately measured
  - No disconnected diagrams
  - Chiral perturbation theory for $g_A(m_\pi^2, V)$
  - Renormalization - 5-d conserved current

Nucleon Axial Charge

- Chiral perturbation theory $g_A(m_\pi^2, V)$
  - Beane and Savage hep-ph/0404131
  - Detmold and Lin hep-lat/0501007
- 1-loop theory has 6 parameters
  - Fix $f_\pi,\, m_\Delta - m_N,\, g_{\Delta N}$ (0.3% error)
  - Fit $g_A,\, g_{\Delta\Delta},\, C$
  - Result $g_A(m_\pi = 140) = 1.212 \pm 0.084$

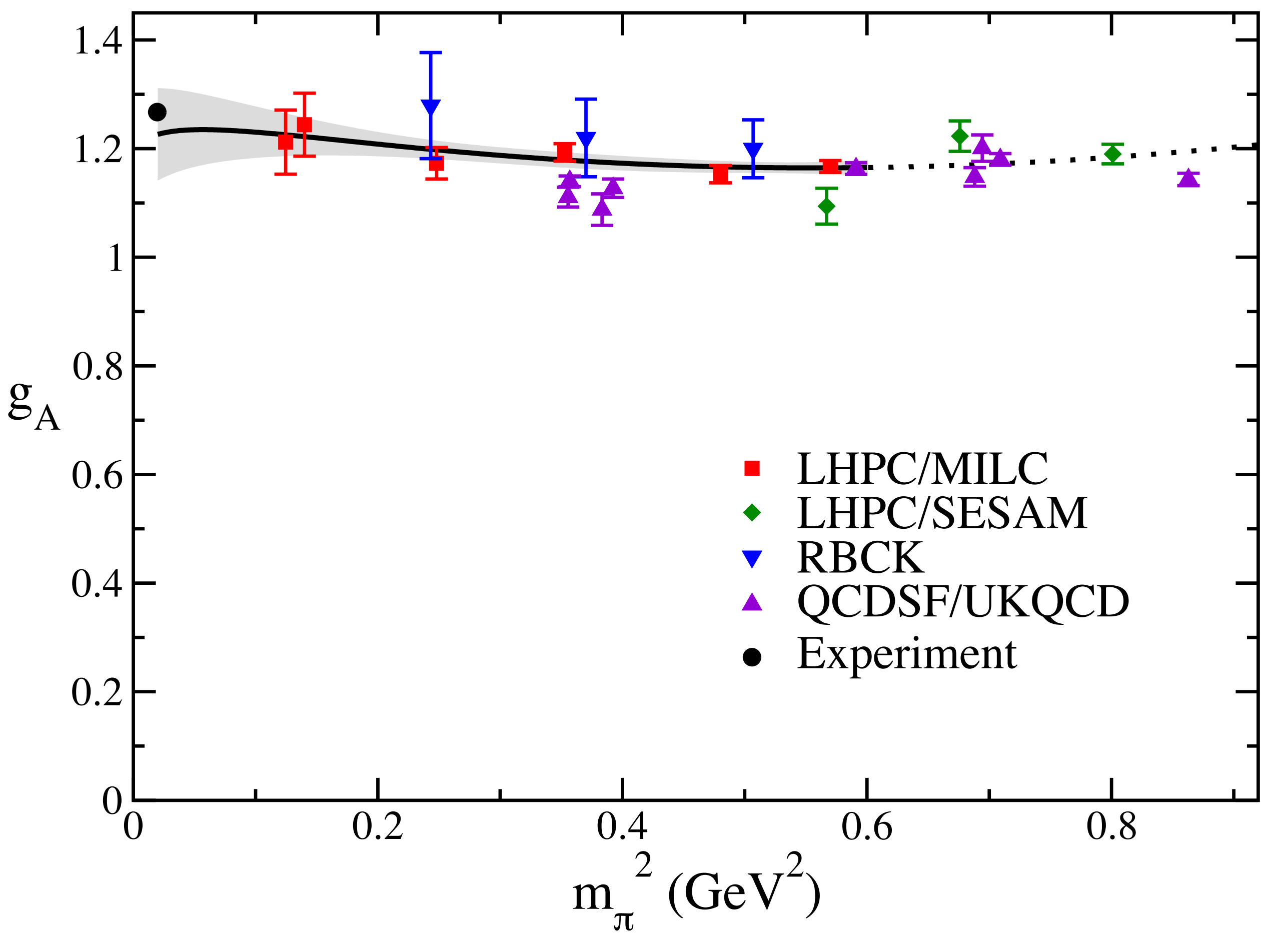


Nucleon axial charge g_A $\langle 1 \rangle_{\Delta q}^{u-d}$



Fit: Beane and Savage hep-ph/0404131

Nucleon axial charge g_A $\langle 1 \rangle_{\Delta q}^{u-d}$



Chiral Extrapolation of Moments

- for example, unpolarized moments

$$\langle x^n \rangle_{u-d} = a_n \left(1 - \frac{(3g_{A,0}^2 + 1)}{(4\pi f_{\pi,0})^2} m_\pi^2 \ln \left(\frac{m_\pi^2}{\mu^2} \right) \right) + b'_n(\mu) m_\pi^2$$

- choose $\mu = f_{\pi,0}$, and at one loop $g_{A,0} \to g_{A,m_\pi}$ and $f_{\pi,0} \to f_{\pi,m_\pi}$

$$\langle x^n \rangle_{u-d} = a_n \left(1 - \frac{(3g_{A,m_\pi}^2 + 1)}{(4\pi)^2} \frac{m_\pi^2}{f_{\pi,m_\pi}^2} \ln \left(\frac{m_\pi^2}{f_{\pi,m_\pi}^2} \right) \right) + b_n \frac{m_\pi^2}{f_{\pi,m_\pi}^2}$$

- self consistently $g_A \to g_{A,\text{lat}}$, $f_\pi \to f_{\pi,\text{lat}}$, $m_\pi \to m_{\pi,\text{lat}}$

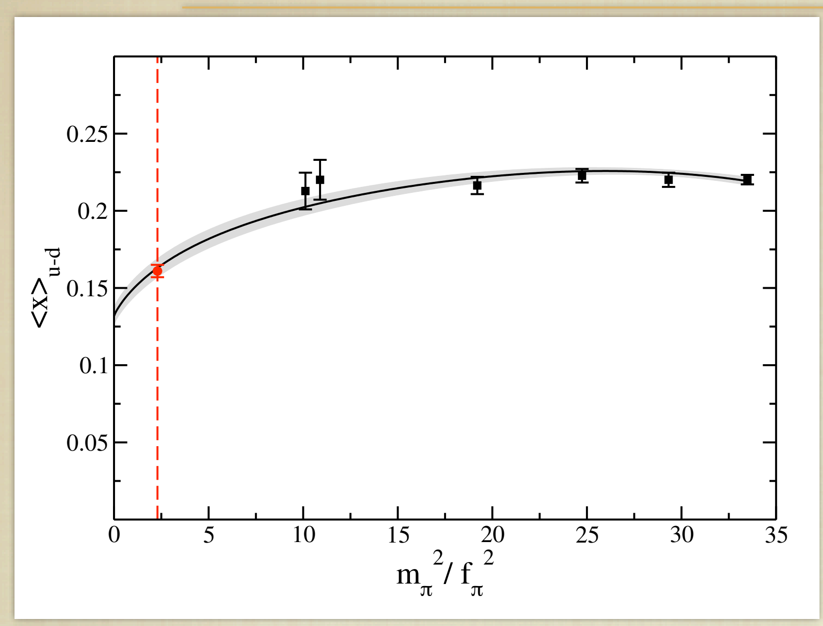
$$\langle x^n \rangle_{u-d} = a_n \left(1 - \frac{(3g_{A,\text{lat}}^2 + 1)}{(4\pi)^2} \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \ln \left(\frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \right) \right) + b_n \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}$$

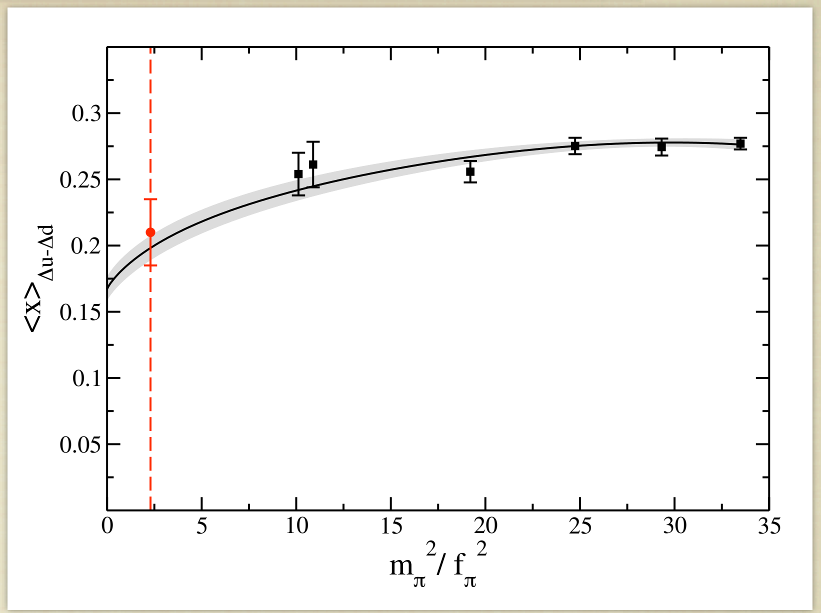
- similarly for the helicity and transversity moments

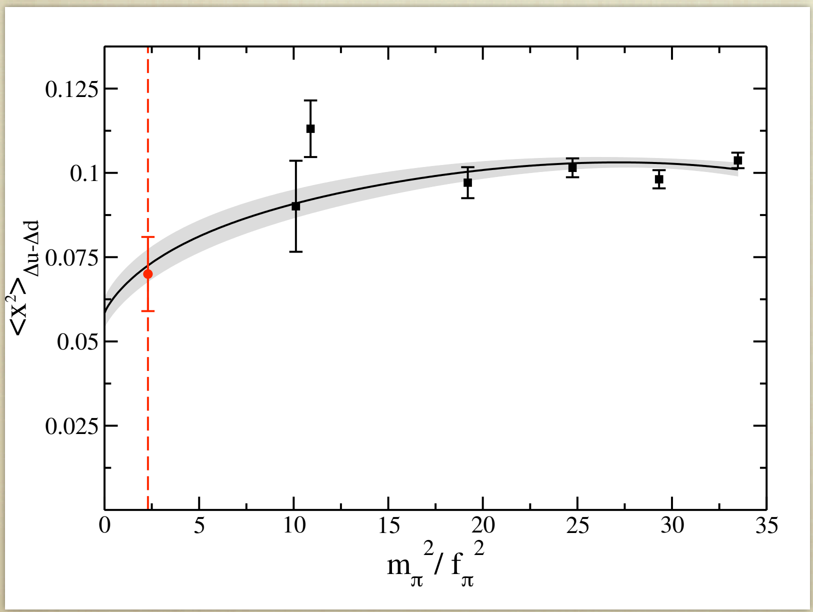
$$\langle x^n \rangle_{\Delta u-\Delta d} = \Delta a_n \left(1 - \frac{(2g_{A,\text{lat}}^2 + 1)}{(4\pi)^2} \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \ln \left(\frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \right) \right) + \Delta b_n \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}$$

$$\langle x^n \rangle_{\delta u-\delta d} = \delta a_n \left(1 - \frac{(4g_{A,\text{lat}}^2 + 1)}{2(4\pi)^2} \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \ln \left(\frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \right) \right) + \delta b_n \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}$$

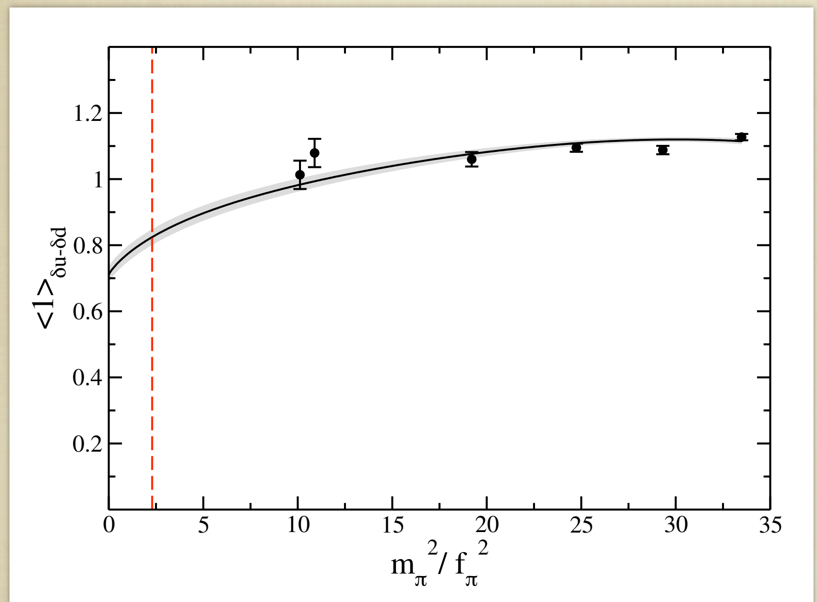
Chiral Extrapolation of Moments

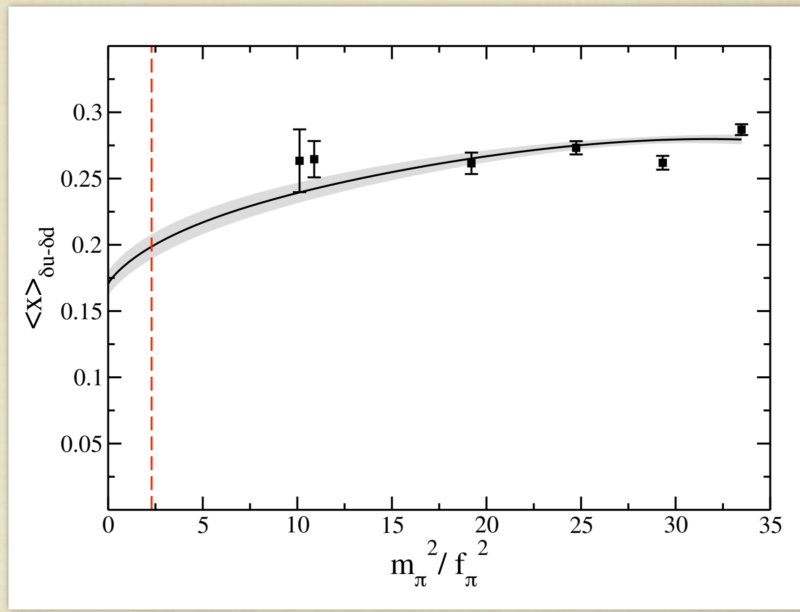




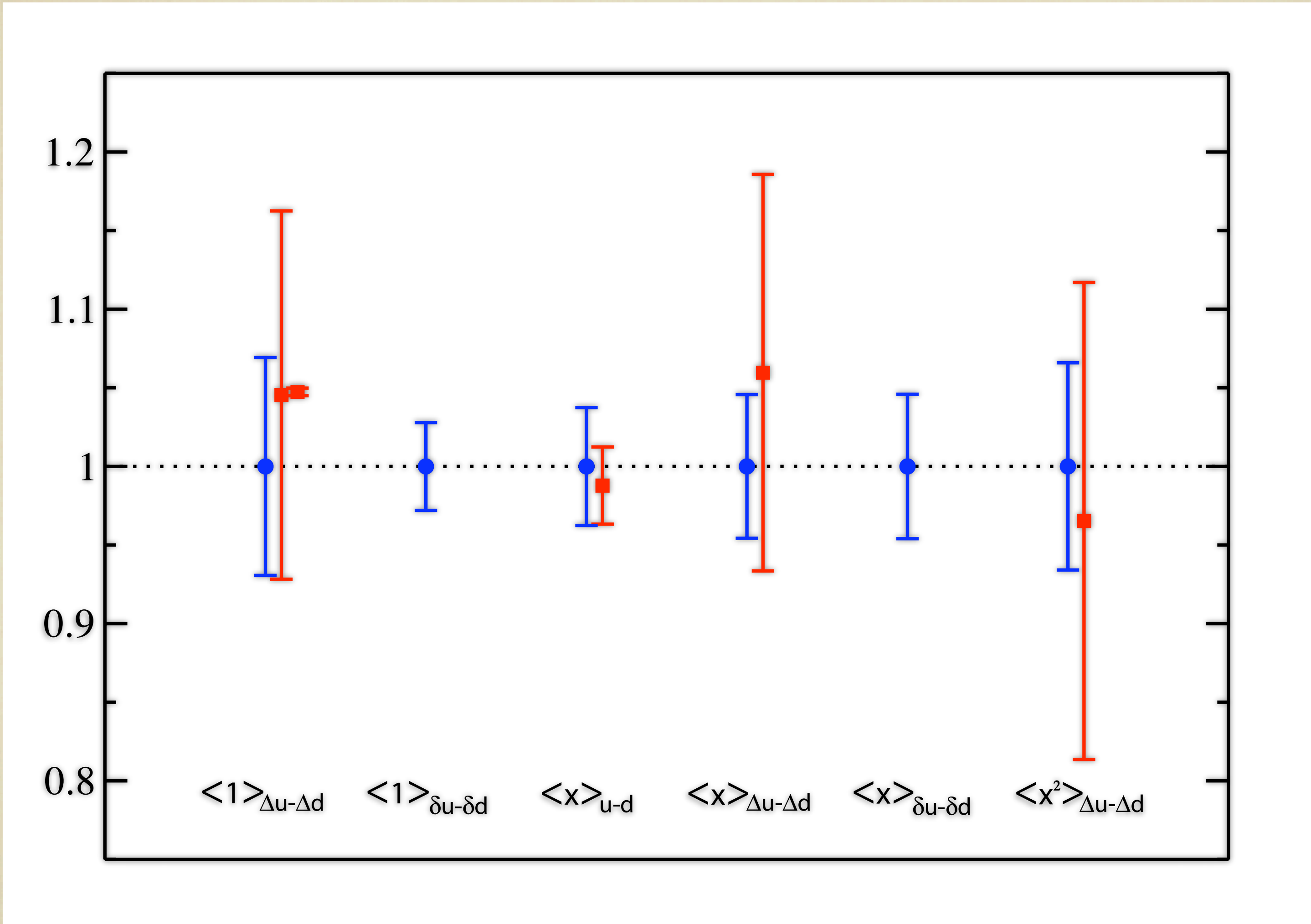


Chiral Extrapolation of Moments





Chiral Extrapolation of Moments



Electromagnetic form factors

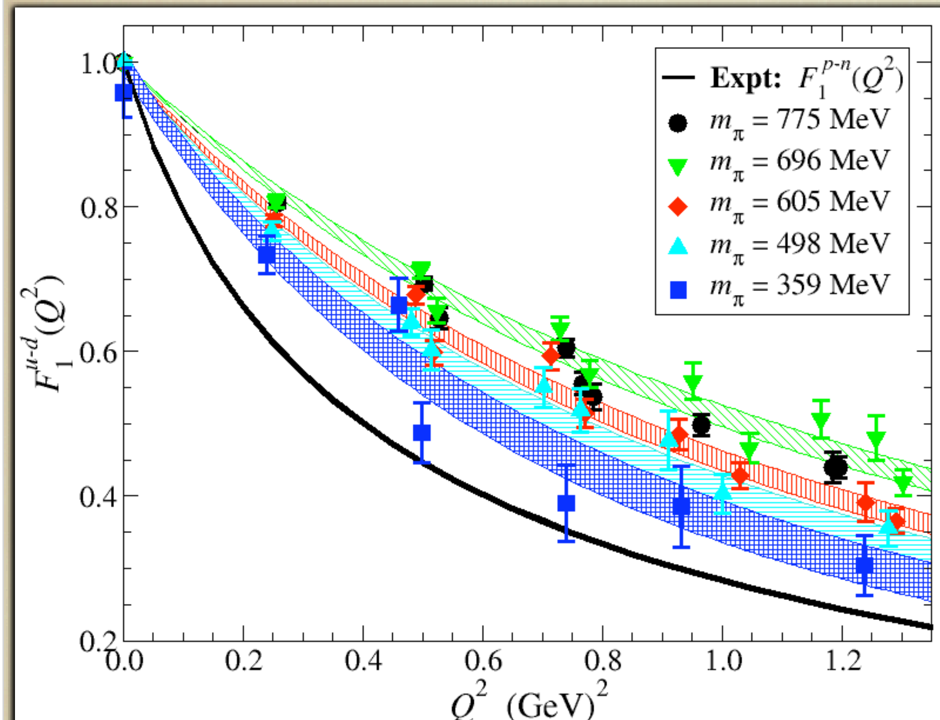
- Simplest off-diagonal matrix element

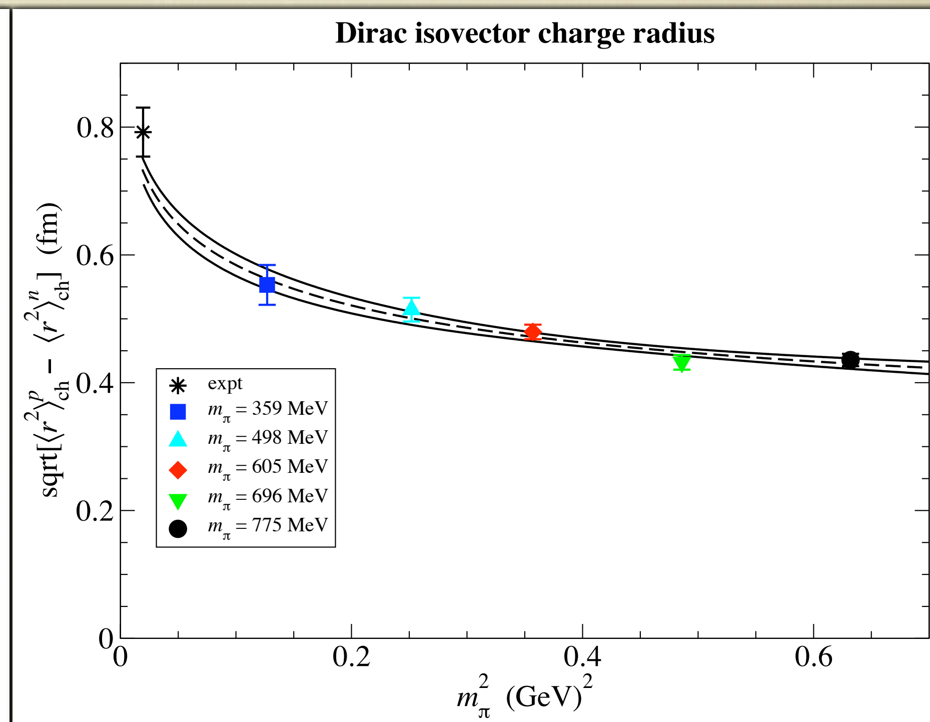
$$\langle p|\bar{\psi}\gamma^{\mu}\psi|p'\rangle = \bar{u}(p)[F_1(q^2)\gamma^{\mu} + F_2(q^2)\frac{i\sigma^{\mu\nu}q_{\nu}}{2m}]u(p')$$

$$G_E(q^2) = F_1(q^2) - \frac{q^2}{4M^2}F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

- Fourier transform of charge density if $\mathrm{L}_{\mathrm{system}} \gg \mathrm{L}_{\mathrm{wavepacket}} \gg \frac{1}{m}$
  - Pb: 5 fm >> 10^{-5} fm, Proton: 0.8 fm ~ 0.2 fm: marginal
  - For transverse Fourier transform of light cone w. f., m → p_+ ~ ∞
- Large q^2: ability of one quark to share q^2 with other constituents to remain in ground state - q^2 counting rules

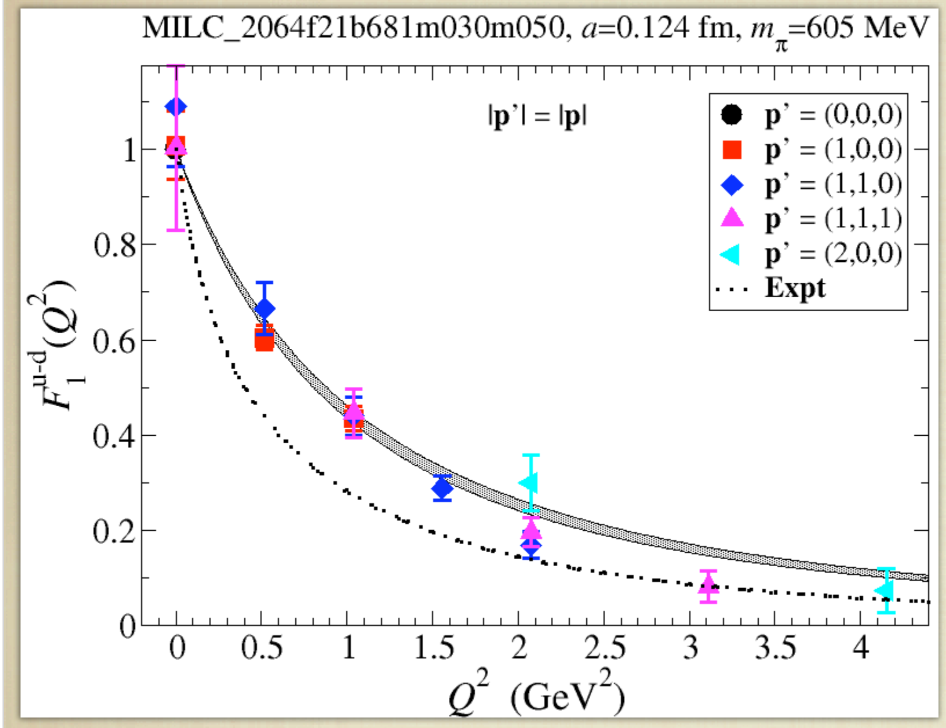
F_1 Isovector Form Factor

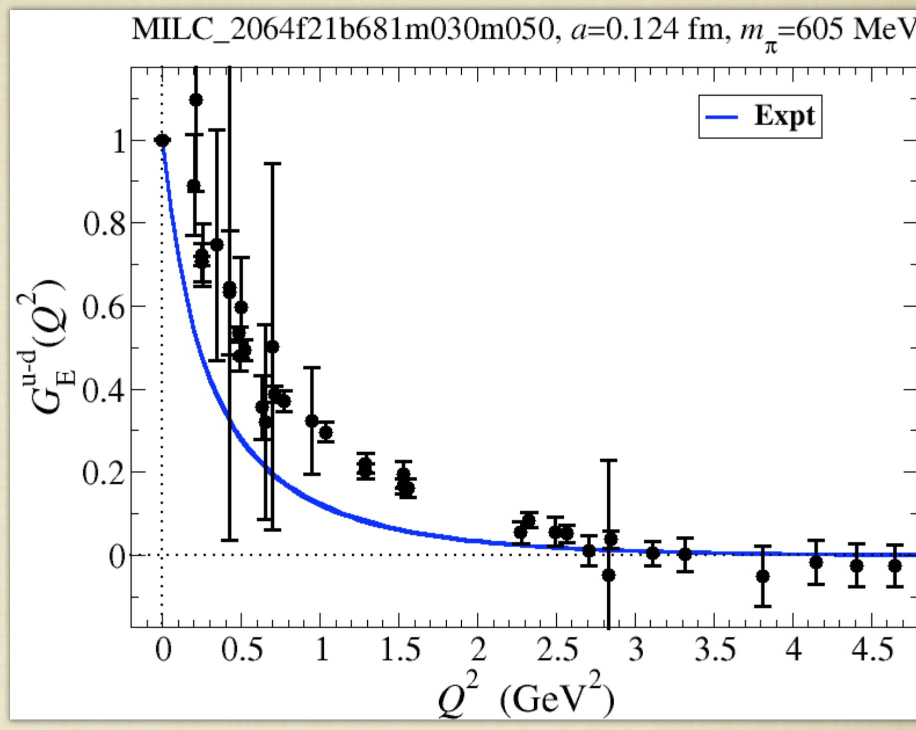




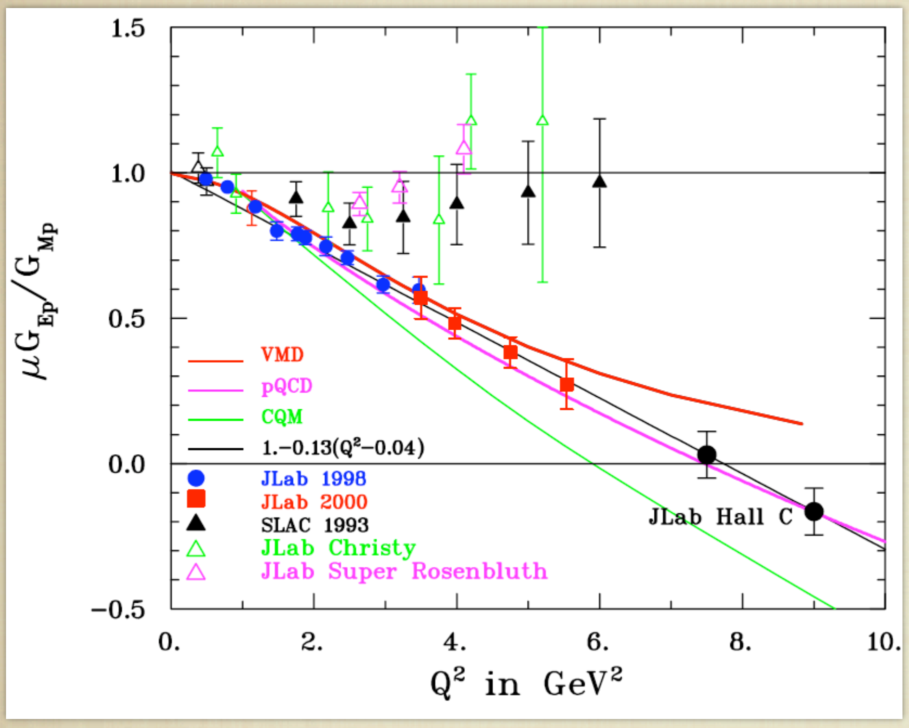
$$\langle r^2 \rangle^{u-d} = a_0 - \frac{(1+5g_A^2)}{(4\pi f_\pi)^2} \log\left(\frac{m_\pi^2}{m_\pi^2 + \Lambda^2}\right)$$

Isovector Form Factors at higher Q^2

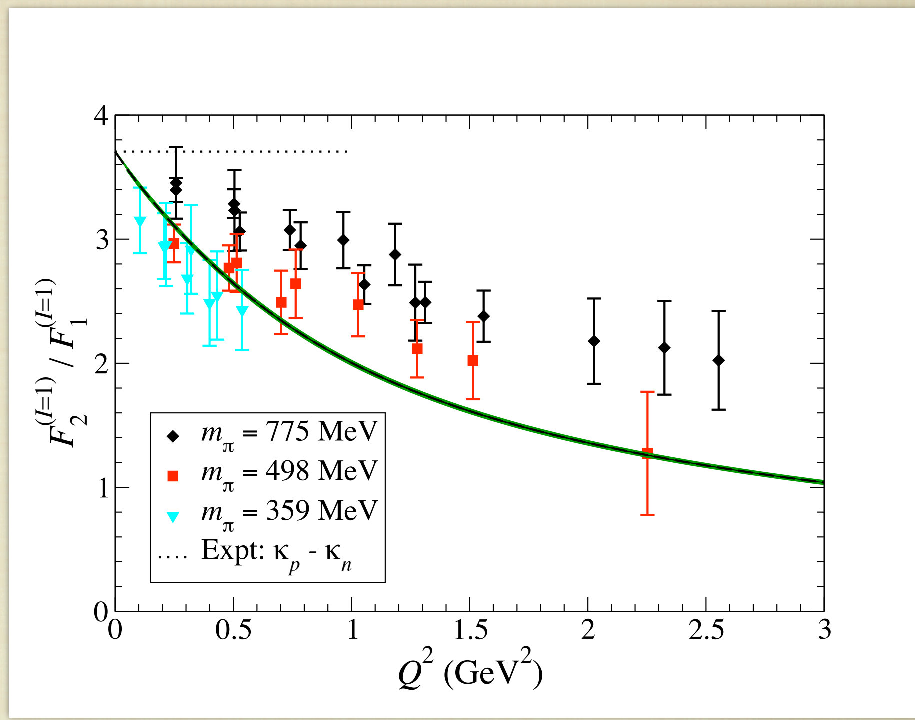




Form factor ratio: F_2 / F_1



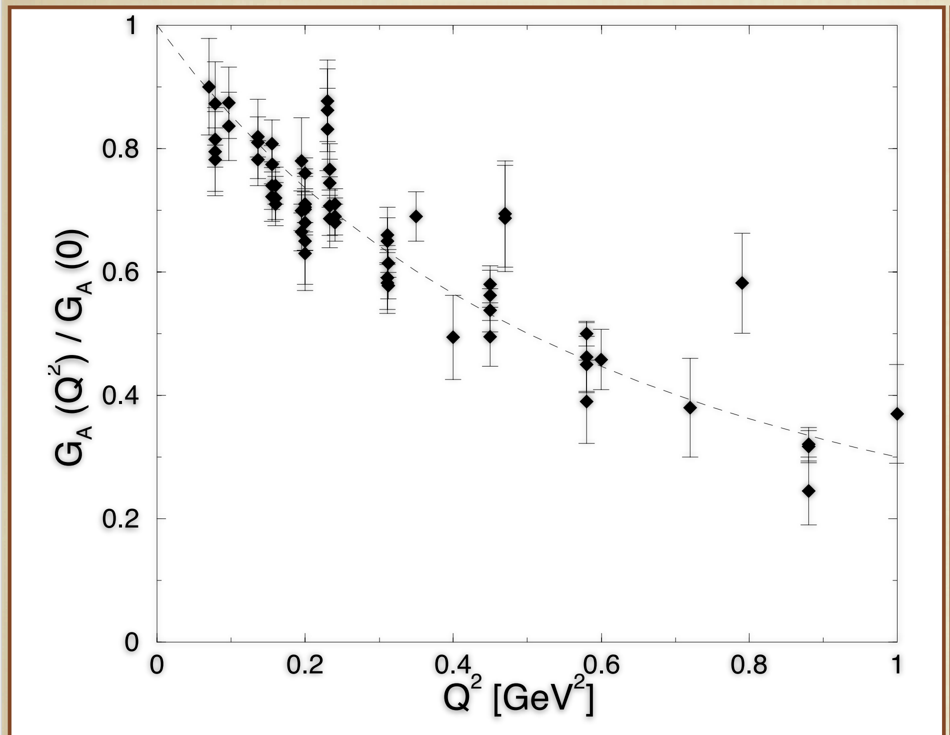
Polarization transfer at JLab

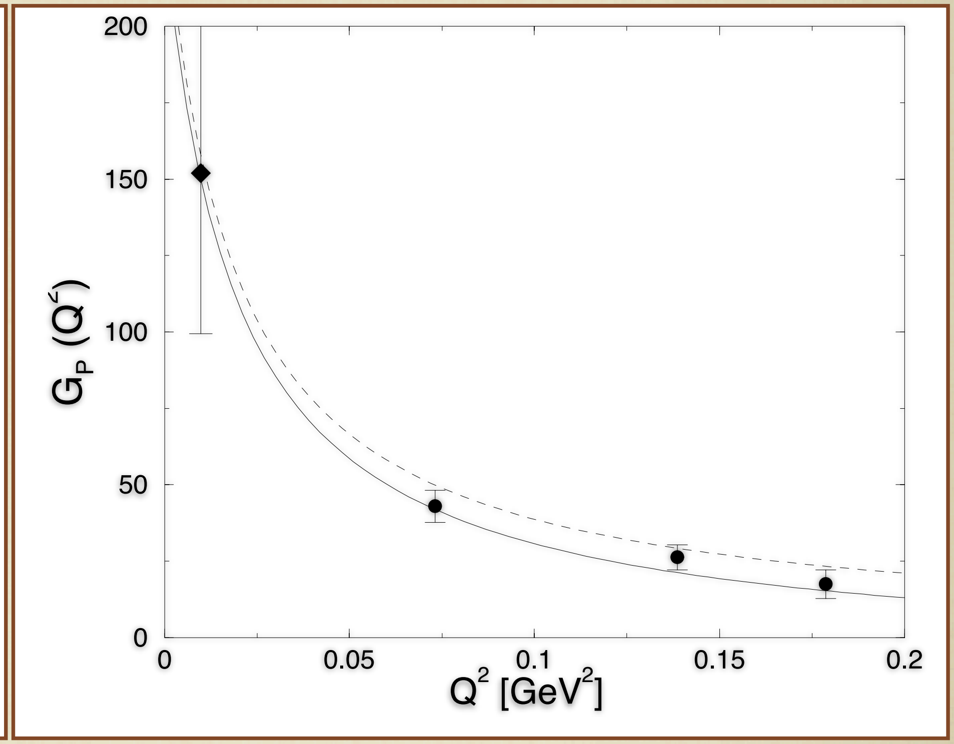


Lattice results

Polarized Nucleon Form Factors G_A and G_P

$$\langle p|\bar{\psi}\gamma^\mu\gamma_5\psi|p'\rangle = \bar{u}(p)[G_A(q^2)\gamma^\mu\gamma_5 + q^\mu\gamma_5 G_P(q^2) + \sigma^{\mu\nu}\gamma_5 q_\nu G_M(q^2)]u(p')$$





Bernard, Elouadrhiri, Meissner, J. Phys. G Nucl. Part. Phys. 2002, R1

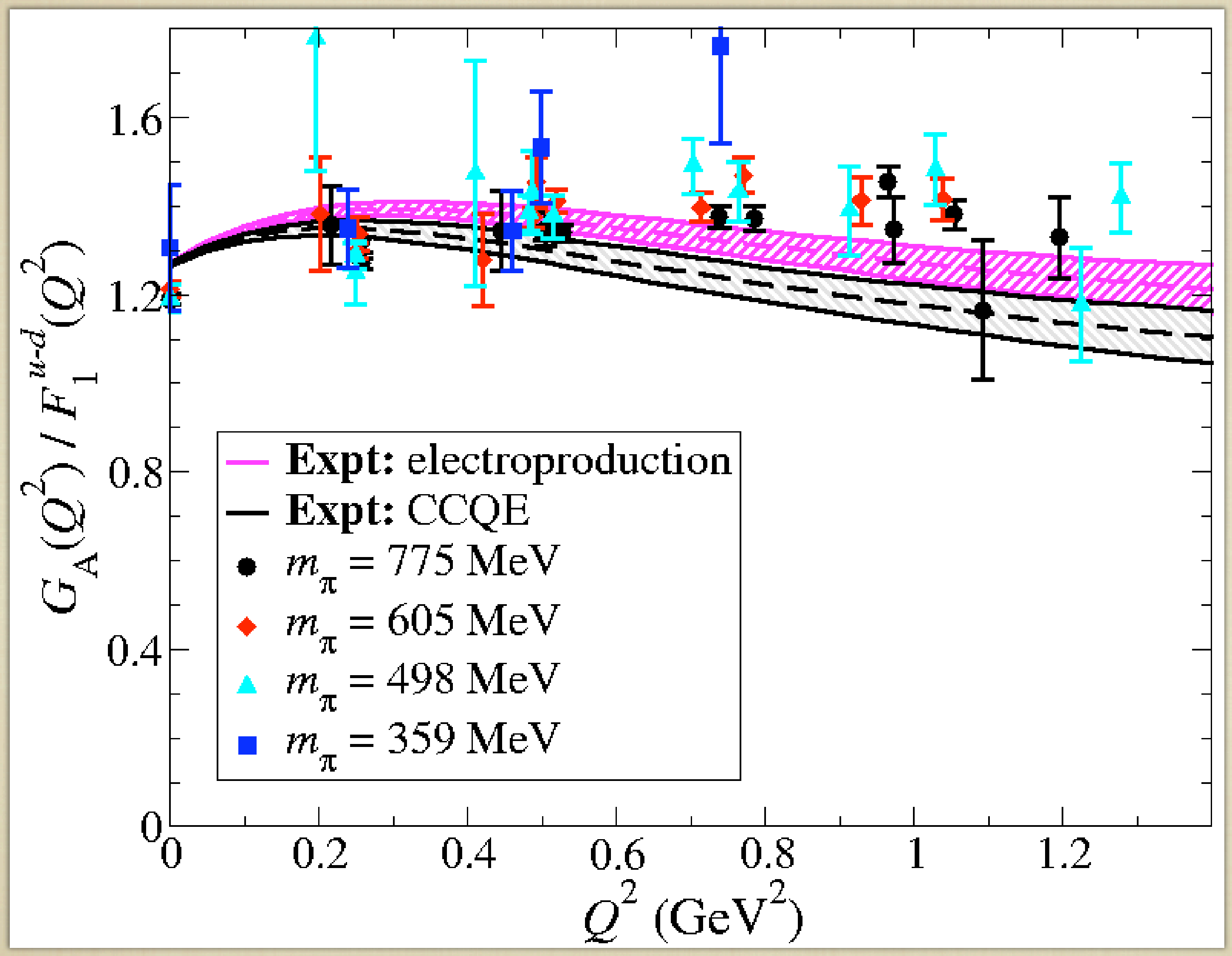
pion electroproduction ◆

$\nu_\mu\, n \rightarrow \mu^-\, p$

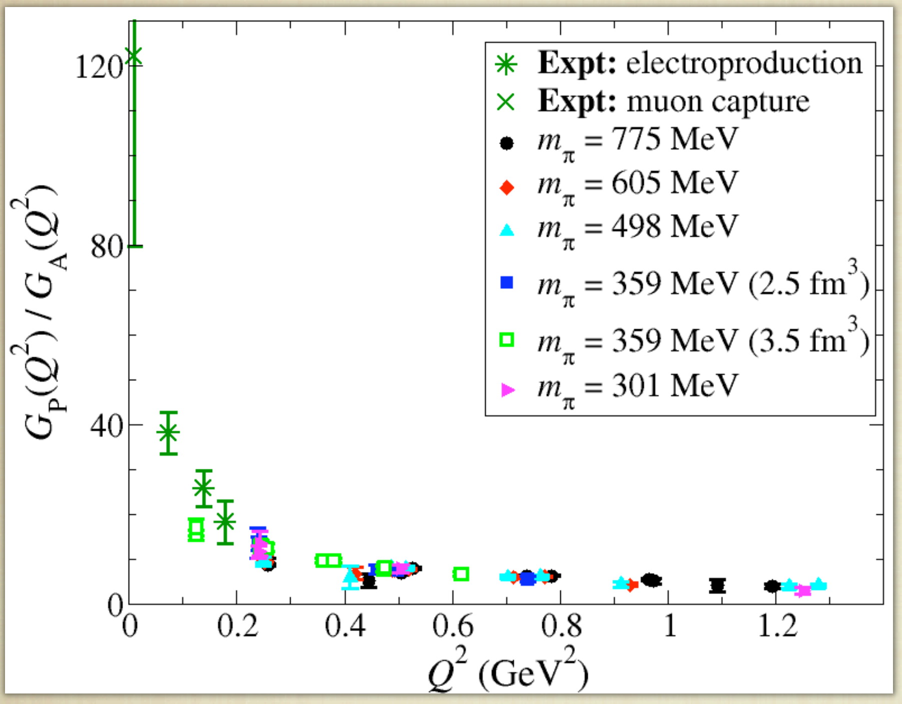
pion electroproduction ●

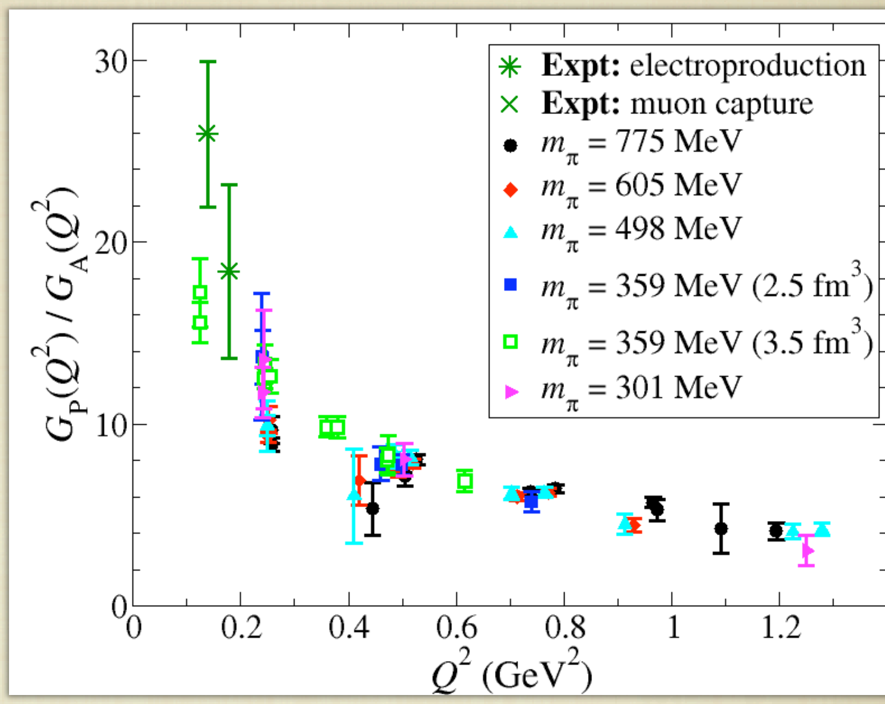
$\mu^-\, p \rightarrow \nu_\mu\, n$ ◆

Form factor ratio: G_A/F_1



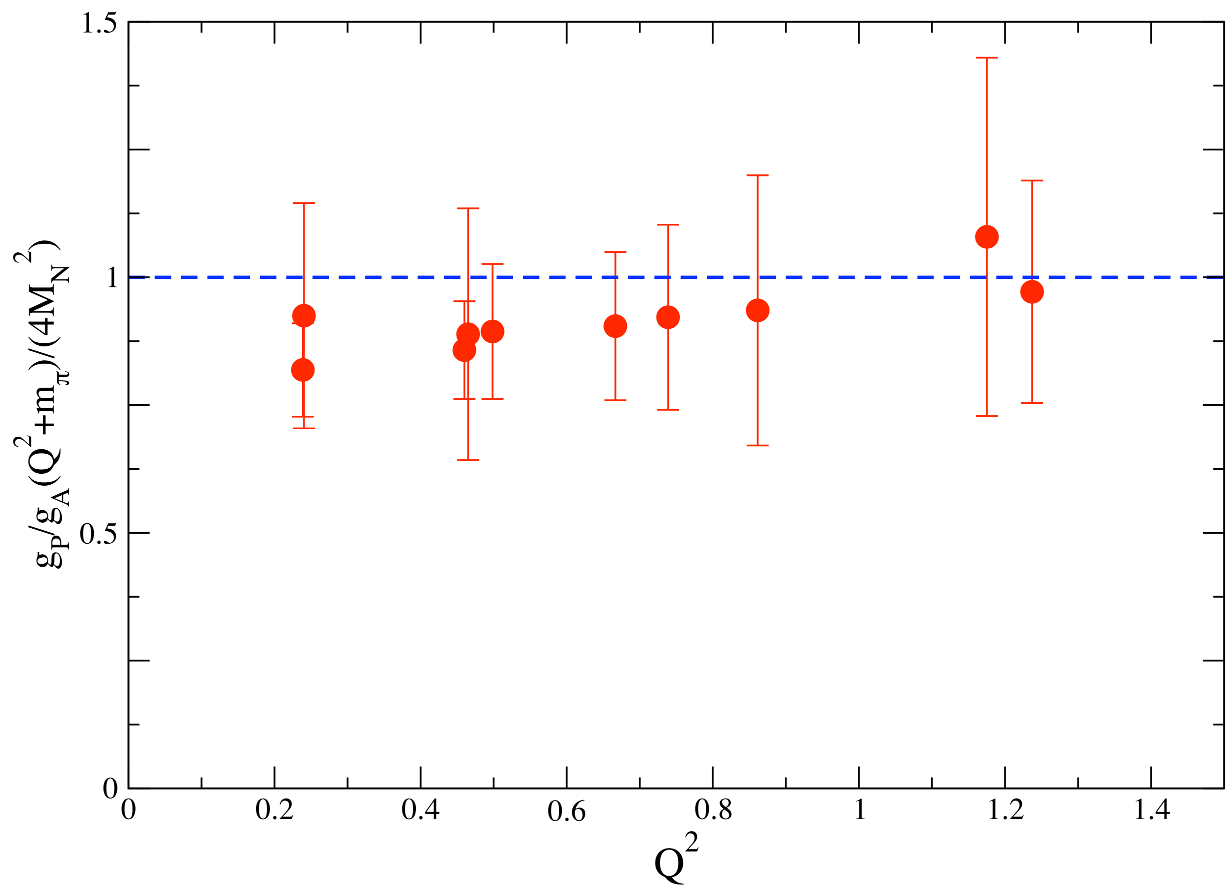
Form factor ratio: G_P/G_A





soft pion pole: $$G_P(q^2) \sim \frac{4M^2 G_A(q^2)}{q^2 - m_\pi^2}$$

Form factor ratio: G_P/G_A



Generalized form factors

$$\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}} = \overline{\psi}_q \gamma^{\{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n\}} \psi_q$$

$$\bar{P} = \tfrac{1}{2}(P' + P)$$

$$\Delta = P' - P$$

$$t = \Delta^2$$

$$\begin{aligned}
\langle P'|\mathcal{O}^{\mu_1}|P\rangle &= \langle\!\langle \gamma^{\mu_1} \rangle\!\rangle A_{10}(t) \\
&+ \frac{i}{2m} \langle\!\langle \sigma^{\mu_1\alpha} \rangle\!\rangle \Delta_\alpha B_{10}(t)\,,
\end{aligned}$$

$$\begin{aligned}
\langle P'|\mathcal{O}^{\{\mu_1\mu_2\}}|P\rangle &= \bar{P}^{\{\mu_1} \langle\!\langle \gamma^{\mu_2\}} \rangle\!\rangle A_{20}(t) \\
&+ \frac{i}{2m} \bar{P}^{\{\mu_1} \langle\!\langle \sigma^{\mu_2\}\alpha} \rangle\!\rangle \Delta_\alpha B_{20}(t) \\
&+ \frac{1}{m} \Delta^{\{\mu_1} \Delta^{\mu_2\}} C_2(t)\,,
\end{aligned}$$

$$\begin{aligned}
\langle P'|\mathcal{O}^{\{\mu_1\mu_2\mu_3\}}|P\rangle &= \bar{P}^{\{\mu_1} \bar{P}^{\mu_2} \langle\!\langle \gamma^{\mu_3\}} \rangle\!\rangle A_{30}(t) \\
&+ \frac{i}{2m} \bar{P}^{\{\mu_1} \bar{P}^{\mu_2} \langle\!\langle \sigma^{\mu_3\}\alpha} \rangle\!\rangle \Delta_\alpha B_{30}(t) \\
&+ \Delta^{\{\mu_1} \Delta^{\mu_2} \langle\!\langle \gamma^{\mu_3\}} \rangle\!\rangle A_{32}(t) \\
&+ \frac{i}{2m} \Delta^{\{\mu_1} \Delta^{\mu_2} \langle\!\langle \sigma^{\mu_3\}\alpha} \rangle\!\rangle \Delta_\alpha B_{32}(t),
\end{aligned}$$

Limits of generalized form factors

- Moments of parton distributions $t \to 0$

$$A_{n0} = \int dx x^{n-1} q(x)$$

- Electromagnetic form factors

$$A_{10} = F_1(t), \quad B_{10} = F_2(t)$$

- Total quark angular momentum

$$J_q = \tfrac{1}{2}[A(0)_{20} + B(0)_{20}]$$

Transverse structure of nucleon

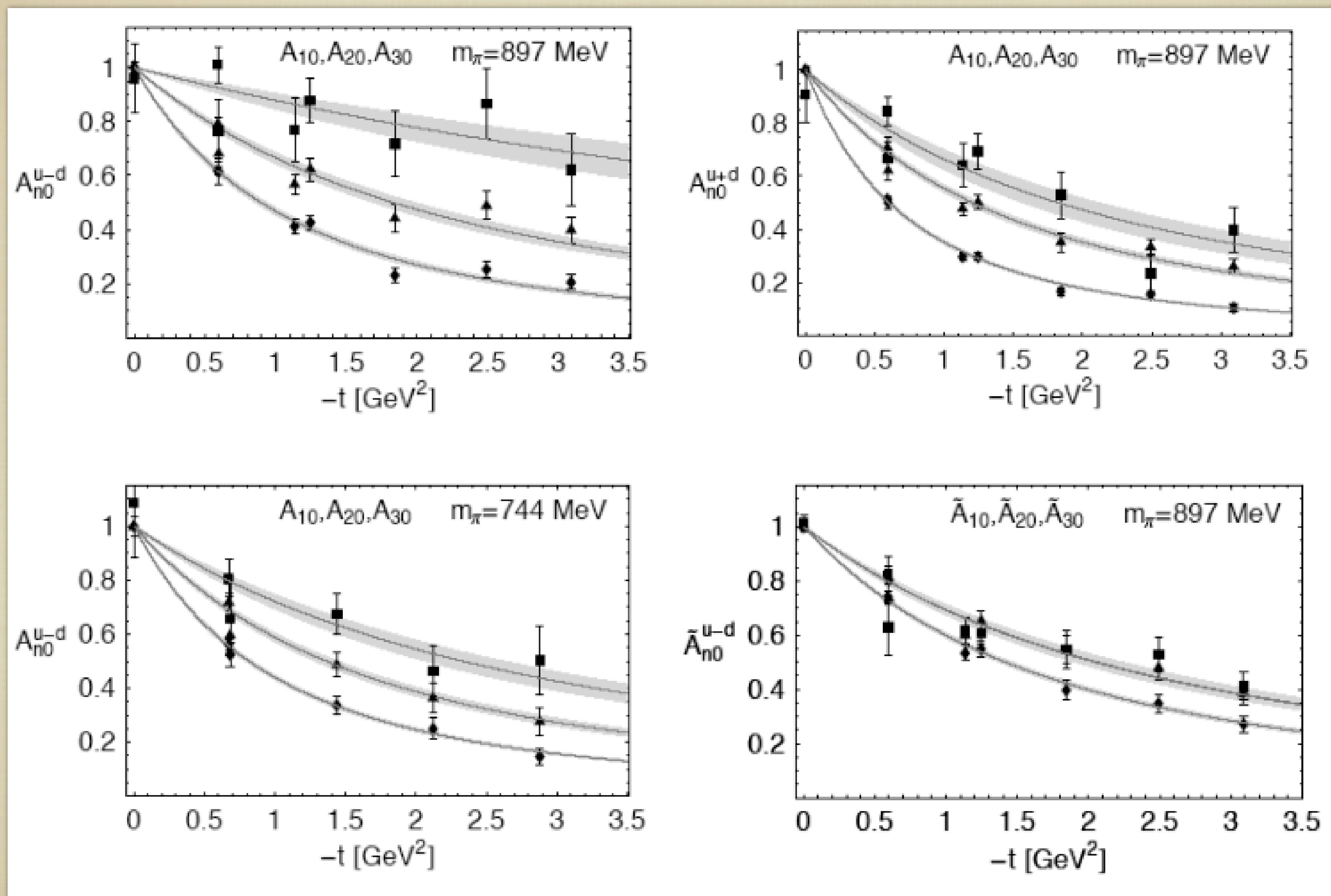
$H(x, 0, -\Delta^2_\perp)$ is transverse Fourier transform of light cone quark distribution $q(x, r_\perp)$ at momentum fraction x

$$q(x, r_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\Delta^2_\perp)\, e^{-ir_\perp \Delta_\perp}$$

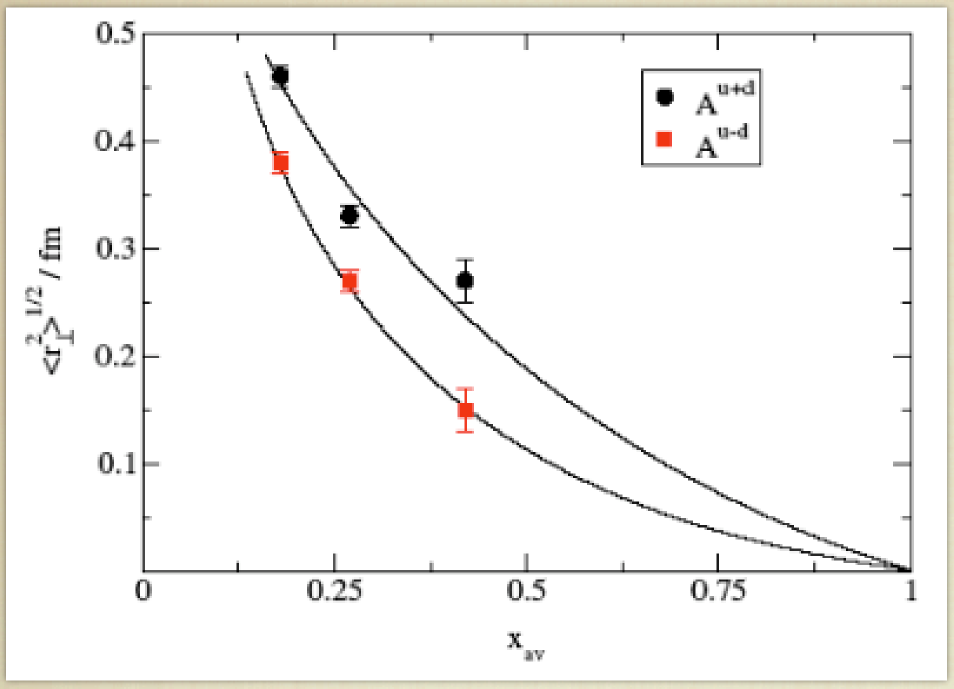
$$\int dx x^{n-1} q(x, r_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} A(-\Delta^2_\perp)\, e^{-ir_\perp \Delta_\perp}$$

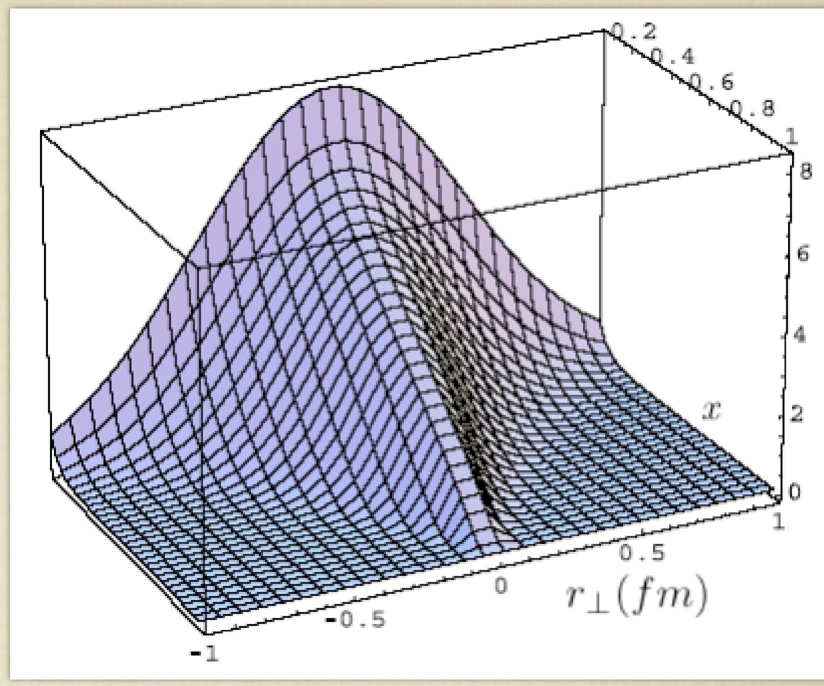
- $x \to 1$: Single Fock space component $\Rightarrow$ slope $\to 0$
- $x \neq 1$: Transverse structure $\Rightarrow$ slope steeper

Generalized form factors from lattice



Transverse size of light-cone wave function

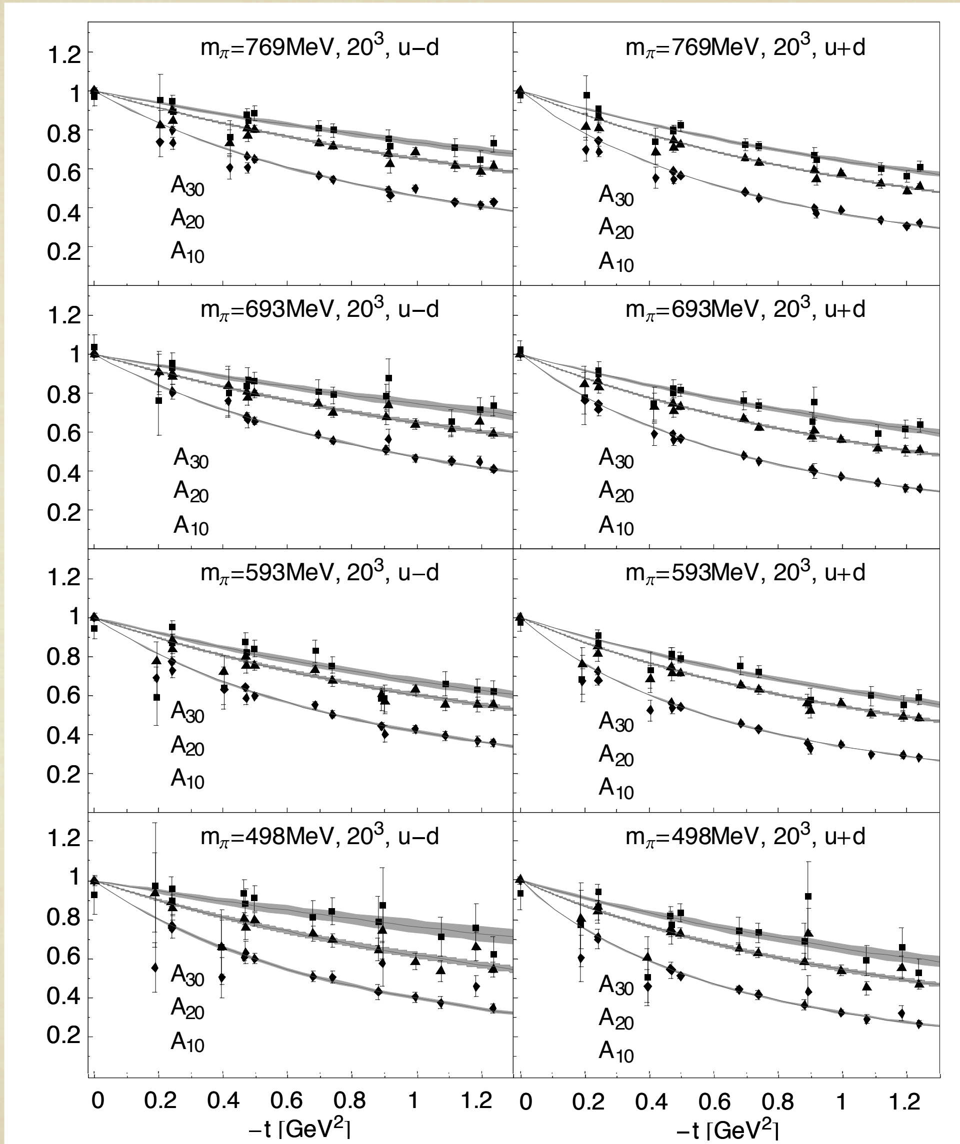




$$x_{\rm av}^n = \frac{\int d^2 r_\perp \int dx\, x \cdot x^{n-1} q(x, \vec{r}_\perp)}{\int d^2 r_\perp \int dx x^{n-1} q(x, \vec{r}_\perp)}$$

$q(x, \vec{r}_\perp)$ model (Burkardt hep-ph/0207047)

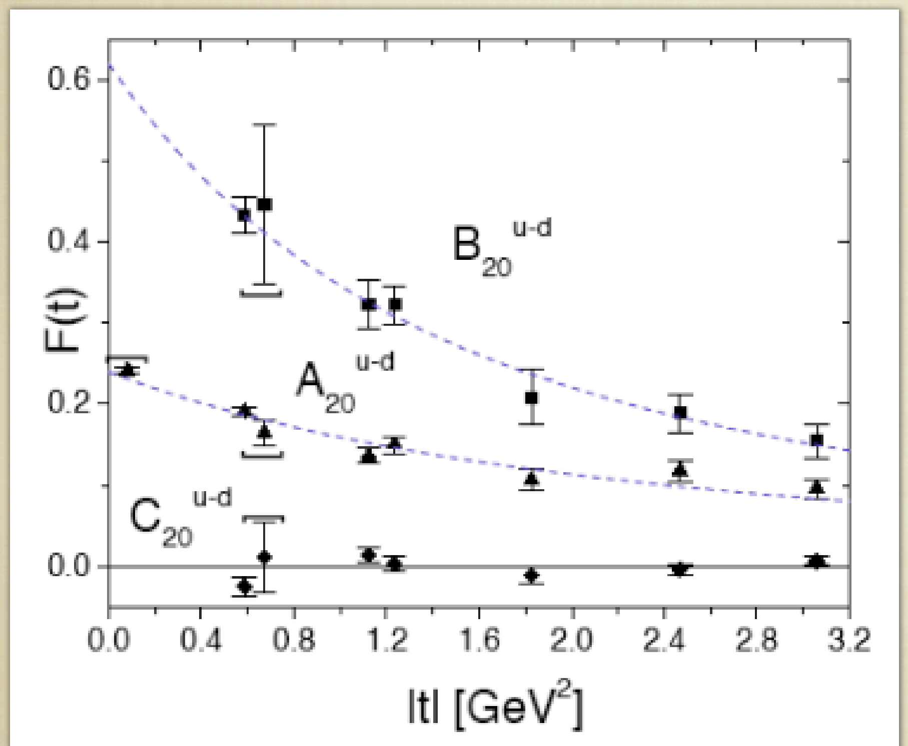
Generalized form factors A_{10}, A_{20}, A_{30}

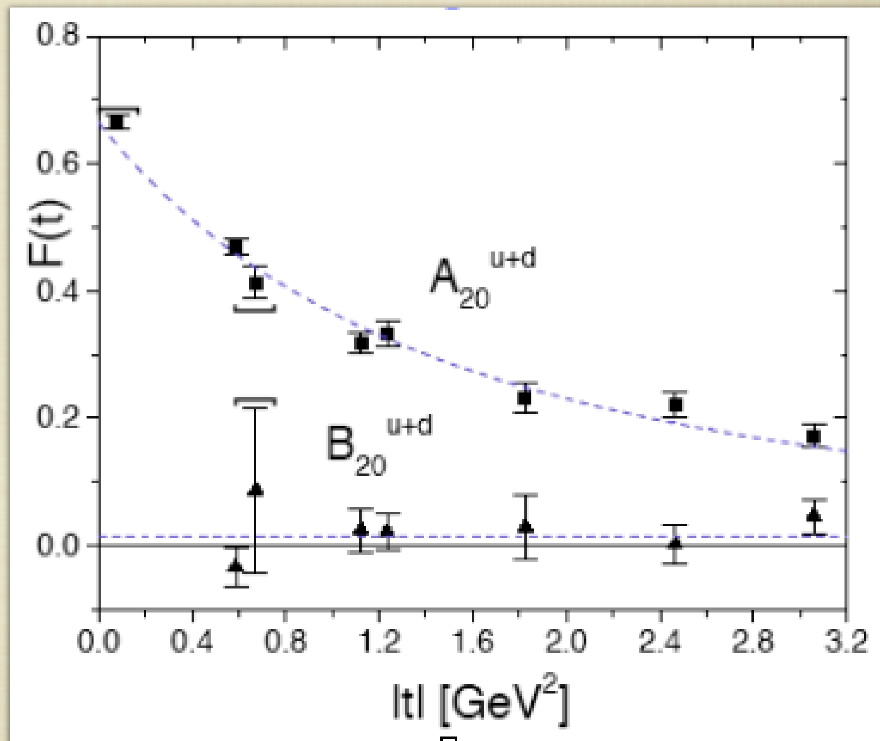


First x moments: A_{20}, B_{20}, C_{20}

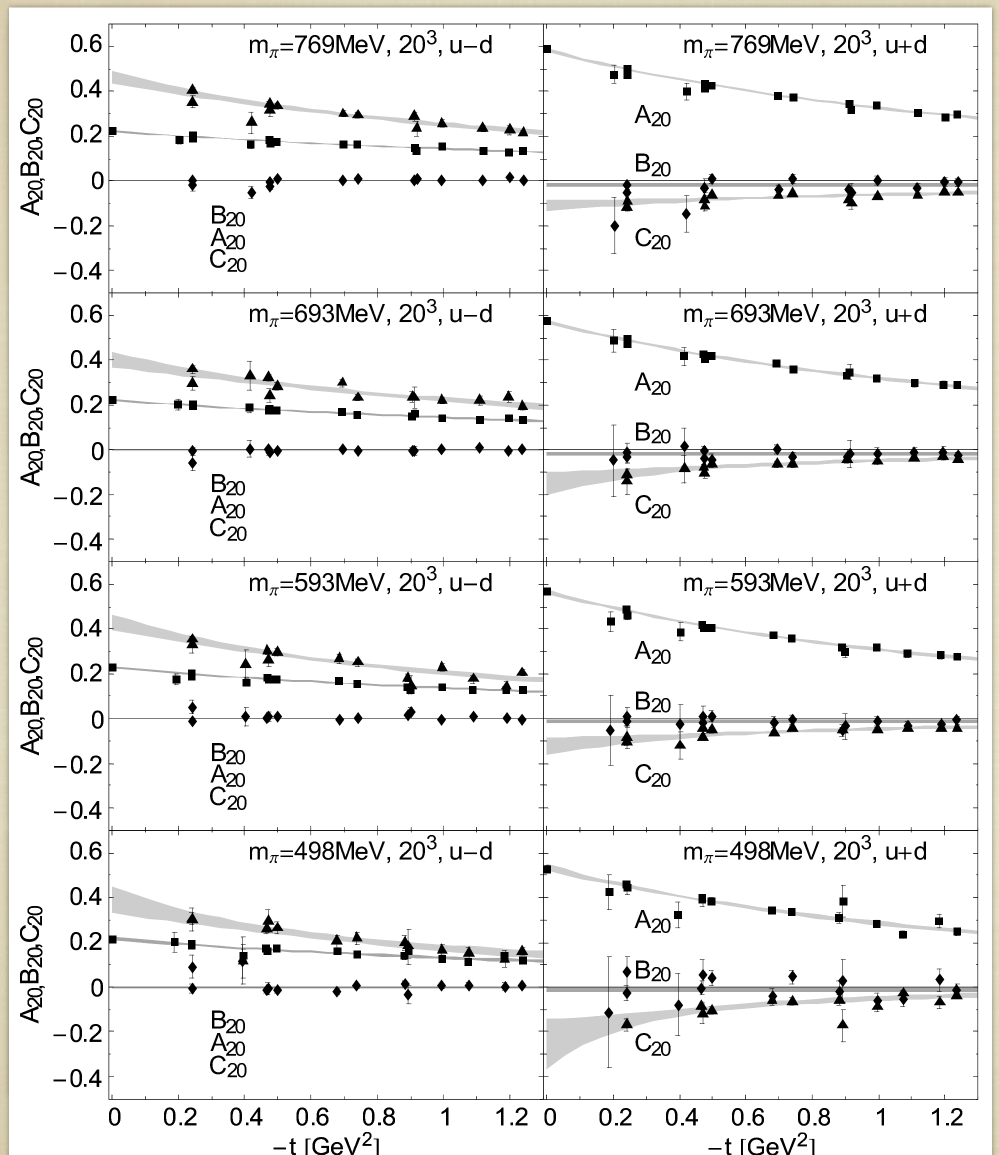
m_π = 897 MeV

LHPC hep-lat/0304018





First x moments: A_{20}, B_{20}, C_{20}



$B_{20}^{u-d} > A_{20}^{u-d}$

$A_{20}^{u+d} > B_{20}^{u+d} \sim 0$

$C_{20}^{u-d} \sim 0$

$C_{20}^{u+d} < 0$

Large N_c behavior

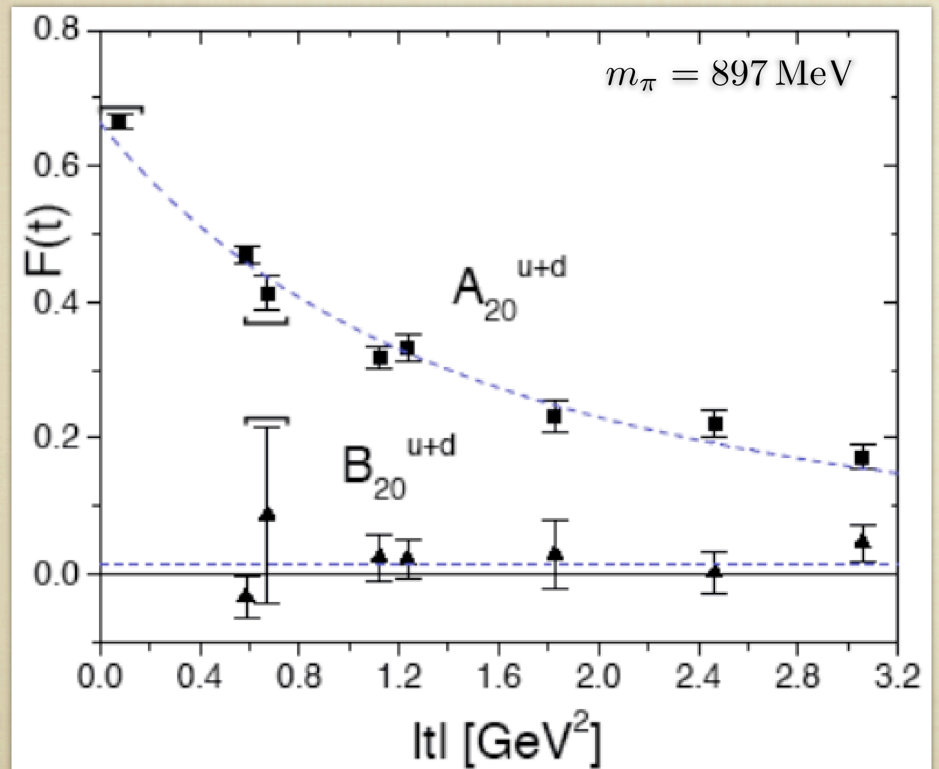
Origin of nucleon spin

"Spin crisis" - only ~ 30% arises from quark spins

quark spin contribution $\frac{1}{2}\Delta\Sigma = \frac{1}{2}\langle 1 \rangle_{\Delta u + \Delta d}$ $\sim \frac{1}{2}0.682(18)$

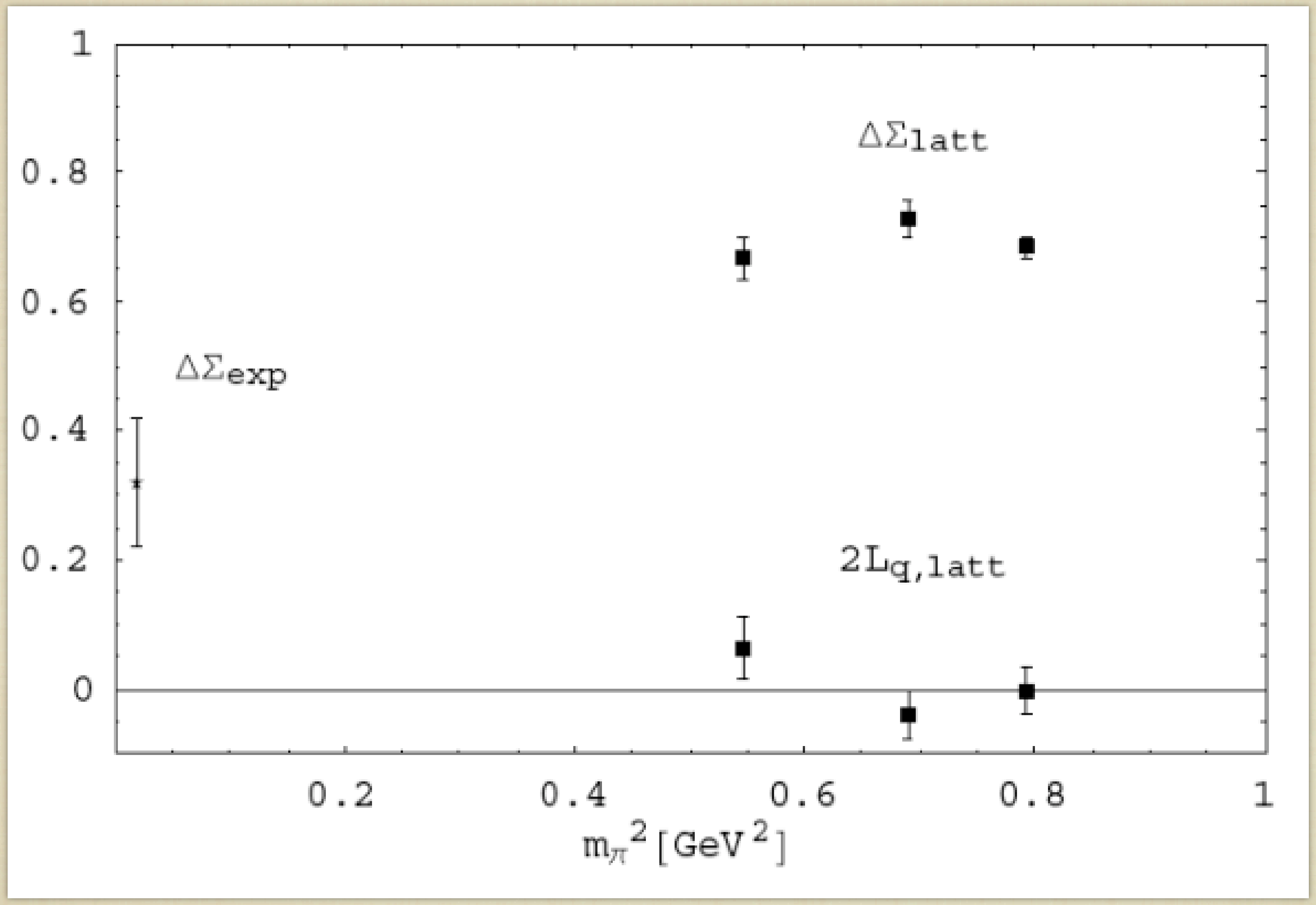
total quark contribution (spin plus orbital)

$$J_q = \frac{1}{2}[A_{20}^{u+d}(0) + B_{20}^{u+d}(0)] = \frac{1}{2}[\langle x \rangle_{u+d} + B_{20}^{u+d}(0)] \quad \sim \frac{1}{2}0.675(7)$$

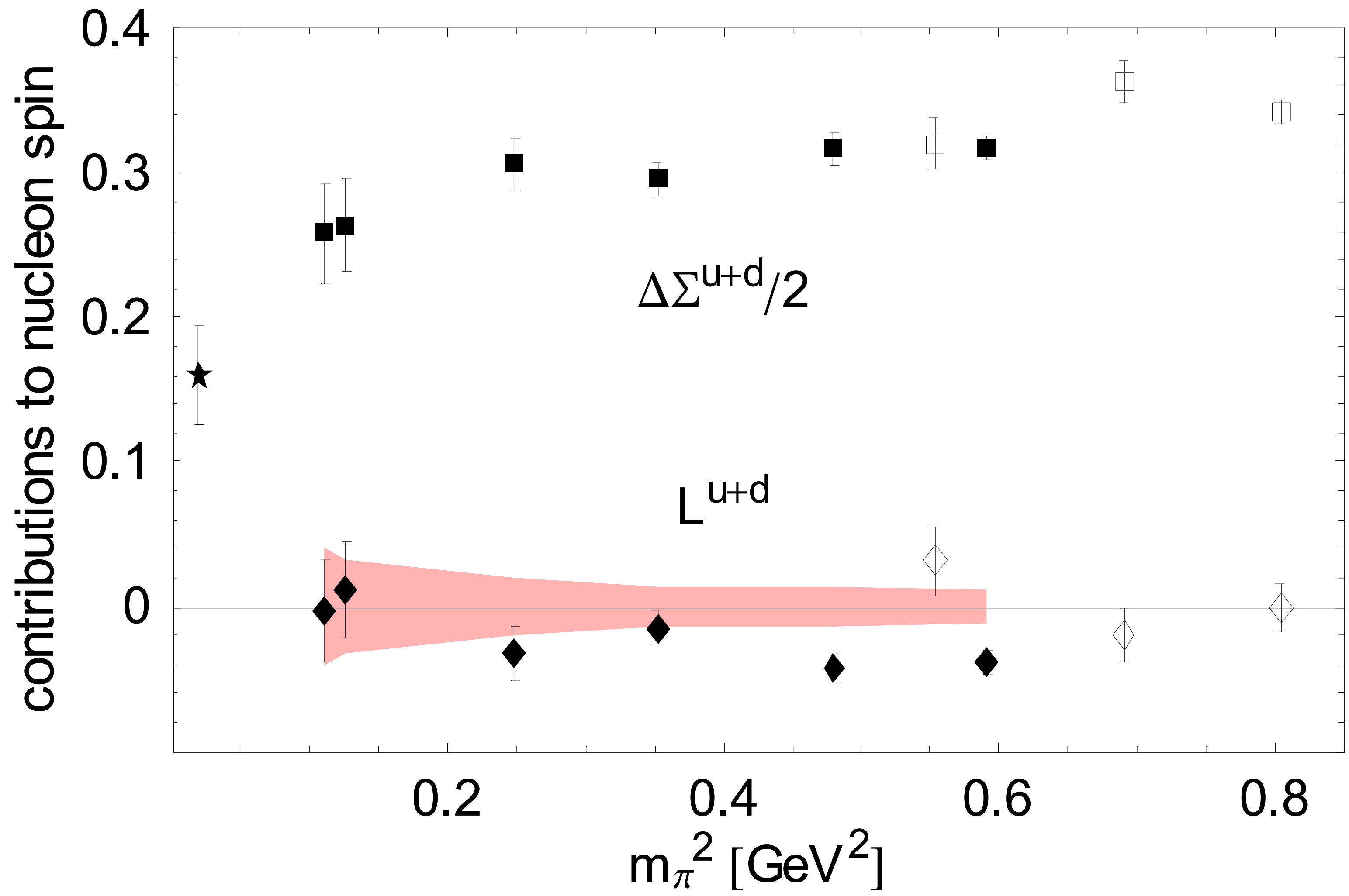


Spin Inventory
68% quark spin
0% quark orbital
32% gluons

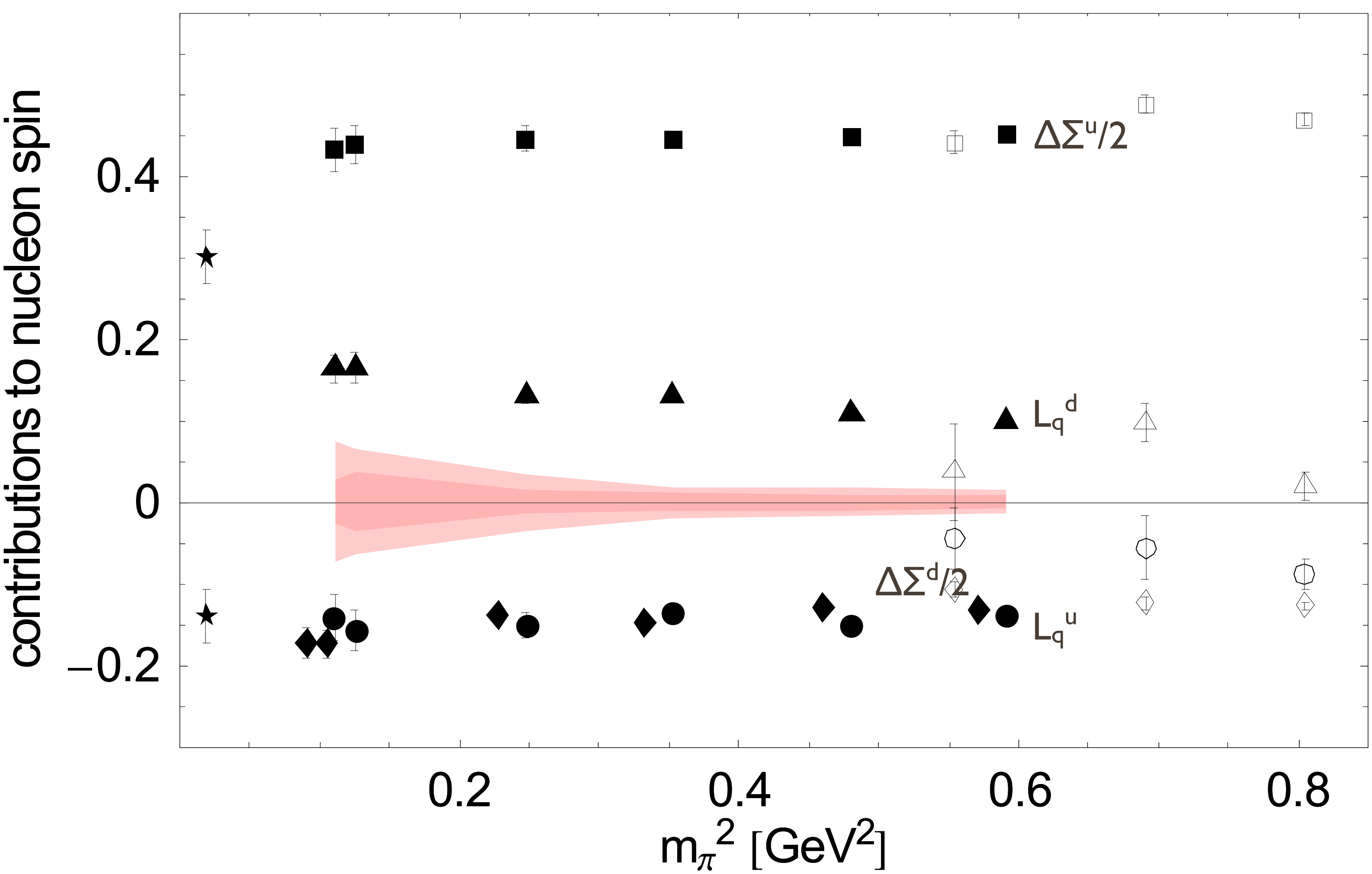
Nucleon spin decomposition



Nucleon spin decomposition



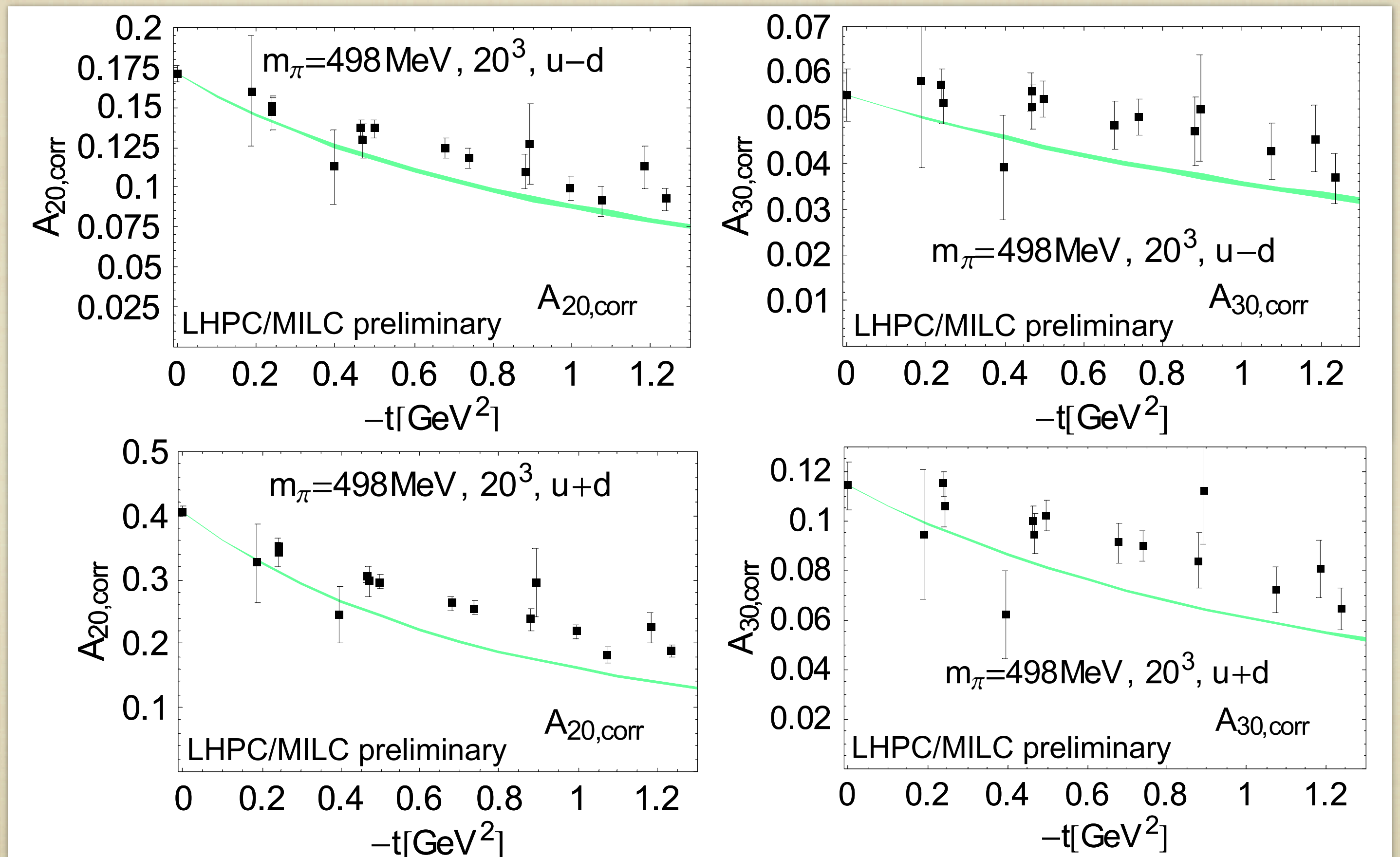
Nucleon spin decomposition



Comparison with Phenomenology

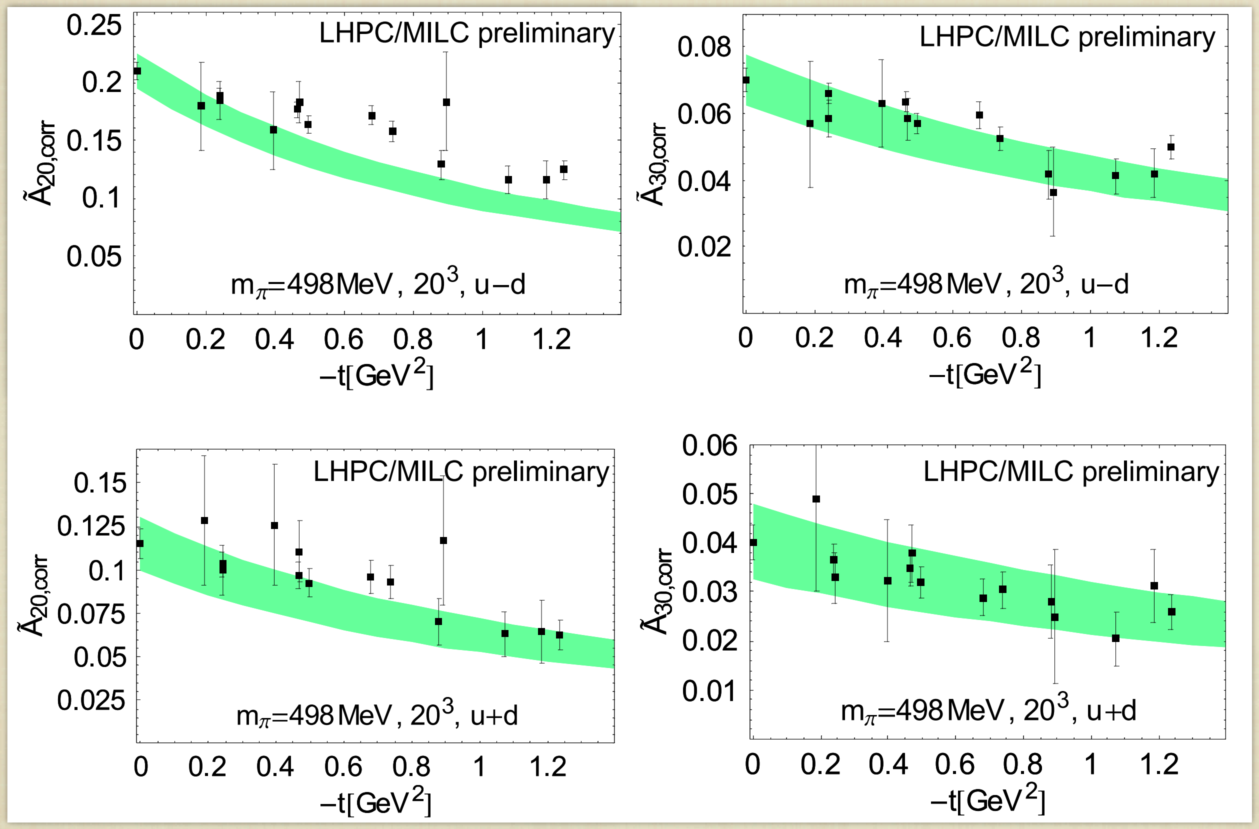
GPD parameterization: Diehl, Feldmann, Jakob, Kroll EPJC 2005
nucleon form factors, CTEQ parton distributions, Regge, Ansatz

$$A_{20} = \int dx\, x\, H(x,0,t) \qquad A_{30} = \int dx\, x^2\, H(x,0,t)$$

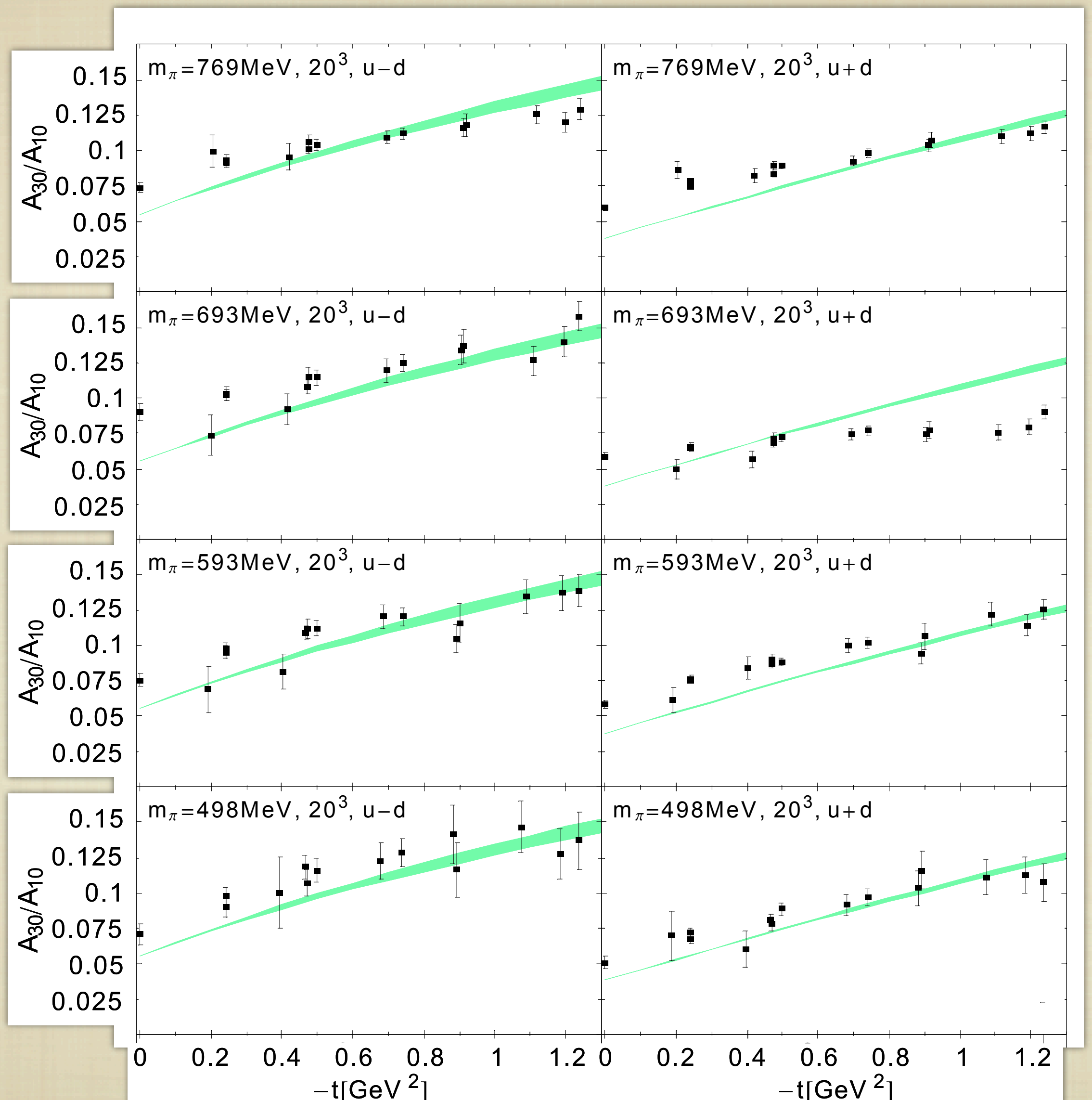


Comparison with Phenomenology

$$\tilde{A}_{20} = \int dx\, x\, \tilde{H}(x,0,t) \qquad \tilde{A}_{30} = \int dx\, x^2\, \tilde{H}(x,0,t)$$

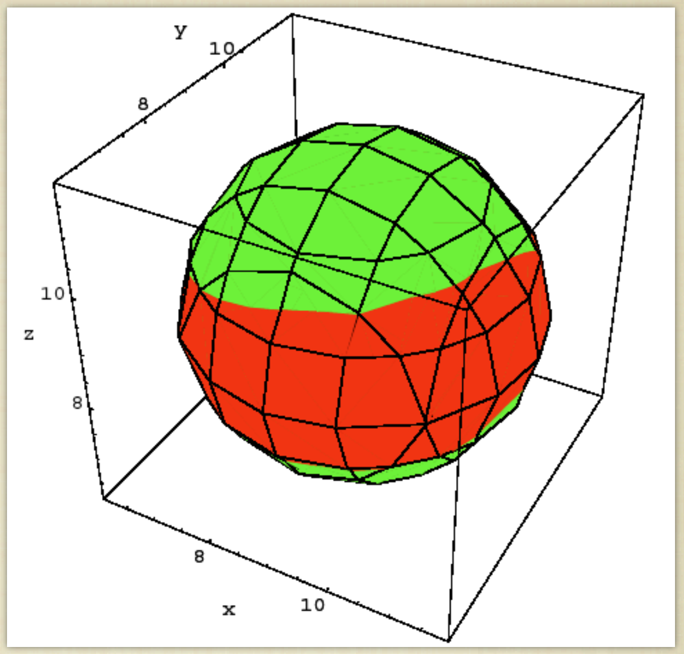


Comparison with Phenomenology



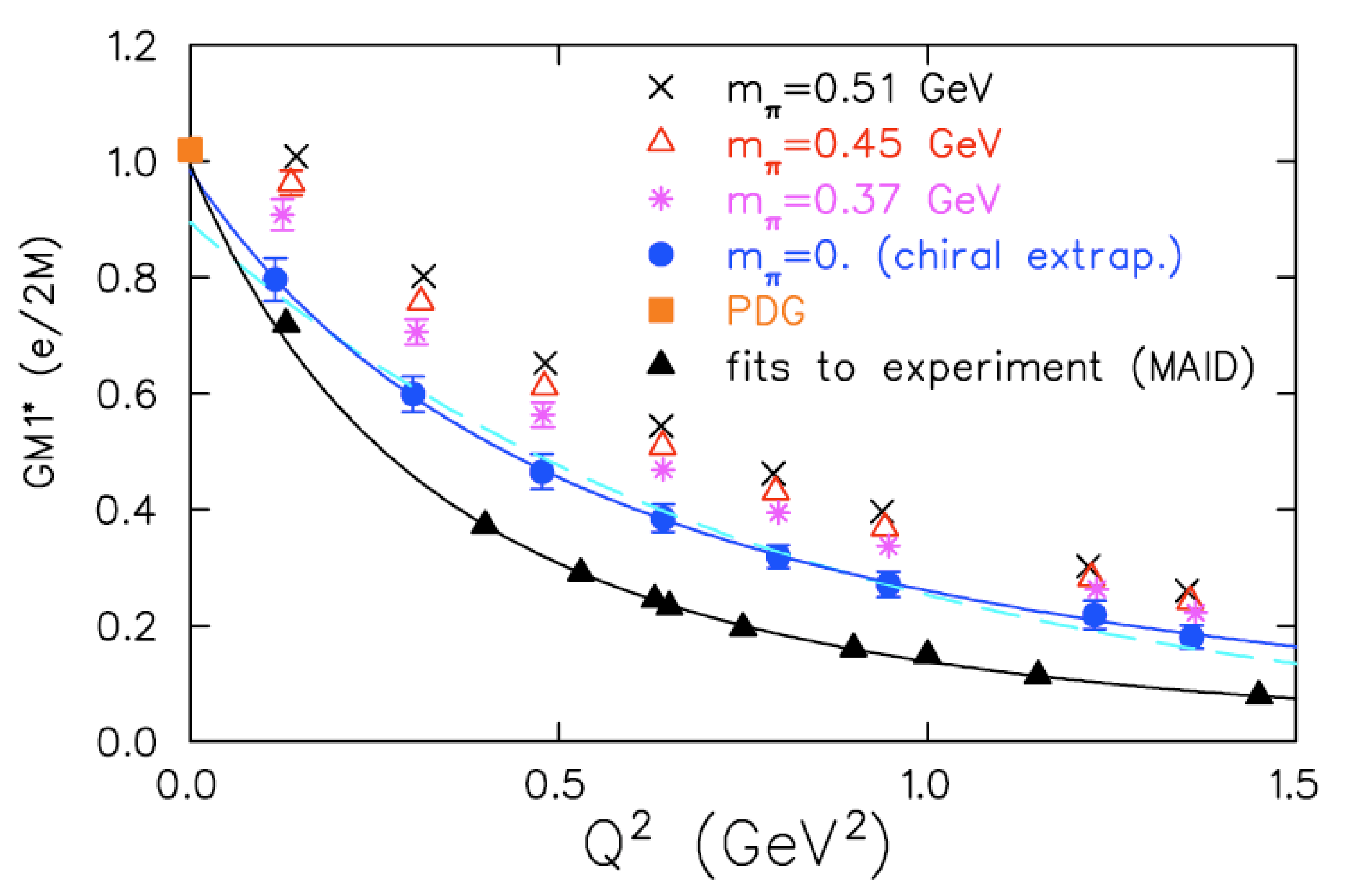
Baryon shapes

- Observe oblate deformation of spin 1/2 Δ directly on lattice from density-density correlation function (Alexandrou, nucl-th/0311007)

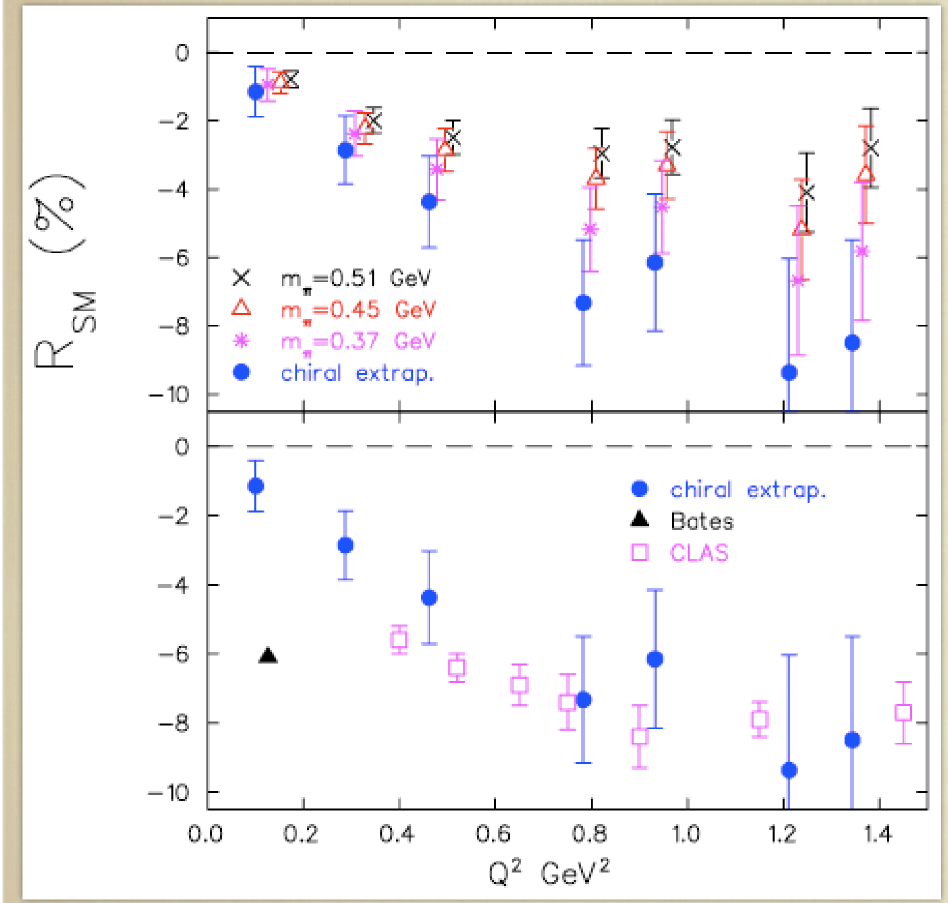


- Infer deformation experimentally from N→Δ transition form factor
  - Dominant transition M1
  - C2 and E2 would vanish if nucleon and Δ spherical

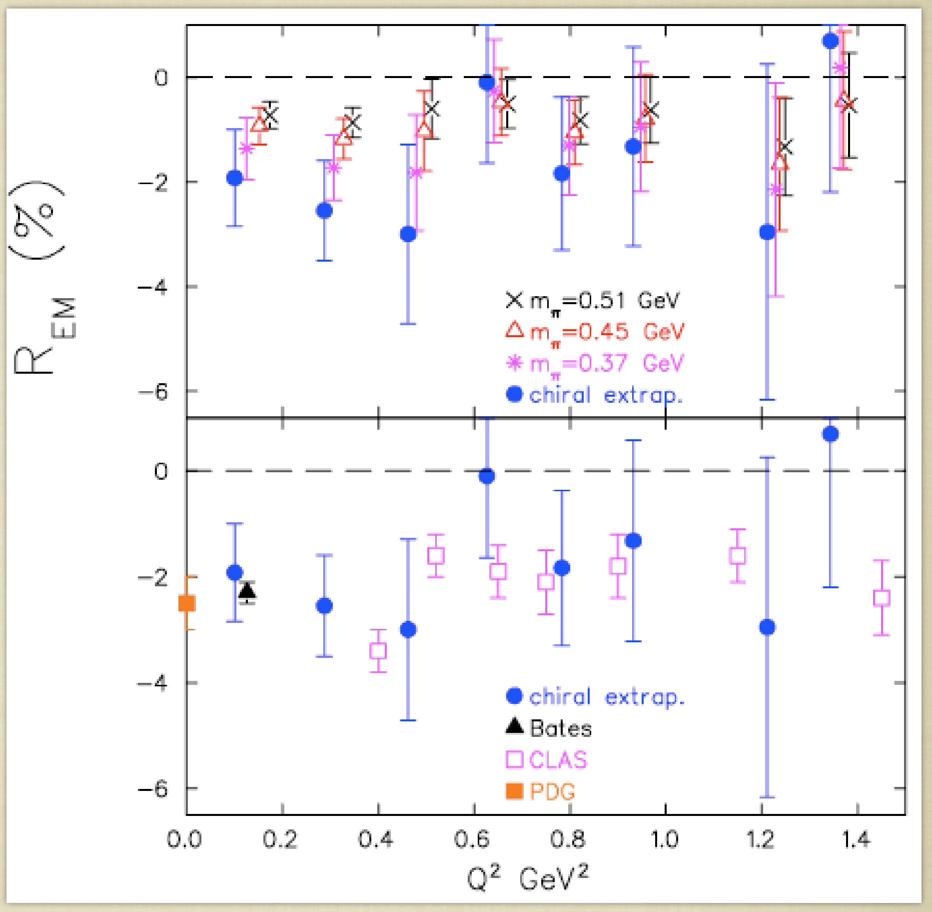
M1 form factor



Electric and Coulomb transitions



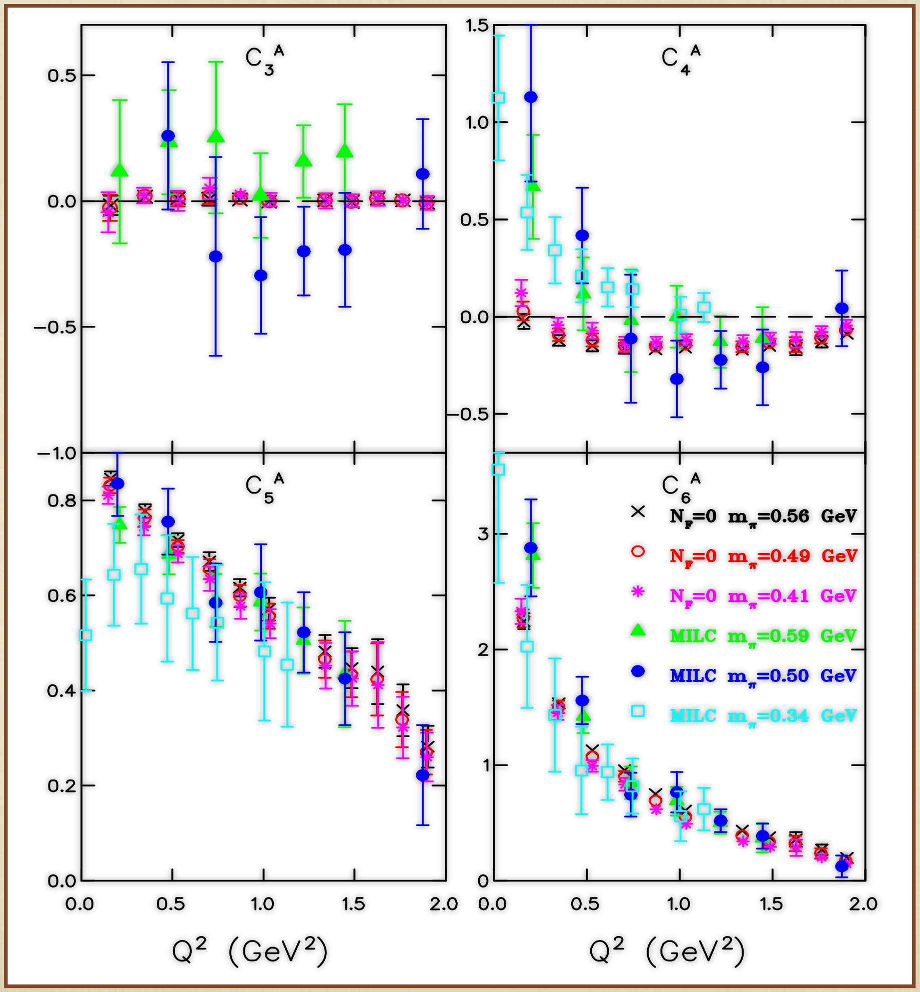
C2/M1



E2/M1

Axial N-Delta transition form factors

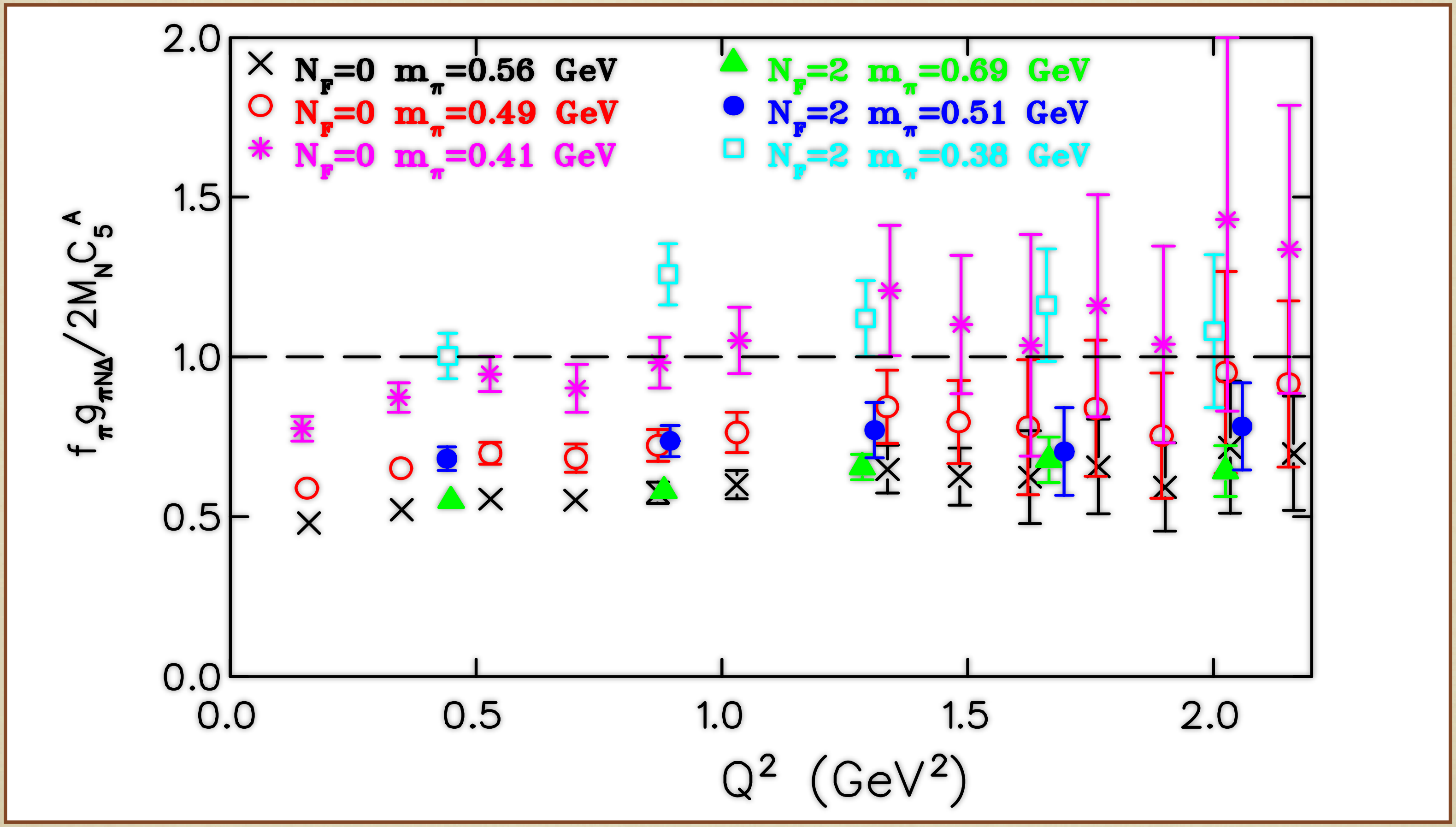
$$\langle \Delta(p',s')|A_\mu|N(p,s)\rangle \propto \bar{u}^\lambda(p',s')\left[\left(\frac{C_3^A(q^2)}{M}\gamma^\nu + \frac{C_4^A(q^2)}{M^2}p'^\nu\right)(g_{\lambda\mu}g_{\rho\nu} - g_{\lambda\rho}g_{\mu\nu})q^\rho + C_5^A(q^2)g_{\lambda\mu} + \frac{C_6^A(q^2)}{M^2}q_\lambda q_\mu\right]u(p,s)$$



Alexandrou, Leontiou, Tsapalis, JN

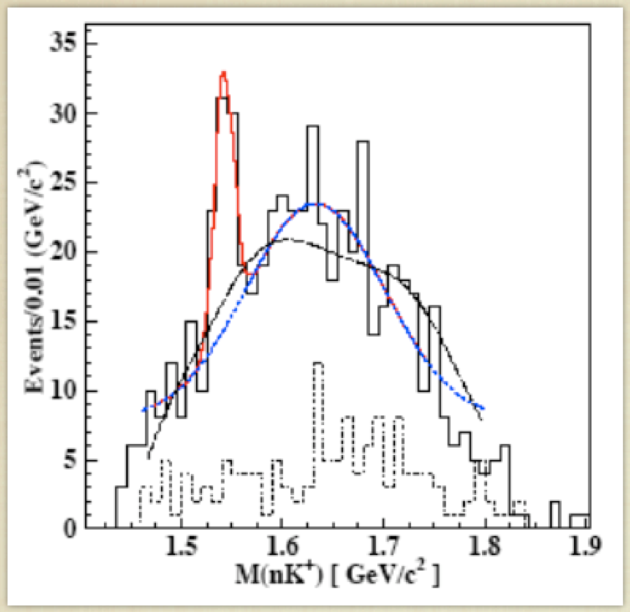
Axial N-Delta transition form factors

Off-diagonal Goldberger-Treiman relation $$C_5^A(q^2) = \frac{F_\pi g_{\pi N\Delta}(q^2)}{2M}$$

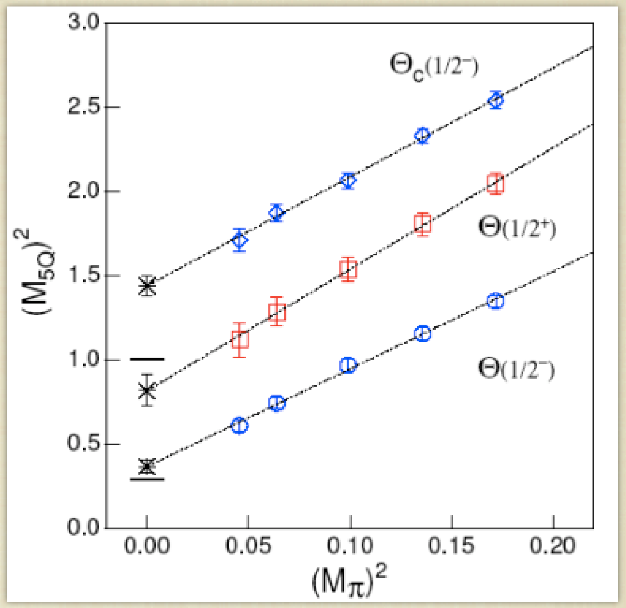


Pentaquarks

Θ(1540) reported at SPRing8, JLab, ...



Stepanyan et.al. hep-ex/0307018



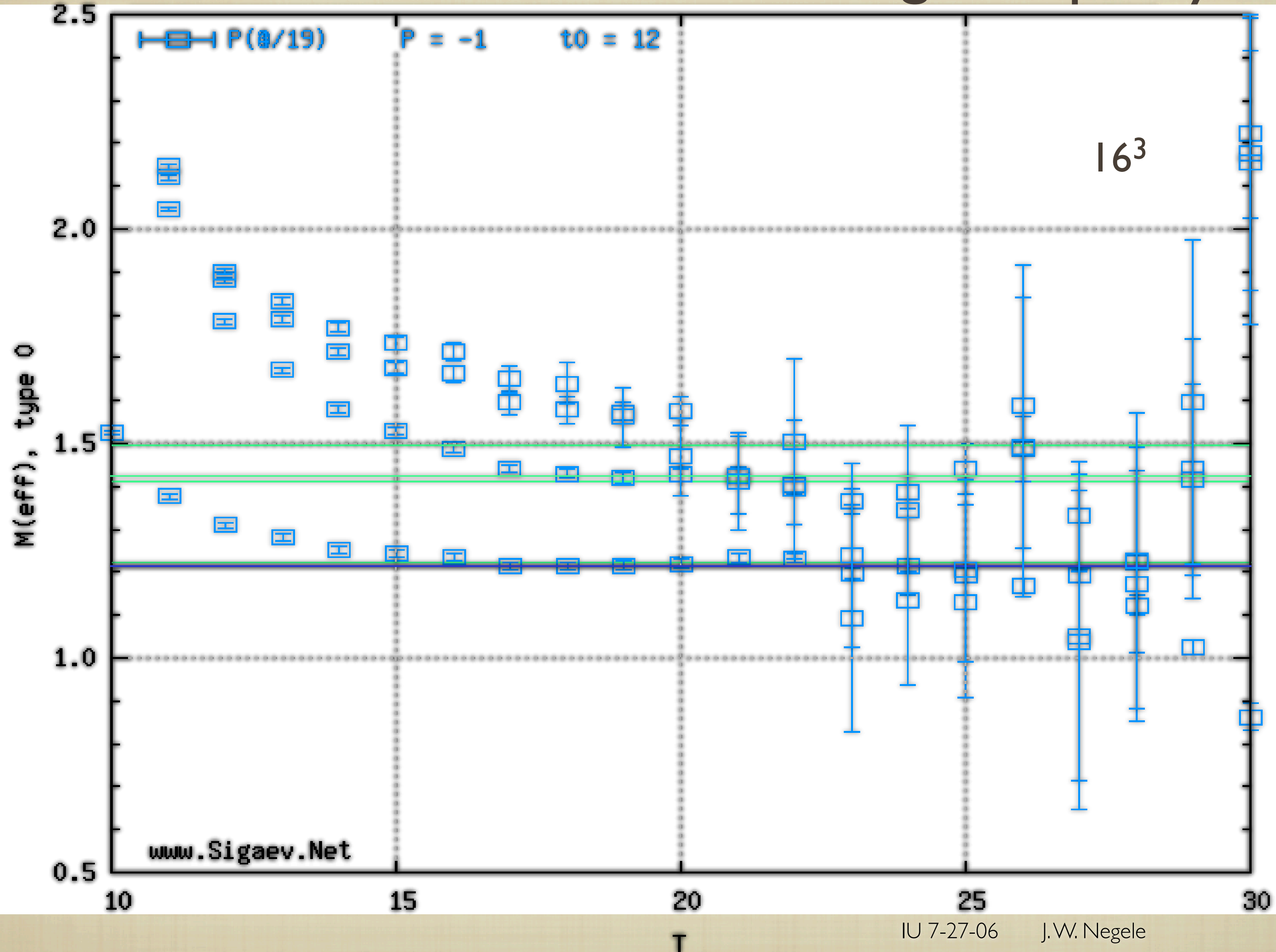
Sasaki hep-lat/0310014

- S = +1 five quark state $uudd\bar{s}$
- Diquark model - positive parity $[ud][ud]\bar{s}$
- Quark model - negative parity
- Lattice calculations in heavy pion world

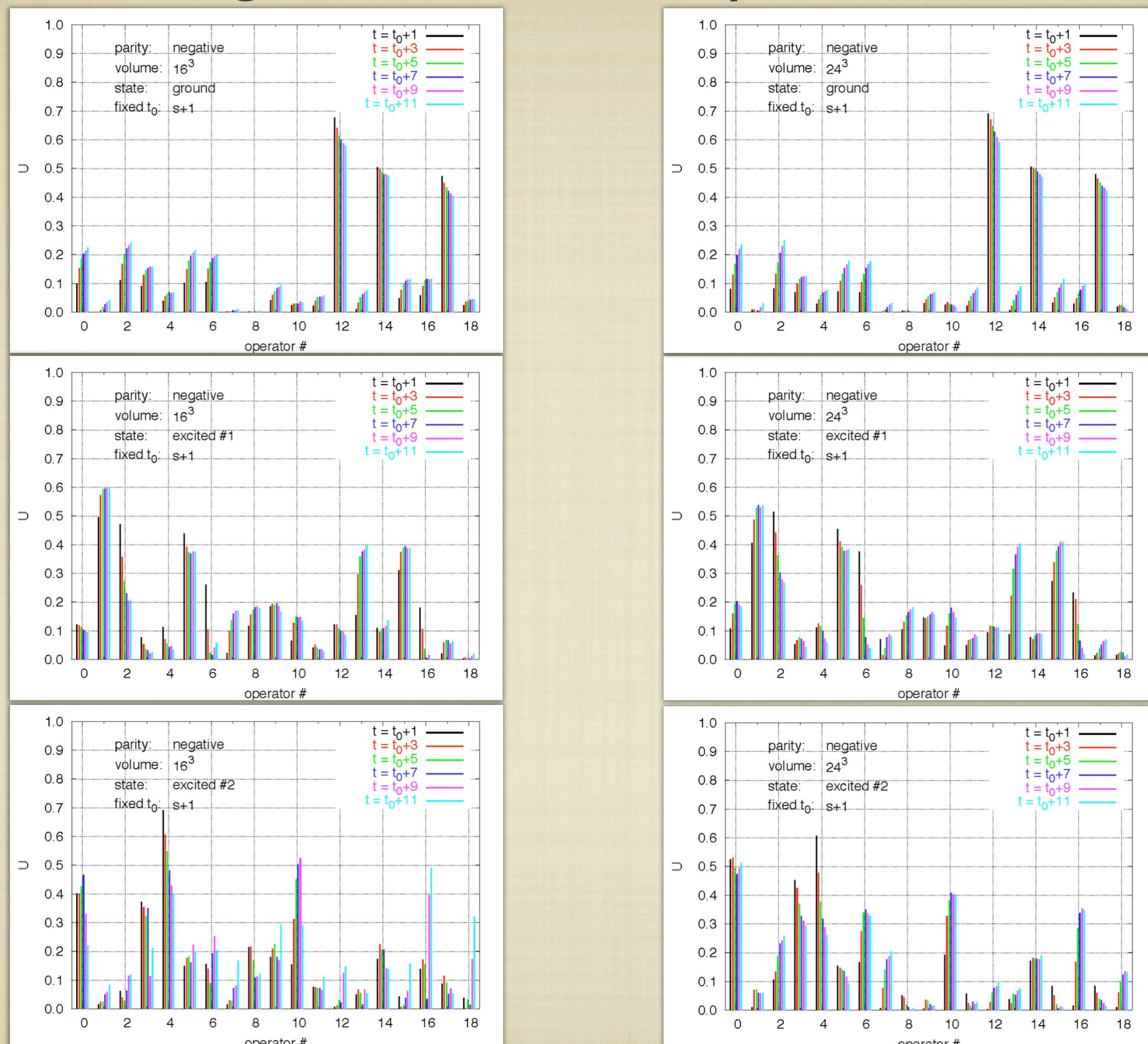
8 local Lorentz covariant spin 1/2 operators

$$
\begin{aligned}
\Pi^{\mathrm{PS}}_{\mathrm{P}\,\varepsilon} &= \epsilon^{efg}\epsilon_{fab}\epsilon_{gcd}\,(q^a C\; \mathrm{i}\tau_2 q^b)\;(q^c C\gamma_5\; \mathrm{i}\tau_2 q^d)\;(\bar{C}\bar{s}^T_e)_\varepsilon \\
\Pi^{\mathrm{SV}}_{\mathrm{V}\,\varepsilon} &= \epsilon^{efg}\epsilon_{fab}\epsilon_{gcd}\,(q^a C\gamma_5\; \mathrm{i}\tau_2 q^b)\;(q^c C\gamma_5\gamma_\mu\; \mathrm{i}\tau_2 q^d)\;(\gamma_5\gamma_\mu\bar{C}\bar{s}^T_e)_\varepsilon \\
\Pi^{\mathrm{PV}}_{\mathrm{A}\,\varepsilon} &= \epsilon^{efg}\epsilon_{fab}\epsilon_{gcd}\,(q^a C\; \mathrm{i}\tau_2 q^b)\;(q^c C\gamma_5\gamma_\mu\; \mathrm{i}\tau_2 q^d)\;(\gamma_\mu\bar{C}\bar{s}^T_e)_\varepsilon \\
\Pi^{\mathrm{VV}}_{\mathrm{T}\,\varepsilon} &= \epsilon_{\mu\nu\rho\lambda}\,\epsilon^{efg}\epsilon_{fab}\epsilon_{gcd}\,(q^a C\gamma_5\gamma_\mu\; \mathrm{i}\tau_2 q^b)\;(q^c C\gamma_5\gamma_\nu\; \mathrm{i}\tau_2 q^d)\;(\sigma_{\rho\lambda}\bar{C}\bar{s}^T_e)_\varepsilon \\
\Pi^{\mathrm{AT}}_{\mathrm{V}\,\varepsilon} &= \epsilon^{efg}\epsilon_{fab}\epsilon_{gcd}\,(q^a C\gamma_\nu\; \tau_2\tau_n q^b)\;(q^c C\sigma_{\nu\mu}\; \tau_2\tau_n q^d)\;(\gamma_5\gamma_\mu\bar{C}\bar{s}^T_e)_\varepsilon \\
\Pi^{\mathrm{AT}}_{\mathrm{A}\,\varepsilon} &= \epsilon_{\mu\nu\rho\lambda}\,\epsilon^{efg}\epsilon_{fab}\epsilon_{gcd}\,(q^a C\gamma_\nu\; \tau_2\tau_n q^b)\;(q^c C\sigma_{\rho\lambda}\; \tau_2\tau_n q^d)\;(\gamma_\mu\bar{C}\bar{s}^T_e)_\varepsilon \\
\Pi^{\mathrm{AA}}_{\mathrm{T}\,\varepsilon} &= \epsilon_{\mu\nu\rho\lambda}\,\epsilon^{efg}\epsilon_{fab}\epsilon_{gcd}\,(q^a C\gamma_\mu\; (\tau_2\tau_n q^b)\;(q^c C\gamma_\nu\; \tau_2\tau_n q^d)\;(\sigma_{\rho\lambda}\bar{C}\bar{s}^T_e)_\varepsilon \\
\Pi^{\mathrm{TT}}_{\mathrm{T}\,\varepsilon} &= \epsilon_{\mu\nu\rho\lambda}\,\epsilon^{efg}\epsilon_{fab}\epsilon_{gcd}\,(q^a C\sigma_{\mu\chi}\; \tau_2\tau_n q^b)\;(q^c C\sigma_{\nu\chi}\; \tau_2\tau_n q^d)\;(\sigma_{\rho\lambda}\bar{C}\bar{s}^T_e)_\varepsilon
\end{aligned}
$$

19 x 19 Correlation matrix - negative parity

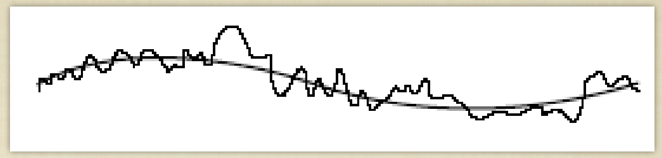


Eigenfunction comparison



Insight into how QCD works: classical solutions

- Stationary phase approximation



$$\int D[A]e^{-\int d^4x S[A]} \sim [\det S'']^{-1} e^{-\int d^4x S[A_{cl}]}$$

- Instanton solutions connect vacuua with different winding numbers

$$A_\mu^a(x) = \frac{2\eta_{a\mu\nu}x_\nu}{x^2+\rho^2}$$

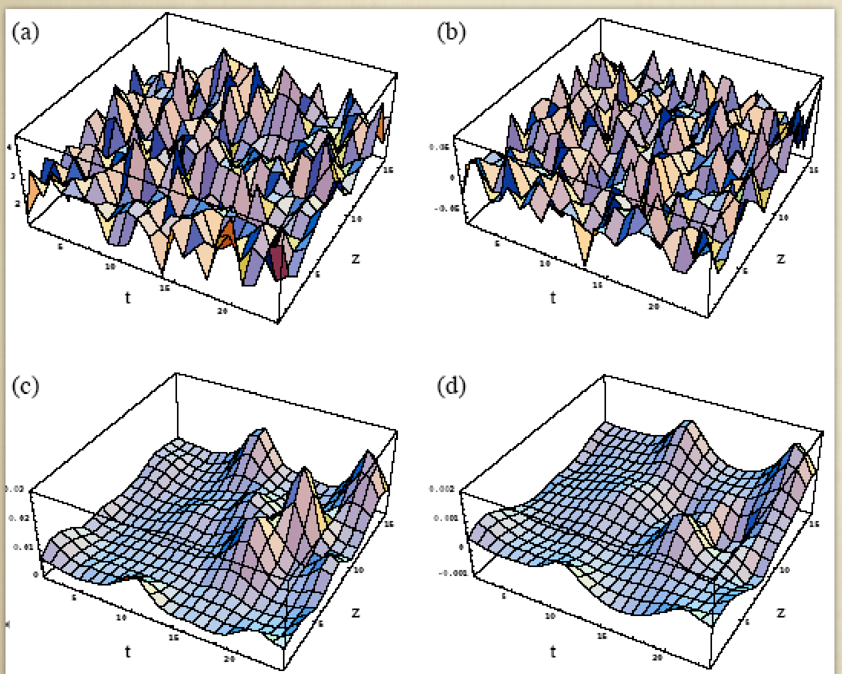
$$S = \frac{1}{4}\int F^2 = \frac{8\pi^2}{g^2}, \quad Q = \frac{g^2}{32\pi^2}\int F\tilde{F} = 1$$

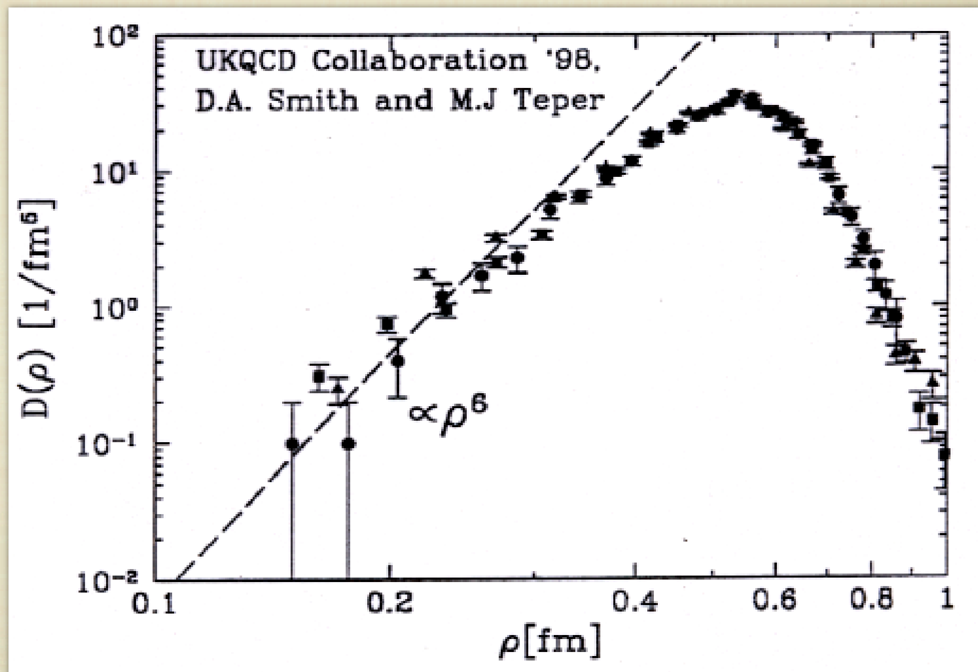
- To what extent are analytic expectations observed on lattice?

Instantons on the lattice

- Cooling (relaxation) reveals lumps with $S \sim \frac{8\pi^2}{g^2}$ and $Q \sim \pm 1$
- For small size ρ, distribution ∝ρ^6

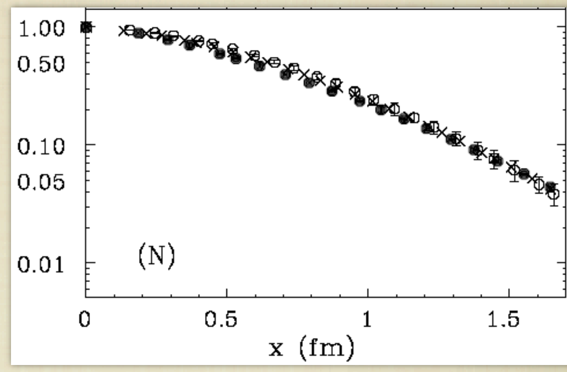
s(x) q(x)

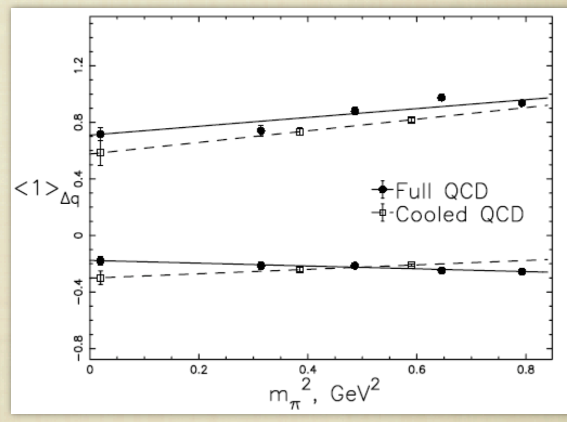




Instantons on the lattice

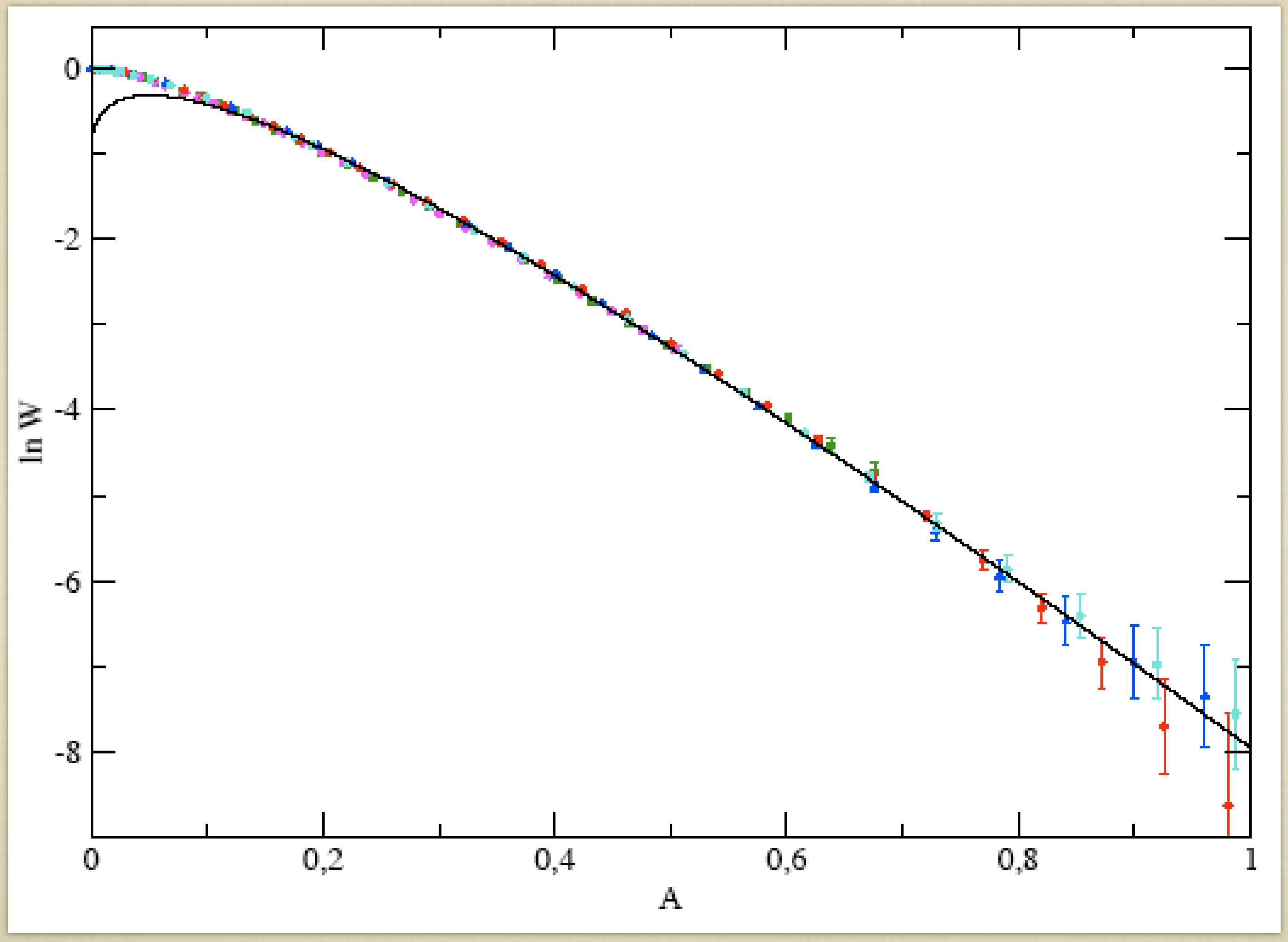
- Observables calculated with only instantons close to those including all gluons
- Observe quark zero modes localized at instantons
- Zero modes from instantons generate and dominate light quark propagators
- Topological susceptibility from instantons, X= (180MeV), yields η′ mass





Confinement from instantons

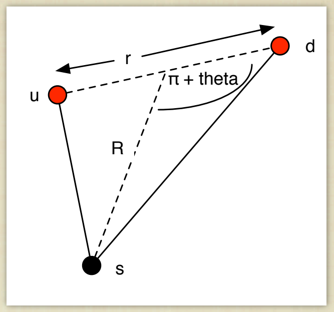
Ensemble of regular gauge instantons yields area law hep-th/0306105



Diquark correlations in heavy light light baryon

Patrick Varilly - senior thesis

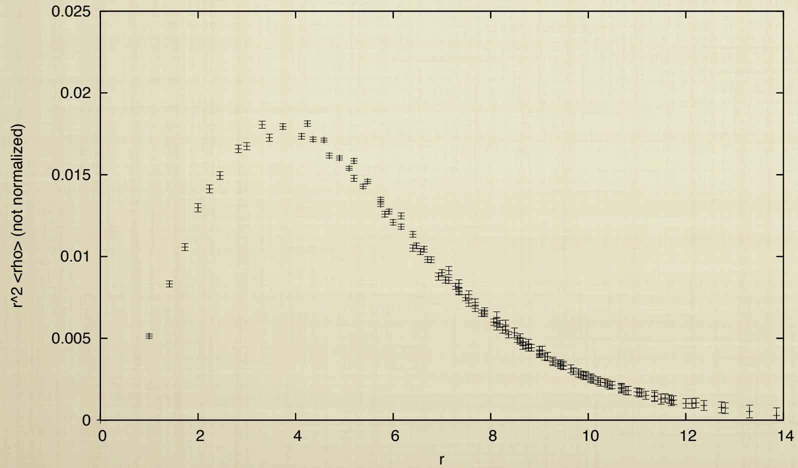
Good diquarks:
color antitriplet
flavor antisymmetric
spin singlet

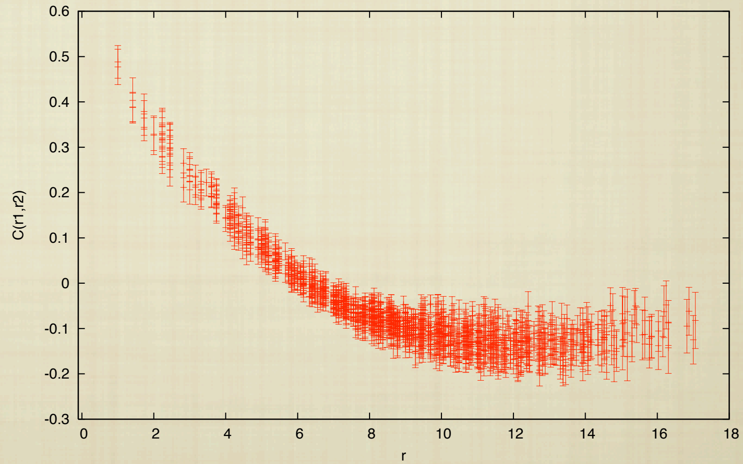


$$(uC\gamma_5 d)h$$

$$\langle \rho(r) \rangle$$

$$C(r_1, r_2) = \frac{\langle \rho(r_1)\rho(r_2) \rangle - \langle \rho(r_1) \rangle \langle \rho(r_2) \rangle}{\langle \rho(r_1) \rangle \langle \rho(r_2) \rangle}$$





Summary

- Lattice Field theory has become powerful tool to solve QCD and understand hadronic physics
  - Substantial theoretical issues and accomplishments
  - Resources now available to solve frontier problems
  - Challenging and rewarding research field
- Entering era of quantitative solution in chiral regime
  - Form factors: F_1 , F_2 , G_A , G_P, $N \to$ Delta
  - Generalized form factors A B C
    - Transverse structure
    - Origin of nucleon spin
  - Insight: instantons, diquarks, dependence on parameters
- Two other major areas:
  - QCD thermodyanmics
  - Weak decays

Current effort and future challenges

- Full QCD with chiral fermions in chiral regime
- Nonperturbative renormalization
- Disconnected diagrams
  - Strangeness content of nucleon
  - Flavor singlet matrix elements
- Gluon observables
- Role of diquarks in hadrons
- Unstable states
- Exotic mesons and baryons
- Evolve quark masses from glueball world to QCD
- Hadron-hadron phase shifts, adiabatic potentials