Physical Nucleon Form Factors From Lattice QCD

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The Plan.

- Introduction.
- Light Front Cloudy Bag Model – Fit to the lattice data and extrapolation to the physical region.
- Conclusions.

Introduction.

- The electromagnetic form factors of the nucleon are an invaluable source of information on its structure.
- The recent advances in the lattice QCD provide us with form factor calculations over wide range of momentum transfer.

Problems:

- Extrapolation of the lattice calculations to the physical quark mass region.
- Extrapolation of $a \to 0$, $V \to \infty$.
- Unquenching.

Introduction.

Our purpose here is three-fold.

- Use the lattice data to investigate whether a particular quark model is capable of describing the properties of the nucleon in this additional dimension of varying m_π.
- Having confirmed that the model is consistent with the lattice data over the range of m_π noted earlier, we use the model to extrapolate to large values of Q^2 (for lattice values of m_π).
- We also use the model to extrapolate the ffs to the physical m_π.

The Light Front Cloudy Bag Model (LFCBM).

[G. Miller, Phys. Rev. C 66, 032201(R) (2002)]

The key features:

- Respects Lorentz Invariance.
- Incorporates pion-cloud effects – long-range structure of the nucleon, also yields correct LNA and NLNA chiral behavior of charge radii.
- Reproduces the four nucleon electromagnetic form factors.
- Is easily adaptable for calculations in high quark mass (“lattice”) regime.

The Light-Front Dynamics.

In light-front dynamics the fields are quantized at a fixed "time“$=\tau=x^0+x^3\equiv x^+$.

The four-vectors are $(x^+,x^-,x_\perp)$, where $x^\pm= x^0\pm x^3$, $x_\perp=(x^1,x^2)$

The relation $p_\mu p^\mu=m^2 \rightarrow p^-=(p_\perp^2+m^2)/p^+$ - is similar to nonrelativistic kinetic energy form.

The Dirac F1 and Pauli F2 form factors given by:

$$\langle N,\lambda' p' \,|J^\mu|\, N,\lambda p\rangle = \overline{u}_{\lambda'}(p')\left[F_1(Q^2)\gamma^\mu + \frac{F_2(Q^2)}{2M_N} i\sigma^{\mu\nu}(p'-p)_\nu\right] u_\lambda(p)$$

The Sachs Form Factors are defined as

$$G_E = F_1 - \frac{Q^2}{4M_N^2}F_2, \quad and \quad G_M = F_1 + F_2$$

The LFCBM FFs.

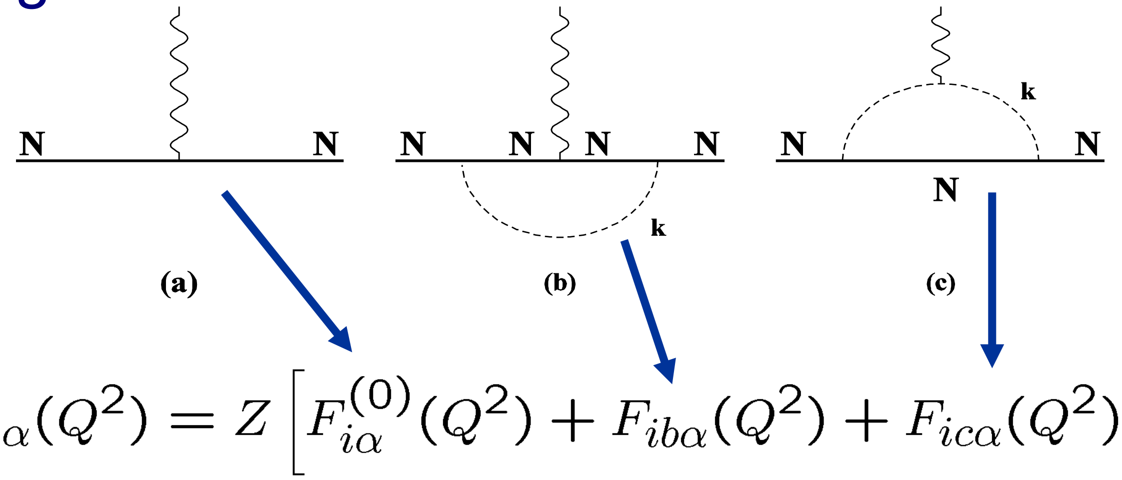
The Quark Core contribution:

Using light-front spinors for the nucleons we can write:

$$F_1(Q^2) = \langle N,\uparrow \left| J^+ \right| N,\uparrow\rangle, \quad \text{and} \quad QF_2(Q^2) = (-2M_N)\langle N,\uparrow \left| J^+ \right| N,\downarrow\rangle\,.$$

The form factors are calculated using the "good" component of the current, J⁺, to suppress the effects of quark-pair terms.

Including the Pion Cloud:



$$F_{i\alpha}(Q^2) = Z\left[F^{(0)}_{i\alpha}(Q^2) + F_{ib\alpha}(Q^2) + F_{ic\alpha}(Q^2)\right]$$

LFCBM Nucleon Wave-Function.

We need to construct the bare (pionless) nucleon wave function , which is antisymmetric, a function of the relative quark momenta and an eigenstate of the canonical spin operator. The commonly used ansatz is:

$$\Psi(p_i)=\Phi(M_0^2)u(p_1)u(p_2)u(p_3)\psi(p_1,p_2,p_3)$$

$$p_i = p_i s_i, \tau_i$$

Where ψ is a spin-isospin amplitude factor, the p_i are expressed in terms of relative coordinates, the $u(p_i)$ are Dirac spinors and Φ is a momentum distribution wave function.

The form of the momentum distribution wave function is taken from by F. Schlumpf, [arXiv:hep-ph/9211255]:

$$\Phi(M_0) = N\left(M_0^2 + \beta^2\right)^{\gamma}$$

with M^2_0 the mass-squared operator for a non-interacting system:

$$M_0^2 = \frac{K_\perp^2}{\eta(1-\eta)} + \frac{k_\perp^2 + M^2}{\eta\xi(1-\xi)} + \frac{M^2}{1-\eta}$$

, where M is the constituent quark mass (assumed constant).

Adaptation of LFCBM to the lattice regime.

The LFCBM is defined by choosing four free parameters: M, β, γ, Λ.

We need to extrapolate:

- The Constituent Quark Mass

$$M = M_\chi + \frac{cm_q^{phys}}{(m_\pi^{phys})^2} m_\pi^2$$

where cm_q^{phys}=5.9 MeV, and M_χ is the constituent quark mass in the chiral limit.

[I. Cloet *et. al*., Phys. Rev. C 65, 062201 (2002)]

Our fitting parameters are

- γ is varied for each m_π.
- M_χ is varied for each a.

The lattice data set used.

We fit the form factor calculations by QCDSF Collaboration.

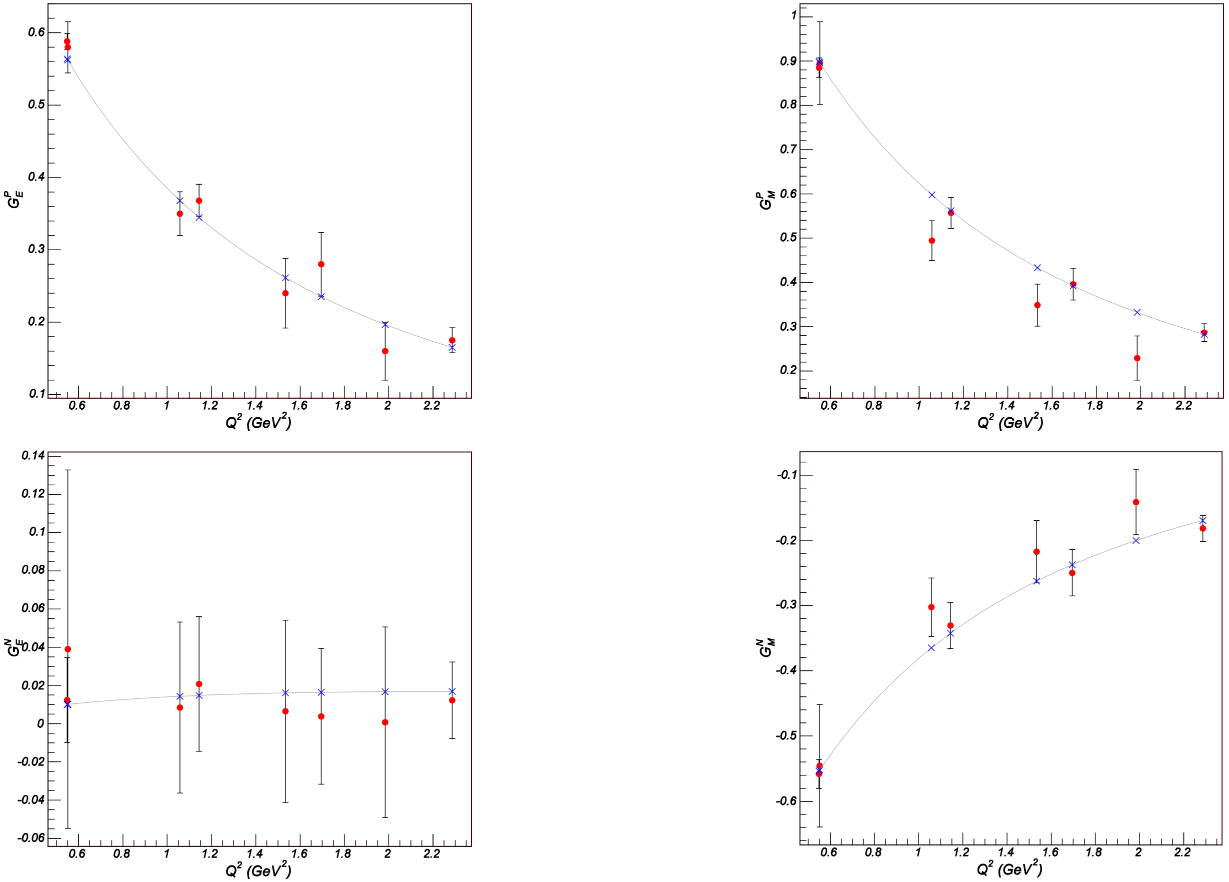
[M. Gockeler et al., arXiv:hep-lat/0303019.]

The calculations were carried out using quenched, non-perturbatively $\mathcal{O}$(a)-improved Wilson fermions (clover fermions).

The data range:

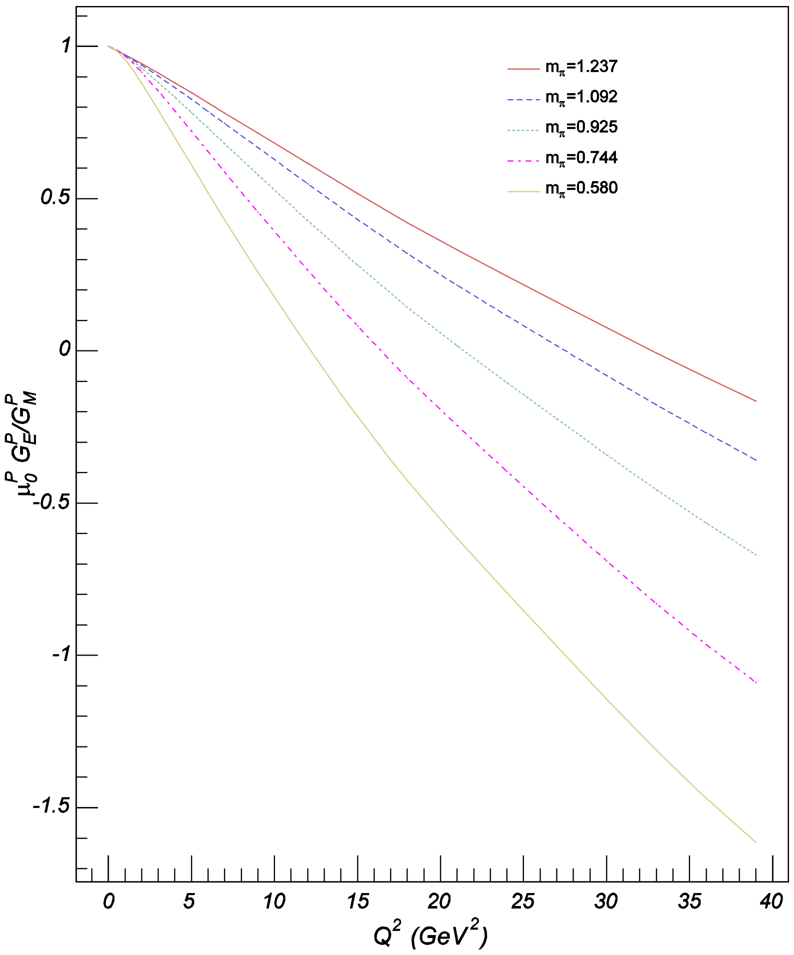
- a - {$0.47\,\mathrm{GeV}^{-1}$, $0.34\,\mathrm{GeV}^{-1}$, $0.26\,\mathrm{GeV}^{-1}$},
  V - {16^3x32, 24^3x48, 32^3x48 },
- $m_\pi \sim$ {1.2GeV,0.6GeV} ,
- $M_N \sim$ {2GeV, 1.5GeV },
- $Q^2 \sim$ {$0.6\,\mathrm{GeV}^2$, $2.3\,\mathrm{GeV}^2$}.

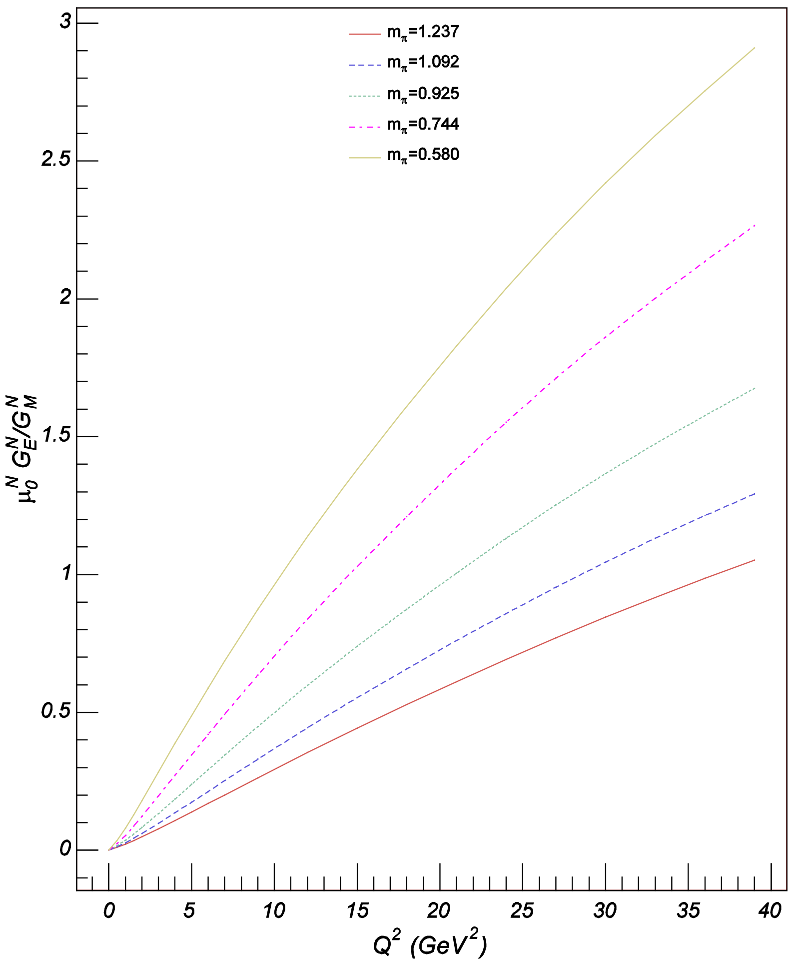
Fits to proton and neutron ffs.



LFCBM fits fit to QCDSF data for nucleon G_E (in units of e) and G_M (in units of $e/2M_N$) for a lattice spacing $a=0.26 GeV^{-1}$,$M_N=1.8$ GeV and $m_\pi=0.93$ GeV.

Extrapolation to high Q^2 of calculated FF for different m_π for $a=0.26\text{GeV}^{-1}$.





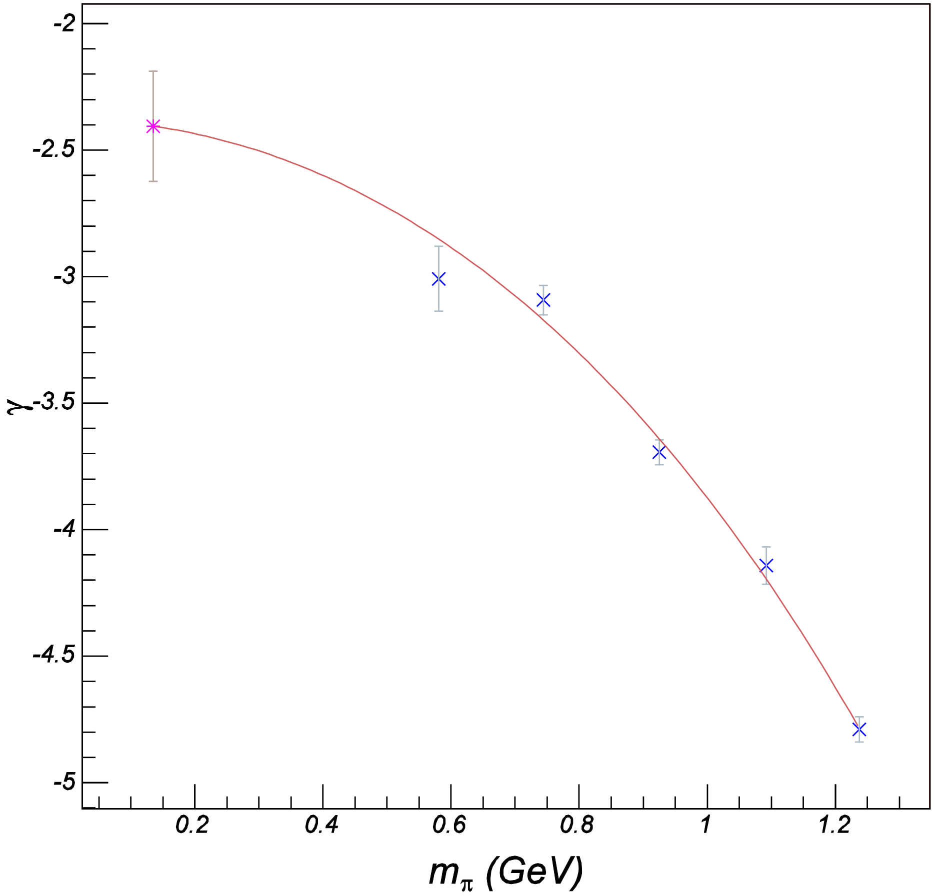
Extrapolation of the fitting parameters to the physical regime.

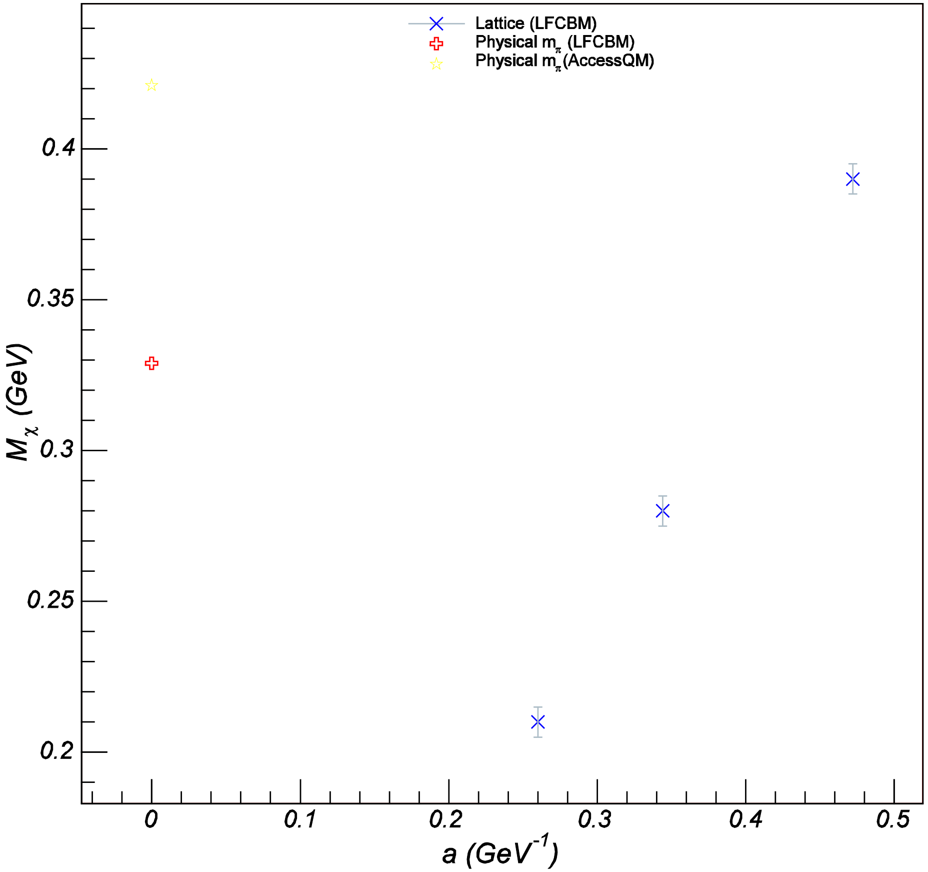
We need to extrapolate the fitting parameters to the physical region. We assume a smooth analytic dependence of the fitting parameters from the quark mass, i.e. m_π^2 :

$$\gamma(m_\pi) = \gamma_0 + \gamma_1 m_\pi^2 + \gamma_2 m_\pi^4$$

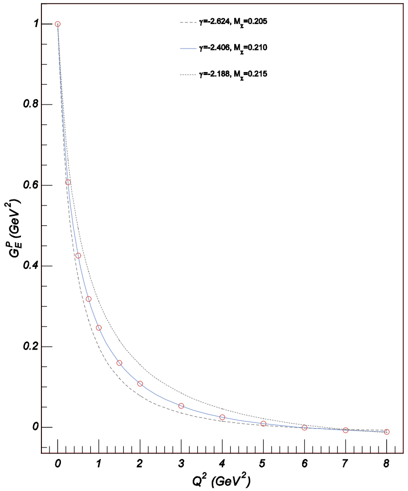
We take all the other parameters from the fits to the experimental data.

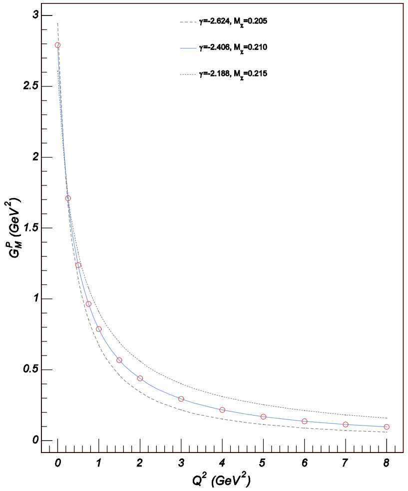
Extrapolation of the fitting parameters of LFCBM for a=0.26GeV^{-1}.

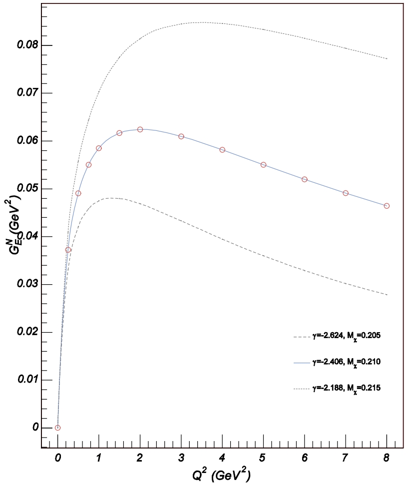


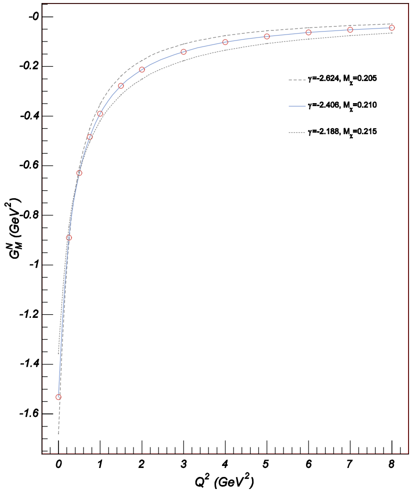


Physical Nucleon FFs from LFCBM extrapolation for a=0.26GeV^{-1}.

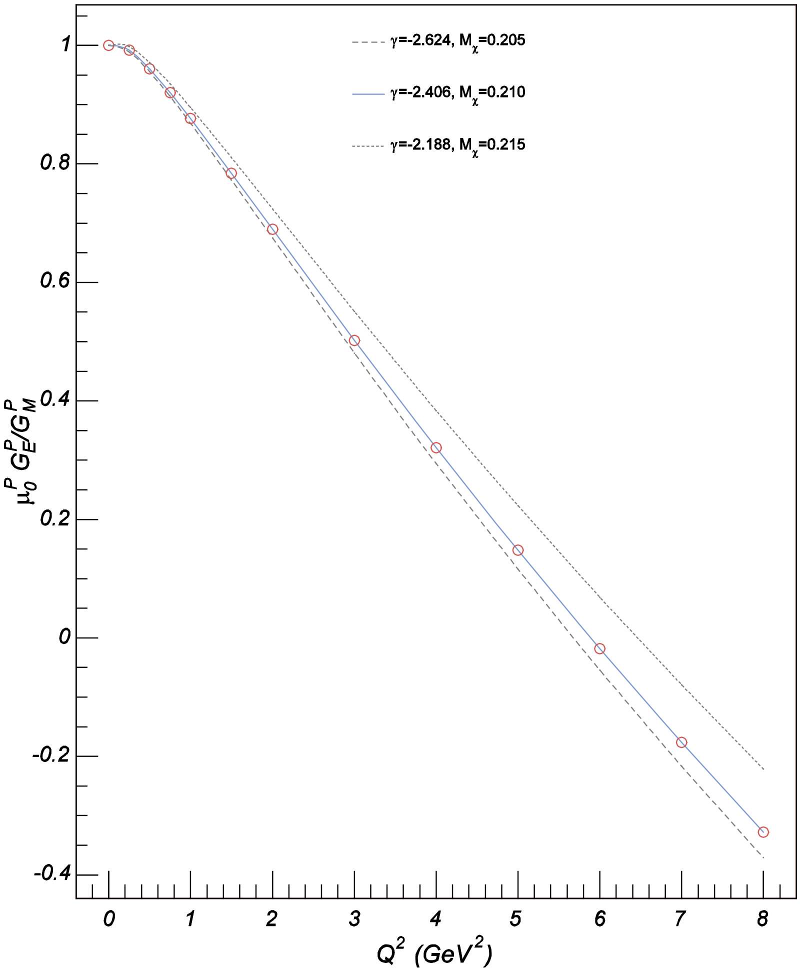


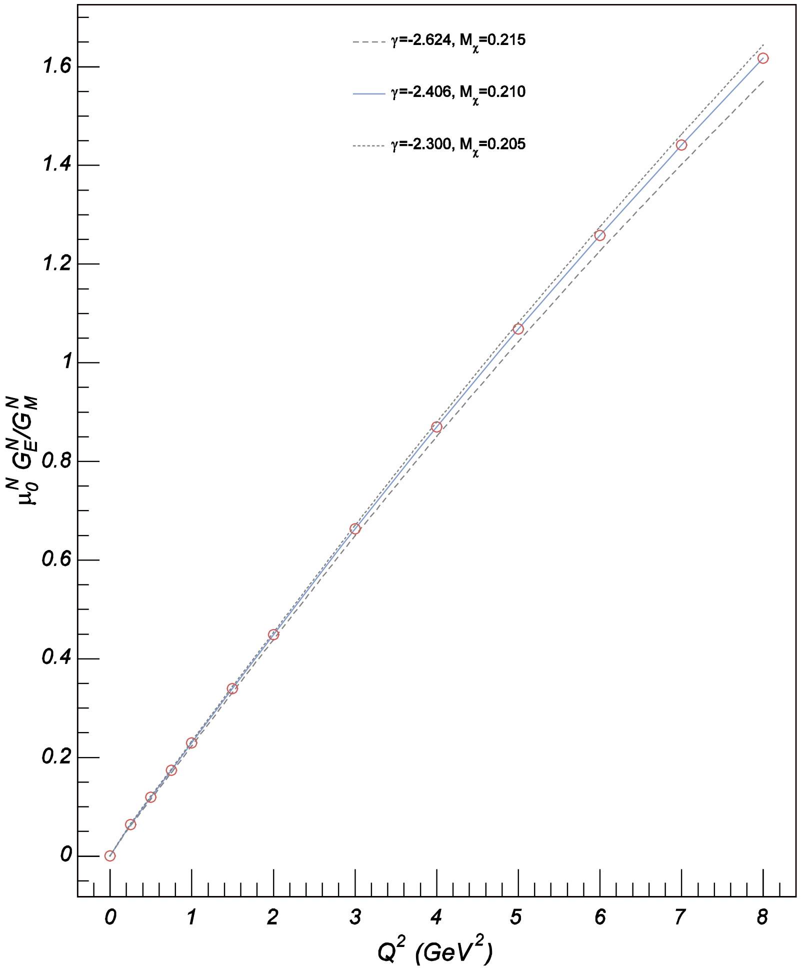




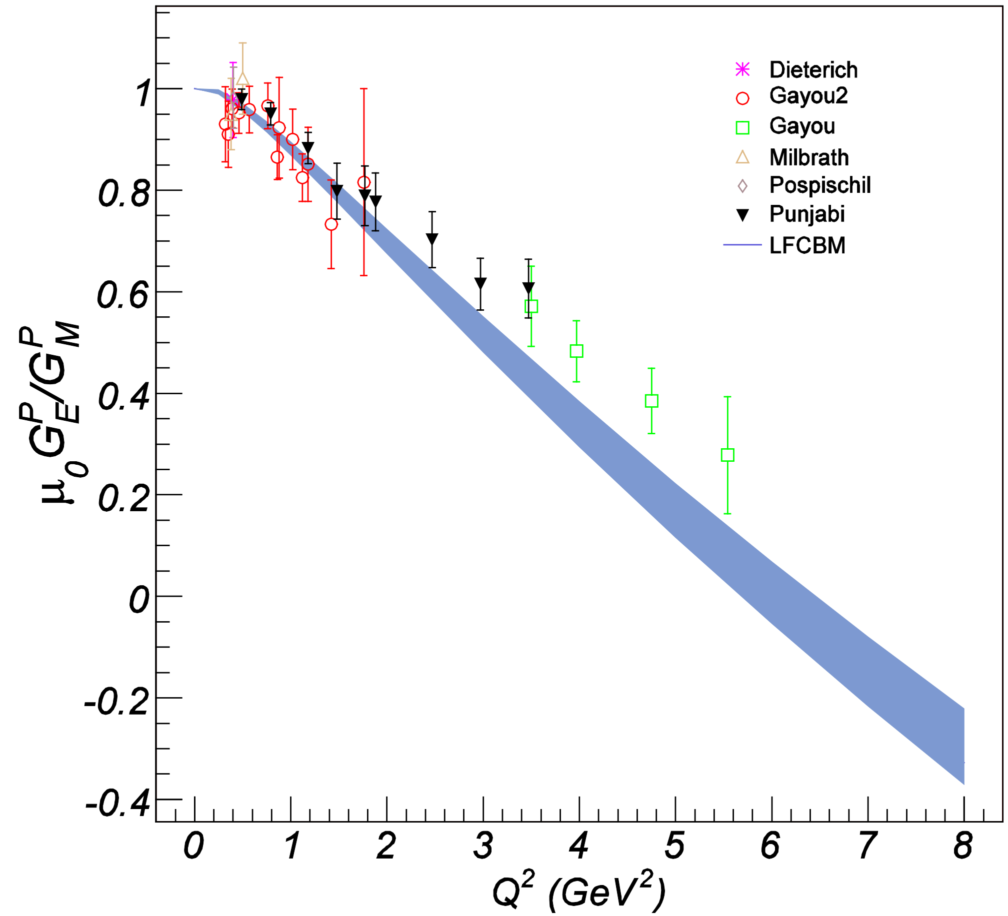


Physical Nucleon FF from LFCBM extrapolation for a=0.26GeV^{-1}.





LFCBM Proton FFs extrapolation for a=0.26GeV^{-1} vs. Experiment



Conclusion.

- We have considered the Light Front Cloudy Bag Model (LFCBM) for extrapolations of lattice data.

- LFCBM gives quite accurate description of the lattice data points. It was used to both extrapolate in Q^2 the calculated form factors, and with smooth quadratic extrapolation of the fitting parameters vs. m_π to extrapolate the form factors to the physical quark mass region.

- We foresee the lattice calculated form factors as an alternative test of validity for any model of the nuclear structure. As new more accurate data at lower m_π and higher Q^2 data will be available in the future, one will be able to have stricter tests and more accurate extrapolations to the physical mass region.