Los Alamos
NATIONAL LABORATORY

Neutrino Oscillations

Or how we know most of what we know

Outline

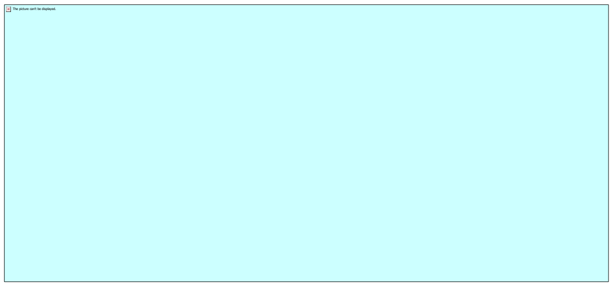
- Two-flavor vacuum oscillations
- Two-flavor matter oscillations
- Three-flavor oscillations
  - The general formalism
  - The “rotation” matrices

Consider Two Mass States

ψ_1 corresponding to m_1
ψ_2 corresponding to m_2

Think of ψ as a Vector

Ψ is a solution of $\mathcal{H}$



The Neutrinos

Consider the weak eigenstates ν_e, ν_μ.
These are not the mass eigenstates, ν_1, ν_2.
The mass eigenstates are propagated via $\mathcal{H}$.

The Mixing Matrix: U

Mixing

Weak eigenstates are a linear superposition of mass eigenstates.

In Vacuum, no potential in $\mathcal{H}$

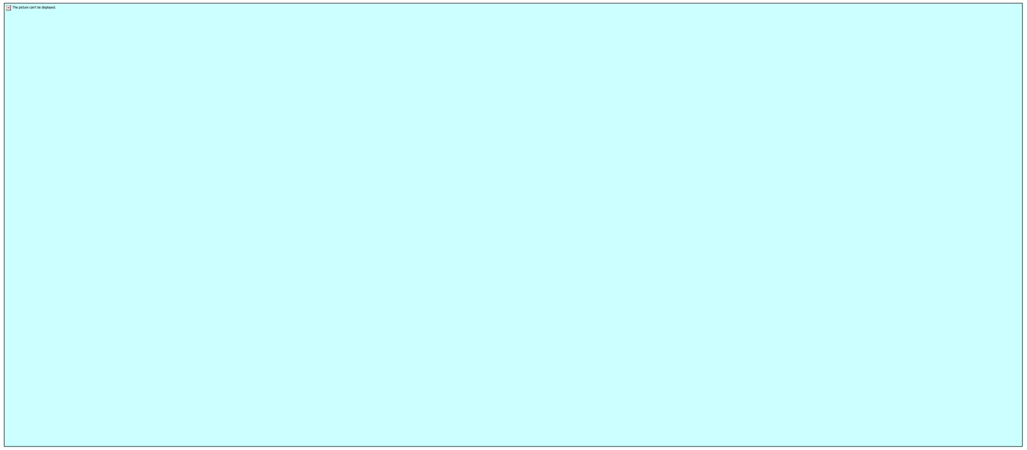
Denote

c = cos θ

s = sin θ

$U\mathcal{H}U^{-1}$

The energy difference (and Trig.)



$U\mathcal{H}U^{-1}$ becomes

The algebra is going to get involved, so lets define A, B, and D such that:

The Diff Eq

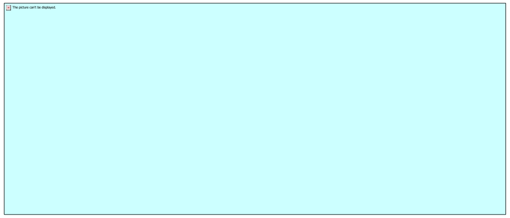
A solution to this equation should have the form:

Insert proposed solution

Two Equations

r_+ solution

r_- solution

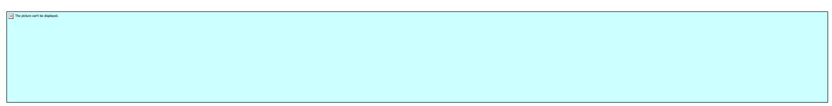


ν_α is a superposition of these 2 solutions

(D+2A) is a constant so we sweep it into a redefinition of the C’s.

The solutions

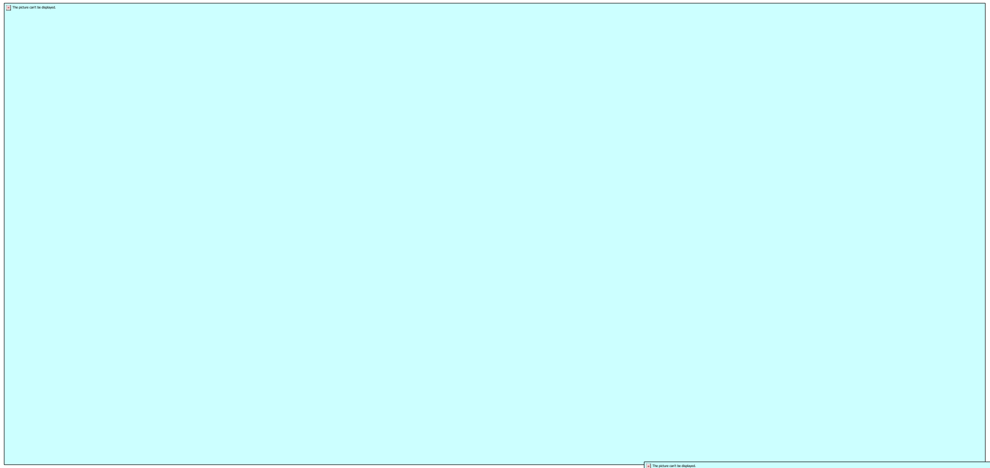
To determine the C's, use $\langle\nu_\alpha|\nu_\alpha\rangle=1$ and assume that at t=0, we have all ν_e.

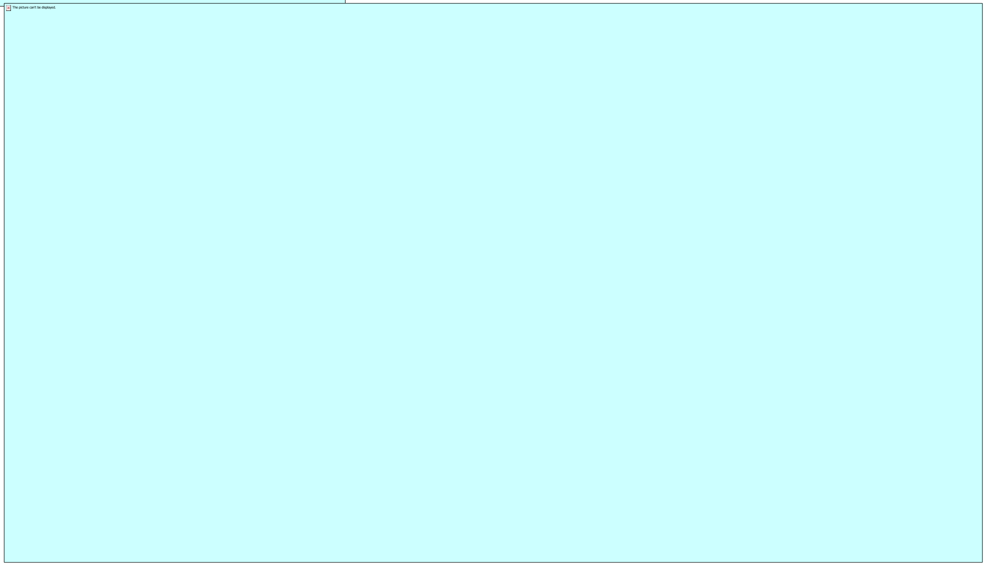


The time dependent solution

What is the probability of finding all ν_μ at time t?

Transition probability





The Answer

Complete mixing: large sin2θ and long R/L would result in an “average”: that is P=1/2.

What about MSW?

The Sun is mostly electrons (not muons).

ν_e can forward scatter from electrons via the charged or neutral current.

ν_μ can only forward scatter via the neutral current.

The ν_e picks up an effective mass term, which acts on the weak eigenstates.

The MSW $\mathcal{H}$ term.

This extra term results in an oscillation probability that can have a resonance. Thus even a small mixing angle, θ, can have a large oscillation probability.

Similar algebra as before

Constant Density Solutions

Note similar form to vacuum Oscillations.

Note that $\sin^2 2\theta_m$ can be 1 even when $\sin^2 2\theta$ is small. That is when:

$L/L_0 = \cos 2\theta$

Variable Density

- Integrate over the changing density (such as in a star).

Three ν Formulism

Transition Probability

Transition Probability

Real U's

Complex U’s

If U is complex, then we have the possibility

Oscillation Experiments

Appearance: look for ν_β when none are expected

Disappearance: look for decrease in flux of ν_α

Neutrino Sources and Oscillations

- **Solar neutrinos**
  - Few MeV, L~10^{11} m
  - Electron neutrinos
  - Most are disappearance expts. (Except SNO NC and SK's slight NC sensitivity)
- **Reactor**
  - Few MeV, L~10m - 300 km
  - Electron neutrino disappearance

Neutrino Sources

- **Accelerator**
  - **30-50 MeV (μ decay)**
  - **DIF sources can be several GeV**
  - **Various appearance and disappearance modes, various baselines**
- **Atmospheric**
  - **π and μ decay**
  - **Various energies**
  - **Baseline from 20 to 10,000 km**

Maki, Nakagawa, Sakata, Pontecorvo

U_{MNSP} Matrix

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \times \begin{pmatrix} \cos\theta_{13} & 0 & e^{-i\delta_{CP}}\sin\theta_{13} \\ 0 & 1 & 0 \\ -e^{i\delta_{CP}}\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix} \times \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha/2} & 0 \\ 0 & 0 & e^{i\alpha/2+i\beta} \end{pmatrix}$$

$\theta_{23} \sim 45°$ $\tan^2\theta_{13} < 0.03$ at 90% CL $\theta_{12} \sim 32°$

Atmospheric Reactor Solar

PDB parameterization

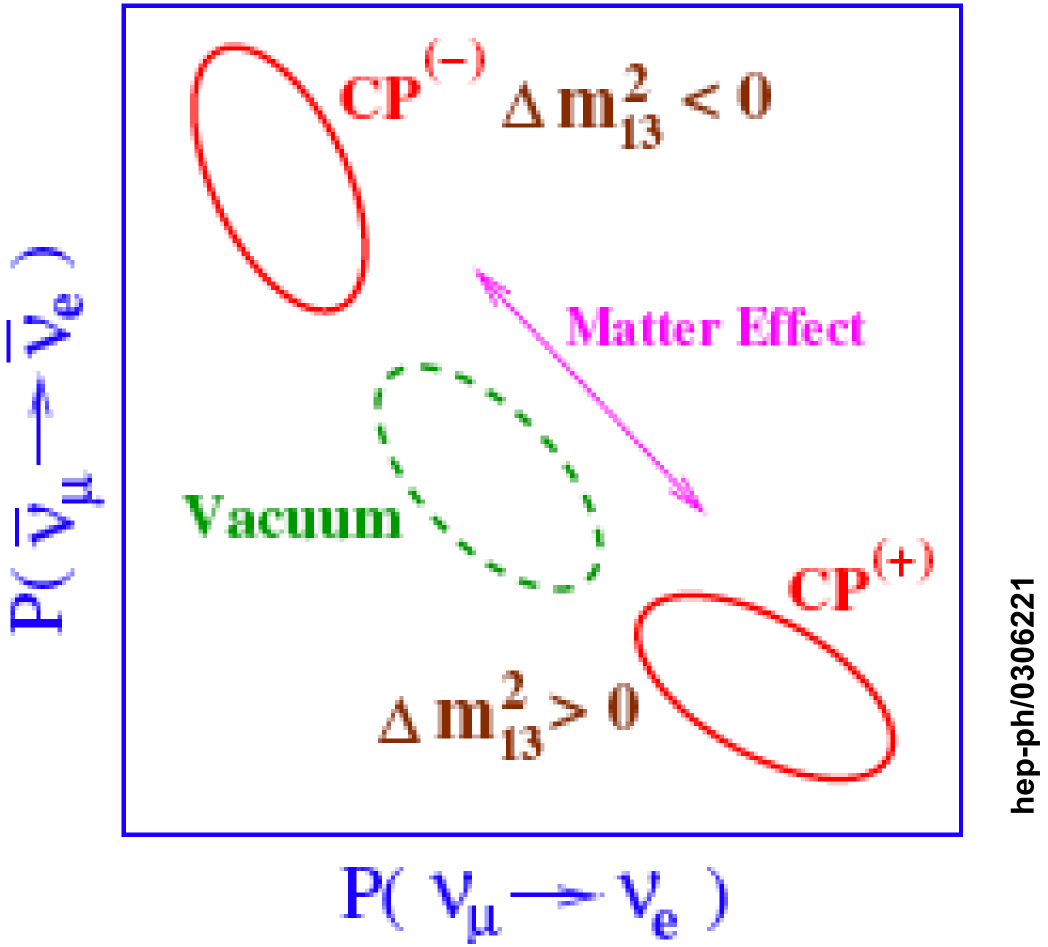
CP violation

The Jarlskog Invariant

Note the product of the sin of all the angles. If any angle is 0, CP violation is not observable.

Note that I have seen different values of the leading constant. (taken to be 1 here)

CP violation



There are only 2 independent δm^2 for 3 ν

This will be important when we discuss LSND.

Resources

- Steve Elliott - UW Phys 558 class notes
- Bahcall Book
- Many phenomenology papers