Molecular Dynamics Simulations of Non-uniform Dense Matter and Neutrino Interactions in Supernovae

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X-Ray image of Crab Nebula Pulsar Credit: NASA/CXC/ASU/J. Hester et al.)

Outline

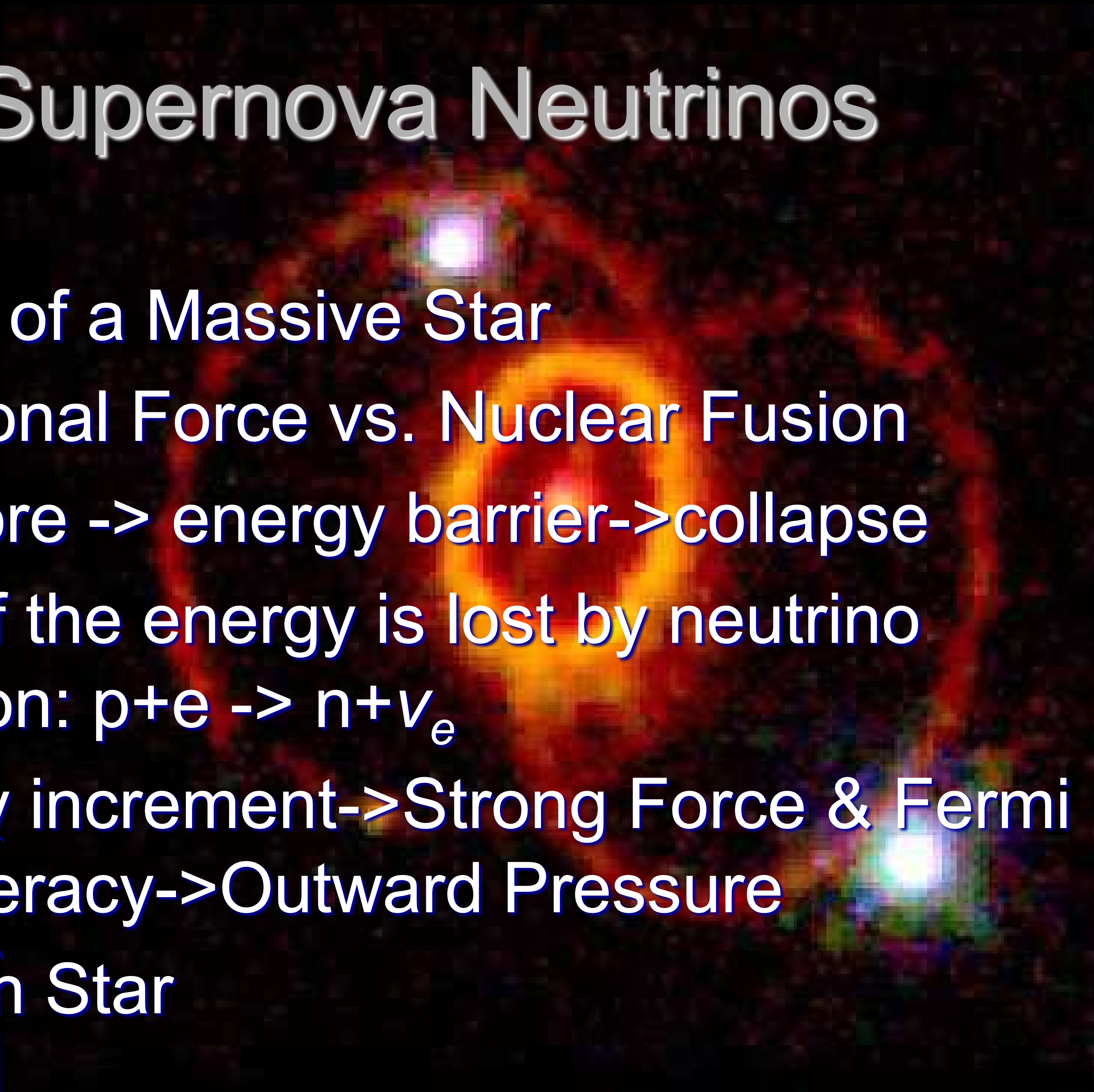
- Introduction: Supernova and Neutron Stars
- Ion Response
  - Neutrino-Nucleus scattering
  - Model
  - Simulation Results
- Future Work

Supernova Neutrinos

Collapse of a Massive Star

Gravitational Force vs. Nuclear Fusion

- Iron Core -> energy barrier->collapse
- Most of the energy is lost by neutrino emission: p+e -> n+ν_e
- Density increment->Strong Force & Fermi Degeneracy->Outward Pressure
- Neutron Star



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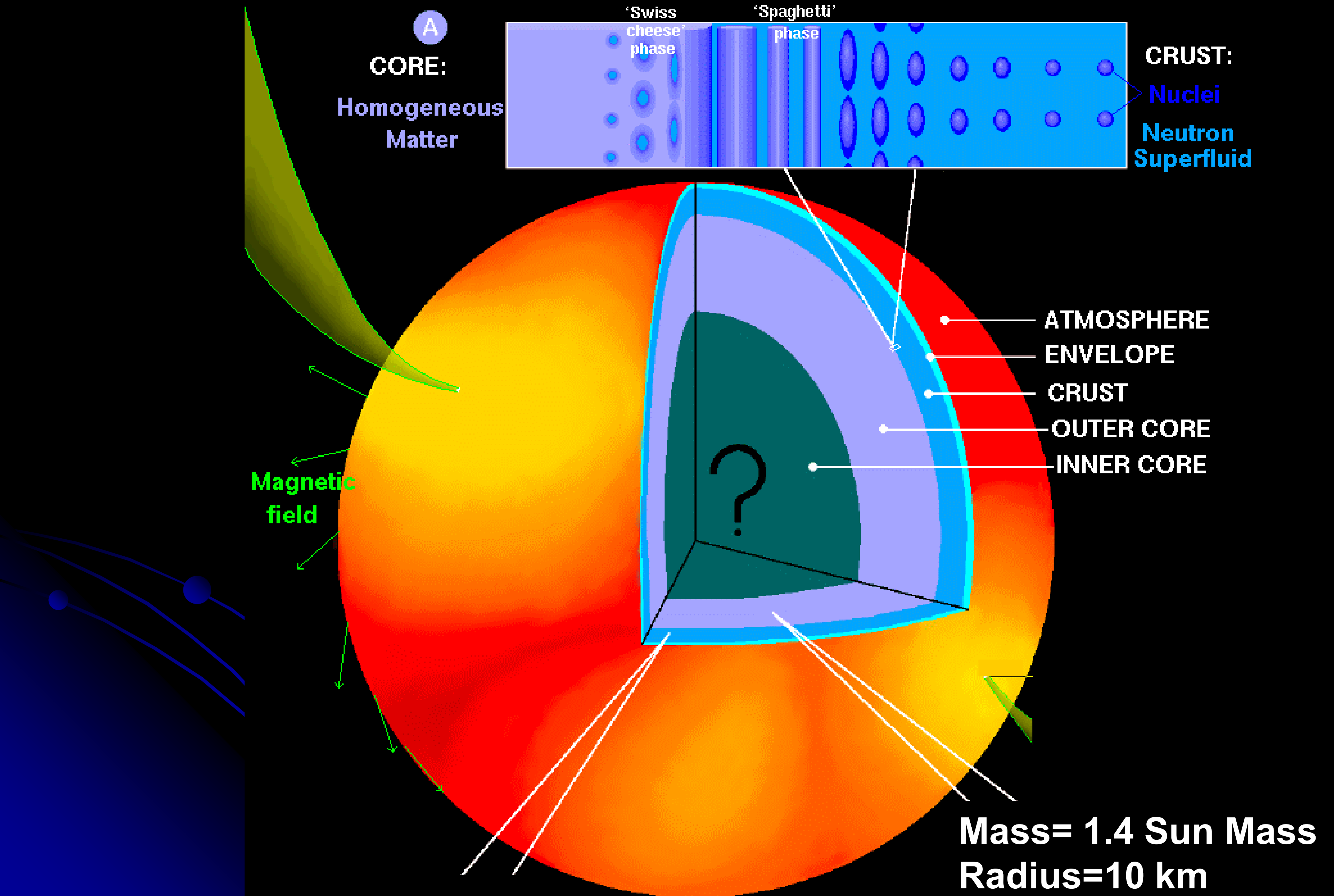
A NEUTRON STAR: SURFACE and INTERIOR



Mass= 1.4 Sun Mass
Radius=10 km

Image taken from Dany Pierre Page, Neutron Star (Theory) Group at UNAM

$\rho \sim 10^{12}$ g/cm^3

- Medium= Plasma Electrons and Nuclei
- Neutrino Trapping in Supernova
  - Neutrino-Nucleus elastic scattering
  - Ion, Electron Screening have important effect

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Ion Response

Ion Response

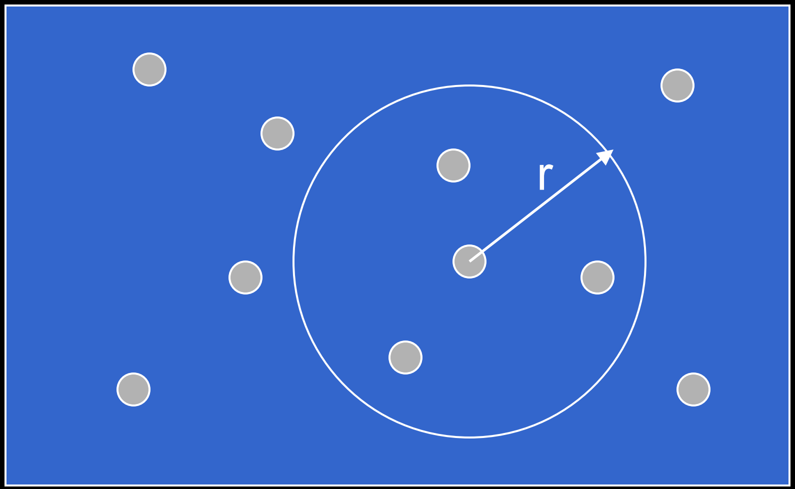
- Linear Response -> Neutrino-Nucleus Elastic Cross Section:

$$\frac{d\sigma}{d\Omega dE} = S(\vec{q},\omega)\frac{d\sigma}{d\Omega dE}\bigg|_{Free}$$

- Correlation Function g(r)
- Static Structure Factor S(q)
- Dynamic Structure Factor S(q,w)

Correlation Function g(r)

Probability of finding another ion a distance r from a given ion.



Static Structure Factor

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$$S(\vec{q}) = 1 + \rho \int d^3 r \left(g(\vec{r}) - 1 \right) \exp(i\vec{q} \cdot \vec{r})$$

$$S(\vec{q}) = \frac{1}{N} \left(\left\langle \Psi_0 \middle| \hat{\rho}^+(\vec{q}) \hat{\rho}(\vec{q}) \middle| \Psi_0 \right\rangle - \left| \left\langle \Psi_0 \middle| \hat{\rho}(\vec{q}) \middle| \Psi_0 \right\rangle \right|^2 \right)$$

Charge Density

$$\rho(\vec{q}) = \sum_{i=1}^{N} \exp(i\vec{q} \cdot \vec{r}_i)$$

Ground State

$$\left| \Psi_0 \right\rangle$$

Model

- Classical Approximation

$$\lambda_B << r_{ij}$$

- Ions interact via screened Coulomb potential

$$V(r_{ij}) = \frac{Z^2 e^2}{4\pi r_{ij}} \exp\left(- {r_{ij}}/{\lambda_e} \right)$$

λ_e electron screening

$$\lambda_e = \frac{\pi}{e k_F}, \alpha = \frac{e^2}{4\pi\hbar c}$$

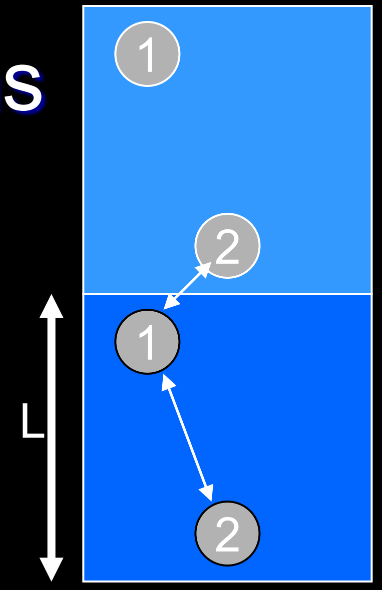
r_{ij} distance between Ions

Monte Carlo Simulation

- T= 1 Mev
- N ions
- ^{56}Fe
- $\rho = 1 \times 10^{12}$ g/cm^3
- Periodic Boundary Conditions

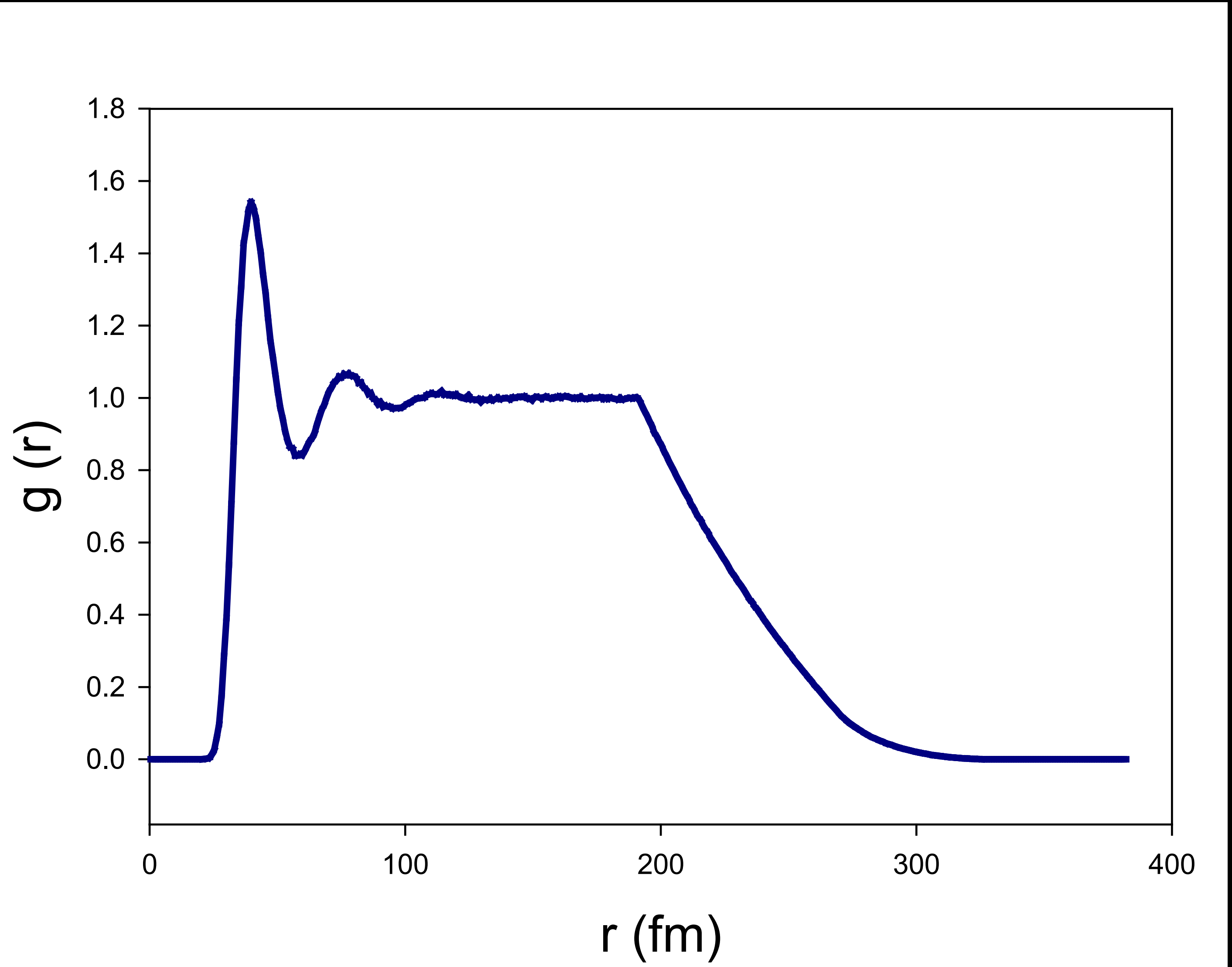
$$x_{ij} = Min\left(\left|x_i - x_j\right|, L - \left|x_i - x_j\right| \right)$$

- r_i at equilibrium, i=1,N



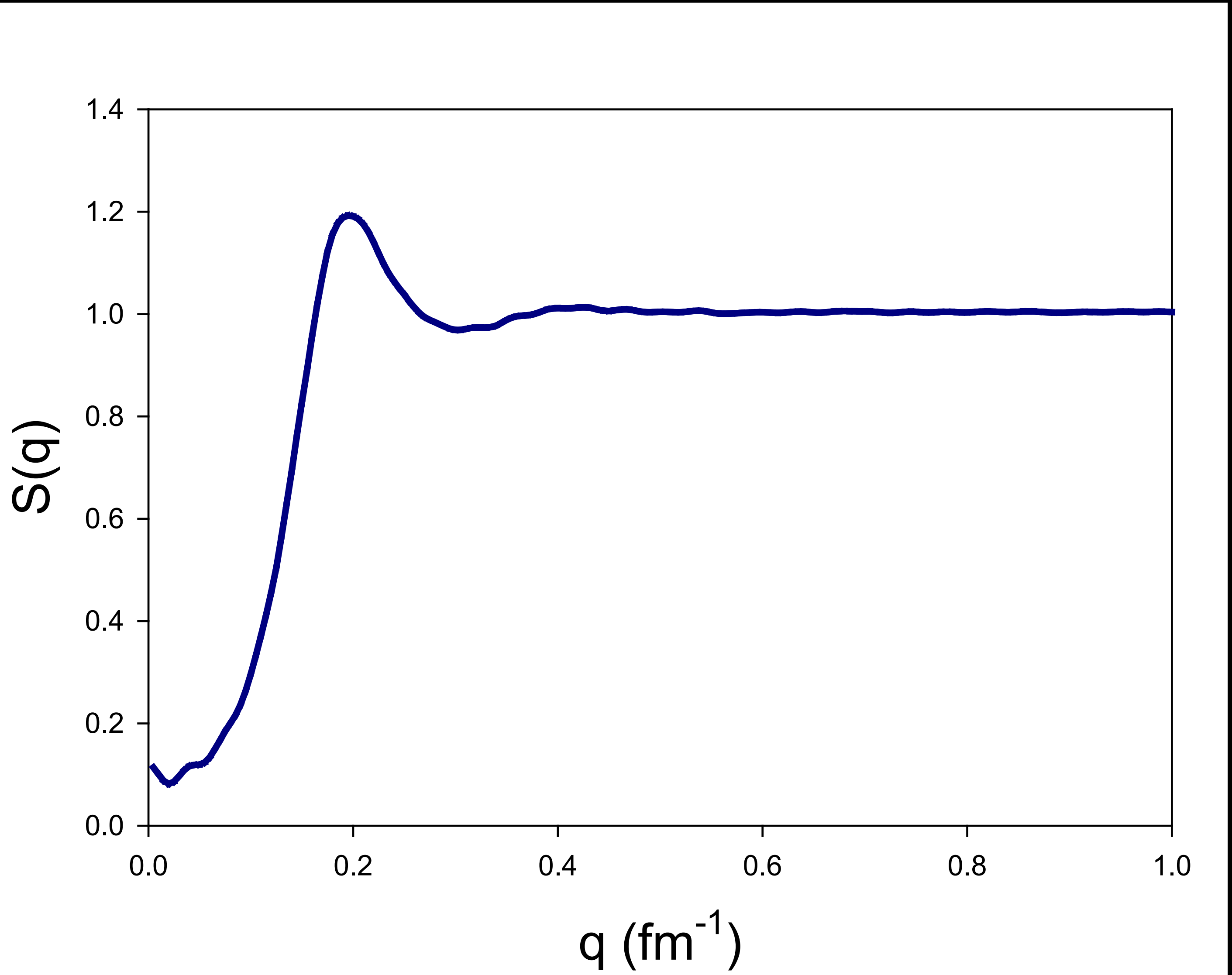
Correlation Function

N=1000 λe=10 fm L=382 fm



Static Structure Factor

N=1000   λe=10 fm   L=382 fm



Dynamic Response

Dynamic Structure Factor

$$S(\vec{q},\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\exp(i\omega t)S(\vec{q},t)dt$$

$$S(\vec{q},t)=\frac{1}{N}\left\{\left\langle\rho(\vec{q},t)\rho(-\vec{q},0)\right\rangle-\left\langle\rho(\vec{q},t)\right\rangle\left\langle\rho(-\vec{q},0)\right\rangle\right\}$$

$$\rho(\vec{q},t)=\sum_{i=1}^{N}\exp(i\vec{q}\cdot\vec{r}_i(t))$$

$$\left\langle\rho(\vec{q},t)\rho(-\vec{q},0)\right\rangle=\frac{1}{\Delta t_1}\int_{0}^{\Delta t_1}\rho(\vec{q},t_1+t)\rho(-\vec{q},t_1)dt_1$$

Molecular Dynamics Algorithm

- Verlet Algorithm

$$\vec{r}(t+\Delta t)=\vec{r}(t)+\vec{v}(t)\Delta t+\frac{1}{2}\vec{a}(t)\Delta t^2$$

$$\vec{v}(t+\Delta t/2)=\vec{v}(t)+\frac{1}{2}\vec{a}(t)\Delta t$$

$$\vec{a}(t+\Delta t)=-\frac{1}{m}\nabla V\left(\vec{r}(t+\Delta t)\right)$$

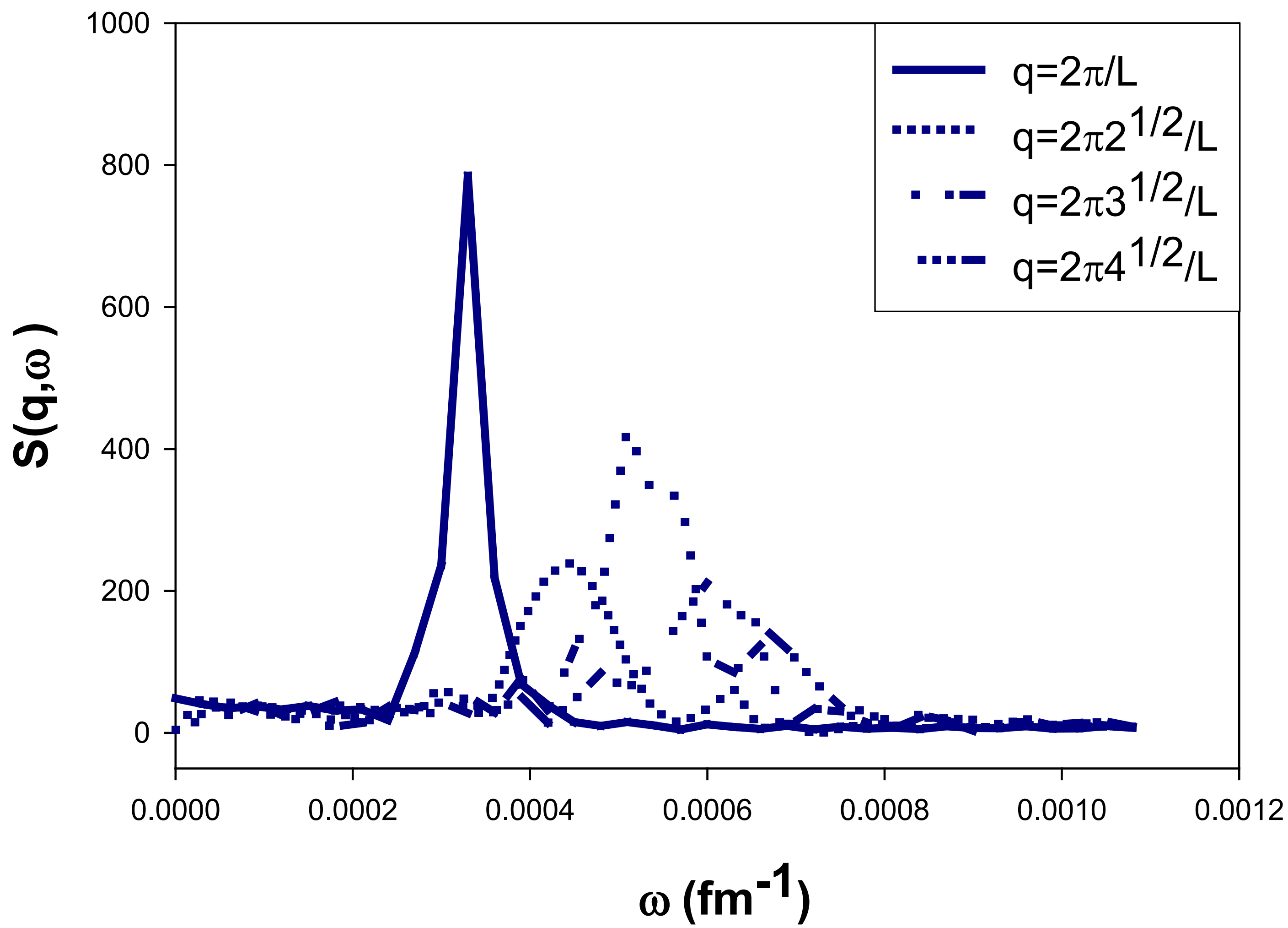
$$\vec{v}(t+\Delta t)=\vec{v}(t+\Delta t/2)+\frac{1}{2}\vec{a}(t+\Delta t)\Delta t$$

Molecular Dynamic Simulation

- T= 1 MeV
- N ions
- ^{56}Fe
- $\rho = 1x10^{12}$ g/cc
- Periodic Boundary Condition
- $r_i(t)$, i=1,N

Dynamic Structure Factor

N=500 λe=10 fm L=304 fm



Peaks

q (fm-1)	ω*10^{-4} (fm-1)	Ω_p*10^{-4}(fm-1)
$q_0=2\pi/L$	3.3	4.2
$q_0 * 2^{1/2}$	4.5	5.7
$q_0 * 3^{1/2}$	5.1	6.9
$q_0 * 4^{1/2}$	6	7.8

$$\Omega_p^2 = 4\pi\hbar c\alpha Z^2 \frac{\rho_i}{M} \frac{q^2}{q^2 + \lambda_e^{-2}}$$

Future Work

- Ion-Ion Screening model break down at large density
- N=100000 nucleons MDGRAPE
- Dynamical Response Pasta