Microscopic interpretation of the excited $K^{\pi} = 0^{+}, 2^{+}$ bands of deformed nuclei

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Outline

- Introduction

What we know about the nucleus
Characteristic energy spectra

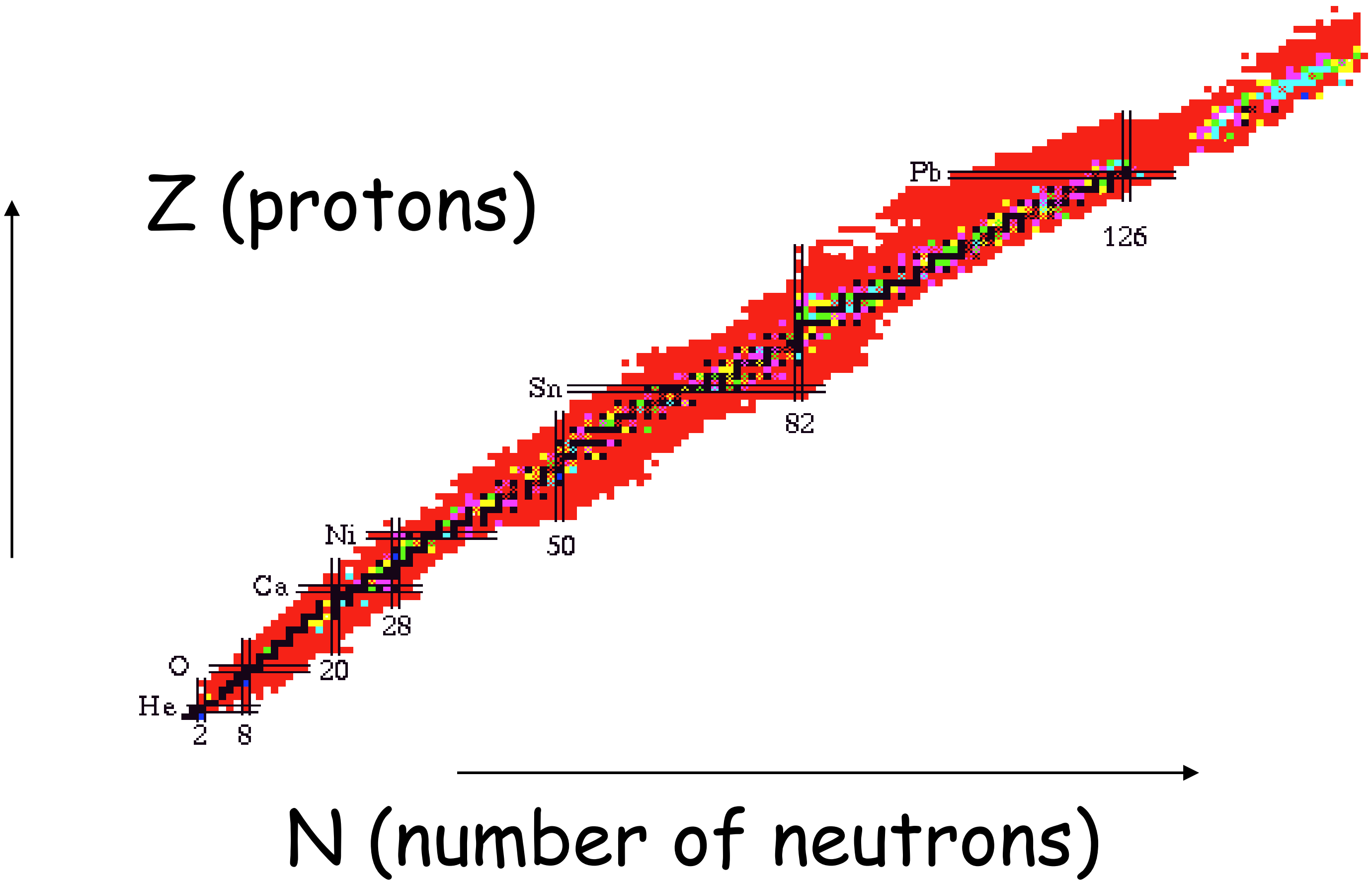
- Theoretical Model

Configuration space
System Hamiltonian

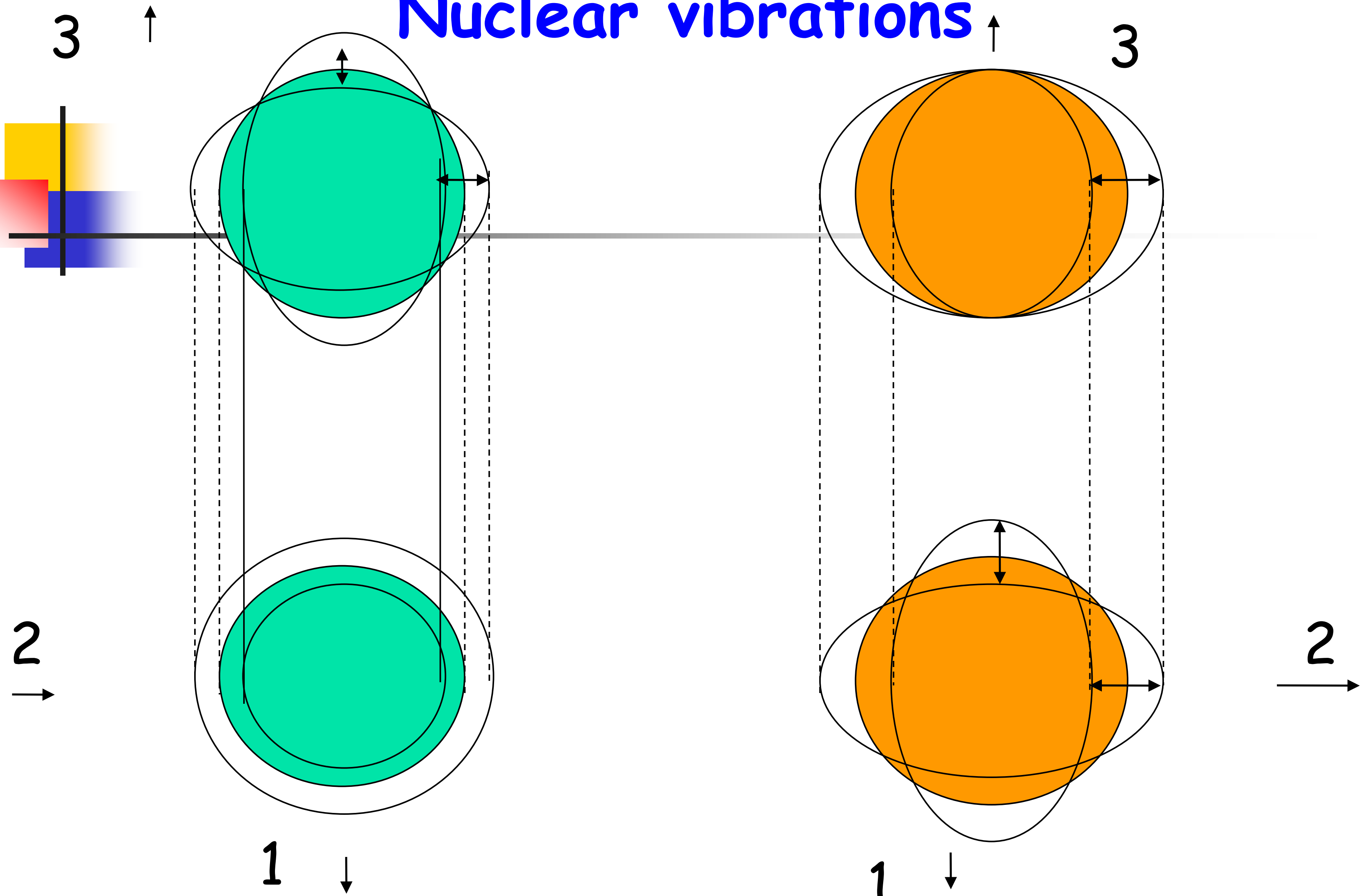
- Results

- Conclusion and future work

Chart of the nuclei



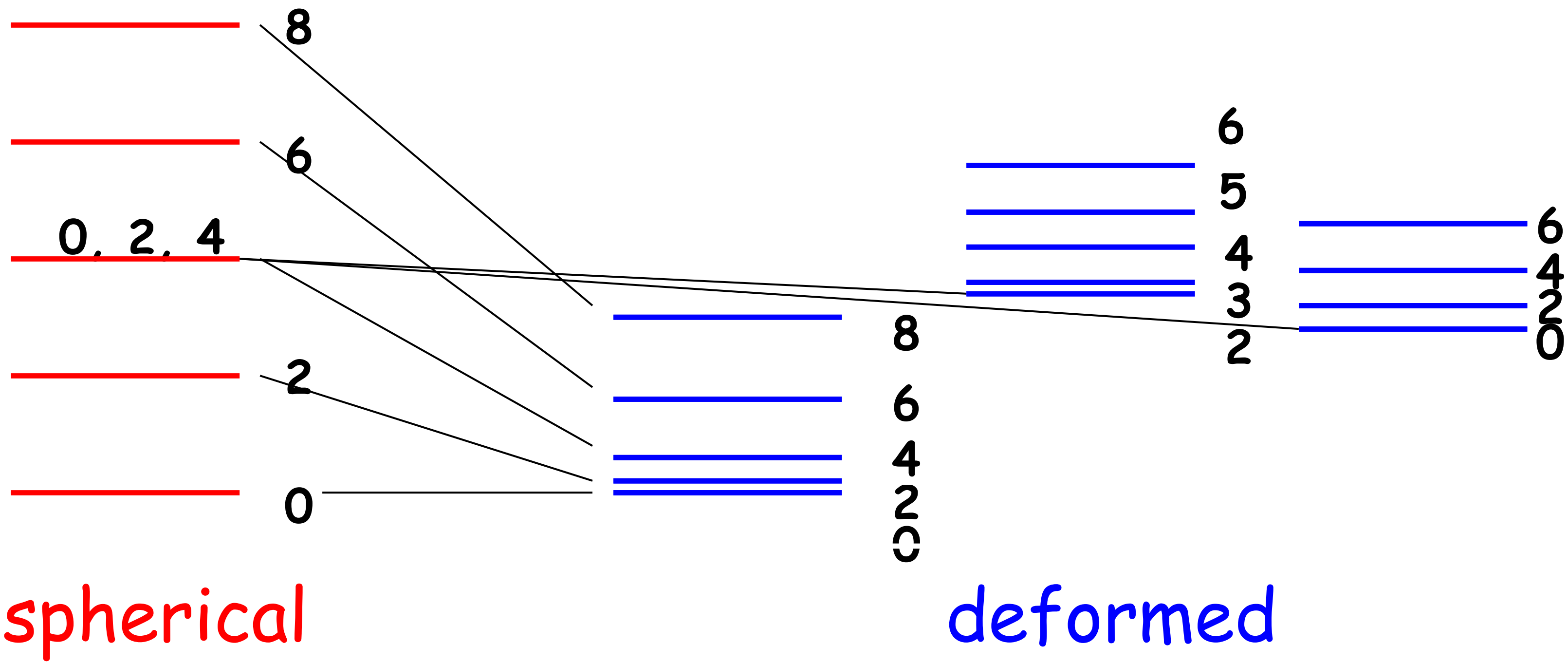
Nuclear vibrations



β-vibration

γ-vibration

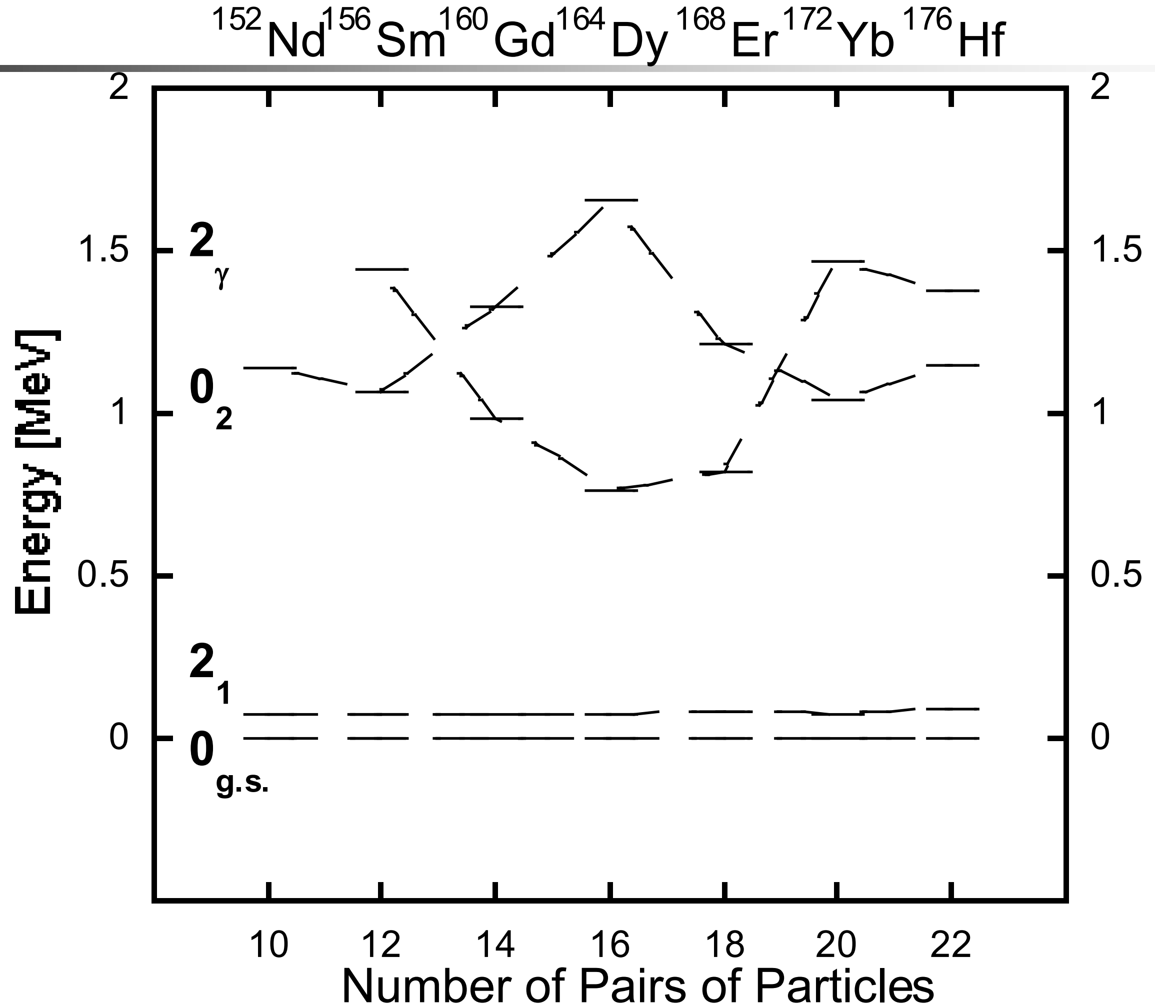
Schematic level schemes of spherical and deformed nuclei



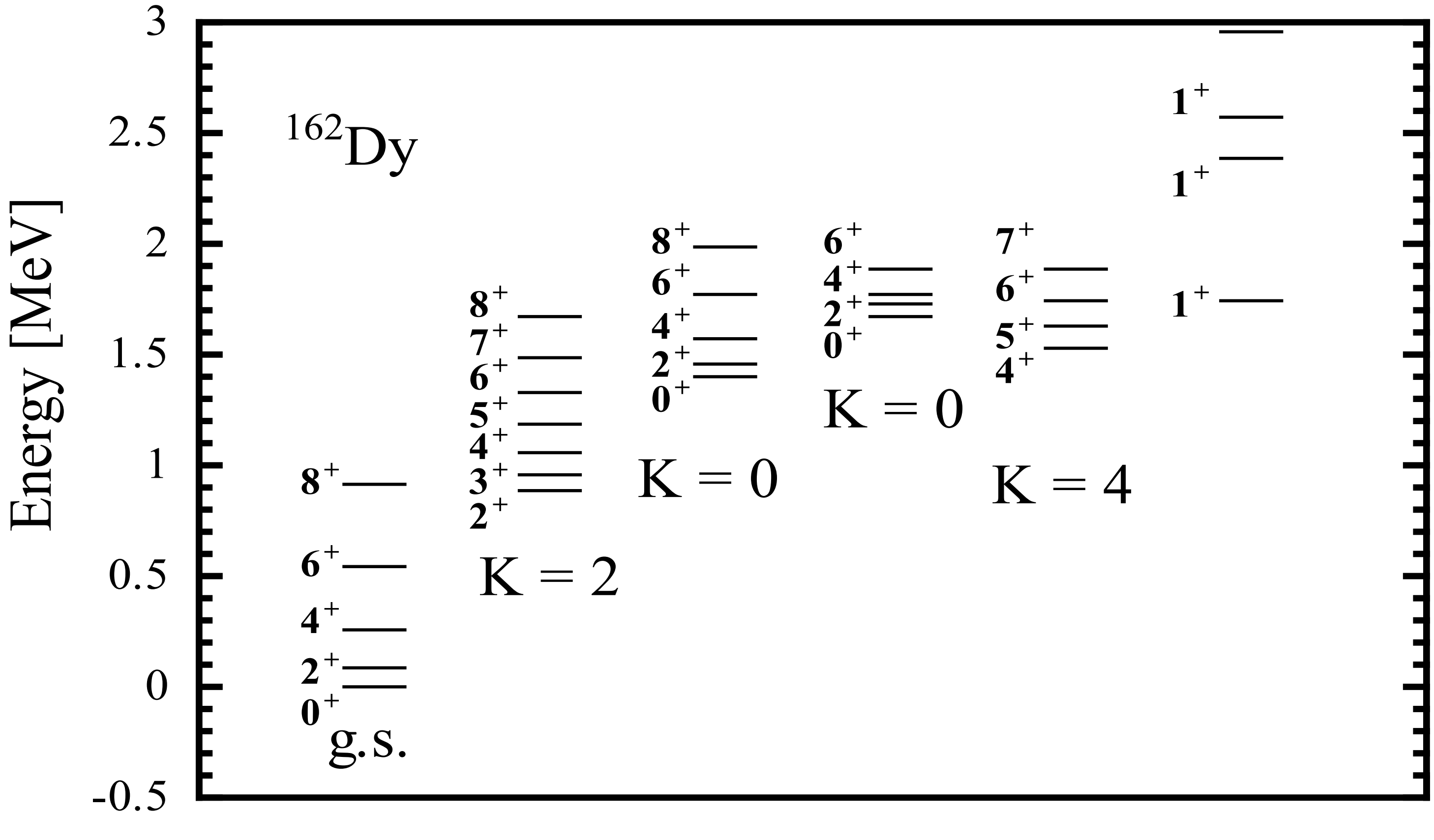
$$E = n\,\hbar\omega$$

$$E = I(I+1)/(2\mathcal{I})$$

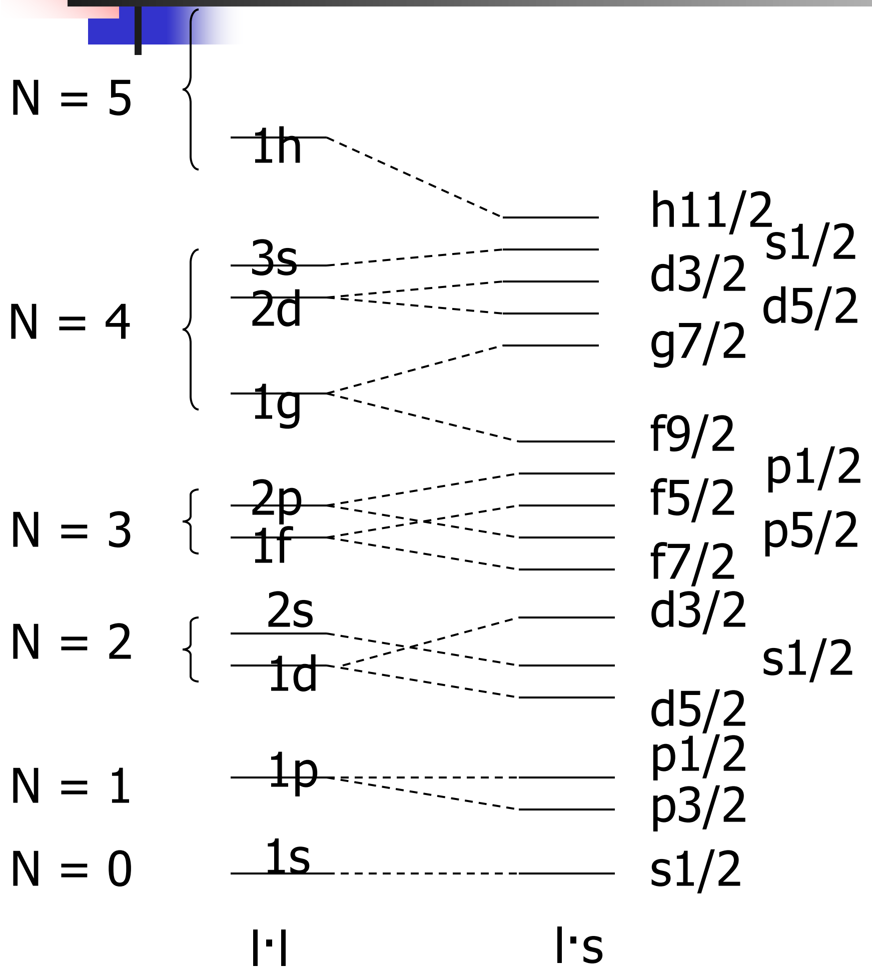
Experimental energy levels



Experimental energy spectra of ^{162}Dy



Single particle energy levels



Particle distribution

Valence space: $U(\Omega_\pi) \otimes U(\Omega_\nu)$

total	normal	unique
$\Omega_\pi = 32$	$\Omega^n_\pi = 20$	$\Omega^u_\pi = 12$
$\Omega_\nu = 44$	$\Omega^n_\nu = 30$	$\Omega^u_\nu = 14$

Particle distributions:

	protons:			neutrons:			(λ,μ) π	ν	total
^{152}Nd	10	6	4	10	6	4	(12,0)	(18,0)	(30, 0)
^{156}Sm	12	6	6	12	6	6	(12,0)	(18,0)	(30, 0)
^{160}Gd	14	8	6	14	8	6	(10,4)	(18,4)	(28, 4)
^{164}Dy	16	10	6	16	10	6	(10,4)	(20,4)	(30, 4)
^{168}Er	18	10	8	18	10	8	(10,4)	(20,4)	(30, 4)
^{172}Yb	20	12	8	20	12	8	(36,0)	(12,0)	(24, 0)
^{176}Hf	22	14	8	22	14	8	(8,30)	(0,12)	(8, 18)

Wave Function

$$|\psi_\gamma\rangle = \sum C^\gamma_i \, |\phi_i\rangle$$

$$|\phi_i\rangle = |\{\alpha_\pi; \alpha_\nu\} \, \rho(\lambda,\mu)KL \, S; JM\rangle$$

$$\alpha_\sigma \, (\sigma = \pi, \nu) = n_\sigma \, [f_\sigma] \, (\lambda_\sigma, \mu_\sigma), S_\sigma$$

$$(\lambda_\pi, \mu_\pi) \otimes (\lambda_\nu, \mu_\nu) = \sum (\lambda, \mu)^\rho$$

Direct Product Coupling

Coupling proton and neutron irreps to total (coupled) SU(3):

$$(\lambda_\pi, \mu_\pi) \otimes (\lambda_\nu, \mu_\nu)$$

$$\rightarrow \quad (\lambda_\pi + \lambda_\nu , \mu_\pi + \mu_\nu)$$

$$+ \quad (\lambda_\pi + \lambda_\nu - 2, \mu_\pi + \mu_\nu + 1)$$

$$+ \quad (\lambda_\pi + \lambda_\nu + 1, \mu_\pi + \mu_\nu - 2)$$

$$+ \quad (\lambda_\pi + \lambda_\nu - 1, \mu_\pi + \mu_\nu - 1)^2$$

$$+ \quad \ldots$$

$$\rightarrow \quad \Sigma_{m,l} \oplus (\lambda_\pi + \lambda_\nu - 2m + l, \mu_\pi + \mu_\nu + m - 2l)^k$$

with the multiplicity denoted by k = k(m,l)

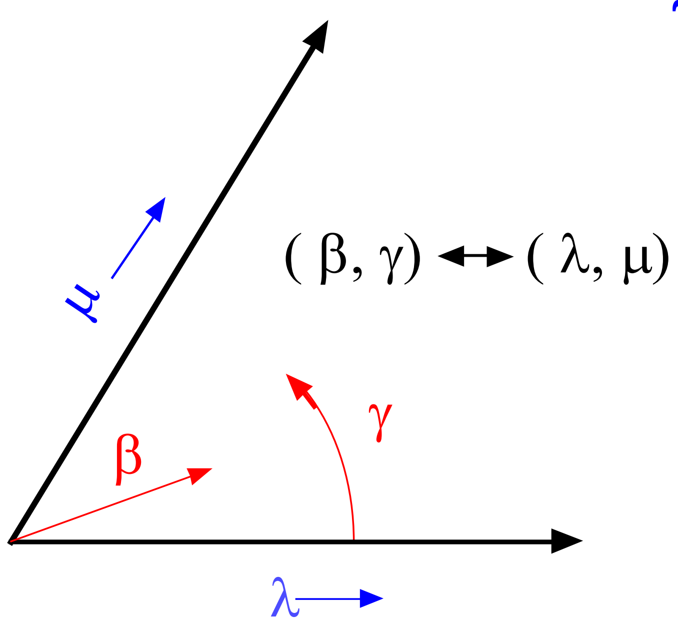
21^{st} SU(3) irreps corresponding to the highest C_2 values were used in ^{160}Gd

(λ_π,μ_π)	(λ_ν,μ_ν)	(λ,μ)						
(10,4)	(18,4)	(28,8)	(29,6)	(30,4)	(31,2)	(32,0)	(26,9)	(27,7)
(10,4)	(20,0)	(30,4)						
(10,4)	(16,5)	(26,9)	(27,7)					
(10,4)	(17,3)	(27,7)						
(12,0)	(18,4)	(30,4)						
(12,0)	(20,0)	(32,0)						
(8,5)	(18,4)	(26,9)	(27,7)					
(9,3)	(18,4)	(27,7)						
(7,7)	(18,4)	(25,11)	(26,9)	(27,7)				
(7,7)	(20,0)	(27,7)						
(4,10)	(18,4)	(22,14)						

Tricks

Invariants	↔	Invariants
Rot(3)		SU(3)
$Tr(Q^2)$	↔	C_2
$Tr(Q^3)$	↔	C_3

$$\beta^2 \sim \lambda^2 + \lambda\mu + \mu + 3(\lambda + \mu + 1)$$
$$\gamma = \tan^{-1}[\sqrt{3}\,\mu / (2\lambda + \mu + 3)]$$



System Hamiltonian

SU(3) conserving Hamiltonian:

$$H = \chi\, Q \cdot Q + a\, L^2 + b\, K_J^2 + a_{sym}\, C_2 + a_3\, C_3$$

+ one-body and two-body terms

$$+ H_{s.p.}^{\pi} + H_{s.p.}^{\nu} + G_{\pi} H_P^{\pi} + G_{\nu} H_P^{\nu}$$

Rewriting

this Hamiltonian becomes ...

$$H = H_{sp}^{\pi} + H_{sp}^{\nu} + G_{\pi} H_P^{\pi} + G_{\nu} H_P^{\nu} + \chi Q \cdot Q$$

$$+ a\, L^2 + b\, K_J^2 + a_{sym}\, C_2 + a_3\, C_3$$

Parameters of the Pseudo-SU(3) Hamiltonian

From systematics:

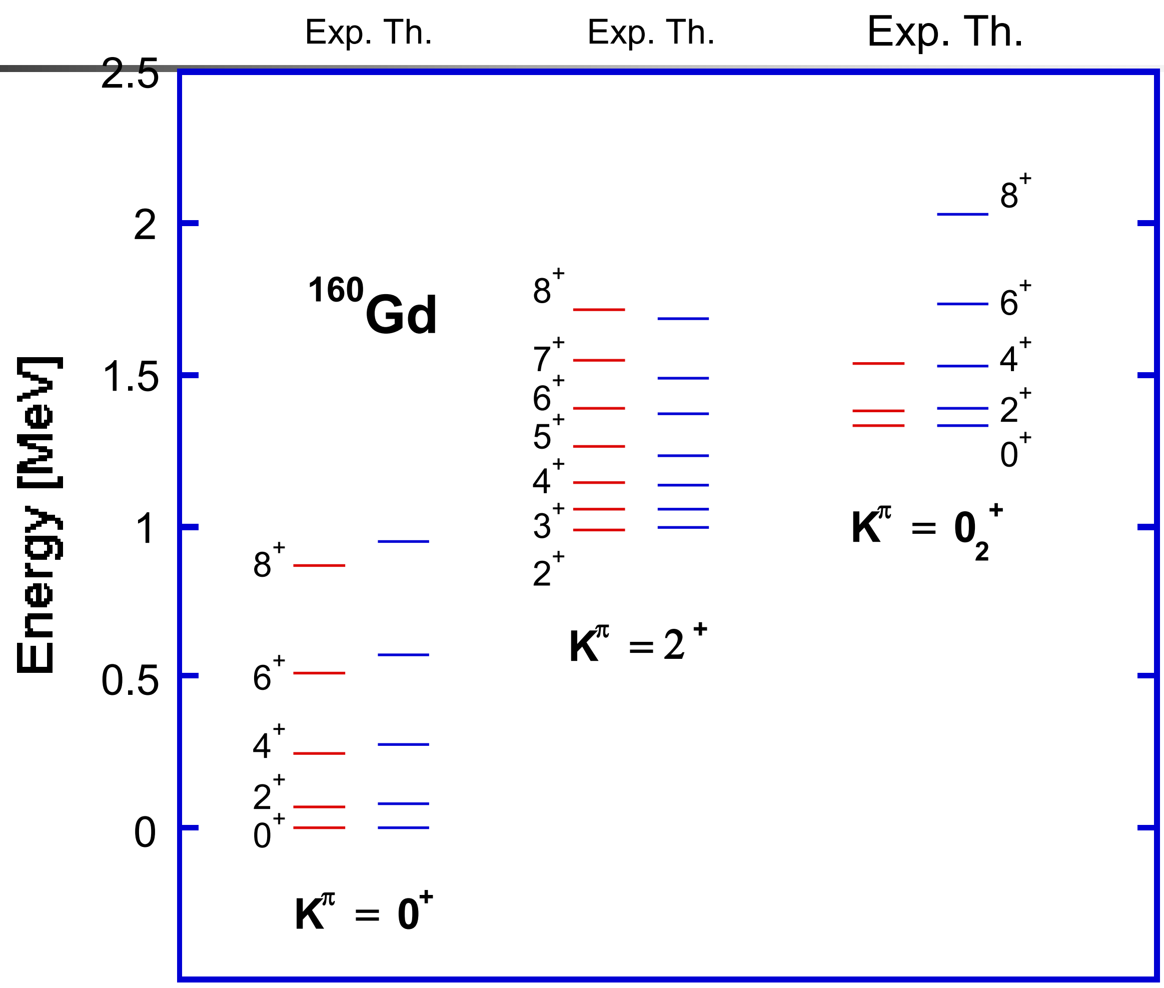
$\chi = 35/A^{5/3}$ MeV
$G_\pi = 21/A$ MeV
$G_\nu = 19/A$ MeV
$\hbar\omega = 41/A^{1/3}$ MeV

$\kappa_\pi = 0.0637 \quad \mu_\pi = 0.60$
$\kappa_\nu = 0.0637 \quad \mu_\nu = 0.60$

Fit to experiment(fine tuning): a, b, a_{sym} and a_3

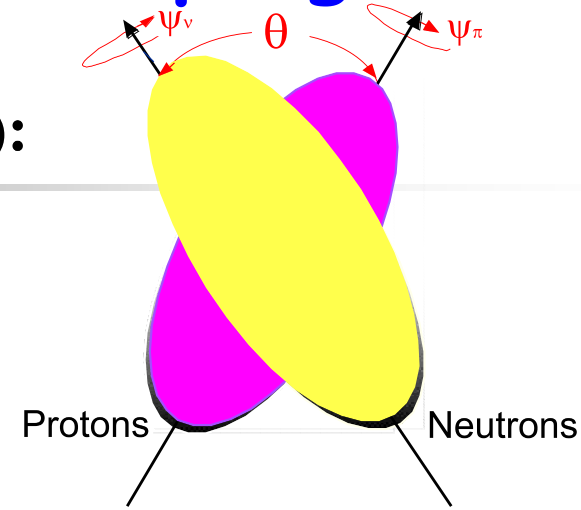
coef./nucleus	^{152}Nd	^{156}Sm	^{160}Gd	^{164}Dy	^{168}Er	^{172}Yb	^{176}Hf
$\chi \times 10^{-3}$	8.0	7.74	7.42	7.12	6.84	6.58	6.33
G_π	0.138	0.135	0.131	0.128	0.125	0.122	0.119
G_ν	0.112	0.109	0.106	0.104	0.101	0.099	0.097
$a_3 \times 10^{-4}$	2.57	2.59	1.93	0.65	0.75	0.31	0.43
a	0.000	0.000	0.001	-0.001	-0.002	-0.001	-0.007
b	0.00	0.55	0.153	0.042	0.022	0.12	0.3
a_s	0.0000	0.0000	0.0035	0.0008	0.0008	0.001	0.006

Energy spectrum for ^{160}Gd



Direct Product Coupling

Coupling proton and neutron irreps to total (coupled) SU(3):



$(\lambda_\pi, \mu_\pi) \otimes (\lambda_\nu, \mu_\nu)$

$\rightarrow \quad (\lambda_\pi + \lambda_\nu, \mu_\pi + \mu_\nu)$

$+ \quad (\lambda_\pi + \lambda_\nu - 2, \mu_\pi + \mu_\nu + 1)$ — **Scissors**

$+ \quad (\lambda_\pi + \lambda_\nu + 1, \mu_\pi + \mu_\nu - 2)$ — **Twist**

$+ \quad (\lambda_\pi + \lambda_\nu - 1, \mu_\pi + \mu_\nu - 1)^2$ — **Scissors + Twist**

$+ \quad \ldots$

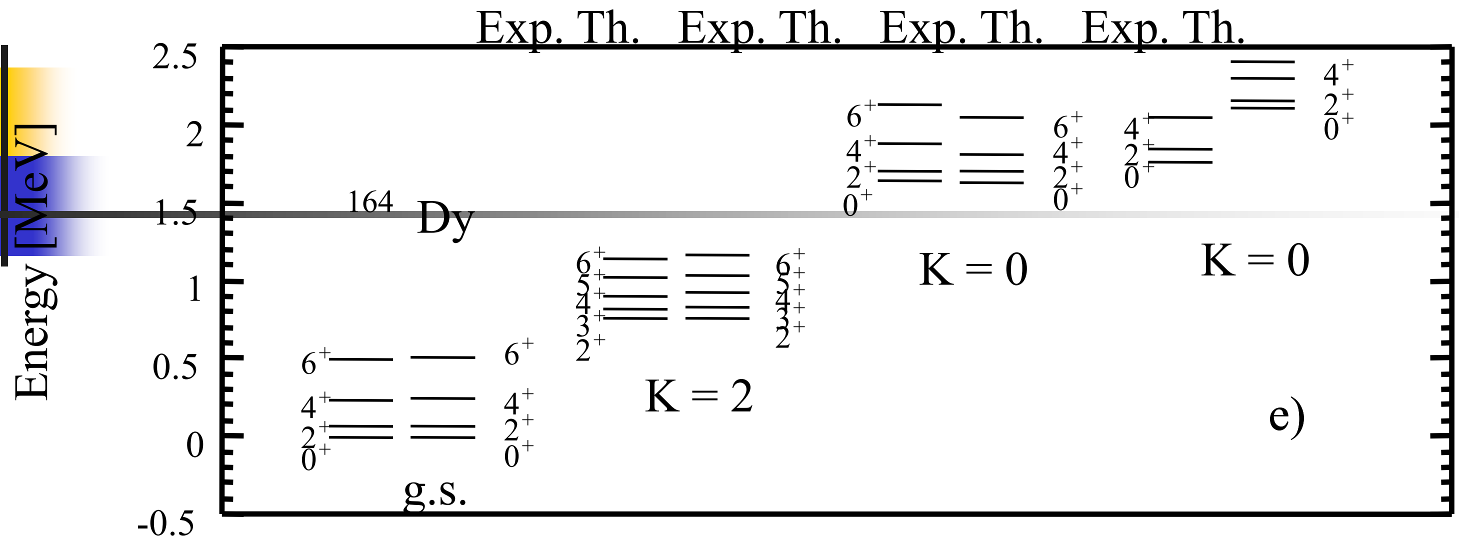
$\rightarrow \quad \sum_{m,l} \oplus (\lambda_\pi + \lambda_\nu - 2m + l, \mu_\pi + \mu_\nu + m - 2l)^k$

★… **Orientation of the π-ν system is quantized with the multiplicity denoted by k = k(m,l)**

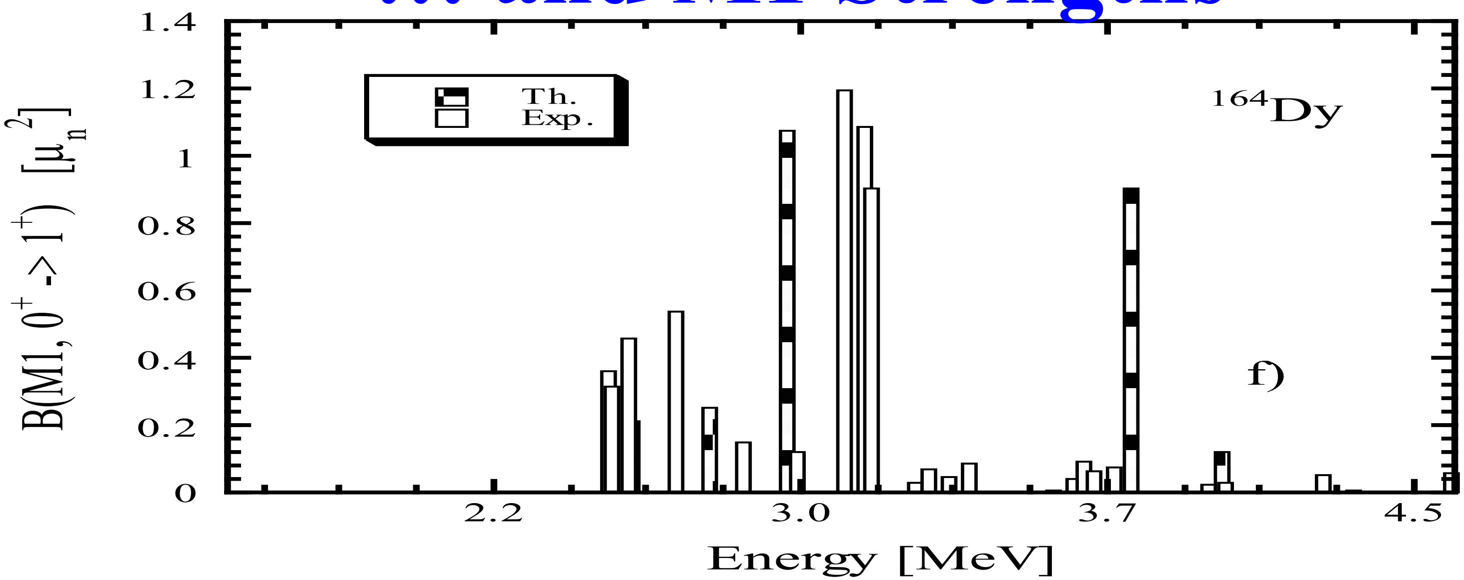
M1 Transition Strengths $[\mu^2_N]$ in the Pure Symmetry Limit of the Pseudo SU(3) Model

Nucleus	$(\lambda_\pi\mu_\pi)$	$(\lambda_\nu\mu_\nu)$	$(\lambda\mu)$	$(\lambda\mu)_{1+}$	B(M1)	mode
^{160,162}Dy	(10,4)	(18,4)	(28,8)	(29, 6)	0.56	t
				(26, 9)	1.77	s
				$(27\ ;\ 7)^1$	1.82	s+t
				$(27\ ;\ 7)^2$	0.083	t+s
^{164} Dy	(10,4)	(20,4)	(30,8)	(31,6)	0.56	t
				(28,9)	1.83	s
				(29,7)	1.88	s+t
				(29,7)	0.09	t+s

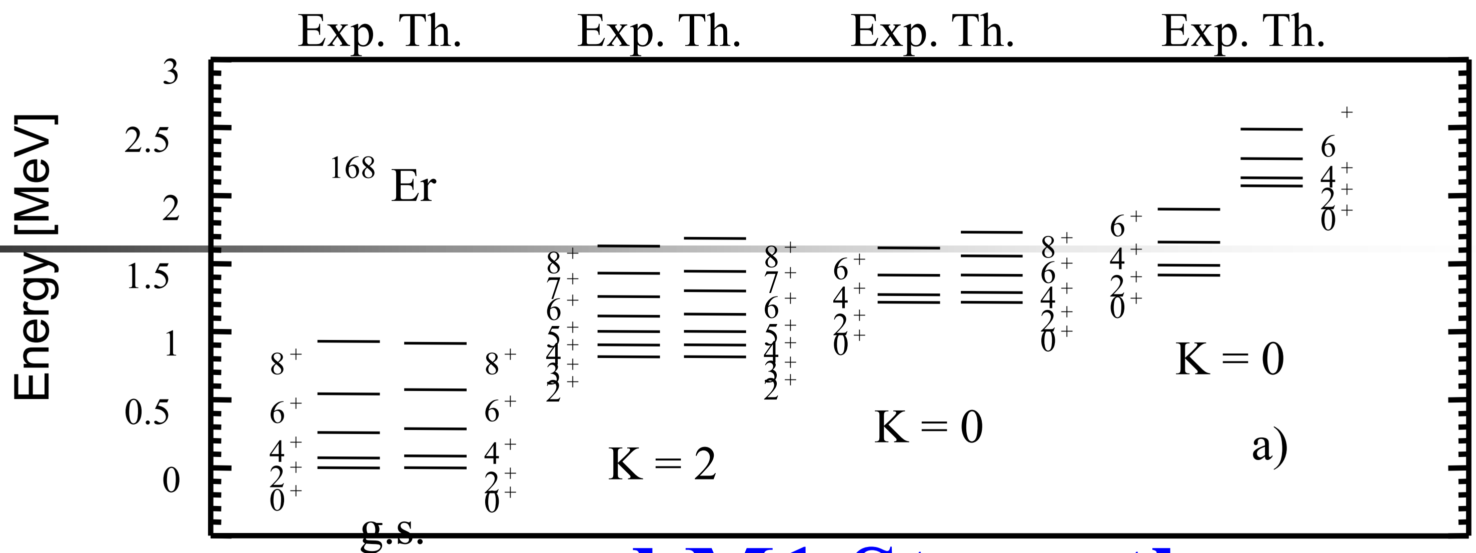
Energy levels of ^{164}Dy



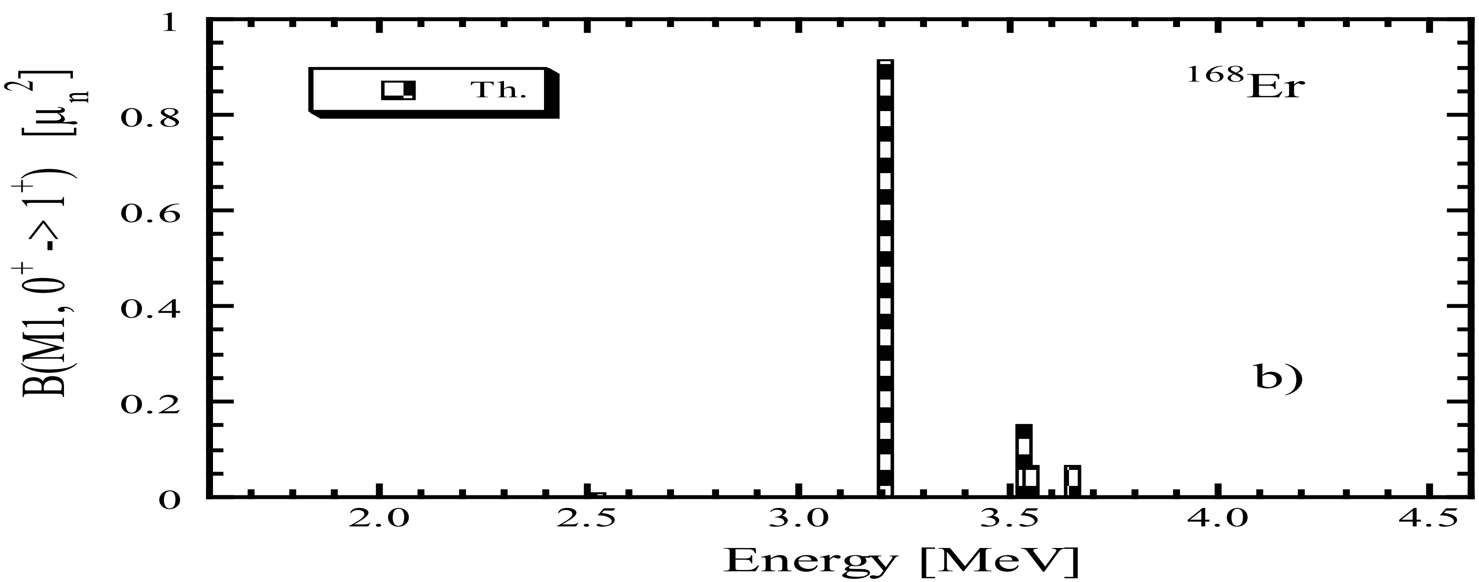
… and M1 Strengths



Energy Levels of ^{168}Er



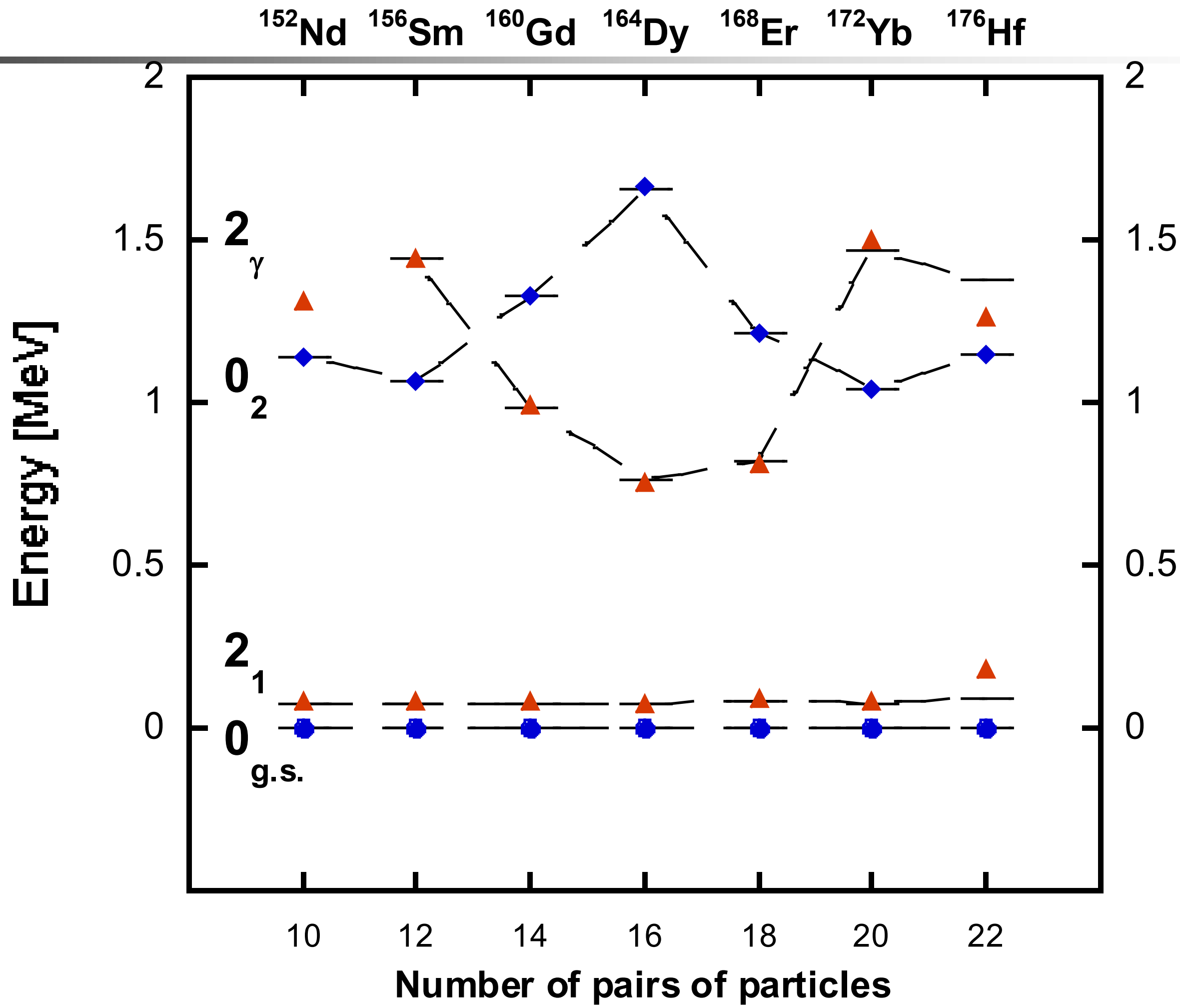
… and M1 Strengths

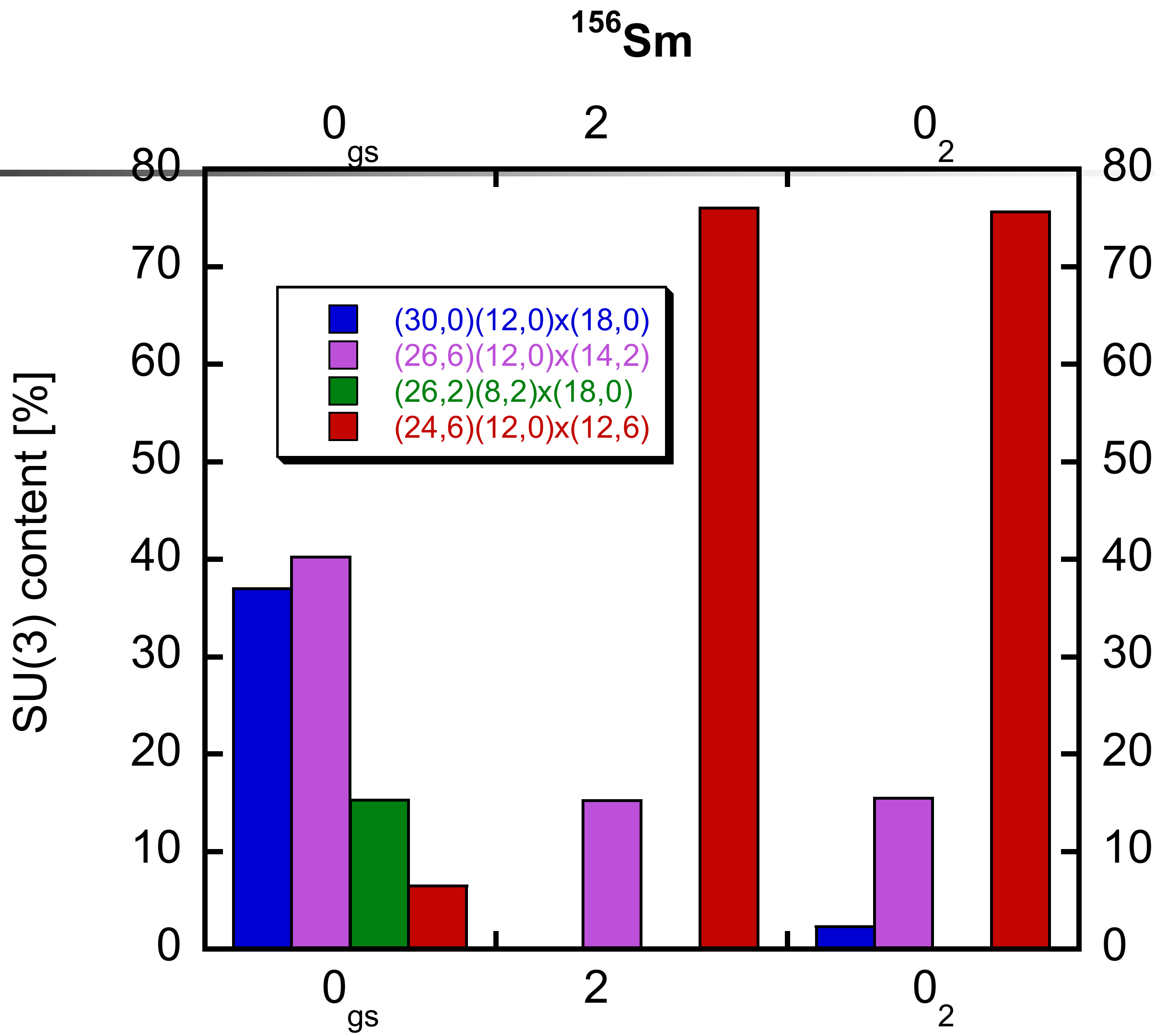


Total B(M1) strength (μ_N^2)

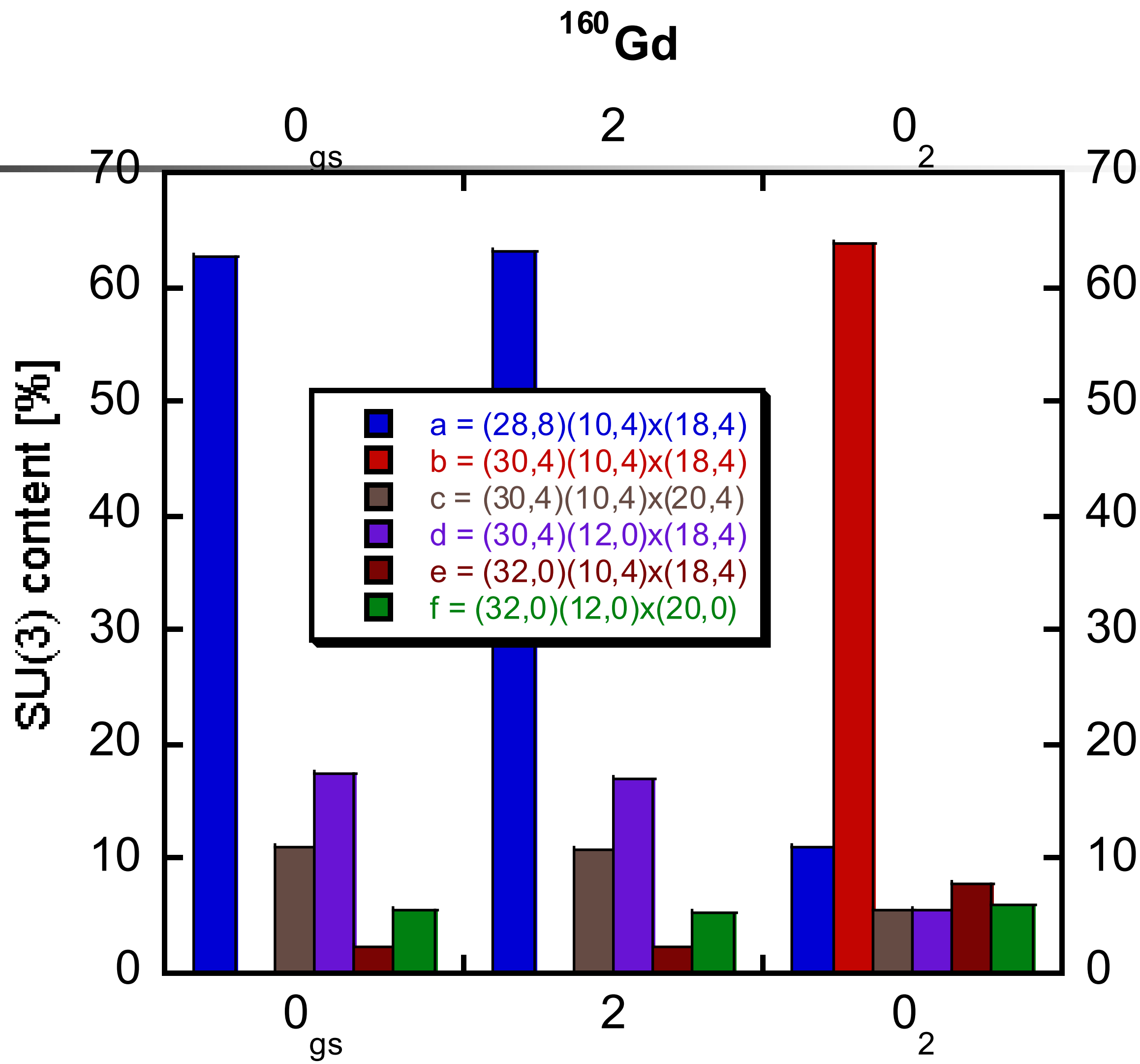
Nucleus	B(M1)[μ_N^2]		
	Experiment	Calculated	
		Pure S U (3)	Theory
^{160}Dy	2.48	4.24	2.32
^{162}Dy	3.29	4.24	2.29
^{164}Dy	5.63	4.36	3.05

First excited K=0⁺ and K=2⁺ states

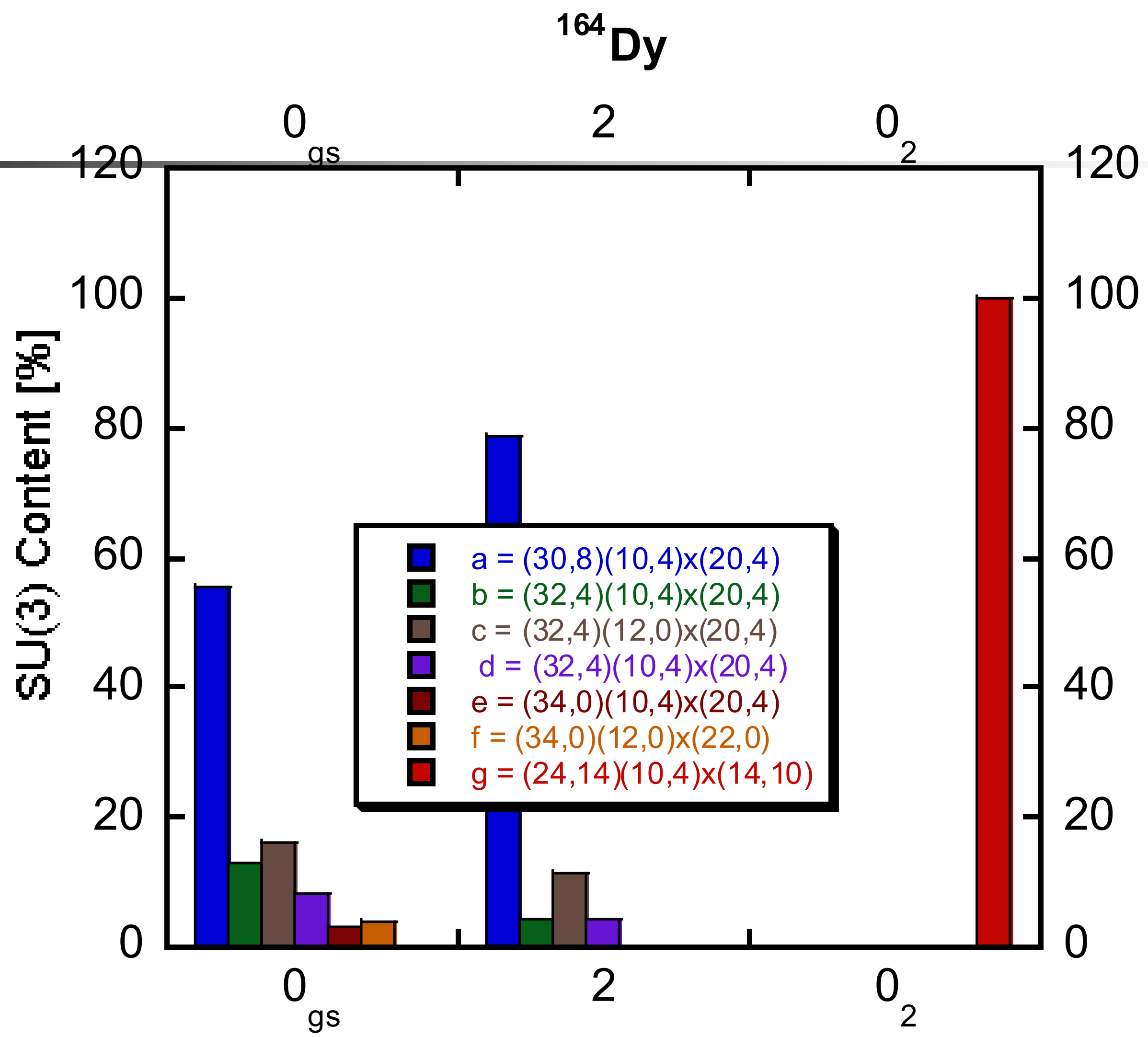




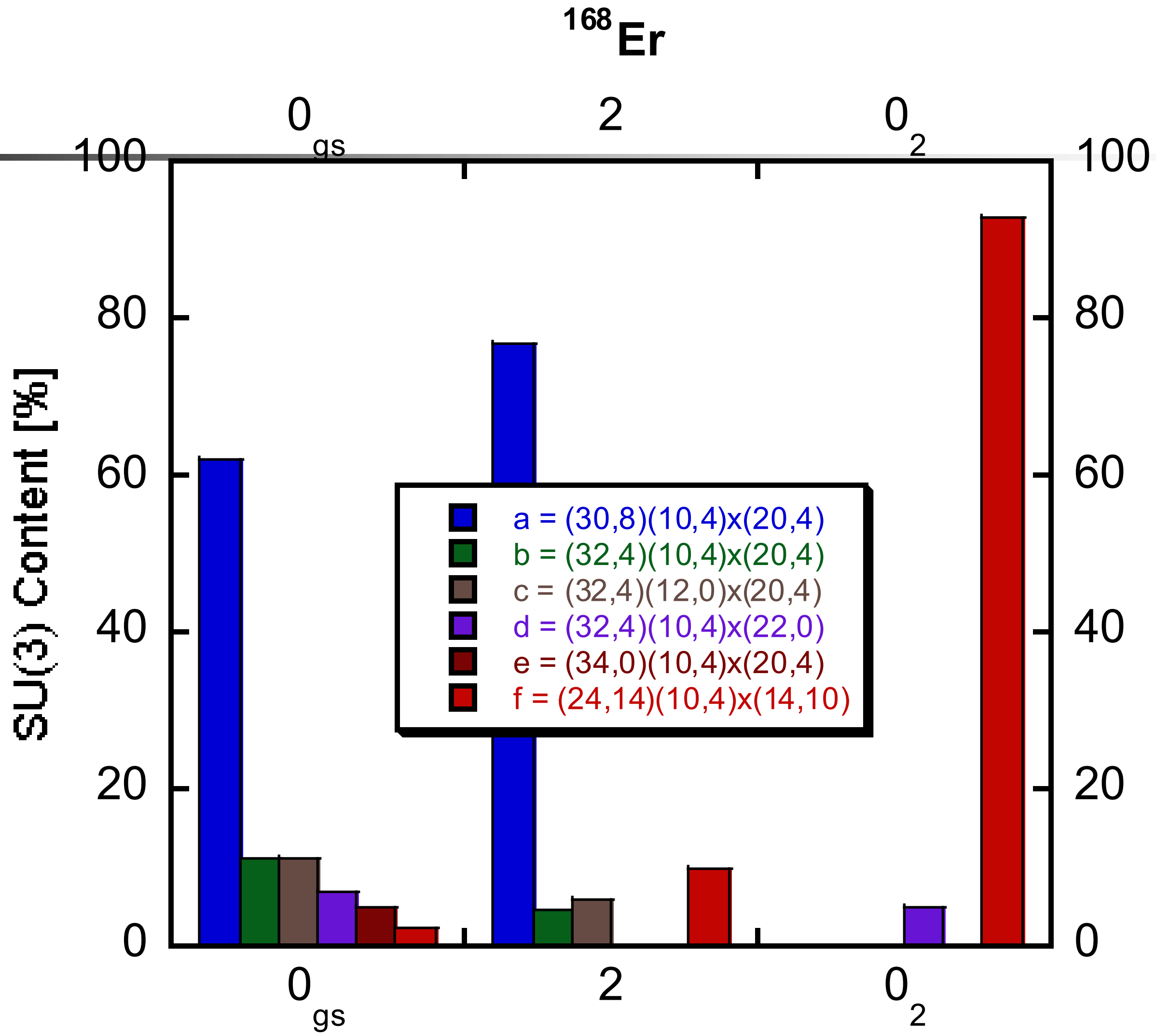
^{156}Sm
0gs
2
02
SU(3) content [%]
(30,0)(12,0)x(18,0)
(26,6)(12,0)x(14,2)
(26,2)(8,2)x(18,0)
(24,6)(12,0)x(12,6)
0
10
20
30
40
50
60
70
80



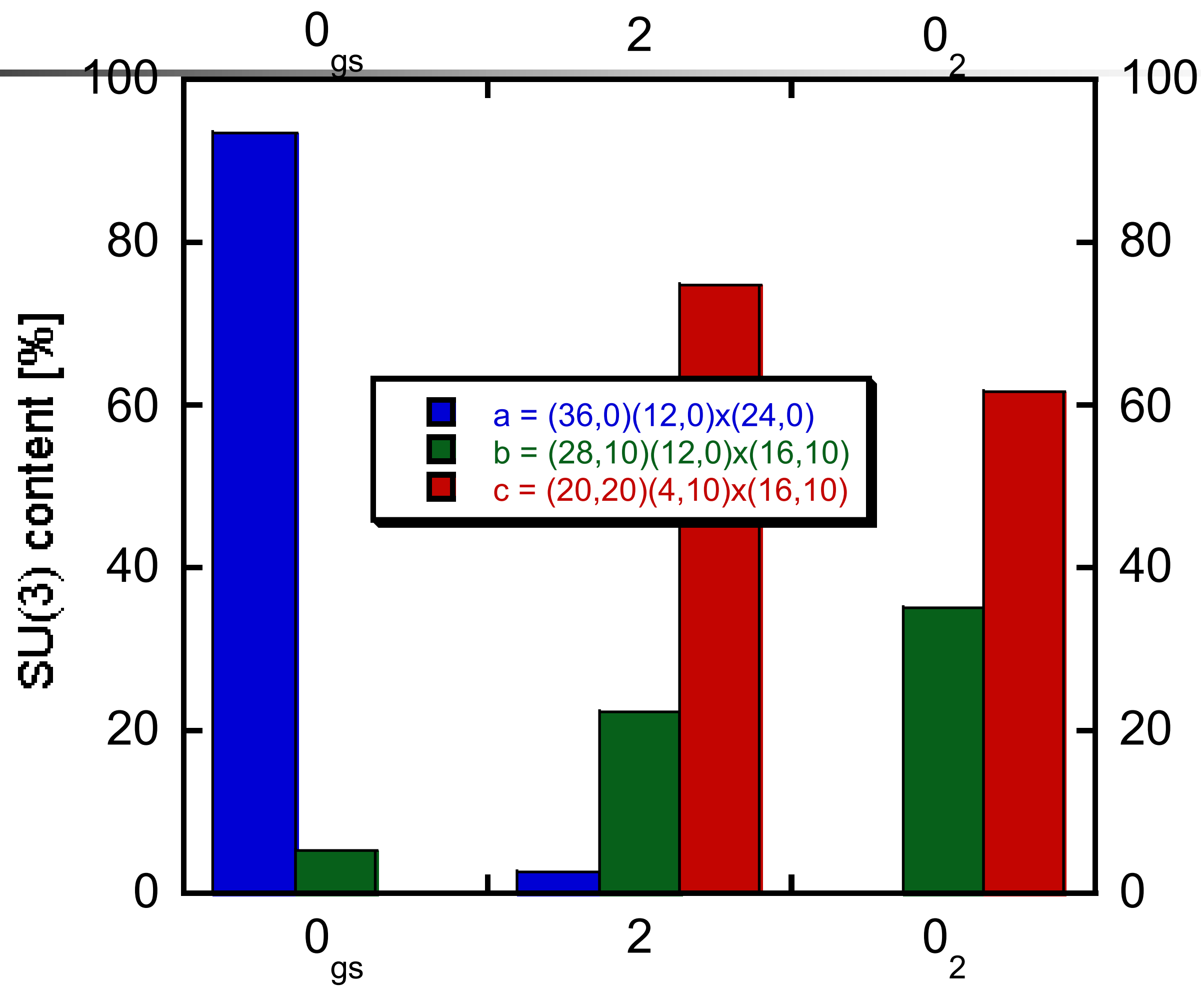
^{160}Gd
0gs
2
02
SU(3) content [%]
0
10
20
30
40
50
60
70
a = (28,8)(10,4)x(18,4)
b = (30,4)(10,4)x(18,4)
c = (30,4)(10,4)x(20,4)
d = (30,4)(12,0)x(18,4)
e = (32,0)(10,4)x(18,4)
f = (32,0)(12,0)x(20,0)



^{164}Dy
SU(3) Content [%]
0 gs
2
0 2
120
100
80
60
40
20
0
a = (30,8)(10,4)x(20,4)
b = (32,4)(10,4)x(20,4)
c = (32,4)(12,0)x(20,4)
d = (32,4)(10,4)x(20,4)
e = (34,0)(10,4)x(20,4)
f = (34,0)(12,0)x(22,0)
g = (24,14)(10,4)x(14,10)


^{168}Er
0_{gs}
2
0_2
100
80
60
40
20
0
SU(3) Content [%]
a = (30,8)(10,4)x(20,4)
b = (32,4)(10,4)x(20,4)
c = (32,4)(12,0)x(20,4)
d = (32,4)(10,4)x(22,0)
e = (34,0)(10,4)x(20,4)
f = (24,14)(10,4)x(14,10)

SU(3) content in ^{172}Yb



Conclusions

- ground state, γ, first and second excited $K = 0^+$ bands well described by a few representations
- •calculated results in good agreement with the low-energy spectra
- •B(E2) transitions within the g.s. band well reproduced
- •1^+ energies fall in the correct energy range

•fragmentation in the B(M1) transition probabilities correctly predicted

Conclusions

- A microscopic interpretation of the relative position of the collective band, as well as that of the levels within the band, follows from an evaluation of the primary SU(3) content of the collective states. The latter are closely linked to nuclear deformation.
  - If the leading configuration is triaxial (nonzero μ), the ground and γ bands belong to the same SU(3) irrep;
  - if the leading SU(3) configuration is axial (μ=0), the K =0 and γ bands come from the same SU(3) irrep.
- A proper description of collective properties of the first excited $K^{\pi} = 0^{+}$ and $K=2^{+}$ states must take into account the mixing of different SU(3)-irreps, which is driven by the Hamiltonian.

Future work

- • In some nuclei total strength of the M1 distribution is larger than the experimental value
- • Consider the states with J = 1 in the configuration space
- • Consider the abnormal parity levels
- • There are new experiments that determine the inter-band B(E2) transitions
  • Improve the model to calculate these transitions
- •Investigate the M1 transitions in light nuclei
- •Investigate the energy spectra in super-heavy nuclei

Thank you!