15thNational Nuclear Physics Summer School

June 15-27, 2003

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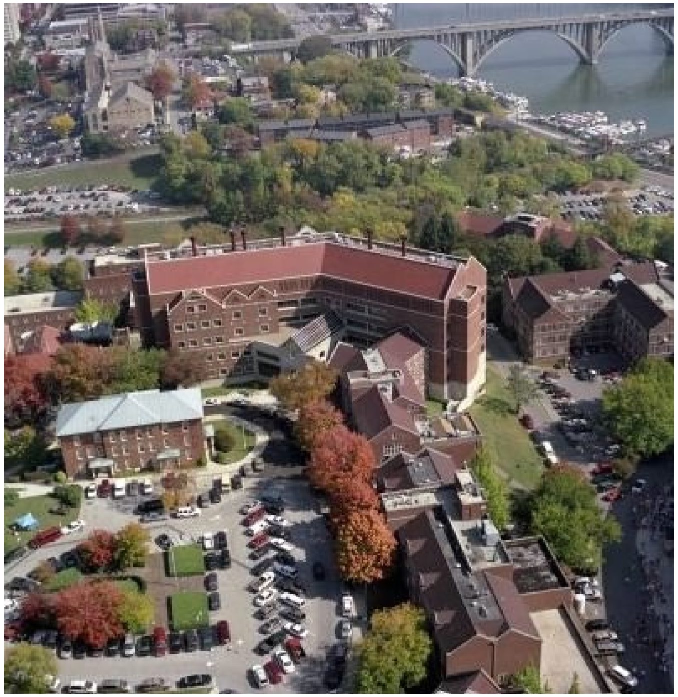

Nuclear Structure

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Lecture #3

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Electro-magnetic transitions

- Ok, now what do I do with the states?
- Well, for one excited states decay!
- Electromagnetic decays
  - Electric multipole (EL)

$$O(ELM) = \sum_{k=1}^{A} e_k r_k^L Y_{LM}(\hat{\mathrm{r}}_k)$$

$$e_p = 1; \qquad e_n = 0 \longleftarrow \boxed{\text{Free charges}}$$

    - ΔJ given by $|J_i - L| \leq J_f \leq J_i + L$
    - Parity change $(-1)^L$
  - Magnetic multipole (ML)

$\longleftarrow$ Free g-factors

    - ΔJ given by $|J_i - L| \leq J_f \leq J_i + L$
    - Parity change $(-1)^{L+1}$

Transition life-times

- Define the "B"-values

Radial wave functions go here!

One-body transition density

- The transition rate is

— **Note the important phase-space factor, E_γ^{2L+1}**

Electromagnetic transitions

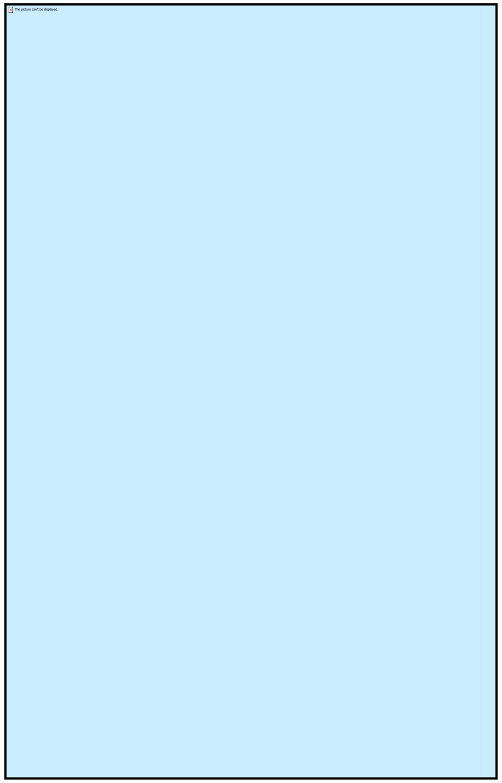
- How well does the shell-model work?
  - Not well at all with free electric charges!
  - Ok, with free g-factors
- So, where did we do wrong?????!!!!!
  - Remember we renormalized the interaction
    - This accounts for excitations not included in the active valence space
  - What about the operators?
    - We also have to renormalize the transition operators!
    - e_p~1.3 and e_n~0.5
    - Free g-factors for M1 transitions are not bad (but some renormalization is needed like adding $[\sigma x Y_2]^1$)
- Only with these renormalized (effective) operators, we can get excellent agreement with experiment

Estimates for electromagnetic transitions

- **Weisskopf estimates**
- **Assume ψ constant over nuclear volume, zero outside**

Estimates for electromagnetic transitions

- **Use the Weisskopf estimates to determine**

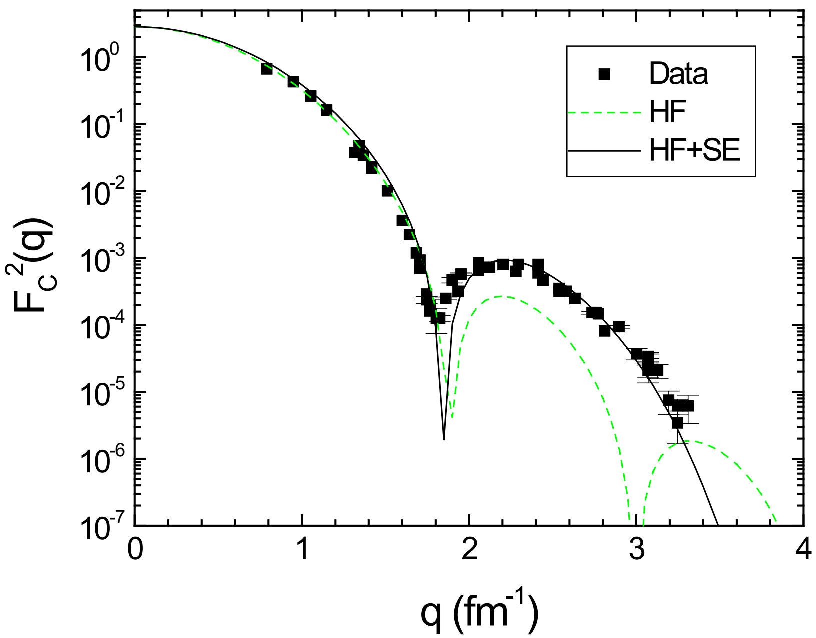


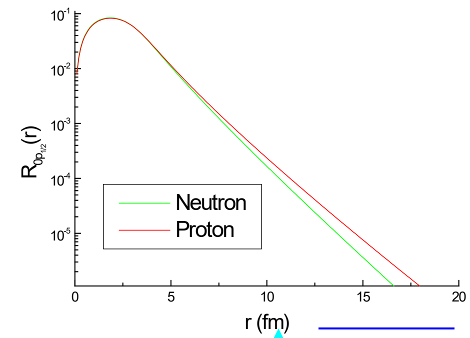
How big are nuclei?

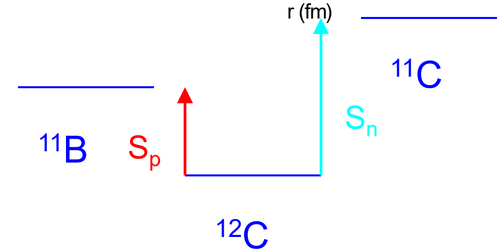
- Electron scattering
  - — Current-current interaction
- Charge form factor

How big are nuclei?

- **Electron scattering**







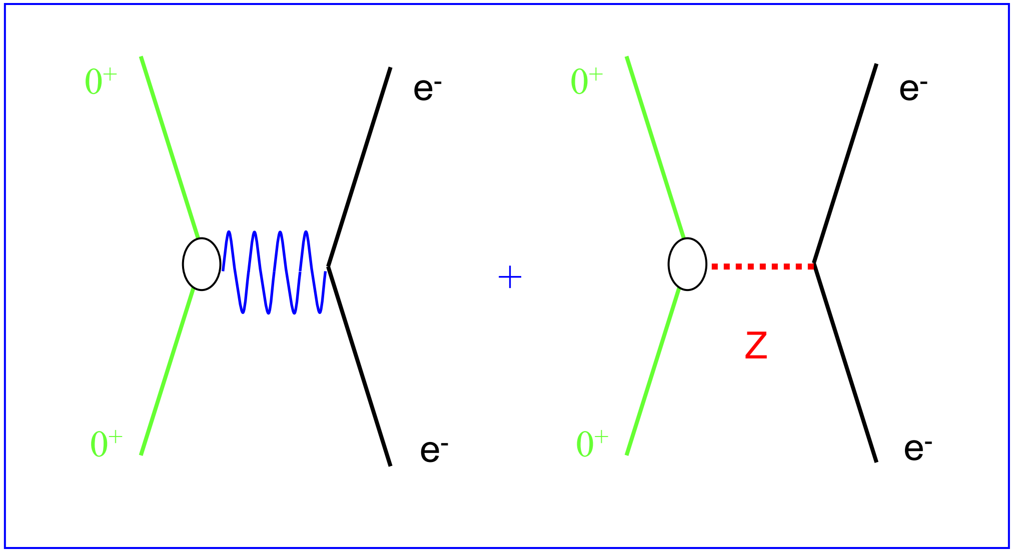
$$n_\mu = \sum_i \langle {}^{12}\mathrm{C} | a_\mu^+ | \psi_i^{A=11} \rangle \langle \psi_i^{A=11} | a_\mu | {}^{12}\mathrm{C} \rangle$$

Separation energy from intermediate state

Determines the asymptotic behavior of the radial wave functions

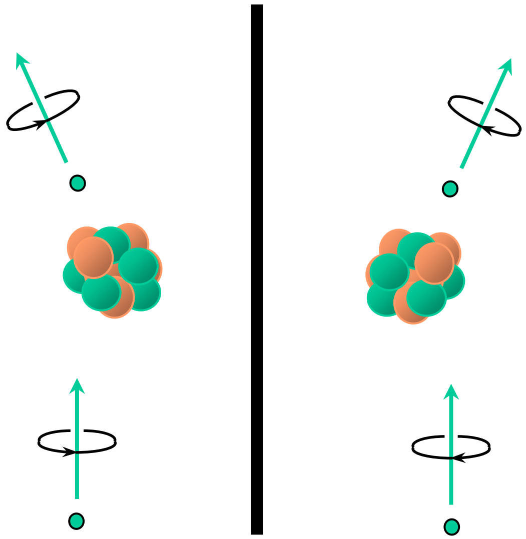
Is there anyway to probe the neutrons?

- Yes, again with electron scattering
  - — But we must look to the parity violating part
  - — Neutral current - Z-boson!



- Parity-violating electron scattering also provides a test of the Standard Model

The Signal



- **PV elastic electron scattering:**

  — **Charges:**

  – **Include intrinsic electric and magnetic form factors**

  – **Neutral current couples more strongly to the neutron distribution**

Nuclear Structure effects

- **With isospin symmetry**

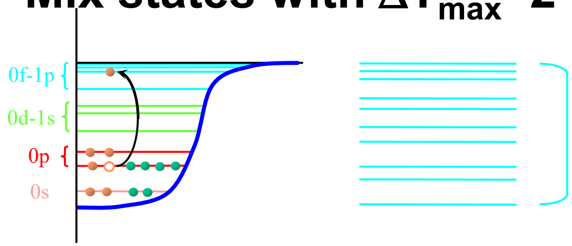
  — **No nuclear structure effects**
    - **low $q \Rightarrow$ 1% measurement of $\sin^2 q_W$**
    - **Deviations are a signature for "new" Physics**
      - **Exotic neutral currents**
      - **Strangeness form factor, $F_C^{(s)}$ (higher q)**

Deviations from q^2 behavior could signal new Physics or be due to Nuclear Physics

Nuclear Structure effects

- **Isospin-symmetry is broken**
  - **— Coulomb interaction (larger)**
  - **— strong interaction (smaller): isotensor or charge-dependent interaction**

    $v^{(pn)} - (v^{(pp)} + v^{(nn)})/2$
- **Mix states with ΔT_{max}=2**



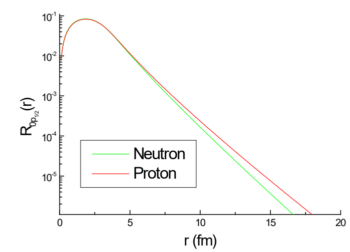
1p-1h Excitations

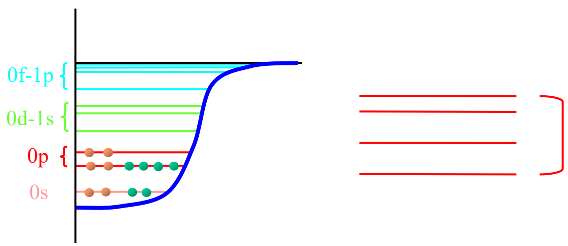
$R_m^{\,p} \neq R_m^{\,n}$

Hartree-Fock

$$V_C(r) = -\frac{1}{2}\frac{(Z-1)e^2}{R^3}r^2 + \frac{3}{2}\frac{(Z-1)e^2}{R}$$

$$V_N(r) = \frac{1}{2}m\omega^2 r^2$$





Valence states

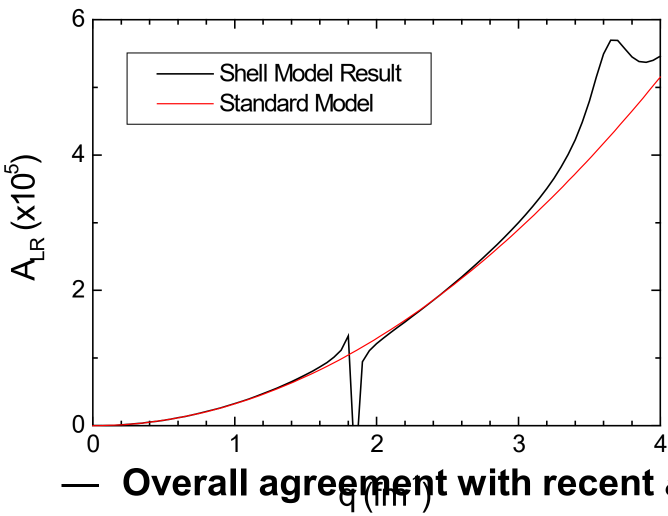
$n_m^{\,p} \neq n_m^{\,n}$

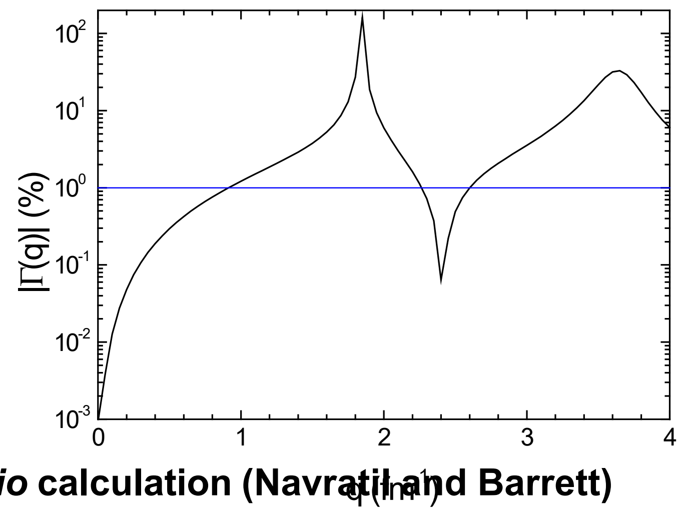
- **Effect on observables:**

Nuclear Structure effects

- **Correction due to breaking of isospin symmetry**

$$A_{LR} = A_{LR}^{0}(q)\left(1+\Gamma(q)\right)$$





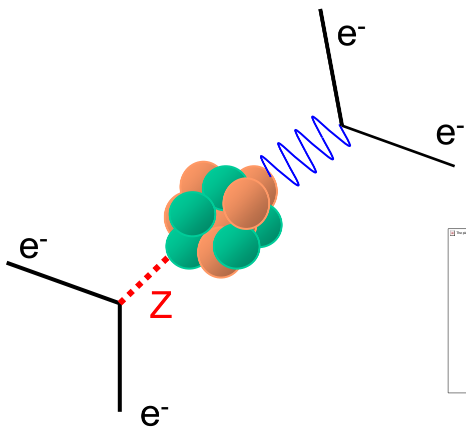
— **Overall agreement with recent *ab initio* calculation (Navratil and Barrett)**

— **G(*q*) < 1% for *q* < 0.9 fm^{-1} (1% measurement for 0.3 < *q* < 1.1 fm^{-1}, Musolf & Donnelly, NPA546, 509 (1992).**

Γ(q) < 1% for q < 0.9 fm^{-1} and q=2.4± 0.1 fm^{-1}

Neutron Radii

- **What good is parity violation?**
  - — **Assume Standard Model correct to 1% level - infer neutron distribution**
- **Very little precision data regarding the distribution of neutrons**
  - — **Useful for mean-field models - improve extrapolation to the drip line**
  - — **Hadron scattering - strong in-medium effects**



Experiments planned for ^{208}Pb at TJNAF

The weak interaction in the shell model

- **β-decay and neutrino absorption**
  - **β-decay**
    - **Partial half-life**
    - **Fermi (F):**
    - **Gamow-Teller (GT):**
      - **g_A=1.2606 $\pm$ 0.0075**
      - **GT is very dependent on model space and shell-model interaction**
        - **Spin-orbit and quasi SU(4) symmetry**
      - **Meson-exchange currents modify B(GT)**
      - **For an effective operator, GT must be renormalized: multiply by ~ 0.75**
    - **Total half life:** **Branching ratio:**

The weak interaction in the shell model
Isospin-symmetry violation

- **Isospin is approximately conserved (~ 1% level)**
- **For transitions, isospin violation enters in two places**
  - **One-body transition density as Ψ no longer has good isospin**

$$\left\langle \Psi_f \left| a_\alpha^+ a_\beta \right| \Psi_i \right\rangle$$

  - **One-body matrix element**

$$\left\langle \alpha \left| \tau_\pm \right| \beta \right\rangle \text{ or } \left\langle \alpha \left| \sigma\tau_\pm \right| \beta \right\rangle$$

    - **Note we have a proton(neutron) converted to a neutron(proton)**
      - **Due to the Coulomb interaction protons and neutrons have different radial wave functions, so we need the overlap:**

$$N_{\alpha\beta} = \int r^2 dr R_\alpha^n R_\beta^p < 1$$

      - **Important for high-precision tests of the vector current (~0.4%)**
      - **For GT this effect can be large, and is essentially contained in the global factor of 0.75 obtained empirically**
      - **Mirror transitions are no longer the same!!!**

Superallowed Fermi β-decay

- **Test of the Standard Model**

- **Cabibbo-Kobayashi-Maskawa matrix**

$$\begin{pmatrix} d_W \\ s_W \\ b_W \end{pmatrix} = \begin{pmatrix} v_{ud} & v_{us} & v_{ub} \\ v_{cd} & v_{cs} & v_{cb} \\ v_{td} & v_{ts} & v_{tb} \end{pmatrix} \begin{pmatrix} d_S \\ s_S \\ b_S \end{pmatrix}$$

$$|v_{ud}|^2 + |v_{us}|^2 + |v_{ub}|^2 = 1$$

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Superallowed Fermi β-decay

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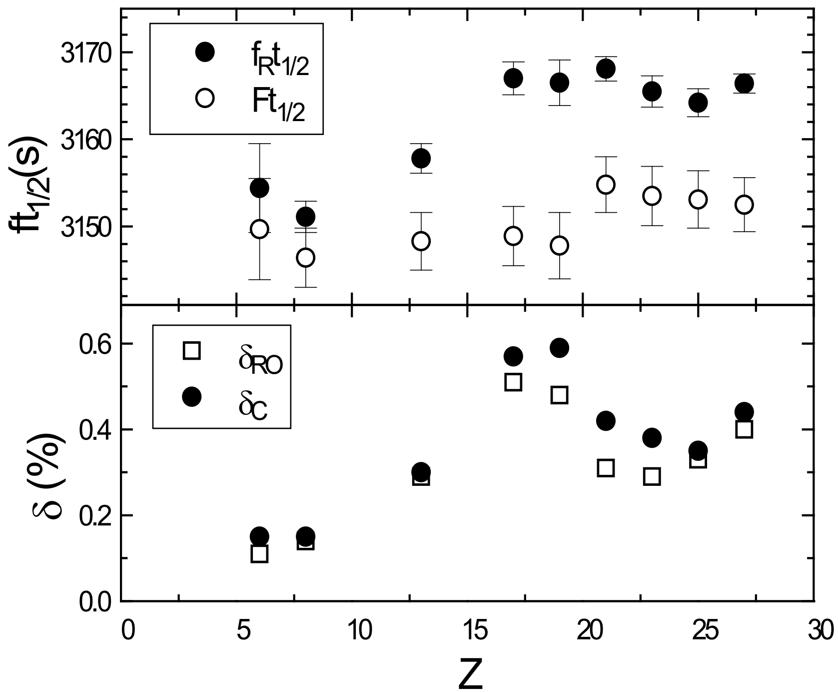
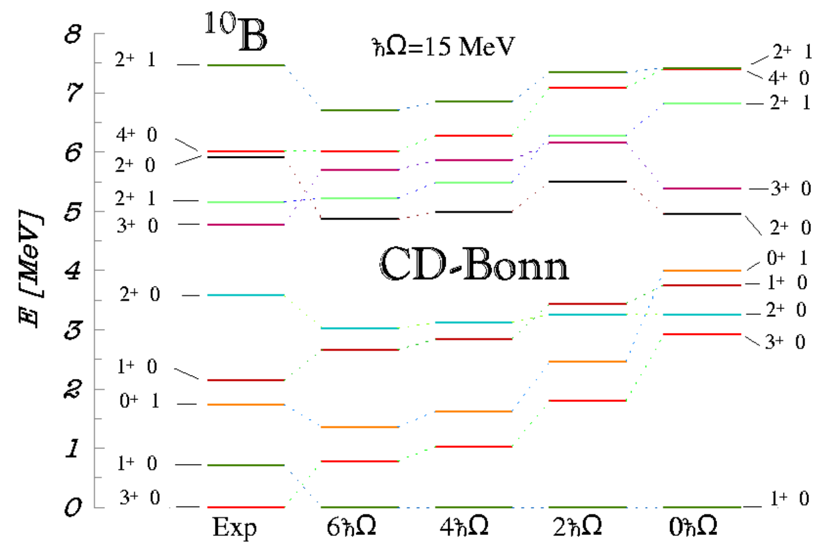
Current Situation

δ_C for ^{10}C: 0.15(9)%



Unitarity condition: 0.9956(8)$_{stat}$(7)$_{sys}$

Ab initio calculation



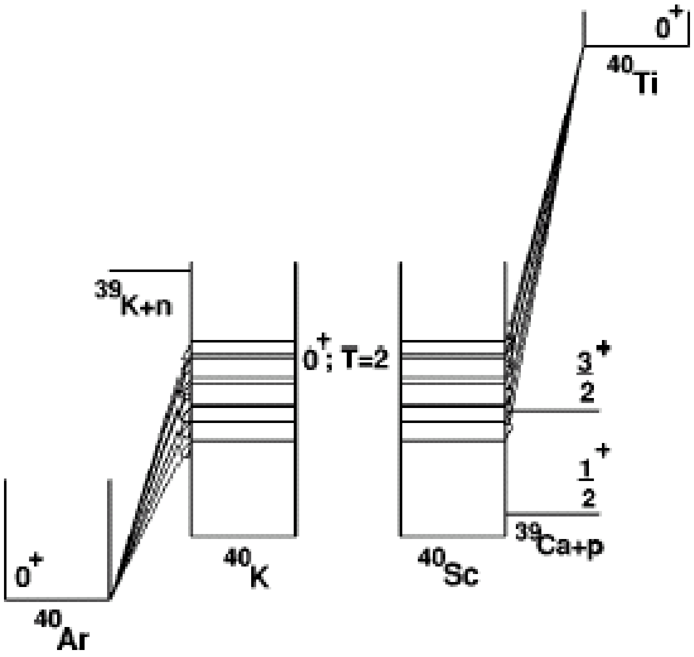
	$0\hbar\Omega$	$2\hbar\Omega$	$4\hbar\Omega$	$6\hbar\Omega$	$8\hbar\Omega$
δ_C(%)	0.0231	0.0466	0.0654	0.0933	0.1211

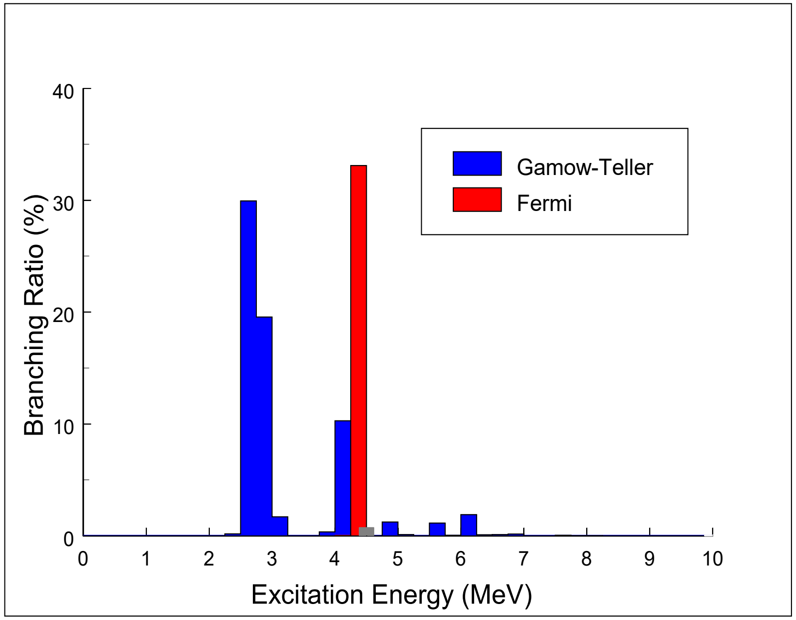
Neutrino absorption

- As for β-decay, neutrino absorption requires Fermi and Gamow-Teller matrix elements
- We carried out calculations for ^{23}Na and ^{40}Ar
  - Ormand et al., PLB308, 207 (1993), Ormand et al., PLB345, 343 (1995)
  - ICARUS and proposed bolometric detectors
- For ^{40}Ar Gamow-Teller is very important
  - Total is twice as large as Fermi contribution
  - Counter to original design assumption
- Can we trust the calculation?
  - β-decay of analog

Checking the calculation

- For ^{40}Ar, look at β-decay of ^{40}Ti
  - β-delayed proton emitter
- Calculated half-life: 55 ± 5 ms
  - Exp: 52.7 ± 1.5 ms and 54 ± 2 ms



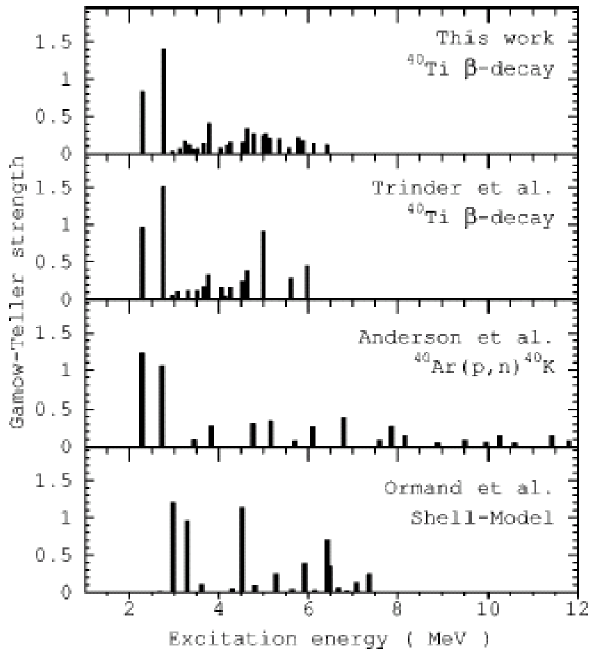


Checking the calculation

- **But there are problems with B(GT) strength**

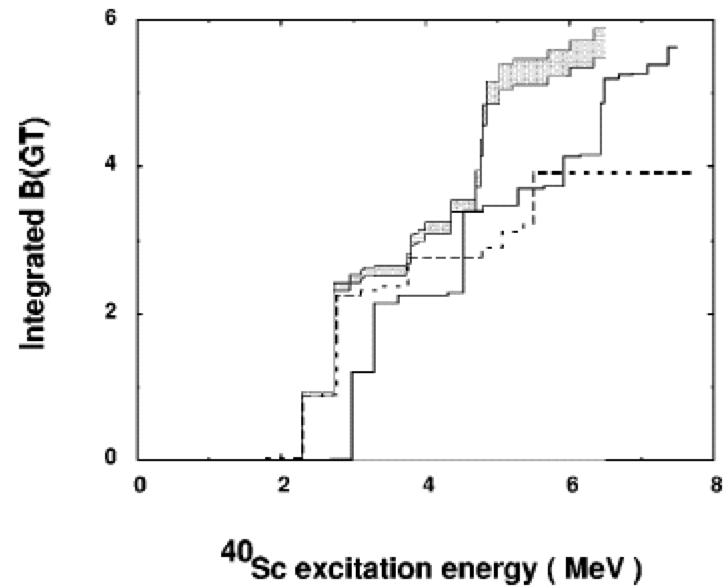
Theory: $\sigma = 11.5 \pm 0.7 \times 10^{-43}$ cm

W. Liu et al., PRC58, 2677 (1998)



$\sigma = 14.3 \pm 0.3 \times 10^{-43}$ cm

M. Bhattacharya et al., PRC58, 3677 (1998)



$\sigma = 14.0 \pm 0.3 \times 10^{-43}$ cm

There is no substitute for experiment when available

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What about heavier nuclei?

- Above A ~ 60 or so the number of configurations just gets to bed too large ~ 10^{10}!
- Here, we need to think of more approximate methods
- The easiest place to start is the mean-field of Hartree-Fock
  - But, once again we have the problem of the interaction
    - Repulsive core causes us no end of grief!!
    - We will, at some point use effective interactions like the Skyrme force

Hartree-Fock

- **There are many choices for the mean field, and Hartree-Fock is one optimal choice**
- **We want to find the best single Slater determinant Φ_0 so that**
- **Thouless' theorem**
  - **Any other Slater determinant Φ not orthogonal to Φ_0 may be written as**
  - **Where i is a state occupied in Φ_0 and m is unoccupied**
  - **Then**

Hartree-Fock

- **Let *i,j,k,l* denote occupied states and *m,n,o,p* unoccupied states**
- **After substituting back we get**

- **This leads directly to the Hartree-Fock single-particle Hamiltonian *h* with matrix elements between any two states α and β**

Hartree-Fock

- We now have a mechanism for defining a mean-field
  - It does depend on the occupied states
  - Also the matrix elements with unoccupied states are zero, so the first order 1p-1h corrections do not contribute
- We obtain an eigenvaule equation (more on this later)
- Energies of A+1 and A-1 nuclei relative to A

$$E_{A+1} - E_A = \varepsilon_m \qquad E_{A-1} - E_A = -\varepsilon_i$$

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Hartree-Fock – Eigenvalue equation

- Two ways to approach the eigenvalue problem
  - Coordinate space where we solve a Schrödinger-like equation
  - Expand in terms of a basis, e.g., harmonic-oscillator wave function
- Expansion
  - Denote basis states by Greek letters, e.g., α
  - From the variational principle, we obtain the eigenvalue equation

Hartree-Fock – Solving the eigenvalue equation

- **As I have written the eigenvalue equation, it doesn't look to useful because we need to know what states are occupied**
- **We use three steps**
  1. **Make an initial guess of the occupied states and the expansion coefficients $C_{i\alpha}$**
     - **For example the lowest Harmonic-oscillator states, or a Woods-Saxon and $C_{i\alpha}=\delta_{i\alpha}$**
  2. **With this ansatz, set up the eigenvalue equations and solve them**
  3. **Use the eigenstates $|i\rangle$ from step 2 to make the Slater determinant Φ_0, go back to step 2 until the coefficients $C_{i\alpha}$ are unchanged**

The Hartree-Fock equations are solved self-consistently

Hartree-Fock – Coordinate space

- **Here, we denote the single-particle wave functions as $\phi_i(\mathrm{r})$**

Direct or Hartree term: U_H

Exchange or Fock term: U_F

- **These equations are solved the same way as the matrix eigenvalue problem before**
  1. **Make a guess for the wave functions $\phi_i(\mathrm{r})$ and Slater determinant Φ_0**
  2. **Solve the Hartree-Fock differential equation to obtain new states $\phi_i(\mathrm{r})$**
  3. **With these go back to step 2 and repeat until $\phi_i(\mathrm{r})$ are unchanged**

Again the Hartree-Fock equations are solved self-consistently

Hartree-Fock

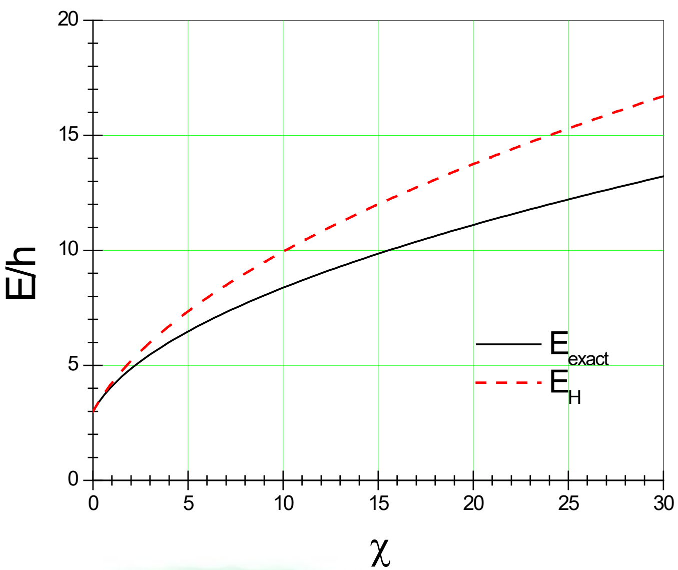
Hard homework problem:

- **M. Moshinsky, Am. J. Phys. 36, 52 (1968). Erratum, Am. J. Phys. 36, 763 (1968).**
- **Two identical spin-1/2 particles in a spin singlet interact via the Hamiltonian**
- **Use the coordinates** $\vec{r} = (\vec{r}_1 - \vec{r}_2)/\sqrt{2}$ **and** $\vec{R} = (\vec{r}_1 + \vec{r}_2)/\sqrt{2}$ **to show the exact energy and wave function are**
- **Note that since the spin wave function (S=0) is anti-symmetric, the spatial wave function is symmetric**

Hartree-Fock

Hard homework problem:

- **The Hartree-Fock solution for the spatial part is the same as the Hartree solution for the S-state. Show the Hartree energy and radial wave function are:**



Hartree-Fock with the Skyrme interaction

- In general, there are serious problems trying to apply Hartree-Fock with realistic NN-interactions (for one the saturation of nuclear matter is incorrect)
- Use an effective interaction, in particular a force proposed by Skyrme

  — P_σ is the spin-exchange operator

- The three-nucleon interaction is actually a density dependent two-body, so replace with a more general form, where α determines the incompressibility of nuclear matter

Hartree-Fock with the Skyrme interaction

- One of the first references: D. Vautherin and D.M. Brink, PRC5, 626 (1972)
- Solve a Shrödinger-like equation

τ_z labels protons or neutrons

— Note the effective mass m^*

— Typically, $m^* < m$, although it doesn't have to, and is determined by the parameters t_1 and t_2

– The effective mass influences the spacing of the single-particle states

– The bias in the past was for m^*/m ~ 0.7 because of earlier calculations with realistic interactions

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Hartree-Fock calculations

- **The nice thing about the Skyrme interaction is that it leads to a computationally tractable problem**
  - **Spherical (one-dimension)**
  - **Deformed**
    - **Axial symmetry (two-dimensions)**
    - **No symmetries (full three-dimensional)**
- **There are also many different choices for the Skyrme parameters**
  - **They all do some things right, and some things wrong, and to a large degree it depends on what you want to do with them**
  - **Some of the leading (or modern) choices are:**
    - **M^*, M. Bartel *et al*., NPA386, 79 (1982)**
    - **SkP [includes pairing], J. Dobaczewski and H. Flocard, NPA422, 103 (1984)**
    - **SkX, B.A. Brown, W.A. Richter, and R. Lindsay, PLB483, 49 (2000)**
    - **Apologies to those not mentioned!**
  - **There is also a finite-range potential based on Gaussians due to D. Gogny, D1S, J. Dechargé and D. Gogny, PRC21, 1568 (1980).**
- **Take a look at J. Dobaczewski *et al*., PRC53, 2809 (1996) for a nice study near the neutron drip-line and the effects of unbound states**

Nuclear structure

- **Remember what our goal is:**
  - **To obtain a quantitative description of all nuclei within a microscopic frame work**
  - **Namely, to solve the many-body Hamiltonian:**

Residual interaction

Perturbation Theory

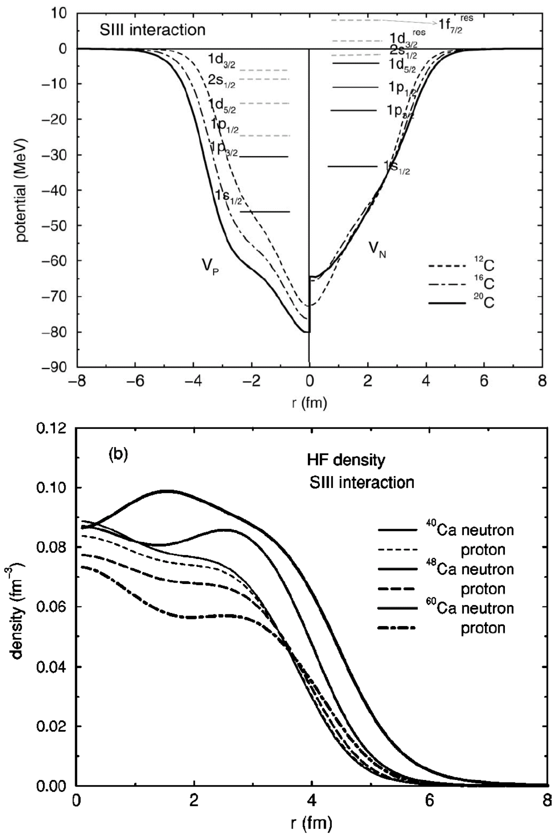
Nuclear structure

- Hartree-Fock is the optimal choice for the mean-field potential *U(r)*!
  - The Skyrme interaction is an "effective" interaction that permits a wide range of studies, e.g., masses, halo-nuclei, etc.
  - Traditionally the Skyrme parameters are fitted to binding energies of doubly magic nuclei, rms charge-radii, the incompressibility, and a few spin-orbit splittings
- One goal would be to calculate masses for all nuclei
  - By fixing the Skyrme force to known nuclei, maybe we can get 500 keV accuracy that CAN be extrapolated into the unknown region
    - This will require some input about neutron densities – parity-violating electron scattering can determine $\langle r^2\rangle_p - \langle r^2\rangle_n$.
  - This could have an important impact

Hartree-Fock calculations

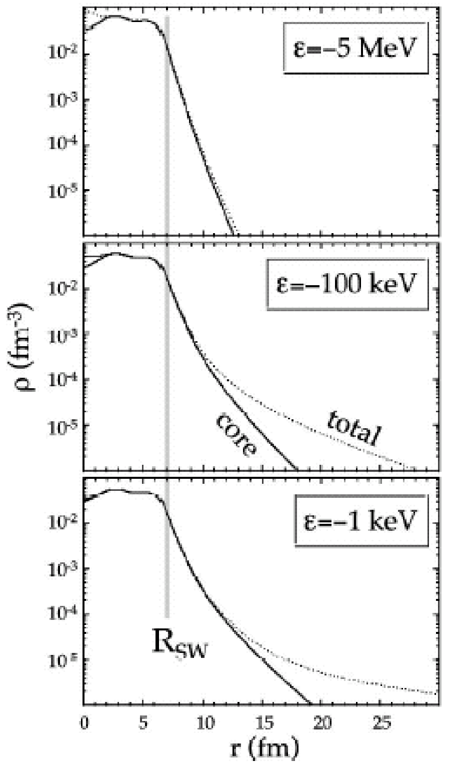
- **Permits a study of a wide-range of nuclei, in particular, those far from stability and with exotic properties, halo nuclei**

H. Sagawa, PRC65, 064314 (2002)



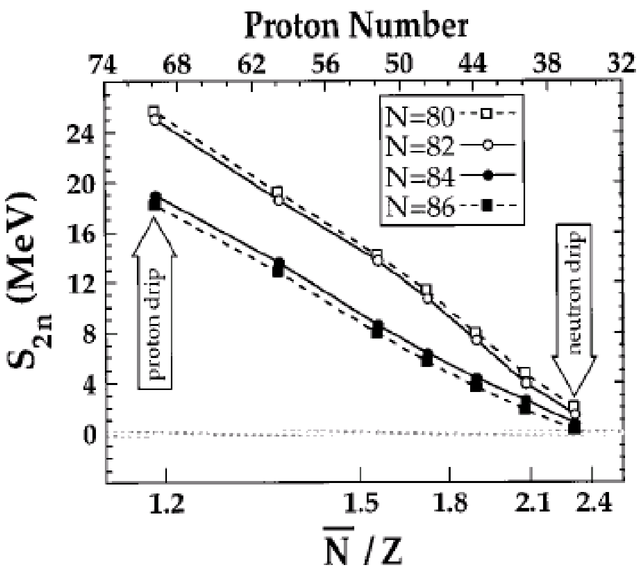
The tail of the radial density depends on the separation energy

S. Mizutori et al. PRC61, 044326 (2000)



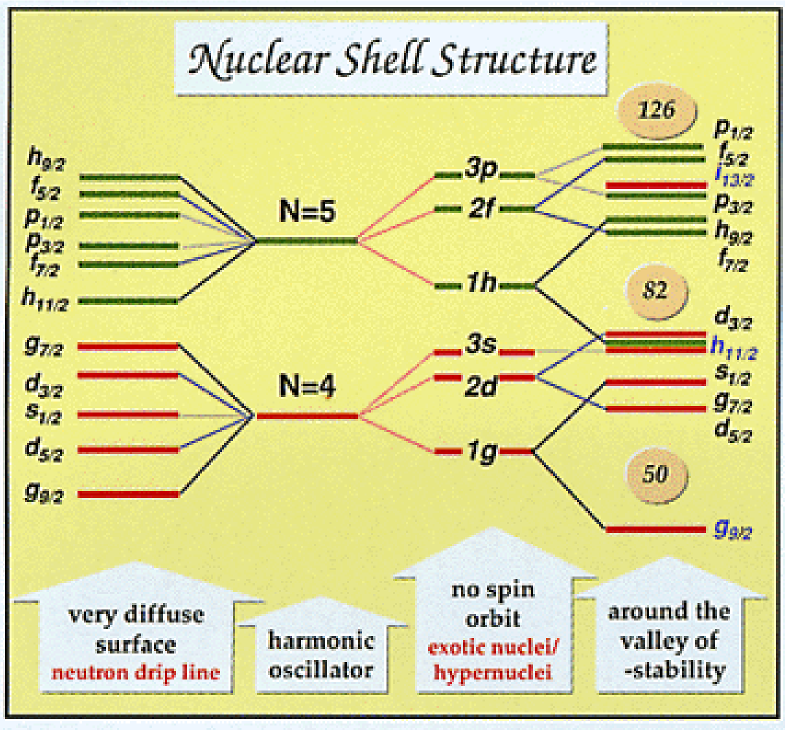
Drip-line studies

J. Dobaczewski *et al.*, PRC53, 2809 (1996)



What can Hartree-Fock calculations tell us about shell structure?

- **Shell structure**
  - **Because of the self-consistency, the shell structure can change from nucleus to nucleus**



J. Dobaczewski *et al.*, PRC53, 2809 (1996)

As we add neutrons, traditional shell closures are changed, and may even disappear!

This is <u>THE</u> challenge in trying to predict the structure of nuclei at the drip lines!

Beyond mean field

- **Hartee-Fock is a good starting approximation**
  - **There are no particle-hole corrections to the HF ground state**

$$\langle m|T|i\rangle + \sum_{j}^{occupied} \langle jm|V|ji\rangle_A = \langle m|h|i\rangle = 0$$

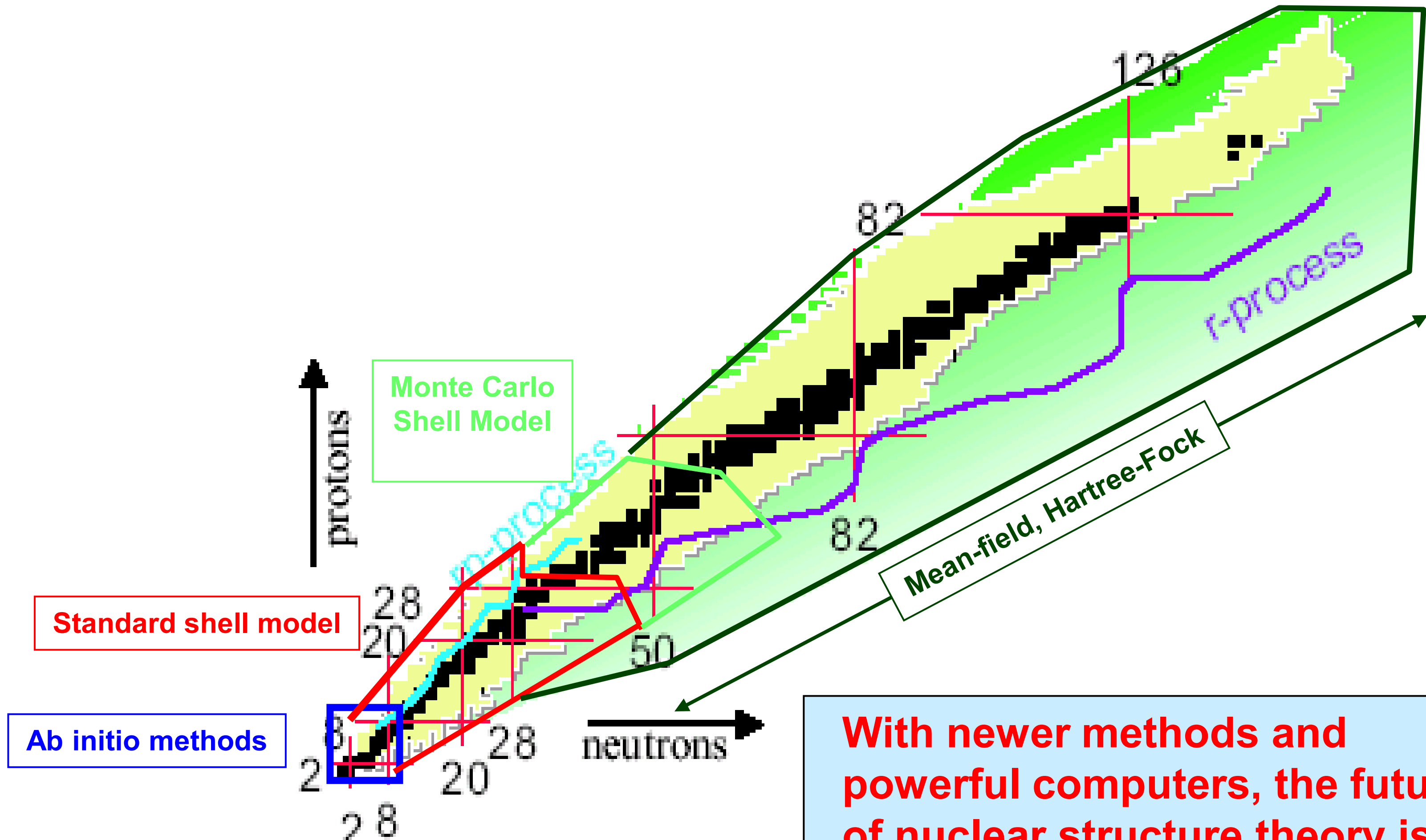
  - **The first correction is**

$$\frac{1}{4}\sum_{ijmn} \frac{\langle ij|V|mn\rangle_A \langle mn|V|ij\rangle_A}{\varepsilon_i + \varepsilon_j - \varepsilon_m - \varepsilon_n}$$

- **However, this doesn't make a lot of sense for Skyrme potentials**
  - **They are fit to closed-shell nuclei, so they effectively have all these higher-order corrections in them!**
- **We can try to estimate the excitation spectrum of one-particle-one-hole states – Giant resonances**
  - **Tamm-Dancoff approximation (TDA)**
  - **Random-Phase approximation (RPA)**

You should look these up!
A Shell Model Description of Light Nuclei, I.S. Towner
The Nuclear Many-Body Problem, Ring & Schuck

Nuclear structure in the future



With newer methods and powerful computers, the future of nuclear structure theory is bright!