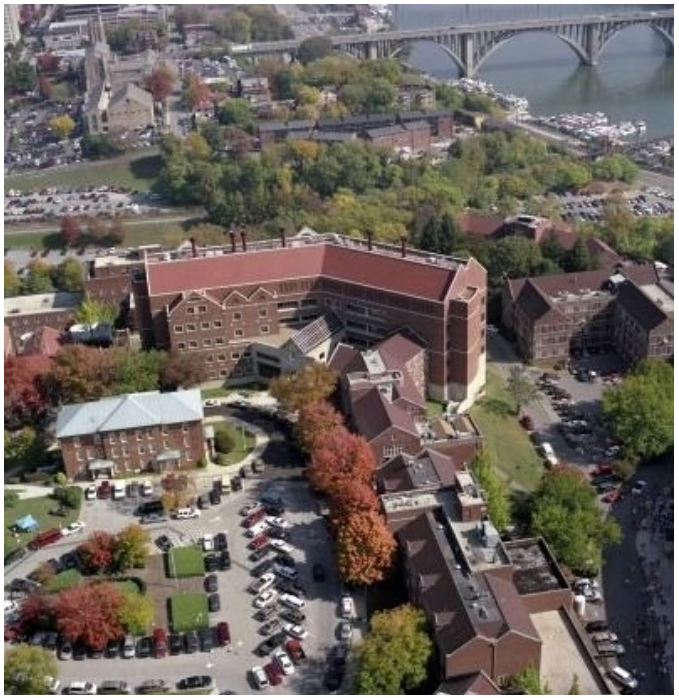
15thNational Nuclear Physics Summer School

June 15-27, 2003



OAK RIDGE NATIONAL LABORATORY

Nuclear Structure

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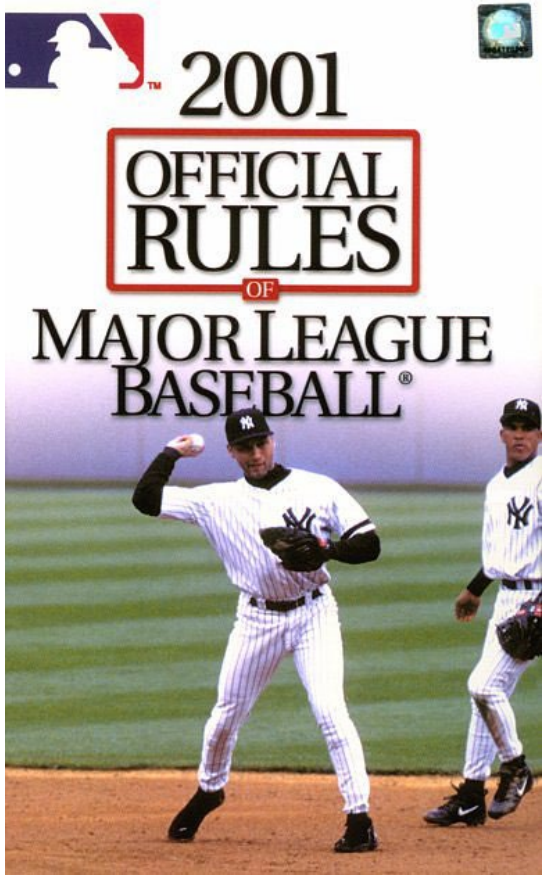
Physics & Advanced Technologies N Division

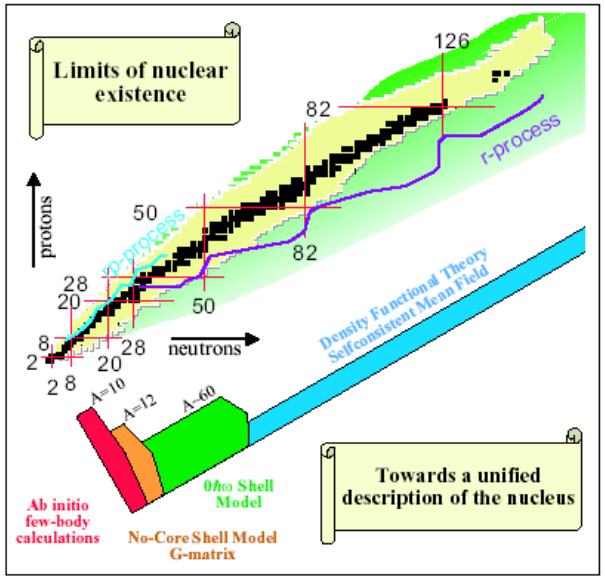
Nuclear Structure

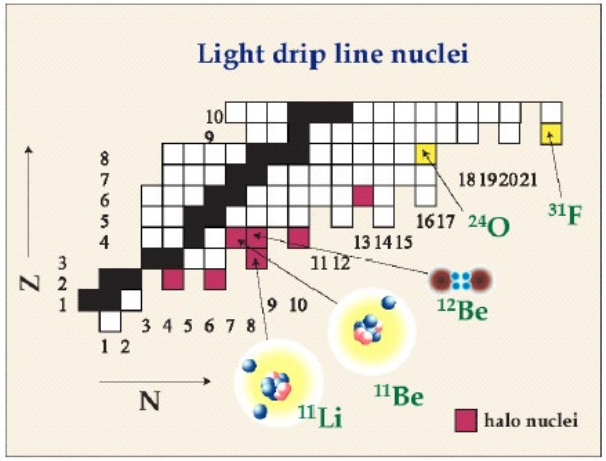
- Of course, I can't cover everything in just five hours
- At least we speak a common language. It could be worse:

I could have to explain the rules to baseball!



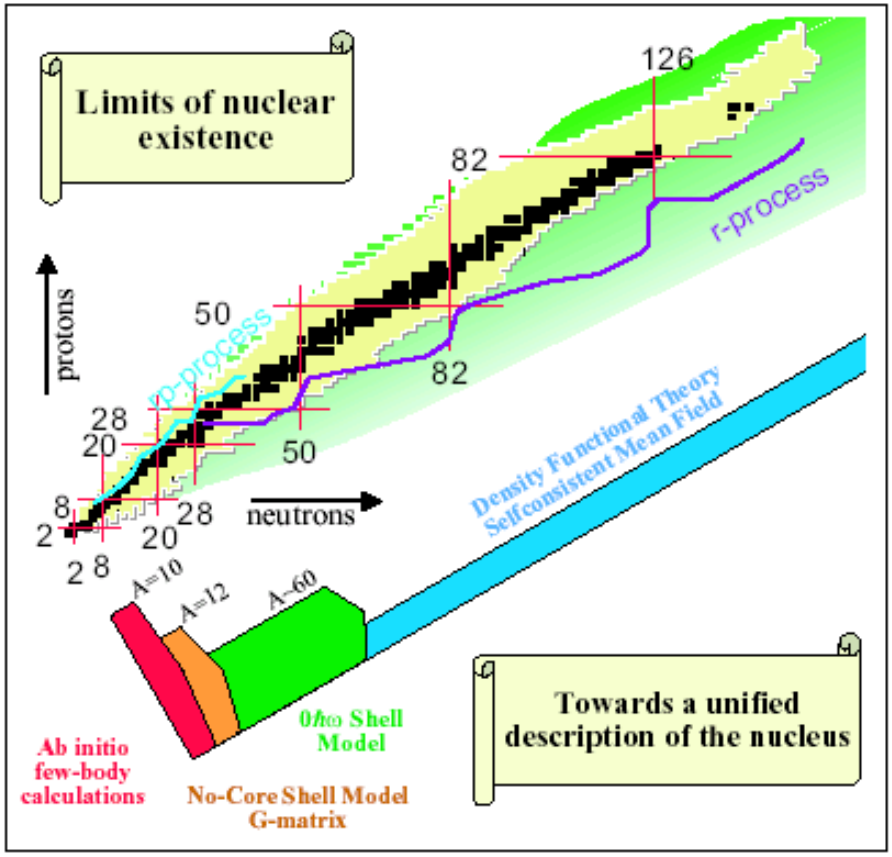
Nuclear Structure





- Nuclear physics is something of a mature field, but there are still many unanswered questions about nuclei.
- The nuclear many-body problem is one of the hardest problems in all of physics!
- Do we really know how they are put together?
  - This is a fundamental question in nuclear physics and we are now getting some interesting answers – for example, three-nucleon forces are important for structure
- Atomic nuclei make up the vast majority of matter that we can see (and touch). How did they (and we) get there?
  - One of the key questions in science
  - Nucleosynthesis
  - Structure plays an important role

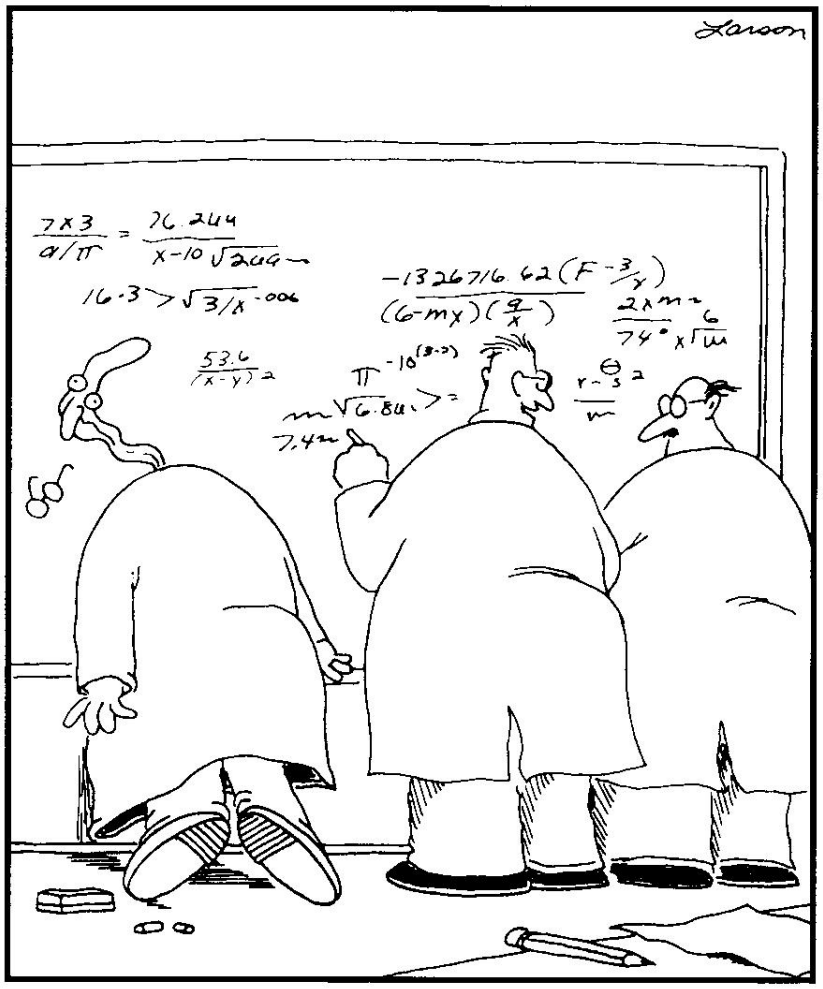
Nuclear Structure



- Exact methods exist up to A=4
- Computationally exact methods for A up to 16
- Approximate many-body methods for A up to 60
- Mostly mean-field pictures for A greater than 60 or so

Nuclear Structure

- This talk will basically be a theory talk
- So, there wil



"Ha! Webster's blown his cerebral cortex."

Before we get started: Some useful reference materials I

General references for nuclear-structure physics:

- ***Angular Momentum in Quantum Mechanics***, **A.R. Edmonds, (Princetion Univ. Press, Princeton, 1968)**
- ***Structure of the Nucleus***, **M.A. Preston and R.K. Bhaduri, (Addison-Wesley,Reading, MA, 1975)**
- ***Nuclear Models***, **W. Greiner and J.A. Maruhn, (Springer Verlag, Berlin, 1996)**
- ***Basic Ideas and Concepts in Nuclear Physics***, **K. Heyde (IoP Publishing, Bristol, 1999)**
- ***Nuclear Structure vols. I & II***, **A. Bohr and B. Mottelson, (W.A. Benjamin, New York, 1969) &**
- ***Nuclear Theory, vols. I-III***, **J.M. Eisenberg and W. Greiner, (North Holland, Amsterdam, 1987)**

Before we get started: Some useful reference materials II

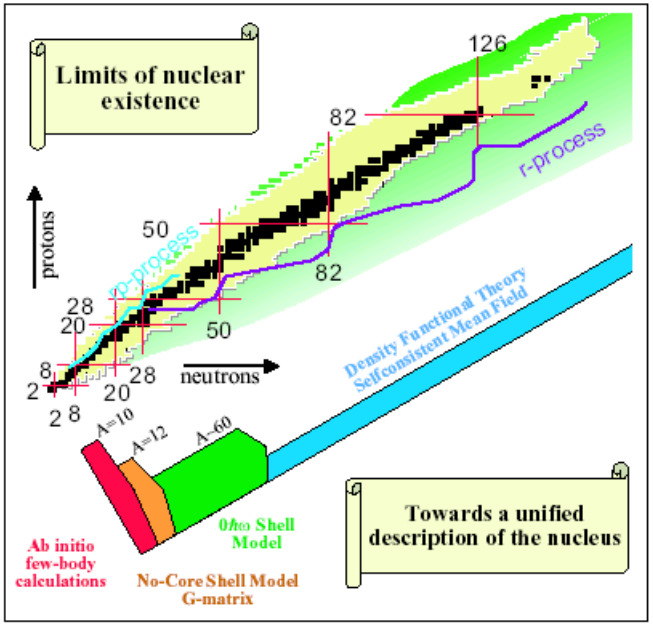
References for many-body problem:

- ***Shell Model Applications in Nuclear Spectroscopy*, P.J. Brussaard and P.W.M. Glaudemans, (North Holland, Amsterdam)**
- ***The Nuclear Many-Body problem,* P. Ring and P. Schuck, (Springer Verlag, Berlin, 1980)**
- ***Theory of the Nuclear Shell Model*, R.D. Lawson, (Clarendon Press, Oxford, 1980)**
- ***A Shell Model Description of Light Nuclei,* I.S. Towner, (Clarendon Press, Oxford, 1977)**

Review for applying the shell model near the drip lines:

- ***The Nuclear Shell Model Towards the Drip Line*, B.A. Brown, Progress in Particle and Nuclear Physics 47, 517 (2001)**

Nuclear masses, what nuclei exist?



One goal in theory is to accurately describe the binding energy

$$M(Z,N,A) = Zm_h + Nm_n - BE(Z,N,A)$$

- **Let's start with the semi-empirical mass formula, Bethe-Weizsäcker formual, or also the liquid-drop model.**

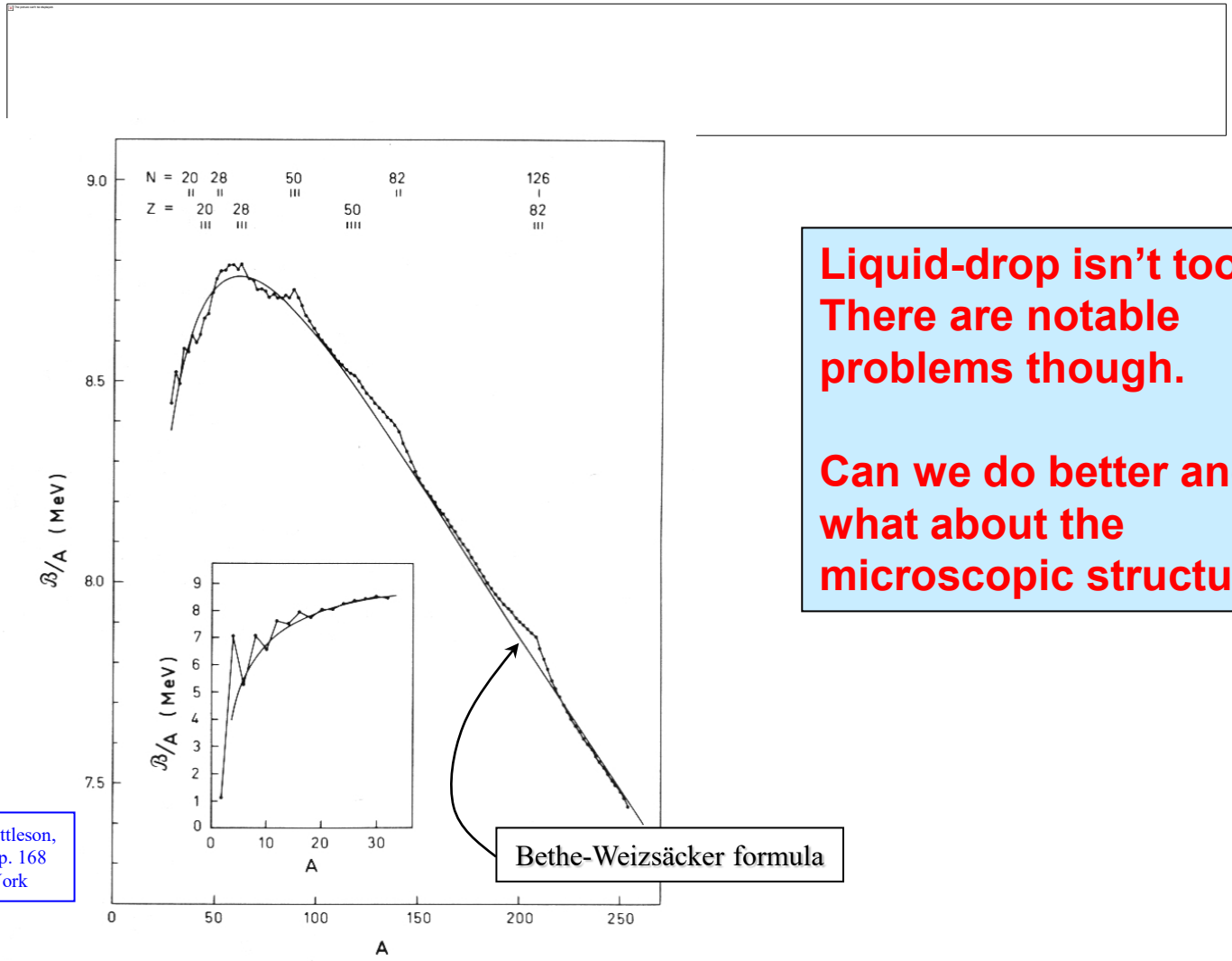
— **There are global Volume, Surface, Symmetry, and Coulomb terms**

— **And specific corrections for each nucleus due to pairing and shell structure**

$$a_V = 15.85\,\text{MeV}$$
$$a_{Surf} = 18.34\,\text{MeV}$$
$$a_{Symm} = 23.21\,\text{MeV}$$
$$a_{Coul} = 0.71\,\text{MeV}$$

Values for the parameters, A.H. Wapstra and N.B. Gove, Nulc. Data Tables 9, 267 (1971)

Nuclear masses, what nuclei exist?



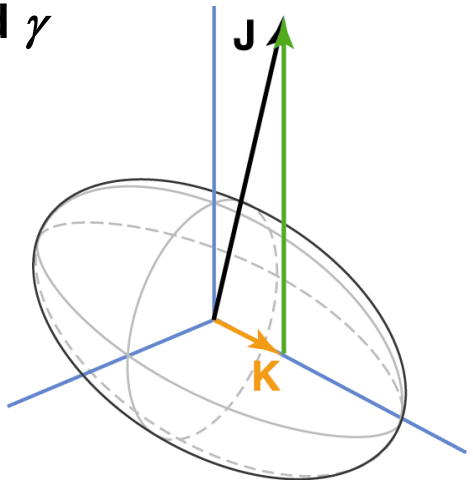
From A. Bohr and B.R. Mottleson, *Nuclear Structure*, vol. 1, p. 168 Benjamin, 1969, New York

Liquid-drop isn't too bad! There are notable problems though.

Can we do better and what about the microscopic structure?

Nuclear masses, what nuclei exist?

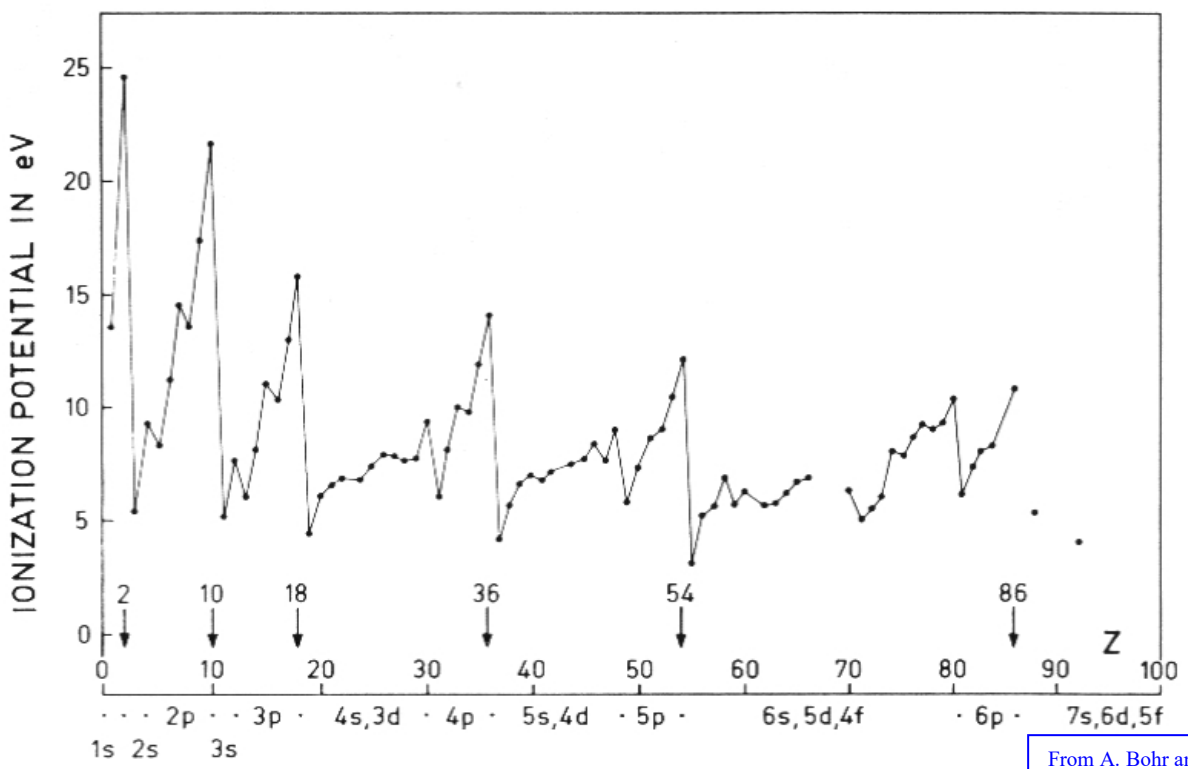
- How about deformation?
- For each energy term, there are also shape factors dependent on the quadrupole deformation parameters β and γ



Note that the liquid drop always has a minimum for a spherical shape!
So, where does deformation come from?

Shell structure - evidence in atoms

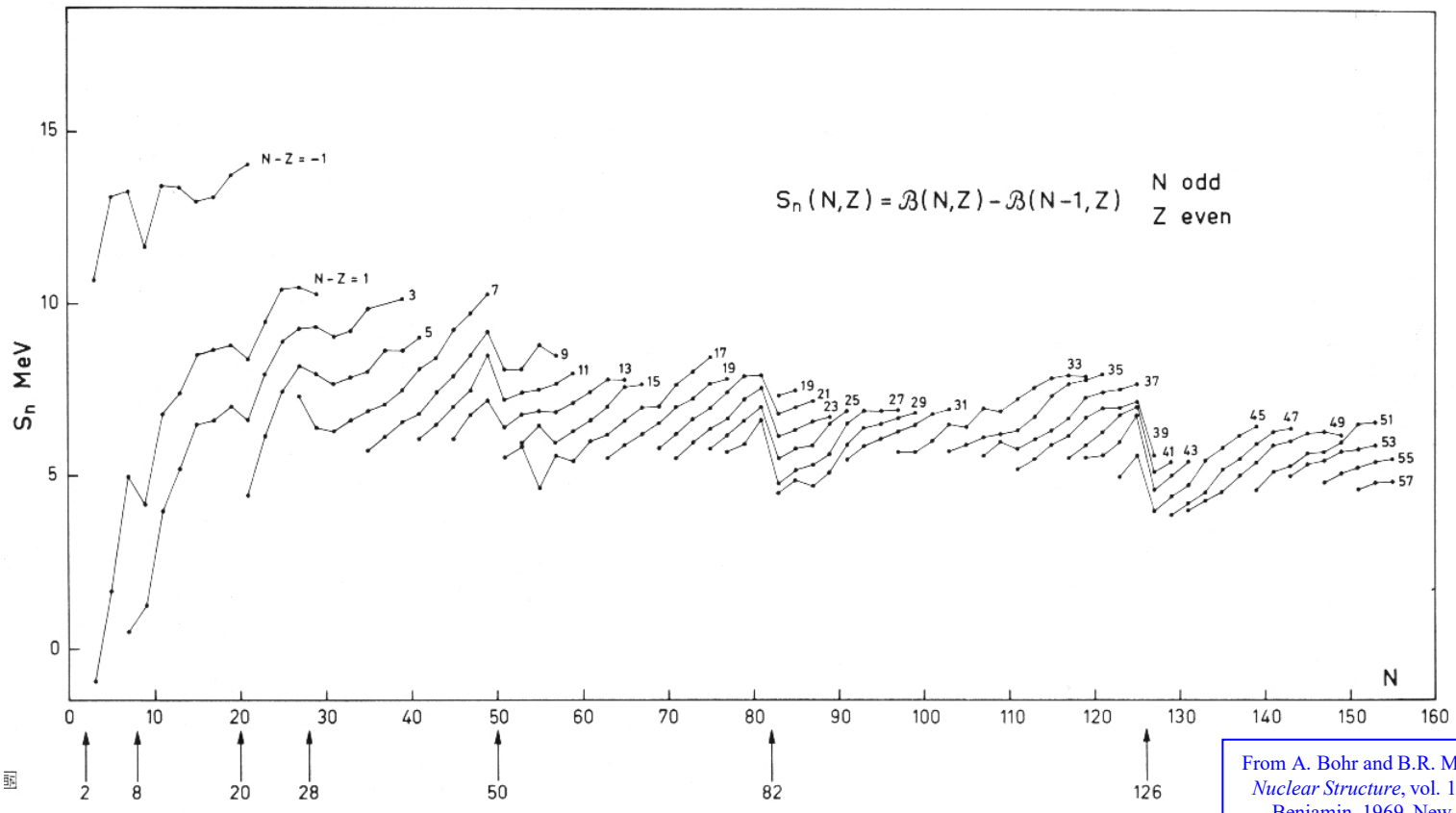
- **Atomic ionization potentials show sharp discontinuities at shell boundaries**



From A. Bohr and B.R. Mottleson, *Nuclear Structure*, vol. 1, p. 191 Benjamin, 1969, New York

Shell structure - neutron separation energies

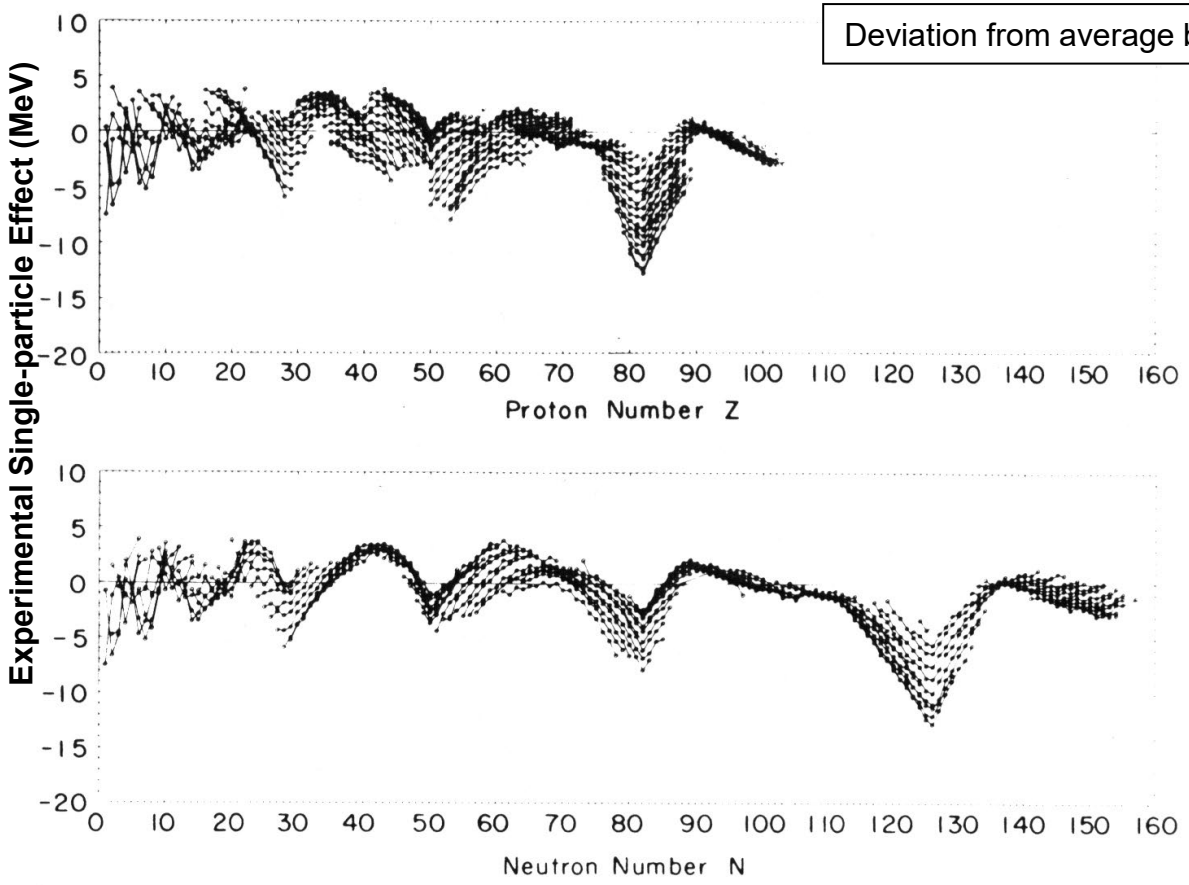
- So do neutron separation energies



From A. Bohr and B.R. Mottleson, *Nuclear Structure*, vol. 1, p. 193 Benjamin, 1969, New York

More evidence of shell structure

- **Binding energies show preferred magic numbers**
  - **2, 8, 20, 28, 50, 82, and 126**

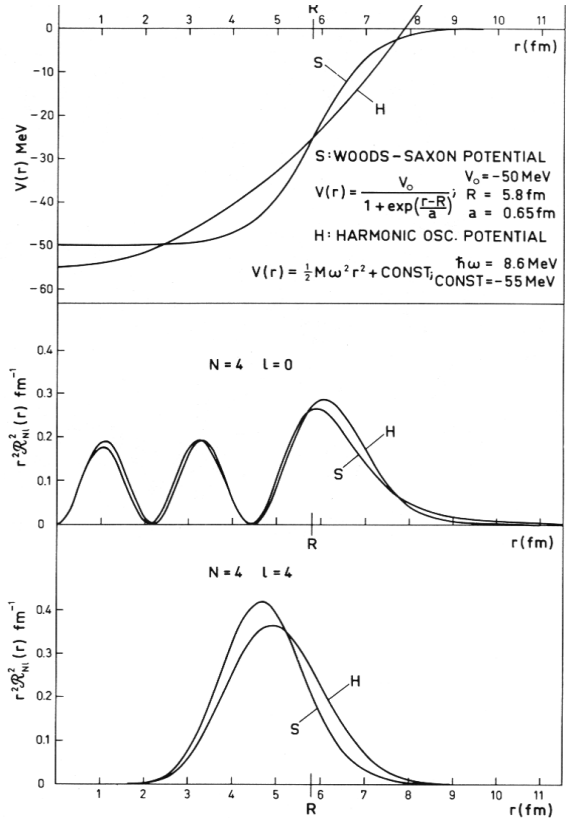


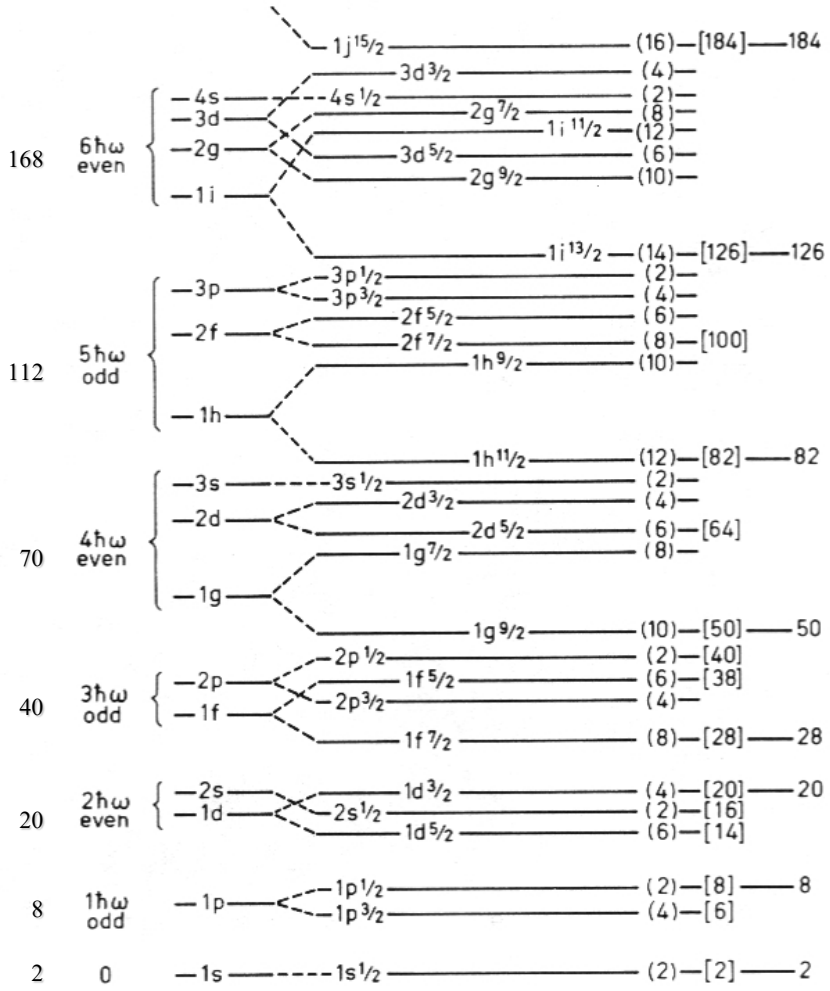
From W.D. Myers and W.J. Swiatecki, Nucl. Phys. **81**, 1 (1966)

Origin of the shell model

- **Goeppert-Mayer and Haxel, Jensen, and Suess proposed the independent-particle shell model to explain the magic numbers**

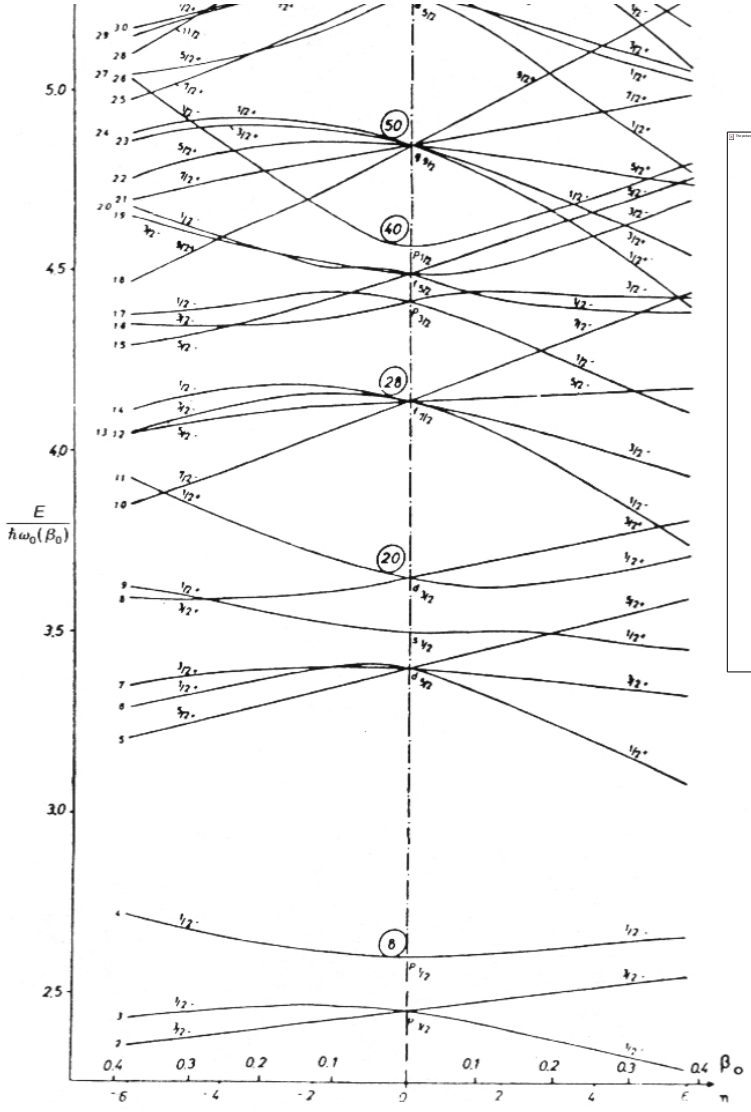
Harmonic oscillator with spin-orbit is a reasonable approximation to the nuclear mean field





M.G. Mayer and J.H.D. Jensen, *Elementary Theory of Nuclear Shell Structure*,p. 58, Wiley, New York, 1955

Nilsson Hamiltonian - Poor man's Hartree-Fock



Oblate, γ=60°

Prolate, γ=0°

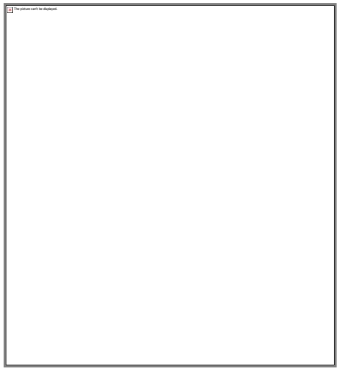
Anisotropic harmonic oscillator

From J.M. Eisenberg and W. Greiner, *Nuclear Models*, p 542, North Holland, Amsterdam, 1987

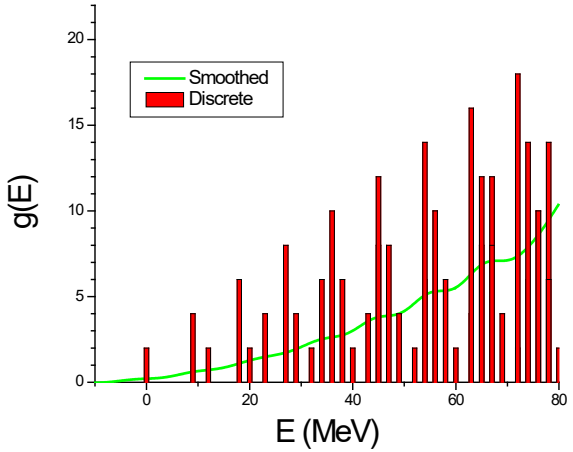
Nuclear masses
Shell corrections to the liquid drop

- **Shell correction**
  - **In general, the liquid drop does a good job on the bulk properties**
    - **The oscillator doesn't!**
  - **But we need to put in corrections due to shell structure**
    - **Strutinsky averaging; difference between the energy of the discrete spectrum and the averaged, smoothed spectrum**

Discrete spectrum



Mean-field single-particle spectrum



Smoothed spectrum

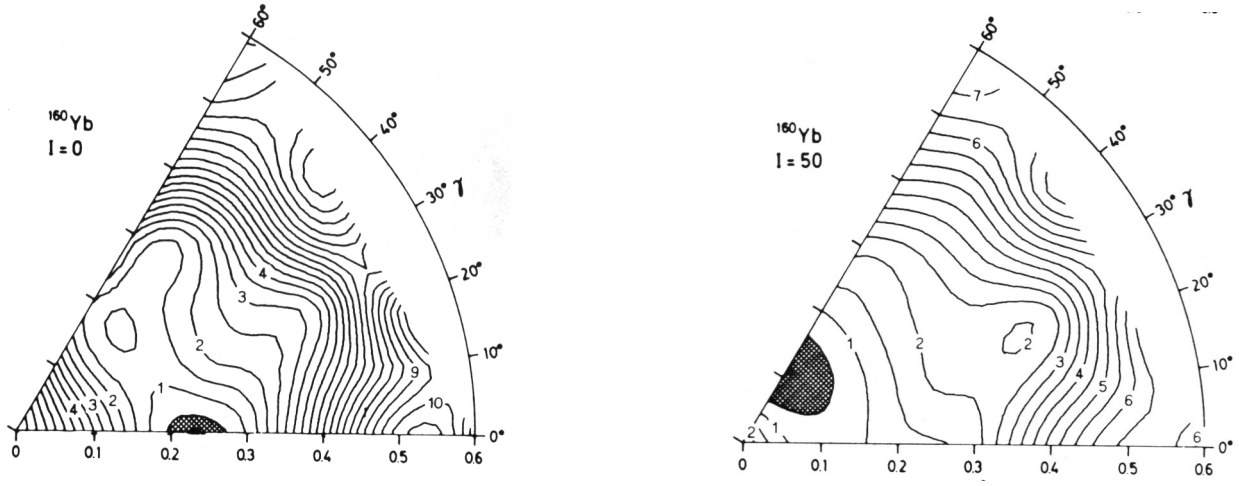
$$\tilde{g}(\varepsilon) = \sum_i f\left(\frac{\varepsilon - \varepsilon_i}{\gamma}\right)\delta(\varepsilon - \varepsilon_i)$$

$$N = \int_{-\infty}^{\tilde{\varepsilon}_F} \tilde{g}(\varepsilon)\, d\varepsilon$$

$$E_{\text{Strutinsky}} = \int_{-\infty}^{\tilde{\varepsilon}_F} \varepsilon \tilde{g}(\varepsilon)\, d\varepsilon$$

Nilsson-Strutinsky and deformation

- **Energy surfaces as a function of deformation**

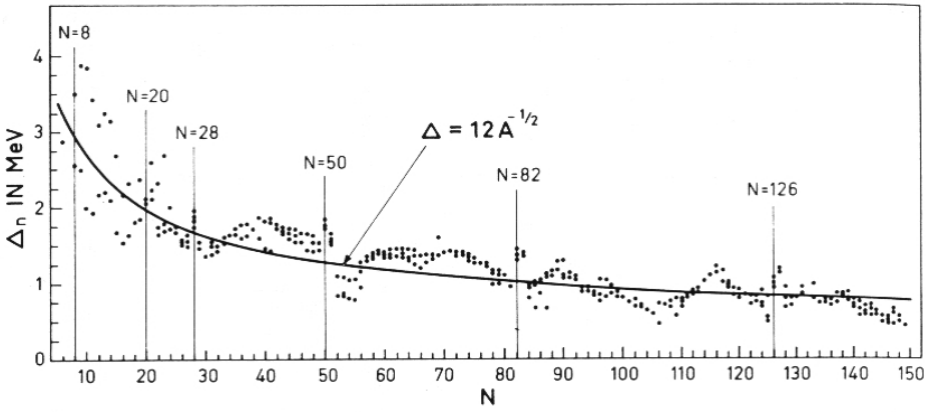


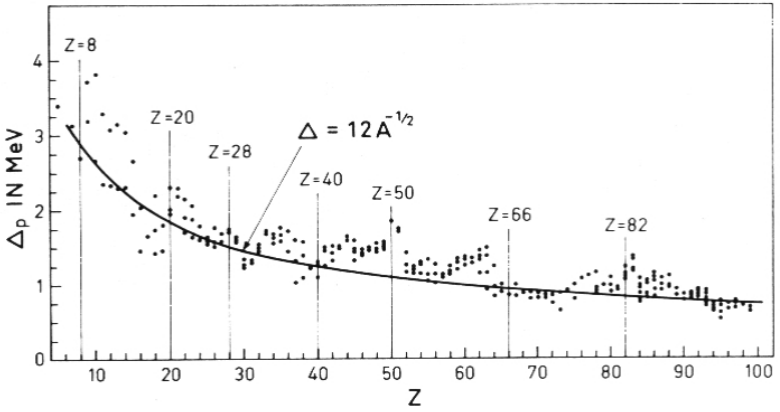
From C.G. Andersson,et al., Nucl. Phys. **A268**, 205 (1976)

Nilsson-Strutinsky is a mean-field type approach that allows for a comprehensive study of nuclear deformation under rotation and at high temperature

Nuclear masses
Pairing corrections to the liquid drop

- **Pairing:**

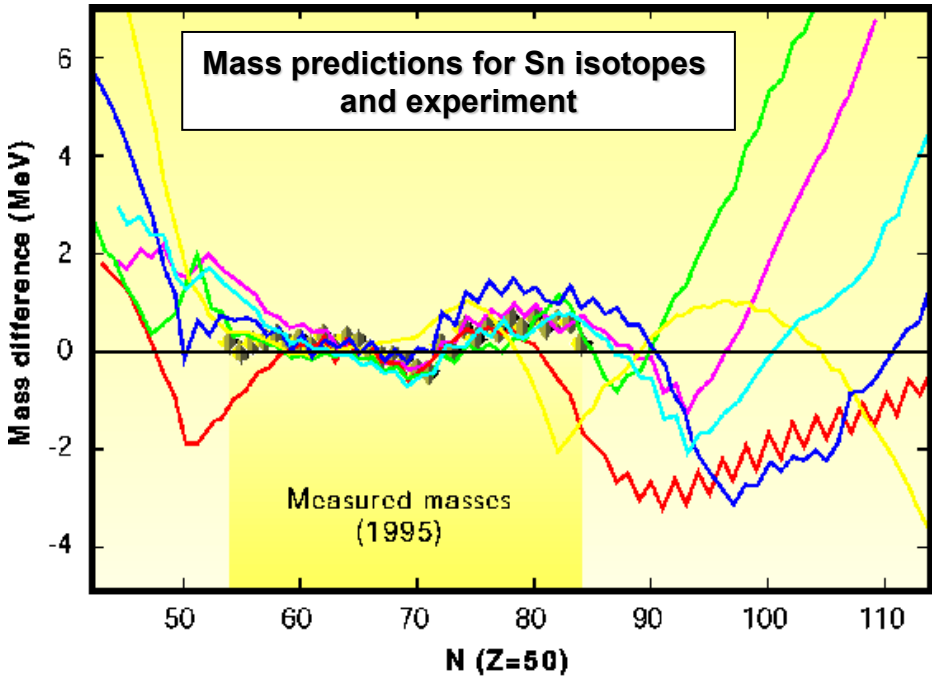




From A. Bohr and B.R. Mottleson, *Nuclear Structure*, vol. 1, p. 170 Benjamin, 1969, New York

How well do mass formulae work?

- **Most formulae reproduce the known masses at the level of ~ 600 keV heavier nuclei, and ~1 MeV for light nuclei**



Predictions towards the neutron-drip line tend to diverge!

Why?

G Audi and AH Wapstra, Nucl Phys. A595 (1995) 409
S Liran and N Zeldes, At Data Nucl. Data Tables 17 (1976) 431
T Tachibana et al, At Data Nucl. Data Tables 39 (1988) 251
Y Aboussir et al, Nucl Phys. A549 (1992) 155
P Moeller et al, At Data Nucl. Data Tables 59 (1995) 185
WD Myers and WJ Swiatecki, Nucl Phys. A601 (1996) 141
H von Groote et al, At Data Nucl. Data Tables 17 (1976) 418

How well do mass formulae work?

- **Where is the neutron drip line?**

What is the limit of stability?
What do we mean by a drip-line?
The neutron separation energy:

$$S_n = BE(Z,N) - BE(Z,N-1)$$

If $S_n > 0$, unbound to neutron emission
The drip line is the point where all nuclei with more neutrons have $S_n > 0$

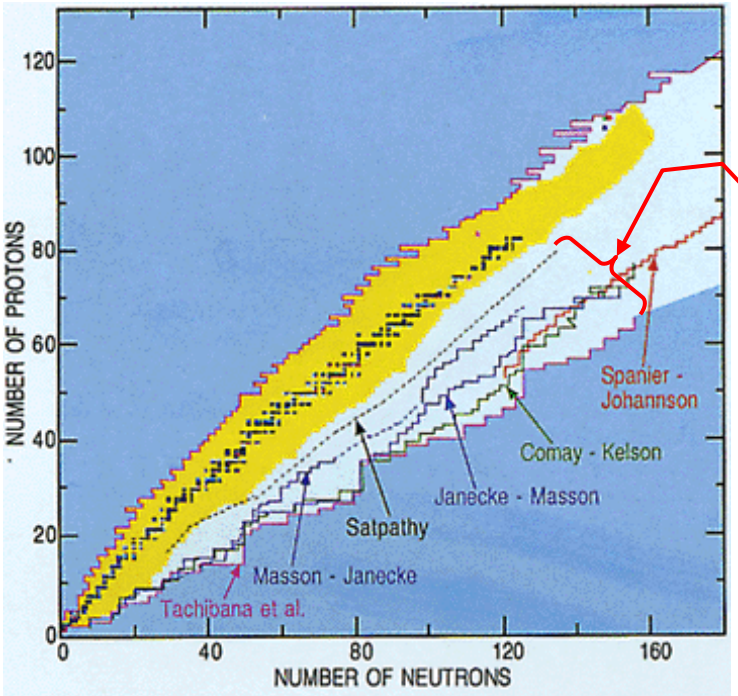


Figure I.4: Model estimates of the neutron-drip line illustrating the large uncertainties involved in precisely predicting where neutron binding in nuclei comes to an end.

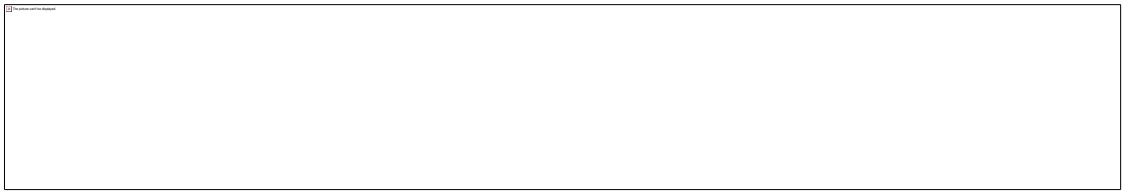
The mass models don't agree as we go away from known masses!

The location of the neutron drip-line in rather uncertain!

Mass Formulae

Homework:

- **Use the Bethe-Weizsäcker fromula to calculate masses, and determine the line of stability (ignore pairing and shell corrections)**
  - **Show that the most stable Z_0 value is**



More advanced homework:

- **Assume symmetric fission and calculate the energy released. Approximately at what A value is the energy released > 0?**

What are the forces in nuclei?

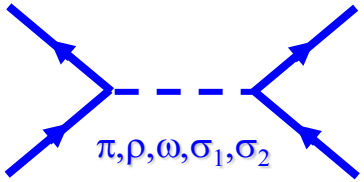
- Srart with the simplest case: Two nucleons
  - NN-scattering
  - The deuteron
- From these we infer the form of the nucleon-nucleon interaction
  - The starting point is, of course, the Yukawa hypothesis of meson exchange
    - Pion, rho, sigma, two pion, etc.
- However, it is also largely phenomenological
  - Deuteron binding energy: 2.224 MeV
  - Deuteron quadrupole moment: 0.282 fm^2
  - Scattering lengths and ranges for pp, nn, and analog pn channels

$a_{pp} = -17.3 \pm 0.4$ fm	$a_{nn} = -18.8 \pm 0.3$ fm	$a_{pn} = -23.75 \pm 0.01$ fm
$r_{pp} = 2.85 \pm 0.04$ fm	$r_{nn} = 2.75 \pm 0.11$ fm	$r_{pn} = 2.75 \pm 0.05$ fm

    - Unbound!
  - Note that $V_{pp} \neq V_{nn} \neq V_{pn}$
- Some of the most salient features are the Tensor force and a strong repulsive core at short distances

NN-interactions

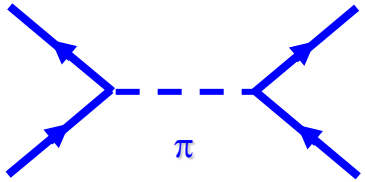
- **Argonne potentials**
  - R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, PRC51, 38 (1995)
  - Coulomb + One pion exchange + intermediate- and short-range
- **Bonn potential**
  - R. Machleidt, PRC63, 024001 (2001)
  - Based on meson-exchange
  - Non-local



- **Effective field theory**
  - C. Ordóñez, L. Ray, U. van Kolck, PRC53, 2086 (1996); E. Epelbaoum, W. Glöckle, Ulf-G. Meißner, NPA637, 107 (1998)
  - Based on Chiral Lagrangians
  - Expansion in momentum relative to a cutoff parameter (~ 1 GeV)
  - Generally has a soft core
- **All are designed to reproduce the deuteron and NN-scattering**

NN-interactions

- **Pion exchange is an integral part of NN-interactions**
  - **Elastic scattering in momentum space**



  - **Or, through a Fourier transform, coordinate space ($\mu = m_\pi c/\hbar$)**

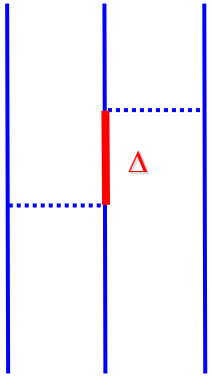
$$V_\pi = \frac{g_\pi^2}{4M^2}\frac{1}{3}m_\pi\left[\sigma_i\cdot\sigma_j + \left(1+\frac{3}{\mu r}+\frac{3}{(\mu r)^2}\right)\underbrace{\left(3\sigma_i\cdot\hat{\mathbf{r}}\,\sigma_i\cdot\hat{\mathbf{r}}-\sigma_i\cdot\sigma_j\right)}_{\text{Tensor operator}}\right]\frac{e^{-\mu r}}{\mu r}$$

- **Off-shell component present in the Bonn potentials**

  - **Non-local (depends on the energies of the initial and final states)**
  - **Plays a role in many-body applications and provides more binding**

Three-Nucleon interactions

- First evidence for three-nucleon forces comes from exact calculations for t and ^{3}He
  - Two-nucleon interactions under bind
    - Note CD-Bonn has a little more binding due to non-local terms
- Further evidence is provided by *ab initio* calculations for ^{10}B
  - NN-interactions give the wrong ground-state spin!
    - More on this later
- Tucson-Melbourne
  - S.A. Coon and M.T. Peña, PRC48, 2559 (1993)
  - Based on two-pion exchange and intermediate Δ's
  - The exact form of NNN is not known



There is mounting evidence that three-body forces are very important

Isospin

- **Isospin is a spectroscopic tool that is based on the similarity between the proton and neutron**
  - **Nearly the same mass, q_p=1, q_n=0**
  - **Heisenberg introduced a spin-like quantity with the z-component defining the electric charge**
  - **Protons and neutrons from an isospin doublet**

$$t_z = q - \frac{1}{2} \qquad |p\rangle = \uparrow \qquad |n\rangle = \downarrow$$

  - **Add isospin using angular momentum algebra, e.g., two particles:**
    - **T=1**

$$|T=1,T_z=1\rangle = \uparrow\uparrow \qquad |T=1,T_z=-1\rangle = \downarrow\downarrow \qquad |T=1,T_z=0\rangle = \frac{1}{\sqrt{2}}\left(\uparrow\downarrow + \downarrow\uparrow\right)$$

    - **T=0**

With T=0, symmetry under p ⇔ n

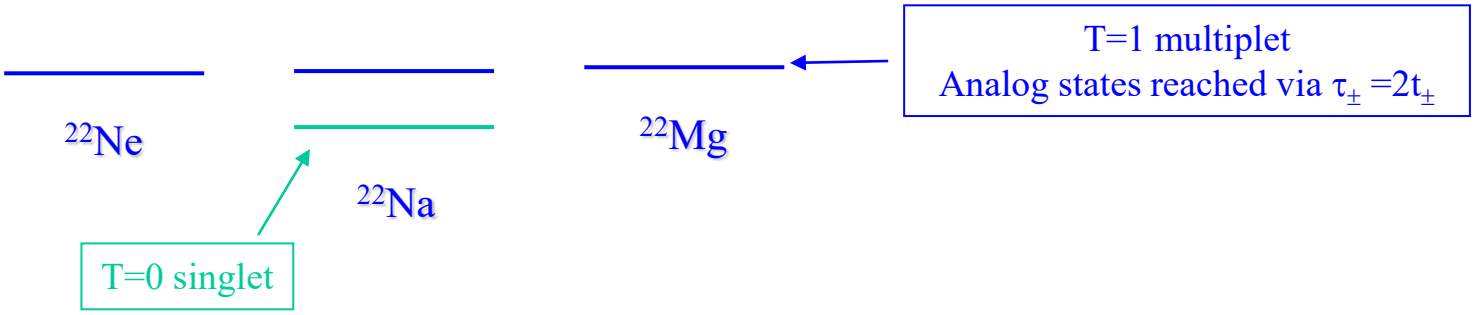
Isospin

- For Z protons and N neutrons

$$T_z = Z - N$$

$$|Z - N| \le T \le Z + N$$

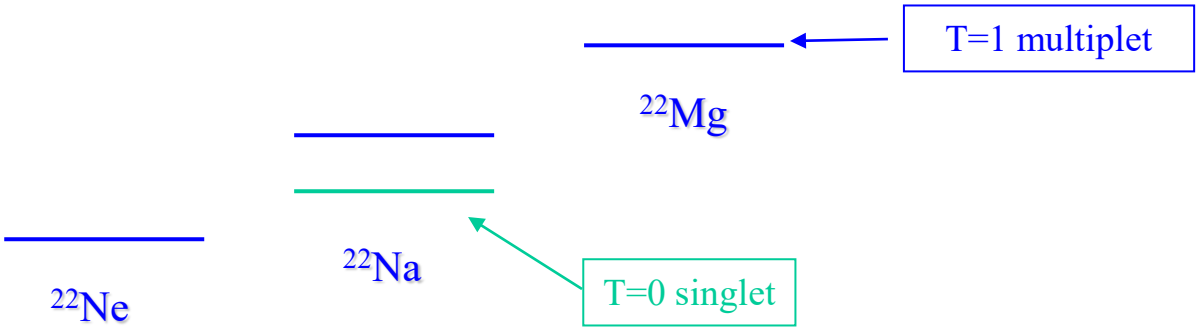
- Even-even N=Z: T=0
- $N \neq Z$: T=T_z
- Odd-odd N=Z: T=0 or T=1 (Above A=22, essentially degenerate)



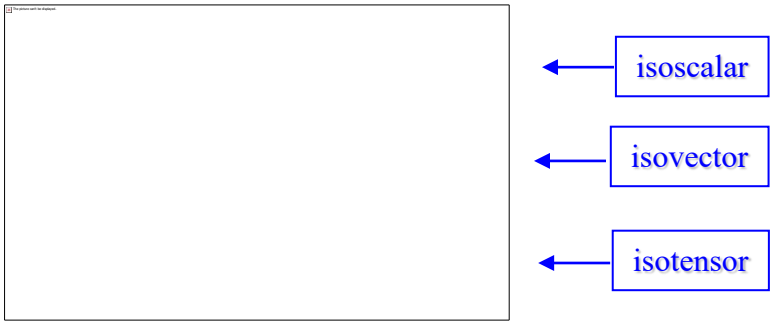
- If V_{pp}=V_{nn}=V_{pn}, isospin-multiplets have the same energy and isospin is a good quantum number

Isospin

- Of course, $V_{pp} \neq V_{nn} \neq V_{pn}$

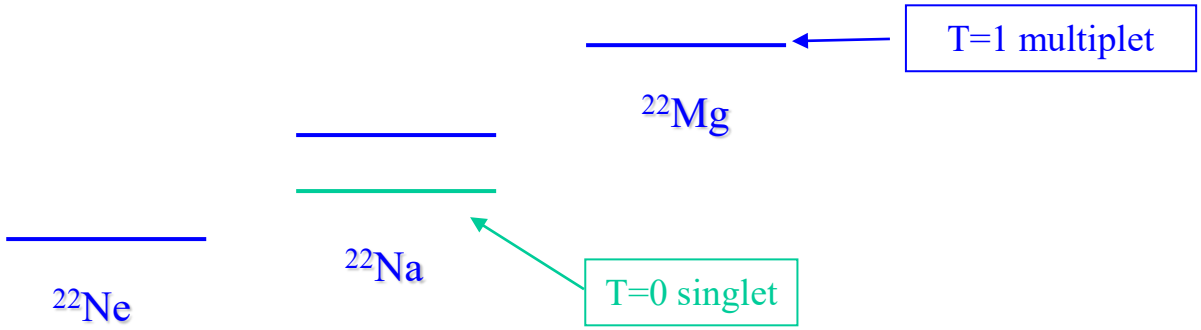


- NN-interaction has scalar, vector, and tensor components in isospin space

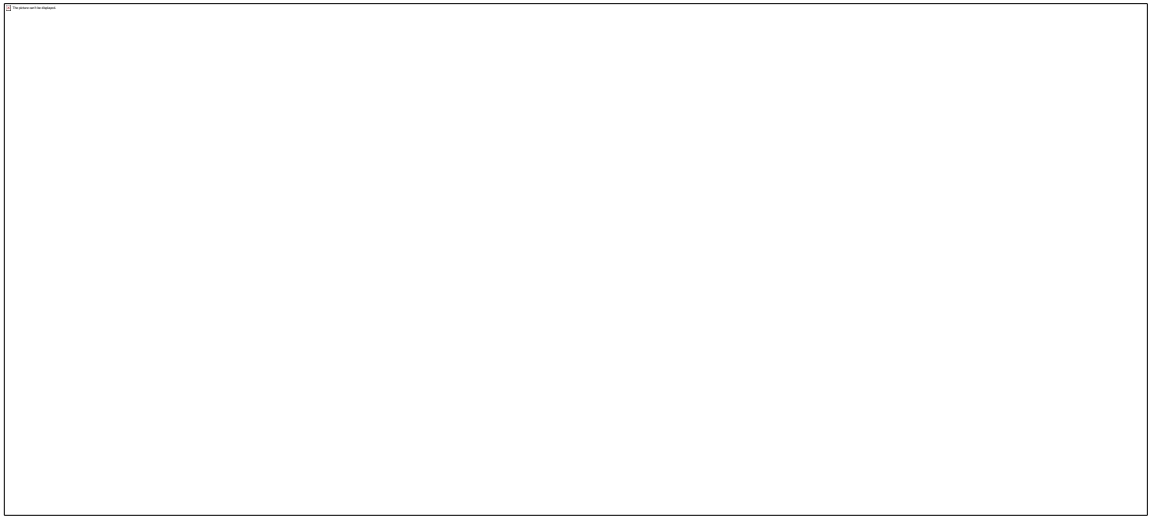


— Note that the Coulomb interaction contributes to each component and is the largest!!!

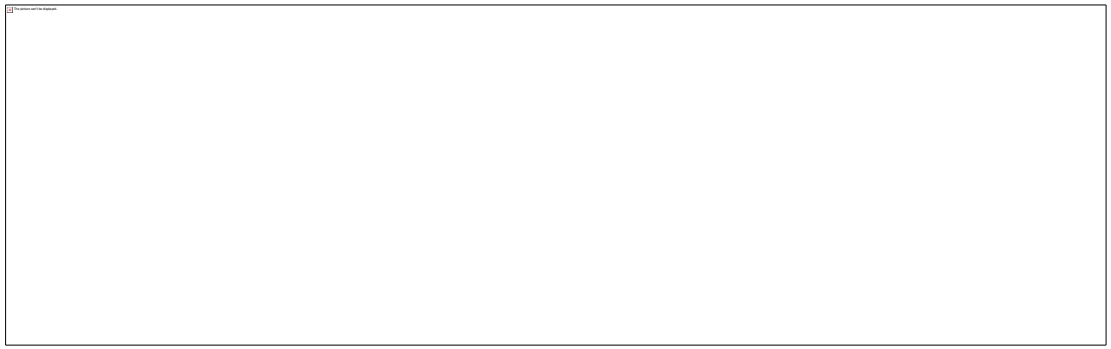
Coulomb-displacement energies



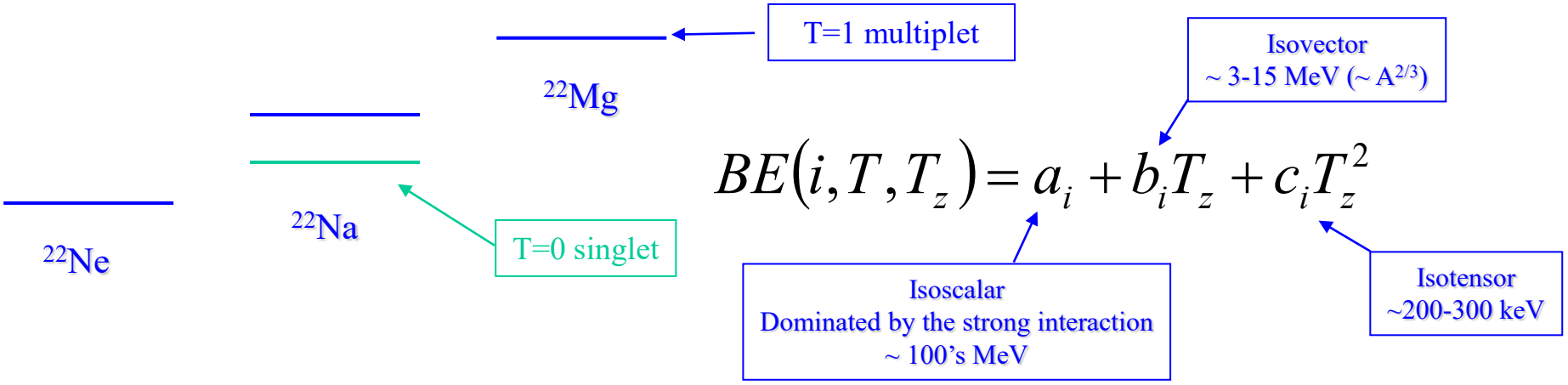
- **We apply the Wigner-Eckart theorem**



Isobaric-Mass-Multiplet Equation



- **The Isobaric-Mass-Multiplet Equation (IMME)**



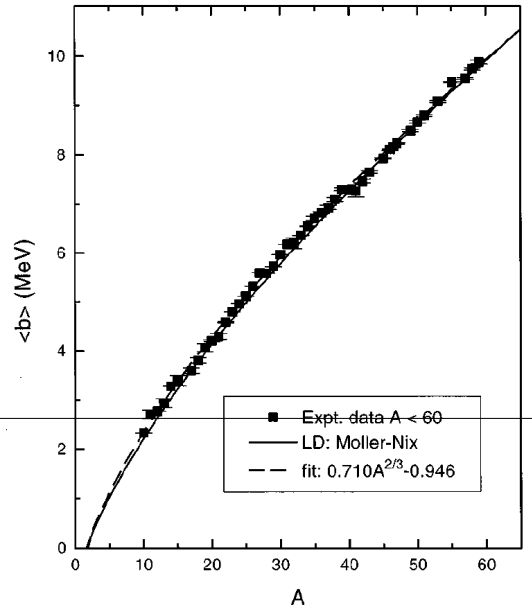
Coulomb-displacement energies

- **Can we use the IMME to predict the proton drip-line?**
  - **Binding energy difference between mirror nuclei**

  - **In a T-multiplet, often the binding energy of the neutron-rich mirror is measured**

Calculated with theory
30 keV uncertainty

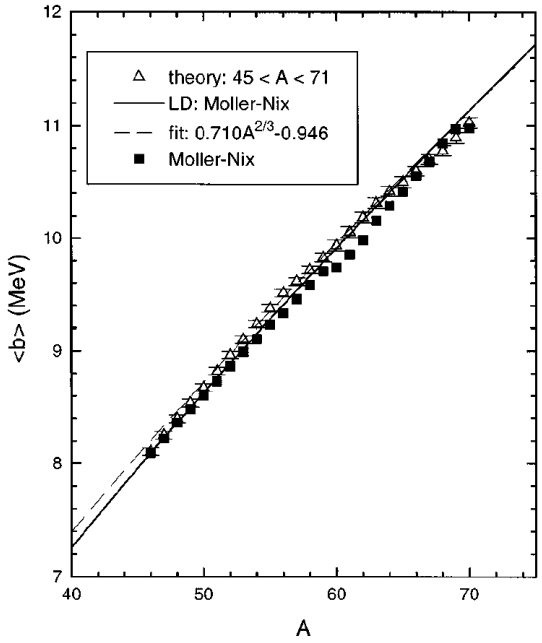
Simple estimate from a charged sphere with radius $r=1.2A^{1/2}$



Coulomb-displacement energies

- **Map the proton drip-line up to A=71 using Coulomb displacement with an error of ~ 100-200 keV on the absolute value**
  - **B.A. Brown, PRC42, 1513 (1991); W.E. Ormand, PRC53, 214 (1996); B.J. Cole, PRC54, 1240 (1996); W.E. Ormand, PRC55, 2407 (1997); B.A. Brown et al., PRC65, 045802 (2002)**

Use nature nature to give us the strong interaction part, i.e., the *a*-coefficient by adding the Coulomb-displacement to the experimental binding energy of the neutron-rich mirror



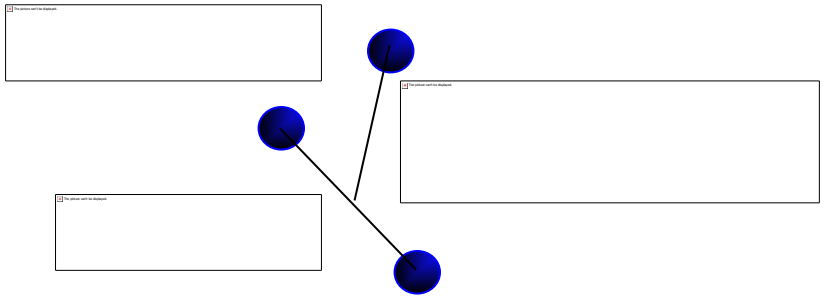
Yes!

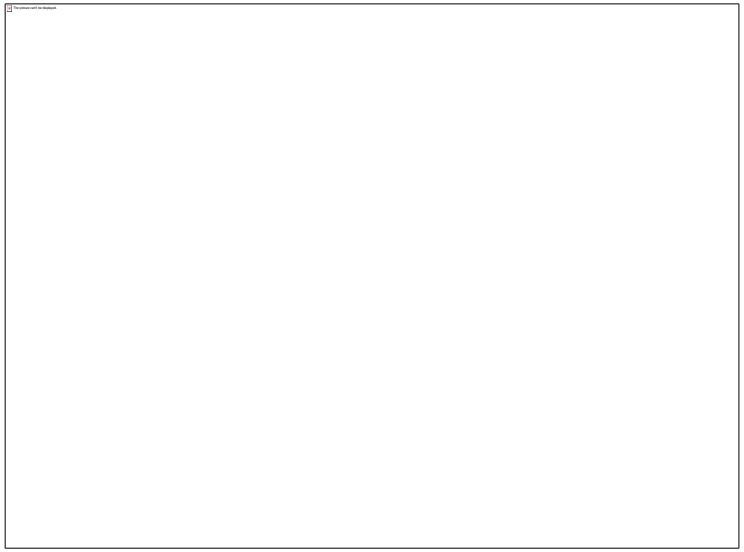
Coulomb displacement energies provide an accurate method to map the proton drip line up to A=71

Now, lets start to get at the structure of nuclei

- For two particles we use Schrodinger equation
- For three and four, we turn to Faddeev and Faddeev-Yakubovsky formulations

Jacobi coordinates





Identical particles

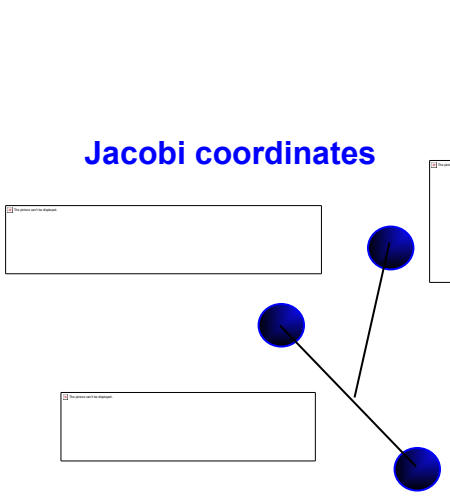
W. Glöcle in *Computational Nuclear Physics*, Springer-Verlag, Berlin, 1991

Exact methods exist for A ≤ 4

Three-particle harmonic oscillator

- For fun, look at the three-particle harmonic oscillator

Jacobi coordinates



Show that we can rewrite *H* as:

Finally, we have to couple L to the spin S and anti-symmetrize

Harmonic oscillators are always a convenient way to start solving the problem

More than four particles

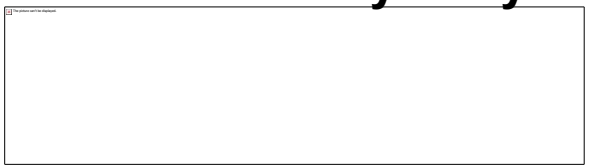
- This is where life starts to get very hard!
- Why?
- Because there are so many degrees of freedom.
- What do we do?
  - Green's Function Monte Carlo

  - Coupled-cluster

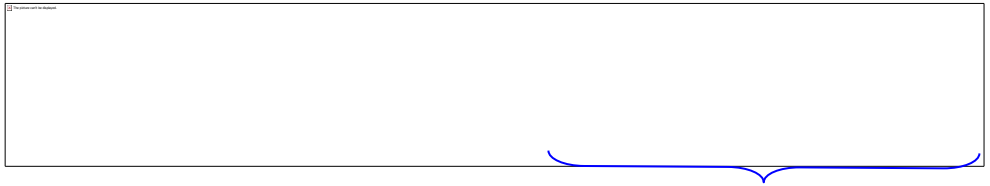
  - Shell model
    - I'll tell you about this approach

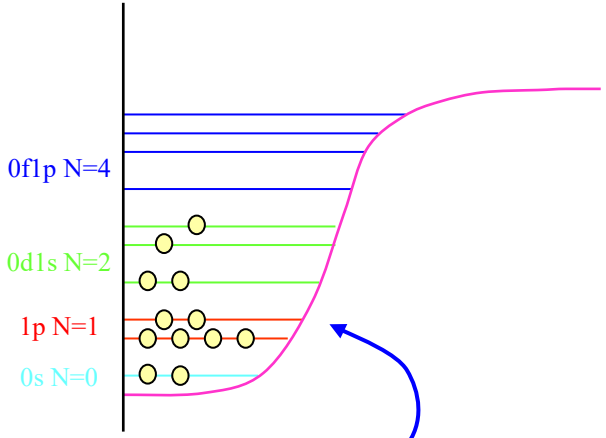
Many-body Hamiltonian

- Start with the many-body Hamiltonian



- Introduce a mean-field U to yield basis





- The mean field determines the shell structure
- In effect, nuclear-structure calculations rely on perturbation theory

The success of any nuclear structure calculation depends on the choice of the mean-field basis and the residual interaction!

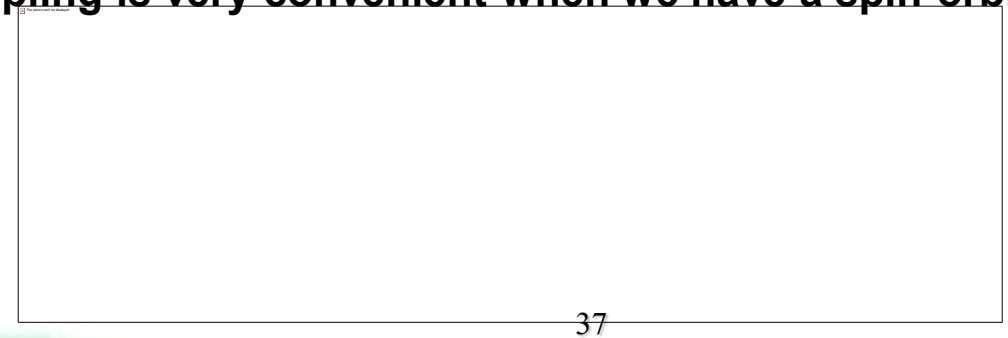
Single-particle wave functions

- With the mean-field, we have the basis for building many-body states
- This starts with the single-particle, radial wave functions, defined by the radial quantum number n, orbital angular momentum l, and z-projection m

  - Now include the spin wave function: $\chi^{S}_{\frac{1}{2}s_z}$

- Two choices, jj-coupling or ls-coupling
  - Ls-coupling

  - jj-coupling is very convenient when we have a spin-orbit ($l \cdot s$) force



Multiple-particle wave functions

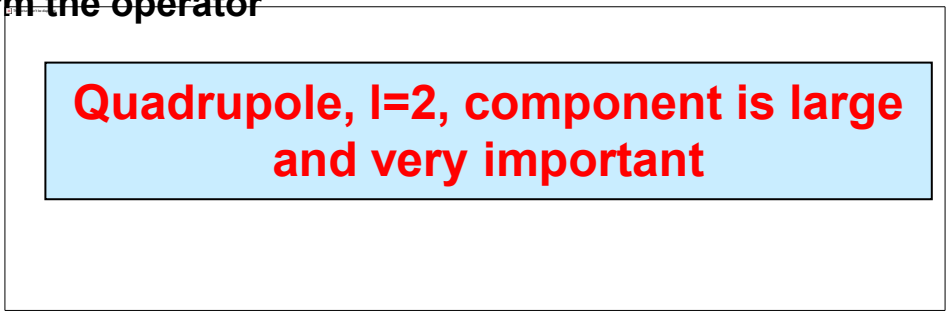
- Total angular momentum, and isospin; $\chi^{T}_{\frac{1}{2}s_z}$
- Anti-symmetrized, two particle, jj-coupled wave function

  - Note $J+T$=odd if the particles occupy the same orbits
- Anti-symmetrized, two particle, LS-coupled wave function

Two-particle wave functions

- Of course, the two pictures describe the same physics, so there is a way to connect them
  - Recoupling coefficients 

- Note that the wave functions have been defined in terms of  and  , but often we need them in terms of the relative coordinate
  - We can do this in two ways
    - Transform the operator



Two-particle wave functions in relative coordinate

- Use Harmonic-oscillator wave functions and decompose in terms of the relative and center-of-mass coordinates, i.e.,

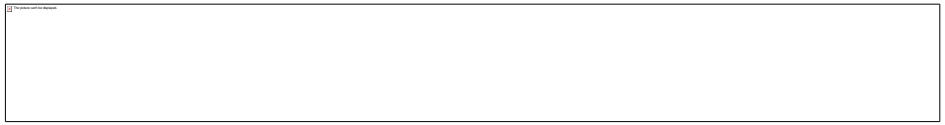
$$r = \left|\vec{r}_1 - \vec{r}_2\right|; \qquad R = \left|\vec{r}_1 + \vec{r}_2\right|/2$$

- Harmonic oscillator wave functions are a very good approximation to the single-particle wave functions
- We have the useful transformation

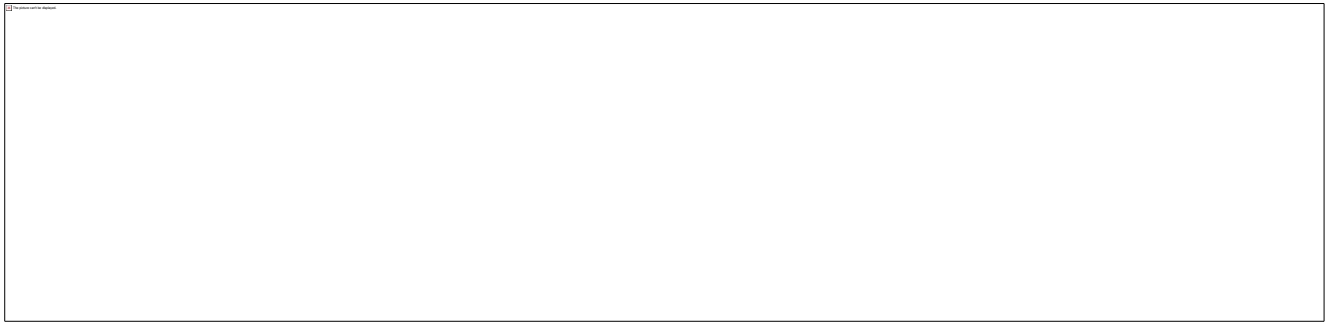
  - $2n_1+l_2+2n_2+l_2=2n+l+2N+L$
  - Where the $M(nlNL;n_1l_1n_2l_2)$ is known as the Moshinksy bracket
  - Note this is where we use the jj to LS coupling transformation
  - For some detailed applications look in *Theory of the Nuclear Shell Model*, R.D. Lawson, (Clarendon Press, Oxford, 1980)

Many-particle wave function

- To add more particles, we just continue along the same lines
- To build states with good angular momentum, we can bootstrap up from the two-particle case, being careful to denote the distinct states
  - This method uses Coefficients of Fractional Parentage (CFP)



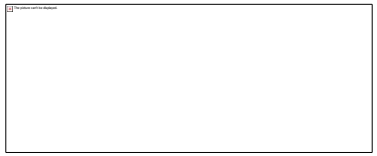
- Or we can make a many-body Slater determinant that has only a specified J_z and T_z and project J and T



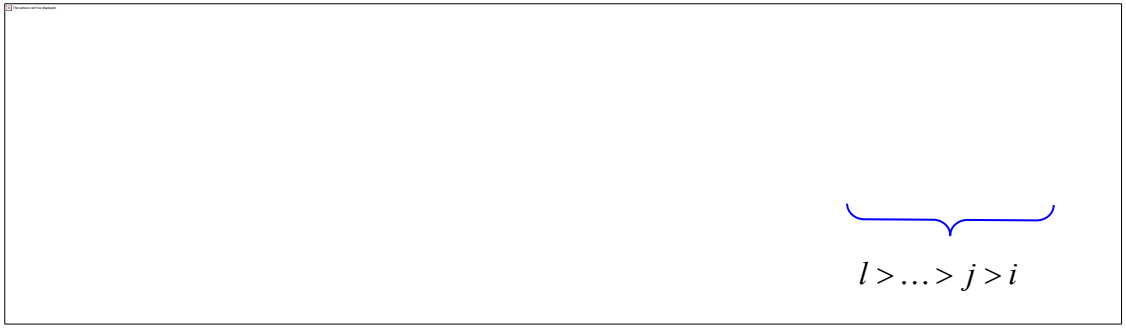
In general Slater determinants are more convenient

Second Quantization

- Second quantization is one of the most useful representations in many-body theory
- Creation and annhilation operators
  - Denote $|0\rangle$ as the state with no particles (the vacuum)
  - a^+_i creates a particle in state i;
  - a_i annhilates a particle in state i;
  - Anticommuntation relations:



- Many-body Slater determinant

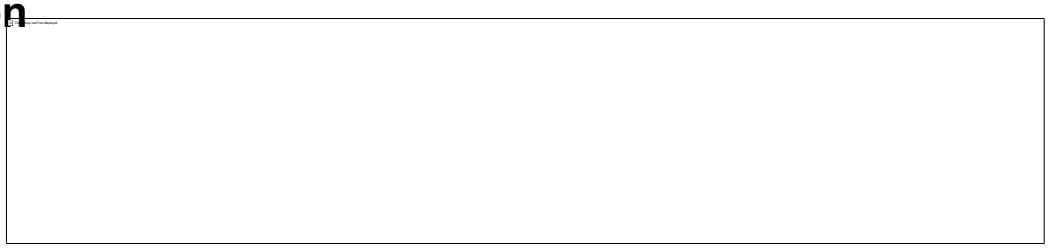


Second Quantization

- **Operators in second-qunatization formalism**
  - **Take any one-body operator O, say quadrupole E2 transition operator $er^2Y_{2\mu}$, the operator is represented as:**

where $\langle j|O|i\rangle$ is the single-particle matrix element of the operator O

  - **The same formalism exists for any n-body operator, e.g., for the NN-interaction**



    - **Here, I've written the two-body matrix element with an implicit assumption that it is anti-symmetrized, i.e.,**

Second Quantization

- **Matrix elements for Slater determinants**

$$\langle acekl|a_c^+ a_m|aeklm\rangle = \langle 0|a_l a_k a_e a_c a_a a_c^+ a_m a_m^+ a_l^+ a_k^+ a_e^+ a_a^+|0\rangle$$

$$= \langle 0|a_l a_k a_e a_c a_a a_c^+ a_l^+ a_k^+ a_e^+ a_a^+|0\rangle = \langle acekl|a_c^+|aekl\rangle$$

$$= -\langle 0|a_l a_k a_e a_c a_a a_l^+ a_k^+ a_e^+ a_c^+ a_a^+|0\rangle = -\langle acekl|acekl\rangle$$

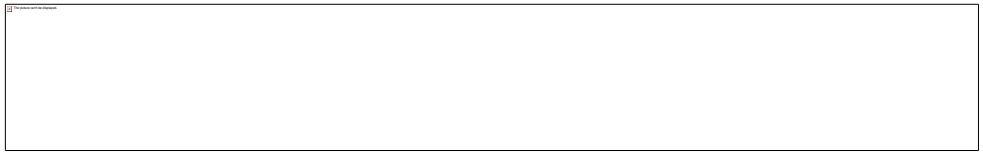
$$= -1$$

Second quantization makes the computation of expectation values for the many-body system simpler

Second Quantization

- **Angular momentum tensors**
  - — **Creation operators rotate as tensors of rank *j***
  - — **Not so for annihilation operators**

- **Anti-symmetrized, two-body state**



The mean field

- One place to start for the mean field is the harmonic oscillator
  - Specifically, we add the center-of-mass potential

  - The Good:
    - Provides a convenient basis to build the many-body Slater determinants
    - Does not affect the intrinsic motion
    - Exact separation between intrinsic and center-of-mass motion
  - The Bad:
    - Radial behavior is not right for large r
    - Provides a confining potential, so all states are effectively bound