Weak Interactions in the Nucleus III

Summer School, Tennessee

June 2003

P, C and T

Unlike the continuous transformations, these are not associated with additive quantum numbers, but multiplicative ones, if any.

$$P: \begin{cases} x \to & -x \\ p \to & -p \\ L \to & L \\ S \to & S \end{cases}$$

exmpl: $P(\pi) = -$ $\quad \Rightarrow \; P(\pi^+ \pi^-) = +$

In L=0 state

P, C and T

Charge conjugation
the Dirac equation with E&M interaction exhibits a symmetry:

$$(i\hbar\gamma^{\mu}\partial_{\mu} - e\gamma^{\mu}A_{\mu} - m)\Psi = 0$$

$$(i\hbar\gamma^{\mu}\partial_{\mu} + e\gamma^{\mu}A_{\mu} - m)\Psi_{c} = 0$$

We now know that all particles have associated antiparticles that have opposite charge and identical other quantum numbers.

How do we obtain the operator *C* such that: $C\Psi^{*} = \Psi_{c}$ →

$$(i\hbar\gamma^{\mu}\partial_{\mu} - e\gamma^{\mu}A_{\mu} - m)\Psi = 0$$

$$(i\hbar\gamma^{\mu*}\partial_{\mu} + e\gamma^{\mu*}A_{\mu} + m)\Psi^{*} = 0$$

↓

We need: $C\gamma^{\mu*}C^{-1} = -\gamma^{\mu}$

← $C = i\gamma^{2}$

P, C and T

Charge conjugation
example:

$$\psi^4 = \frac{1}{N}\begin{pmatrix}0\\0\\0\\1\end{pmatrix} e^{+imt}$$

E<0
Spin-down e⁻

$$i\gamma^2\psi^{4*} = \frac{i}{N}\begin{pmatrix}0&0&0&-i\\0&0&i&0\\0&i&0&0\\-i&0&0&0\end{pmatrix}\begin{pmatrix}0\\0\\0\\1\end{pmatrix} e^{-imt} = \frac{1}{N}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} e^{-imt} = \psi^1$$

E>0
Spin-up e⁺

P, C and T

Charge conjugation symmetry
clearly broken by Weak Interaction

$$H(e^-) \neq H(e^+)$$

P, C and T

Time Reversal

$$T: \begin{cases} x \to & x \\ p \to & -p \\ L \to & -L \\ S \to & -S \end{cases}$$

Time Reversal
Wigner showed that it needs to be anti-unitary

$$T = UK$$

Example of construction:

$$TpT^{-1} = -p$$
$$TxT^{-1} = x$$
$$\Rightarrow T \equiv K?$$

but how about *S*?

$$K(s_x, s_y, s_z)K^{-1} = (s_x, -s_y, s_z) \neq -S$$

Need to include in addition

$$e^{i\pi S_y/\hbar}(s_x, s_y, s_z)e^{-i\pi S_y/\hbar} = (-s_x, s_y, -s_z)$$

CPT conserved for all local, Lorentz-invariant interactions

So CP-violation implies T-violation

Properties of Time-Reversal Operator

$$T\vec{p}\,T^{-1} \rightarrow -\vec{p}$$

$$T\vec{J}\,T^{-1} \rightarrow -\vec{J}$$

$$|\Psi'\rangle = T\,|\Psi\rangle$$

$$|\Phi'\rangle = T\,|\Phi\rangle \qquad \Rightarrow \qquad \langle\Phi'|\Psi'\rangle = \langle\Psi|\Phi\rangle^{*}$$

T-violation observables are related to phases in the interaction

Time Reversal Symmetry Violation

CP-violation observed in K decay (circa 1964)

$C(K^\circ) = \overline{K}^\circ$ $C(\overline{K}^\circ) = K^\circ$

$P(K^\circ) = -K^\circ$ $P(\overline{K}^\circ) = -\overline{K}^\circ$

$CP(K^\circ) = -\overline{K}^\circ$ $CP(\overline{K}^\circ) = -K^\circ$

Not eigen-states of *CP*

Instead:

$CP(K^\circ - \overline{K}^\circ) = (K^\circ - \overline{K}^\circ)$

$CP(K^\circ + \overline{K}^\circ) = -(K^\circ + \overline{K}^\circ)$

$K1 = (K^\circ - \overline{K}^\circ)$ has *CP*=+; should decay to $\pi^+\pi^-$

$K2 = (K^\circ + \overline{K}^\circ)$ has *CP*=−; should decay to $\pi^+\pi^-\pi^\circ$

In a beam of K°'s one expects two components:

K1 (short lived) $\rightarrow \pi^+\pi^-$

K2 (long lived) $\rightarrow \pi^+\pi^-\pi^\circ$

Instead, experiment shows:

K1 (short lived) $\rightarrow \pi^+\pi^- + \pi^+\pi^-\pi^\circ$

K2 (long lived) $\rightarrow \pi^+\pi^-\pi^\circ + \pi^+\pi^-$

CP-violation and matter-anti-matter asymmetry

CP-violation so far observed in K decays could be explained by a (non-trivial) phase in the CKM matrix

But this is not enough to explain the matter-anti-matter asymmetry presently observed in Universe.

CP-violation and supersymmetry

Supersymmetry predicts CP violation observables larger than predicted by SM.

For example, in the scenario that supersymmetry will appear at the next run in the LHC: unless large suppressions are at work, the EDM should be on the order of 100 below the present limit.

Time-Reversal Invariance Violation

$$H = -G_W/\sqrt{2}\,(H_{V,A})$$

$$H_{V,A} = (\bar{\Psi}_p \gamma^\mu \Psi_n)\,(\bar{\Psi}_e (C_V \gamma_\mu + C'_V \gamma_\mu \gamma_5)\,\Psi_\nu) + (\bar{\Psi}_p \gamma^\mu \gamma_5 \Psi_n)\,(\bar{\Psi}_e (C'_A \gamma_\mu + C_A \gamma_\mu \gamma_5)\,\Psi_\nu)$$

Consequence: Decay Rate

$$dW = dW_0\left(1 + a\frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + b\frac{\Gamma m_e}{E_e} + \langle \vec{J} \rangle \left[A\frac{\vec{p}_e}{E_e} + B\frac{\vec{p}_\nu}{E_\nu} + D\frac{\vec{p}_e \times \vec{p}_\nu}{E_e E_\nu}\right]\right)$$

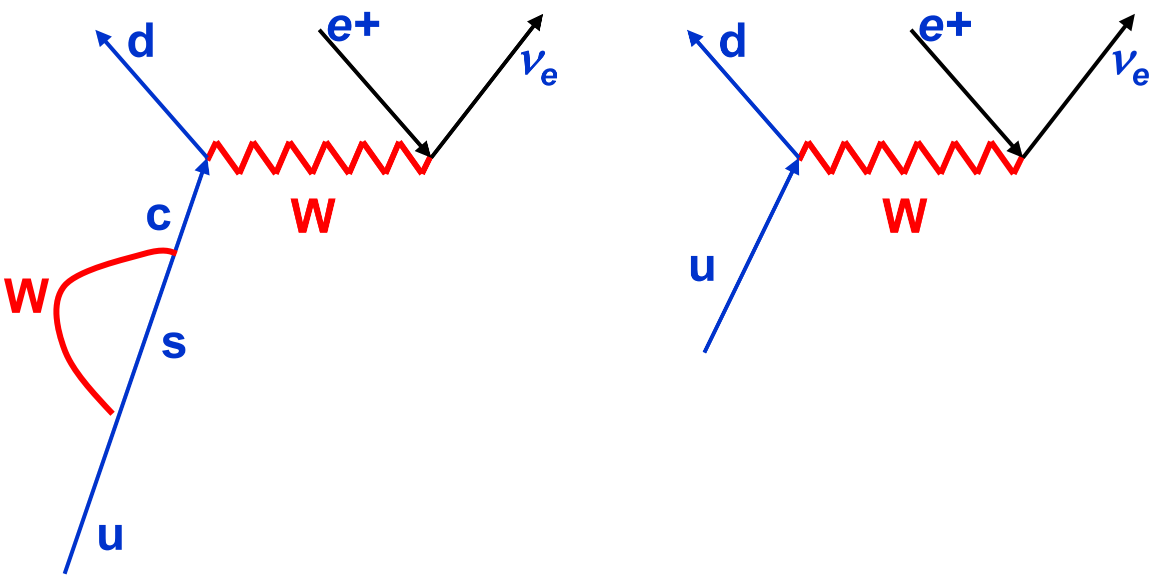
$$D = \frac{-2\,Im[\lambda\,\{C_V C_A^* + C'_V C'^*_A\}]}{C_V^2 + C'^2_V + \lambda^2\,\{C_A^2 + C'^2_A\}}$$

$\lambda = \langle GT \rangle / \langle F \rangle$

A phase between C_V and C_A produces 'Time Reversal' Violation

Expensive to pick up a phase in the Standard Model:

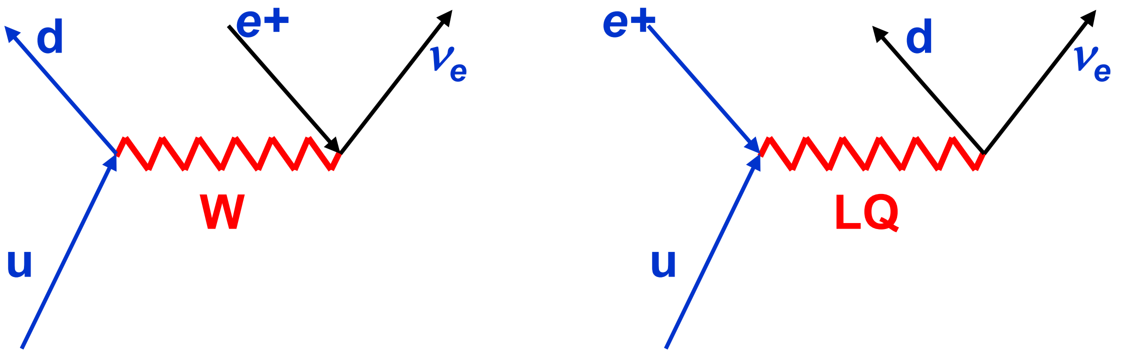
$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} \quad \begin{pmatrix} t \\ b' \end{pmatrix}$$



According to the Standard Model it is very hard to observe TRIV in nuclear Weak decays

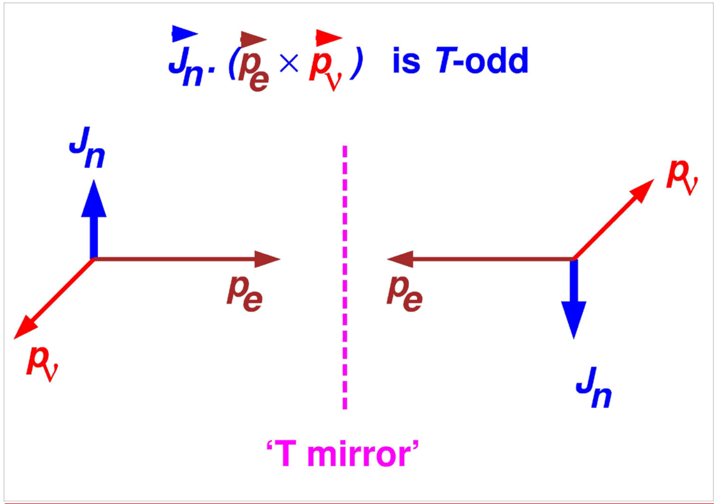
Cheaper to pick up a phase in extensions of the Standard Model:

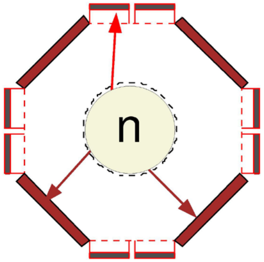
Example: Lepto-Quarks



emiT: Time Reversal Violation in Neutron β Decay

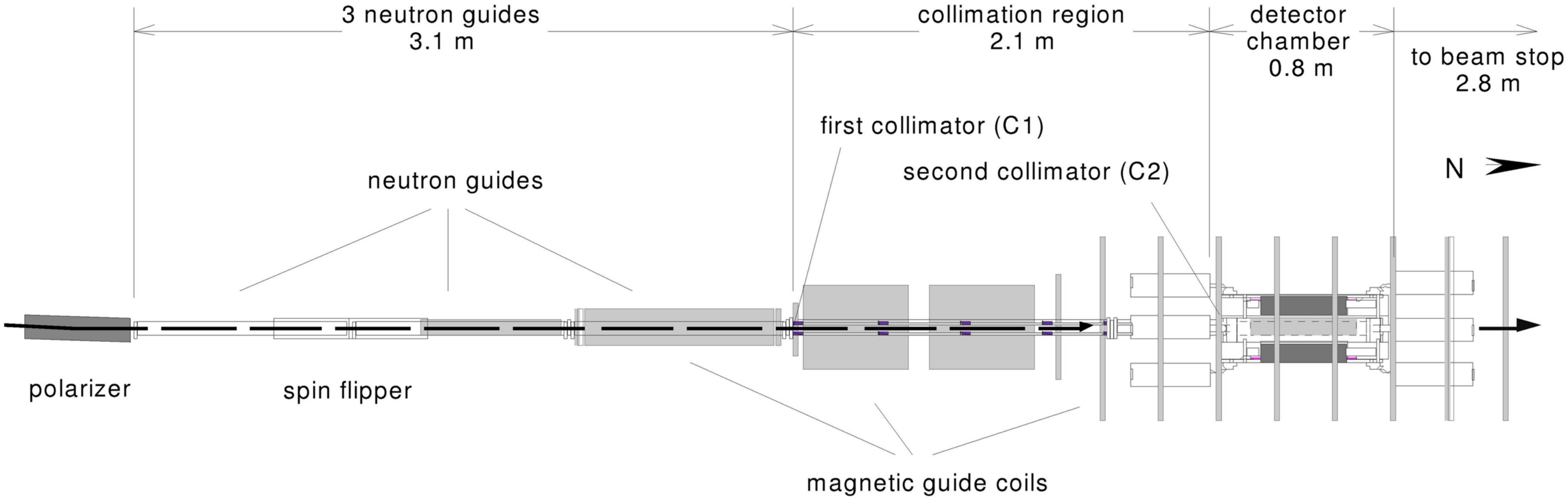
Institutions: LBNL, LANL, NIST, U. of Michigan, U. of Washington...
Publications: Phys. Rev. C **62**, 055551 (2000).





Predictions for *D* in neutron decay

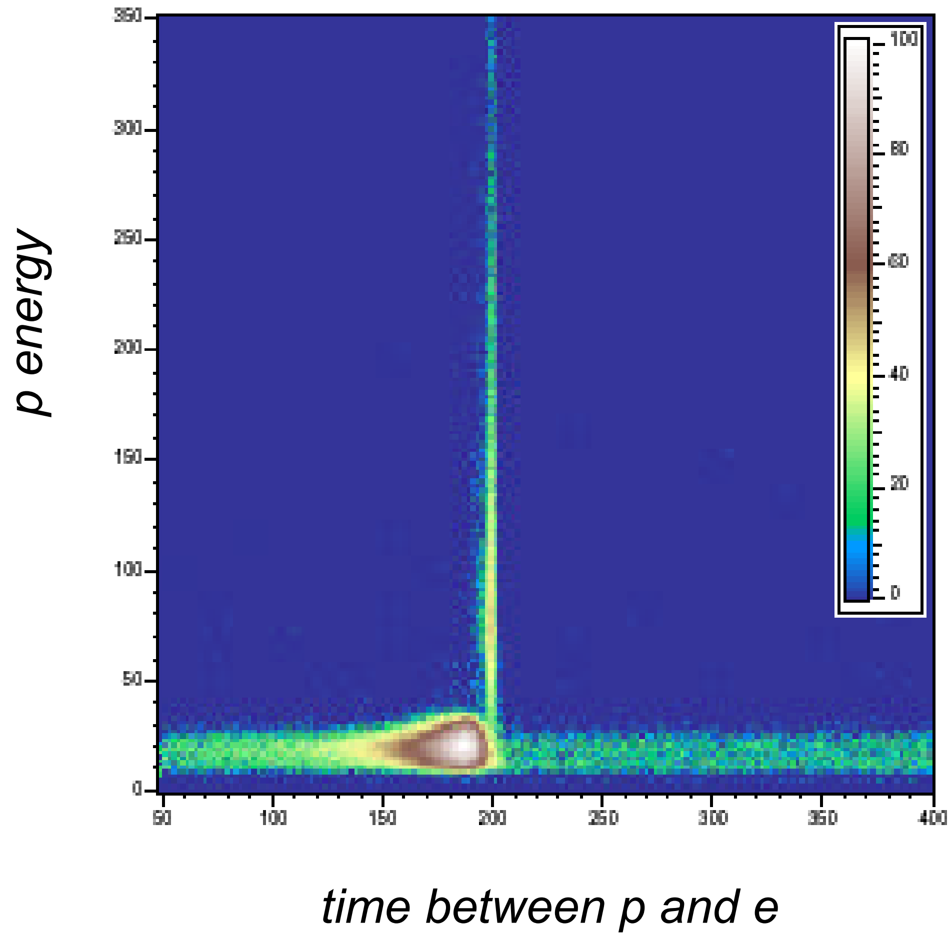
Kobayashi-Maskawa phase	$\leq 10^{-12}$
Theta-QCD	$\leq 10^{-14}$
Supersymmetry	$\leq 10^{-6}$
Left-Right Symmetry	$\leq 10^{-4}$
Lepto-quarks	$\leq 10^{-3}$


3 neutron guides
3.1 m
collimation region
2.1 m
detector
chamber
0.8 m
to beam stop
2.8 m
first collimator (C1)
second collimator (C2)
N
neutron guides
polarizer
spin flipper
magnetic guide coils

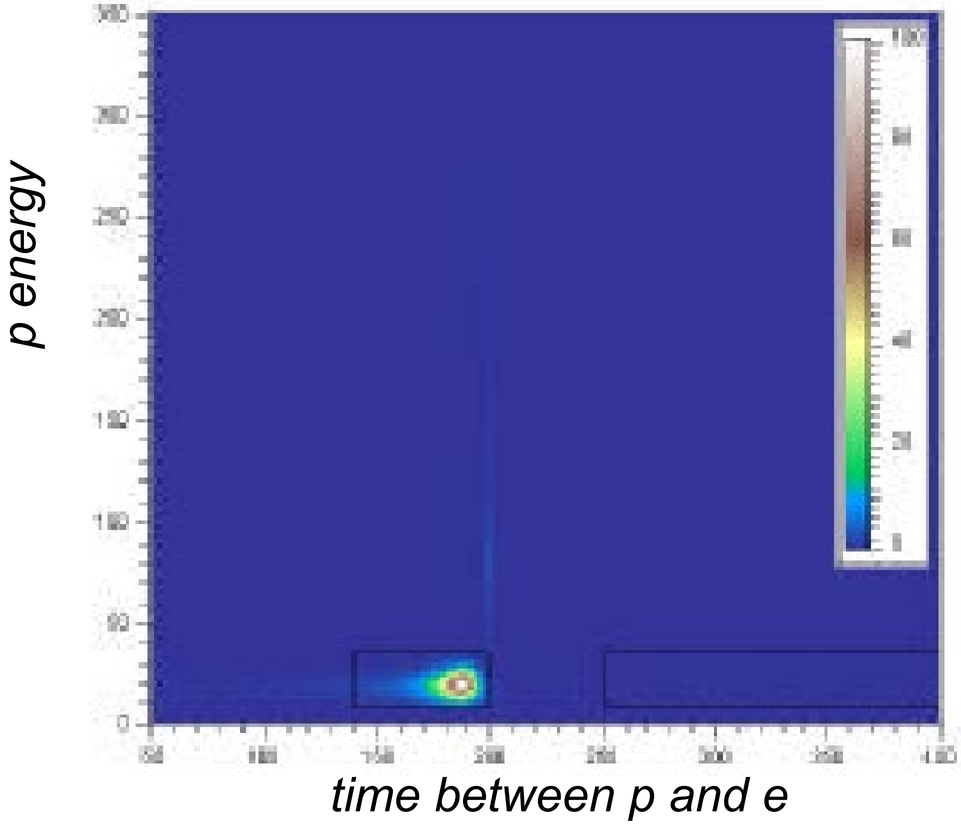
Detecting D: problems and solutions

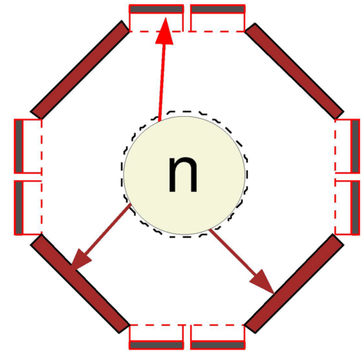
Problem	Solution
1) neutrino too hard to detect	1) Detect proton instead $\vec{p}_\nu = -\vec{p}_e - \vec{p}_p \quad \Rightarrow$ $\vec{J}.\vec{p}_e \times \vec{p}_\nu = -\vec{J}.\vec{p}_p \times \vec{p}_\nu$
2) proton energy less than 1 keV: too hard to detect	2) accelerate protons to 30 keV.
3) potential systematic effects may require information on position of electron	3) use plastic scintillator and detect amplitude and time on both ends.

First phase of emiT: PRC 62, 055551 (2000).



Second phase of emiT: presently running.





emiT published first results in 2000 that yielded
$D=0.6\pm1.2(stat)\pm0.5(syst)\ x10^{-3}$
and is now running an upgraded version
that should yield $D<310^{-4}$

Electric Dipole Moment

Existence of EDM violates Time-Reversal symmetry:

$$T: \begin{cases} x \to x \\ p \to -p \\ L \to -L \\ S \to -S \end{cases} \qquad T: \begin{cases} q \to q \\ j \to -j \\ E \to E \\ B \to -B \end{cases}$$

EDM operator does not flip sign under T:

$$\hat{d} = \sum_i r_i q_i$$

But for a non-degenerate system:

$$\vec{d} = g_d \vec{J}$$
$$\vec{\mu} = g_M \vec{J}$$

Electrons, nuclei and atoms are non-degenerate systems, consequently a non-zero *d* implies violation of T.

Electric Dipole Moment

Existence of EDM violates Time-Reversal and P symmetries:

$$P: \begin{cases} x \to & -x \\ p \to & -p \\ L \to & L \\ S \to & S \end{cases}$$

$$P: \begin{cases} q \to & q \\ j \to & j \\ E \to & -E \\ B \to & B \end{cases}$$

EDM operator does flip sign under P:

$$\hat{d} = \sum_i r_i q_i$$

But for a non-degenerate system:

$$\vec{d} = g_d \vec{J}$$
$$\vec{\mu} = g_M \vec{J}$$

Electrons, nuclei and atoms are non-degenerate systems, consequently a non-zero *d* implies violation of P.

Electric Dipole Moment

The operator for EDM: $\hat{d} = \sum_i r_i q_i$ **does not flip sign under T.**

But the eigenvalue does:

$$\begin{cases} \hat{d}\left|\psi\right\rangle = d\left|\psi\right\rangle \\ \hat{d}\left|T\psi\right\rangle = -d\left|T\psi\right\rangle \end{cases}$$

Naively: EDMs on composite neutral systems (e.g. atoms) cancel out.

1) Schiff showed that this is **incorrect** because of the finite size of the atom
nuclear charge density ≠ dipole-moment density.
Suppression factor = (Z nuclear size/atom size)^2

2) Sandars (relativistic effects; contribution from electrons):
EDM(atom) α $Z^3\alpha^2$ EDM(e)
enhancement ≈ 10^3 for Tl

3) Sandars : In polar molecules field at atom can be much larger than external field.
Enhancements ≈ 10^3

Neutron EDM; Ramsey,

Dress et al. Phys. Rep. 43, 410 (1978).

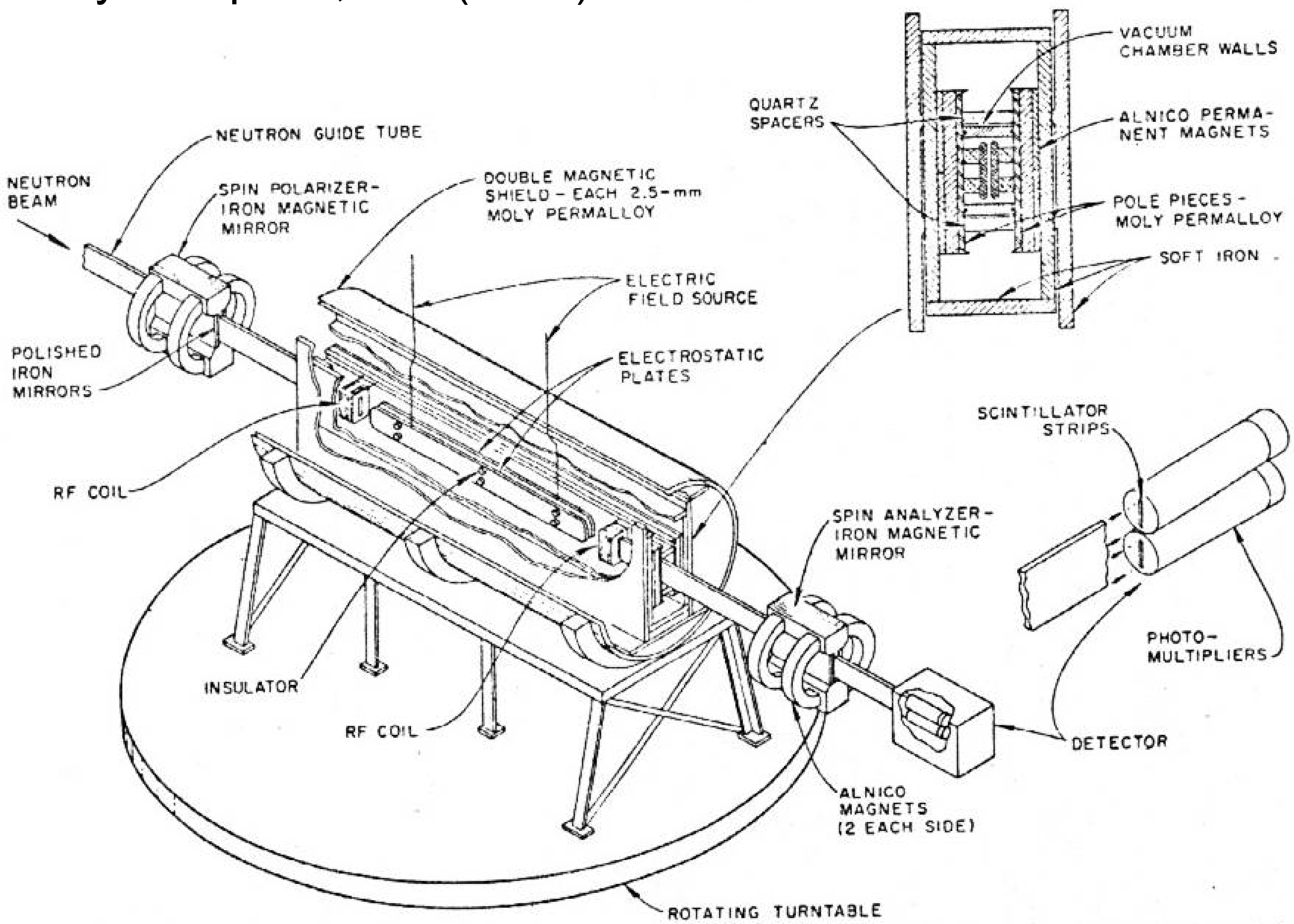
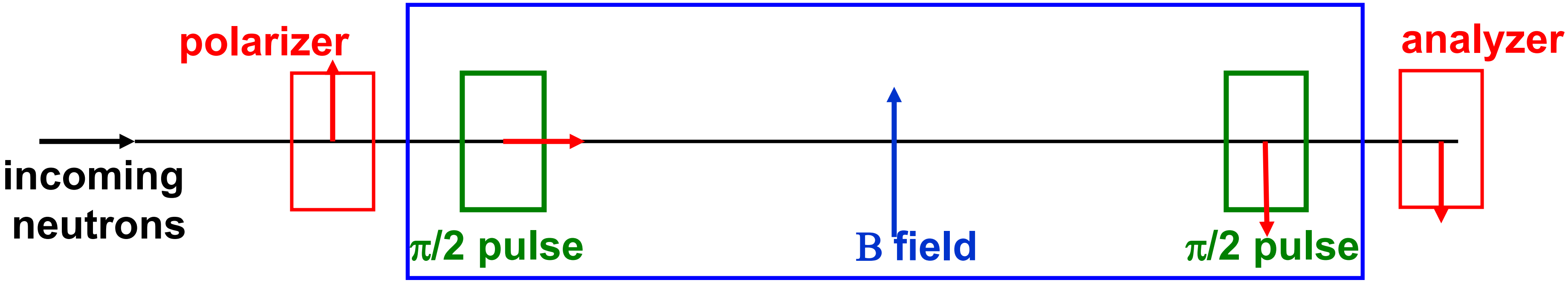


FIGURE 1 Pictorial representation of the neutron-beam spectrometer. The insert, upper right, is a cross-sectional view through the midpoint of the apparatus indicating various materials used in construction. The gap between the magnetic poles in 9 cm.

Neutron EDM; Ramsey,

Dress et al. Phys. Rep. 43, 410 (1978).

Simplified version



Neutron EDM; Ramsey,

Dress et al. Phys. Rep. 43, 410 (1978).

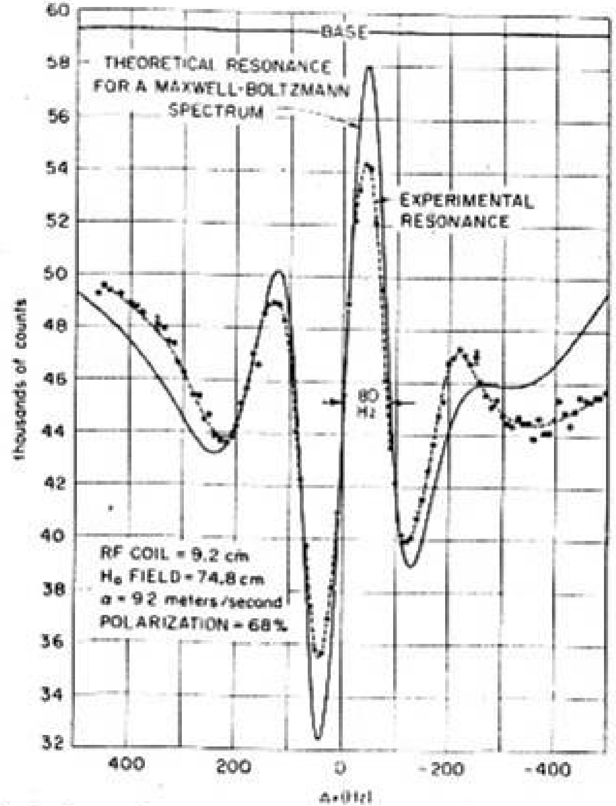


FIGURE 2 Typical magnetic resonance with the neutron-beam apparatus for a phase shift of $\pi/2$ between the two oscillatory fields. The calculated transition probability for a Maxwell-Boltzmann distribution characterized by a temperature of 1 K is shown by the solid curve. The departure of the experimental curve from theory when far from resonance is to be expected from the known departure of the beam velocity from a Maxwell-Boltzmann distribution.

*From Rosenberry and Chupp, PRL **86**, 22 (2001).*

d(129Xe) = 0.7±3.3(stat)±0.1(syst) x 10^{-27} *e* cm.

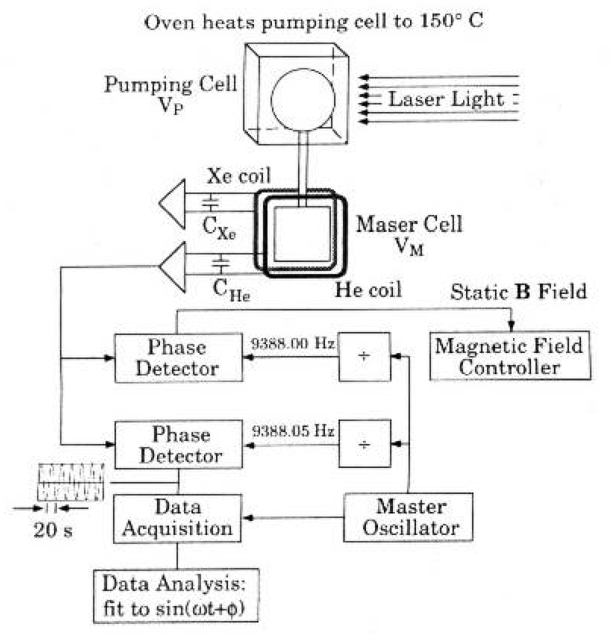


FIG. 1. The noble gas Zeeman maser apparatus. The two-cell system necessary for dual species operation is illustrated. For the experimental results presented in this paper, we used a single cell in the measurement cell position. The laser optically pumps Rb atoms that transfer spin to the noble gas nuclei. Each pickup coil is part of a tank circuit tuned to the noble gas precession frequency. The magnetic energy stored in the tank circuit is oscillating at the precession frequency and causes Rabi precession, which rotates the spins toward the lower energy state.

Present limits on EDM's

d(n)<6.3 10^{-26} e cm, PRL **82**, 904

d(e) <1.6 10^{-27} e cm, PRL **88**, 071805

d(199Hg)<2.1 10^{-28} e cm PRL **86**, 2506

Large potential enhancement in nuclei

Haxton, Henley, PRL 51, 1937 (1983).

Spevak, Auerbach, Flambaum, PRC 56, 1357 (1997).

Engel, Friar and Hayes, PRC 61, 035502 (2000).

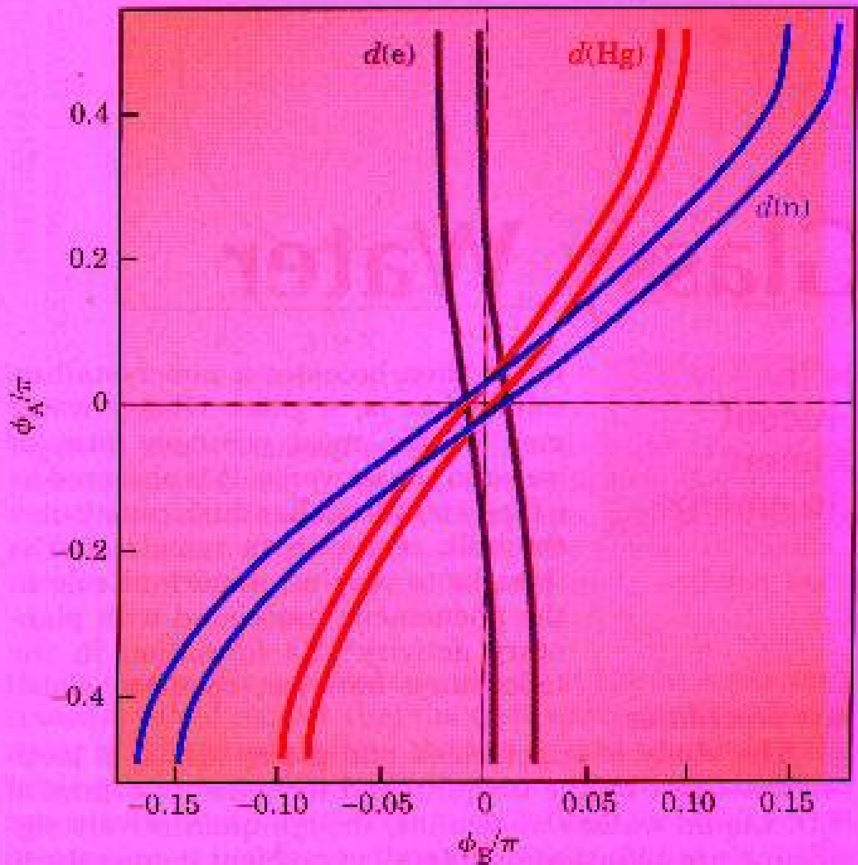


Figure 4. The phase angles that lead to *CP* (charge conjugation–parity) violations in minimal supersymmetric extensions of the standard model are severely constrained by electric dipole moment experiments on neutrons, mercury-199 atoms, and electrons. The angles ϕ_A and ϕ_B allowed by different experiments overlap only in a small region about zero, assuming a minimum superpartner mass of 500 GeV. (See ref. 8; updated figure courtesy of Maxim Pospelov, University of Victoria, Canada.)

The End