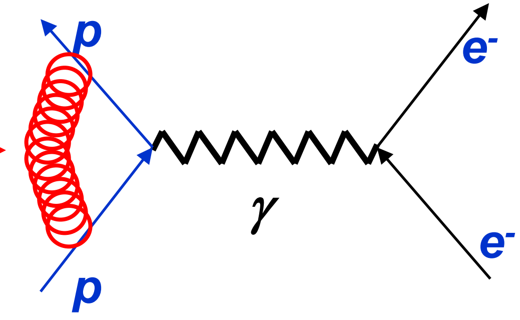
Weak Interactions in the Nucleus II

Summer School, Tennessee

June 2003

Recoil effects in E&M

Hadrons exchange **gluons**
so need to include most general
Lorentz-invariant terms in
interaction



Recoil effects

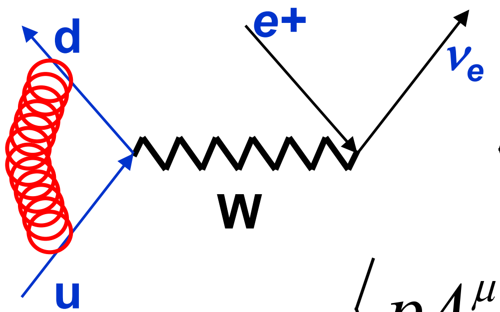
$$\left\langle p V_{E\&M}{}^{\mu} p \right\rangle \Leftrightarrow \left\langle p\left(g_V \gamma^{\mu} + g_{WM}\sigma^{\mu\nu}q_{\nu} + g_S q^{\mu}\right)p\right\rangle$$

Anomalous magnetic moment

Has to be zero to allow conservation of charge:

$$\left\langle \partial_{\mu} V_{E\&M}{}^{\mu} \right\rangle = \left\langle\left(g_V q_{\mu}\gamma^{\mu} + g_{WM} q_{\mu}\sigma^{\mu\nu}q_{\nu} + g_S q_{\mu}q^{\mu}\right)\right\rangle = g_S q^2 \neq 0$$

Recoil effects in Weak decays



Recoil effects

$$\langle p V^{\mu} n \rangle \Leftrightarrow \langle p \left(g_V \gamma^{\mu} + g_{WM} \sigma^{\mu\nu} q_{\nu} + g_S q^{\mu} \right) n \rangle$$

$$\langle p A^{\mu} n \rangle \Leftrightarrow \langle p \left(g_A \gamma^{\mu} \gamma_5 + g_T \sigma^{\mu\nu} q_{\nu} \gamma_5 + g_{PS} q^{\mu} \gamma_5 \right) n \rangle$$

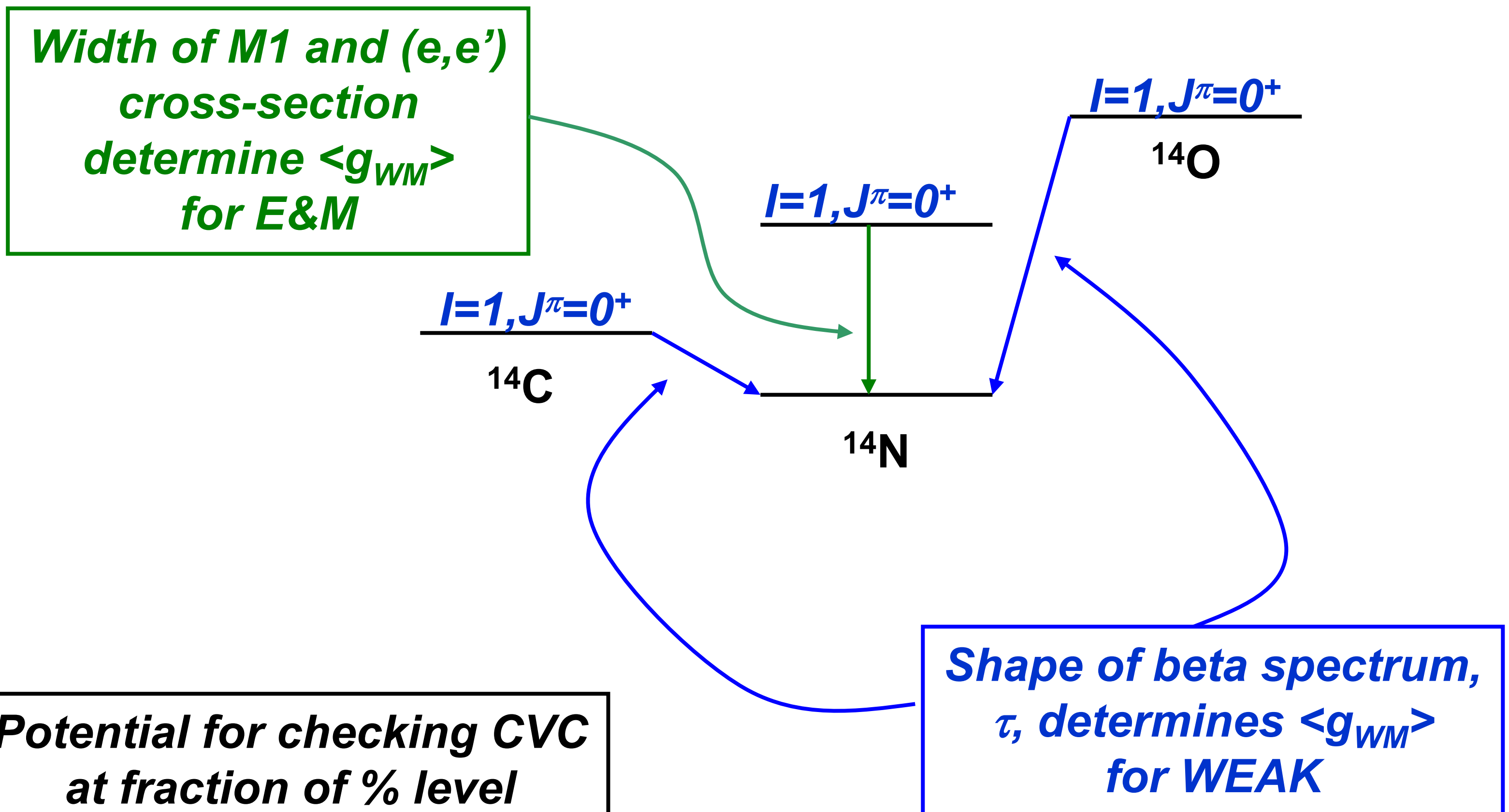
g_S and g_{WM}: Conservation of the Vector Current:
***I*=1 form factors in $V_{E\&M}$ are identical to form factors in V_{WEAK}**

g_{PS}: Partial Conservation of the Axial Current (plus pion-pole dominance):

$$g_{PS} = -\frac{2i\, m_p\, g_A}{q^2 + m_{\pi}^2}$$

g_T: Second Class Currents: breaking of G-parity

Conservation of the Vector Current:
I=1 form factors in $V_{E\&M}$ are identical to form factors in V_{WEAK}



Potential for checking CVC at fraction of % level

A. Garcia and B.A. Brown, Phys Rev. C 52, 3416 (1995).

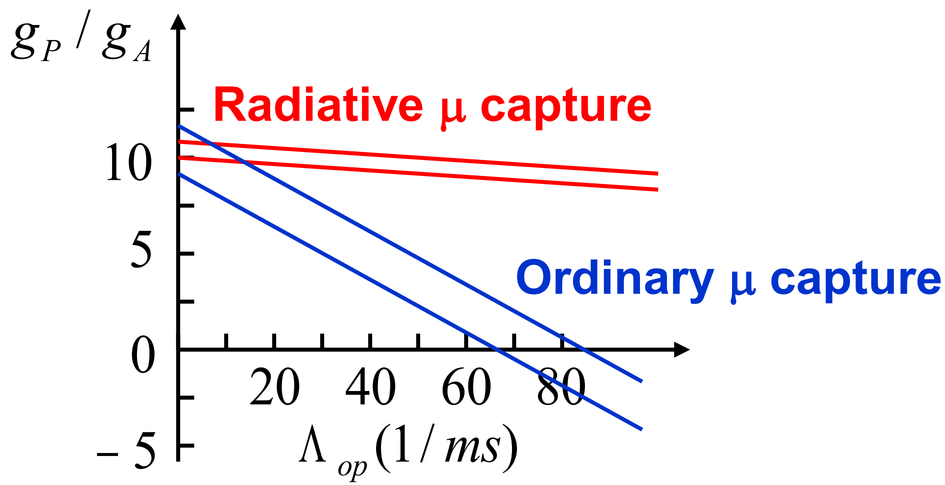
g_{PS}: Partial Conservation of the Axial Current (plus pion-pole dominance):

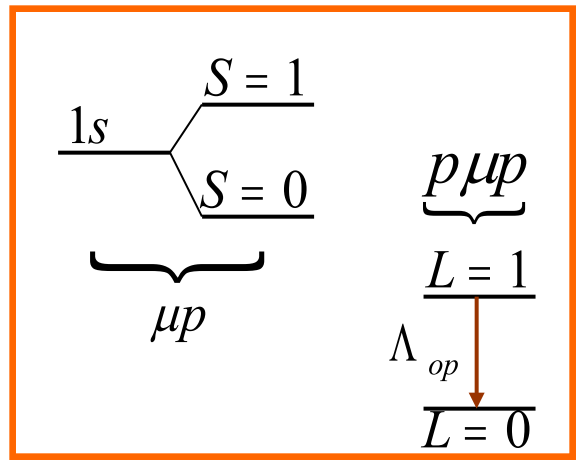
$$g_{PS} = -\frac{2i\, m_p\, g_A}{q^2 + m_\pi^2}$$

Approximation should hold very well : *u* and *d* quarks are very light chiral symmetry: V. Bernard et al. *Phys. Rev. D 50, 6899 (1994).*

Measure intensity of $\mu + p = \nu + n + \gamma$

$$g_P = m_\mu g_{PS}(0.88 m_\mu^2) = (6.7 \pm 0.2) g_A$$





g_T: Second Class Currents: breaking of G-parity

In 1970's evidence that $(ft)^+/(ft)^-$ changed linearly with end-point energy

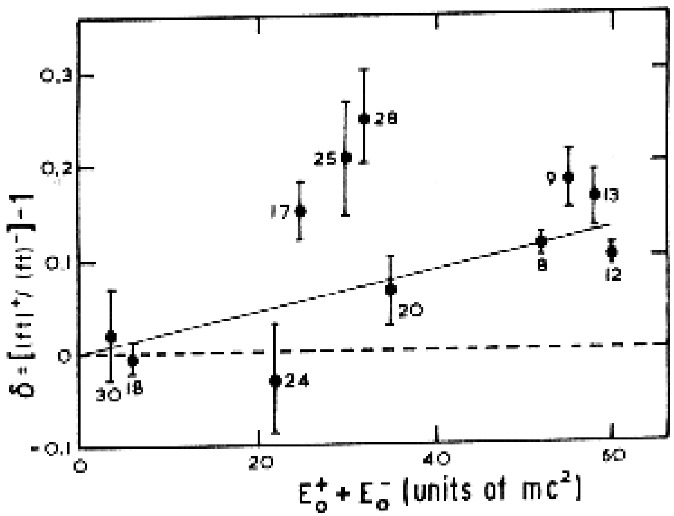


Fig. V-1. The asymmetry $\delta = [(ft)^+/(ft)^-] - 1$ in mirror axial vector β-decays. The sloping straight line corresponds to $f_T \approx 2 \times 10^{-3}$.

In the 70's Wilkinson pointed out that $ft(\beta^-) \neq ft(\beta^+)$ could originate in: $f_t \neq 0$.

Ist-class currents	II-class currents
$G\, V_\mu^I\, G^{-1} = +V_\mu^I$	$G\, V_\mu^{II} G^{-1} = -V_\mu^{II}$
$G\, A_\mu^I\, G^{-1} = -A_\mu^I$	$G\, A_\mu^{II}\, G^{-1} = +A_\mu^{II}$

where:

$G = C\, e^{-i\pi T_2}$

f_t and f_S represent the couplings for II Class Curr.

Large 2nd-Class Curr. are strongly disfavored by 1st-Class theorists

But: 1) we heard that before (Parity, Time-Reversal...)

2) even in the absence of new physics the differences in the u-d quark wave functions yields:

$f_t / f_A \approx 0.03$

Present upper limits are: $f_t / f_A \leq 0.4$

Observables

1) Angular distribution of $\beta^{\pm}$ from oriented nuclei.
For example, for $1^+ \rightarrow 0^+$:

$$dW/d\Omega \propto 1 \mp P(1+E\,\alpha_{\mp})\,P_1(\cos\theta) + A\,E\,\alpha_{\mp}\,P_2(\cos\theta)$$

$$\alpha_{\mp} = \frac{\pm F_{WM} - F_t^{I} \pm F_t^{II}}{3\,M\,F_A}$$

Alignment can be obtained (analyzed) by:
a) delicate manipulation of populations using external magnetic fields in crystals; (Minamisono et al. PRL **80,** 1998)
b) measuring β-γ angular correlations (Rosa et al. PRC **37,** 1988 Tribble et al. PRC **23,** 1981)

2) β-ν correlation:.
For example, for $1^+ \rightarrow 0^+$:

$$dW/d\Omega \propto 1 + X\,v/c\,P_1(\cos\theta) + (v/c)^2 E/M\;P_2(\cos\theta)$$

$$X = -1/3 + F_{WM}\;\ldots\; + F_t\;\ldots$$

From R. D. McKeown, et al.
Phys. Rev. C **22**, 738-749 (1980)

$$N_{\mp}(\theta,E,E_0)=F(E,E_0)[1+a_{\mp}(E,E_0)\cos\theta + p_{\mp}(E,E_0)\cos^2\theta],$$

$$p_{\mp}(E,E_0)=\frac{E}{2Mc}\left((c-d\pm b)+\frac{1}{(14)^{1/2}}\left\{\left[\pm 3f\pm(\tfrac{3}{2})^{1/2}g\frac{E_0-E}{M}+3j_2\frac{E_0-2E}{2M}\right]\right\}-3/(35)^{1/2}j_3E/M\right),$$

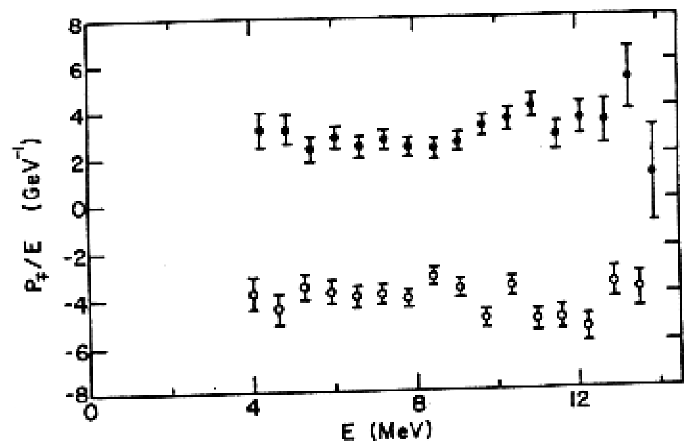


FIG. 16. The slopes of the $p_{\mp}$ coefficients vs beta energy when the data are analyzed without regard for final-state energy. The solid dots are p_-/E (^{8}Li decay) and the open circles are p_+/E (^{8}B decay).

The ratio $R(E)$ of β-ray counts detected in region II by the up (down) counter with alignment A_+ to the counts with alignment A_- is given as

$$R(E) - 1 = N(E, A_+, P_+)/N(E, A_-, P_-) - 1$$
$$= (-1)^{\lambda+1/2}\Delta P(B_1/B_0) + \Delta A(B_2/B_0), \quad (2)$$

From Minamisono et al.
Phys. Rev. Lett. 80, 4132(1998).

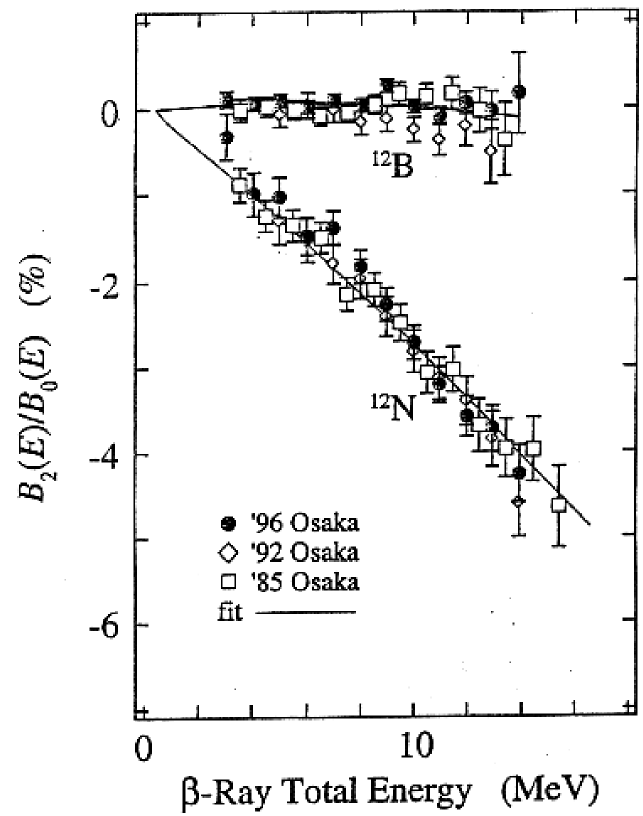


FIG. 2. Values of B_2/B_0 from aligned ^{12}B (^{12}N). Three sets of $(B_2/B_0)_\mp$ obtained at different times are shown (circles: 1996; diamonds: 1992; squares: 1985). A $2M(f_T/f_A)$ value is extracted from the best fit of the theoretical curve to each set of data. The solid lines are the theoretical curves with the weighted mean values given in Table II.

The ratio is given as $B_2(E)/B_0(E)/E =$

Angular momentum and Rotations

Rotating the coordinate system:

$$\vec{x}^R = R_z(\alpha)\vec{x}$$

$$\psi^R(\vec{x}) = U_z(\alpha)\psi(\vec{x})$$

Invariance: $$\psi(\vec{x}) = \psi^R(\vec{x}^R) = (1 + \delta\alpha\frac{\partial}{\partial\varphi})\psi^R(\vec{x})$$

$$(1 - \delta\alpha\frac{\partial}{\partial\varphi})\psi(\vec{x}) = \psi^R(\vec{x}) \qquad U_z(\alpha) = (1 - \delta\alpha\frac{\partial}{\partial\varphi})$$

For any rotation: $$U_{\vec{n}}(\alpha) = e^{-i\alpha\vec{J}.\vec{n}/\hbar}$$

Invariance under rotations imply conservation of J

Isospin

Notice *n+n, n+p, p+p* hadronic interactions are very similar.

Use spin formalism to take into account Pauli exclusion principle etc.

$$\left|p\right\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad \left|n\right\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$I^{+}\left|n\right\rangle = \left|p\right\rangle; \quad I^{-}\left|p\right\rangle = \left|n\right\rangle;$$

$$\hat{q} \equiv e\left(\frac{1}{2} + \hat{I}^{z}\right).$$

$$I^{z}\left|n\right\rangle = -\frac{1}{2}\left|n\right\rangle; \quad I^{z}\left|p\right\rangle = +\frac{1}{2}\left|p\right\rangle;$$

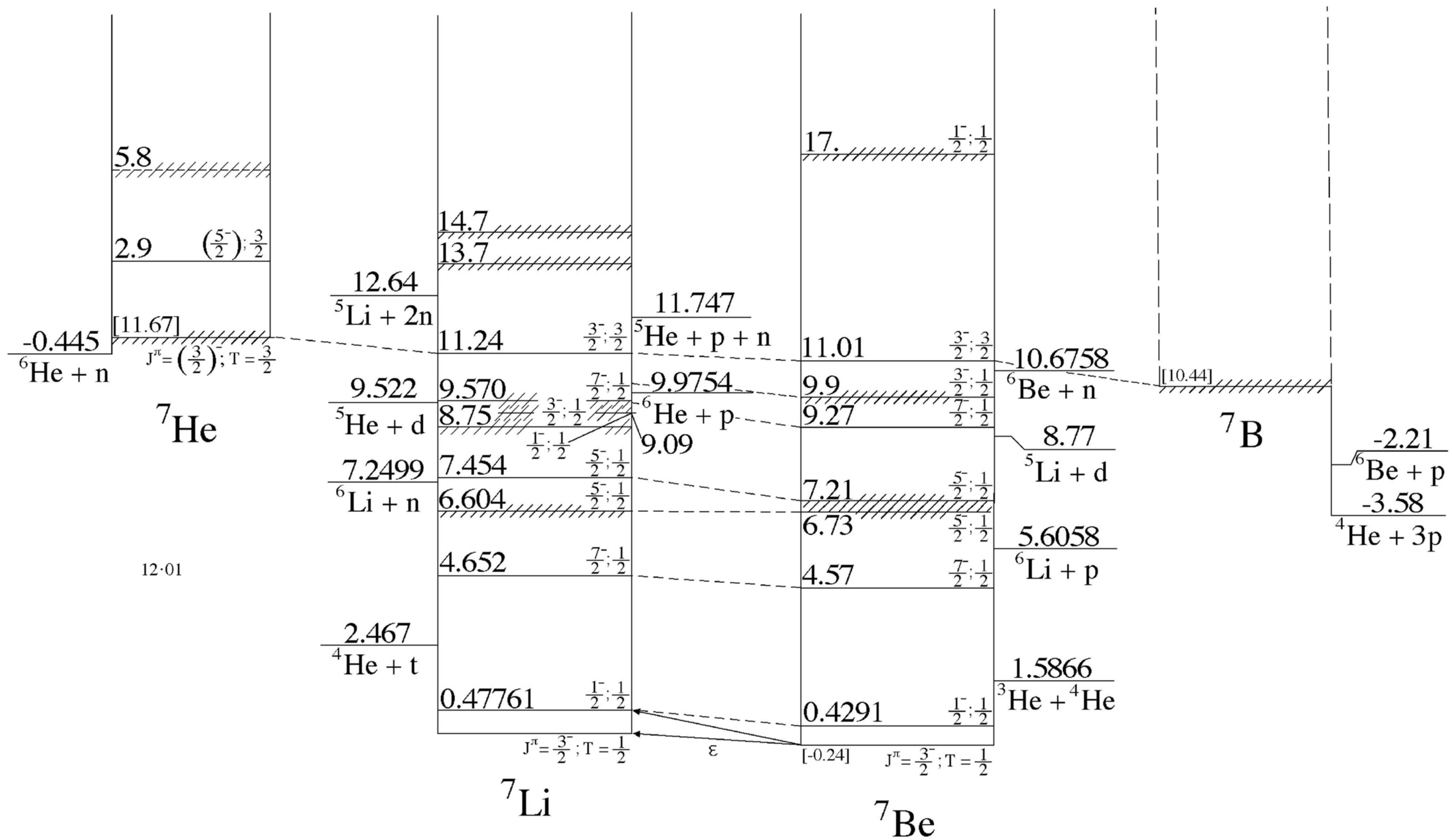


Figure 11: Isobar diagram, $A = 7$. For notation see Fig. 3.

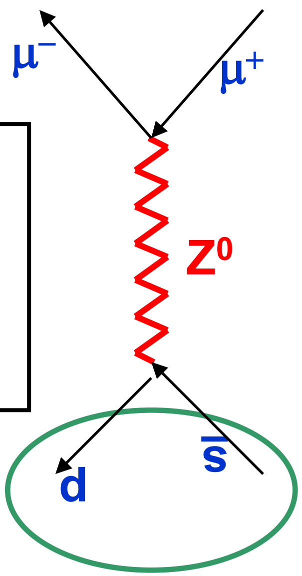
Weak Decays of Quarks

Once upon a time: $u \; d \; s$

Weak interactions needed neutral component to render *g sin*θ = *e.*

But the process
$K_L^0 \rightarrow \mu^+\mu^-$
could not be observed.

Why not??



Weak Decays of Quarks

$$\begin{pmatrix} u \\ d\cos\vartheta + s\sin\vartheta \end{pmatrix} \quad \begin{pmatrix} c \\ s\cos\vartheta - d\sin\vartheta \end{pmatrix}$$

G.I.M. proposed

The neutral weak currents go like:

$$\langle d\cos\vartheta + s\sin\vartheta | J^{nc} | d\cos\vartheta + s\sin\vartheta \rangle +$$
$$\langle d\cos\vartheta - s\sin\vartheta | J^{nc} | d\cos\vartheta - s\sin\vartheta \rangle =$$
$$\langle d | J^{nc} | d \rangle + \langle s | J^{nc} | s \rangle$$

No strangeness-changing Neutral Currents

Weak Decays of Quarks

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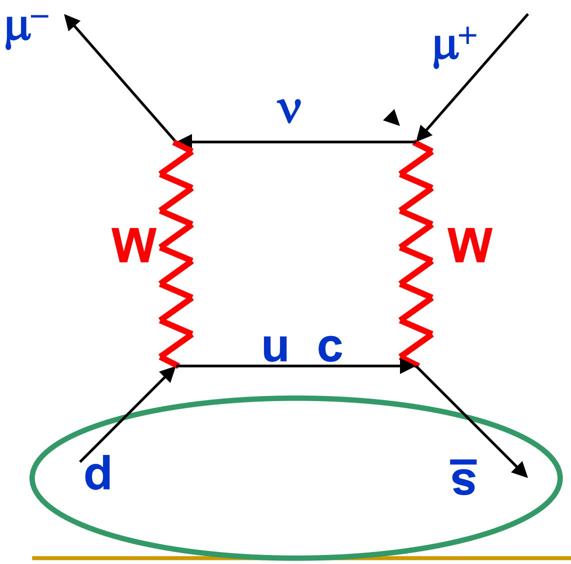


The process $K_L^0 \rightarrow \mu^+\mu^-$ actually goes through this diagram

Finding J/Psi confirmed the existence of the c quark and gave validity to the G.I.M. hypothesis

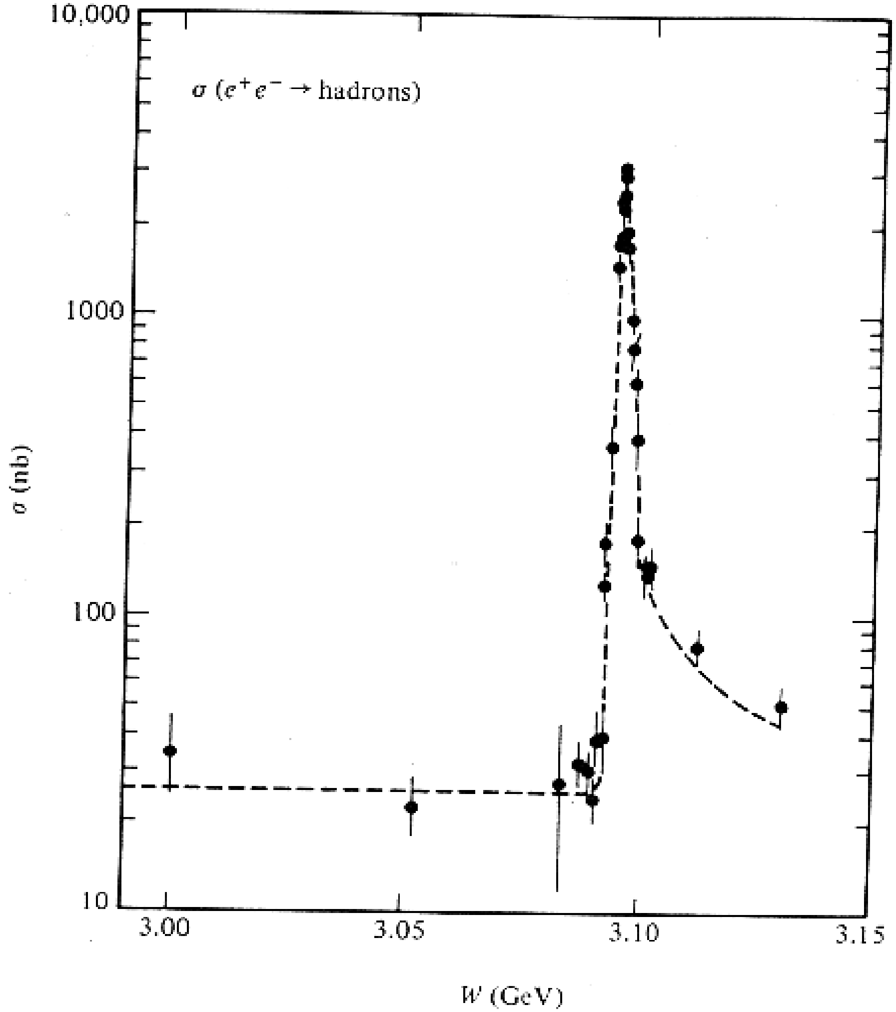
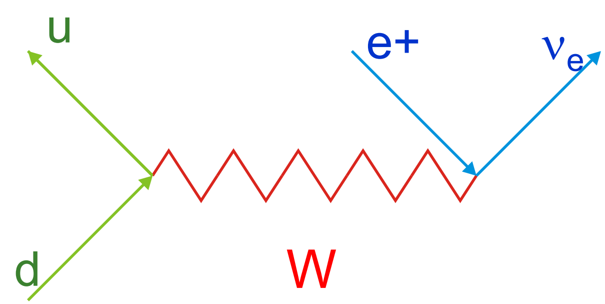


Fig. 10.23. Total hadron production cross section in e^+e^- collisions near 3.1 GeV and the J/ψ peak. [From A. M. Boyarski et al., *Phys. Rev. Lett.* **34**, 1357 (1975).]

Weak decays in the Standard Model

			Q	I
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$	0 -1	1/2
$\begin{pmatrix} u \\ d' \end{pmatrix}$	$\begin{pmatrix} c \\ s' \end{pmatrix}$	$\begin{pmatrix} t \\ b' \end{pmatrix}$	+2/3 -1/3	1/2



From nuclear 0.974 (→ V_{ud})

Ke3 0.220 (→ V_{us})

$b \to u l \nu$ 0.080 (→ V_{ub})

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} Vud & Vus & Vub \\ Vcd & Vcs & Vcb \\ Vtd & Vts & Vtb \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

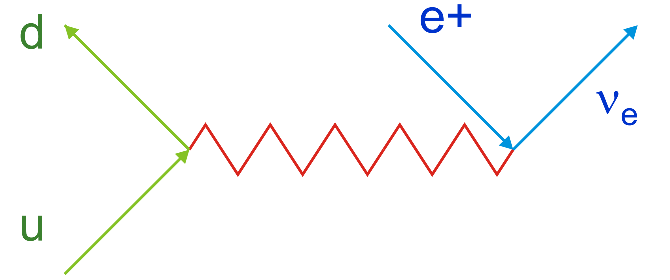
CKM matrix: Is it really Unitary?

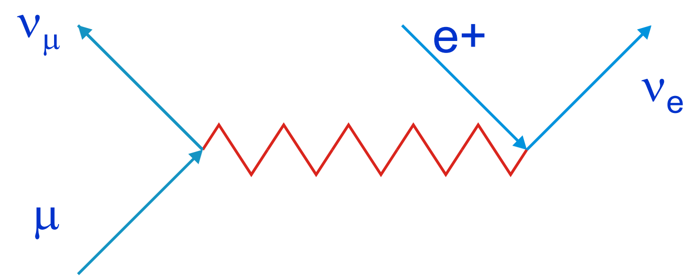
In order to measure V_{ud} we compare intensities for semi-leptonic to purely leptonic decays

Fermi's Golden rule:
$\tau^{-1}\ \alpha\ |\langle f\ H\ i\rangle|^2\ f(E)$

Then:
$|\langle \boldsymbol{f\ I\ i} \rangle|^2\ V_{ud}^2 \approx f\tau_{\mu}\ / f\tau_{quarks}$

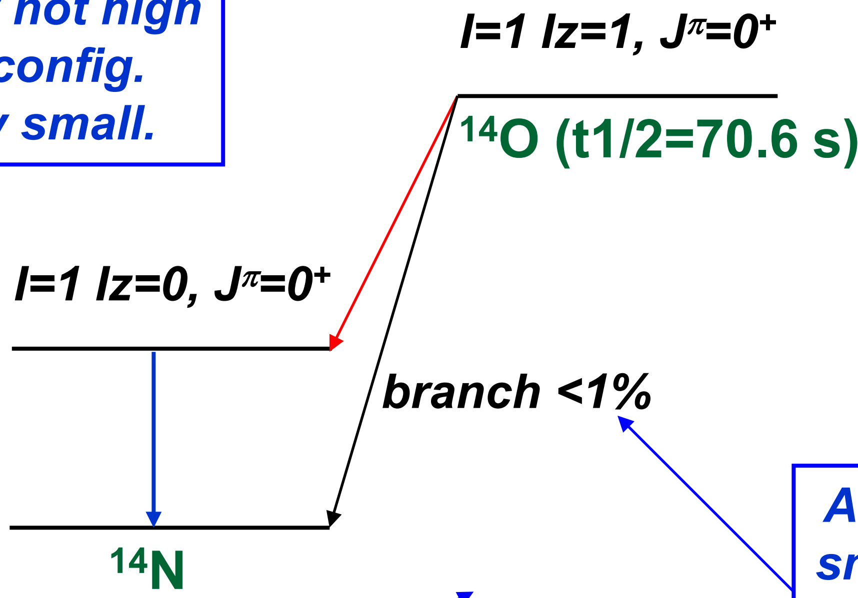
$$\langle I, I_z - 1|\hat{I}^-|I, I_z\rangle = \sqrt{(I + I_z)(I - I_z + 1)}$$





Example: decay of ^{14}O

Level density not high so Isospin config. mixing very small.



^{14}O easy to produce and half-life convenient for separating from other radioactivity.

Additional branches so small that beta intensity can almost be obtained directly from ^{14}O half-life

Many features contribute to allowing a precise determination of ft

Nuclear weak decays are driven by two currents : V_μ and A_μ.
V_μ is conserved (in the same sense that the electromagnetic current is conserved).

Initially most precise results came from decays for which only V_μ can contribute:

J^π(Initial nucleus)=0^+ → J^π(Final nucleus)=0^+

From Hardy et al, Nucl. Phys. A509, 429 (1990).

Note this range is only 0.5%

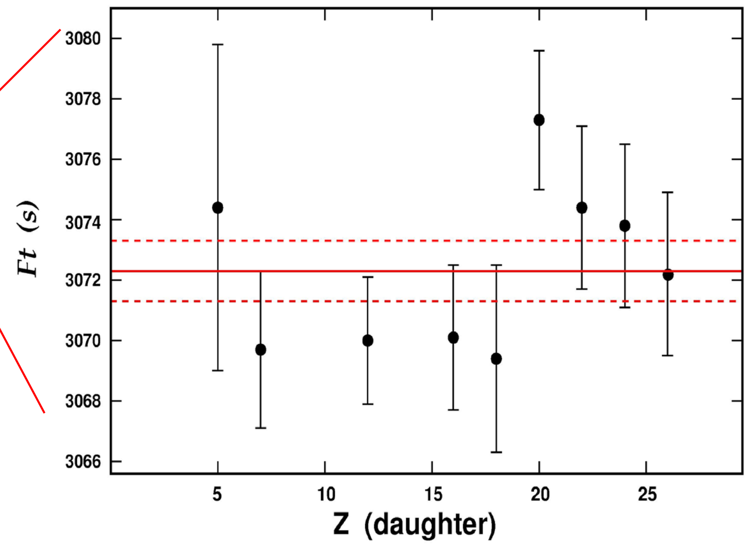


Figure 2. $\mathcal{F}t$ values for the nine well-known cases and the best least-squares one-parameter fit.

Nuclear 0+ → 0+ decays:

$$|Vud|^2 + |Vus|^2 + |Vub|^2 = 0.9968 \pm 0.0014$$

2.3σ away from 1

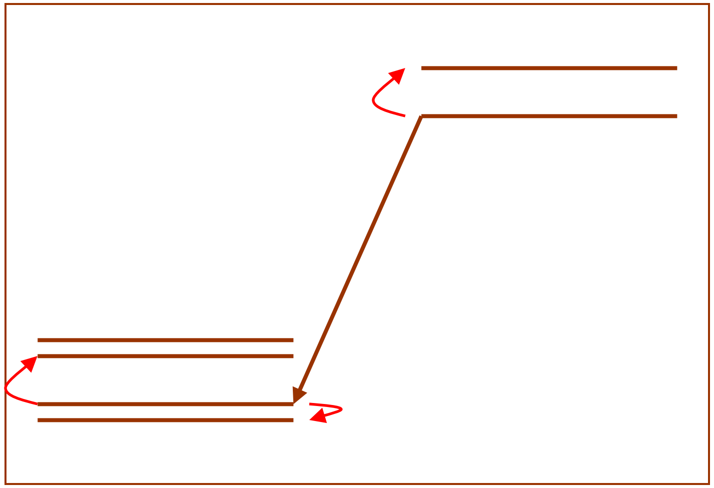
Complication: Isospin symmetry breaking

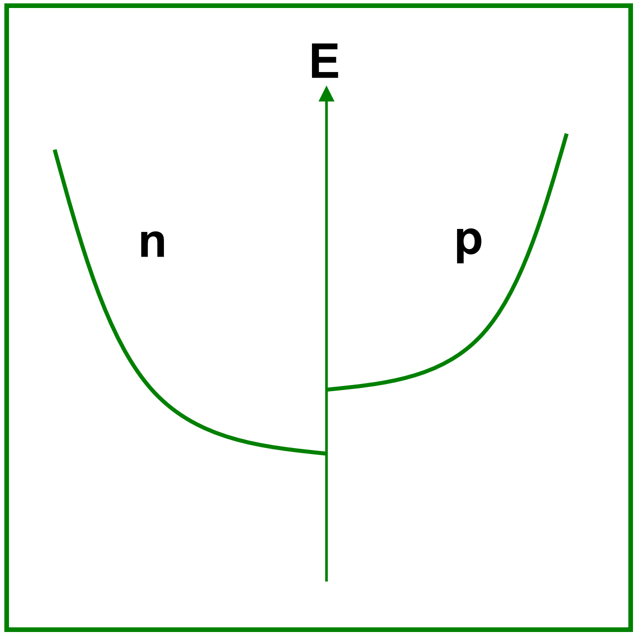
Nuclei do have charge and $\left|\psi\right\rangle = \left|I, I_z\right\rangle + \varepsilon\left|I \pm 1, I_z\right\rangle$

To understand it we can separate it into effects at two different levels:

1) decaying proton and new-born neutron sample different mean fields.

2) shell-model configurations are mixed





Radiative and isospin breaking corrections have to be taken into account.

Table 1. Experimental results (Q_{EC}, $t_{1/2}$ and branching ratio, R) and calculated corrections (δ_C and δ_R) for $0^+ \rightarrow 0^+$ transitions.

	Q_{EC} (keV)	$t_{1/2}$ (ms)	R (%)	ft (s)	δ_C (%)	δ_R (%)	$\mathcal{F}t$ (s)
^{10}C	1907.77(9)	19290(12)	1.4638(22)	3040.1(51)	0.16(3)	1.30(4)	3074.4(54)
^{14}O	2830.51(22)	70603(18)	99.336(10)	3038.1(18)	0.22(3)	1.26(5)	3069.7(26)
^{26m}Al	4232.42(35)	6344.9(19)	$\geq$ 99.97	3035.8(17)	0.31(3)	1.45(2)	3070.0(21)
^{34}Cl	5491.71(22)	1525.76(88)	$\geq$ 99.988	3048.4(19)	0.61(3)	1.33(3)	3070.1(24)
^{38m}K	6043.76(56)	923.95(64)	$\geq$ 99.998	3047.9(26)	0.62(3)	1.33(4)	3069.4(31)
^{42}Sc	6425.58(28)	680.72(26)	99.9941(14)	3045.1(14)	0.41(3)	1.47(5)	3077.3(24)
^{46}V	7050.63(69)	422.51(11)	99.9848(13)	3044.6(18)	0.41(3)	1.40(6)	3074.4(27)
^{50}Mn	7632.39(28)	283.25(14)	99.942(3)	3043.7(16)	0.41(3)	1.40(7)	3073.8(27)
^{54}Co	8242.56(28)	193.270(63)	99.9955(6)	3045.8(11)	0.52(3)	1.39(7)	3072.2(27)

From Hardy et al, Nucl. Phys. A509, 429 (1990).

The complication of isospin-breaking corrections

can be circumvented by looking at:

1) $\pi^+ \rightarrow \pi^0 e^+ \nu$

2) $n \rightarrow p\, e^- \nu$

1) $\pi^+ \rightarrow \pi^0 e^+ \nu$ **has a very small branch (≈10^{-8});**

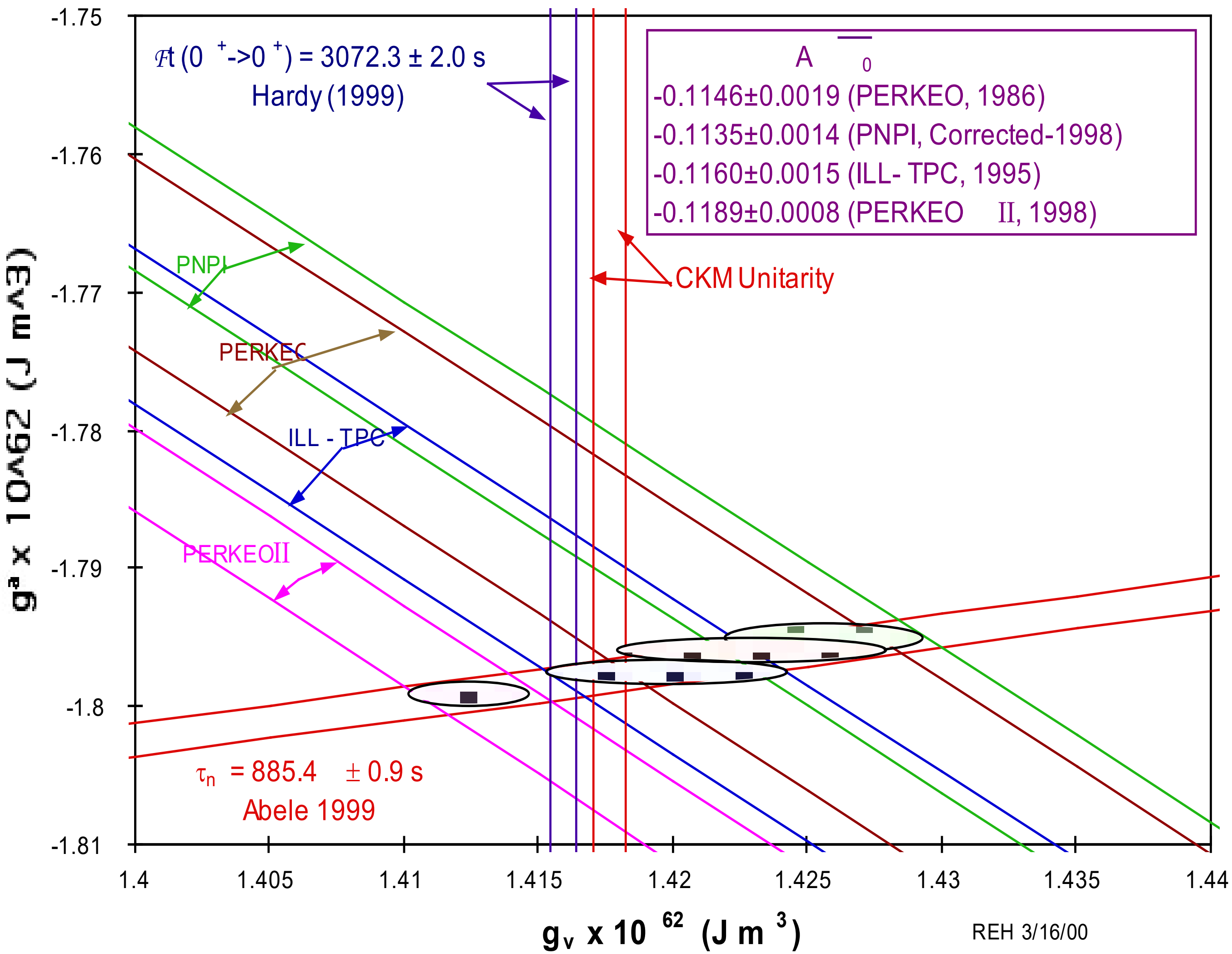
2) $n \rightarrow p\, e^- \nu$ **is a mixed transition (*V*μ and *A*μ contribute).**

Determinnig V_{ud} from neutron β decay

- **Disadvantage**: V_μ and A_μ contribute to this $J^\pi=1/2^+ \rightarrow 1/2^+$ decay. Consequently need to measure 2 quantities with precision.
- **Advantage**: Simplest nuclear decay. No isospin-breaking corrections.

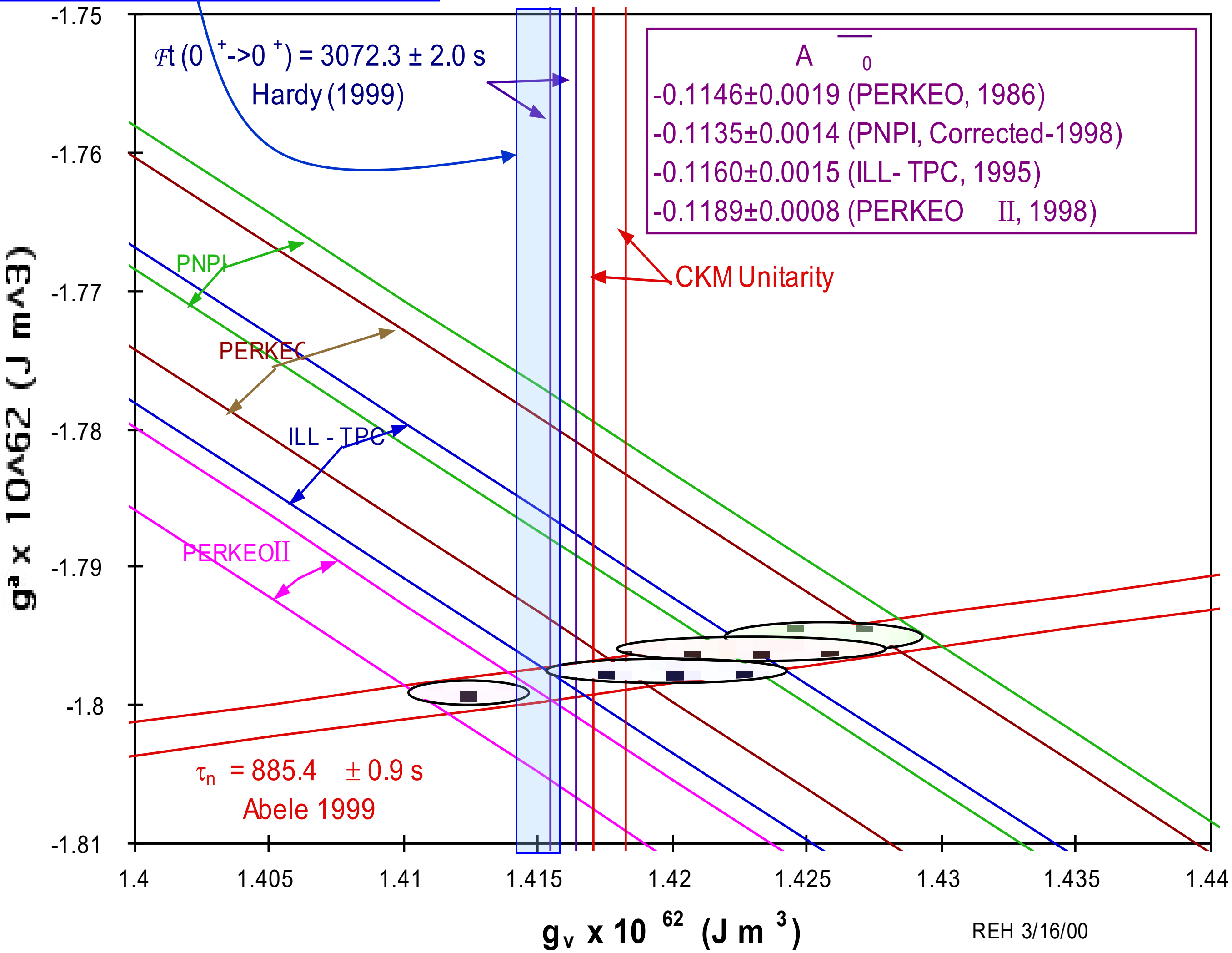
Neutron τ already well known: need to determine β asymmetry (e^- angular distribution) from polarized neutrons.

Weak Coupling Constants

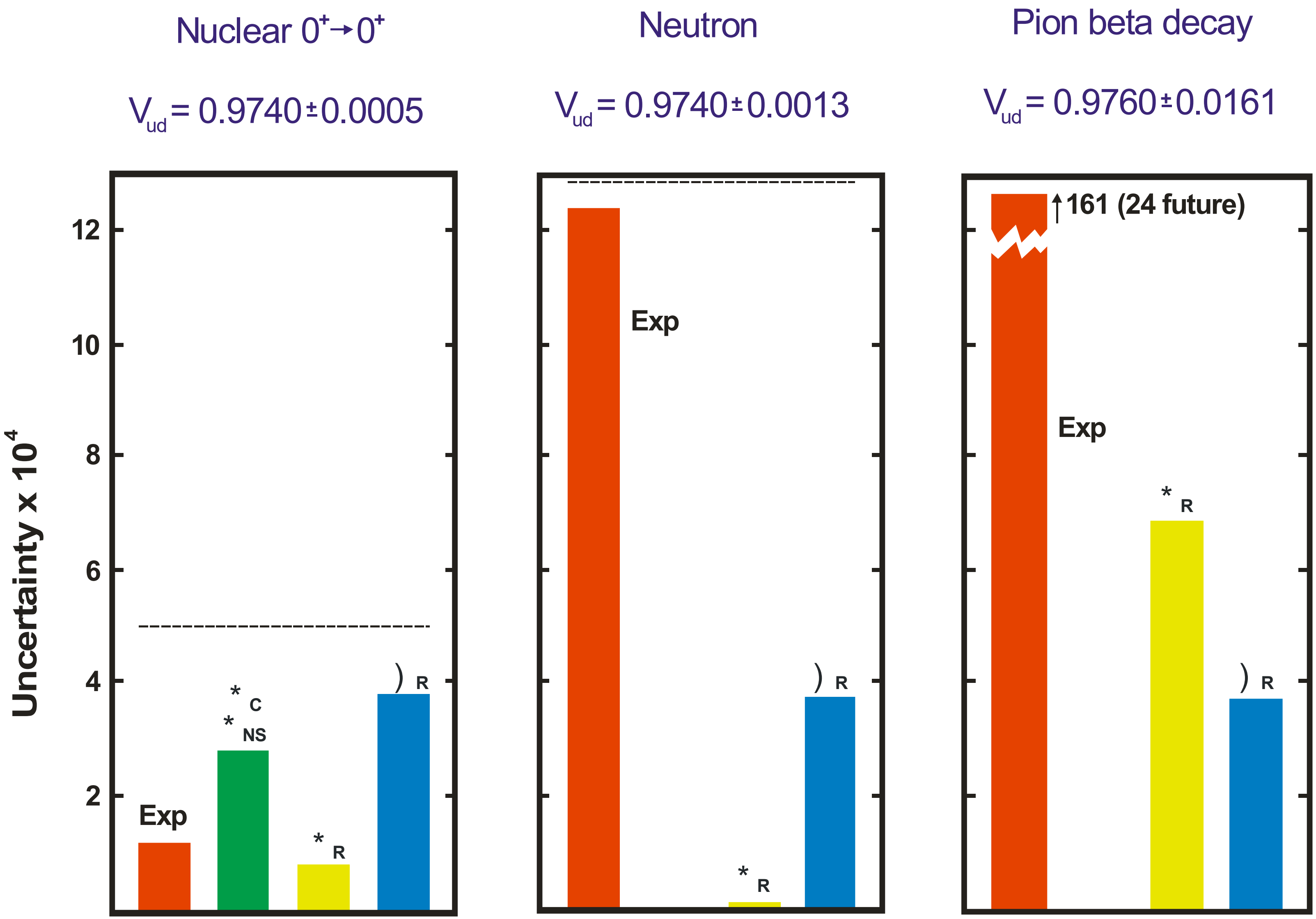


Weak Coupling Constants

Cold-Neutron Decay

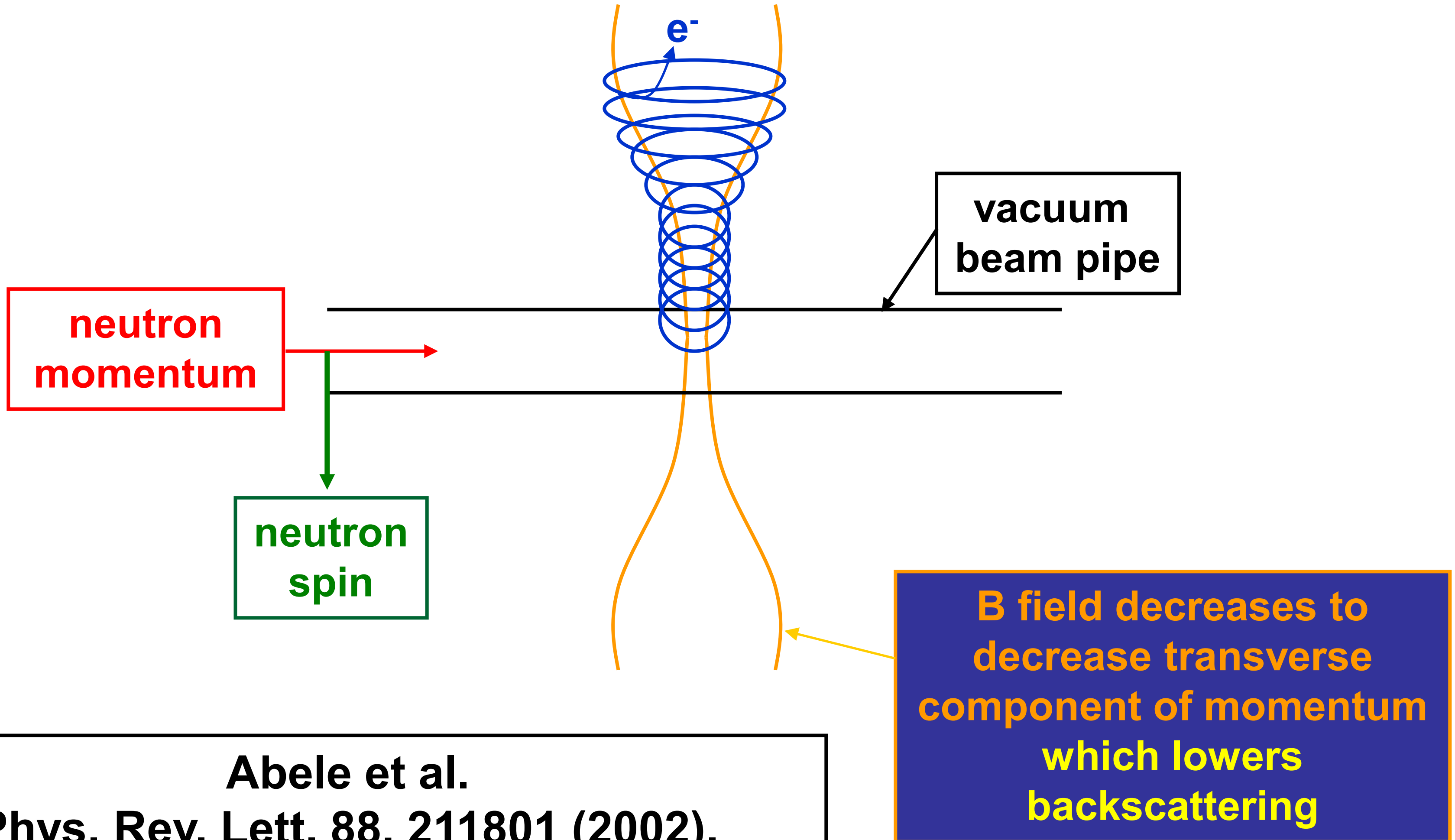


CONTRIBUTIONS TO V_{ud} UNCERTAINTY



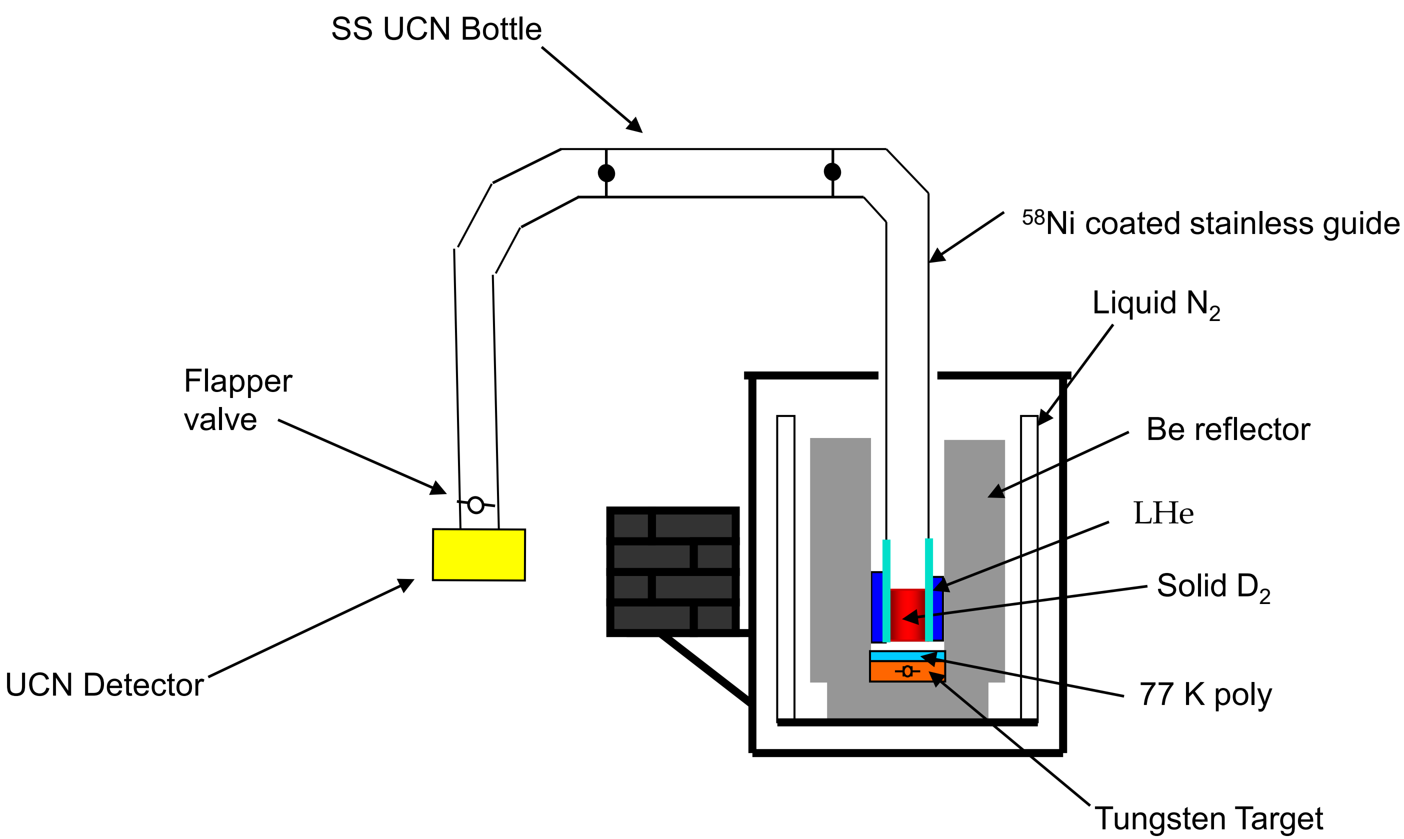
J.C. Hardy, 2003

Beta asymmetry with beam of Cold Neutrons ($v \approx 500$ m/s)



Abele et al.
Phys. Rev. Lett. 88, 211801 (2002).

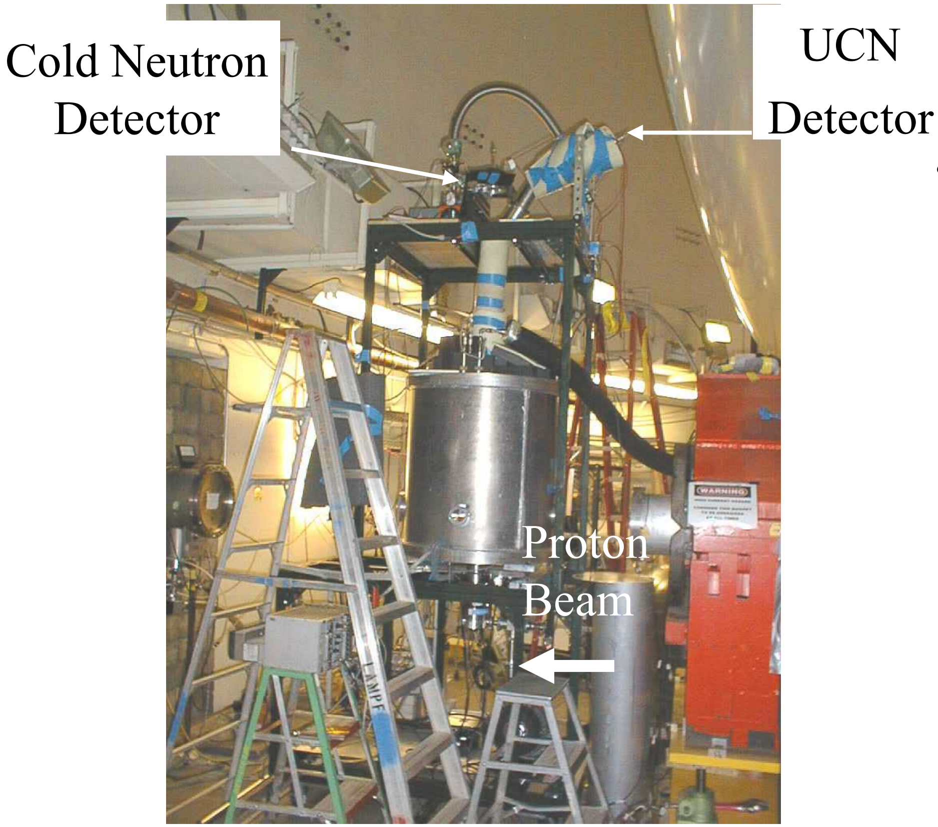
Ultra-Cold Neutron Source Layout (LANL)

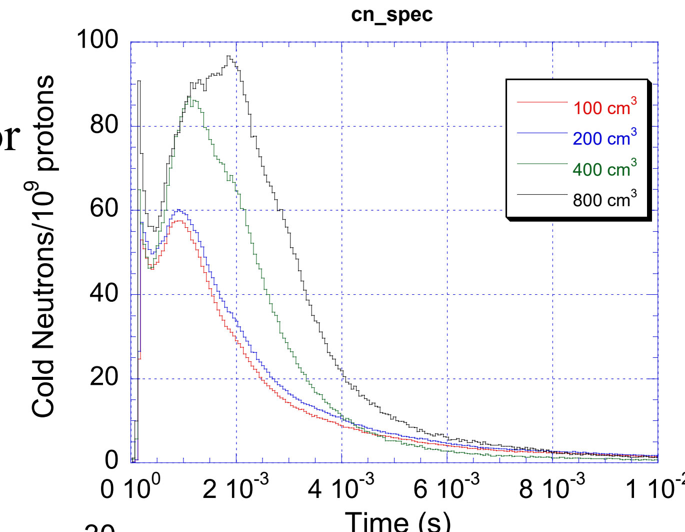


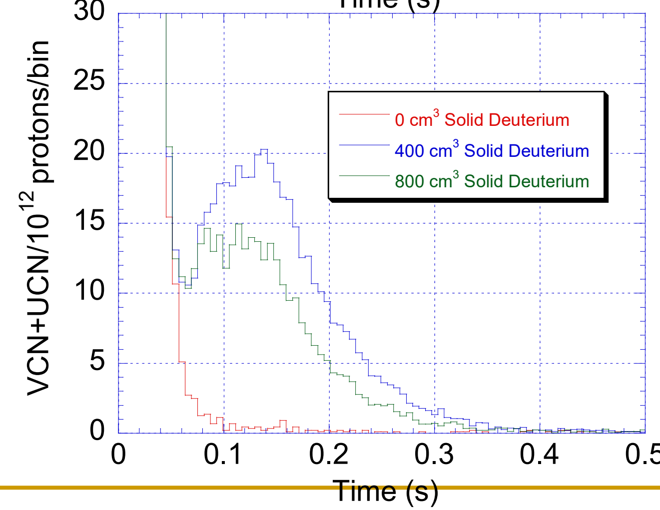
A. Saunders, 2003
C. Morris et al, PRL 89, 272501

Line C Measurements

A. Saunders, 2003
C. Morris et al, PRL 89, 272501



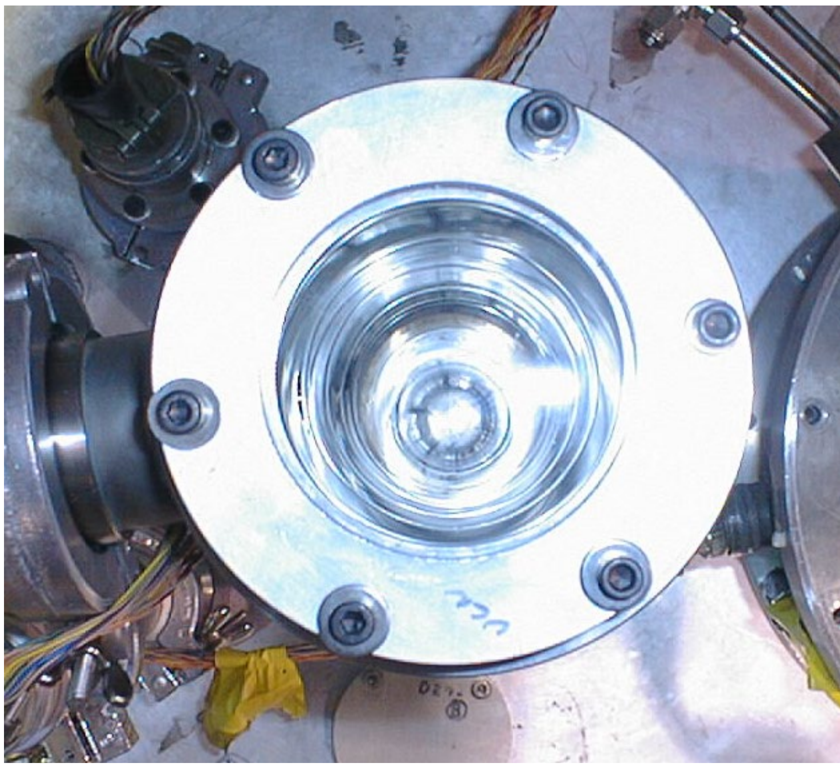




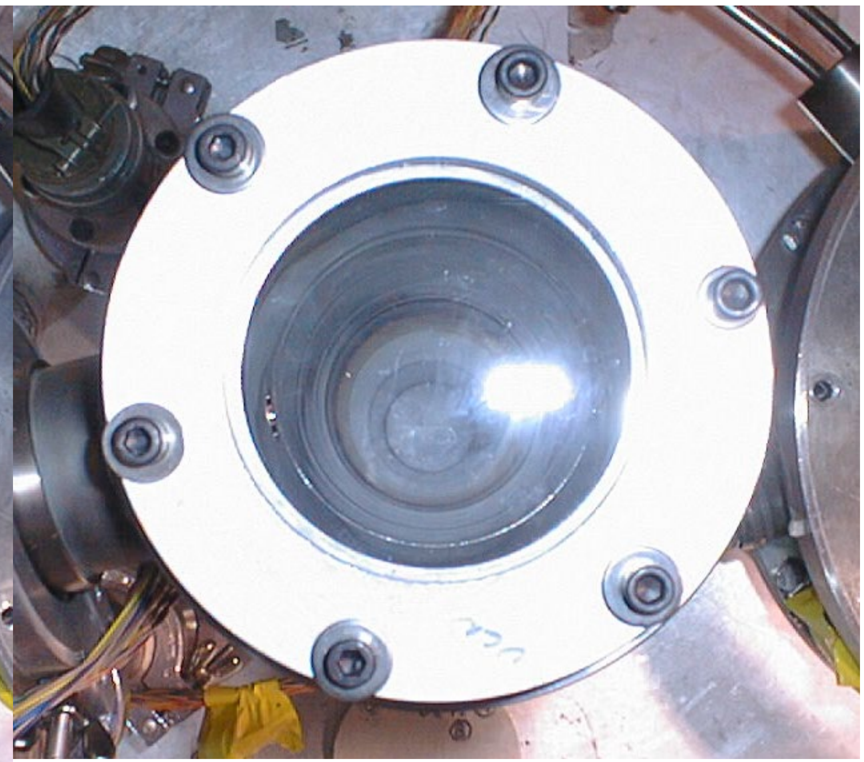
- Line C results show reduced (÷100) UCN production.
  - D_2 frost on guide windows and walls.
  - Gravity+Aluminum detector window

Solid D_2 in a "windowless" container

Grown from a gas phase at 50 mbar

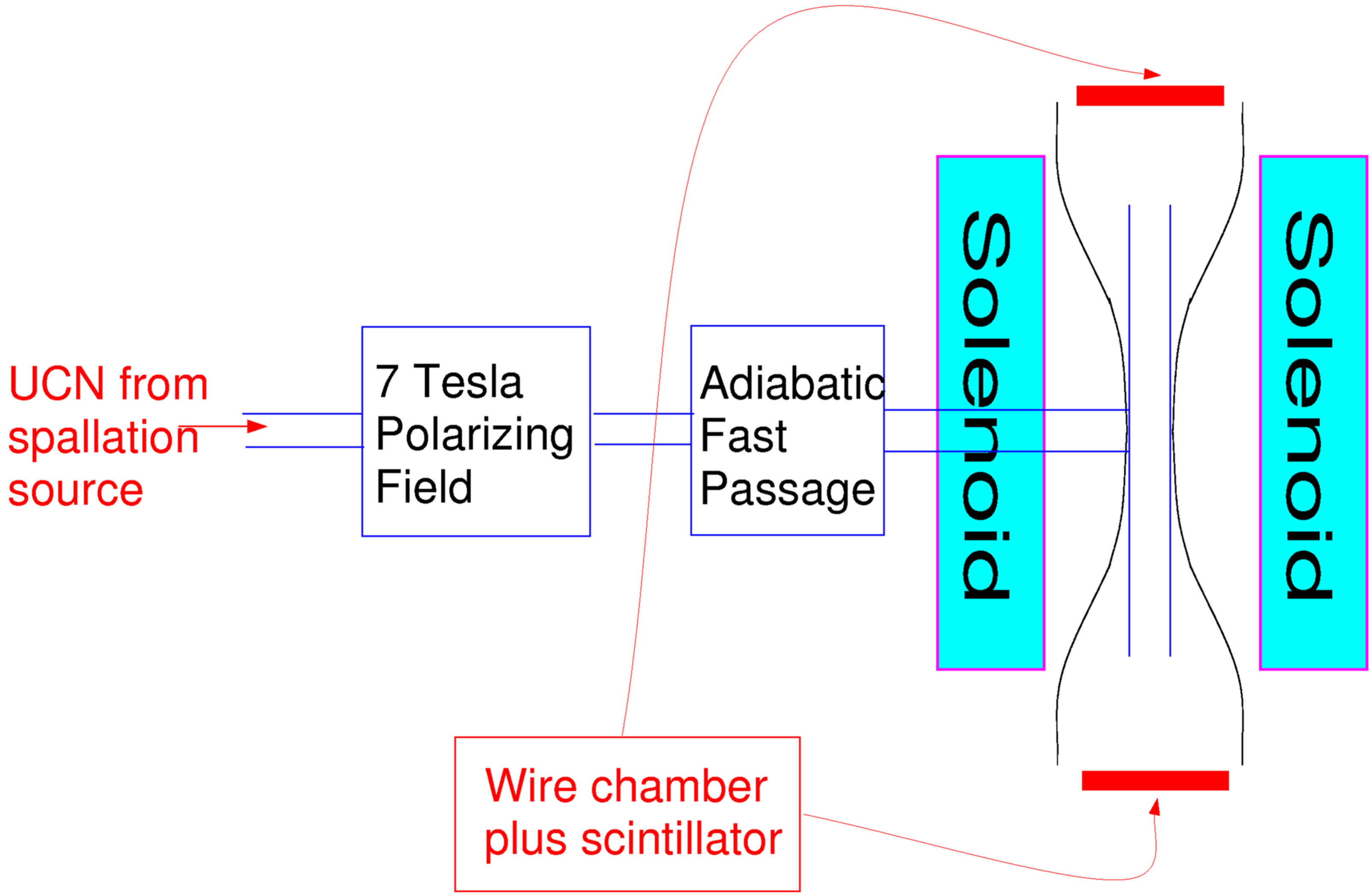


Cooled through the triple point



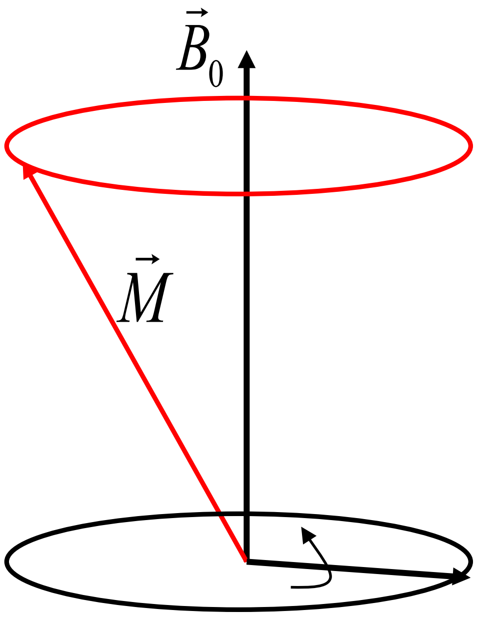
A. Saunders, 2003
C. Morris et al, PRL 89, 272501

β asymmetry from neutron decay using Ultra Cold Neutrons



Neutrons in Magnetic Field

$$\frac{d\vec{M}}{dt} = \gamma\, \vec{M} \times \vec{B} \qquad \vec{M} = \gamma\, \vec{J}$$



In a frame rotating with freq. ω:

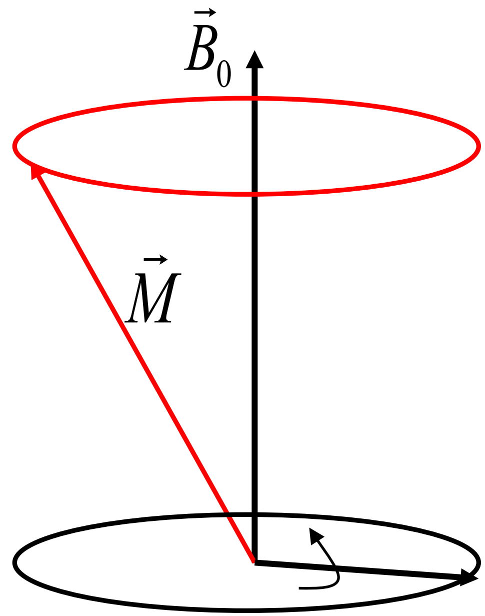
$$\frac{d\vec{M}}{dt} = \frac{\partial\vec{M}}{\partial t} + \vec{\omega} \times \vec{M}$$

$$\frac{\partial\vec{M}}{\partial t} = \gamma\, \vec{M} \times (\vec{B} + \frac{\vec{\omega}}{\gamma})$$

$$\vec{B}_e = \vec{B} + \frac{\vec{\omega}}{\gamma}$$

Neutrons in Magnetic Field

$$\frac{d\vec{M}}{dt} = \gamma\, \vec{M} \times \vec{B} \qquad \vec{M} = \gamma\, \vec{J}$$



Field B_1 rotating with freq. ω around B_0.

$$\vec{B} = \hat{k} B_0 + \vec{B}_1$$

In a frame rotating with freq. ω:

$$\frac{d\vec{M}}{dt} = \frac{\partial\vec{M}}{\partial t} + \vec{\omega} \times \vec{M}$$

$$\frac{\partial\vec{M}}{\partial t} = \gamma\, \vec{M} \times (\vec{B} + \frac{\vec{\omega}}{\gamma})$$

$$\vec{B}_e = \hat{k}(B_0 + \frac{\omega}{\gamma}) + \hat{i}\, B_1 \;\longrightarrow\; \vec{B}_e = \hat{k}(-\frac{\omega_0}{\gamma} + \frac{\omega}{\gamma}) - \hat{i}\,\frac{\omega_1}{\gamma}$$

strength of B_0

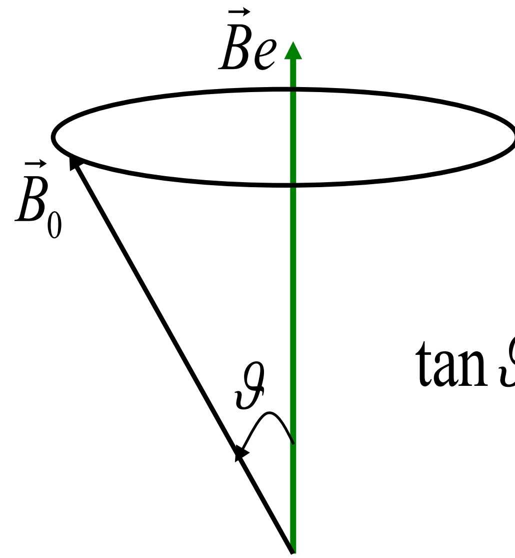
freq. of B_1

strength of B_1

Neutrons in Magnetic Field

$$\vec{B}_e = \hat{k}\left(-\frac{\omega_0}{\gamma} + \frac{\omega}{\gamma}\right) - \hat{i}\,\frac{\omega_1}{\gamma}$$

$$\left|\vec{B}_e\right| = \sqrt{\left(-\frac{\omega_0}{\gamma} + \frac{\omega}{\gamma}\right)^2 + \left(\frac{\omega_1}{\gamma}\right)^2} = -\frac{a}{\gamma}$$



$$\tan\vartheta = \frac{\omega_1}{\omega_0 - \omega}$$

In S' motion is precession around $\boldsymbol{B_e}$ with angular velocity $\boldsymbol{a = -\omega B_e}$

$$\cos\alpha = \frac{\vec{B}_0 \cdot \vec{M}}{\left|B_0\right|\left|M\right|}$$

$$\vec{M} = (M\sin\vartheta\sin(at), -M\sin\vartheta\cos(at), M\cos\vartheta)$$

$$\vec{B}_0 = (0, -B_0\sin\vartheta, B_0\cos\vartheta)$$

$$\cos\alpha = \cos^2\vartheta + \sin^2\vartheta\cos(at)$$